Assignment 2: Auctioning with Levelled Commitment

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ABSTRACT

We present an implementation of a bidding model based on the Vickrey system which attempts to analyze how a bidding market develops across multiple rounds given different parameters. We examine the effects these modifications have in regards to maintaining price stability, maximizing profits of both the buyers and the sellers and producing a fair, non-biased auction. Our results show that an ideal auction is only achievable by allowing the bidders to constantly increase their bids across multiple rounds, which is unrealistic, and that providing additional penalties or decrease factors will only lead to faster profit convergence for both the buyers and the sellers.

1 INTRODUCTION

In modern days, auctions have started to play a bigger role in our society. While auction mechanisms themselves are usually a part of economics research, auctioning can be used in the context of multiagent systems as well [2], where task assignment for different agents can be done based on auctioning.

Therefore, modelling an auction to compare how well an auction works for both the buyers and the sellers is an interesting and relevant topic to explore the possibilities within the auction domain. Other auctioning systems exist, such as the English (or absolute) auction, Dutch auction (which works in a descending manner), or the Vickrey-Clarke-Groves auction [4].

For this assignment we are tasked to implement a modified second-price sealed-bid auction that is organized in rounds. Second-price means that the buyer that places the highest bid doesn't actually pay his own bid, but the second highest bid. This type of auction is also known as a Vickrey auction and it incentives buyers to bid according to their true value and not under- or overbid [1]. Sealed-bid means that buyers won't be able to see what other buyers have proposed as a bid, which prevents them from basing their bid on the bids of others [3]. We will consider a list of modifications to the auction system with the aim of producing a fair auction setting, where the profits of both buyers and sellers are maximized and the price of the items remains stable across multiple rounds.

2 METHODOLOGY

2.1 Simulation Description

We first select K sellers and $N \geq K$ buyers in one simulation. The simulation will last R rounds and each round will have K auctions. The order in which the actions take place within each round is always random. In each round, every seller offers one item of his own particular type, and every buyer can only be the winner of only one auction.

In the beginning of the simulation, we set a universal maximum starting price S_{max}, and every seller k sets a random starting price for the item put on the auction, which is denoted as $S_k \in (0, S_{max})$. At the same time, every buyer set his bid with a unique bidding factor: $B_{n,k} = \alpha_{n,k} S_k$, $1 \leq \alpha$.
Subsequently we know the market price of one type of goods: $\Xi_k = \sum_n \frac{B_{n,k}}{N}$. According to the Vickrey mechanism, a buyer with the highest bid (lower than Ξ_k) is the winner of the auction, which then pays the second-highest bid to the seller. In case only one (winning) bid is below the market price, the second highest bid is calculated as the average between the winning bid and the auction's starting price.

An example of the simulation software interface with arbitrary values and the corresponding outputs is given in the Appendix, together with a basic guide on how to utilize the software.

2.1.1 *Auction.* We consider two auctioning schemes:

- *"Pure" Auctioning*: in "pure" auctioning, the buyers, which win an auction in a round, cannot participate in any further auctions during the round. Therefore, the profits of the sellers s_k and buyers b_n during round i will be: $p_{s_k} = \sum b_i - S_i$ and $p_{b_n} = \sum \Xi_i - b_i$, respectively.
- *Auctioning with leveled commitment*: if a buyer, which already won an auction in a round, wins another auction in the same round, the buyer must decommit from the auction result which brings it the least profit. During the decommitment, buyer n pays seller k (which sold the item the buyer is decommiting from) an annulling penalty fee, $f_{n,k} = \epsilon b_i$. Therefore, the profits of the sellers s_k and buyers b_n during round i will be: $p_{s_k} = (\sum b_i - S_i) + \sum f_j$ and $p_{b_n} = \sum(\Xi_i - b_i) - \sum f_j$, respectively.

