THEORY OF MECHANISMS

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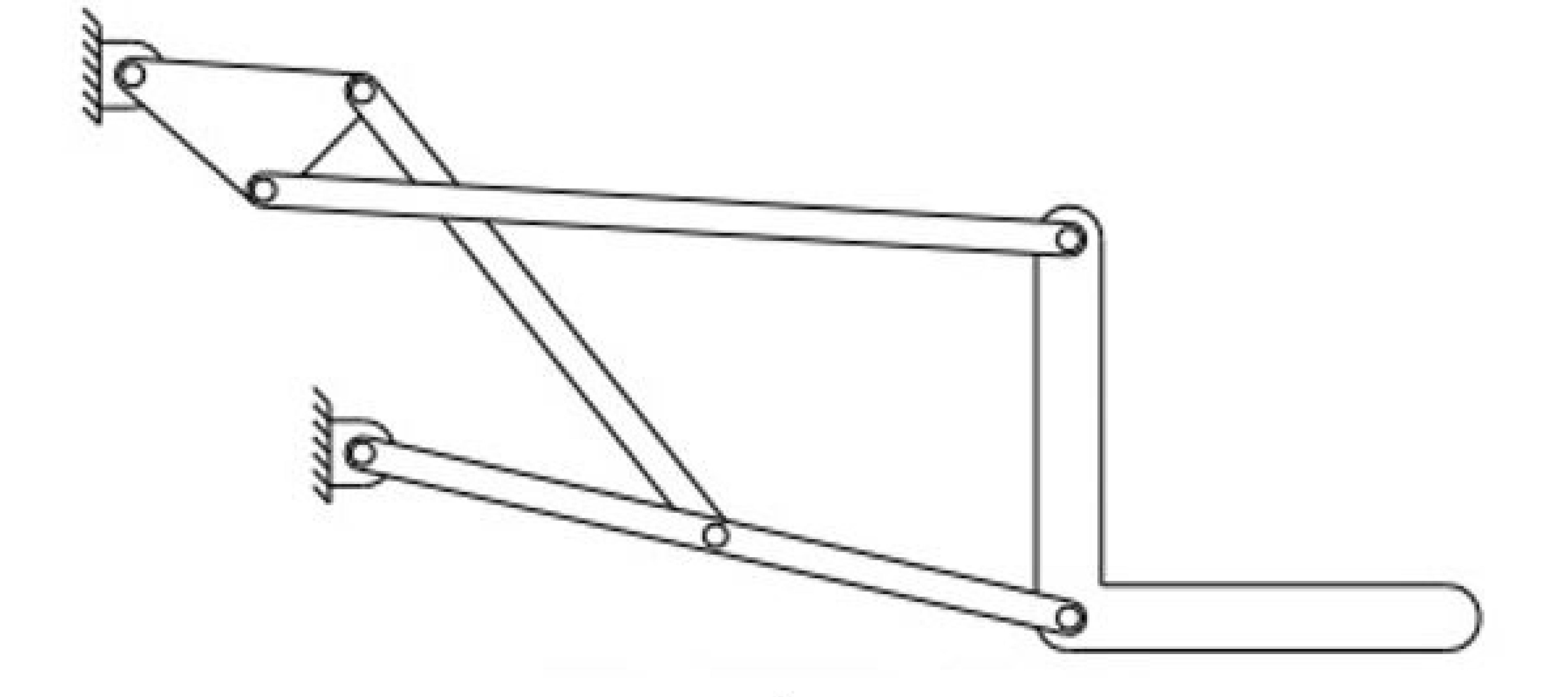
OUR MECHANISM

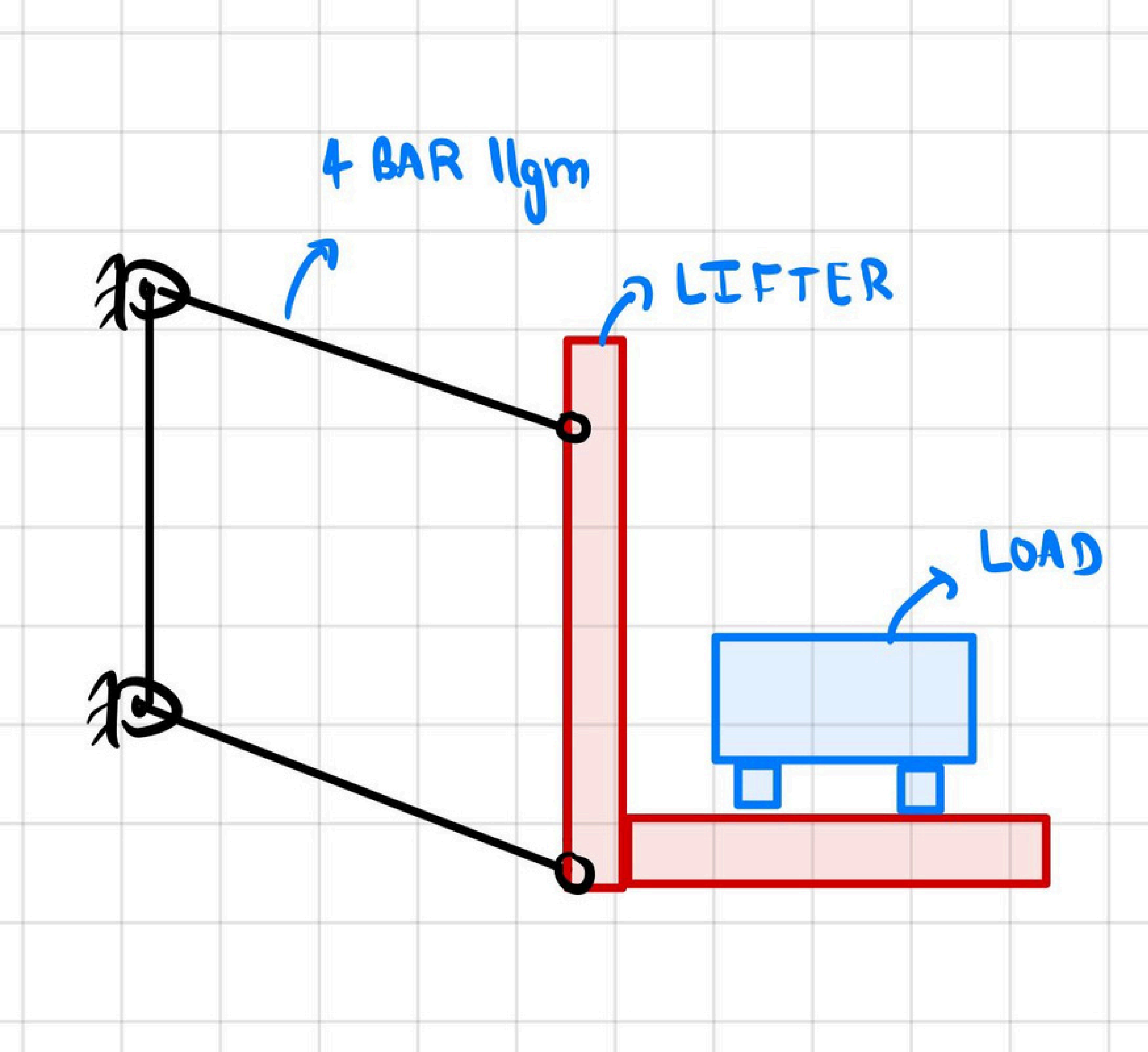


6 BAR LIFTER

Vertically lifts a load using a rotary input.

The linkage is in fact an inversion of Stephenson's mechanism, and one of the ternary links acts as our input.

Why are we not using a simple parallelogram?



