# THW Modelling v1.0

Asad Mohiuddin

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# 1 Ideal Analytical Model

To start the simplest model we apply the following assumptions:

- The thermo-physical properties of the medium are always constant.
- Hot wire has negligible heat capacity and a large thermal conductivity.
- No thermal resistance exists between the contact of the medium and surface of the hot wire.
- Pure conductive heat transfer from an infinity long wire that has a infinitely thin radius to an infinite medium.

Primarily there is heat transfer to the medium where the temperature of the hot wire varies with time. The heat transfer can be treated as a boundary condition as we are relative to the medium. The basic problem is driven by the non-stationary heat diffusion of Fourier's law

$$\rho C_p \frac{\partial T}{\partial t} = \lambda \nabla^2 T$$

$\lambda$ is the thermal conductivity of the medium

$\rho$ is the density of the medium

$C_p$ is the specific heat capacity of the medium

Substituting in for the constants

$$\frac{\partial T}{\partial t} = \alpha \nabla^2 T$$

Where $\alpha = \frac{\lambda}{\rho C_p}$ the thermal diffusivity of the medium. The ability of the medium to conduct thermal energy, relative to its ability to store it.

## 1.1 Boundary conditions

**(1)** For any radial distance $r$ at $t \leq 0$

$$\Delta T_{ideal}(r, t) = 0$$

This boundary condition states that at the start the wire is in thermal equilibrium with the surrounding medium.

**(2)** At $r = 0$ and $t \geq 0$

$$\lim_{r \to \infty} \left( r \frac{\delta T}{\delta r} \right) = -\frac{q}{2\pi\lambda}$$

This describes the radial temperature distribution as we get closer to the wire.

**(3)** At $r = \infty$ and $t \geq 0$

$$\lim_{r \to \infty} \Delta T_{ideal}(r,t) = 0$$

A long distance away from the wire the temperature profile remains constant.

The solution to this problem is well known and was analytically proven by Carlslaw H S and Jaeger J C in 1959, they found

$$\Delta T_{ideal}(r,t) = \frac{q}{4\pi\lambda} E_1\left(\frac{4\alpha t}{r^2}\right)$$

Where $q$ is the power input per unit length of wire. The expression $E_1\left(\frac{4\alpha t}{r^2}\right)$ is the exponential integral and the series expansion of this exponential integral is

$$\Delta T_{ideal}(r,t) = \frac{q}{4\pi\lambda}\left[\ln\left(\frac{4\alpha t}{r^2 C}\right) + \frac{\left(\frac{r^2}{4\alpha t}\right)}{1(1!)} + \frac{\left(\frac{r^2}{4\alpha t}\right)}{2(2!)} + ...\right]$$

Where $C = e^{\gamma} = 1.781$ is the exponential of Euler's constant.

For sufficiently long times i.e. small values of $\frac{r^2}{4\alpha t}$ where $t \gg \frac{r^2}{4\alpha}$ ($10ms \leq t \leq 100ms$). Also the surface of the hot wire imposes a uniform temperature which is equal to that in the conducting medium at $r = r_0$. The ideal equation describing the temperature history of the wire

$$\Delta T_{ideal}(r_0,t) = \frac{q}{4\pi\lambda} \ln\left(\frac{4\alpha t}{r^2 C}\right)$$

That can also be expressed as:

$$\Delta T_{ideal}(r_0,t) = \frac{q}{4\pi\lambda} \ln\left(\frac{4\alpha}{r^2 C}\right) + \frac{q}{4\pi\lambda} \ln(t)$$