ME6221: Theory of Mechanisms
6-Bar Mechanical Lift

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Problem Statement

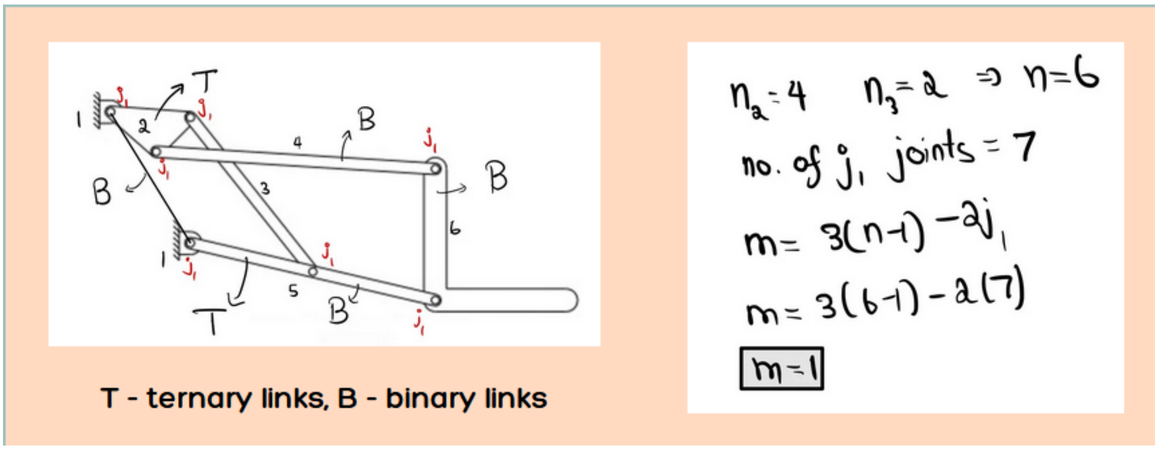
This project aims to synthesise a 6-bar Stephenson's chain to be used as a Mechanical Lift. A mechanism that is designed with the intent to lift objects from a lower location to a higher location. While 4-bar mechanisms can be used as mechanical lifts, the 6-bar Stephenson's chain maintains the output at a constant orientation while still having a better mechanical advantage.

Applications

The 6-bar mechanism can be used for an important task in a factory setting where heavy objects need to be lifted, or on the front of vehicles that need to pick and place objects that are not feasible to people to do otherwise. The mechanism converts the rotational motion of the crank to translatory motion of the output link, providing a vertical lift to carry loads. This linkage moves objects to different heights, using a simple range of input. Some of its uses include a wheelchair lifter and positioning heavy weight objects onto raised platforms.

Mobility Analysis

Our lifter has 6 links, of which 2 are tertiary and 4 are binary. Using Grubler's criteria.



Method

For synthesis, we used a combination of methods, namely dyad form synthesis, then following up with iterating these results to simulate our required model. We realised our mechanism is an inversion of Stephenson's chain, and thus we were able to find a 4 bar which drives a 5 bar, originating from the ground link.

Writing loop equations for the 4 and 5 bar, and using dyad equations for loops in this linkage, we arrive at a set of equations governing the position and orientation of the links.

Our procedure involves relating the input link with the output link, i.e., function generation. Taking three positions into consideration, assuming a set of free choices and using all necessary constraints, a set of equations will be obtained, upon which solving, the link lengths can be found. This completes our synthesis.

Mathematical Formulation

Step 1: The following equations can be arrived at by considering all loops and dyad equations.

