Numerical Analysis Project 2

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Justification of Method

I elected to use cubic splines due to the discrete nature of the data set - because we have no guarantee of consistent, polynomial-like behavior at the data points, it is likely that a single interpolating polynomial would suffer from erratic behavior in between points to accommodate inconsistencies. Although Bezier Curves would likely provide a smooth, appealing interpolation of the data, because they do not necessarily agree with the data points, we would have less fine control over their behavior which is problematic for precise determination of ideal lighting. With cubic splines, we can make adjustments solely through manipulation of the number, spacing, etc of data points without the need to also tweak control points to get expected behavior.

Additionally, to find y values corresponding to a given x value on a Bezier curve, we would need to find the roots of $x = (1-t)^3 * x_0 + 3*(1-t)^2 * t * x_1 + 3*(1-t)*t^2*x_2$, giving us the t corresponding to that x value, and then use that to calculate $y(t)$. Although Newton's method is fast and reliable for cubic equations, it is preferable to not do more work.

Computation of Splines

Raw Output for Data Set 1

```
Coefficients for Data Set 1

0.000000 0.892633 0.923469 1.172533 0.000000

 Values of Cubic Interpolant for Data Set 1
(-0.952381, -0.404201)
(-0.857143, -0.487113)
(-0.761905, -0.567712)
(-0.666667, -0.644455)
(-0.571429, -0.715801)
(-0.476190, -0.780210)
(-0.380952, -0.836615)
(-0.285714, -0.884856)
```

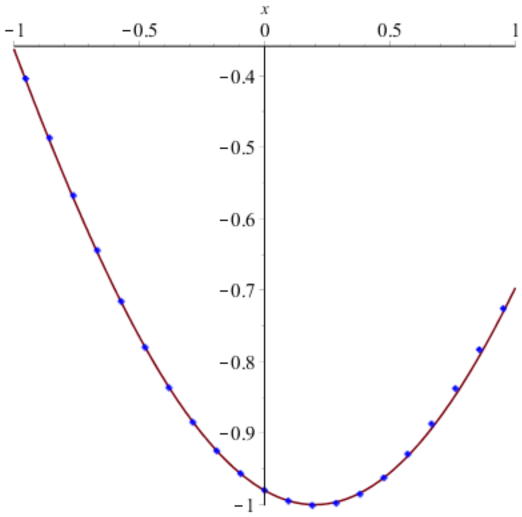
```
(-0.190476, -0.924882)
(-0.095238, -0.956637)
(0.000000, -0.980070)
(0.095238, -0.995064)
(0.190476, -1.001251)
(0.285714, -0.998202)
(0.380952, -0.985485)
(0.476190, -0.962672)
(0.571429, -0.929503)
(0.666667, -0.887225)
(0.761905, -0.837856)
(0.857143, -0.783424)
(0.952381, -0.725952)
```

Raw Output for Data Set 2

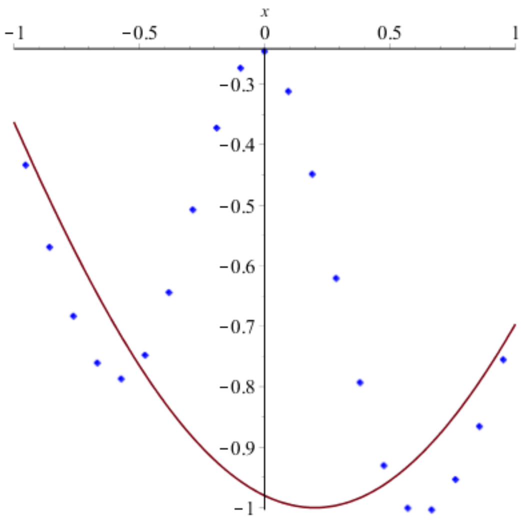
```
Coefficients for Data Set 2

0.000000 8.444919 -11.663674 8.724819 0.000000

 Values of Cubic Interpolant for Data Set 2
(-0.952381, -0.433899)
(-0.857143, -0.569682)
(-0.761905, -0.683580)
(-0.666667, -0.761003)
(-0.571429, -0.787360)
(-0.476190, -0.748191)
(-0.380952, -0.644578)
(-0.285714, -0.507793)
(-0.190476, -0.372577)
(-0.095238, -0.273673)
(0.000000, -0.245820)
(0.095238, -0.312099)
(0.190476, -0.448947)
(0.285714, -0.621138)
(0.380952, -0.793448)
(0.476190, -0.930652)
(0.571429, -1.001062)
(0.666667, -1.003772)
(0.761905, -0.953724)
(0.857143, -0.865992)
(0.952381, -0.755650)
```

