North Carolina A&T State University

Final Project for ECEN 668: Theory of Linear Control Systems

Control System Design for Active Bus Suspension System

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Abstract

The ride comfort and handling of an automobile depends on the performance of its suspension system. In this project, we have designed a controller for an active suspension system using pole placement technique that avoids large oscillations and dissipates the oscillations quickly. While modeling the system, the hydraulic actuator and the road profile were included as an input and disturbance respectively. The two approaches used to design a controller are pole placement with feed-forward gain and pole placement with integral controller. Based on the performance evaluation of these two approaches, we recommend using the later to control this system.

Contents

List of figures

Introduction

A suspension system is a part of an automobile that connects the axle and wheel assemblies of a vehicle to the frame of the vehicle. The main function of suspension system in the vehicle is to improve the riding comfort and the road-holding ability[1]. The two basic components of a suspension system are spring and damper. The role of the spring is to support the static weight of the vehicle. Whereas, the role of the damper is to dissipate vibrational energy. There are two types of suspension systems: passive suspension system and active suspension system. The main difference between these two is that passive suspension system is one in which the main components (spring and damper) are fixed. In an active suspension, the damper or both the damper and the actuator are replaced with a force actuator[2].

Our objective in this project is to design a controller for an active suspension system using a pole placement technique. The controller is expected to avoid large oscillations and dissipate the oscillations quickly. To achieve this objective we first develop a mathematical model of our system. The developed mathematical model is of 4^{th} order with 4 states. We also incorporated the actuator force as input and the road profile as a disturbance. The next step is analyzing the open loop system, which includes the impulse and step response of the system along with the stability of the system.

After analyzing the open loop system, we then proceed to the design of controller. To design a controller for our system, we first checked the controllability of the system using the controllability matrix. Once we check the controllability of our system, since we are using a pole placing technique to design the controller, we used Bessel’s prototype transfer function to select the desired poles. Based on these poles we designed a control law that includes the feed-forward gain. Next to that we designed a control law that has an integral controller, which performs well when the system is subject to disturbance.

System Modelling

We can represent our system using mass, spring, and damper as follows.

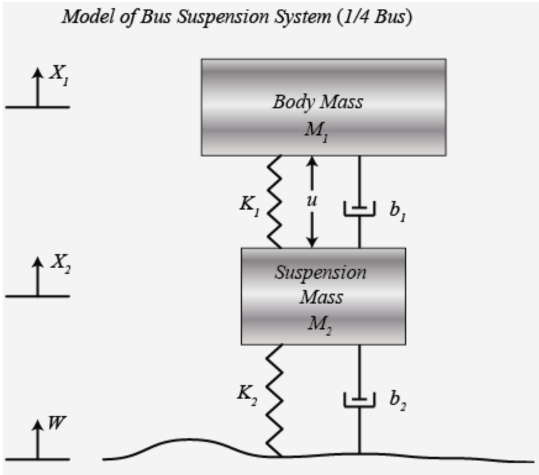

Figure 1: Physical Model of the System

Considering each mass separately, we can write differential equations to describe the model of the system as:

$$M_1\ddot{x_1} = -b_1\left(\dot{x}_1 - \dot{x}_2\right) - k_1(x_1 - x_2) + u$$

$$M_2\ddot{x_2} = b_1\left(\dot{x}_1 - \dot{x}_2\right) + k_1(x_1 - x_2) - b_2\left(\dot{x}_2 - \dot{w}\right) - k_2(x_2 - w) - U$$

From the above differential equations we can get the state space representation of the system as

$$\begin{bmatrix} \dot{x_1} \\ \ddot{x_1} \\ \dot{y_1} \\ \dot{y_2} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-b_1b_2}{m_1m_2} & 0 & \frac{b_1}{m_1}\left(\frac{b_1}{m_1}+\frac{b_1}{m_2}+\frac{b_2}{m_2}\right) - \frac{k_1}{m_1} & \frac{-b_1}{m_1} \\ \frac{b_2}{m_2} & 0 & -\left(\frac{b_1}{m_1}+\frac{b_1}{m_2}+\frac{b_2}{m_2}\right) & 1 \\ \frac{k_2}{m_2} & 0 & -\left(\frac{k_1}{m_1}+\frac{k_1}{m_2}+\frac{k_2}{m_2}\right) & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ \dot{x_1} \\ y_1 \\ y_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \frac{1}{m_1} & \frac{b_1b_2}{m_1m_2} \\ 0 & \frac{-b_2}{m_2} \\ \left(\frac{1}{m_2}+\frac{1}{m_2}\right) & \frac{-k_2}{m_2} \end{bmatrix} \begin{bmatrix} u \\ w \end{bmatrix}$$

$$y = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ \dot{x_1} \\ y_1 \\ y_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \end{bmatrix} \begin{bmatrix} u \\ w \end{bmatrix}$$

¼ bus body mass	2500 kg
Suspension system	320 kg
Spring constant of suspension system	80,000 N/m
Spring constant of wheel and tire	500,000 N/m
Damping constant of suspension system	350 N.s/m

