Modern Computer Vision
Programming Assignment 1

Image mosaicing

1 Problem statement

Image mosaicing is the alignment and stitching of a collection of images having overlapping regions into a single image. In this assignment, you have been given three images which were captured by panning the scene left to right. These images (`img1.pgm`, `img2.pgm` and `img3.pgm`) capture overlapping regions of the same scene from different viewpoints. The task is to determine the geometric transformations (homographies) between these images and stitch them into a single image.

2 Steps

1. Take `img2.pgm` as the reference image.
2. Determine homography H_{21} between $I_2 =$ `img2.pgm` and $I_1 =$ `img1.pgm` such that $I_1 = H_{21}I_2$.
3. Determine homography H_{23} between $I_2 =$ `img2.pgm` and $I_3 =$ `img3.pgm` such that $I_3 = H_{23}I_2$.
4. Create an empty canvas. For every pixel in the canvas, find corresponding points in I_1, I_2 and I_3 using H_{21}, identity matrix and H_{23} respectively (target-to-source mapping). Blend the three values by averaging them. Employ the values in blending only if it falls within the corresponding image bounds. Choose the origin so as to get a full mosaic.

2.1 Determining homography between two images

1. Determine SIFT features of the two images and determine correspondences between them. File `sift_corresp.m` returns the SIFT correspondences between two images (see Section **??**). Now to find H such that:

$$\begin{bmatrix} x_i' \\ y_i' \\ 1 \end{bmatrix} \sim H \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}$$

2. Run RANSAC on matched points (correspondences) to remove outliers (wrong matches), and find the homography between the two images.

(a) Input: Matched points (x_i, y_i) and (x'_i, y'_i) with $i \in \mathcal{M}$.

(b) Randomly pick four correspondences (so that we can form eight equations), i.e. (x_i, y_i) and (x'_i, y'_i) with $i \in \mathcal{R} \subset \mathcal{M}$ and $|\mathcal{R}| = 4$, where $|\cdot|$ denotes the cardinality of the set.

(c) Calculate the homography H using the above four correspondences (see Section 2.2).

(d) For each of the remaining correspondences (x_i, y_i) and (x'_i, y'_i) with $i \in \mathcal{P} = \mathcal{M} \backslash \mathcal{R}$, check whether they satisfy the homography (within an error bound). If yes, add the index of that correspondence to the consensus set.

$$\begin{bmatrix} x''_i \\ y''_i \\ z''_i \end{bmatrix} \leftarrow H \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}, \text{ and normalize so that } z''_i = 1,$$

i.e. $x''_i \leftarrow x''_i / z''_i$ and $y''_i \leftarrow y''_i / z''_i$

If $\sqrt{(x'_i - x''_i)^2 + (y'_i - y''_i)^2} < \epsilon(= 10)$, then update consensus set $\mathcal{C} \leftarrow \mathcal{C} \cup \{i\}$.

(e) If the consensus set is large enough i.e. if $|\mathcal{C}| > d(= 0.8|\mathcal{P}|)$, then return this homography H, else go to step (b).

(f) Output: Homography H.

2.2 Calculating homography

1. Consider a correspondence (x, y) and (x', y'),

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \\ h_7 & h_8 & h_9 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}.$$

Upon normalizing z',

$$\begin{aligned} x' &= h_1 x + h_2 y + h_3 / h_7 x + h_8 y + h_9, \\ y' &= h_4 x + h_5 y + h_6 / h_7 x + h_8 y + h_9. \end{aligned}$$

Form two equations for each correspondence (corresponding to two rows of matrix A).

$$\begin{aligned} &(x)h_1 + (y)h_2 + (1)h_3 + (0)h_4 + (0)h_5 + (0)h_6 \\ &\qquad\qquad - (x'x)h_7 - (x'y)h_8 - (x')h_9 = 0 \\ &(0)h_1 + (0)h_2 + (0)h_3 + (x)h_4 + (y)h_5 + (1)h_6 \\ &\qquad\qquad - (y'x)h_7 - (y'y)h_8 - (y')h_9 = 0 \end{aligned}$$

2. Solve the system,

$$A_{8\times 9}\begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_9 \end{bmatrix} = 0$$

i.e., find the null space of A.

3. Homography matrix

$$H = \begin{bmatrix} h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \\ h_7 & h_8 & h_9 \end{bmatrix}.$$