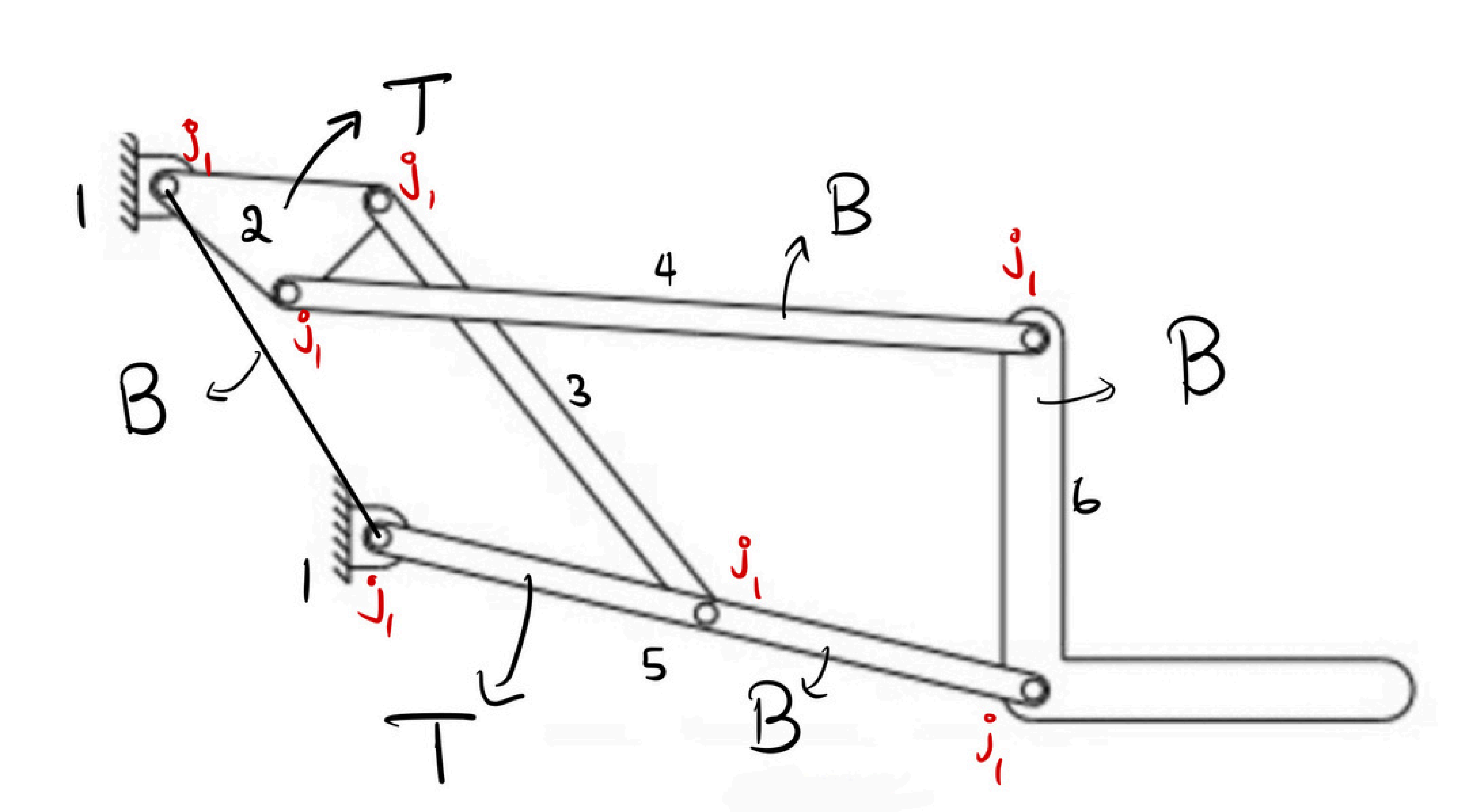
This limits the lifter's range of motion and its payload capacity.

This is harder to control.

MA is better with a 6 bar.

MOBILITY ANALYSIS

Our lifter has 2 ternary links, 4 binary links, and a mobility of 1.