2.1.2 *Buyer Strategy.* In order to maximize their profits, and knowing the mechanisms of the auction, the buyers will implement the following strategies in case of a "pure" auction and a "leveled" auction:

- *Standard strategy*: when the buyer b_i wins an auction or bids higher than the market price, they will decrease their corresponding bid to items from that particular seller by a bidding factor $\underline{\Delta}\alpha_{n,k}$. If the buyer bids lower than the winner of the auction, then they will increase their bid by a corresponding bidding factor $\overline{\Delta}\alpha_{n,k}$. We will assume the buyer can implement this strategy for both types of auction described above.

- *Advanced strategy*: in the case of leveled commitment, the buyer will adjust their bids by taking into account the annulling fee into their profits and adjust their bid correspondingly according to the mechanism used in the standard strategy. This formulation ensures that the buyer is eager to annul a result of another auction won in the same round if they stand to gain more by bidding on the new item than they lose by decommitting from the previously won item.

2.1.3 Seller Strategy. In a similar manner to the buyers, we consider three strategies for the sellers, based on whether they can adjust the starting price of the item between rounds of auctioning or not.

- *Random*: if no strategy is specified for the sellers, they will select a random starting price in the given range at the beginning of the auction. The starting price in future rounds of the auction will be the evaluated market price at the end of the previous round.
- *Own good*: in the case the seller wants to maximize their profits, it is not sufficient that the seller sets the starting price to the maximum starting price. Given that the seller profit is calculated as the difference between the market price and second-bid paid to the seller. We define overbidder as bidders who lost the auction by bidding over the market price, and underbidders as bidders who lost the auction by bidding below the winning bid. Sellers need to adjust the starting price to ensure that there are more overbidders than underbidders, thus driving the market price, and therefore their profits upwards. This can be done by setting the starting price to the market value of the previous round and subsequently increasing it by a factor $\overline{\Delta}\beta_{n,k}$ each round when there are more overbidders than underbidders.
- *Common good*: in order to ensure the "common" good of the market, the seller should adjust the starting price by a factor $\overline{\Delta}\beta_{n,k}$ in cases where there were more underbidders u_k than overbidders o_k and $\underline{\Delta}\beta_{n,k}$ otherwise. Specifically, the starting price S_k in a round is calculated by:

$$S_k = \begin{cases} \overline{\Delta}\beta_{n,k}\Xi_{k-1}, & \text{if } u_k > o_k \\ \underline{\Delta}\beta_{n,k}\Xi_{k-1}, & \text{if } o_k > u_k \\ \Xi_{k-1} & \text{otherwise.} \end{cases}$$

2.2 Experiments

2.2.1 Naive scenario. As described in the assignment document we started our experiments from an auction containing one seller, two buyers and 100 rounds. The auctioning type was set to be pure and they buyers were "asked" not to change their buying factors α from round to round.

2.2.2 Advanced bidding strategy. In this scenario, the bidders were asked to adjust their bidding factors based on whether they under- or overbid.

2.2.3 Varying penalty factor. For this experiment we changed the auction type to "leveled" in order to introduce the penalty factors. The penalty factors are of use when a buyer refuses to buy a previously won item, instead choosing to pay a penalty fee in order to increase his profits. We also show the results for both standard and advanced bidding strategies here.

2.2.4 Sellers strategies. Sellers consider three strategies. For the first one, they set the initial price to random and then simply reuse the previous market price in future rounds. The second strategy, "own good", prioritizes each seller's own profit by increasing the starting price between rounds depending on the ratio of overbidders to underbidders, up to the maximum starting price. The third strategy, "common good", allows the seller to adjust the starting price both up and down depending on whether there are more overbidders or underbidders. Furthermore, we vary the price increase and decrease factors simultaneously for each strategy.

2.2.5 Varying buyer increase/decrease factors. For this experiment, we consider changing the bid increase/decrease factors as well as the overall initialization of the bidding factor. The bidding factor represents how much the buyer is willing to spend to purchase a particular item. For the bidding factor, we consider two initialization strategies: either constant value for all items or randomly initialized by sampling from a uniform distribution between 1 and a user-defined value. The bid increase factor $\overline{\Delta}$ and bid decrease factor $\underline{\Delta}$ are the values by which the bidder changes their bid between rounds depending on the results of the previous round (either increasing or decreasing), and can be set separately.

