Index Number:.....

UNIVERSITY OF GHANA

BSc/BA, FIRST SEMESTER EXAMINATIONS: 2021/2022

DEPARTMENT OF MATHEMATICS

MATH 121: ALGEBRA AND TRIGONOMETRY (3 credits)

INSTRUCTION:

Answer all questions in Sections A and B. Submit both the question paper and the answer booklet.

TIME ALLOWED: 2 Hours

Section A: Multiple Choice Questions (40 marks)

- This section is composed of **20** multiple choice questions requiring **20** answers. Each correct answer is worth **2** marks.
- For each question, indicate your answer by **circling** True or False, or the letter that matches your answer on **this question paper**.

EXAMINERS: S. Sarfo, B. V. Normenyo, R. Twum, C. P. N. Ogbogbo, A. Asare-Tuah, J. S. Bansah, A. Abubakar, D. Adu-Gyamfi

Index Number:.....

1. For any bijective function f, $f \circ f^{-1}(x)$ is equal to ___.
   (a) x^2
   (b) 1
   (c) x
   (d) $2x$

2. When to prove that $P \implies Q$ is true, we proceed with the assumption that P is true and the conclusion Q false, that type of proof is known as ___.
   (a) Direct proof
   (b) Vacuous proof
   (c) Proof by contraposition
   (d) Proof by contradiction

3. If A is a subset of B and B is a subset of C, then the cardinality of $A \cup B \cup C$ is ___.
   (a) Cardinality of A
   (b) Cardinality of B
   (c) Cardinality of C
   (d) Cardinality of $A + B + C$

4. Let $A = \{1, 2, 3, \{1, 4\}\}$. All of the following are true except ___,
   (a) $2 \in A$
   (b) $4 \in A$
   (c) $\{1, 4\} \in A$
   (d) $\{1, 2, 3\} \subseteq A$

5. Let p be the proposition, "high speed driving is dangerous" and q the proposition, "Kofi was a wise man". Which of the following is the appropriate logical representation of the statement, "High speed driving is not dangerous or Kofi was a wise man."?
   (a) $p \vee q$
   (b) $p \wedge q$
   (c) $\neg(p \vee q)$
   (d) $\neg p \vee q$

6. What is the inverse of the conditional statement "A positive integer is a composite only if it has divisors other than 1 and itself."?
   (a) If a positive integer has no divisors other than 1 and itself, then it is not composite.
   (b) If a positive integer is not composite, then it has no divisors other than 1 and itself.
   (c) A positive integer is a composite if it has divisors other than 1 and itself.
   (d) None of the above.

7. $A = \{1, 3, 5\}, B = \{2, 4, 6\}$, and $C = \{0, 2, 4, 6, 8\}$. Which of the following may be considered as a universal set for all of the three sets A, B and C?

   (a) $\{0, 1, 2, 3, 4, 5, 6\}$

   (b) $\{1, 2, 3, 4, 5, 6, 7, 8\}$

   (c) $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

   (d) None of the above.

8. Which of the following quantifier notation best represents the statement?

   There is an integer n for which $n^2 = 2$

   (a) $\exists n \in n^2, n = 2$

   (b) $\forall n \in \mathbb{Z}, n^2 = 2$

   (c) $\exists n \in \mathbb{Z}, n^2 = 2$

   (d) $\exists n \in n^2 = 2$

9. The set difference of the set A with null set is ___.

   (a) *Set A*

   (b) *Null set*

   (c) *Universal set*

   (d) *Set B*

10. The converse of the statement $P \implies Q$ is ___.

    (a) $Q \implies P$

    (b) $\neg Q \implies P$

    (c) $\neg P \implies \neg Q$

    (d) $\neg Q \implies \neg P$

11. All the following are odd functions except___.

    (a) $f(x) = x^5 + x^3$

    (b) $t(x) = 3x^4 - 5$

    (c) $k(x) = x^3 - x$

    (d) $r(x) = x + \frac{1}{x}$

12. Which of the following statements is the negation of the statement, "4 is odd or -9 is positive."

    (a) 4 is odd or -9 is not negative.

    (b) 4 is even or -9 is not negative.

    (c) 4 is odd and -9 is not negative.

    (d) 4 is even and -9 is negative.

