Numerical Analysis Project 3

Margaret Dorsey

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Lane Emden Equations

More data can be found in the corresponding .txt files in the outputs directory.

n = .5

$\Xi = 2.753100$
$-\left(\frac{\partial\theta}{\partial\xi}\right)_{\xi=\Xi} = 0.500242$

ξ	θ	$\hat{M}$	$\hat{I}$	$\hat{\Omega}$
0	1	0	0	0
.5	.958594	.517034	.013161	3.307733
1.0	0.837851	3.965218	0.052130	1.662054
1.5	0.646511	12.497775	0.115720	1.115660
2.0	0.402580	26.388160	0.199817	0.849346
2.5	0.132636	42.431575	0.292766	0.702564
2.753100	0.000349	47.608767	0.325830	0.666770

n = 1

$\Xi = 3.142100$
$-\left(\frac{\partial\theta}{\partial\xi}\right)_{\xi=\Xi} = 0.318430$

ξ	θ	$\hat{M}$	$\hat{I}$	$\hat{\Omega}$
0	1	0	0	0
.5	0.958851	0.510625	0.010080	3.779649
1.0	0.841772	3.774032	0.039630	1.906359
1.5	0.664997	11.201527	0.086815	1.288487
2.0	0.454649	21.885479	0.146994	0.991357
2.5	0.239389	32.689292	0.211071	0.829708
3.0	0.047040	39.095204	0.257711	0.754742
3.142100	0.000189	39.478411	0.261297	0.750121

n = 2

$\Xi = 4.353100$
$-\left(\frac{\partial\theta}{\partial\xi}\right)_{\xi=\Xi} = 0.127300$

ξ	θ	$\hat{M}$	$\hat{I}$	$\hat{\Omega}$
0	1	0	0	0
.5	0.959353	0.498253	0.005227	5.248850
1.0	0.848929	3.440920	0.020269	2.666237
1.5	0.695367	9.277393	0.043458	1.823020
2.0	0.529836	16.404083	0.071768	1.422937
2.5	0.374739	22.793235	0.101328	1.204538
3.0	0.241824	27.213646	0.127545	1.083075
3.5	0.133969	29.506644	0.145944	1.022508
4.0	0.048840	30.241656	0.154024	1.001799
4.353100	0.000111	30.298098	0.154833	1.000052

n = 3

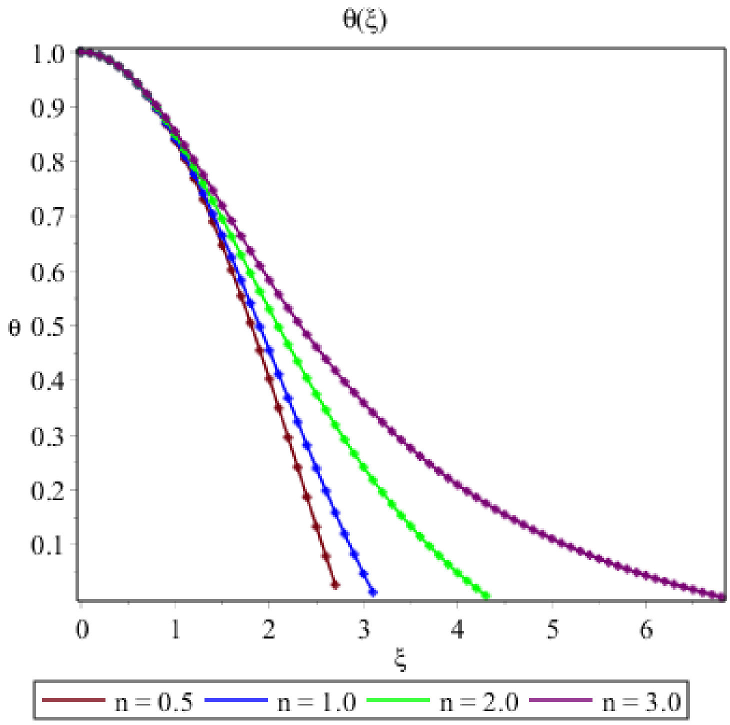
$\Xi = 6.897200$
$-\left(\frac{\partial\theta}{\partial\xi}\right)_{\xi=\Xi} = 0.042440$