3 RESULTS

3.1 Naive scenario

In this scenario, the profits of the seller increased linearly over the course of rounds. For buyers however, only the profits of a single buyer increased. This trend continued for cases where the sellers outnumber the buyers. One or more buyers simply could never win. This observation is consistent with our expectations considering the bidder's strategy, namely to never change one's own bidding factor. Meaning that from round to round, the price of an item might've changed, but the buyer closest to the market price never did.

When the opposite was true, and the sellers outnumbered the buyers, some of the sellers never received the profit. While in the previous scenario the buyer who's profit remained at 0 was random for each run of the simulation, the id of the seller who experienced the same was always consistent. The reason for this occurrence lies in the implementation of the simulation. Sellers' items are sold in the sequence of their id's. The seller with id of 0 always goes first. So in the event that there were less buyers than sellers, the sellers with the higher id value never received a chance to sell their item. This pattern is consistent across different numbers of buyers and sellers.

3.2 Advanced bidding strategy

Advanced bidding strategy has caused both the buyer and seller round profits to go down to 0. Meaning that the strategy had forced buyers to set their sellers factors (by which the starting price is multiplied to determine the actual bid). The bid increase factor was 1.2 and the decrease was 0.9. The number of sellers and buyers was 10 and 4 respectively and the auction ran for 100 rounds. The evolution of these profits can be seen in Figures 1, 2. This development is within our expectations as the buyers attempted to get as close to the market price as possible in order to win their bet. After a while, this wish caused them all to have their bid equal to that of the starting price. Therefore not buyers nor sellers could make any profit anymore.

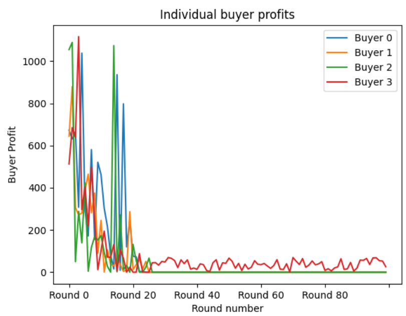

Figure 1: Profits per round of buyers using advanced bidding strategies. Ten sellers, four buyers, 100 rounds.

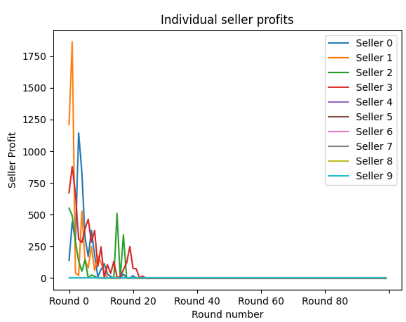

Figure 2: Profits per round of sellers when buyers are using advanced bidding strategies. Ten sellers, four buyers, 100 rounds.

3.3 Varying penalty factors

Introducing the penalty factors has had a noticeable effect on the profits of the sellers. Their average profits per round rose linearly with the value of the penalty factor. This is not an unexpected results as in addition to selling their item, the sellers can now collect additional fees from buyers refusing to buy their product. This trend can be seen by comparing Figures 3 and 4. The profit is simply higher on average. The buyer profits when using the standard bidding strategy did not change. Therefore we only show the Figures of buyers when advanced strategy was applied.

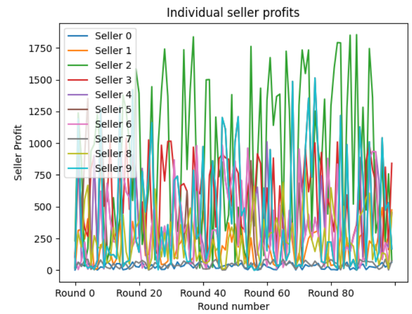

Figure 3: Profits per round of sellers in a leveled auction. Ten sellers, four buyers, 100 rounds. Penalty factor is equal to 0.5