T - ternary links, B - binary links

$n_2 = 4 \quad n_3 = 2 \Rightarrow n = 6$

no. of j_1 joints $= 7$

$m = 3(n-1) - 2j_1$

$m = 3(6-1) - 2(7)$

$\boxed{m = 1}$

POSITION ANALYSIS

The ground link supports a 4 bar and a 5 bar

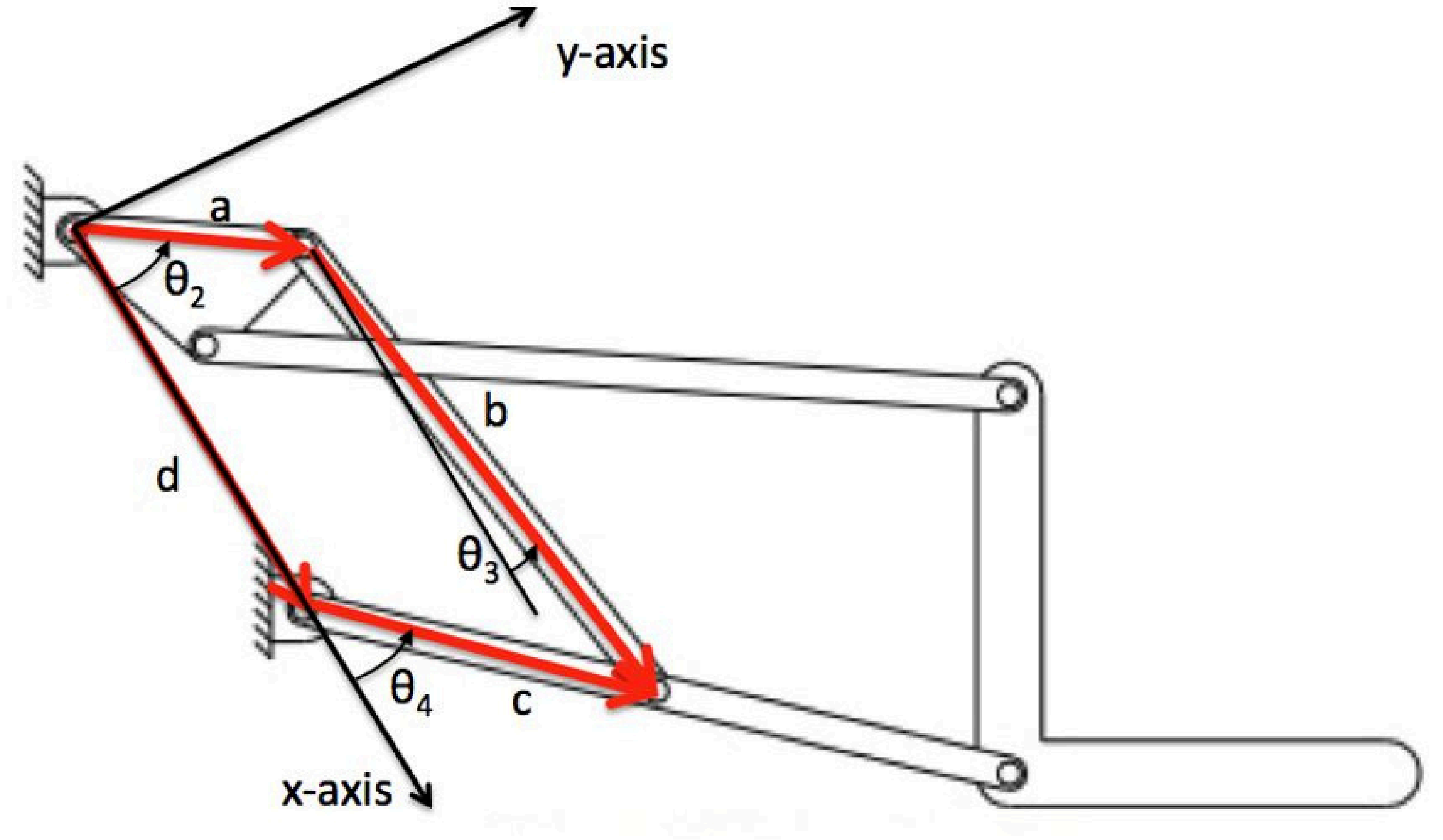


Figure 2

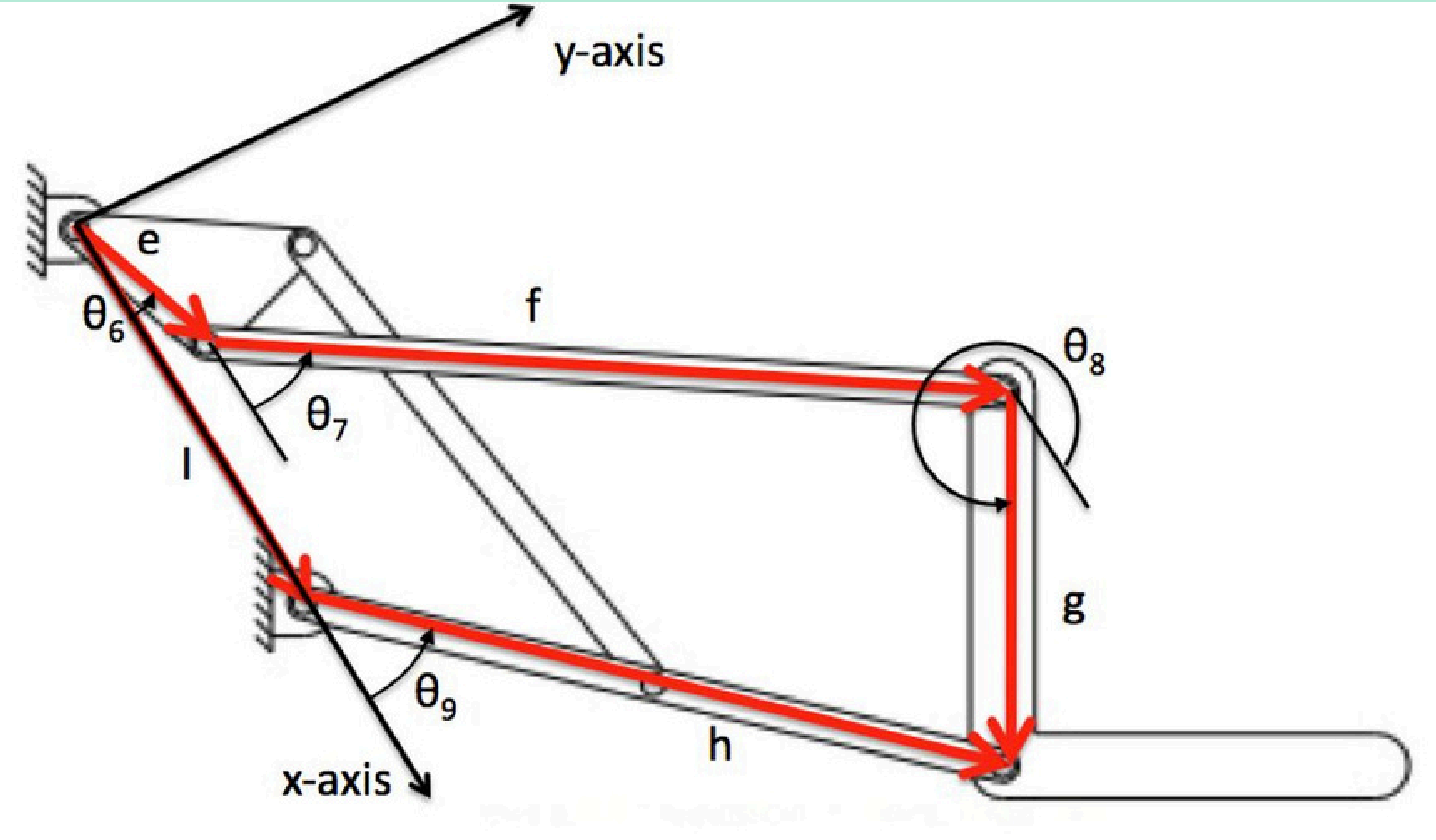


Figure 3

the 4 bar drives the 5 bar

PART 1

DYAD FORM SYNTHESIS

Synthesis using dyad form equations and free choices.

Steps Involved

Dyad Form - write the dyad form and loop closure equations.

Constraints and Free Choices - the lifter's orientation does not change

Results - solving equations with our choices and constraints

DYAD FORM EQUATIONS

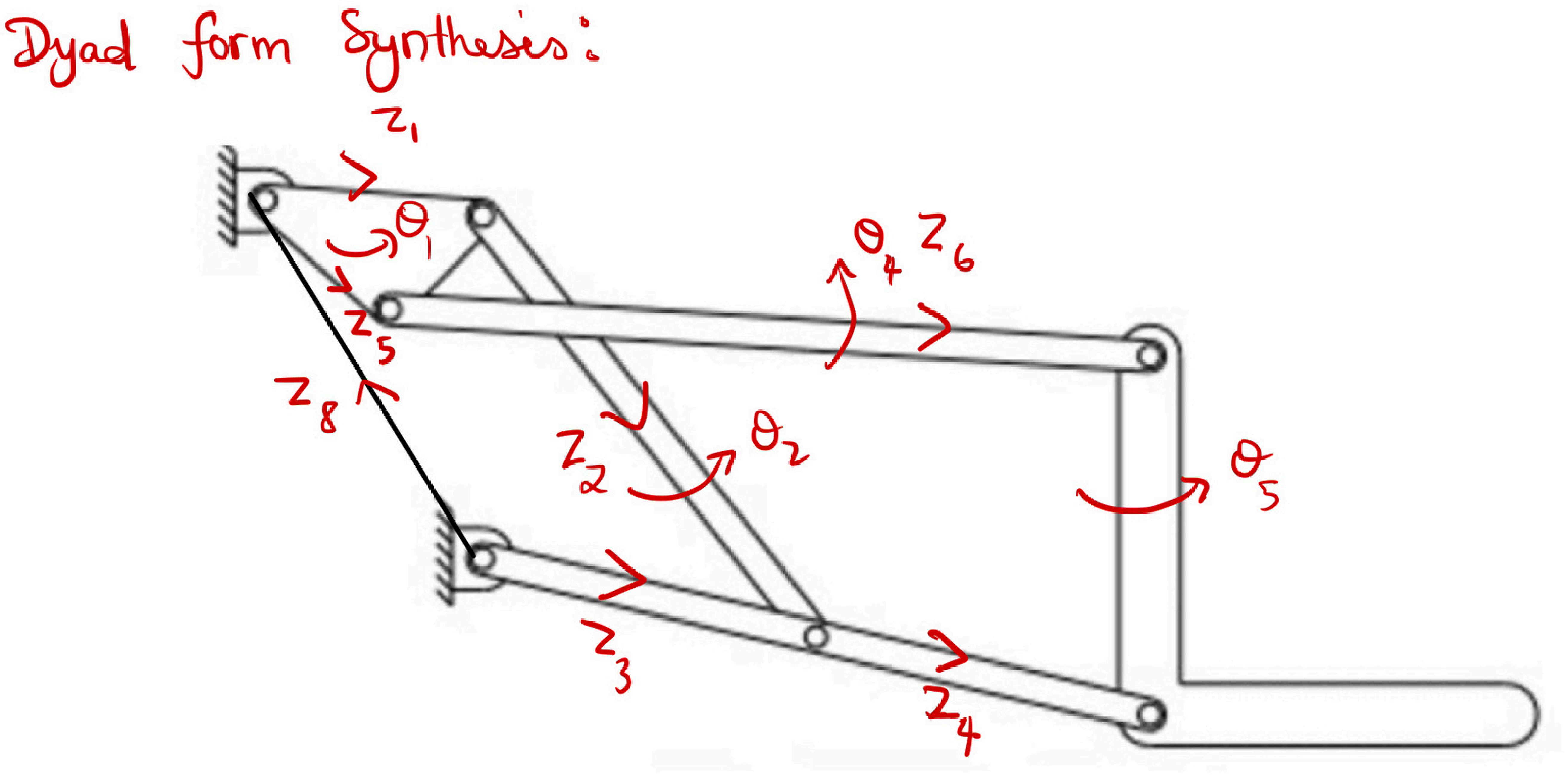


Figure 1

DYAD FORM EQUATIONS

Dyad form Equations:

loop 1 : $Z_1(e^{i\theta_1}-1)+Z_2(e^{i\theta_2}-1)+Z_4(e^{i\theta_3}-1) = \delta$

loop 2 : $Z_5(e^{i\theta_1}-1)+Z_6(e^{i\theta_4}-1)+Z_7(e^{i\theta_5}-1) = \delta$

loop 3 : $Z_3(e^{i\theta_3}-1)+Z_4(e^{i\theta_3}-1) = \delta$ * → Redundant

Loop Closure Equation:

loop 1 : $Z_1+Z_2-Z_3+Z_8=0$

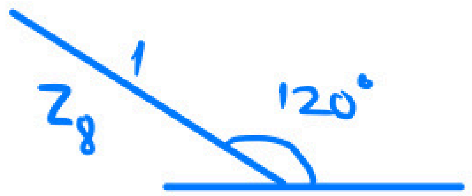
loop 2 : $Z_1e^{i\theta_1}+Z_2e^{i\theta_2}-Z_3e^{i\theta_3}+Z_8=0$

