Numerical Analysis Homework 1

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Problem 4

a.) $10.5d = 1010.1b = 01000000001001010\ldots0$

$10d = 1010d,\ .5d * 2d = 1.0d = 1.0b = .1b * 2b$. So $(10 + .5)d = (1010 + .1)b$.

$1010.1 = 1.0101x2^3 = 1.0101 \times 2^{1026-1023}$

b.) $\frac{1}{3}d = .\overline{01}b = 0011111111101010101\ldots01$

$\frac{1}{3} * 2 = \frac{2}{3},\ \frac{2}{3} * 2 = 1 + \frac{1}{3} \to \frac{1}{3} * 2 = \ldots = .\overline{01}b$

$.\overline{01} = 1.\overline{01}x2^{-2} = 1.\overline{01} \times 2^{1021-1023}$

c.) $\frac{22}{7}d = 11.\overline{001}b = 01000000000000100100100\ldots001$

$\frac{22}{7} = 3 + \frac{1}{7} \to \frac{1}{7} * 2 = \frac{2}{7} \to \frac{4}{7} \to 1 + 17\ldots = (11.\overline{001})b$

$11.\overline{001} = 1.1\overline{001} \times 2 = 1001001\ldots001$

Problem 5

Bisection

Raw Output

```
i: 0 a: 1.000000000 b: 2.000000000 value: -1.000000000
i: 1 a: 1.000000000 b: 1.500000000 value: 1.375000000
i: 2 a: 1.250000000 b: 1.500000000 value: -0.046875000
i: 3 a: 1.250000000 b: 1.375000000 value: 0.599609375
i: 4 a: 1.250000000 b: 1.312500000 value: 0.260986328
i: 5 a: 1.250000000 b: 1.281250000 value: 0.103302002
i: 6 a: 1.250000000 b: 1.265625000 value: 0.027286530
i: 7 a: 1.257812500 b: 1.265625000 value: -0.010024548
i: 8 a: 1.257812500 b: 1.261718750 value: 0.008573234
i: 9 a: 1.259765625 b: 1.261718750 value: -0.000740074
i: 10 a: 1.259765625 b: 1.260742188 value: 0.003912973
i: 11 a: 1.259765625 b: 1.260253906 value: 0.001585548
```

```
i: 12 a: 1.259765625 b: 1.260009766 value: 0.000422512
i: 13 a: 1.259887695 b: 1.260009766 value: -0.000158837
i: 14 a: 1.259887695 b: 1.259948730 value: 0.000131823
i: 15 a: 1.259918213 b: 1.259948730 value: -0.000013510
i: 16 a: 1.259918213 b: 1.259933472 value: 0.000059156
i: 17 a: 1.259918213 b: 1.259925842 value: 0.000022822
i: 18 a: 1.259918213 b: 1.259922028 value: 0.000004656
i: 19 a: 1.259920120 b: 1.259922028 value: -0.000004427
i: 20 a: 1.259920120 b: 1.259921074 value: 0.000000114
i: 21 a: 1.259920597 b: 1.259921074 value: -0.000002156
i: 22 a: 1.259920835 b: 1.259921074 value: -0.000001021
i: 23 a: 1.259920955 b: 1.259921074 value: -0.000000453
i: 24 a: 1.259921014 b: 1.259921074 value: -0.000000169
i: 25 a: 1.259921044 b: 1.259921074 value: -0.000000028
i: 26 a: 1.259921044 b: 1.259921059 value: 0.000000043
i: 27 a: 1.259921044 b: 1.259921052 value: 0.000000008
i: 28 a: 1.259921048 b: 1.259921052 value: -0.000000010
i: 29 a: 1.259921050 b: 1.259921052 value: -0.000000001
```

Analysis

The bisection method took 29 iterations to successfully find the root within an acceptable tolerance.

