UNIVERSITY OF CALGARY
FACULTY OF SCIENCE
DEPARTMENT OF MATHEMATICS & STATISTICS

FINAL EXAMINATION

LINEAR METHODS I – MATH 211
ALL SECTIONS – FALL 2015

Exam version: 0

Date: December 18, 2015 **Time: 3 hours**

SECTION NUMBER	INSTRUCTOR NAME	STUDENT I.D. NUMBER	FIRST NAME	LAST NAME

EXAMINATION RULES

1. This is a closed book examination.
2. No aids are allowed for this examination.
3. Provide all answers on the scantron sheet.
4. Scantron sheets must be filled in during the exam time period. No additional time will be granted to fill in the scantron form.
5. The use of personal electronic or communication devices is prohibited.
6. A University of Calgary Student ID card is required to write the Final Examination and could be requested for midterm examinations. If adequate ID isn't presented, the student must complete an Identification Form.
7. Students late in arriving will not be permitted after one-half hour of the examination time has passed.
8. No student will be permitted to leave the examination room during the first 30 minutes, nor during the last 15 minutes of the examination. Students must stop writing and hand in their exam immediately when time expires.
9. All inquiries and requests must be addressed to the exam supervisor.
10. Students are strictly cautioned against:
    (a) communicating to other students;
    (b) leaving answer papers exposed to view;
    (c) attempting to read other students' examination papers.
11. During the final examination, if a student becomes ill or receives word of domestic affliction, the student must report to the Invigilator, hand in the unfinished paper and request that it be cancelled. If ill, the student must report immediately to a physician/counselor for a medical note to support a deferred examination application.
12. Once the examination has been handed in for marking, a student cannot request that the examination be cancelled. Retroactive withdrawals from the course will be denied.
13. Failure to comply with these regulations will result in rejection of the examination paper.

Part A: Mark your answers on the scantron sheet provided. **Exam version: 0**

Each question is worth 2 points.

1. Consider the following augmented matrix in which $*$ denotes arbitrary numbers and $\blacksquare$ denotes **nonzero** numbers.

$$\left[\begin{array}{cccc|c} \blacksquare & * & * & * & * \\ 0 & \blacksquare & * & * & * \\ 0 & 0 & \blacksquare & * & * \\ 0 & 0 & 0 & \blacksquare & * \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

What can you say about the corresponding system of linear equations?

(a) It has no solution.
(b) It has a unique solution.
(c) The general solution has exactly one parameter.
(d) The general solution has exactly two parameters.
(e) There is insufficient information to determine the answer.

2. For which values of h and k does the system

$$\begin{array}{rcrcl} x & + & 3y & = & k \\ 4x & + & hy & = & h \end{array}$$

have infinitely many solutions?

(a) $h \neq 12$ and $k = 3$.
(b) $h \neq 12$ and $k \neq 3$.
(c) $h = 12$ and $k = 3$.
(d) $h = 12$ and $k \neq 3$.
(e) None of the above.

3. Let A be an $n \times n$ matrix, and I the $n \times n$ identity matrix. Which of the following statements is always true?

(a) If $A^2 = A$, then $A = 0$ or $A = I$.
(b) If $A^2 = A$ and $A \neq 0$, then A is invertible.
(c) If $A^2 = A$, then det $A = 0$ or det $A = 1$.
(d) If det $A = 0$ or det $A = 1$, then $A^2 = A$.
(e) None of the above.

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4. Suppose $A = \begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix}$ and $AB = \begin{bmatrix} 1 & 1 & 0 & 3 \\ -1 & 4 & 2 & 5 \end{bmatrix}$. The $(1,1)$-entry of B is

   (a) 1.
   (b) -1.
   (c) -4.
   (d) 5.
   (e) 7.

5. If A is an $n \times n$ matrix that satisfies $A^3 - 4A - I = 0$, then:

   (a) $A^{-1} = A - 4I$.
   (b) $A^{-1} = A^2 - 4I$.
   (c) $A^{-1} = A^2 - 4A$.
   (d) $A^{-1} = A^3 - 4A$.
   (e) None of the above.

6. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 3 \end{bmatrix}$. Then $\det(\text{adj } A)$ is equal to:

   (a) 6.
   (b) 36.
   (c) $1/6$.
   (d) $1/36$.
   (e) None of the above.

