Numerical Analysis Project 3

Margaret Dorsey

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Lane Emden Equations

More data can be found in the corresponding .txt files in the outputs directory.

n = .5

$\Xi = 2.753100$
$-\left(\frac{\partial\theta}{\partial\xi}\right)_{\xi=\Xi} = 0.500242$

ξ	θ	$\hat{M}$	$\hat{I}$	$\hat{\Omega}$
0	1	0	0	0
.5	.958594	.517034	.013161	3.307733
1.0	0.837851	3.965218	0.052130	1.662054
1.5	0.646511	12.497775	0.115720	1.115660
2.0	0.402580	26.388160	0.199817	0.849346
2.5	0.132636	42.431575	0.292766	0.702564
2.753100	0.000349	47.608767	0.325830	0.666770

n = 1

$\Xi = 3.142100$
$-\left(\frac{\partial\theta}{\partial\xi}\right)_{\xi=\Xi} = 0.318430$

ξ	θ	$\hat{M}$	$\hat{I}$	$\hat{\Omega}$
0	1	0	0	0
.5	0.958851	0.510625	0.010080	3.779649
1.0	0.841772	3.774032	0.039630	1.906359
1.5	0.664997	11.201527	0.086815	1.288487
2.0	0.454649	21.885479	0.146994	0.991357
2.5	0.239389	32.689292	0.211071	0.829708
3.0	0.047040	39.095204	0.257711	0.754742
3.142100	0.000189	39.478411	0.261297	0.750121

n = 2

$\Xi = 4.353100$
$-\left(\frac{\partial\theta}{\partial\xi}\right)_{\xi=\Xi} = 0.127300$

ξ	θ	$\hat{M}$	$\hat{I}$	$\hat{\Omega}$
0	1	0	0	0
.5	0.959353	0.498253	0.005227	5.248850
1.0	0.848929	3.440920	0.020269	2.666237
1.5	0.695367	9.277393	0.043458	1.823020
2.0	0.529836	16.404083	0.071768	1.422937
2.5	0.374739	22.793235	0.101328	1.204538
3.0	0.241824	27.213646	0.127545	1.083075
3.5	0.133969	29.506644	0.145944	1.022508
4.0	0.048840	30.241656	0.154024	1.001799
4.353100	0.000111	30.298098	0.154833	1.000052

n = 3

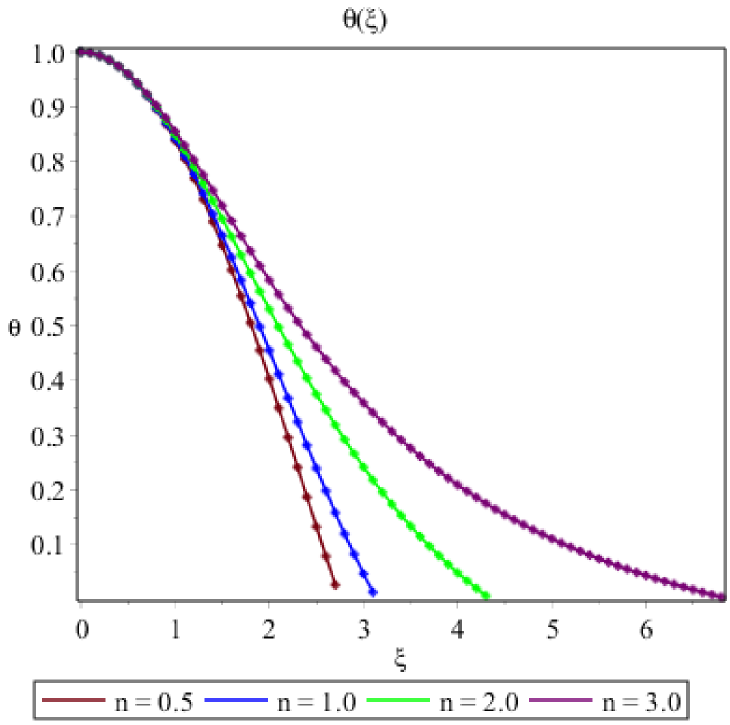
$\Xi = 6.897200$
$-\left(\frac{\partial\theta}{\partial\xi}\right)_{\xi=\Xi} = 0.042440$