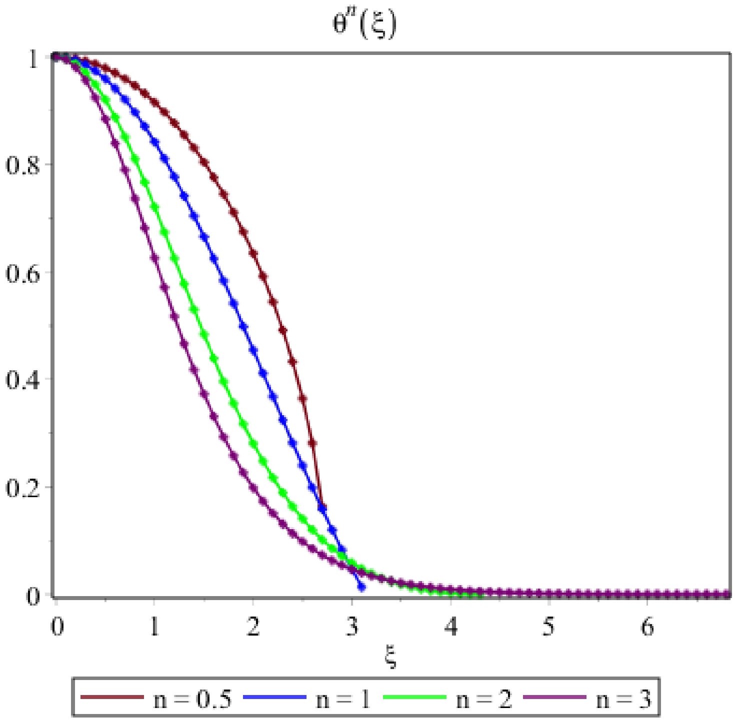
ξ	θ	$\hat{M}$	$\hat{I}$	$\hat{\Omega}$
0	1	0	0	0
.5	0.959839	0.486443	0.002072	8.335976
1.0	0.855310	3.160498	0.007936	4.262277
1.5	0.719502	7.914317	0.016735	2.941765
2.0	0.582851	13.143968	0.027220	2.318066
2.5	0.461127	17.590476	0.038177	1.973579
3.0	0.359227	20.815551	0.048537	1.770052
3.5	0.276263	22.913024	0.057505	1.647475
4.0	0.209282	24.161423	0.064605	1.574866
4.5	0.155069	24.840997	0.069681	1.533989
5.0	0.110900	25.171911	0.072868	1.513033
5.5	0.074353	25.309843	0.074544	1.503789
6.0	0.043794	25.353617	0.075200	1.500692
6.5	0.017914	25.361601	0.075344	1.500099
6.897200	0.000036	25.361901	0.075350	1.500076

Constant Factor K

$K \approx 12.56$, based on an average of the values of $\hat{M}$ for $n = 0.5, 1, 2, 3$.

Plots


θ(ξ)
1.0
0.9
0.8
0.7
0.6
θ 0.5
0.4
0.3
0.2
0.1
0
1
2
3
4
5
6
ξ
n = 0.5
n = 1.0
n = 2.0
n = 3.0