SOLVING...

loop 2 − loop 1 = $z_1(e^{i\theta_1}-1) + z_2(e^{i\theta_2}-1) - z_3(e^{i\theta_3}-1) = 0$

Assuming $z_8 = e^{i\frac{2\pi}{3}}$

$z_3 = z_4$



$\delta' = z_3(e^{i\theta_3}-1)$

$z_1(e^{i\theta_{1i}}-1) + z_2(e^{i\theta_{2i}}-1) = \delta'_i$ → for i^{th} position

On Representing as a matrix:

$$\begin{pmatrix} z_1 & z_2 & 1 \end{pmatrix} \begin{pmatrix} e^{i\theta_{11}}-1 & e^{i\theta_{21}}-1 & 1 \\ e^{i\theta_{12}}-1 & e^{i\theta_{22}}-1 & 1 \\ -\delta'_1 & -\delta'_2 & -z_3+z_8 \end{pmatrix} = 0$$

$\therefore \quad AZ = 0$

$|A| = 0$

SOLVING...

$$\begin{vmatrix} e^{i\theta_{11}}-1 & e^{i\theta_{21}}-1 & 1 \\ e^{i\theta_{12}}-1 & e^{i\theta_{22}}-1 & 1 \\ -\delta_1' & -\delta_2' & -Z_3+Z_8 \end{vmatrix} = 0$$

Assumptions:

$\theta_{11} = 20°$ $\quad \theta_{31} = 18°$ $\quad \Rightarrow \delta_1' = Z_3(e^{i18°}-1) = 0.156 + 0.215i$

$\theta_{12} = 30°$ $\quad \theta_{32} = 27°$ $\quad \Rightarrow \delta_2' = Z_3(e^{i27°}-1) = 0.208 + 0.339i$

$Z_3 = 0.853\, e^{-i\frac{\pi}{4}}$

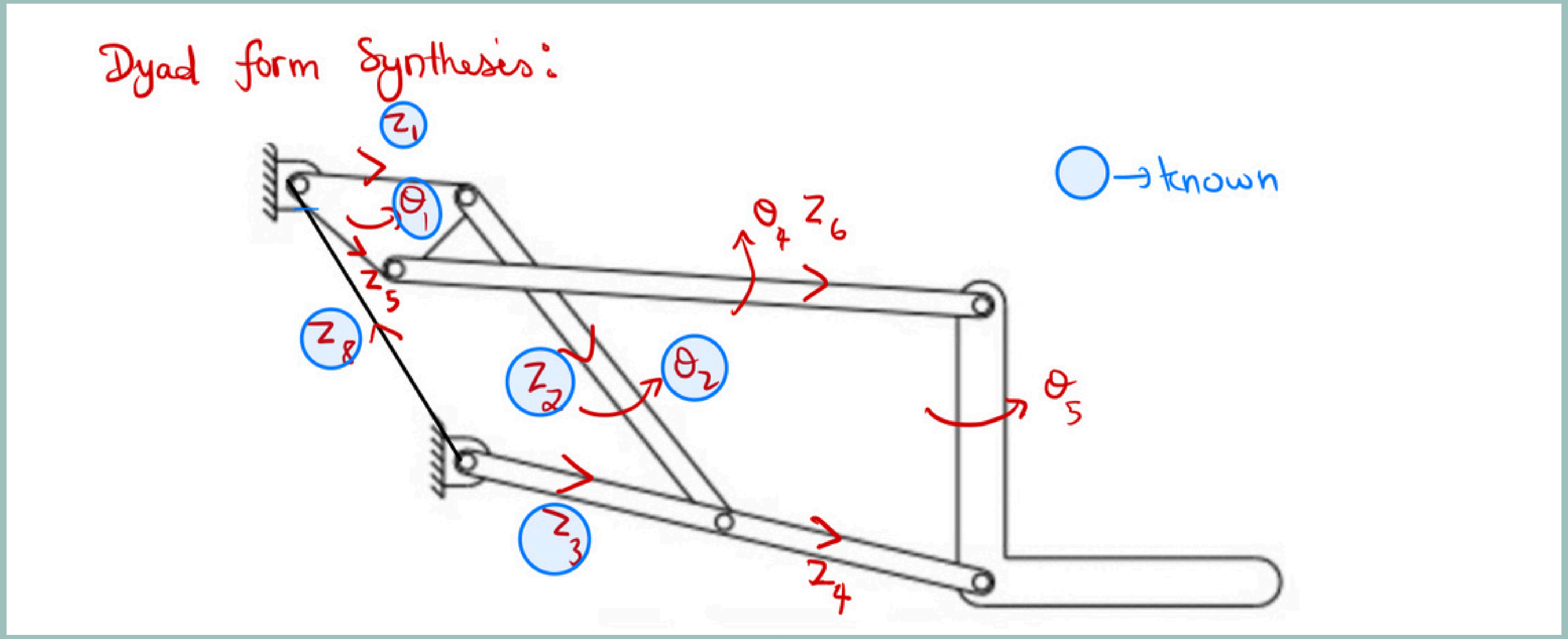
On solving (2 eqn's 2 unknowns) we get: $\theta_{21} = 3.194°$ $\quad \theta_{22} = 25.383°$

$$\Rightarrow Z_1(e^{i20°}-1) + Z_2(e^{i\theta_{21}}-1) = \delta_1' = 0.156 + 0.215i$$

$$Z_1(e^{i30°}-1) + Z_2(e^{i\theta_{22}}-1) = \delta' = 0.208 + 0.339i$$

On Solving (4 eqn's 4 unknowns) we get: $Z_1 = 0.532\, e^{i(-8°)}$ $\quad Z_2 = 1.386\, e^{i(-52°)}$

SOLVING...



Similar to the previous method, we solve for the remaining unknowns using the following determinant:

$$\begin{pmatrix} e^{i\theta_{11}}-1 & e^{i\theta_{41}}-1 & 1 \\ e^{i\theta_{12}}-1 & e^{i\theta_{42}}-1 & 1 \\ -\delta_1 & -\delta_2 & Z_7-Z_3-Z_4+Z_8 \end{pmatrix} = 0$$

RESULTS

We took the ground link to be 30° to vertical

Scaling to a reasonable level

Z1=25.76

Z2=67.075

Z3=41.303

Z3+Z4=82.587 (link 5)