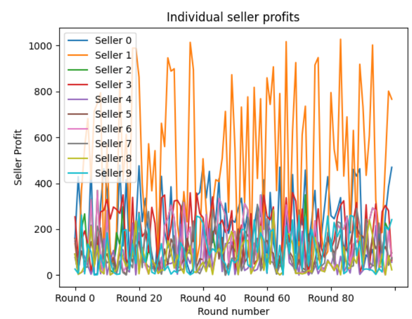

Figure 4: Profits per round of sellers in a leveled auction. Ten sellers, four buyers, 100 rounds. Penalty factor is equal to 0.1

By introducing a leveled auction we manage to escape the plateau of per round profits that otherwise occurs after a certain number of rounds. As can be seen in Figure 6, introducing a low penalty factor of 0.1 makes the profits of buyers more stable over a longer period of time.

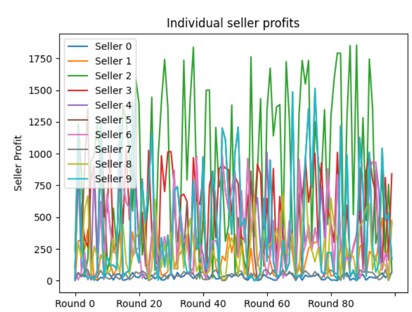

Figure 5: Profits per round of buyers in a leveled auction. Ten sellers, four buyers, 100 rounds. Penalty factor is equal to 0.5

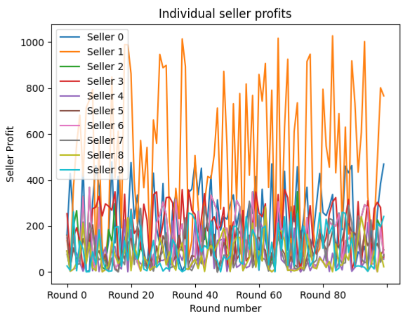

Figure 6: Profits per round of buyers in a leveled auction. Ten sellers, four buyers, 100 rounds. Penalty factor is equal to 0.1

3.4 Seller strategies

The different seller strategies proposed do not seem to produce any significant changes to one another for the stability of the market. The seller strategies themselves did not affect the overall behaviour of the market even with a parameter sweep over a range of values for the price increase / decrease factors. The full experimental results are contained in the Appendix.

3.5 Varying buyer increase/decrease factors

The buyer increase factor was varied between 1.1 and 1.7. This, because an increase factor of 1 does not make sense, and the buyer profits started to become nonsensical around an increase factor of 1.7. The buyer decrease factor was varied between 0.6 and 0.9 with the same motivation as the increase factor. 0.6 created very slanted graphs, while higher than 0.9 makes no sense as a decrease factor.

While the increase factor was varied, the decrease factor was kept at 0.9, and during the experiments of the decrease factor, the increase factor was kept at 1.3. Varying these factors showed that even a small change can have a huge impact over the course of multiple rounds.

3.5.1 Increase factor. With an increase factor lower than 1.3, the buyers would not increase their bids enough and their bids would slowly converge to the market price, resulting in almost no profit. With an increase factor higher than 1.3, but especially after increasing beyond 1.4, the buyers would increase their bids so much per round that the buyer profit skyrocketed. During one experiment, a buyer reached a profit of $1.6 * 10^7$ in a single round. Hence, it seems that an increase factor that lies in the interval [1.3; 1.4] results in the most fair distribution of profits among sellers which can be seen in Figure 7. There was no notable difference between a pure auction and a leveled auction while varying the increase factor.

3.5.2 Decrease factor. With a decrease factor lower than 0.9, the buyers would decrease their bids so much that it would diminish the market price, and hence their profit over multiple rounds. While a decrease factor too close to 1.0 will not allow the decrease factor to keep the market price in check, resulting in the same effect observed with a high increase factor. The desired interval for the decrease factor to distribute the profits fairly over all buyers therefore lies between 0.85 and 0.95 as can be seen in Figure 8