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7. Find the matrix C so that $\mathrm{adj}(C) = \begin{bmatrix} 1 & 1 \\ 2 & 4 \end{bmatrix}$.

   (a) $C = \begin{bmatrix} 4 & 1 \\ 2 & 1 \end{bmatrix}$.

   (b) $C = \frac{1}{2}\begin{bmatrix} 4 & 1 \\ 2 & 1 \end{bmatrix}$.

   (c) $C = \begin{bmatrix} 4 & -1 \\ -2 & 1 \end{bmatrix}$.

   (d) $C = \frac{1}{2}\begin{bmatrix} 4 & -1 \\ -2 & 1 \end{bmatrix}$.

   (e) None of the above.

8. Let L be the line with parametric equations

$$L : \begin{cases} x &= 1+t \\ y &= 2 \\ z &= 3+t \end{cases}.$$

   Let P_1 and P_2 be the planes with equations $x+y-z=3$ and $x-y-z=5$ respectively. Which of the following is true?

   (a) L lies in P_1.

   (b) L lies in P_2.

   (c) P_1 is parallel to P_2.

   (d) L is parallel to the intersection of P_1 and P_2.

   (e) None of the above.

9. Let $\vec{u} = [3\sqrt{3},\, 3,\, 0]^T$ be a vector in $\mathbb{R}^3$. Which of the following vector makes an included angle of $\pi/3$ radians with $\vec{u}$?

   (a) $\vec{v} = [\sqrt{2},\, \sqrt{2},\, 0]^T$.

   (b) $\vec{v} = [\sqrt{6},\, 0,\, \sqrt{2}]^T$.

   (c) $\vec{v} = [\sqrt{3},\, 0,\, \sqrt{3}]^T$.

   (d) $\vec{v} = [\sqrt{3},\, 0,\, \sqrt{6}]^T$.

   (e) None of the above.

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10. The shortest distance between the plane P with equation $x - y + 2z = 2$ and the line L with parametric equations

$$L : \begin{cases} x &= 1+t \\ y &= 1-t \\ z &= 3-t \end{cases} .$$

is:

(a) 0.
(b) $2/3$.
(c) $\frac{2}{3}\sqrt{6}$.
(d) $\dfrac{2}{3\sqrt{6}}$.
(e) None of the above.

11. Let $\vec{u} = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. Then $\|\vec{u} - \text{proj}_{\vec{w}}\vec{u}\|$ is equal to:

(a) $\sqrt{3}$.
(b) $2\sqrt{3}$.
(c) $\sqrt{2}$.
(d) $2\sqrt{2}$.
(e) None of the above.

12. Let $S : \mathbb{R}^2 \to \mathbb{R}^2$ and $T : \mathbb{R}^2 \to \mathbb{R}^2$ be linear transformations such that

$$S\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \end{bmatrix}, \quad S\begin{bmatrix} 0 \\ -2 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}, \quad T\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad T\begin{bmatrix} 0 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

Find $(S \circ T)\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right)$.

(a) $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ (c) $\begin{bmatrix} 0 \\ -2 \end{bmatrix}$ (d) $\begin{bmatrix} 0 \\ 3 \end{bmatrix}$ (e) $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

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13. Let $z = 1 + i$ and $w = 2 - i$ be complex numbers. Then $\left|\dfrac{w - \overline{z}}{z}\right|$ equals:

    (a) $\sqrt{2}$.

    (b) $1/\sqrt{2}$.

    (c) $3\sqrt{2}$.

    (d) $3/\sqrt{2}$.

    (e) None of the above.

14. Suppose $p(x) = x^2 - ix + c$ is a polynomial with $-i$ as a root. What is the value of c?

    (a) $c = 0$.

    (b) $c = 2$.

    (c) $c = -2$.

    (d) $c = 2i$.

    (e) $c = -2i$.

15. The roots of the quadratic equation $z^2 + 6z + 10 = 0$ are

    (a) $3 + i$ and $3 - i$.

    (b) $3 + 2i$ and $3 - 2i$.

    (c) $-3 + 2i$ and $-3 - 2i$.

    (d) $-3 + i$ and $-3 - i$.

    (e) None of the above

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16. The values of i^{2015} is:
    (a) 1.
    (b) -1.
    (c) i.
    (d) $-i$.
    (e) None of the above.