Damping constant of wheel and tire	15,020 N.s/m

Substituting the above values into the state space representation gives:

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \\ \dot{X}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -6.5713 & 0 & -25.256 & -1.14 \\ 46.9375 & 0 & -48.17 & 1 \\ 1562.5 & 0 & -1844.5 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0.0004 & 6.5713 \\ 0 & -46.937 \\ 0.0035 & -1562.5 \end{bmatrix} [U\ W]$$

$$Y = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} + \begin{bmatrix} 0 & 0 \end{bmatrix} [U\ W]$$

Open Loop Analysis of the System

Now let us analyze the above system without applying any feedback controller. Analyzing the open loop system is of high importance because:

> A linear time-invariant control system is stable if the output eventually comes back to its equilibrium state when the system is subjected to an initial condition. A linear time-invariant control system is critically stable if oscillations of the output continue forever. It is unstable if the output diverges without bound from its equilibrium state when the system is subjected to an initial condition[3].

Impulse response

We can represent non-zero initial conditions of a system using the impulse response. So, if the impulse response of the system approaches zero, then the system is stable. However, if the impulse response of the system is unbounded, then it is unstable.

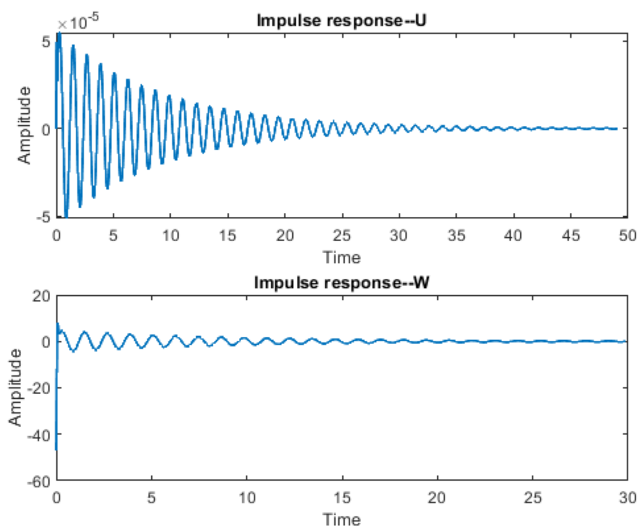

Figure 2: Impulse Responses of the System

As we can see from the above plot, the impulse response approaches zero as time goes. Therefore, our system is stable.

Step response

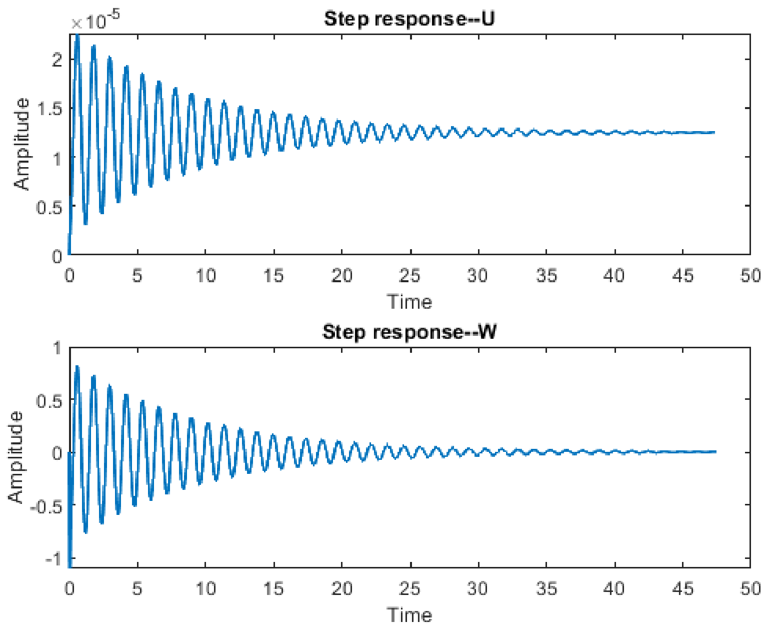

Figure 3: Step Responses of the System

As we can see from the above figure, the step response is stable but this is not a response that we want to get from our system. Therefore, we need to design a controller that makes the suspension system behave in a way that we want.

Stability analysis of the system based on the Eigen Values

Another way to study the stability of a system is by studying its Eigen values, which can be calculated using $|A - \lambda I| = 0$.

λ 1 = -(23.9758 +35.1869i)

λ 2 = -(23.9758 -35.1869i)

λ 3 = (-0.1098 + 5.2504i)

λ 4 = (-0.1098 - 5.2504i)

As we can see, each real part of the Eigen value is less than zero. Therefore, the system is

- **BIBO stable**
  - **Stable in the sense of Lyapunov**, and
  - **Asymptotically stable in the sense of Lyapunov**

Control System Design

Controller design with feed-forward gain

A) Let $w = 0$ and $r(t) = 2\ for\ t \geq 0$. Design a state-feedback control in the form of $u(t) = -kTx(t) + Fr(t)$ to place the eigenvalues of the closed-loop system at appropriate places and track the reference.

