Assignment 1: Strategic Voting

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ABSTRACT

We present an implementation of a Tactical Voting Analyst agent (TVA). The agent analyzes the risk of strategic voting by individual voters in different voting situations, particularly plurality, voting for two, veto voting and Borda voting. We consider compromising, burying and bullet voting as the primary tactical voting strategies used in the context of an election. The key findings made during this assignment are:

- Plurality voting is sensitive to increased numbers of candidates.
- Strategic voting with a plurality scenario decreased the happiness significantly.
- In a four party election, no strategy has influence on the outcome.
- In an unequally divided voter scenario, veto voting withstands any strategy and results in the highest happiness.
- Bullet voting strategy results in a very low happiness.

KEYWORDS

Voting Strategy, Tactical Voting, Social Simulation

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1 INTRODUCTION

Being able to predict the outcome of an election is very difficult. The winner of an election depends on various factors based on their own popularity within their own voter base. Sometimes candidates in an election can win by a landslide, sometimes they can go neck-to-neck with he outcome in the hands of a select few tactical voters [1].

There are multiple different voting situations describing various scenarios. The simplest one would technically be plurality voting or voting for one, used by many democracies across the world. In plurality voting, the voters each vote for their single top preference. This can be easily expanded to allow the voters to vote for their top two choices, called multiple non-transferable voting with 2 options, which we will simply call voting for two. In both situations, the candidate with the most votes is declared the winner. Voters can also cast a vote for who they don't want to win an election, effectively vetoing a choice [6]. In the case of two candidates, this can basically be reduced to plurality voting, but with more candidates, this can mean that the least hated option wins the election.

The voting schemes mentioned so far assume that all candidates that receive a vote from a voter are treated equally. A fairer way to take into account the voters' preferences is to allow them to rank the candidates, where the option that is, on average, preferred by the most voters is declared the winner [5]. This is essentially what Borda voting entails [3]. Another option would be Condorcet voting, where the candidates are paired up for elections with each of the other candidates and the candidate who wins most matchups is declared the winner [2].

For this project, we will only consider the first four voting schemes described so far: plurality, voting for two, veto voting and Borda voting. Other voting methods, such as Condorcet voting, will not be implemented.

In many popular elections, you often have voters voting tactically when they believe that their top preference is unlikely to win [7]. Tactical voting usually entails voting for the candidate that is more likely to win than their top preference but is still better than their least preferred option, however multiple different methods to vote tactically exist [4]. For this project, we will consider the following methods: compromising, burying and bullet voting. Compromising entails the voters voting for or ranking any other candidate higher rather than their most preferred candidate to see if they can sway the results of the election. Burying entails ranking a choice lower than others. Finally, bullet voting entails voting just for fewer than the maximum number of candidates allowed. In this context, we will consider bullet voting as voting for a single candidate.

To show and experiment with the different voting schemes and tactical voting situations, we implement a Tactical Voting Analyst (TVA) agent or engine. The aim of this agent is to provide and analyze the risk of strategic voting for each of the candidates by taking into account their happiness and strategic voting options. In order to guide this research, we will utilize an election-like situation with several different candidates or parties.

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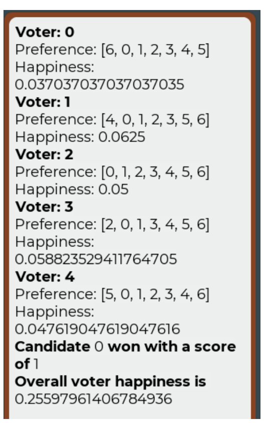
2 METHODOLOGY

2.1 Description of the engine and computational limitations

Our engine accepts arbitrary numbers of voters and their preferences and calculates a result based on either a random or explicitly specified voter-preference matrix.

Figure 1 shows the example output. The aforementioned example uses vote-for-one as its scheme as it's the simplest possible case. Each voter is an object containing an *id* and a *list* of existing candidate object references sorted in the order of their preference. The voter also knows the current voting scheme and whether the strategy is to bullet-vote. This allows us to simply ask each voter in a list to cast a vote, therefore individually increasing the score of their preferred candidate based on the voting requirements.