17. If $z = -1 + \sqrt{3}i$, then an argument of z^{90} is:
    (a) 0.
    (b) $\pi/3$.
    (c) $2\pi/3$.
    (d) $4\pi/3$.
    (e) none of the above.

18. If $z = \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i$ is a fourth root of w, then which of the following is a cube root of w?
    (a) $\frac{1}{2} - \frac{\sqrt{3}}{2}i$.
    (b) $\frac{\sqrt{3}}{2} - \frac{1}{2}i$.
    (c) $-\frac{1}{2} + \frac{\sqrt{3}}{2}i$.
    (d) $-\frac{1}{2} - \frac{\sqrt{3}}{2}i$.
    (e) None of the above.

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19. The matrix A can be written in the form

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}.$$

Which of the following statements **is false**?

(a) The matrix A is diagonalizable.

(b) 2 is an eigenvalue of multiplicity two, and is the only eigenvalue.

(c) For any integer $n \geq 2$, we can write $A^n = 2^n I$.

(d) The general solution of the system $(2I - A)X = 0$ has exactly one parameter.

(e) The characteristic polynomial of A is $c_A(x) = (x-2)^2$.

20. The eigenvalues of $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ -2 & -2 & -1 \end{bmatrix}$ are

(a) 1, -1, and -3.

(b) 1, $1-2i$ and $1+2i$.

(c) 3 and -1, with -1 having multiplicity two.

(d) 1 and -1, with 1 having multiplicity two.

(e) none of the above.

21. Consider an arbitrary $n \times n$ matrix A. Which of the following statements is ALWAYS true?

(a) A always has n distinct complex eigenvalues.

(b) If A does not have n distinct complex eigenvalues, then A is not diagonalizable.

(c) If A has a single complex eigenvalue of multiplicity n, then A is diagonalizable.

(d) If all complex eigenvalues have multiplicity one, then A is diagonalizable.

(e) None of the above.

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22. Let $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. Which of the following statements is true?

   (a) $A^2 = A$.
   (b) A is not diagonalizable.
   (c) A is not invertible.
   (d) adj $A = A$.
   (e) All of the above statements are false.

23. Consider the Markov Chain $S_{n+1} = PS_n$ where the transition matrix is given by:

$$P = \begin{bmatrix} 1/2 & 1/3 \\ 1/2 & 2/3 \end{bmatrix}.$$

   Which of the following is the steady state vector of this Markov Chain?

   (a) $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$.
   (b) $\begin{bmatrix} 1/3 \\ 2/3 \end{bmatrix}$.
   (c) $\begin{bmatrix} 2/5 \\ 3/5 \end{bmatrix}$.
   (d) $\begin{bmatrix} 2/3 \\ 1 \end{bmatrix}$.
   (e) There are no steady state vectors for the Markov Chain.

24. Consider the matrix

$$A = \begin{bmatrix} 0 & 1 \\ 3 & 2 \end{bmatrix}.$$

   Which of the following matrices P diagonalizes A?

   (a) $P = \begin{bmatrix} 1 & 3 \\ -1 & 1 \end{bmatrix}$.
   (b) $P = \begin{bmatrix} -1 & -1 \\ -3 & 1 \end{bmatrix}$.
   (c) $P = \begin{bmatrix} 1 & 3 \\ -1 & -1 \end{bmatrix}$.
   (d) $P = \begin{bmatrix} 1 & 1 \\ 3 & 1 \end{bmatrix}$.
   (e) A is not diagonalizable.

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25. Consider the matrix

$$A = \begin{bmatrix} 0 & 1 \\ 1/3 & 2/3 \end{bmatrix}.$$

For large values of n, the entries of the matrix power A^n are closest to the entries of which of the following matrices?

(a) $\begin{bmatrix} 1 & 1 \\ 3 & 3 \end{bmatrix}$.

(b) $\begin{bmatrix} 1 & 3 \\ 1 & 3 \end{bmatrix}$.

(c) $\frac{1}{4}\begin{bmatrix} 1 & 1 \\ 3 & 3 \end{bmatrix}$.

(d) $\frac{1}{4}\begin{bmatrix} 1 & 3 \\ 1 & 3 \end{bmatrix}$.

(e) $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$.