Z5=20.244

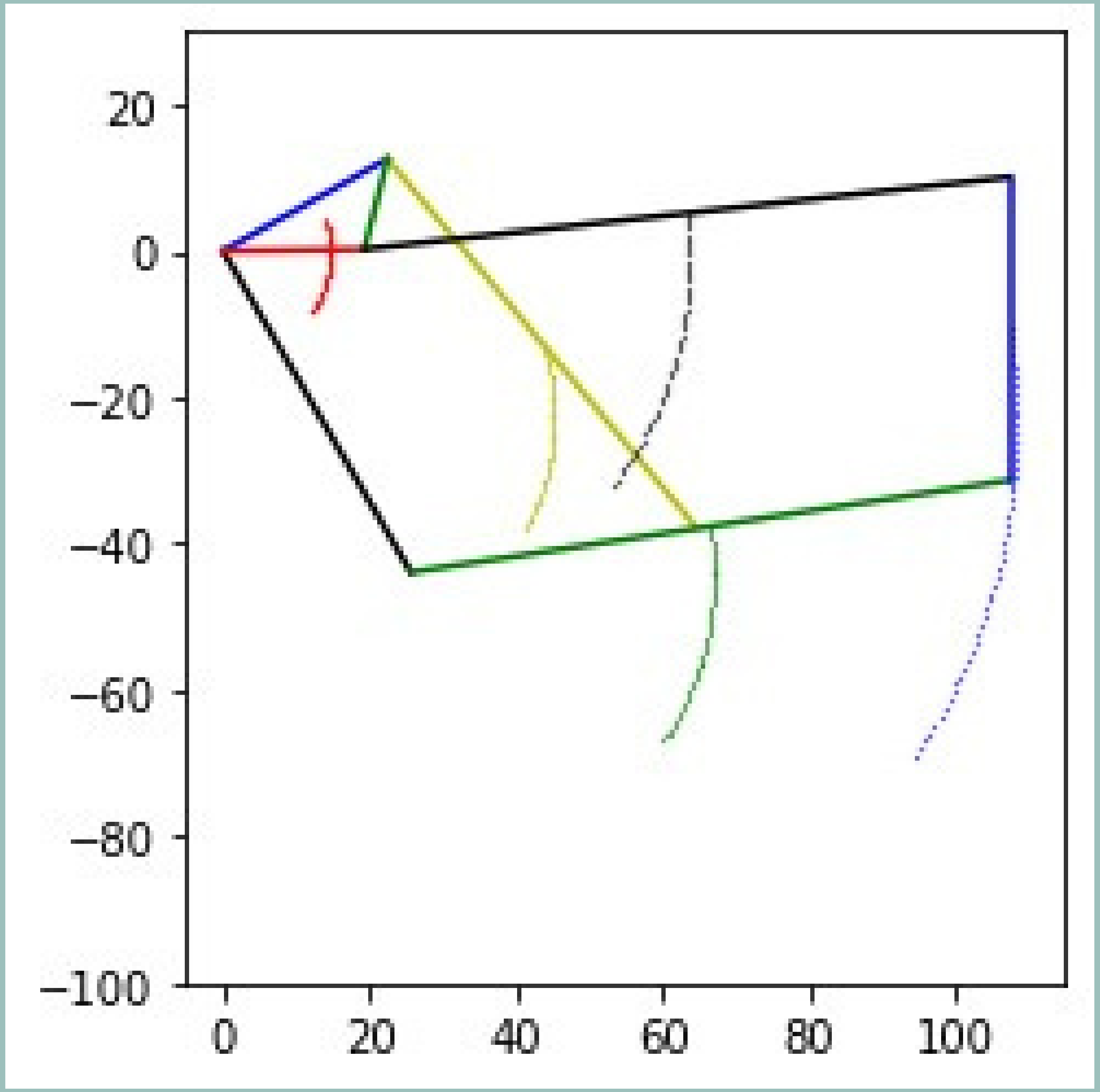
Z6=86.755

Z7=42.414

Z8=48.393

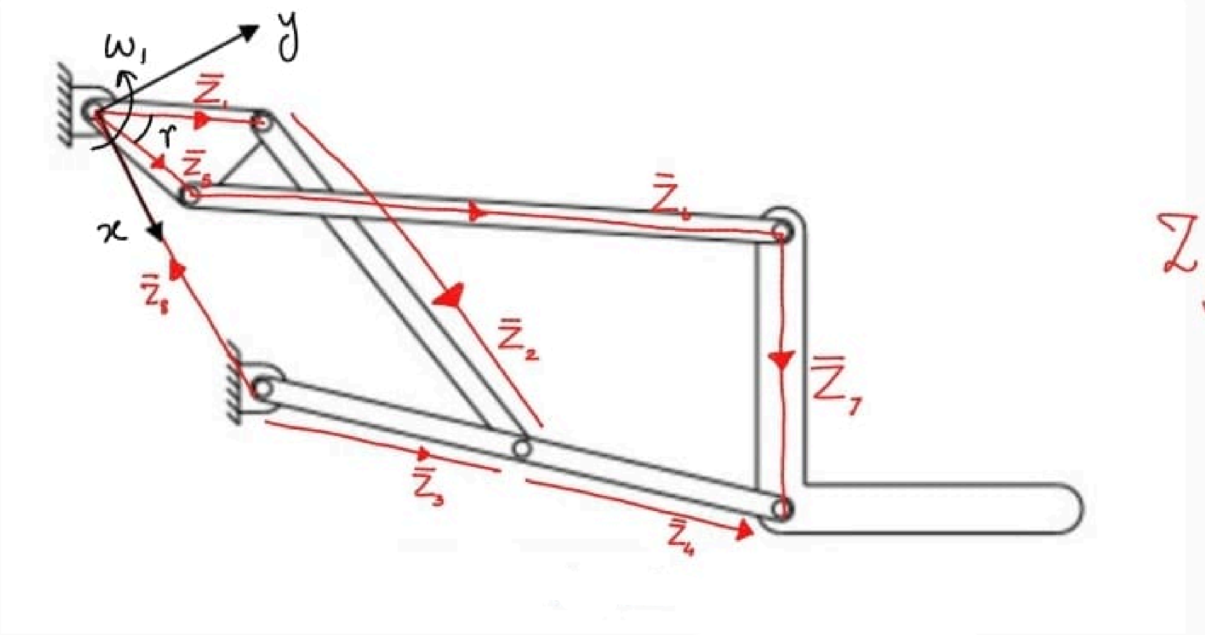
(all units are in cm)

COG TRAJECTORIES



VELOCITY ANALYSIS

Velocity Analysis



$$Z_j = r_j e^{i\theta_j}$$

loop-1

$$Z_1 + Z_2 - Z_3 + Z_8 = 0$$

$$r_1 e^{i\theta_1} + r_2 e^{i\theta_2} - r_3 e^{i\theta_4} + r_8 = 0$$

$$(\theta_3 = \theta_4)$$

d. w.r.t

$$i r_1 \omega_1 e^{i\theta_1} + i r_2 \omega_2 e^{i\theta_2} - i r_3 \omega_4 e^{i\theta_4} + 0 = 0 \quad \text{—(1)}$$

loop-2

$$Z_5 + Z_6 + Z_7 - Z_4 - Z_3 + Z_8 = 0$$

$$r_5 e^{i\theta_5} + r_6 e^{i\theta_6} + r_7 e^{i\theta_7} - (r_4 + r_3) e^{i\theta_4} + r_8 = 0$$

differentiate with respect to time

$$i r_5 \omega_5 e^{i\theta_5} + i r_6 \omega_6 e^{i\theta_6} + i r_7 \omega_7 e^{i\theta_7} - i(r_4 + r_3) \omega_4 e^{i\theta_4} = 0$$

$$i r_5 \omega_1 e^{i(\theta_1 - \gamma)} + i r_6 \omega_6 e^{i\theta_6} + i r_7 \omega_7 e^{i\theta_7} - i(r_3 + r_4) \omega_4 e^{i\theta_4} = 0 \quad \text{—(2)}$$

VELOCITY ANALYSIS

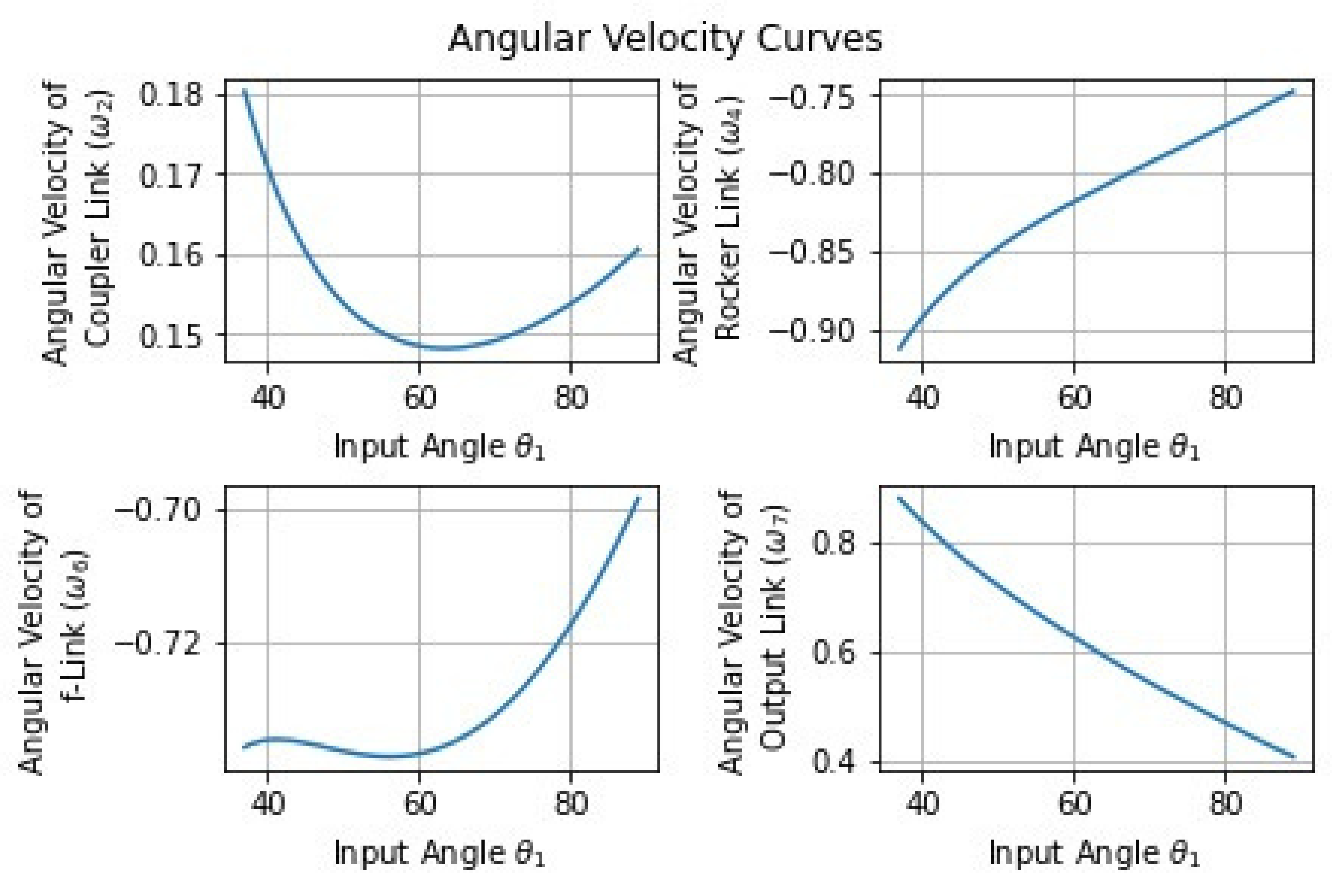
Known : ω_1 unknown $\omega_2\ \omega_4\ \omega_6\ \omega_7$