Plots of Output

Data Set 1



Data Set 2

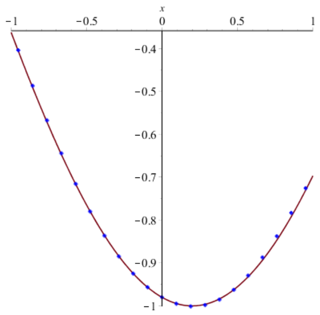


Propagation of Error

In order to maintain agreement in the first and second derivative, all of the data points changed, at least slightly, when the center input point was changed signficantly (from $-.980070$ to $-.245820$). However, the change in a given point becomes less and less the further away from the incorrect data the point is, so the error does not propagate completely throughout, although it does effect the entire output set.

Testing and Maximum Error Bound

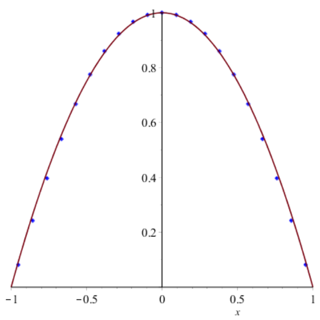
$-\cos(x-.2)$



$$y(x) = \begin{cases} -.94416178 + 0.01328678x + .297544285714286 * x^3 + .892632857x^2 & x < -.5 \\ -.9800700 - .20216250x + .4617342857x^2 + 0.01027857x^3 & x < 0 \\ -.98007000 - .20216250x + .4617342857x^2 + 0.083021x^3 & x < 0.5 \\ -.9208367857 - .5575617857x + 1.172532857x^2 - .3908442857x^3 & x \geq .5 \end{cases}$$

The maximum error on the interval $[-1,1]$, $|-\cos(x-.2)-y(x)|$, is 0.00877438285370491 at $x = .809398143755132$.

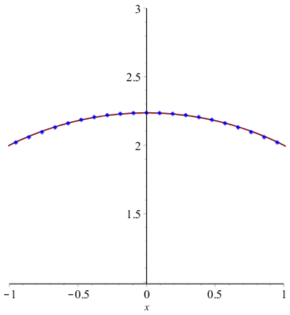
$1-x^2$



$$y(x) = \begin{cases} -.857142857142857 * x + .857142857142857 - .857142857142857 * x^3 - 2.57142857142857 * x^2 \\ 1.00000000000000 - .857142857142857 * x^2 + .285714285714286 * x^3 \\ 1. - .857142857142857 * x^2 - .285714285714286 * x^3 \\ .857142857142857 + .857142857142857 * x - 2.57142857142857 * x^2 + .857142857142857 * x^3 \end{cases}$$

The maximum error on the interval $[-1,1]$, $|1-x^2-y(x)|$, is 0.0240658925870171 at $x = -.811419518230411$.

$\sqrt{5-x^2}$



$$y(x) = \begin{cases} 2.19686285714286 - .235230857142857 * x - .216046857142857 * x^3 - .648140571428571 * x^2 \\ 2.23606800000000 - .177678857142857 * x^2 + 0.975942857142857e - 1 * x^3 \\ 2.23606800000000 + 1.38777878078145 * 10^(- 17) * x - .177678857142857 * x^2 - 0.975942857142 \\ 2.19686285714286 + .235230857142857 * x - .648140571428571 * x^2 + .216046857142857 \end{cases}$$

The maximum error on the interval $[-1,1]$, $|-\sqrt{5-x^2} - y(x)|$, is 0.00723274707416888 at $x = -.812835264258024$.

Conclusions

In short, the program approximated all three functions fairly well, producing convincing graphs and relatively small maximum errors on the interval.

Accuracy of Derivative Values

$-\cos(x - .2)$

$1 - x^2$

$\sqrt{5-x^2}$