The first thing that we need to make sure before getting into the design of a controller is that we need to know if the system is controllable or not. By definition, our system is controllable if the rank of the controllability matrix is full.

$$C = [B \quad AB \quad A^2B \quad A^3B]$$

$$C = \begin{bmatrix} 0 & 0.0004 & -0.0005 & -0.0917 \\ 0.0004 & -0.0005 & -0.0917 & 4.6404 \\ 0 & 0.0035 & -0.151 & 1.3752 \\ 0.0035 & 0 & -5.8769 & 277.8013 \end{bmatrix}$$

Rank(C) = 4, therefore our system is controllable.

Now, we can go ahead and design the controller.

To design a controller using pole placement technique, we need to have the desired poles first. This desired poles are the ones that will determine the nature of our controlled system. In this project, I selected these desired poles from Bessel filter prototype and the desired poles will be:

λ 1 = -(0.6573+0.8302i)

λ 2 = -(0.6573-0.8302i)

λ 3 = -(0.9047+0.2711i)

λ 4 = -(0.9047-0.2711i)

To place the poles of the given system to the above poles, I used the Ackerman method, which is given by the equation:

$$K^T = [0\ 0\ 0\ 1]C^{-1}\Delta d(A)$$

$$\Delta d(S) = S^4 + 3.124S^3 + 4.3919S^2 + 3.2014S + 1.0002$$

$$\Delta d(A) = A^4 + 3.124A^3 + 4.3919A^2 + 3.2014A + 1.0002I$$

$$\Delta d(A) = 1 \times 10^8 \begin{bmatrix} 0.0003 & 0 & -0.0002 & 0 \\ 0.0126 & 0.0003 & -0.0159 & -0.0002 \\ 0.0361 & -0.0006 & -0.0159 & 0.0007 \\ 0.9962 & 0.0101 & -1.1959 & -0.0064 \end{bmatrix}$$

$$C^{-1} = \begin{bmatrix} 2710.8 & 2927.5 & 102.24 & -48.51 \\ 3020.8 & -13.418 & -60.978 & 1.52 \\ 86.94 & -0.807 & -9.978 & 0.0916 \\ 1.8 & -0.0542 & -0.212 & 0.0062 \end{bmatrix}$$

$$K^T = [0\ 0\ 0\ 1] \times \begin{bmatrix} 2710.8 & 2927.5 & 102.24 & -48.51 \\ 3020.8 & -13.418 & -60.978 & 1.52 \\ 86.94 & -0.807 & -9.978 & 0.0916 \\ 1.8 & -0.0542 & -0.212 & 0.0062 \end{bmatrix} \times 1 \times 10^8 \begin{bmatrix} 0.0003 & 0 & -0.0002 & 0 \\ 0.0126 & 0.0003 & -0.0159 & -0.0002 \\ 0.0361 & -0.0006 & -0.0159 & 0.0007 \\ 0.9962 & 0.0101 & -1.1959 & -0.0064 \end{bmatrix}$$

$$K^T = 1 \times 10^5 [-1.7591 \quad 0.1403 \quad 1.1364 \quad -0.1437]$$

Now, let's find the feed-forward gain F using

$$F = [C[-A + bK^T]^{-1} b]^{-1}$$

$$F = 1.6003$$

Finally, our control law, which is given by the form

$$U = -K^T X + Fr$$

$$U(t) = -\left[1 \times 10^5 [\ -1.7591 \quad 0.1403 \quad 1.1364 \quad -0.1437]\right] \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} + [1.6003] r(t)$$

b) For part (a), find the state space representation of the closed-loop system and then, use MATLAB to plot the response of the system to see whether y(t) asymptotically tracks the reference signal $r(t) = 2\ for\ t \geq 0$.

The closed loop state space representation of the system has a form

$$\dot{X} = (A + BK^T)X + (BF)U$$

$$Y = CX + DU$$

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \\ \dot{X}_4 \end{bmatrix} = 1x10^3 \begin{bmatrix} 0 & 0.001 & 0 & 0 \\ 0.0638 & -0.0056 & -0.0707 & 0.0056 \\ 0.0469 & 0 & -0.0482 & 0.001 \\ 2.1826 & -0.0494 & -2.2451 & 0.0507 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0.0006 \\ 0 \\ 0.0056 \end{bmatrix} [U]$$