Dyad form Synthesis:

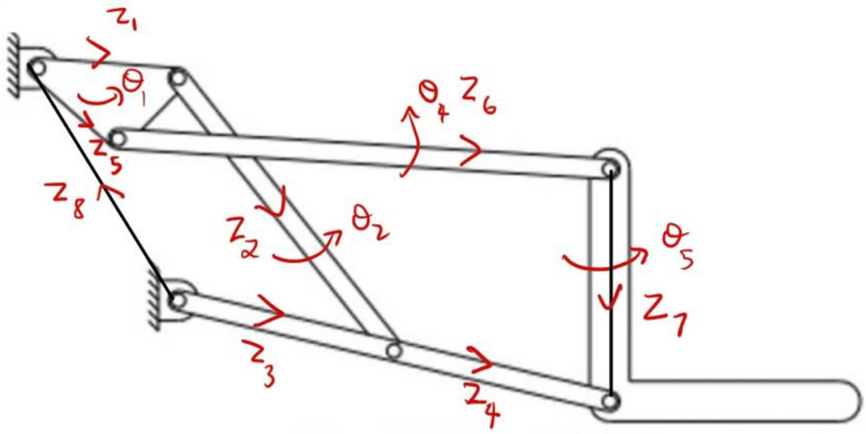


Figure 1

Dyad form Equations:

loop 1 : $z_1(e^{i\theta_1}-1)+z_2(e^{i\theta_2}-1)+z_4(e^{i\theta_3}-1)=\delta$

loop 2 : $z_5(e^{i\theta_1}-1)+z_6(e^{i\theta_4}-1)+z_7(e^{i\theta_5}-1)=\delta$

loop 3 : $z_3(e^{i\theta_3}-1)+z_4(e^{i\theta_3}-1)=\delta$ * → Redundant

Loop Closure Equation:

loop 1 : $z_1+z_2-z_3+z_8=0$

loop 2 : $z_1e^{i\theta_1}+z_2e^{i\theta_2}-z_3e^{i\theta_3}+z_8=0$

Step 2: Solving for $\mathbf{z_1}$, $\mathbf{z_2}$, $\mathbf{z_3}$, $\mathbf{z_4}$ using 4 bar dyad equations.

$$\text{loop } 2 - \text{loop } 1 = z_1\left(e^{i\theta_1}-1\right) + z_2\left(e^{i\theta_2}-1\right) - z_3\left(e^{i\theta_3}-1\right) = 0$$

Assuming $z_8 = e^{i\frac{2\pi}{3}}$

$z_3 = z_4$

$$\delta' = z_3\left(e^{i\theta_3}-1\right)$$

$$z_1\left(e^{i\theta_{1i}}-1\right) + z_2\left(e^{i\theta_{2i}}-1\right) = \delta_i' \quad \rightarrow \text{ for } i^{th} \text{ position}$$

On Representing as a matrix:

$$\begin{pmatrix} z_1 & z_2 & 1 \end{pmatrix} \begin{pmatrix} e^{i\theta_{11}}-1 & e^{i\theta_{21}}-1 & 1 \\ e^{i\theta_{12}}-1 & e^{i\theta_{22}}-1 & 1 \\ -\delta_1' & -\delta_2' & -z_3+z_8 \end{pmatrix} = 0$$

$$\therefore \quad A Z = 0$$

$$|A| = 0$$

$$\begin{vmatrix} e^{i\theta_{11}}-1 & e^{i\theta_{21}}-1 & 1 \\ e^{i\theta_{12}}-1 & e^{i\theta_{22}}-1 & 1 \\ -\delta_1' & -\delta_2' & -z_3+z_8 \end{vmatrix} = 0$$

Assumptions:

$\theta_{11} = 20^\circ \qquad \theta_{31} = 18^\circ \qquad \Rightarrow \delta_1' = z_3\left(e^{i18^\circ}-1\right) = 0.156 + 0.215i$

$\theta_{12} = 30^\circ \qquad \theta_{32} = 27^\circ \qquad \Rightarrow \delta_2' = z_3\left(e^{i27^\circ}-1\right) = 0.208 + 0.339i$

$z_3 = 0.853\, e^{-i\frac{\pi}{4}}$

On solving (2 eqn's 2 unknowns) we get: $\theta_{21} = 3.194^\circ \qquad \theta_{22} = 25.383^\circ$

$$\Rightarrow z_1\left(e^{i20^\circ}-1\right) + z_2\left(e^{i\theta_{21}}-1\right) = \delta_1' = 0.156 + 0.215i$$

$$z_1\left(e^{i30^\circ}-1\right) + z_2\left(e^{i\theta_{22}}-1\right) = \delta' = 0.208 + 0.339i$$

On Solving (4 eqn's 4 unknowns) we get: $z_1 = 0.532\, e^{i(-8^\circ)} \quad z_2 = 1.386\, e^{i(-52^\circ)}$

Step 3: Solving for $\mathbf{z_5}$, $\mathbf{z_6}$, $\mathbf{z_7}$ using remaining dyad equations.