Secant

Raw Output

```
i: 0 a: 2.000000000 b: 0.500000000 value: -1.875000000
i: 1 a: 0.500000000 b: 0.857142857 value: -1.370262391
i: 2 a: 0.857142857 b: 1.826714801 value: 4.095540811
i: 3 a: 1.826714801 b: 1.100211976 value: -0.668230377
i: 4 a: 1.100211976 b: 1.202120997 value: -0.262821088
i: 5 a: 1.202120997 b: 1.268187169 value: 0.039623768
i: 6 a: 1.268187169 b: 1.259531737 value: -0.001853412
i: 7 a: 1.259531737 b: 1.259918506 value: -0.000012113
i: 8 a: 1.259918506 b: 1.259921051 value: 0.000000004
i: 9 a: 1.259921051 b: 1.259921050 value: -0.000000000
```

False Position

Raw Output

```
i: 0 a: 0.500000000 b: 2.000000000 value: -1.875000000
i: 1 a: 0.857142857 b: 2.000000000 value: -1.370262391
```

```
i: 2 a: 1.069620253 b: 2.000000000 value: -0.776260854
i: 3 a: 1.176200769 b: 2.000000000 value: -0.372787106
i: 4 a: 1.224390317 b: 2.000000000 value: -0.164477725
i: 5 a: 1.245084774 b: 2.000000000 value: -0.069824644
i: 6 a: 1.253768993 b: 2.000000000 value: -0.029154523
i: 7 a: 1.257377460 b: 2.000000000 value: -0.012088652
i: 8 a: 1.258870669 b: 2.000000000 value: -0.004997956
i: 9 a: 1.259487511 b: 2.000000000 value: -0.002063890
i: 10 a: 1.259742146 b: 2.000000000 value: -0.000851855
i: 11 a: 1.259847230 b: 2.000000000 value: -0.000351525
i: 12 a: 1.259890591 b: 2.000000000 value: -0.000145047
i: 13 a: 1.259908483 b: 2.000000000 value: -0.000059848
i: 14 a: 1.259915865 b: 2.000000000 value: -0.000024693
i: 15 a: 1.259918910 b: 2.000000000 value: -0.000010189
i: 16 a: 1.259920167 b: 2.000000000 value: -0.000004204
i: 17 a: 1.259920686 b: 2.000000000 value: -0.000001734
i: 18 a: 1.259920900 b: 2.000000000 value: -0.000000716
i: 19 a: 1.259920988 b: 2.000000000 value: -0.000000295
i: 20 a: 1.259921024 b: 2.000000000 value: -0.000000122
i: 21 a: 1.259921039 b: 2.000000000 value: -0.000000050
i: 22 a: 1.259921046 b: 2.000000000 value: -0.000000021
i: 23 a: 1.259921048 b: 2.000000000 value: -0.000000009
i: 24 a: 1.259921049 b: 2.000000000 value: -0.000000004
i: 25 a: 1.259921050 b: 2.000000000 value: -0.000000001
i: 26 a: 1.259921050 b: 2.000000000 value: -0.000000001
```

