# Numerical Analysis Homework 3

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## Problem 1

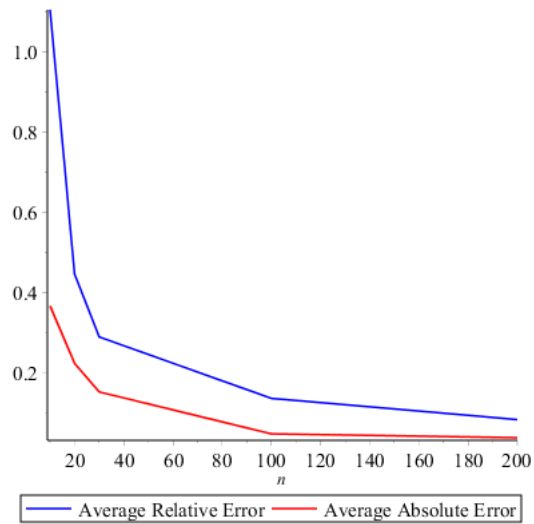Computation

Verifying Correctness

## Problem 2

5 Point Stencils

First Derivative

| n | Average Absolute Error | Average Relative Error |
|---|---|---|
| 10 | 0.367324 | 1.105306 |
| 20 | 0.223943 | 0.445905 |
| 30 | 0.152942 | 0.289964 |
| 100 | 0.048564 | 0.136987 |
| 200 | 0.038970 | 0.084145 |

## Second Derivative

| n | Average Absolute Error | Average Relative Error |
|---|---|---|
| 10 | 0.745691 | 82151.805057 |
| 20 | 2.315927 | 22255.208023 |
| 30 | 0.312447 | 16943.067463 |
| 100 | 1.473741 | 10594.331845 |
| 200 | 6.611830 | 40690.086618 |
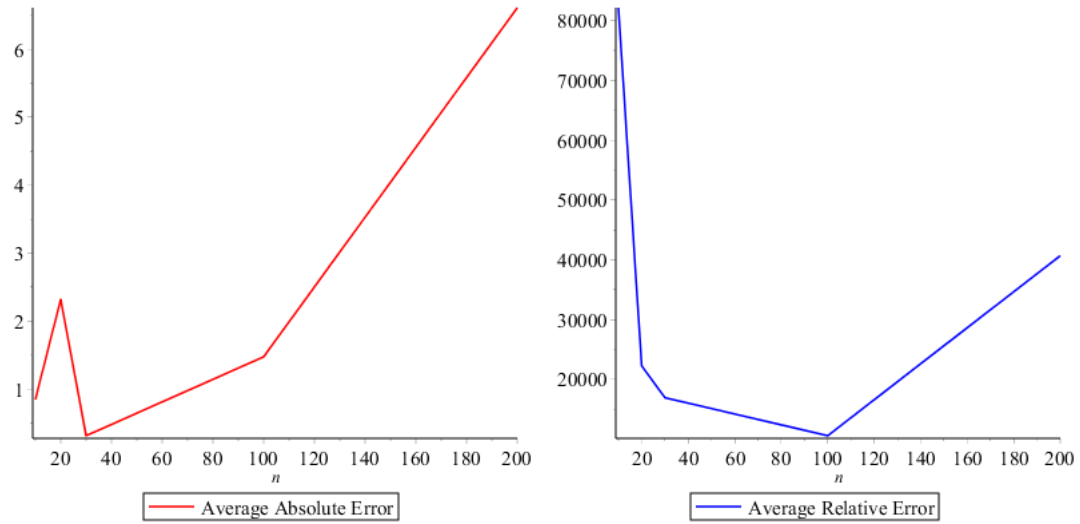


## Simpson's Rule

Actual value of $\int_0^\pi \sin x \cdot e^{\cos x}\,dx$ : 2.350402.

| n | Simpson's Result | Absolute Error |
|---|---|---|
| 4 | 1.606199 | .744203 |
| 10 | 2.215001 | .135401 |
| 20 | 2.315927 | .034475 |
| 30 | 2.334574 | .015828 |
| 100 | 2.349178 | .001224 |
| 200 | 2.350407 | .000005 |
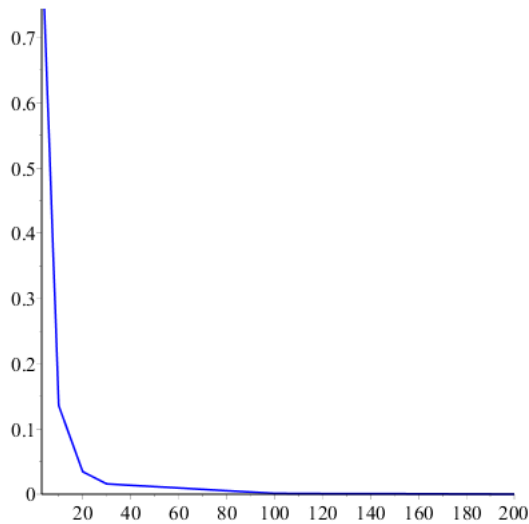
## Analysis

Simpson's method remained fairly stable despite the noise, with the error showing a clear exponential decay as $n$ increased, and achieving $10^{-5}$ accuracy at $n = 200$.

The first derivative using stencils did a little worse, with the error not only not decreasing as quickly with increasing $n$, but also seeming to level out in its decay as $n$ becomes large.

The second derivative suffered from huge relative error as the true value of the second derivative became small, and regardless of $n$, it seems as though the error in the second derivative approximation stayed consistent with (and amplified) the behavior of the noise.

Raw data for this problem can be found in the outputs directory.

## Problem 3

| | |
|---|---|
| Simpson's Method Integration | 0.316200 |
| Trapezoid Method Integration | 0.318500 |
| Total Emitted Energy from Magnitude Spline | $64.469777 \cdot L_\odot$ |
| Total Emitted Energy from Luminosity Spline | $64.476557 \cdot L_\odot$ |

## Problem 4

Median Photon Energy

Mean Photon Energy

Standard Deviation in Wavelength

## Problem 5

a.) $\int_{-1}^{1} \cos^2 x \, dx$
   Actual value: 1.4546
   Romberg 3,3 Value: .868757

b.) $\int_{-\frac{3}{4}}^{\frac{3}{4}} x \ln(x+1) dx$
   Actual value: .324332
   Romberg 3,3 Value: .270399

c.) $\int_{1}^{4} \sin^2 x - 2x \sin x + 1 dx$
   Actual value: 1.3668
   Romberg 3,3 Value:

d.) $\int_{e}^{2e} \frac{1}{x \ln x} dx$
   Actual value: .52659
   Romberg 3,3 Value: .382850

## Problem 6