Dyad form Synthesis:

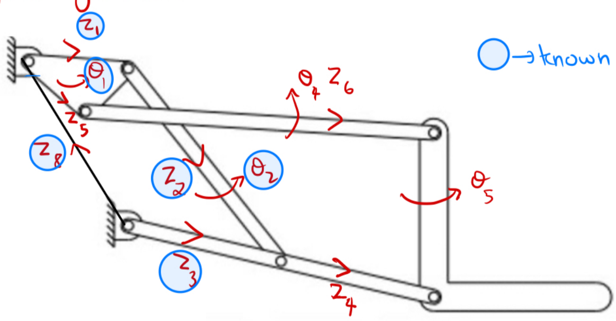


Figure 1

$$z_5(e^{i\theta_1}-1) + z_6(e^{i\theta_4}-1) + z_7(e^{i\theta_5}-1) = \delta$$

$\theta_5 = 0$ (since lifter orientation is constant)

$z_5(e^{i\theta_{1i}}-1) + z_6(e^{i\theta_{4i}}-1) = \delta_i$ → for i^{th} position.

from geometry, $\delta_i = 2\delta_i'$

loop closure Eq^n: $z_5 + z_6 + z_7 - z_3 - z_4 + z_8 = 0$

On Representing as a matrix:

$$(z_5 \quad z_6 \quad 1)\begin{pmatrix} e^{i\theta_{11}}-1 & e^{i\theta_{41}}-1 & 1 \\ e^{i\theta_{12}}-1 & e^{i\theta_{42}}-1 & 1 \\ -\delta_1 & -\delta_2 & z_7 - z_3 - z_4 + z_8 \end{pmatrix} = 0$$

$\therefore \; Bz = 0$

$\Rightarrow |B| = 0$

Assumptions:

$z_7 = 0.876 e^{i\pi}$

And using previous assumptions and the fact that $\delta_i = 2\delta_i'$

On solving (2 eq^n's 2 unknowns) we get: $\theta_{41} = 11.837°$ $\quad \theta_{42} = 17.263°$

$$\Rightarrow z_5(e^{i20°}-1) + z_6(e^{i\theta_{41}}-1) = \delta_1 = 0.312 + 0.43i$$

$$z_5(e^{i30°}-1) + z_6(e^{i\theta_{42}}-1) = \delta_2 = 0.416 + 0.678i$$

On Solving (4 eq^n's 4 unknowns) we get: $z_5 = 0.418 e^{i(-53°)}$ $\quad z_6 = 1.79 e^{i(-7°)}$