$$Y = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} + [0][U]$$

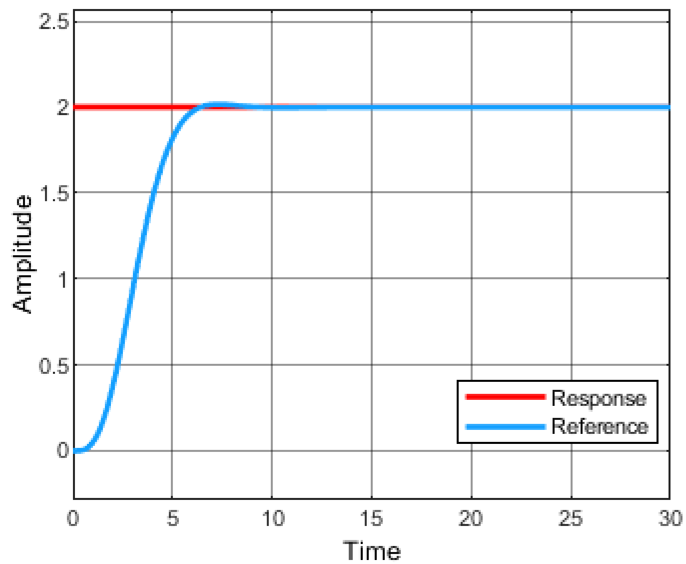

Figure 4: Closed Loop System Response

The role of the feed forward gain is to compensate the constant gain that is developed by the controller. The following figures explain this role of the feed-forward gain.

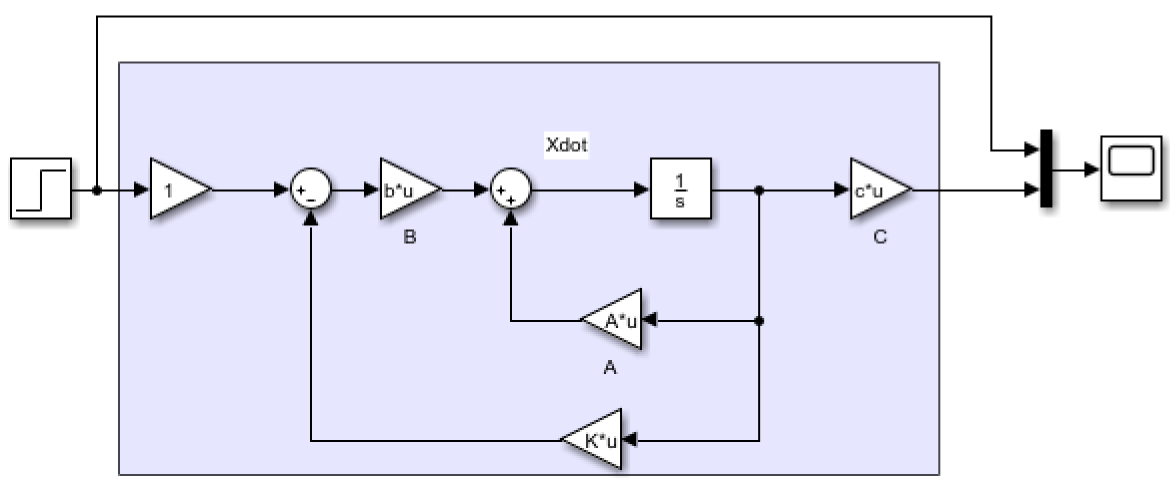

Figure 5: The Closed Loop System without Disturbance

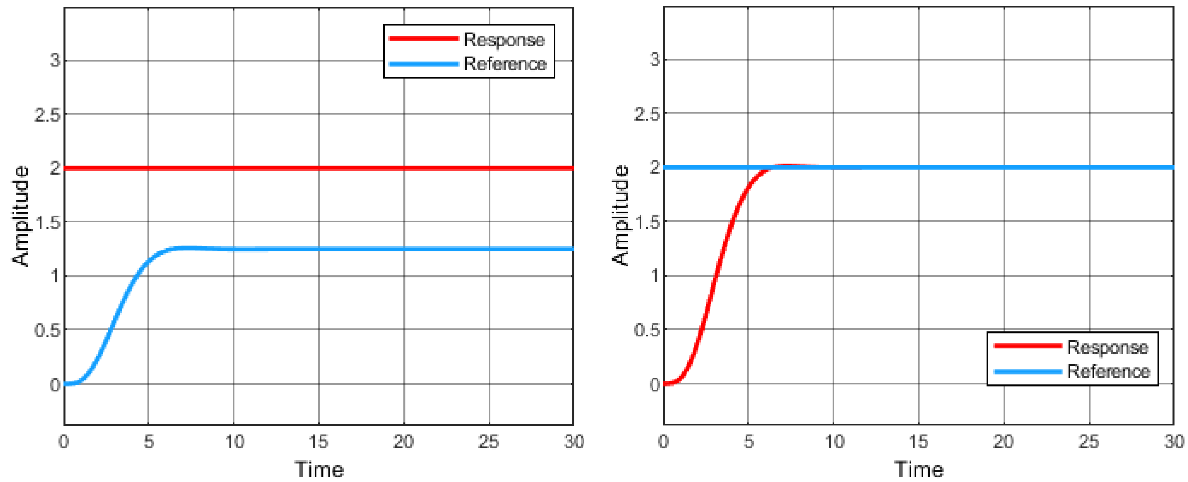

Figure 6: The effect of the feedforward gain

C) In part (b), let $w(t) = 1 \; for \; t \geq 10$. Use MATLAB to plot the response of the system to see whether y(t) asymptotically tracks the reference signal $r(t) = 2 \; for \; t \geq 0$ in the presence of disturbance.