FPI

1. $g(x) = \frac{x}{2} + \frac{1}{x^2}$

   Raw Output

   ```
   i: 0 x: 1.000000000 value: 1.500000000
   i: 1 x: 1.500000000 value: 1.194444444
   i: 2 x: 1.194444444 value: 1.298141638
   i: 3 x: 1.298141638 value: 1.242482157
   i: 4 x: 1.242482157 value: 1.269009360
   i: 5 x: 1.269009360 value: 1.255474294
   i: 6 x: 1.255474294 value: 1.262168081
   i: 7 x: 1.262168081 value: 1.258803531
   i: 8 x: 1.258803531 value: 1.260481298
   i: 9 x: 1.260481298 value: 1.259641299
   i: 10 x: 1.259641299 value: 1.260061018
   ```

```
i: 11 x: 1.260061018 value: 1.259851089
i: 12 x: 1.259851089 value: 1.259956036
i: 13 x: 1.259956036 value: 1.259903558
i: 14 x: 1.259903558 value: 1.259929796
i: 15 x: 1.259929796 value: 1.259916677
i: 16 x: 1.259916677 value: 1.259923236
i: 17 x: 1.259923236 value: 1.259919957
i: 18 x: 1.259919957 value: 1.259921597
i: 19 x: 1.259921597 value: 1.259920777
i: 20 x: 1.259920777 value: 1.259921187
i: 21 x: 1.259921187 value: 1.259920982
i: 22 x: 1.259920982 value: 1.259921084
i: 23 x: 1.259921084 value: 1.259921033
i: 24 x: 1.259921033 value: 1.259921058
i: 25 x: 1.259921058 value: 1.259921046
i: 26 x: 1.259921046 value: 1.259921052
i: 27 x: 1.259921052 value: 1.259921049
i: 28 x: 1.259921049 value: 1.259921050
```

2. $g(x) = \frac{2x}{3} + \frac{2}{3x^2}$

Raw Output

```
i: 0 x: 1.000000000 value: 1.333333333
i: 1 x: 1.333333333 value: 1.263888889
i: 2 x: 1.263888889 value: 1.259933493
i: 3 x: 1.259933493 value: 1.259921050
```

3. $g(x) = x - \alpha(x^3 - 2)$

Raw Output

```
i: 0 x: 1.000000000 value: 1.250000000
i: 1 x: 1.250000000 value: 1.261718750
i: 2 x: 1.261718750 value: 1.259575441
i: 3 x: 1.259575441 value: 1.259986793
i: 4 x: 1.259986793 value: 1.259908518
i: 5 x: 1.259908518 value: 1.259923438
i: 6 x: 1.259923438 value: 1.259920595
i: 7 x: 1.259920595 value: 1.259921137
i: 8 x: 1.259921137 value: 1.259921033
i: 9 x: 1.259921033 value: 1.259921053
i: 10 x: 1.259921053 value: 1.259921049
```

Problem 6

The functions $g_i(x)$ that I constructed are

$$g_1(x) = \frac{-1}{14.5}f(x) + x$$

$$g_2(x) = \frac{1}{8}f(x) + x$$

$$g_3(x) = \frac{1}{16}f(x) + x$$

where constants were obtained by taking the negative reciprocal of the approximate derivative of $f(x) = 2x^3 - 8x - 1$ at the roots.