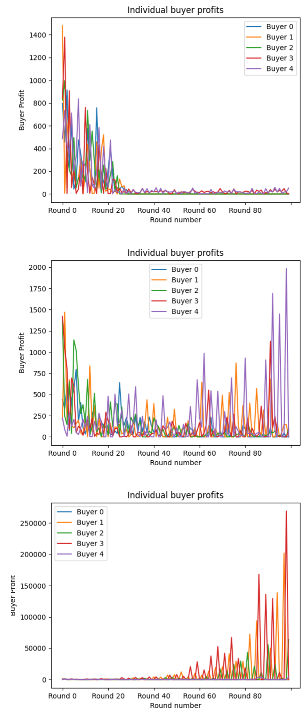

Figure 7: Profits per round of buyers. 1.1, 1.3, and 1.5 bid increase factor. Pure auction

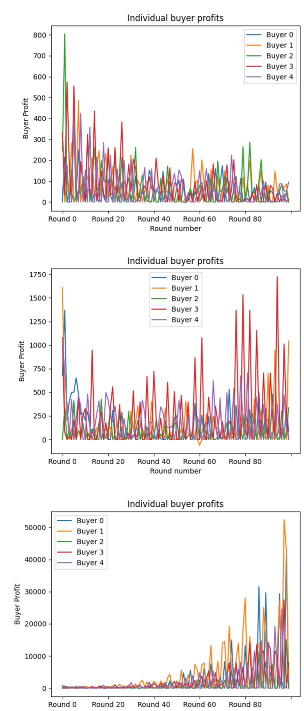

Figure 8: Profits per round of buyers. 0.85, 0.9, and 0.95 bid decrease factor. Leveled auction

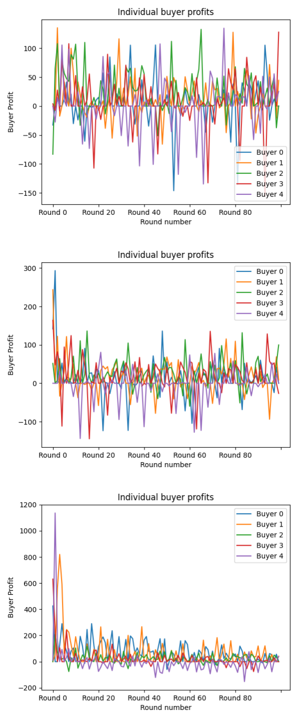

Figure 9: Profits per round of buyers. Bidding Factor 1.1, 2.0, 5.0. Leveled auction

3.6 Varying values of bidding factors

The bidding factor for a buyer is sampled from a uniform distribution between one and the bidding factor that is set in our interface. The factor was increased from 1.1 upwards until a noticeable change for the buyers could be observed in Figure 9. With a leveled auction and a too low bidding factor, the buyers experience a lot of losses during the rounds which is not desirable. Increasing the bidding factor makes the profits of the buyers converge faster. This change can be noticed with a bidding factor larger than 3.

4 DISCUSSION

The results obtained in the experiments were expected. Generally speaking, more advanced, dynamic strategies oriented that "learned" from previous rounds of an auction produced better results for everyone. Allowing buyers to change their bidding factors meant that every buyer could participate in the auction and actually earn a profit. Making the auction leveled introduced a more fair distribution of profit across rounds and a low penalty factor allowed the buyers to experiment more with their bids and profits. In this leveled auction where each buyer tries to maximize it's profit we had come close to the ideal auction setting.

An ideal auction, as mentioned previously, must aim to maximize the profits of the buyers and sellers alike while maintaining the stability of the prices on the auctioned items. Furthermore, it must maintain fairness and unbiasness at all times. In order for the proposed auctioning scenario to be considered an ideal auction, it must satisfy all of these criteria.

We simulated an auction environment through some initial randomness in terms of market price values and bidder initial bids. While real auctions rarely function this way, it can be claimed that the proposed system has several points of equilibrium dependant on a careful balance between the bid increase and decrease factors, as well as the overall bidding factors for each buyer. Other parameters, such as the penalty factor, number of buyers and sellers and the seller strategy parameters rarely had an effect on the overall auction.

