

# Numerical Analysis Project 3

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## Lane Emden Equations

More data can be found in the corresponding .txt files in the outputs directory.

$n = .5$

$$\Xi = 2.753100$$
$$-\left(\frac{\partial \theta}{\partial \xi}\right)_{\xi=\Xi} = 0.500242$$

$\xi$	$\theta$	$\hat{M}$	$\hat{I}$	$\hat{\Omega}$
0	1	0	0	0
.5	.958594	.517034	.013161	3.307733
1.0	0.837851	3.965218	0.052130	1.662054
1.5	0.646511	12.497775	0.115720	1.115660
2.0	0.402580	26.388160	0.199817	0.849346
2.5	0.132636	42.431575	0.292766	0.702564
2.753100	0.000349	47.608767	0.325830	0.666770

$n = 1$

$$\Xi = 3.142100$$
$$-\left(\frac{\partial \theta}{\partial \xi}\right)_{\xi=\Xi} = 0.318430$$

$\xi$	$\theta$	$\hat{M}$	$\hat{I}$	$\hat{\Omega}$
0	1	0	0	0
.5	0.958851	0.510625	0.010080	3.779649
1.0	0.841772	3.774032	0.039630	1.906359
1.5	0.664997	11.201527	0.086815	1.288487
2.0	0.454649	21.885479	0.146994	0.991357
2.5	0.239389	32.689292	0.211071	0.829708
3.0	0.047040	39.095204	0.257711	0.754742
3.142100	0.000189	39.478411	0.261297	0.750121

$n = 2$

$\Xi = 4.353100$

$-\left(\frac{\partial\theta}{\partial\xi}\right)_{\xi=\Xi} = 0.127300$

$\xi$	$\theta$	$\hat{M}$	$\hat{I}$	$\hat{\Omega}$
0	1	0	0	0
.5	0.959353	0.498253	0.005227	5.248850
1.0	0.848929	3.440920	0.020269	2.666237
1.5	0.695367	9.277393	0.043458	1.823020
2.0	0.529836	16.404083	0.071768	1.422937
2.5	0.374739	22.793235	0.101328	1.204538
3.0	0.241824	27.213646	0.127545	1.083075
3.5	0.133969	29.506644	0.145944	1.022508
4.0	0.048840	30.241656	0.154024	1.001799
4.353100	0.000111	30.298098	0.154833	1.000052

$n = 3$

$\Xi = 6.897200$

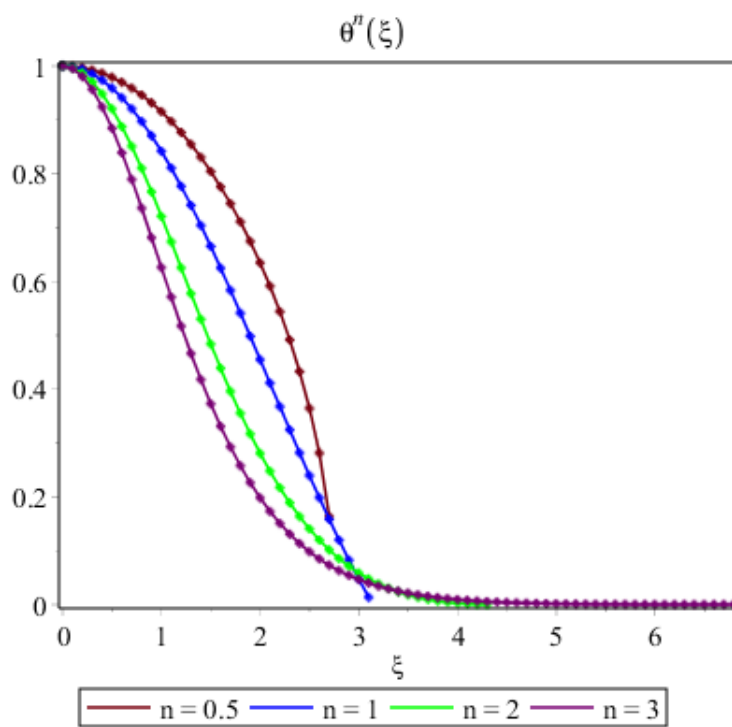
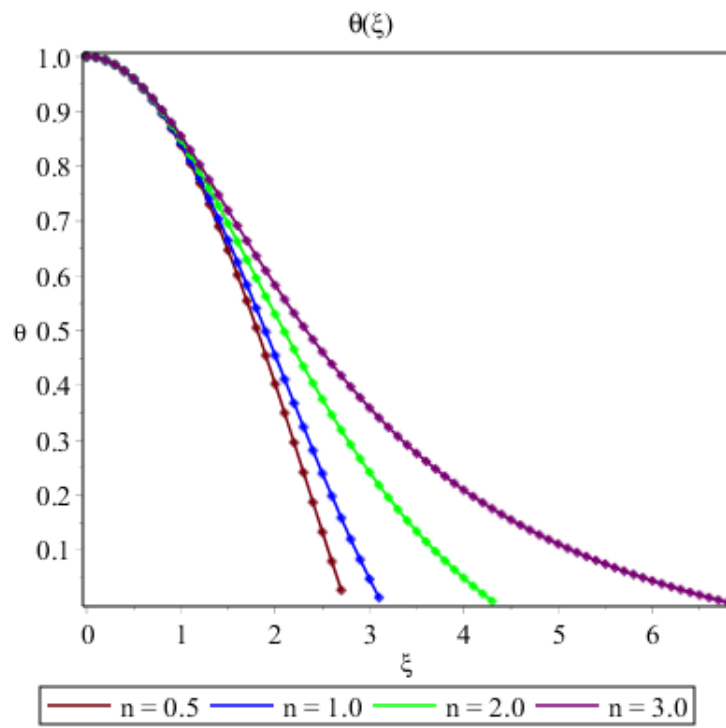
$-\left(\frac{\partial\theta}{\partial\xi}\right)_{\xi=\Xi} = 0.042440$