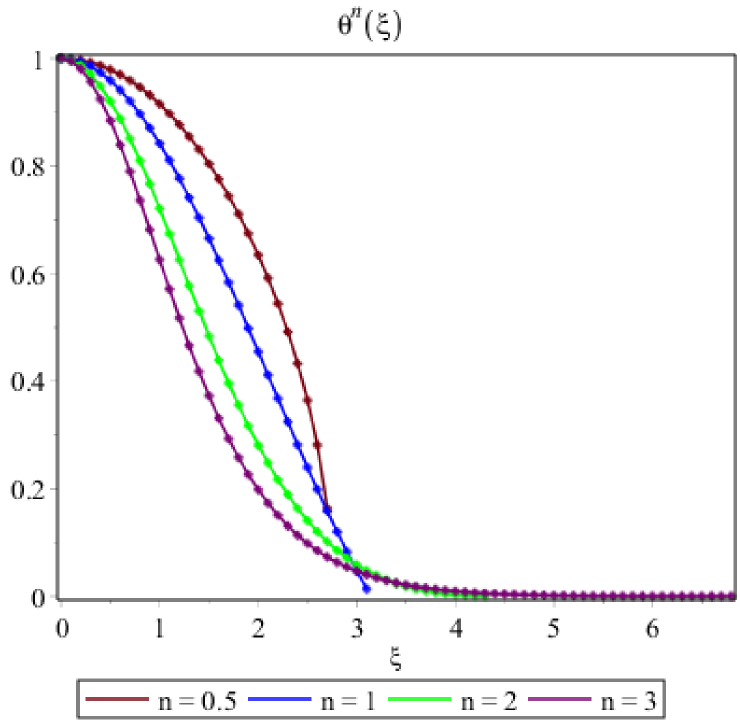
ξ	θ	$\hat{M}$	$\hat{I}$	$\hat{\Omega}$
0	1	0	0	0
.5	0.959839	0.486443	0.002072	8.335976
1.0	0.855310	3.160498	0.007936	4.262277
1.5	0.719502	7.914317	0.016735	2.941765
2.0	0.582851	13.143968	0.027220	2.318066
2.5	0.461127	17.590476	0.038177	1.973579
3.0	0.359227	20.815551	0.048537	1.770052
3.5	0.276263	22.913024	0.057505	1.647475
4.0	0.209282	24.161423	0.064605	1.574866
4.5	0.155069	24.840997	0.069681	1.533989
5.0	0.110900	25.171911	0.072868	1.513033
5.5	0.074353	25.309843	0.074544	1.503789
6.0	0.043794	25.353617	0.075200	1.500692
6.5	0.017914	25.361601	0.075344	1.500099
6.897200	0.000036	25.361901	0.075350	1.500076

Constant Factor K

$K \approx 12.56$, based on an average of the values of $\hat{M}$ for $n = 0.5, 1, 2, 3$.

Plots


θ(ξ)
1.0
0.9
0.8
0.7
0.6
θ 0.5
0.4
0.3
0.2
0.1
0
1
2
3
4
5
6
ξ
n = 0.5
n = 1.0
n = 2.0
n = 3.0


$\theta^n(\xi)$
1
0.8
0.6
0.4
0.2
0
0
1
2
3
4
5
6
ξ
n = 0.5
n = 1
n = 2
n = 3

Modelling Earth's Sun

$$\begin{aligned}
\rho_{unit} &= \frac{\Xi^3}{\hat{M}} \frac{M_\odot}{R_\odot} \\
&= \frac{6.897200^3}{25.361901} \frac{1.989 \times 10^{30}\mathbf{kg}}{695700\mathbf{km}} \\
&= 3.6987 \times 10^{25} \frac{\mathbf{kg}}{\mathbf{km}} = \rho_c
\end{aligned}$$

$$P_c = \kappa \rho_c^{1+1/n}$$

$$\begin{aligned}
\kappa &= \frac{4\pi G \alpha^2}{n+1} \rho_c^{n-1/n} \\
\alpha &= \frac{R_\odot}{\Xi} = \frac{695700\mathbf{km}}{6.897} = 1.00869 \times 10^5
\end{aligned}$$

$$\begin{aligned}
\kappa &= \frac{4\pi (6.67 \times 10^{-14} \frac{\mathbf{km}}{\mathbf{s^2}})(1.00869 \times 10^5 \mathbf{km})}{4} \rho_c^{2/3} \\
&= 2.113652 \times 10^-8 \rho_c^{2/3} \\
&= 2.346387 \times 10^9
\end{aligned}$$

$$\begin{aligned}
P_c &= 2.346387 \times 10^9 \rho_c^{4/3} \\
&= 2.891560562 * 10^4 3
\end{aligned}$$

White Dwarfs

$\frac{\partial V}{\partial s}$

$$\begin{aligned}
\frac{1}{s^2}\frac{d}{ds}\left(s^2 V G(\theta)\right) &= -\theta \\
\frac{1}{s^2}\left[2sVG + s^2\frac{d}{ds}(VG)\right] &= -\theta \\
\frac{1}{s^2}\left[2sVG + s^2\frac{dV}{ds}G + s^2V^2G'(\theta)\right] &= -\theta \\
\frac{2VG}{s} + \frac{dV}{ds}G + V^2G' &= -\theta \\
&\frac{dV}{ds} = \frac{2V}{s} - \frac{V^2G'}{G} - \frac{\theta}{G}
\end{aligned}$$

Integrations for Selected Values of $\theta(0)$

Dimensionless Mass

Mass-Radius Relation