θ^n(ξ)
1
0.8
0.6
0.4
0.2
0
0
1
2
3
4
5
6
ξ
n = 0.5
n = 1
n = 2
n = 3

Modelling Earth's Sun

$$
\begin{aligned}
\rho_{unit} &= \frac{\Xi^3}{\hat{M}} \frac{M_\odot}{R_\odot} \\
&= \frac{6.897200^3}{25.361901} \frac{1.989 \times 10^{30}\mathbf{kg}}{695700\mathbf{km}} \\
&= 3.6987 \times 10^{25} \frac{\mathbf{kg}}{\mathbf{km}} = \rho_c
\end{aligned}
$$

$$
P_c = \kappa \rho_c^{1+1/n}
$$

$$
\begin{aligned}
\kappa &= \frac{4\pi G \alpha^2}{n+1} \rho_c^{n-1/n} \\
\alpha &= \frac{R_\odot}{\Xi} = \frac{695700\mathbf{km}}{6.897} = 1.00869 \times 10^5
\end{aligned}
$$

$$
\begin{aligned}
\kappa &= \frac{4\pi (6.67 \times 10^{-14} \frac{\mathbf{km}}{\mathbf{s^2}})(1.00869 \times 10^5 \mathbf{km})}{4} \rho_c^{2/3} \\
&= 2.113652 \times 10^{-}8 \rho_c^{2/3} \\
&= 2.346387 \times 10^9
\end{aligned}
$$

$$
\begin{aligned}
P_c &= 2.346387 \times 10^9 \rho_c^{4/3} \\
&= 2.891560562 * 10^4 3
\end{aligned}
$$

White Dwarfs

$\frac{\partial V}{\partial s}$

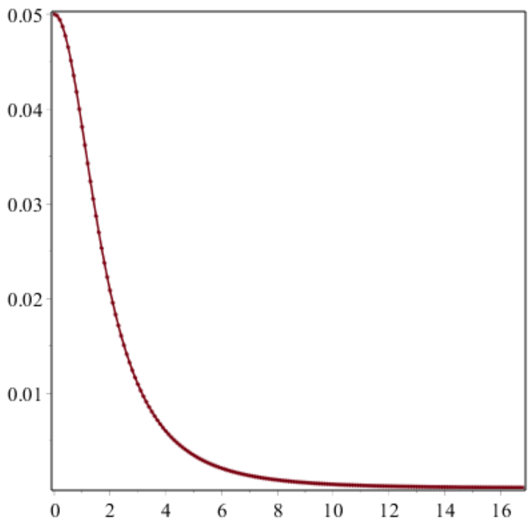
$$\begin{aligned}
\frac{1}{s^2}\frac{d}{ds}\left(s^2 V G(\theta)\right) &= -\theta \\
\frac{1}{s^2}\left[2sVG + s^2\frac{d}{ds}(VG)\right] &= -\theta \\
\frac{1}{s^2}\left[2sVG + s^2\frac{dV}{ds}G + s^2V^2G'(\theta)\right] &= -\theta \\
\frac{2VG}{s} + \frac{dV}{ds}G + V^2G' &= -\theta \\
\frac{dV}{ds} = \frac{2V}{s} - \frac{V^2G'}{G} - \frac{\theta}{G}
\end{aligned}$$

Integrations for Selected Values of $\theta(0)$

Full data outputs are available in the outputs directory.

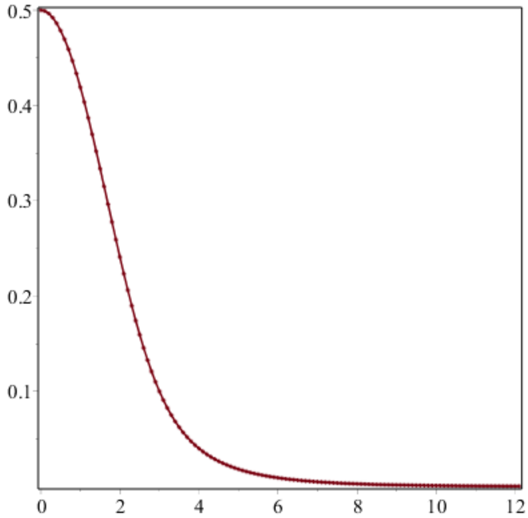
$\theta(0) = .05$

s	θ	$\hat{M}$
0	.05	0
5	0.003474	4.530548
10	0.000394	8.354221
15	0.000080	10.060809
16.8	0.000050	10.421411