The preference vector of voter 0 in Figure 1 depicts the order of *candidate id*'s which is then used for voting. Voter 0 received a preference vector of [0, 0, 0, 0, 0, 0, 1] therefore his preference vector has the last candidate at the first place, followed by other candidates sorted in an ascending order by their *id*. The *candidate id* is also used to break ties in favour of the nominee with the lowest digit. Therefore, while candidates 0, 2, 4, 5 and 6 all received the same score of 1, the candidate 0 became the actual winner as shown in Figure 1.



Voter: 0
Preference: [6, 0, 1, 2, 3, 4, 5]
Happiness: 0.037037037037037035
Voter: 1
Preference: [4, 0, 1, 2, 3, 5, 6]
Happiness: 0.0625
Voter: 2
Preference: [0, 1, 2, 3, 4, 5, 6]
Happiness: 0.05
Voter: 3
Preference: [2, 0, 1, 3, 4, 5, 6]
Happiness: 0.058823529411764705
Voter: 4
Preference: [5, 0, 1, 2, 3, 4, 6]
Happiness: 0.047619047619047616
Candidate 0 **won with a score of** 1
Overall voter happiness is 0.25597961406784936

Figure 1: Example output of the engine. The example uses 5 voters and 7 candidates. The scheme used in the image is vote-for-one. No tactical voting strategy was used.

2.2 Voting situations

We consider voting in the context of a single-round presidential election. Here, we assume an abstract 2D-grid to which each voter assigns themselves to the candidate closest to them in the form of a preference vector. Since the space is 2D, we use the Euclidean distance. Figure 2 roughly depicts such a situation with two candidates and multiple voters.

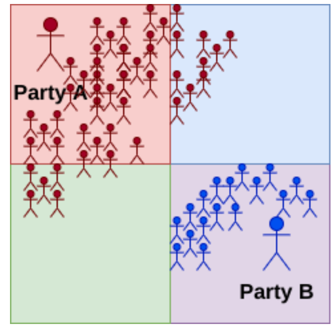


Figure 2: Distribution of voters on a political compass. Red candidate gets 70% of the vote.

With plurality voting, each voter can only vote for one candidate, so Figure 2 is an accurate representation for a two candidate election. Voting-for-two in the described system is trivial, as both candidates would receive an equal share of the votes, so additional candidates are added in accompanying experiments. The same can be said for Veto voting and Borda voting.

In each voting situation where it makes sense, we consider cases where compromising, burying and bullet voting are applied, each time utilizing randomized or semi-randomized voting vectors. For experiments, we utilize two main metrics. Firstly, the happiness of a voter is defined as the inverse of the distance between the outcome of the election and the voter's preference vector, normalized on a [0,1] scale. We also consider the risk of strategic voting as the ratio between the number of strategic voting options to the number of voters for each type of tactical voting situation.

3 RESULTS

3.1 Plurality voting

3.1.1 The original situation. In general, we consider a voting system where the total number of voters is 100. However, for different experiments different multiples of 10 are used as a way of representing cooperating groups. For example, changing the number of voters to 10 gives each voter 10 times more voting power than in the previously mentioned scenario. The reported happiness will also be scaled by a factor to match the happiness of a comparable 100 voters election.

Following the depiction of the current standings, it can easily be concluded that the red-square candidate is winning, as he has far more supporters. We use Figure 2 as the base for all our experiments with plurality voting. In our scenario, the voters vote for a specific candidate, not the party as a whole. Our goal is to sway the election by changing the parameters of the situation such as the number candidates, voting scheme and voting strategy. We assume that the distribution of the votes in Figure 2 is 70/30 in favour of the red candidate. The happiness of the voters following the election of the red candidate is 85.0.

3.1.2 Changing the situation. Perhaps the easiest way to sway the results of a plurality voting system is to add more candidates. Provided no strategic voting takes place, it is assumed that each voter will choose the candidate closest to them. Therefore splitting the vote in such a way that the blue candidate is able to win. Figure 3 depicts just such situation, where the brown and green candidates each split the red's candidate vote equally. The brown, red and green candidates therefore each have 23% of the vote, losing the majority to the blue party. This situation produces the overall voter happiness of 39.5.