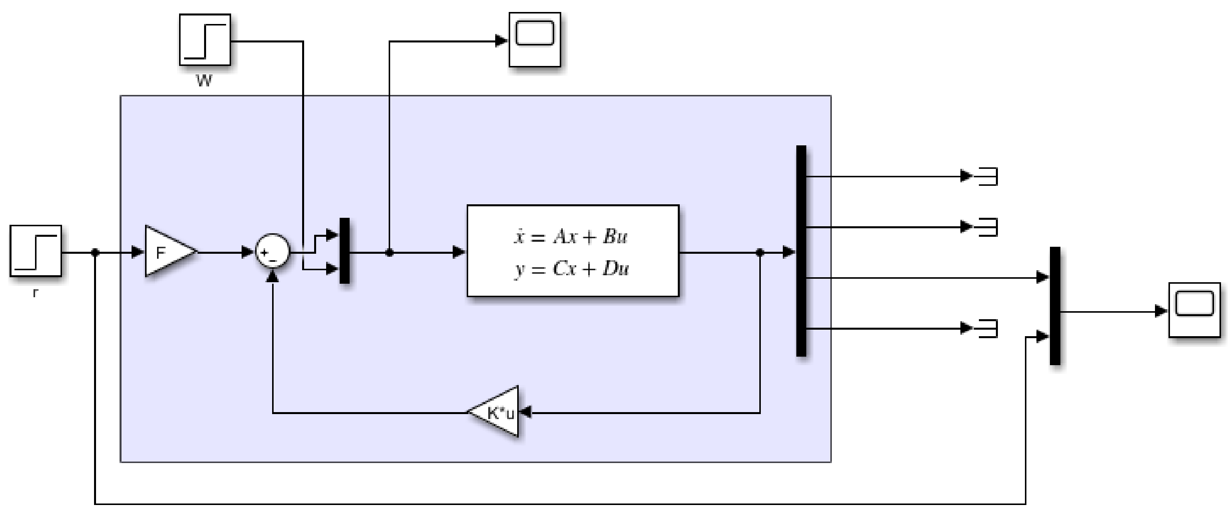

Figure 7: The closed loop system with disturbance

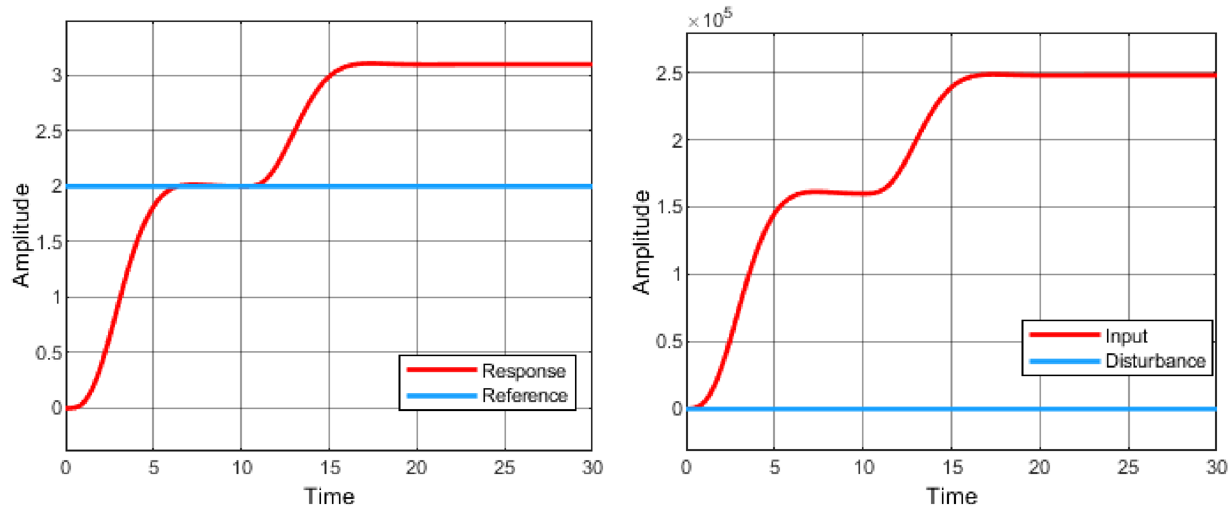

Figure 8: The response and the input of the system including the disturbance

Controller Design with Integral Controller

D) Instead of feed-forward gain F in part (a), design an integral control combined with the state-feedback controller to reject the constant disturbance $w(t)$. You can use MATLAB command place, to find the feedback and integral gains.