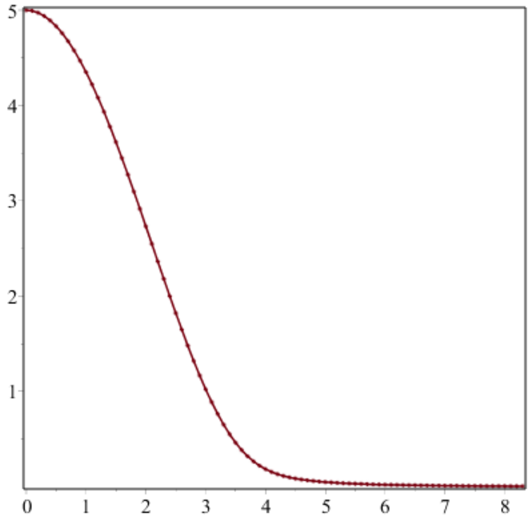
$\theta(0) = .5$

s	θ	$\hat{M}$
0	.5	0
3	0.099983	23.313870
6	0.009359	44.747602
9	0.001821	53.285835
12.1	0.000491	57.345664



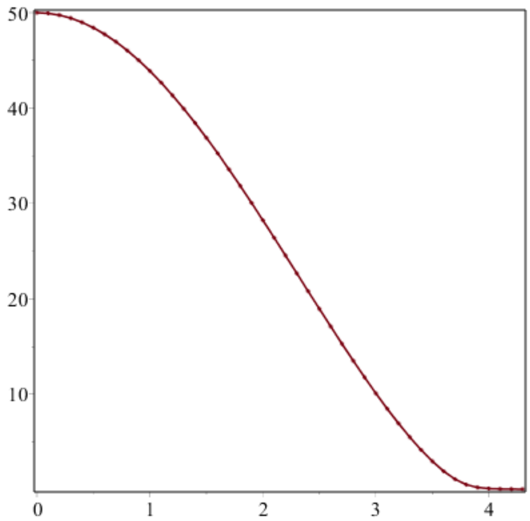
$\theta(0) = 5$

s	θ	$\hat{M}$
0	5	0
2	2.731982	119.423255
4	0.184593	44.747602
6	0.021044	365.125754
8.3	0.004759	379.679845



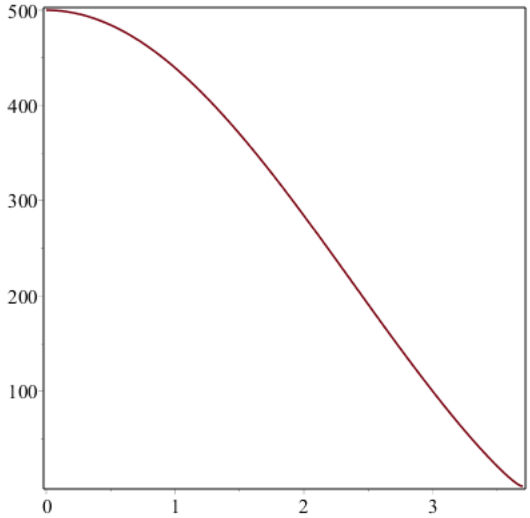
$\theta(0) = 50$

s	θ	$\hat{M}$
0	50	0
1	43.888692	193.326200
2	28.244552	1217.559635
3	10.097859	2635.815212
4.3	0.045212	3135.770481