In order to maximize the profits of both buyers and sellers as well as maintain market price stability, the buyers must be willing to increase their bids significantly more in case they underbid. This is in contrast to the overbidders, who must decrease their bids by significantly less in future rounds to obtain the same results. These results imply that, in order for a auctioning system to be ideal, the overall values of the bids must increase such that the profits each round are constantly increasing. If, however, the goal of the auction is to minimize but stabilize the market price, then a careful balance between overbidders and underbidders is necessary.

The fairness of an auction

5 CONCLUSION

We have presented an auctioning simulation software based on a modification of the Vickrey auctioning scheme with sealed bids, and commented on the effectiveness of the modifications on producing an ideal auction setting, where the profits of the sellers and buyers are maximized, the prices of the items remain stable across multiple rounds, and the auction is conducted in a fair and unbiased manner. By carefully tuning the parameters of an auction, we can ensure the stability of the price across multiple items and distribute the profits among sellers and buyers equally.
However, these parameters depend not only on the context of the auction but also assume that the buyers place their bids honestly and that the sellers have common good in their mind.

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APPENDIX

Software Description

Figure 10 contains a screenshot of the software with the arbitrarily selected "default" inputs. The numbers of buyers and sellers are limited to 20, and the number of rounds to 1000. Higher values are possible by changing the code, but those values were selected for stability purposes. All of the hyperparameter default values can be seen in table 1.

There are multiple auction settings options for each category. For auction type, the auction can be either "pure" (default) or "leveled". The price type (or seller strategy) can also be adjusted between "random" (default), "own good" and "common good", representing the various seller strategies. The bidding strategies can be adjusted between "standard" and "advanced". All of these are described in the Methodology section of the report.

Other parameters for the auction are also adjustable, but their inputs are limited to a maximum of 5 characters to deal with floating point precision accuracy (in the case of the multiplier factors) and to prevent the program values from becoming too large (in the case of the maximum starting price). The maximum starting price is also an integer, but the bids/profits can include floating point values to 2 decimal places (to represent cents). In case the auctioning type is set to "leveled", the "Refund Penalty Factor" input will also appear, allowing the user to adjust its input as well. The "Bidding factor" represents a multiplier used to determine how much a buyer is willing to bid for an individual item. It can be either set to a constant value across all items, or to be sampled on a uniform distribution ranging from 1 to the user specified value. Note that setting the bidding factor to lower than 1 will result in negative profits for the buyers and leads to a maximally unstable auction.

Once all parameters are set, the "execute" button will run the simulation and output the results. In case the user wants to check different results of the simulation, a dropdown option has been added allowing the user to change between all outputs, buyer profits, seller profits, etc. After selecting an option, the user must press the output button. In some cases, the displayed graph will also change.