$\xi$	$\theta$	$\hat{M}$	$\hat{I}$	$\hat{\Omega}$
0	1	0	0	0
.5	0.959839	0.486443	0.002072	8.335976
1.0	0.855310	3.160498	0.007936	4.262277
1.5	0.719502	7.914317	0.016735	2.941765
2.0	0.582851	13.143968	0.027220	2.318066
2.5	0.461127	17.590476	0.038177	1.973579
3.0	0.359227	20.815551	0.048537	1.770052
3.5	0.276263	22.913024	0.057505	1.647475
4.0	0.209282	24.161423	0.064605	1.574866
4.5	0.155069	24.840997	0.069681	1.533989
5.0	0.110900	25.171911	0.072868	1.513033
5.5	0.074353	25.309843	0.074544	1.503789
6.0	0.043794	25.353617	0.075200	1.500692
6.5	0.017914	25.361601	0.075344	1.500099
6.897200	0.000036	25.361901	0.075350	1.500076

Constant Factor  $K$

$K \approx 12.56$ , based on an average of the values of  $\hat{M}$  for  $n = 0.5, 1, 2, 3$ .

## Plots



## Modelling Earth's Sun

$$\begin{aligned}
 \rho_{unit} &= \frac{\Xi^3}{\hat{M}} \frac{M_{\odot}}{R_{\odot}} \\
 &= \frac{6.897200^3}{25.361901} \frac{1.989 \times 10^{30} \mathbf{kg}}{695700 \mathbf{km}} \\
 &= 3.6987 \times 10^{25} \frac{\mathbf{kg}}{\mathbf{km}} = \rho_c
 \end{aligned}$$

$$P_c = \kappa \rho_c^{1+1/n}$$

$$\begin{aligned}
 \kappa &= \frac{4\pi G \alpha^2}{n+1} \rho_c^{n-1/n} \\
 \alpha &= \frac{R_{\odot}}{\Xi} = \frac{695700 \mathbf{km}}{6.897} = 1.00869 \times 10^5
 \end{aligned}$$

$$\begin{aligned}
 \kappa &= \frac{4\pi(6.67 \times 10^{-14} \frac{\mathbf{km}}{\mathbf{s}^2})(1.00869 \times 10^5 \mathbf{km})}{4} \rho_c^{2/3} \\
 &= 2.113652 \times 10^{-8} \rho_c^{2/3} \\
 &= 2.346387 \times 10^9
 \end{aligned}$$

$$\begin{aligned}
 P_c &= 2.346387 \times 10^9 \rho_c^{4/3} \\
 &= 2.891560562 * 10^4 3
 \end{aligned}$$