4 eqn 4 unknown

$$\begin{bmatrix} i\omega_1 r_1 \cos\theta_1 \\ i\omega_1 r_1 \sin\theta_1 \\ i\omega_1 r_5 \cos(\theta_1-\gamma) \\ i\omega_1 r_5 \sin(\theta_1-\gamma) \end{bmatrix}$$

$$= \begin{bmatrix} -ir_2\cos\theta_2 & -ir_3\cos\theta_4 & 0 & 0 \\ -ir_2\sin\theta_2 & -ir_3\sin\theta_4 & 0 & 0 \\ 0 & +i(r_3+r_4)\cos\theta_4 & -ir_6\cos\theta_6 & -ir_7\cos\theta_7 \\ 0 & +i(r_3+r_4)\sin\theta_4 & -ir_6\sin\theta_6 & -ir_7\sin\theta_7 \end{bmatrix} \begin{bmatrix} \omega_2 \\ \omega_4 \\ \omega_6 \\ \omega_7 \end{bmatrix}$$

VELOCITY ANALYSIS



ACCELERATION ANALYSIS

Acceleration Analysis :

Loop closure :

① $$Z_1 + Z_2 - Z_3 + Z_8 = 0$$

$$r_1 e^{i\theta_1} + r_2 e^{i\theta_2} - r_3 e^{i\theta_4} + r_8 = 0$$

$$(\theta_3 = \theta_4)$$

② $$Z_5 + Z_6 + Z_7 - Z_4 - Z_3 + Z_8 = 0$$

$$r_5 e^{i\theta_5} + r_6 e^{i\theta_6} + r_7 e^{i\theta_7} - (r_4 + r_3) e^{i\theta_4} + r_8 = 0$$

Velocity eqns :

① $$i r_1 \omega_1 e^{i\theta_1} + i r_2 \omega_2 e^{i\theta_2} - i r_3 \omega_4 e^{i\theta_4} + 0 = 0$$

② $$i r_5 \omega_5 e^{i\theta_5} + i r_6 \omega_6 e^{i\theta_6} + i r_7 \omega_7 e^{i\theta_7} - i(r_4 + r_3)\omega_4 e^{i\theta_4} = 0$$

$$i r_5 \omega_1 e^{i(\theta_1 - \gamma)} + i r_6 \omega_6 e^{i\theta_6} + i r_7 \omega_7 e^{i\theta_7} - i(r_4 + r_3)\omega_4 e^{i\theta_4} = 0$$

Acceleration eqns

① $$(i\alpha_1 - \omega_1^2) r_1 e^{i\theta_1} + (i\alpha_2 - \omega_2^2) r_2 e^{i\theta_2} - (i\alpha_4 - \omega_4^2) r_3 e^{i\theta_4} = 0$$

② $$(i\alpha_1 - \omega_1^2) r_5 e^{i(\theta_1 - \gamma)} + (i\alpha_6 - \omega_6^2) r_6 e^{i\theta_6} + (i\alpha_7 - \omega_7^2) r_7 e^{i\theta_7}$$

$$- (i\alpha_4 - \omega_4^2)(r_4 + r_3) e^{i\theta_4} = 0$$

ACCELERATION ANALYSIS

Knowns α_1 Unknowns $\rightarrow$ $\alpha_2 \quad \alpha_4 \quad \alpha_6 \quad \alpha_7$

$$(i\alpha_1 - \omega_1^2)\, r_1 e^{i\theta_1} = (i\alpha_4 - \omega_4^2)\, r_3 e^{i\theta_4} - (i\alpha_2 - \omega_2^2)\, r_2 e^{i\theta_2}$$

$$(i\alpha_1 - \omega_1^2)\, r_1 e^{i\theta_1} + \omega_4^2 r_3 e^{i\theta_4} - \omega_2^2 r_2 e^{i\theta_2} = i\alpha_4 r_3 e^{i\theta_4} - i\alpha_2 r_2 e^{i\theta_2}$$

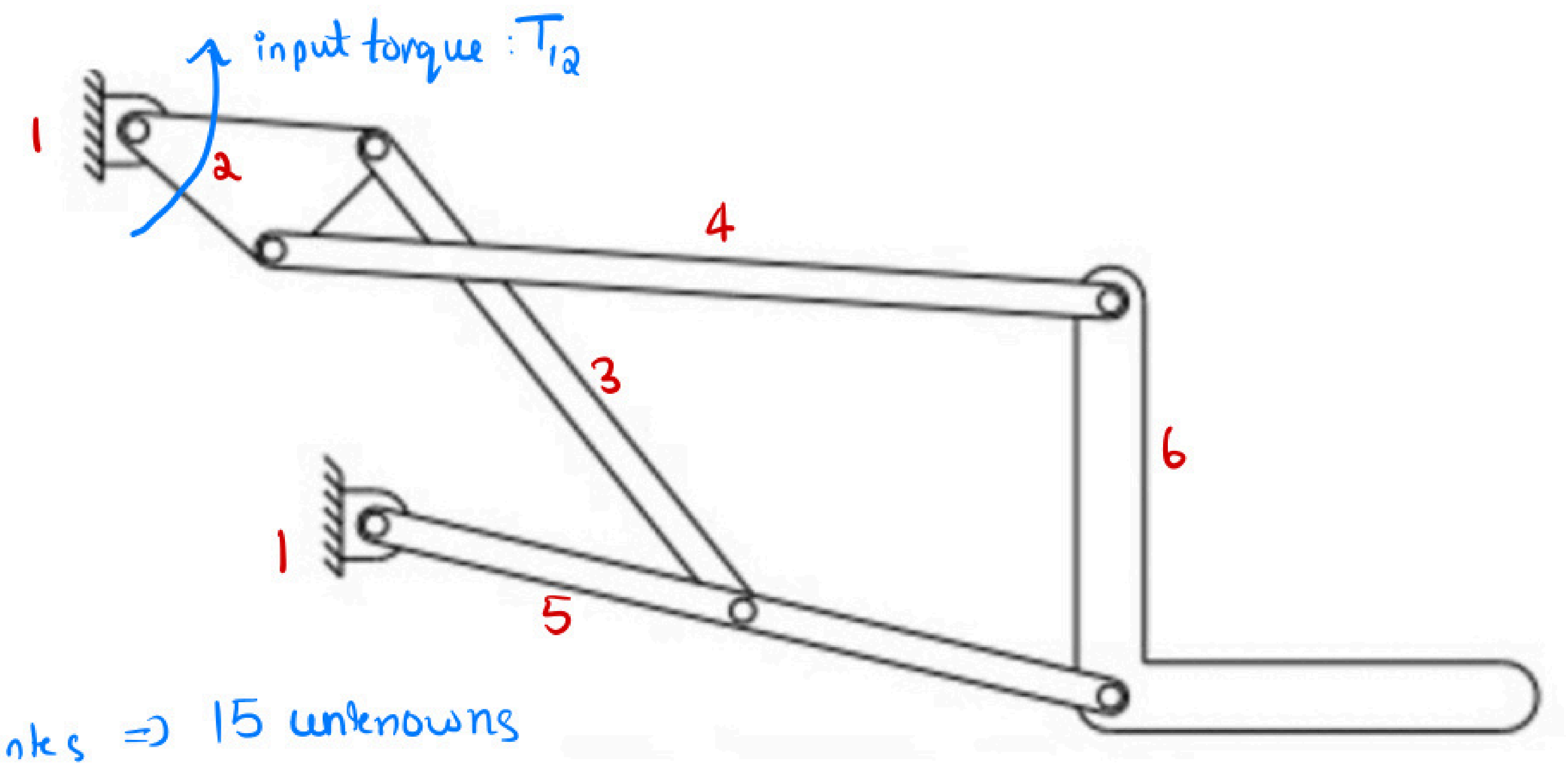
$$(i\alpha_1 - \omega_1^2)\, r_5 e^{i(\theta_1 - r)} = (i\alpha_4 - \omega_4^2)(r_3 + r_4) e^{i\theta_4} - (i\alpha_6 - \omega_6^2)\, r_6 e^{i\theta_6} - (i\alpha_7 - \omega_7^2)\, r_7 e^{i\theta_7}$$

