The final element in the philosophical picture of the mind is the connection between knowledge and action. This question is vital to AI because intelligence requires action as well as reasoning. Moreover, only by understanding how actions are justified can we understand how to build an agent whose actions are justifiable (or rational). Aristotle argued (in *De Motu Animalium*) that actions are justified by a logical connection between goals and knowledge of the action's outcome (the last part of this extract also appears on the front cover of this book, in the original Greek):

But how does it happen that thinking is sometimes accompanied by action and sometimes not, sometimes by motion, and sometimes not? It looks as if almost the same thing happens as in the case of reasoning and making inferences about unchanging objects. But in that case the end is a speculative proposition ... whereas here the conclusion which results from the two premises is an action. ... I need covering; a cloak is a covering. I need a cloak. What I need, I have to make; I need a cloak. I have to make a cloak. And the conclusion, the "I have to make a cloak," is an action.

In the *Nicomachean Ethics* (Book III. 3, 1112b), Aristotle further elaborates on this topic, suggesting an algorithm:

We deliberate not about ends, but about means. For a doctor does not deliberate whether he shall heal, nor an orator whether he shall persuade, ... They assume the end and consider how and by what means it is attained, and if it seems easily and best produced thereby; while if it is achieved by one means only they consider *how* it will be achieved by this and by what means *this* will be achieved, till they come to the first cause, ... and what is last in the order of analysis seems to be first in the order of becoming. And if we come on an impossibility, we give up the search, e.g., if we need money and this cannot be got; but if a thing appears possible we try to do it.

Aristotle's algorithm was implemented 2300 years later by Newell and Simon in their GPS program. We would now call it a regression planning system (see Chapter 10).

Goal-based analysis is useful, but does not say what to do when several actions will achieve the goal or when no action will achieve it completely. Antoine Arnauld (1612–1694) correctly described a quantitative formula for deciding what action to take in cases like this (see Chapter 16). John Stuart Mill's (1806–1873) book *Utilitarianism* (Mill, 1863) promoted the idea of rational decision criteria in all spheres of human activity. The more formal theory of decisions is discussed in the following section.

## 1.2.2 Mathematics

- What are the formal rules to draw valid conclusions?
- What can be computed?
- How do we reason with uncertain information?

Philosophers staked out some of the fundamental ideas of AI, but the leap to a formal science required a level of mathematical formalization in three fundamental areas: logic, computation, and probability.

The idea of formal logic can be traced back to the philosophers of ancient Greece, but its mathematical development really began with the work of George Boole (1815–1864), who