## White Dwarfs

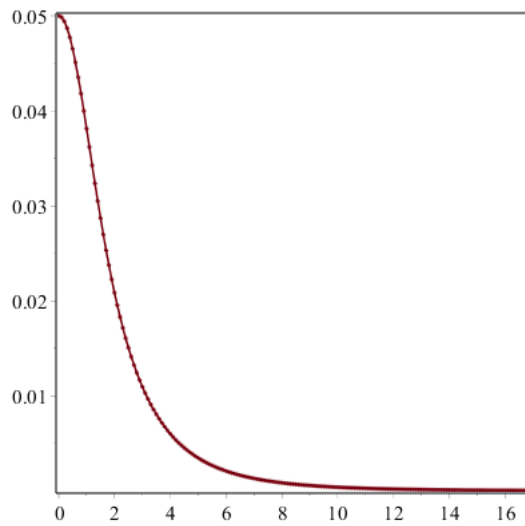
$$\frac{\partial V}{\partial s}$$

$$\begin{aligned}\frac{1}{s^2} \frac{d}{ds} (s^2 V G(\theta)) &= -\theta \\ \frac{1}{s^2} \left[ 2s V G + s^2 \frac{d}{ds} (V G) \right] &= -\theta \\ \frac{1}{s^2} \left[ 2s V G + s^2 \frac{dV}{ds} G + s^2 V^2 G'(\theta) \right] &= -\theta \\ \frac{2VG}{s} + \frac{dV}{ds} G + V^2 G' &= -\theta \\ \frac{dV}{ds} &= \frac{2V}{s} - \frac{V^2 G'}{G} - \frac{\theta}{G}\end{aligned}$$

Integrations for Selected Values of  $\theta(0)$

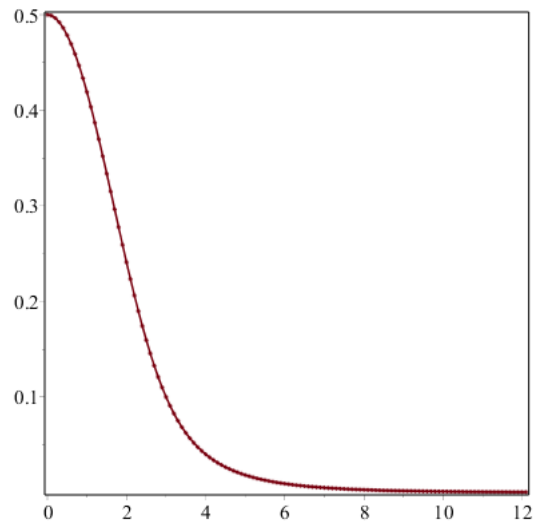
$$\theta(0) = .05$$

s	$\theta$	$\hat{M}$
0	.05	0
5	0.003474	4.530548
10	0.000394	8.354221
15	0.000080	10.060809
16.8	0.000050	10.421411



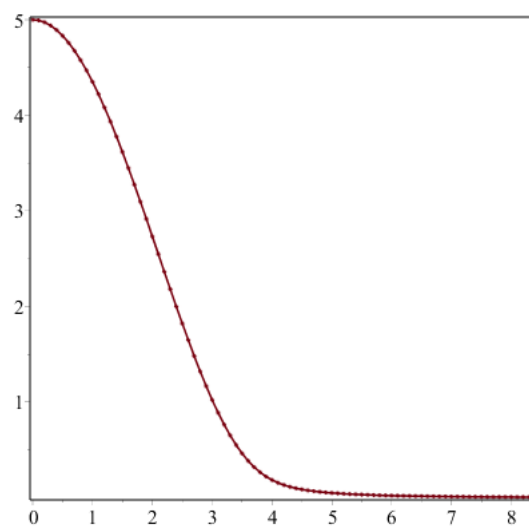
$$\theta(0) = .5$$

s	$\theta$	$\hat{M}$
0	.5	0
3	0.099983	23.313870
6	0.009359	44.747602
9	0.001821	53.285835
12.1	0.000491	57.345664



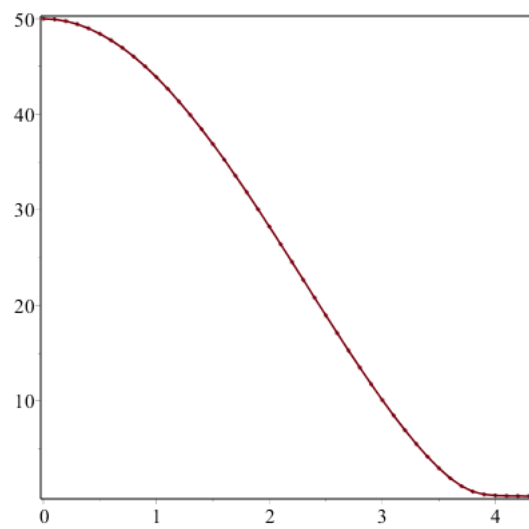
$$\theta(0) = 5$$

s	$\theta$	$\hat{M}$
0	5	0
2	2.731982	119.423255
4	0.184593	44.747602
6	0.021044	365.125754
8.3	0.004759	379.679845



