Numerical Analysis Homework 2

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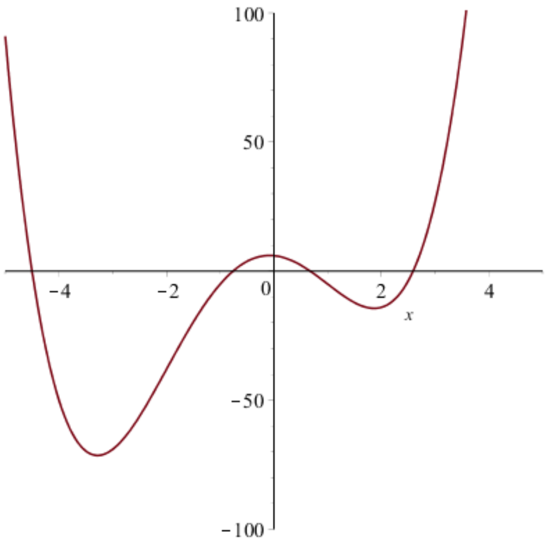
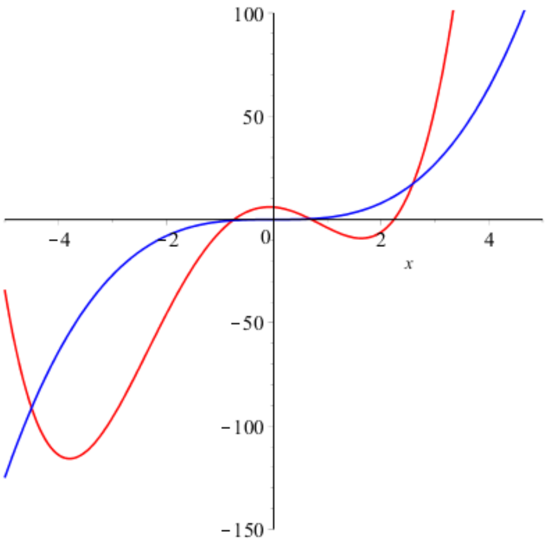
Problem 1

No, a degree 3 polynomial cannot intersect a degree 4 polynomial in exactly 5 points - let $f(x)$ be a degree 3 polynomial, and $g(x)$ be degree 4. Any intersection of f and g must be a root of $f-g$ and $g-f$, which are degree 4 polynomials, and therefore may have at most 4 real roots.

It is, however, possible for f and g to intersect at exactly four points. Consider $f(x)=x^3$, and $g(x)=x^4+3x^3-12x^2-2x+6$. Their difference, $x^4+2x^3-12x^2-2x+6$, is plotted below and clearly has exactly 4 distinct real roots. f and g can intersect only at those roots, and must intersect at them, so thus f and g intersect exactly 4 times, as shown on the second plot.





Problem 2

Using the formula

$$P_n(x) - \sum_{j=1}^{n} y_j \prod_{k=1,k\neq j}^{n} \frac{x-x_k}{x_j-x_k}$$

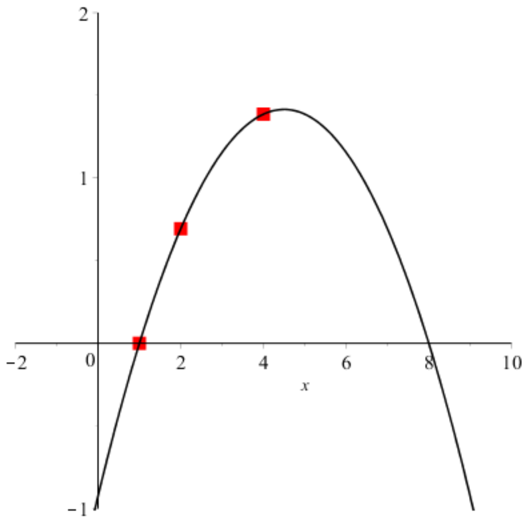
we obtain

$$P_2(x) = \frac{(x-2)(x-4)}{3} \cdot 0 + \frac{(x-1)(x-4)}{-2} \cdot \ln 2 + \frac{(x-1)(x-2)}{6} \cdot \ln 4$$

$$P_2(x) = \frac{-(x-1)(x-4)}{2} \cdot \ln 2 + \frac{(x-1)(x-2)}{3} \cdot \ln 2$$

$$P_2(x) = \frac{-\ln 2}{6} x^2 + \frac{3\ln 2}{2} x - \frac{4\ln 2}{3}$$

The graph of this polynomial fit is shown below.