Velocity Analysis

Velocity Analysis

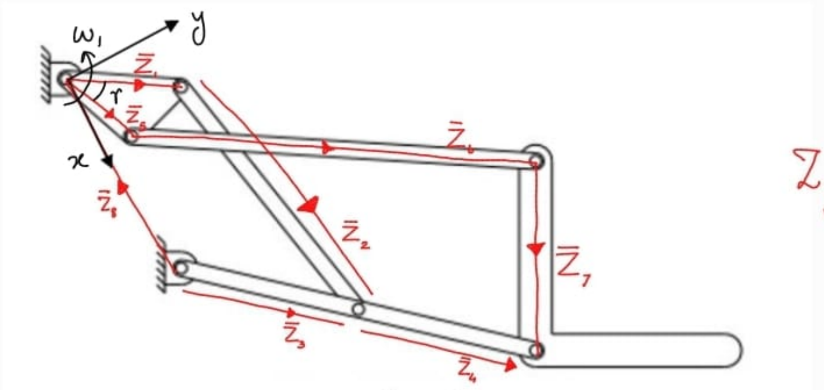


Figure 1

$Z_j = r_j e^{i\theta_j}$

loop-1

$$Z_1 + Z_2 - Z_3 + Z_8 = 0$$

$$r_1 e^{i\theta_1} + r_2 e^{i\theta_2} - r_3 e^{i\theta_4} + r_8 = 0$$

$(\theta_3 = \theta_4)$

d. w.r.t

$$i r_1 \omega_1 e^{i\theta_1} + i r_2 \omega_2 e^{i\theta_2} - i r_3 \omega_4 e^{i\theta_4} + 0 = 0 \quad (1)$$

loop-2

$$Z_5 + Z_6 + Z_7 - Z_4 - Z_3 + Z_8 = 0$$

$$r_5 e^{i\theta_5} + r_6 e^{i\theta_6} + r_7 e^{i\theta_7} - (r_4 + r_3) e^{i\theta_4} + r_8 = 0$$

differentiate with respect to time

$$i r_5 \omega_5 e^{i\theta_5} + i r_6 \omega_6 e^{i\theta_6} + i r_7 \omega_7 e^{i\theta_7} - i(r_4 + r_3)\omega_4 e^{i\theta_4} = 0$$

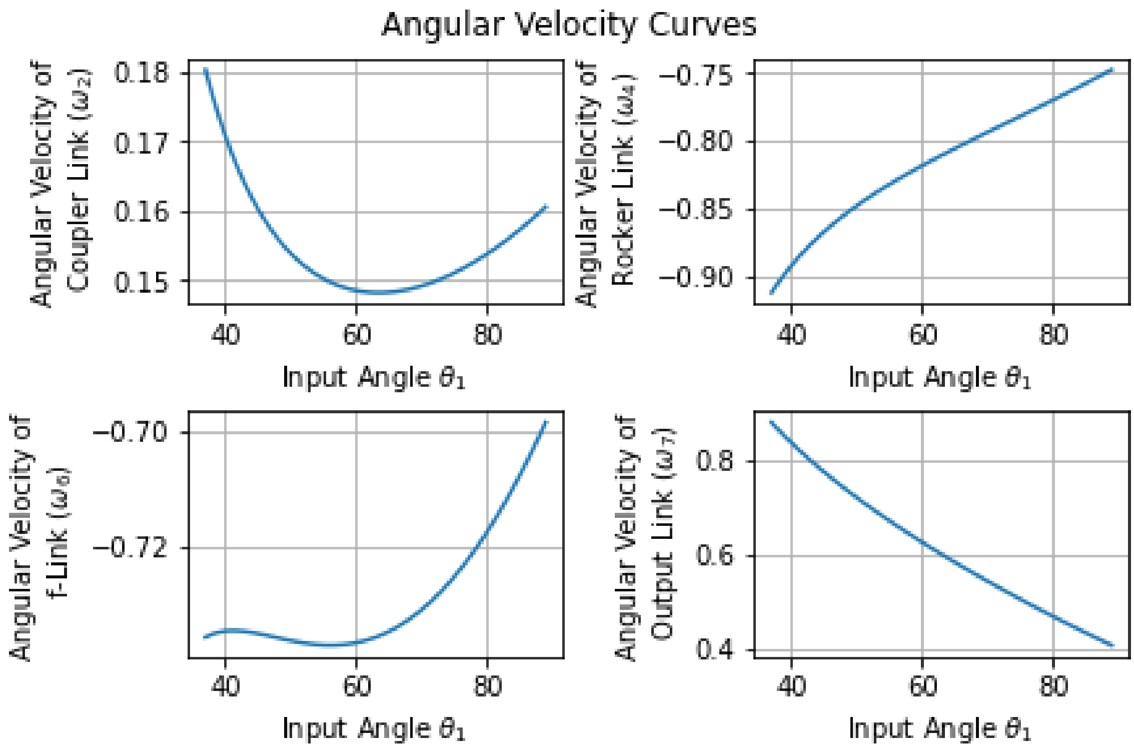
$$i r_5 \omega_1 e^{i(\theta_1 - \gamma)} + i r_6 \omega_6 e^{i\theta_6} + i r_7 \omega_7 e^{i\theta_7} - i(r_3 + r_4)\omega_4 e^{i\theta_4} = 0 \quad \text{— (2)}$$