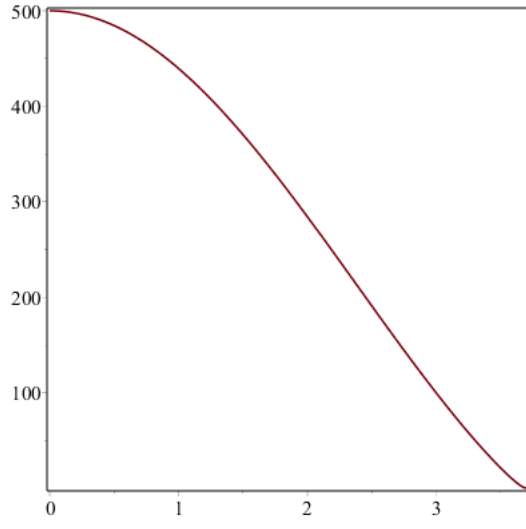
$$\theta(0) = 50$$

s	$\theta$	$\hat{M}$
0	50	0
1	43.888692	193.326200
2	28.244552	1217.559635
3	10.097859	2635.815212
4.3	0.045212	3135.770481



$$\theta(0) = 500$$

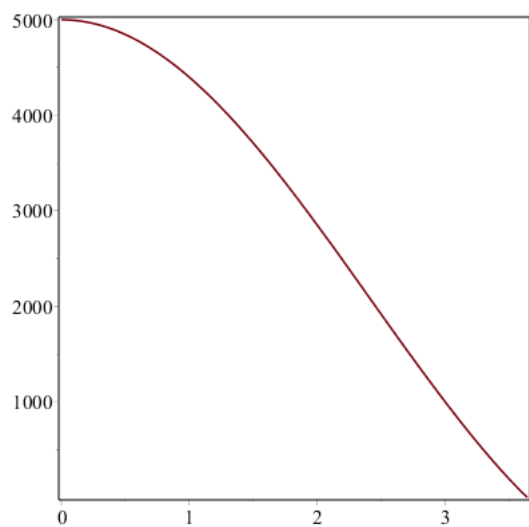
s	$\theta$	$\hat{M}$
0	500	0
1	439.582357	1940.891340
2	284.402935	12224.924810
3	100.083901	26471.219489
3.7	0.274937	30576.933377



$$\theta(0) = 5000$$

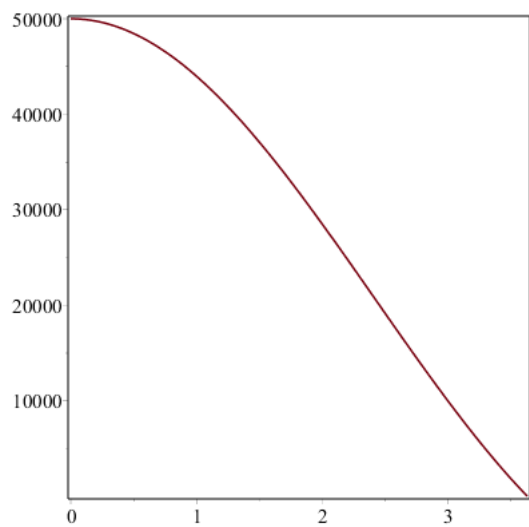
s	$\theta$	$\hat{M}$
0	5000	0
1	4397.550817	19413.417938
2	2848.169588	122353.877586
3	997.874672	264958.303355
3.64	3.295236	304300.467510





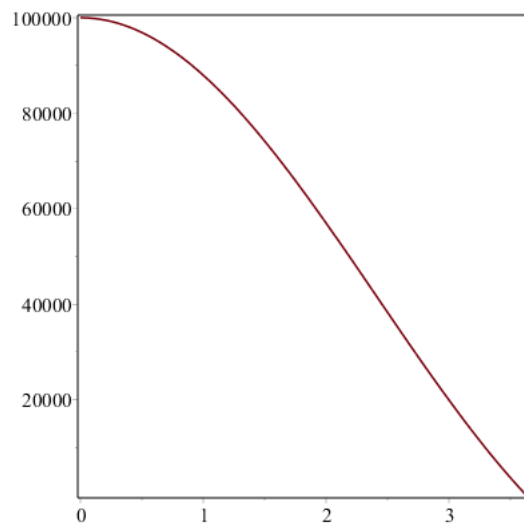
$$\theta(0) = 50000$$

s	$\theta$	$\hat{M}$
0	50000	0
1	43979.215785	194143.848806
2	28490.570788	1223763.244462
3	9971.996493	2650101.145882
3.63	22.034888	3039898.580706



$$\theta(0) = 100000$$

s	$\theta$	$\hat{M}$
0	500000	0
1	87959.184507	388289.661272
2	56982.943351	2447572.068240
3	19942.606275	5300307.090000
3.63	16.402276	6079172.237637



Dimensionless Mass

Mass-Radius Relation