Using this approximation, we obtain $\ln 3 \approx P_2(3) = \frac{-9\ln 2}{6} + \frac{9\ln 2}{2} - \frac{4\ln 2}{3} = \frac{5}{3}\ln 2 \approx 1.1552453$. Using the formula

$$|f(x) - P(x)| \leq \max_{[2,4]} \left| \frac{f^{(n+1)}(\xi)}{(n+1)!} \right| \cdot \max_{[2,4]} \left| \prod_{i=0}^{n} (x - x_i) \right|$$

with $f(x) = \ln x$, $f^{(3)}(x) = \frac{2}{x^3}$, we obtain

$$|f(x) - P(x)| \leq \max_{[2,4]} \left| \frac{1}{3(\xi)^3} \right| \cdot \max_{[2,4]} \left| x^3 - 7x^2 + 14x - 8 \right|$$

Because $f^{(3)}(\xi)$ is strictly decreasing and non-negative on our interval, we know the maximum on $[2, 4]$ is $\frac{1}{24}$ at 2. Using the derivative of $x^3 - 7x^2 + 14x - 8$ with the quadratic formula, we find that the maximum absolute value on $[2, 4]$ is $-\frac{2}{27}(10 + 7\sqrt{7})$ at $x = \frac{7}{3} + \frac{\sqrt{7}}{3}$, or approximately 2.11261.

Multiplying these values, we obtain $|f(x) - P(x)| \leq .0880255$.

Comparing our value of 1.1552453 to the actual value of $\ln 3 \approx 1.0986123$, we have $|f(x) - P_2(x)| = .056630 < .0880255$, as expected.

Problem 3

Using the formulas $B(0) = P_0$ and $B(1) = P_3$, we obtain $P_0 = (1,1)$ and $P_3 = (9,1)$. We then calculate $B'(t) = (x'(t), y'(t)) = (12t + 6t^2, 3t^2 - 1)$, and note that because in general $B'(t) = 3(1-t)^2(P_1 - P_0) + 6(1-t)t(P_2 - P_1) + 3t^2(P_3 - P_2)$, $B'(0) = 3(P_1 - P_0)$ and $B'(1) = 3(P_3 - P_2)$. Thus we have $P_1 = (1, \frac{2}{3})$ and $P_2 = (3, \frac{1}{3})$. This gives us the complete set of control points for the curve, namely

$$\{(1,1), (1, \frac{2}{3}), (3, \frac{1}{3}), (9,1)\}$$

Problem 4

We first note that

$$\begin{aligned}
A_1 &= \tfrac{x_2 - x}{x_2 - x_1} &= 1 - x \\
A_2 &= \tfrac{x_3 - x}{x_3 - x_2} &= 2 - x \\
A_3 &= \tfrac{x_4 - x}{x_4 - x_3} &= 3 - x
\end{aligned}$$

$$\begin{aligned}
B_1 &= \tfrac{x - x_1}{x_2 - x_1} &= x - 0 \\
B_2 &= \tfrac{x - x_2}{x_3 - x_2} &= x - 1 \\
B_3 &= \tfrac{x - x_3}{x_4 - x_3} &= x - 2
\end{aligned}$$