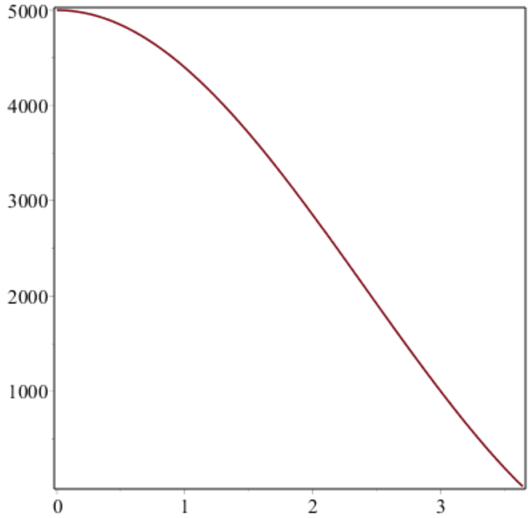
$\theta(0) = 500$

s	θ	$\hat{M}$
0	500	0
1	439.582357	1940.891340
2	284.402935	12224.924810
3	100.083901	26471.219489
3.7	0.274937	30576.933377



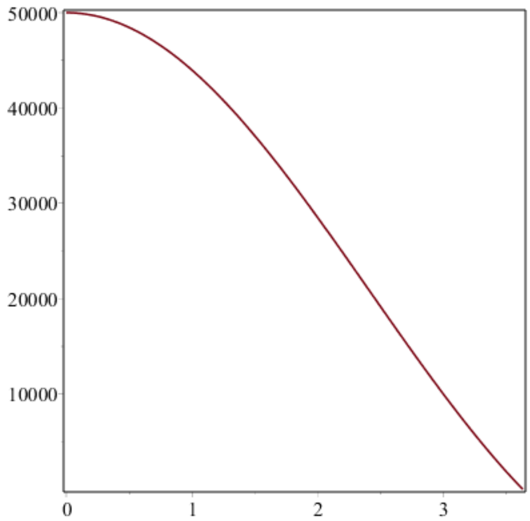
$\theta(0) = 5000$

s	θ	$\hat{M}$
0	5000	0
1	4397.550817	19413.417938
2	2848.169588	122353.877586
3	997.874672	264958.303355
3.64	3.295236	304300.467510



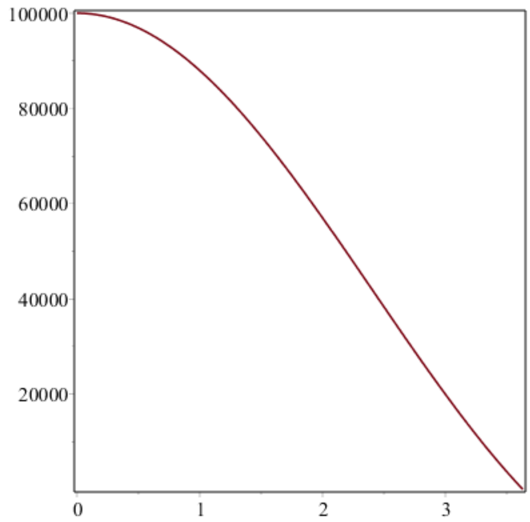
$\theta(0) = 50000$

s	θ	$\hat{M}$
0	50000	0
1	43979.215785	194143.848806
2	28490.570788	1223763.244462
3	9971.996493	2650101.145882
3.63	22.034888	3039898.580706



