Numerical Analysis (MATH-411)
Homework #4: Due Thursday, 12/8/2016

1.-3.) For the following differential equations, determine approximately how many steps you need to evolve the solution for 1 unit of time, resulting in an error of no more that 10^{-6}, by using Euler's method, RK2, and RK4 integration. The true answers may be found using Wolfram Alpha.

1. $y' = te^{3t} - 2y,\ y(0) = 0;\ \ 0 \leq t \leq 1$
2. $y' = 1 + \frac{y}{t},\ y(1) = 2;\ \ 1 \leq t \leq 2$
3. $y' = t^{-2}(\sin(2t) - 2ty),\ y(1) = 2;\ \ 1 \leq t \leq 2$
4. Solve the second-order equation $y'' - 2ty' + 2y = 0$ with initial data $y(0) = 1,\ y'(0) = 1$ by writing it as a pair of first-order equations and solving using RK4. Use enough steps to determine $y(5)$ with a *relative* error of no more that 10^{-8}.
5. Use Five-step Adams-Bashforth to solve $y' = te^{3t} - 2y, y(0) = 0$ on the interval $0 \leq t \leq 2$, using a stepsize $\delta t = 0.05$. Compare the error from using RK4 to generate the initial values to using the actual function values, $y(t) = \frac{1}{5}te^{3t} - \frac{1}{25}e^{3t} + \frac{1}{25}e^{-2t}$.
6. Same as the previous problem, but use the four-step Adams-Moulton method and compare the errors generated by the initial assumptions.