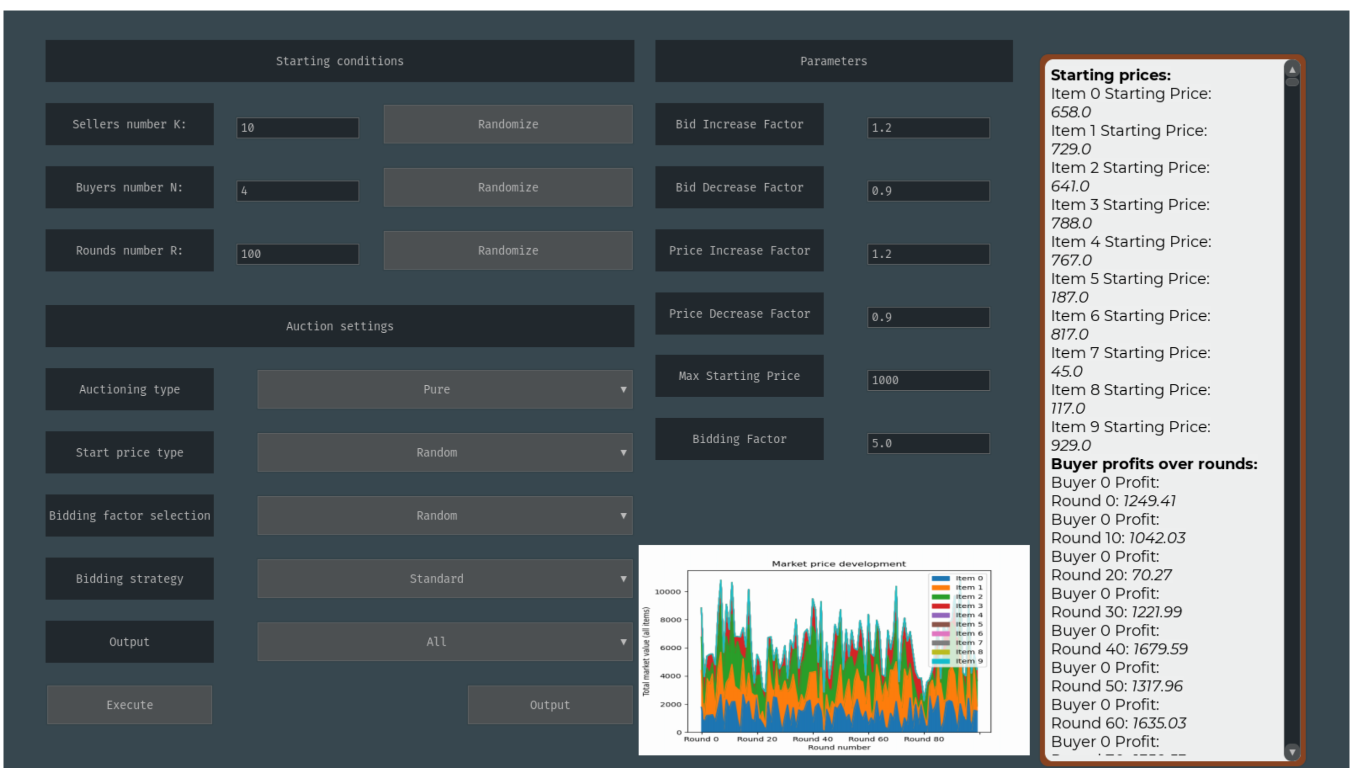

Figure 10: Example image of the software with corresponding output on arbitrary "default" values.

Parameter Type	Value
Bid Increase Factor	1.2
Bid Decrease Factor	0.9
Price Increase Factor	1.2
Price Decrease Factor	0.9
Max Starting Price	1000
Bidding Factor	5.0
Number of Buyers	4
Number of Sellers	10
Number of Rounds	100

Table 1: Default hyperparameters values based on the "best" observed results.

Seller Strategies

The chosen seller strategies were devised with the intention of reducing the randomness involved in the stability of the auctions by allowing the sellers to "push-back" against the continuous convergence observed across multiple experiments. Additional (example) results for all three seller strategies implemented can be seen below. The hyperparameters for these results can be seen in table 2.

Note that these experiments were run on a "pure" auction, with random bidding factor initialization. The bidding strategy does not matter, but was set to advanced for these experiments. However, note that the choice of type of auction and bidding factor initialization did not produce any variation in the "randomness" of the results, and neither did a hyperparameter sweep for the decrease factors over {0.1,0.5,0.7,0.8,0.9,0.99,1} and the increase factors over {1,1.05,1.1,1.2,1.3,1.5,2}, so these results are representative of the effects the seller strategies have by themselves in regards to the stability of the market.