The closed loop state space representation of the system including the integral control element looks like

$$\dot{\bar{X}} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} \begin{bmatrix} X \\ X_I \end{bmatrix} + \begin{bmatrix} b_1 \\ 0 \end{bmatrix} U + \begin{bmatrix} b_2 \\ 0 \end{bmatrix} W + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r$$

$$\mathrm{Y} = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} X \\ X_I \end{bmatrix}$$

$$\mathrm{U} = \begin{bmatrix} K^T & -K_I \end{bmatrix} \begin{bmatrix} X \\ X_I \end{bmatrix}$$

Now, the system becomes a 5th order system and to get the control gains KT and KI, we need to place the poles of the new system in appropriate places. To do that I selected the desired poles from Bessel's prototype and the new desired poles will be

λ 1 = -(0.9264)

λ 2 = - (0.5906+0.9072i)

λ 3 = -(0.5906-0.9072i)

λ 4 = -(0.8516+0.4427i)

λ 5 = -(0.8516-0.4427i)

By using MATLAB's place command, I the following control law

$$K^T = 1x10^5[-1.6635 \quad 0.1381 \quad 1.0382 \quad -0.1415]$$

$$K_I = 1.6$$

$$\text{U}=1\text{x}10^5[-1.6635 \quad 0.1381 \quad 1.0382 \quad -0.1415 \quad -0.000016]\begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_I \end{bmatrix}$$

e) In part (d), let $w(t) = 1 \; for \; t \geq 10$. Use MATLAB to plot the response of the system to see whether y(t) asymptotically tracks the reference signal $r(t) = 2 \; for \; t \geq 0$ in the presence of disturbance.

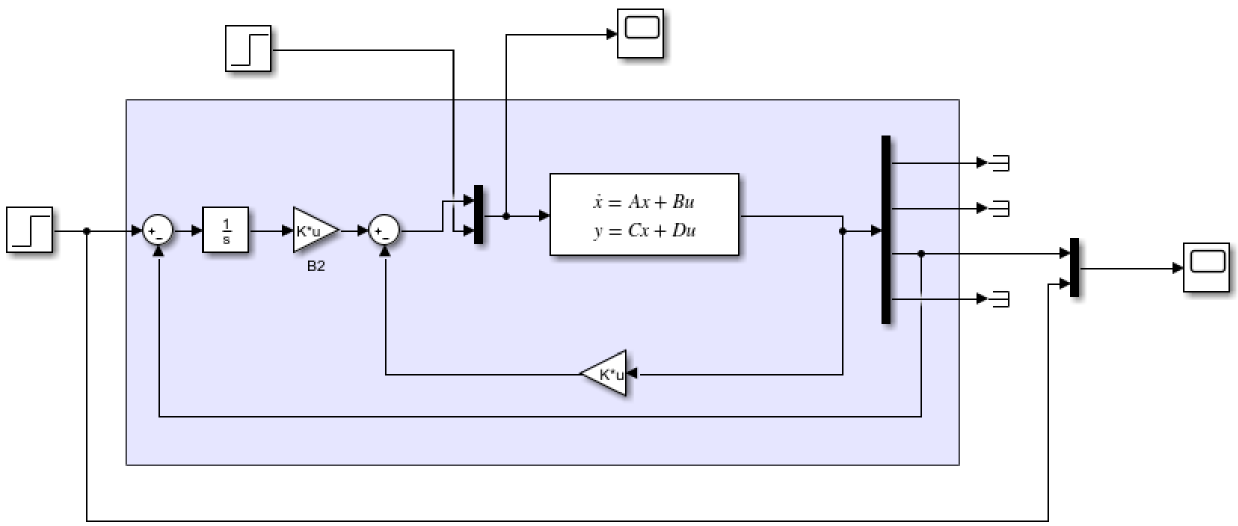

Figure 9: The closed loop system with the integral controller

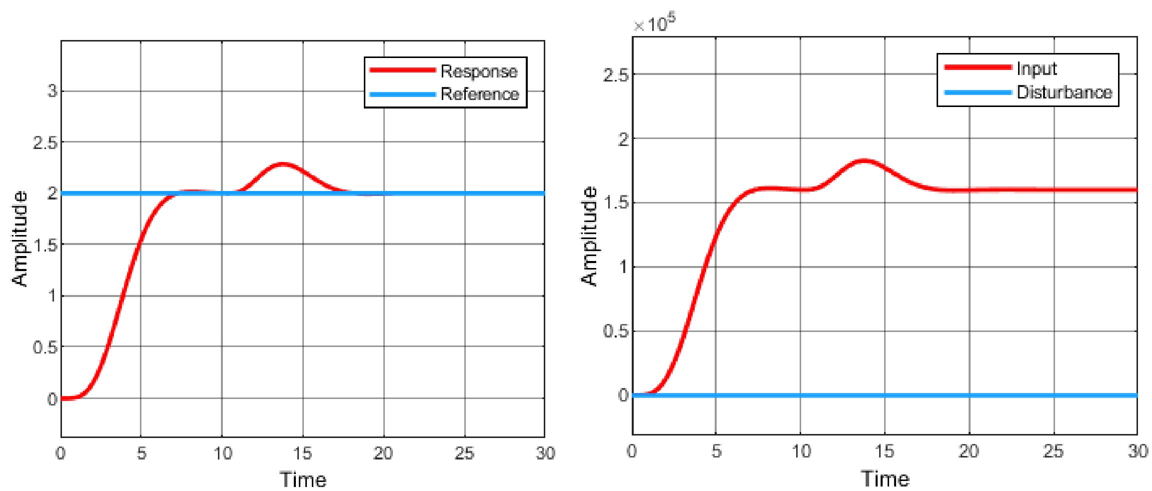

Figure 10: The response and input of the system with integral control

As, we can see from the above figure, the integral controller playes a huge role in rejecting the constant disturbance.