Known : ω_1 unknown $\omega_2\ \omega_4\ \omega_6\ \omega_7$

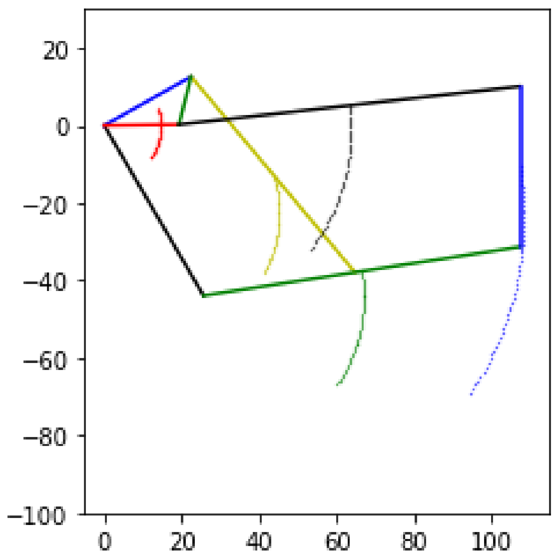
4 eqn 4 unknown

$$\begin{bmatrix} i\omega_1 r_1 \cos\theta_1 \\ i\omega_1 r_1 \sin\theta_1 \\ i\omega_1 r_5 \cos(\theta_1 - \gamma) \\ i\omega_1 r_5 \sin(\theta_1 - \gamma) \end{bmatrix}$$

$$= \begin{bmatrix} -ir_2\cos\theta_2 & -ir_3\cos\theta_4 & 0 & 0 \\ -ir_2\sin\theta_2 & -ir_3\sin\theta_4 & 0 & 0 \\ 0 & +i(r_3+r_4)\cos\theta_4 & -ir_6\cos\theta_6 & -ir_7\cos\theta_7 \\ 0 & +i(r_3+r_4)\sin\theta_4 & -ir_6\sin\theta_6 & -ir_7\sin\theta_7 \end{bmatrix} \begin{bmatrix} \omega_2 \\ \omega_4 \\ \omega_6 \\ \omega_7 \end{bmatrix}$$



Centre of Gravity Trajectories



Acceleration Analysis

Acceleration Analysis :

Loop closure :

(1) $$Z_1 + Z_2 - Z_3 + Z_8 = 0$$

$$r_1 e^{i\theta_1} + r_2 e^{i\theta_2} - r_3 e^{i\theta_4} + r_8 = 0$$

$(\theta_3 = \theta_4)$

(2) $$Z_5 + Z_6 + Z_7 - Z_4 - Z_3 + Z_8 = 0$$

$$r_5 e^{i\theta_5} + r_6 e^{i\theta_6} + r_7 e^{i\theta_7} - (r_4 + r_3) e^{i\theta_4} + r_8 = 0$$

Velocity eqns :

(1) $$i r_1 \omega_1 e^{i\theta_1} + i r_2 \omega_2 e^{i\theta_2} - i r_3 \omega_4 e^{i\theta_4} + 0 = 0$$

(2) $$i r_5 \omega_5 e^{i\theta_5} + i r_6 \omega_6 e^{i\theta_6} + i r_7 \omega_7 e^{i\theta_7} - i(r_4 + r_3)\omega_4 e^{i\theta_4} = 0$$

$$i r_5 \omega_1 e^{i(\theta_1 - \gamma)} + i r_6 \omega_6 e^{i\theta_6} + i r_7 \omega_7 e^{i\theta_7} - i(r_4 + r_3)\omega_4 e^{i\theta_4} = 0$$