$$\begin{aligned}
C_1 &= \tfrac{(x_2 - x_1)^2}{6}[A_1^3 - A_1] &= \tfrac{1}{6}(-x^3 + 3x^2 - 2x) \\
C_2 &= \tfrac{(x_3 - x_2)^2}{6}[A_2^3 - A_2] &= \tfrac{1}{6}(-x^3 + 6x^2 - 11x + 6) \\
C_3 &= \tfrac{(x_4 - x_3)^2}{6}[A_3^3 - A_3] &= \tfrac{1}{6}(-x^3 + 9x^2 - 26x + 24)
\end{aligned}$$

$$\begin{aligned}
D_1 &= \tfrac{(x_2 - x_1)^2}{6}[B_1^3 - B_1] &= \tfrac{1}{6}(x^3 - x) \\
D_2 &= \tfrac{(x_3 - x_2)^2}{6}[B_2^3 - B_2] &= \tfrac{1}{6}(x^3 - 3x^2 + 2x) \\
D_3 &= \tfrac{(x_4 - x_3)^2}{6}[B_3^3 - B_3] &= \tfrac{1}{6}(x^3 - 6x^2 + 11x - 6)
\end{aligned}$$

Natural Endpoint Condition

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ x_2 - x_1 & 2(x_3 - x_1) & x_3 - x_2 & 0 \\ 0 & x_3 - x_2 & 2(x_4 - x_2) & x_4 - x_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_1'' \\ y_2'' \\ y_3'' \\ y_4'' \end{bmatrix} = \begin{bmatrix} 0 \\ 6\left(\frac{y_3 - y_2}{x_3 - x_2} - \frac{y_2 - y_1}{x_2 - x_1}\right) \\ 6\left(\frac{y_4 - y_3}{x_4 - x_3} - \frac{y_3 - y_2}{x_3 - x_2}\right) \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 - 0 & 2(2 - 0) & 2 - 1 & 0 \\ 0 & 2 - 1 & 2(3 - 1) & 3 - 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_1'' \\ y_2'' \\ y_3'' \\ y_4'' \end{bmatrix} = \begin{bmatrix} 0 \\ 6\left(\frac{2-5}{2-1} - \frac{5-3}{1-0}\right) \\ 6\left(\frac{1-2}{3-2} - \frac{2-5}{2-1}\right) \\ 0 \end{bmatrix}$$

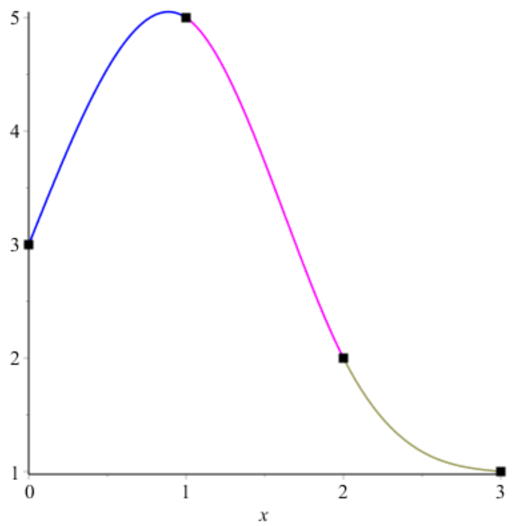
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 4 & 1 & 0 \\ 0 & 1 & 4 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_1'' \\ y_2'' \\ y_3'' \\ y_4'' \end{bmatrix} = \begin{bmatrix} 0 \\ -30 \\ 12 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} y_1'' \\ y_2'' \\ y_3'' \\ y_4'' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\frac{4}{15} & \frac{4}{15} & -\frac{1}{15} & \frac{1}{15} \\ \frac{1}{15} & -\frac{1}{15} & \frac{4}{15} & -\frac{4}{15} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -30 \\ 12 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} y_1'' \\ y_2'' \\ y_3'' \\ y_4'' \end{bmatrix} = \begin{bmatrix} 0 \\ -\frac{44}{5} \\ \frac{26}{5} \\ 0 \end{bmatrix}$$