The next thing that we need to consider after designing a controller that provides the desired response is that we should check how each state in the state space is behaving. This is important because in order to make sure the system is behaving in an appropriate manner and not dying inside, we should have a proper understanding of how each state is behaving and this can be done using state trajectories. And from the figures below we can see that all the states behave normally and there is no internal problem in our system.

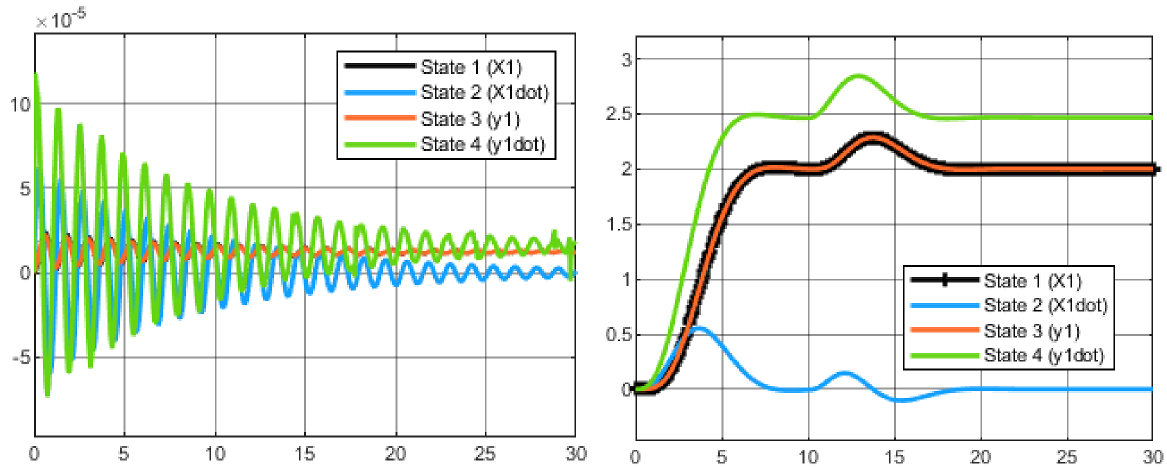

Figure 11: Open loop and closed loop state trajectories

Comparison between Feed-forward gain and Integral controller

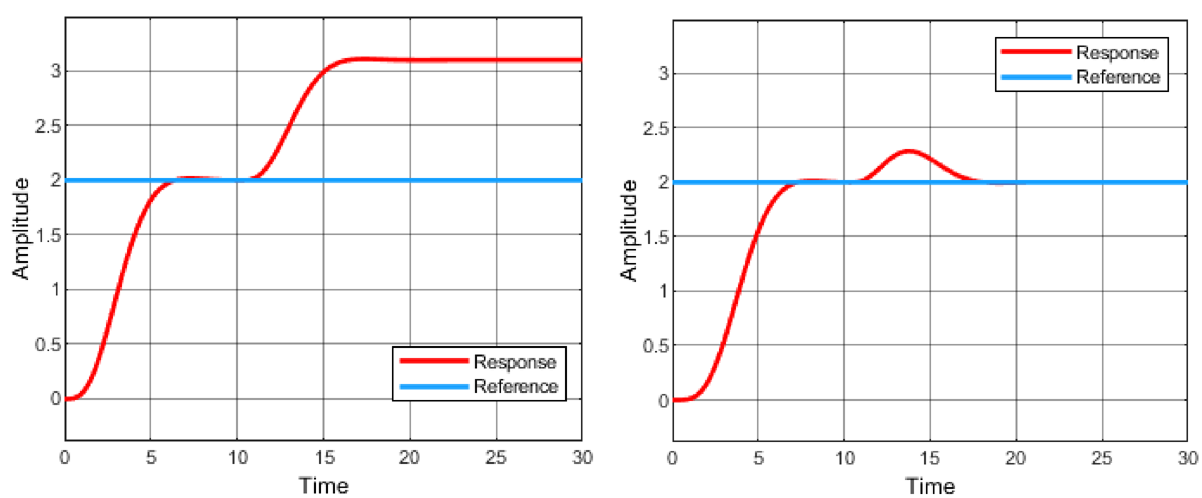

Figure 12: Comparison between feed-forward and integral controller

Further comparison

In the above section, we have seen that the integral control performs very well in the presence of disturbance which is of a step signal type. However, to compare the performance of the two control design approaches, I applied different shapes of disturbances, which represent different road profiles.

Comparison of feedforward and integral controls when a square waveform is applied.