Acceleration eqns

(1) $$(i\alpha_1 - \omega_1^2) r_1 e^{i\theta_1} + (i\alpha_2 - \omega_2^2) r_2 e^{i\theta_2} - (i\alpha_4 - \omega_4^2) r_3 e^{i\theta_4} = 0$$

(2) $$(i\alpha_1 - \omega_1^2) r_5 e^{i(\theta_1 - \gamma)} + (i\alpha_6 - \omega_6^2) r_6 e^{i\theta_6} + (i\alpha_7 - \omega_7^2) r_7 e^{i\theta_7}$$

$$- (i\alpha_4 - \omega_4^2)(r_4 + r_3) e^{i\theta_4} = 0$$

Knowns α_1 Unknowns $\rightarrow$ α_2 α_4 α_6 α_7

$$(i\alpha_1 - \omega_1^2)\, r_1 e^{i\theta_1} = (i\alpha_4 - \omega_4^2)\, r_3 e^{i\theta_4} - (i\alpha_2 - \omega_2^2)\, r_2 e^{i\theta_2}$$

$$(i\alpha_1 - \omega_1^2)\, r_1 e^{i\theta_1} + \omega_4^2 r_3 e^{i\theta_4} - \omega_2^2 r_2 e^{i\theta_2} = i\alpha_4 r_3 e^{i\theta_4} - i\alpha_2 r_2 e^{i\theta_2}$$

$$(i\alpha_1 - \omega_1^2)\, r_5 e^{i(\theta_1 - r)} = (i\alpha_4 - \omega_4^2)(r_3 + r_4) e^{i\theta_4} - (i\alpha_6 - \omega_6^2)\, r_6 e^{i\theta_6} - (i\alpha_7 - \omega_7^2)\, r_7 e^{i\theta_7}$$

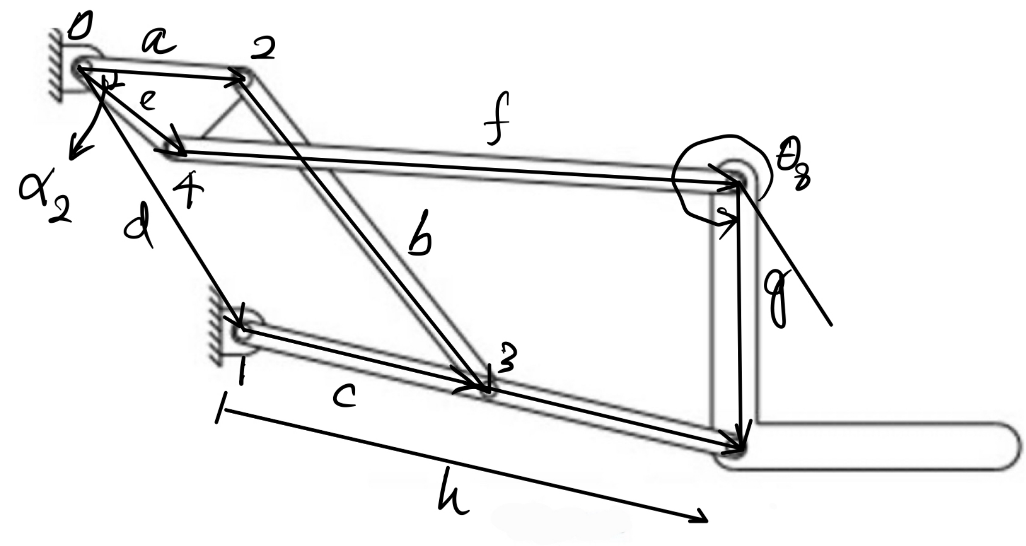
$$(i\alpha_1 - \omega_1^2)\, r_5 e^{i(\theta_1 - r)} + \omega_4^2 (r_4 + r_3) e^{i\theta_4} - \omega_6^2 r_6 e^{i\theta_6} - \omega_7^2 r_7 e^{i\theta_7} = i\alpha_4 (r_3 + r_4) e^{i\theta_4} - i\alpha_6 r_6 e^{i\theta_6} - i\alpha_7 r_7 e^{i\theta_7}$$