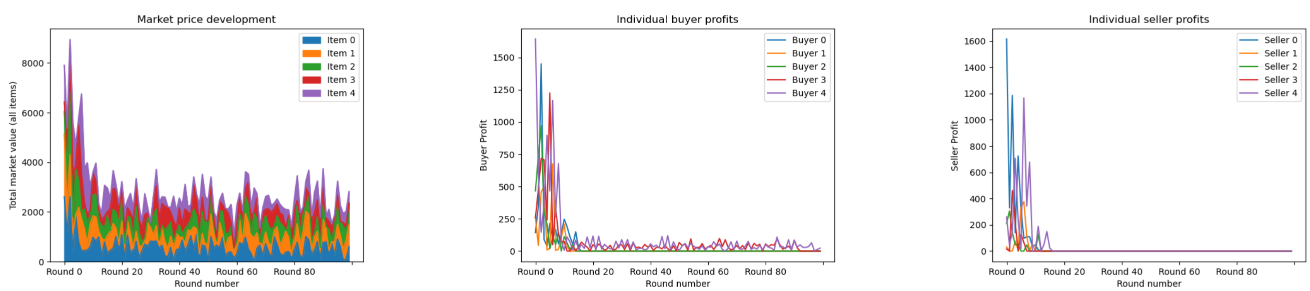

Figure 11: Results of the random seller strategy ("start price type") in terms of the profits each round for the sellers and buyers, as well as the development of the market price. Note that the market prices converge to stable values between 500 to 1000.

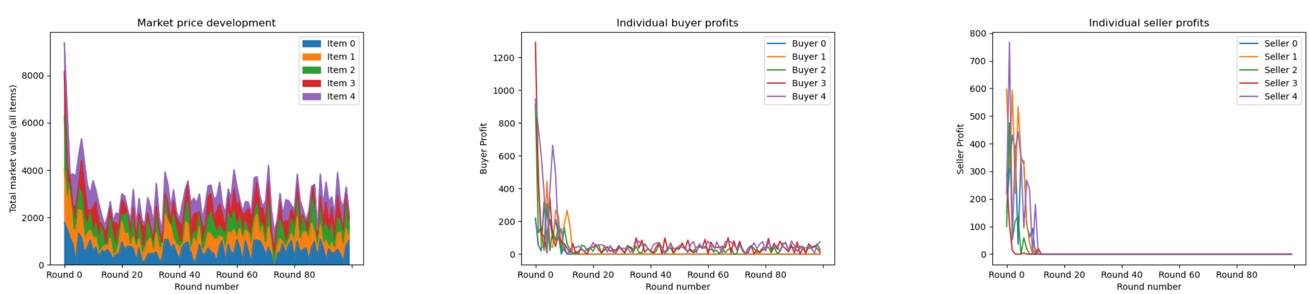

Figure 12: Results of the "own good" seller strategy ("start price type") in terms of the profits each round for the sellers and buyers, as well as the development of the market price. Note that the market prices converge to stable values between 500 to 1000.

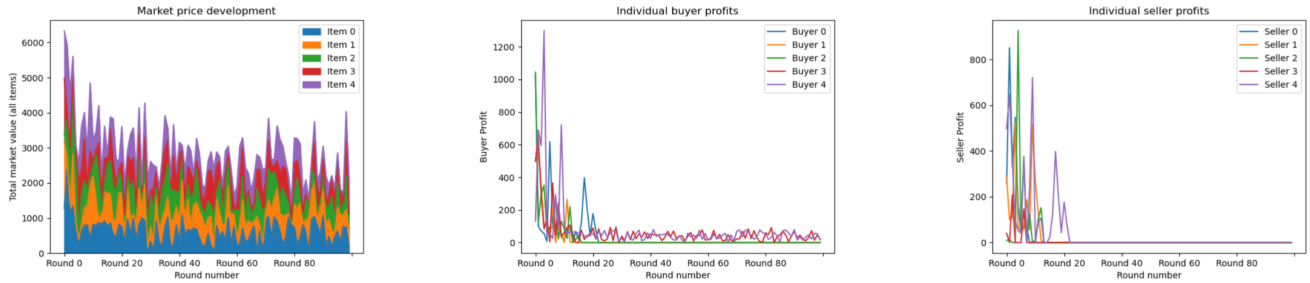

Figure 13: Results of the "common good" seller strategy ("start price type") in terms of the profits each round for the sellers and buyers, as well as the development of the market price. Note that the market prices converge to stable values between 500 to 1000.

Parameter Type	Value
Bid Increase Factor	1.2
Bid Decrease Factor	0.8
Price Increase Factor	1.2
Price Decrease Factor	0.8
Max Starting Price	1000
Bidding Factor	5.0
Number of Buyers	5
Number of Sellers	5

Table 2: Hyperparameters used for the seller strategy experiments.