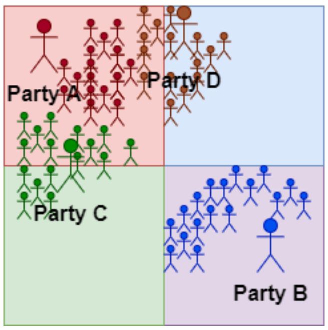


Figure 3: Distribution of voters on a political compass. Green and brown candidates split red candidate's vote causing the blue candidate to reach majority in the absence of strategic voting.

3.1.3 Voting strategically. Assuming the polls had been released indicating that the brown party has the support of 29% of the voters and blue party of 30%, the voters of red and green party might choose to vote for the brown candidate. In such a scenario, these voters will be tactically voting for their second or third preference, in a sense, voting against the blue party in an anti-plurality voting. Following our experiments, 10% of the total voters previously belonging to either the red or the green party can sway the election. Provided they voted for the polls-favourite, in this case the brown party.

The scenario mentioned above (29% brown, 30% blue, 20% red, 21% green), without any strategic voting, produces an overall voter happiness of 52.85 and results in the blue party victory. However, if 10% of the total voters from either green or red party decides to compromise their selection, choosing the brown party instead, they can sway the election, resulting in brown's victory. This decision however decreases the overall voting happiness as the true choice of the strategizing party has been pushed back, resulting in a happiness of 31.50.

3.2 Voting for two

Following the example provided in Section 2.2, we allow the voters to vote for their second choice. We assume here that the second choice of the voters from red, green and brown party is one of the candidates from the aforementioned party. The blue voters then choose either the red, brown or green party as their second choice randomly.

During our testing, the brown candidate won with a score of 50 and the overall voter happiness of 69.0.

In the voting for two situation, no (rational) strategy applied by any voter, or any group of the voters would result in a outcome different from the brown party taking the win. A different outcome is only possible if a significant group of voters from either green, red or brown parties chooses to vote for the blue party. No other outcome will also improve the happiness of the voters.

3.3 Veto Voting

As shown in Figure 3, non-blue voters clearly prefer a non-blue candidate as they are closest to them on the political spectrum. Therefore, we change the situation in such a way that the blue voters always vote for the blue candidate and two other random candidates. Other voters on the other hand always vote for the remaining three candidates, therefore voting against blue. This setup results in the highest overall happiness recorded so far, equal to 73.67. No voting strategy can be applied to change the election results, even when the voters collude in groups of 10% each. No other voting strategy also can result in a higher happiness than voting honestly.

3.4 Borda Voting

Here we take the normal situation used in the previous examples, namely the one showed in Figure 3. During this experiment, much like in Veto voting, we assume that the blue voters vote for the blue candidate as their most favourite party and randomly sort other parties. Other, non-blue parties, vote for their own party and put the other two non-blue parties on random positions while putting the blue candidate last. Such a setup results in a red party victory a voter happiness of 41.17. The red party winning while not being the favourite can be attributed to random chance of other voters placing the red party more favourably on average.

By applying the compromising strategy and settings the voter collaboration to groups of 10%, we can generate some valid strategies which not only improve the happiness of the group but also the overall happiness. For example, a voter group voting for the brown party can choose to place the red party as their last choice, choosing the blue party to be their number 3 preference. This results in the brown party taking the win and improving the happiness of the 29% voters, as well as the voters of other parties who consider the brown party to be favourable. This change brings the happiness up to the value of 42.59.

Burying does not produce similar results, failing to increase the individual happiness of the voters. This holds true for even when the voters are asked to coordinate in groups of 10%.

3.5 Bullet voting

The previously discussed Borda election containing 100 voters and 4 candidates of course results in a blue loss and a happiness of 41.17. If a group of brown voters compromised their vote, then the overall happiness was improved from 41.17 to 42.59. When asked to vote for their first choice, the election ends with the blue candidate winning, gaining 30% of all the votes. This unfortunate outcome brings the happiness down to 17.93. Asking voters or groups of 10% voters to vote for their first candidate does not produce any valid voting strategies.