Root 1: $x \approx -1.9343$

Raw Output - Initial Guess -2

```
--------------------------
 FALSE POSITION METHOD G_1
 ----------------------


i: 0 x: -2.000000000 value: -1.931034483
i: 1 x: -1.931034483 value: -1.934277890
i: 2 x: -1.934277890 value: -1.934297805
i: 3 x: -1.934297805 value: -1.934297876


--------------------------
 FALSE POSITION METHOD G_2
 ----------------------


i: 0 x: -2.000000000 value: -2.125000000
i: 1 x: -2.125000000 value: -2.523925781
i: 2 x: -2.523925781 value: -4.144478854
i: 3 x: -4.144478854 value: -17.922122637
i: 4 x: -17.922122637 value: -1439.282558733
i: 5 x: -1439.282558733 value: -745380791.274546146
i: 6 x: -745380791.274546146 value: -1035319987915346794376069912.000000000
i: 7 x: -1035319987915346794376069912.000000000 value: -2774366325311505457578237345539754
i: 8 x: -27743663253115054575782373455339754203596759078544148181726442812163260447445728.0
value: -533864966103796174913924902921888412004402109774161279458134099385561857535971830
```

```
i: 9 x: -5338649661037961749139249029218884120044021097741612794581340993855618575359718:
value: -inf



--------------------------
 FALSE POSITION METHOD G_3
 ----------------------


i: 0 x: -2.000000000 value: -2.062500000
i: 1 x: -2.062500000 value: -2.190460205
i: 2 x: -2.190460205 value: -2.471490348
i: 3 x: -2.471490348 value: -3.185309780
i: 4 x: -3.185309780 value: -5.695003010
i: 5 x: -5.695003010 value: -25.998297789
i: 6 x: -25.998297789 value: -2209.630166633
i: 7 x: -2209.630166633 value: -1348556479.545314789
i: 8 x: -1348556479.545314789 value: -306561373512356053857075200.000000000
i: 9 x: -306561373512356053857075200.000000000 value: -36013249732803240464435759871431327
i: 10 x: -3601324973280324046443575987143132735405031185648250586653515602256206860648448
value: -5838441740431293736846375112159843378479548958498592641326713918840415288458319612
i: 11 x: -5838441740431293736846375112159843378479548958498592641326713918840415288458319
value: -inf



Root 2: $x \approx -0.12549$

Raw Output - Initial Guess 0


--------------------------
 FALSE POSITION METHOD G_1
 ----------------------


i: 0 x: 0.000000000 value: 0.068965517
i: 1 x: 0.068965517 value: 0.175935731
i: 2 x: 0.175935731 value: 0.341218093
i: 3 x: 0.341218093 value: 0.592962151
i: 4 x: 0.592962151 value: 0.960322243
i: 5 x: 0.960322243 value: 1.436965242
i: 6 x: 1.436965242 value: 1.889477770
i: 7 x: 1.889477770 value: 2.070475964
i: 8 x: 2.070475964 value: 2.057516112
```

```
i: 9 x: 2.057516112 value: 2.060251590
i: 10 x: 2.060251590 value: 2.059698076
i: 11 x: 2.059698076 value: 2.059811107
i: 12 x: 2.059811107 value: 2.059788068
i: 13 x: 2.059788068 value: 2.059792766
i: 14 x: 2.059792766 value: 2.059791808
i: 15 x: 2.059791808 value: 2.059792003
i: 16 x: 2.059792003 value: 2.059791963
i: 17 x: 2.059791963 value: 2.059791971
i: 18 x: 2.059791971 value: 2.059791970


--------------------------
 FALSE POSITION METHOD G_2
 ----------------------


i: 0 x: 0.000000000 value: -0.125000000
i: 1 x: -0.125000000 value: -0.125488281
i: 2 x: -0.125488281 value: -0.125494026
i: 3 x: -0.125494026 value: -0.125494094


--------------------------
 FALSE POSITION METHOD G_3
 ----------------------


i: 0 x: 0.000000000 value: -0.062500000
i: 1 x: -0.062500000 value: -0.093780518
i: 2 x: -0.093780518 value: -0.109493356
i: 3 x: -0.109493356 value: -0.117410765
i: 4 x: -0.117410765 value: -0.121407700
i: 5 x: -0.121407700 value: -0.123427541
i: 6 x: -0.123427541 value: -0.124448813
i: 7 x: -0.124448813 value: -0.124965332
i: 8 x: -0.124965332 value: -0.125226603
i: 9 x: -0.125226603 value: -0.125358772
i: 10 x: -0.125358772 value: -0.125425635
i: 11 x: -0.125425635 value: -0.125459461
i: 12 x: -0.125459461 value: -0.125476573
i: 13 x: -0.125476573 value: -0.125485230
i: 14 x: -0.125485230 value: -0.125489610
```

```
i: 15 x: -0.125489610 value: -0.125491826
i: 16 x: -0.125491826 value: -0.125492947
i: 17 x: -0.125492947 value: -0.125493514
i: 18 x: -0.125493514 value: -0.125493801
i: 19 x: -0.125493801 value: -0.125493946
i: 20 x: -0.125493946 value: -0.125494019
i: 21 x: -0.125494019 value: -0.125494056
i: 22 x: -0.125494056 value: -0.125494075
i: 23 x: -0.125494075 value: -0.125494085
i: 24 x: -0.125494085 value: -0.125494089
i: 25 x: -0.125494089 value: -0.125494092
i: 26 x: -0.125494092 value: -0.125494093
```