13. Let $A = \{1, 2, 3, 4\}$, and U be the set of all natural numbers. Then $U \setminus A'$ is given by set.
    (a) $\{1, 2, 3, 4, 5, 6, \ldots\}$
    (b) $\{5, 6, 7, 8, 9, \ldots\}$
    (c) $\{1, 2, 3, 4\}$
    (d) All of the above.

14. The function $f(x) = x + 1$ from the set of integers to itself is *onto*.
    (a) True
    (b) False
    (c) Neither of the above.

15. Let f and g be functions from the set of integers to itself, defined by $f(x) = 2x + 1$ and $g(x) = 3x + 4$ respectively. Then the composition $f(g(x))$ is ___.
    (a) $6x + 6$
    (b) $6x + 7$
    (c) $6x + 8$
    (d) $6x + 9$

16. What is the domain of the function $f(x) = x^{\frac{1}{2}}$?
    (a) $(2, \infty)$
    (b) $(-\infty, 1)$
    (c) $[0, \infty)$
    (d) None of the above

17. If $A = \{x : x = 4n + 1, 2 \leq n \leq 5 \text{ where } n \in \mathbb{Z}\}$, then the number of subsets of A is
    (a) 2
    (b) 4
    (c) 8
    (d) 16

18. If A has 4 elements and B has 8 elements then the minimum and maximum possible number of elements in $A \cup B$ are ___.
    (a) $4, 8$
    (b) $8, 12$
    (c) $4, 12$
    (d) None of the above.

19. Which of the following two sets are *disjoint*?
    (a) $\{1, 3, 5\}$ and $\{1, 3, 6\}$
    (b) $\{1, 2, 3\}$ and $\{1, 2, 3\}$
    (c) $\{1, 3, 5\}$ and $\{2, 3, 4\}$
    (d) $\{1, 3, 5\}$ and $\{2, 4, 6\}$

20. Let C and D be two sets. Which of the following statements is/are true?
    i) $C \cup D = D \cup C$
    ii) $C \cap D = D \cap C$

    (a) Statement i.

    (b) Statement ii.

    (c) Both statements

    (d) None of the statements.

Index Number:.....

Section B: Written Answer Questions (60 marks)

- This section is composed of 3 questions, **Answer ALL**. Each question is worth **20** marks.
- You are to write complete solutions to all questions in the answer booklet provided by the invigilators.
- You must fasten this test paper together with your answer booklet, and then turn in the fastened papers to the invigilators at the end of the examination.

1. a) Prove by mathematical induction that for all $n \geq 1$.

$$\sum_{r=1}^{n} r(r+1) = \frac{1}{3}n(n+1)(n+2).$$

[10 MARKS]

b) Find a counterexample to disprove each of the following statements.

i. For every positive integer n, $n^2 + n + 1$ is always a prime. **[5 MARKS]**

ii. If n is an integer and n^2 is divisible by 4, then n is divisible by 4. **[5 MARKS]**

2. (a) i. Prove $f : (-2, \infty) \to \mathbb{R}^+$ given by $f(x) = \frac{2}{\sqrt{x+2}}$ is injective. **[5 MARKS]**

ii. Find the inverse function f^{-1} of f above. **[7 MARKS]**

iii. State the range of f. **[4 MARKS]**

(b) Evaluate each expression using the functions f and g respectively defined by $f(x) = 2 - x$, and

$$g(x) = \begin{cases} 2x + 1, & \text{if } x < -2 \\ x^2 - 1, & \text{if } -2 \leq x \leq 3 \\ 4 - x, & \text{if } x > 3 \end{cases}$$

i. $f(g(-2))$ **[2 MARKS]**

ii. $f(g(-4))$ **[2 MARKS]**

3. (a) Suppose $A = \{1, 2, 3\}$ and $B = \{a, b\}$, list all elements of $A \times B$. **[3 MARKS]**

(b) Let $I = \{1, 2, 3\}$, and for each $i \in I$ let $A_i = \{i, i+1, i+2, i+3\}$. Find

i. A_i for each $i \in I$. **[3 MARKS]**

ii. $\bigcap_{i \in I} A_i$. **[2 MARKS]**

iii. $\bigcup_{i \in I} A_i$. **[2 MARKS]**

c) Let A, B and D be sets with $B \subset D$.

i. Prove that $A \times B \subseteq A \times D$. **[5 MARKS]**

ii. Is $A \times B = A \times D$? Justify your answer. **[5 MARKS]**