3.6 Comparing voting situations

To compare the different voting situations, a random Borda voting matrix was generated with 1000 voters and 5 options (preferences), the matrices for plurality, voting for two, and veto voting could be deduced from this Borda voting. These matrices are then used to compute the overall voter happiness. To provide a fair comparison, this process was repeated 100 times. The mean of the results was recorded. The results of this experiment can be found in Table 1.

Voting scheme	Happiness
Plurality	**144.00**
For two	136.01
Borda	136.05
Veto	130.99

Table 1: Voting situation table

As was expected, the best performing voting situation is plurality as the only one getting a point will be the number one preference of each voter, thus the preference with the most points will be the first choice for the majority of the voters, resulting in a high happiness. On the other side is veto voting, which awards a point to every preference but the one that is at the end of each voters preference list. This will mean that for a randomly generated preference matrix, the chance that the top preference of each voter will win using this voting situation is very low.

4 DISCUSSION

From the experiments that were conducted an analysis can be made about the effectiveness of voting situations, and how they can be influenced.

4.1 Voting situation comparison with same matrix

The effectiveness of each voting situation was discussed in section 3.6. This comparison did not introduce any surprising differences, as it was expected that whenever voters only assign a point to the preference they like the most, the overall happiness will be high as well. The opposite of this is the veto voting, where every preference gets a point except the one a voter dislikes most. Intuitively this will result in a low happiness as voters cannot express what their favorite option is. All of these outcomes were expected with a random generated preference matrix. In the other experiments a more elaborate approach to manipulating the preference matrix and hence, tactical voting was performed.

4.2 Voting tactics discussed per voting situation

4.2.1 Candidate increase. The first strategy that is discussed is adding more candidates to be voted for. In the example it can be seen that by adding more candidates, the voters that would vote for a candidate in a 2 party system can be divided into smaller groups by introducing more candidates, and with this division, the majority is split until the candidate that lost in the 2 party system suddenly has a majority over the smaller divided groups that originated from the original winner. As was expected, this divided the first preference of the voters over multiple candidates, resulting in a lower happiness.

4.2.2 Voting for two stalemate. The second experiment shows that with the 'voting for two' situation in a scenario with 4 parties, for every vote that is cast, half of the amount of candidates receives a point (vote). A candidate that appears most on the second place of voters while candidates that appear on the first place for some voters, might be third or forth for other voters, will therefore have more points overall, and thus will be the winner. In the experiment this effect can be seen for the 'brown' party. Most 'red' voters will have brown as their second option, same goes for the majority of blue voters. With this voting situation it is not possible for a voter or group of voters to change the outcome using voting strategies. The only way this can be achieved is when a significant group of voters completely changes their preference to not include brown in their top 2 and instead chose for blue. This outcome was to be expected as well when intuitively looking at the scenario in Figure 2. With voting for two, a shared second choice between the two major competing parties will get a share of the votes of both competitors and therefore becomes virtually unbeatable.

4.2.3 Veto voting happiness. The third experiment shows that when a majority of voters dislikes one candidate (blue) veto voting is a good way to combat the division tactic discussed in the first strategy experiment. With plurality, introducing more candidates will decrease the amount of voters per candidate. This does not hold for veto voting as more candidates will also increase the amount of 'points' to be given, making this division impossible. When using a random matrix like the comparison discussed before, veto voting is very inefficient which makes sense, but in the example used in our experiments it was also to be expected that the majority of voters would be happy when any candidate won but the one they didn't want to win (in this case blue).

4.2.4 Bullet voting. Bullet voting is a voting tactic to be applied with approval voting. Voters get to vote for any candidate they deem worthy, which might be only one, every candidate and any number in between. Approval voting when all voters vote honestly has a lot of upsides as it allows voters to be more expressive, showing support for multiple candidates if they want, a voter can never get a worse result by voting for their favorite, and it tends to elect candidates who would beat all rivals head-to-head. This can however be countered by dishonest voters using the *Bullet Voting* tactic, where they only vote for their number one preference which will push their favorite higher up, while leaving their favorites' competition behind as these are not awarded any votes. From our experiment is shows that when using this tactic the overall voter happiness fell to just 17.92. This result was no surprise as a voter that is voting for just one candidate will not convey their preference very well, so the outcome will not match with the preferences of many voters, decreasing the happiness significantly.