Root 3: $x \approx 2.0598$

Raw Output - Initial Guess 2

```
--------------------------
 FALSE POSITION METHOD G_1
 ----------------------
```

```
i: 0 x: 2.000000000 value: 2.068965517
i: 1 x: 2.068965517 value: 2.057849709
i: 2 x: 2.057849709 value: 2.060184771
i: 3 x: 2.060184771 value: 2.059711749
i: 4 x: 2.059711749 value: 2.059808321
i: 5 x: 2.059808321 value: 2.059788636
i: 6 x: 2.059788636 value: 2.059792650
i: 7 x: 2.059792650 value: 2.059791831
i: 8 x: 2.059791831 value: 2.059791998
i: 9 x: 2.059791998 value: 2.059791964
i: 10 x: 2.059791964 value: 2.059791971
i: 11 x: 2.059791971 value: 2.059791970
```

```
--------------------------
 FALSE POSITION METHOD G_2
 ----------------------
```

```
i: 0 x: 2.000000000 value: 1.875000000
```

8

```
i: 1 x: 1.875000000 value: 1.522949219
i: 2 x: 1.522949219 value: 0.758072328
i: 3 x: 0.758072328 value: -0.016088951
i: 4 x: -0.016088951 value: -0.125001041
i: 5 x: -0.125001041 value: -0.125488293
i: 6 x: -0.125488293 value: -0.125494026
i: 7 x: -0.125494026 value: -0.125494094
```

```
--------------------------
 FALSE POSITION METHOD G_3
 ----------------------
```

```
i: 0 x: 2.000000000 value: 1.937500000
i: 1 x: 1.937500000 value: 1.815399170
i: 2 x: 1.815399170 value: 1.593070099
i: 3 x: 1.593070099 value: 1.239411117
i: 4 x: 1.239411117 value: 0.795194169
i: 5 x: 0.795194169 value: 0.397950600
i: 6 x: 0.397950600 value: 0.144352965
i: 7 x: 0.144352965 value: 0.010052482
i: 8 x: 0.010052482 value: -0.057473632
i: 9 x: -0.057473632 value: -0.091260547
i: 10 x: -0.091260547 value: -0.108225281
i: 11 x: -0.108225281 value: -0.116771092
i: 12 x: -0.116771092 value: -0.121084575
i: 13 x: -0.121084575 value: -0.123264197
i: 14 x: -0.123264197 value: -0.124366209
i: 15 x: -0.124366209 value: -0.124923550
i: 16 x: -0.124923550 value: -0.125205468
i: 17 x: -0.125205468 value: -0.125348081
i: 18 x: -0.125348081 value: -0.125420226
i: 19 x: -0.125420226 value: -0.125456724
i: 20 x: -0.125456724 value: -0.125475189
i: 21 x: -0.125475189 value: -0.125484530
i: 22 x: -0.125484530 value: -0.125489256
i: 23 x: -0.125489256 value: -0.125491646
i: 24 x: -0.125491646 value: -0.125492856
i: 25 x: -0.125492856 value: -0.125493468
i: 26 x: -0.125493468 value: -0.125493777
i: 27 x: -0.125493777 value: -0.125493934
i: 28 x: -0.125493934 value: -0.125494013
```

```
i: 29 x: -0.125494013 value: -0.125494053
i: 30 x: -0.125494053 value: -0.125494074
i: 31 x: -0.125494074 value: -0.125494084
i: 32 x: -0.125494084 value: -0.125494089
i: 33 x: -0.125494089 value: -0.125494092
i: 34 x: -0.125494092 value: -0.125494093
```

Problem 7

Problem 8

Problem 9

Problem 10

   i.)

   ii.)

   iii.)

   iv.)

   v.)