$\theta(0) = 100000$

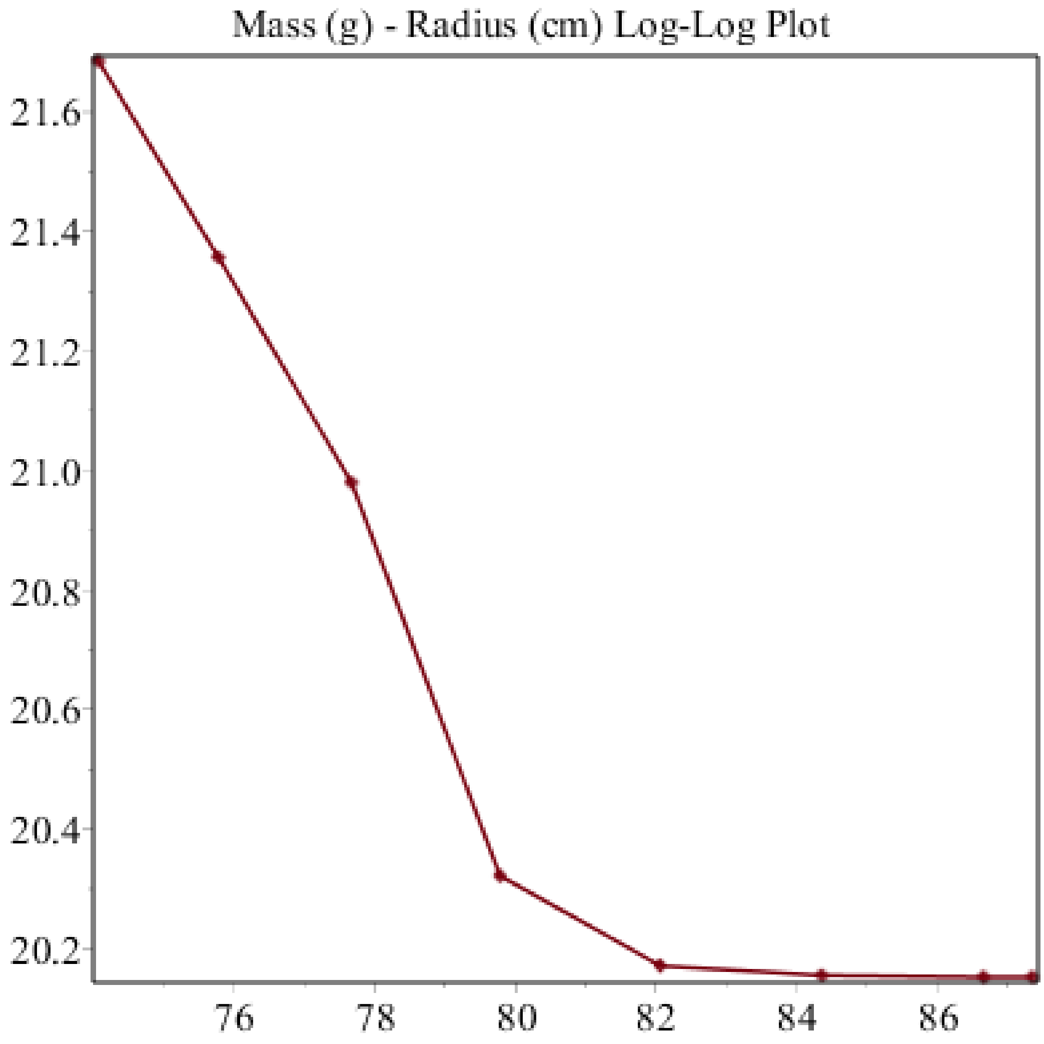
s	θ	$\hat{M}$
0	500000	0
1	87959.184507	388289.661272
2	56982.943351	2447572.068240
3	19942.606275	5300307.090000
3.63	16.402276	6079172.237637



Mass-Radius Relation

Note: The exact values here might be slightly off, due to s_{max} being found with relatively low precision. This should not effect overall behavior.

$\theta(0)$	$\ln(M)$	$\ln(R)$
.05	74.07760638	21.68482052
.5	75.78284118	21.35664709
5	77.67307233	20.97969715
50	79.78437413	20.32205666
500	82.06174514	20.17177446
5000	84.35951482	20.15542532
50000	86.66107865	20.15267429
500000	87.35412304	20.15267429


Mass (g) - Radius (cm) Log-Log Plot
21.6
21.4
21.2
21.0
20.8
20.6
20.4
20.2
76
78
80
82
84
86