

Numerical Analysis Project 3

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Lane Emden Equations

More data can be found in the corresponding .txt files in the outputs directory.

$n = .5$

$\Xi = 2.753100$

$-\left(\frac{\partial \theta}{\partial \xi}\right)_{\xi=\Xi} = 0.500242$

| ξ | θ | \hat{M} | \hat{I} | $\hat{\Omega}$ |
|----------|----------|-----------|-----------|----------------|
| 0 | 1 | 0 | 0 | 0 |
| .5 | .958594 | .517034 | .013161 | 3.307733 |
| 1.0 | 0.837851 | 3.965218 | 0.052130 | 1.662054 |
| 1.5 | 0.646511 | 12.497775 | 0.115720 | 1.115660 |
| 2.0 | 0.402580 | 26.388160 | 0.199817 | 0.849346 |
| 2.5 | 0.132636 | 42.431575 | 0.292766 | 0.702564 |
| 2.753100 | 0.000349 | 47.608767 | 0.325830 | 0.666770 |

$n = 1$

$\Xi = 3.142100$

$-\left(\frac{\partial \theta}{\partial \xi}\right)_{\xi=\Xi} = 0.318430$

| ξ | θ | \hat{M} | \hat{I} | $\hat{\Omega}$ |
|----------|----------|-----------|-----------|----------------|
| 0 | 1 | 0 | 0 | 0 |
| .5 | 0.958851 | 0.510625 | 0.010080 | 3.779649 |
| 1.0 | 0.841772 | 3.774032 | 0.039630 | 1.906359 |
| 1.5 | 0.664997 | 11.201527 | 0.086815 | 1.288487 |
| 2.0 | 0.454649 | 21.885479 | 0.146994 | 0.991357 |
| 2.5 | 0.239389 | 32.689292 | 0.211071 | 0.829708 |
| 3.0 | 0.047040 | 39.095204 | 0.257711 | 0.754742 |
| 3.142100 | 0.000189 | 39.478411 | 0.261297 | 0.750121 |

$n = 2$

$\Xi = 4.353100$

$-\left(\frac{\partial\theta}{\partial\xi}\right)_{\xi=\Xi} = 0.127300$

| ξ | θ | \hat{M} | \hat{I} | $\hat{\Omega}$ |
|----------|----------|-----------|-----------|----------------|
| 0 | 1 | 0 | 0 | 0 |
| .5 | 0.959353 | 0.498253 | 0.005227 | 5.248850 |
| 1.0 | 0.848929 | 3.440920 | 0.020269 | 2.666237 |
| 1.5 | 0.695367 | 9.277393 | 0.043458 | 1.823020 |
| 2.0 | 0.529836 | 16.404083 | 0.071768 | 1.422937 |
| 2.5 | 0.374739 | 22.793235 | 0.101328 | 1.204538 |
| 3.0 | 0.241824 | 27.213646 | 0.127545 | 1.083075 |
| 3.5 | 0.133969 | 29.506644 | 0.145944 | 1.022508 |
| 4.0 | 0.048840 | 30.241656 | 0.154024 | 1.001799 |
| 4.353100 | 0.000111 | 30.298098 | 0.154833 | 1.000052 |

$n = 3$

$\Xi = 6.897200$

$-\left(\frac{\partial\theta}{\partial\xi}\right)_{\xi=\Xi} = 0.042440$

| ξ | θ | \hat{M} | \hat{I} | $\hat{\Omega}$ |
|----------|----------|-----------|-----------|----------------|
| 0 | 1 | 0 | 0 | 0 |
| .5 | 0.959839 | 0.486443 | 0.002072 | 8.335976 |
| 1.0 | 0.855310 | 3.160498 | 0.007936 | 4.262277 |
| 1.5 | 0.719502 | 7.914317 | 0.016735 | 2.941765 |
| 2.0 | 0.582851 | 13.143968 | 0.027220 | 2.318066 |
| 2.5 | 0.461127 | 17.590476 | 0.038177 | 1.973579 |
| 3.0 | 0.359227 | 20.815551 | 0.048537 | 1.770052 |
| 3.5 | 0.276263 | 22.913024 | 0.057505 | 1.647475 |
| 4.0 | 0.209282 | 24.161423 | 0.064605 | 1.574866 |
| 4.5 | 0.155069 | 24.840997 | 0.069681 | 1.533989 |
| 5.0 | 0.110900 | 25.171911 | 0.072868 | 1.513033 |
| 5.5 | 0.074353 | 25.309843 | 0.074544 | 1.503789 |
| 6.0 | 0.043794 | 25.353617 | 0.075200 | 1.500692 |
| 6.5 | 0.017914 | 25.361601 | 0.075344 | 1.500099 |
| 6.897200 | 0.000036 | 25.361901 | 0.075350 | 1.500076 |

Constant Factor K

$K \approx 12.56$, based on an average of the values of \hat{M} for $n = 0.5, 1, 2, 3$.

Modelling Earth's Sun

1 White Dwarfs

$$\frac{\partial V}{\partial s}$$

Integrations for Selected Values of $\theta(0)$

Dimensionless Mass

Mass-Radius Relation