4.3 Voter collusion

The engine could be expanded to consider situations with voter collusion, where different voters try to fulfil their own agendas by agreeing to vote tactically alongside other voters. Specifically, if voters 1 and 2 had different first preferences but the same second preference in a 4 candidate election, they might both agree to vote for their second preference to guarantee their second preference winning, thus maximizing the overall happiness of the voter base while guaranteeing their lowest ranked choices don't win. In situations with many voters, using the happiness metric and considering different pairings of colluding voters, the engine could potentially calculate the effects of voter collusion. This would greatly increase the number of tactical voting scenarios to be considered, as each of the voters can collude with any number of the remaining voters. An implementation of this would involve allowing the voters to coalesce into a group with a higher voting power that is capable of splitting their votes strategically. For example, if voters 1-3 all had the same interest in making a specific candidate win, they could be considered as a single voting block with 3 voting ballots with a modified preference order depending on the average individual preferences of voters 1-3.

4.4 Counter-strategic voting

Another option would be to expand the engine to consider counter-strategic voting, where a voter modifies his own votes based on the assumptions that any other voter would vote strategically. This is fairly trivial to do if each voter only tries to perform counter-strategic voting by considering each other individual voter and their strategic voting options. An example would be that, in a election between 3 or more candidates and multiple voters, voter 1 decides to strategically vote for candidate 2 since they do not believe their own top preference, candidate 1, would win. Voter 2 could attempt to counter this specifically by voting for candidate 1 themselves in order to increase the chances of candidate 3 winning. Of course, if the engine needs to consider each voter trying to perform counter-strategic voting or if each counter-strategic voter tries to counter multiple tactical voters simultaneously, the situations becomes more complex. Both of these situations can be implemented by providing the user with the possibility to input the number of counter-strategic voters and number of strategic voters in any given scenario. Furthermore, the situations can become more complex yet if we look deeper and consider situations where each voter tries to counter-strategic voting by counter-strategic voting themselves and so on, so an additional input asking for the depth of counter-strategic voting might also be beneficial.

If we consider the possibility of voter collusion and counter strategic voting simultaneously, where several voters might coalesce into a group with a combined preference and at least one other voter tries to prevent this group from affecting the election negatively, the situation becomes yet more complex. Voter collusion might be impossible to counter in certain situations where there aren't enough counter-strategic voters, so ideally there should also be the option to coalesce counter-strategic voters into a single common-interest group.

4.5 Imperfect information

The main assumption made was that the engine and, therefore, the voters had access to perfect information - all voters knew the true preferences of all other voters. Without this assumption, the TVA agent cannot come to a conclusive decisions on who will win and instead has to either consider a factorial number of options for each of the voters. This can be improved my making use of a probabilistic engine (such as using Markov Decision Processes) to determine the outcome of a voting situation. Another option for situations with round-based voting is to include a hidden agenda and n-dimensional political compass and placing the candidates on this compass. The voters would then likely vote for candidates nearer to their chosen candidates in this n-dimensional space, so a probabilistic preference vector can be constructed for future voting rounds.

5 CONCLUSION

This paper presented an implementation of a Tactical Voting Analyst agent, responsible for analyzing different voting scenarios where voters may or may not vote tactically. The current implementation considers plurality voting, voting for two, veto voting and Borda voting as the main voting schema, and analyzes single-round elections with respect to compromising, burying and bullet-voting tactical voting options. We have shown that the election could be manipulated dependent on voting schemes. Some of the potential election manipulations include adding additional candidates thereby splitting the vote of similarly-minded people or convincing groups of voters to vote for their second choice.

The agent can be extended to consider multiple-round voting and, therefore, also considering push-over tactical voting, as well as Condorcet voting. Other extensions of the implemented tool might be the option to experiment with voter collusion, counter-strategic voting, and imperfect information. For future work these extensions might allow for more extensive research in the efficiency of voting situations and how malicious voters might try to influence these elections. Comparing all of these tools will allow researchers to make even better conclusions about what types of voting are suitable for certain situations.