$$\begin{bmatrix} (i\alpha_1 - \omega_1^2)\, r_1 \cos\theta_1 + \omega_4^2 r_3 \cos\theta_4 - \omega_2^2 r_2 \cos\theta_2 \\ (i\alpha_1 - \omega_1^2)\, r_1 \sin\theta_1 + \omega_4^2 r_3 \sin\theta_4 - \omega_2^2 r_2 \sin\theta_2 \\ (i\alpha_1 - \omega_1^2)\, r_5 \cos(\theta_1 - r) + \omega_4^2 (r_3 + r_4) \cos\theta_4 - \omega_6^2 r_6 \cos\theta_6 - \omega_7^2 r_7 \cos\theta_7 \\ (i\alpha_1 - \omega_1^2)\, r_5 \sin(\theta_1 - r) + \omega_4^2 (r_3 + r_4) \sin\theta_4 - \omega_6^2 r_6 \sin\theta_6 - \omega_7^2 r_7 \sin\theta_7 \end{bmatrix}$$

$$= \begin{bmatrix} -ir_2 \cos\theta_2 & ir_3 \cos\theta_4 & 0 & 0 \\ -ir_2 \sin\theta_2 & ir_3 \sin\theta_4 & 0 & 0 \\ 0 & i(r_3 + r_4)\cos\theta_4 & -ir_6 \cos\theta_6 & -ir_7 \cos\theta_7 \\ 0 & i(r_3 + r_4)\sin\theta_4 & -ir_6 \sin\theta_6 & -ir_7 \sin\theta_7 \end{bmatrix} \begin{bmatrix} \alpha_2 \\ \alpha_4 \\ \alpha_6 \\ \alpha_7 \end{bmatrix}$$

Code

The code starts with a set of 8 link lengths (a, b, c, d, e, f, g and h) and generates a new set of link lengths within 5 units of each of the original lengths. It then validates the link lengths by performing position synthesis for the entire range of motion. The code then repeats this process until it obtains 100 sets of link lengths, i.e., 100 different solutions for the 6-bar linkage. Finally, the mechanism that gives the highest range of motion for the output link is chosen as the optimum solution.
Position synthesis is done by emulating the geometric method: Each joint's coordinates are obtained in a sequence by performing vector rotation/scaling and by cutting arcs using known centres and radii.



Joint 0 is fixed at the origin and the ground link is assumed to be horizontal, hence fixing the second fixed pivot (**Joint 1**). The joint on the crank link (**Joint 2**) is obtained as $(a\cos\theta_2, a\sin\theta_2)$. Now that joints 1 and 2 are known, **Joint 3** is obtained by finding the point of intersection of a circle centred at joints 1 and 2 and with radii equal to b and c. The other joint on the crank (**Joint 4**) is obtained by rotating vector **a** by α_2 and scaling it by the ratio e/a. **Joint 5** is obtained by scaling vector **c** by h/c. **g** is known since θ_8 is constant and hence, **Joint 6** is obtained by **g - Joint 5**.

Summary

We have designed an inversion of the Stephenson's mechanism (dof=1) which serves to lift a load vertically, with a better mechanical advantage compared to a parallelogram 4 bar lifter. We synthesized the link lengths using dyad form synthesis by taking constraints and free choices as required, thereby obtaining the results mentioned below. We performed velocity analysis to find the relation between input and output angular velocity and plotted the corresponding graphs.We did the same for Angular acceleration. We wrote a code that starts with the set of link lengths and randomly generates more sets of link lengths around the original set. The sets were validated using position analysis (emulating geometric synthesis) and we finally chose the link lengths that give the highest range of motion. We then perform velocity and acceleration analysis by differentiating the loop closure equations and solving the resulting matrix equations.

Conclusion

Thus, we have synthesised a 6 bar lifter mechanism by the method of dyad form synthesis and iterative link length generation. We have obtained the following results: Z1=25.76 Z2=67.075 Z3=41.303 Z3+Z4=82.587 (link 5) Z5=20.244 Z6=86.755 Z7=42.414 Z8=48.393 (all units are in cm)