$$(i\alpha_1 - \omega_1^2)\, r_5 e^{i(\theta_1 - r)} + \omega_4^2 (r_4 + r_3) e^{i\theta_4} - \omega_6^2 r_6 e^{i\theta_6} - \omega_7^2 r_7 e^{i\theta_7} = i\alpha_4 (r_3 + r_4) e^{i\theta_4} - i\alpha_6 r_6 e^{i\theta_6} - i\alpha_7 r_7 e^{i\theta_7}$$

$$\begin{bmatrix} (i\alpha_1 - \omega_1^2)\, r_1 \cos\theta_1 + \omega_4^2 r_3 \cos\theta_4 - \omega_2^2 r_2 \cos\theta_2 \\ (i\alpha_1 - \omega_1^2)\, r_1 \sin\theta_1 + \omega_4^2 r_3 \sin\theta_4 - \omega_2^2 r_2 \sin\theta_2 \\ (i\alpha_1 - \omega_1^2)\, r_5 \cos(\theta_1 - r) + \omega_4^2 (r_3 + r_4) \cos\theta_4 - \omega_6^2 r_6 \cos\theta_6 - \omega_7^2 r_7 \cos\theta_7 \\ (i\alpha_1 - \omega_1^2)\, r_5 \sin(\theta_1 - r) + \omega_4^2 (r_3 + r_4) \sin\theta_4 - \omega_6^2 r_6 \sin\theta_6 - \omega_7^2 r_7 \sin\theta_7 \end{bmatrix}$$

$$= \begin{bmatrix} -i r_2 \cos\theta_2 & i r_3 \cos\theta_4 & 0 & 0 \\ -i r_2 \sin\theta_2 & i r_3 \sin\theta_4 & 0 & 0 \\ 0 & i(r_3 + r_4)\cos\theta_4 & -i r_6 \cos\theta_6 & -i r_7 \cos\theta_7 \\ 0 & i(r_3 + r_4)\sin\theta_4 & -i r_6 \sin\theta_6 & -i r_7 \sin\theta_7 \end{bmatrix} \begin{bmatrix} \alpha_2 \\ \alpha_4 \\ \alpha_6 \\ \alpha_7 \end{bmatrix}$$

FORCE ANALYSIS



5 links $\Rightarrow$ 15 unknowns

Each link : $\sum F_x = \sum F_y = \sum \tau = 0 \Rightarrow$ 3 eqn's

$\therefore$ 15 unknowns   15 equations

- N_x, N_y from ground $\Rightarrow$ 4
- F_{ij} & $-F_{ji}$ b/w i^{th} & j^{th} link. $\Rightarrow$ 10
- $T_{12} \Rightarrow$ 1

PART 2

SIMULATION

Synthesis using iterative methods.

Steps Involved

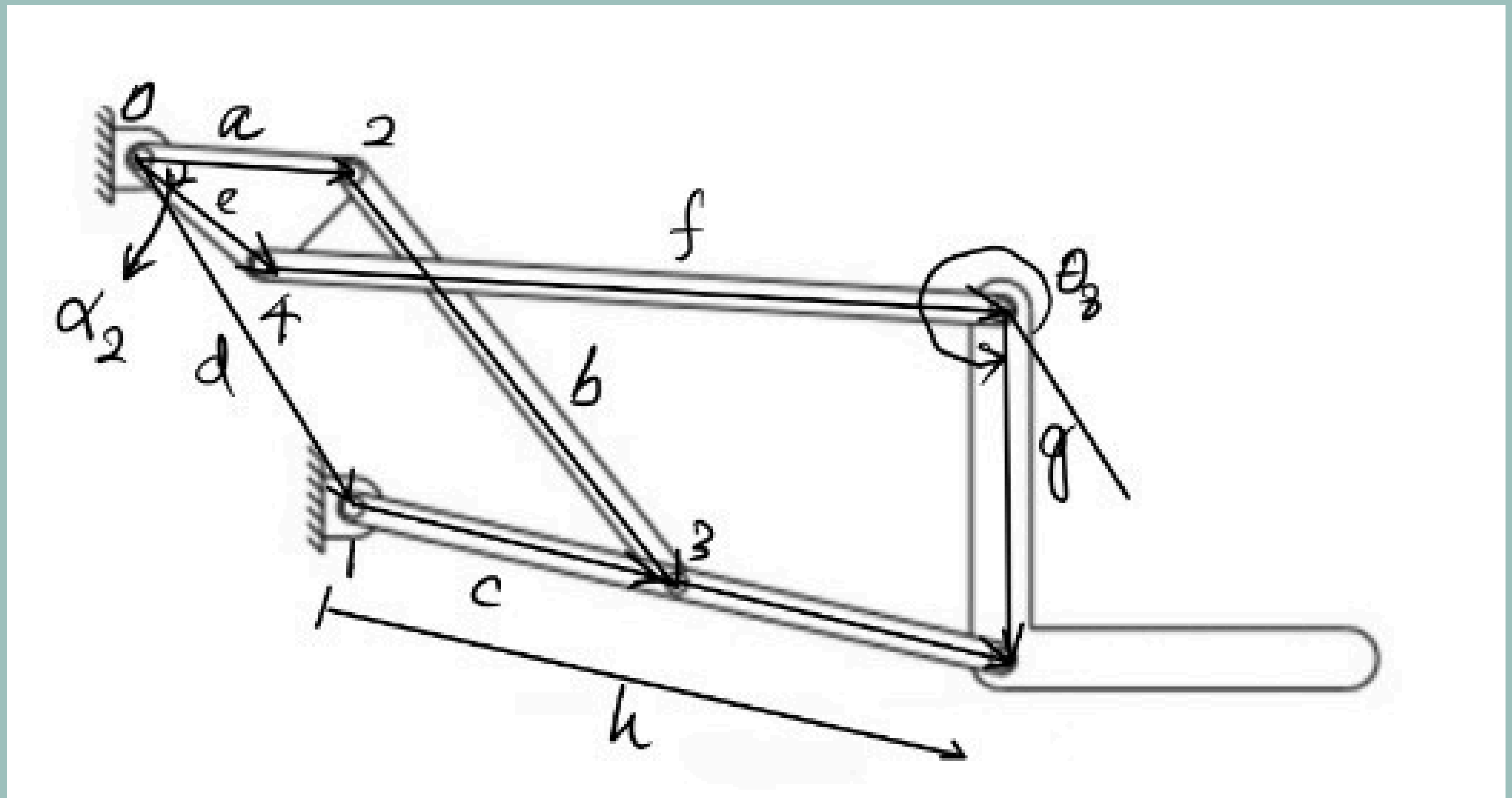
Geometrical Analysis - understanding how the mechanism is spaced out