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APPENDIX

Strategic voting risk with number of candidates.

The following figures show the resulting strategic voting risk by varying the number of candidates. All experiments were run 10 times for each data point with randomized preference vectors, and the data spread is given in the shaded regions in the figure. For all experiments, there were 20 voters and between 2 and 10 candidates.

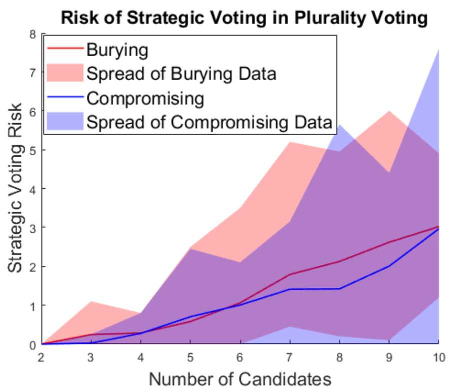

Figure 4: Increase in strategic voting risk with number of candidates under the plurality voting scheme. The risk of strategic voting with compromising and burying strategic options is shown.

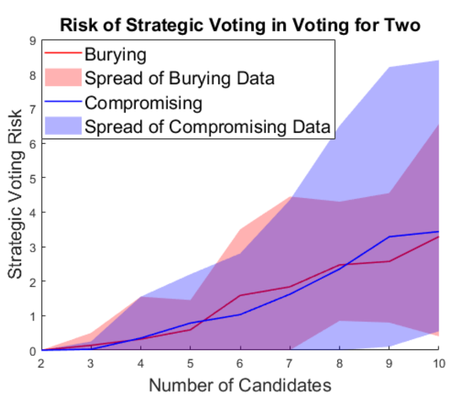

Figure 5: Increase in strategic voting risk with number of candidates under the voting for two voting scheme. The risk of strategic voting with compromising and burying strategic options is shown.

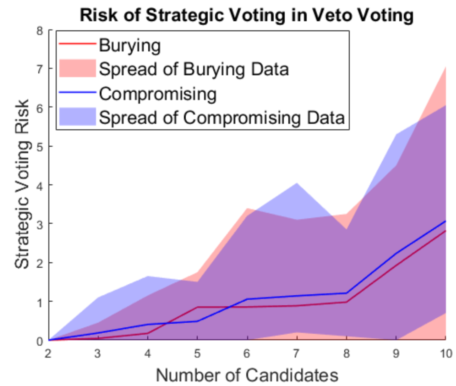

Figure 6: Increase in strategic voting risk with number of candidates under the veto voting scheme. The risk of strategic voting with compromising and burying strategic options is shown.

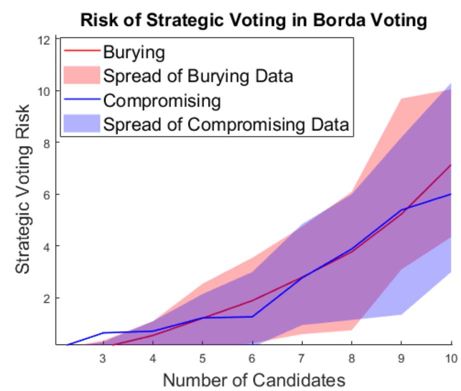

Figure 7: Increase in strategic voting risk with number of candidates under the Borda voting scheme. The risk of strategic voting with compromising and burying strategic options is shown.

In most situations, the risk of strategic voting for burying and compromising under the different voting schemes match very closely, so there is very little difference in the number of strategic voting options between both types of strategic voting. However, between different schema, we can see that both options are much more risky in Borda voting than in any of the other types of voting. The more complex the type of choices voters have, the higher the risk, hence both Borda voting and voting for two have higher risk with higher number of candidates than either plurality of veto voting. Furthermore, as the number of candidates increase, the influence each individual voter has over the election also increases, thus increasing the risk of strategic voting. By keeping the number of candidates small relative to the number of voters, this risk can be minimized.