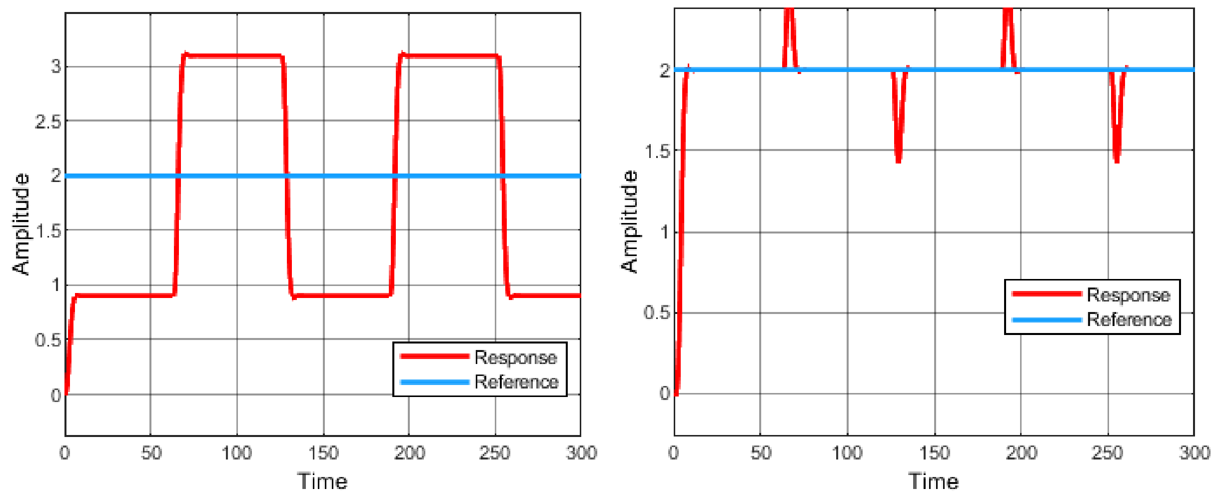

Figure 13: Comparison of feedforward and integral controllers when the road profile is of a square waveform

Comparison of feedforward and integral controls when a sinusoidal waveform is applied.

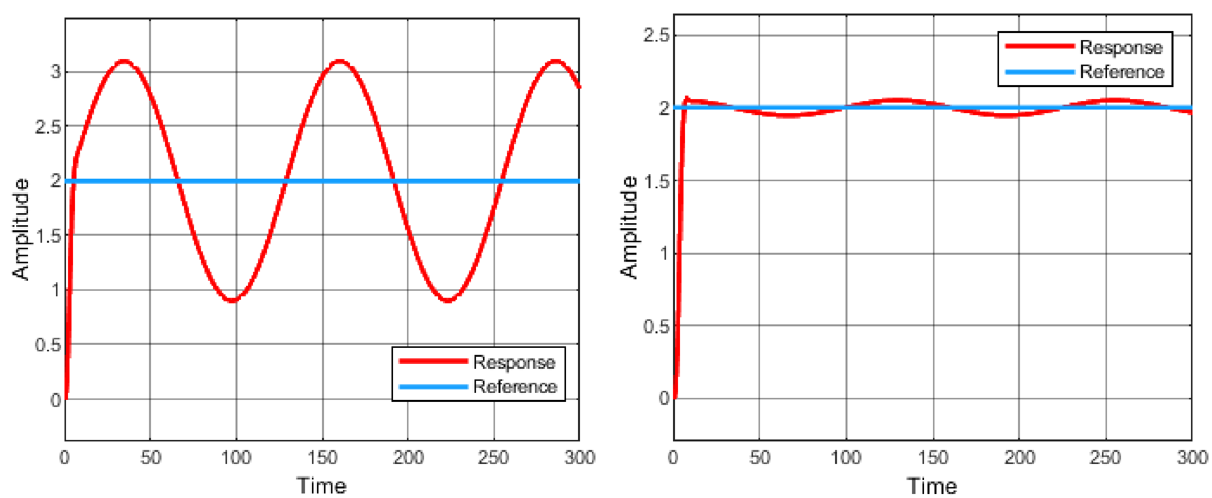

Figure 14: Comparison of feedforward and integral controllers when the road profile is of a sinusoidal waveform

As one can observe from the above figures, the integral controller performs very well in the presence of different forms of disturbances (different road profiles) by avoiding large oscillations and dissipating the effect of the disturbance quickly.

Conclusion

In this project, I had the chance to design a controller for an active suspension system using pole placement technique, where the desired poles were selected from the Bessel prototype transfer functions. The two approaches I used to design the controller are using feed-forward gain and using integral control. As indicated in fig   the role of the feed-forward gain is to compensate the constant gain which is introduced to the system by the controller. However, when the system is subjected to a constant disturbance, the feed forward gain cannot reject this disturbance. In this case we need an integral controller that accumulates the past error elements to reject the effect of the disturbance on the system. As it can be seen from fig 13 and figure 14, the integral controller avoid large oscillations and dissipates the oscillations quickly. Hence, the integral controller with the specified parameters should be implemented.

References

[1] R. Bai and D. Guo, "Sliding-Mode Control of the Active Suspension System with the Dynamics of a Hydraulic Actuator," *Complexity*, 2018. [Online]. Available: https://www.hindawi.com/journals/complexity/2018/5907208/. [Accessed: 07-Dec-2019].
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