Constraints - the lifter's orientation is constant

Python Code - iteratively solving equations with the conditions

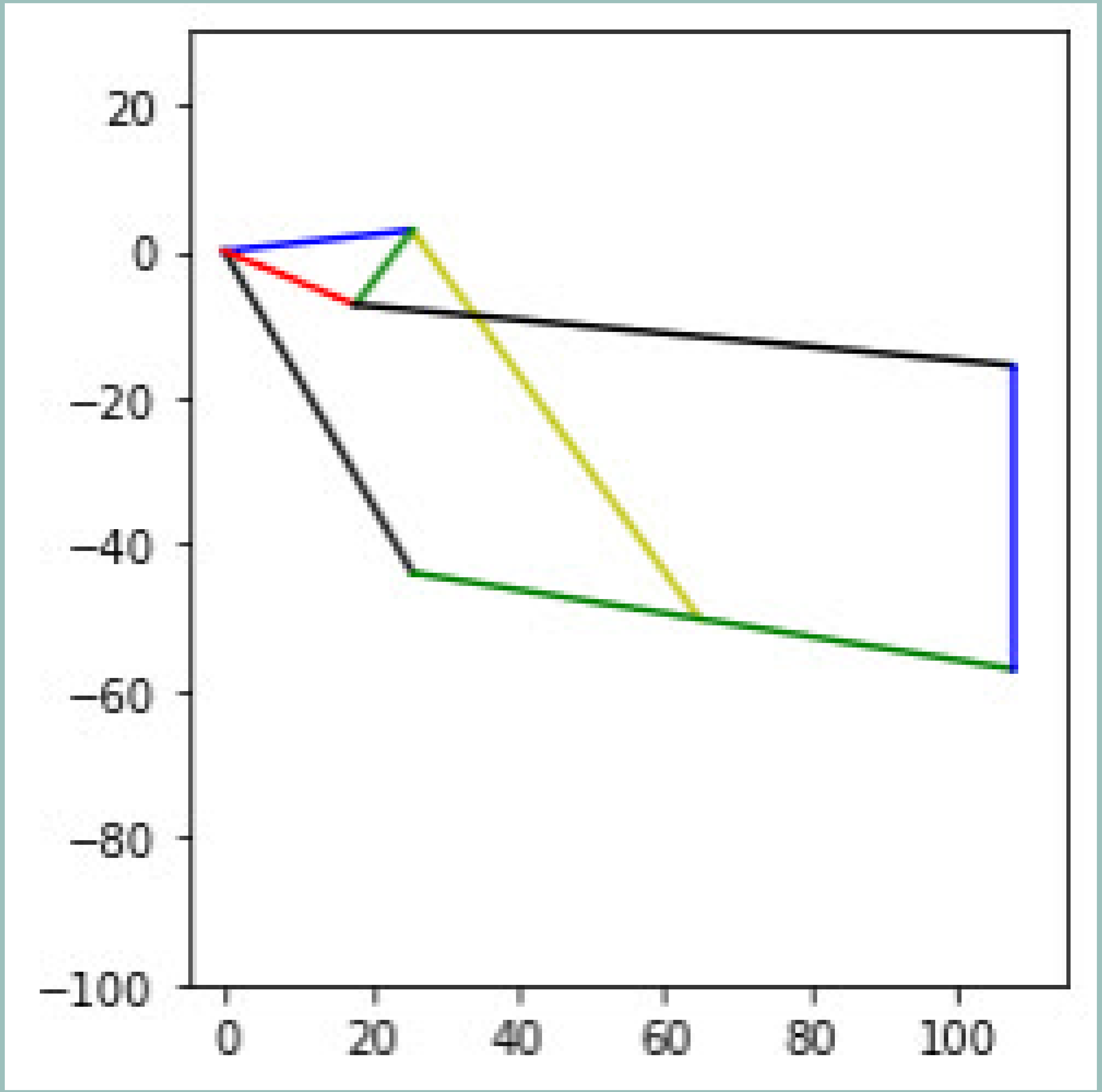
METHOD OF SIMULATION

The code iteratively generates 100 sets of 6-bar linkage link lengths within 5 units of the original lengths, validates them through position synthesis, and selects the configuration with the highest output link range of motion as the optimal solution.

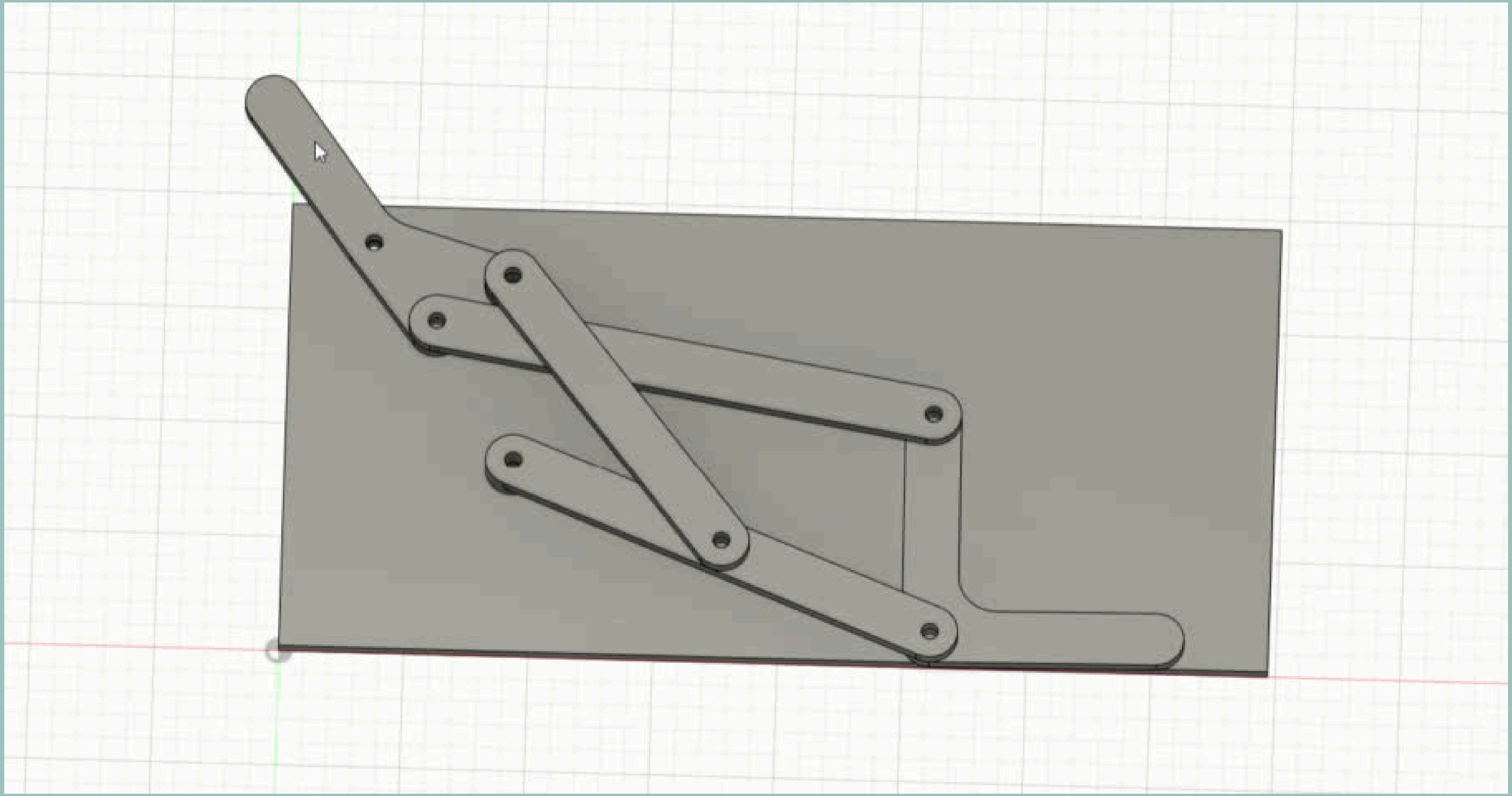


SIMULATION

With the results obtained in dyad form synthesis we used an iterative process to simulate the mechanism as shown in the figure.



CAD MODEL



Thank You