Using these derived values of y_i'' with our A,B,C, and D equations above, we have

$$\begin{aligned} y_1^{(cubic)} &= y_1A_1 + y_2B_1 + y_1''C_1 + y_2''D_1 &= -\tfrac{22}{15}x^3 + \tfrac{52}{15}x + 3 \\ y_2^{(cubic)} &= y_2A_2 + y_3B_2 + y_2''C_2 + y_3''D_2 &= \tfrac{7}{3}x^3 - \tfrac{57}{5}x^2 + \tfrac{223}{15}x - \tfrac{4}{5} \\ y_3^{(cubic)} &= y_3A_3 + y_4B_3 + y_3''C_3 + y_4''D_3 &= -\tfrac{13}{15}x^3 + \tfrac{39}{5}x^2 - \tfrac{353}{15}x + \tfrac{124}{5} \end{aligned}$$



Curvature-Adjusted Endpoint Condition

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ x_2 - x_1 & 2(x_3 - x_1) & x_3 - x_2 & 0 \\ 0 & x_3 - x_2 & 2(x_4 - x_2) & x_4 - x_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_1'' \\ y_2'' \\ y_3'' \\ y_4'' \end{bmatrix} = \begin{bmatrix} k_1 \\ 6\left(\frac{y_3-y_2}{x_3-x_2} - \frac{y_2-y_1}{x_2-x_1}\right) \\ 6\left(\frac{y_4-y_3}{x_4-x_3} - \frac{y_3-y_2}{x_3-x_2}\right) \\ k_n \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1-0 & 2(2-0) & 2-1 & 0 \\ 0 & 2-1 & 2(3-1) & 3-2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_1'' \\ y_2'' \\ y_3'' \\ y_4'' \end{bmatrix} = \begin{bmatrix} k_n \\ 6\left(\frac{2-5}{2-1} - \frac{5-3}{1-0}\right) \\ 6\left(\frac{1-2}{3-2} - \frac{2-5}{2-1}\right) \\ k_n \end{bmatrix}$$

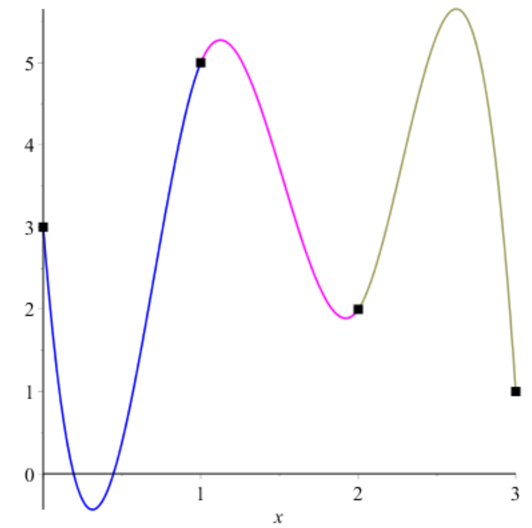
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 4 & 1 & 0 \\ 0 & 1 & 4 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_1'' \\ y_2'' \\ y_3'' \\ y_4'' \end{bmatrix} = \begin{bmatrix} k_1 \\ -30 \\ 12 \\ k_n \end{bmatrix}$$

Choosing $k_1 = 100$, $k_2 = -100$, we have

$$\begin{bmatrix} y_1'' \\ y_2'' \\ y_3'' \\ y_4'' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\frac{4}{15} & \frac{4}{15} & -\frac{1}{15} & \frac{1}{15} \\ \frac{1}{15} & -\frac{1}{15} & \frac{4}{15} & -\frac{4}{15} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 100 \\ -30 \\ 12 \\ -100 \end{bmatrix}$$

$$\begin{bmatrix} y_1'' \\ y_2'' \\ y_3'' \\ y_4'' \end{bmatrix} = \begin{bmatrix} 100 \\ -\frac{632}{15} \\ \frac{578}{15} \\ -100 \end{bmatrix}$$

$$\begin{aligned} y_1^{(cubic)} &= -\frac{1066}{45} + 50x^2 - \frac{1094}{45}x + 3 \\ y_2^{(cubic)} &= \frac{121}{9}x^3 - \frac{307}{5}x^2 + \frac{3919}{45}x - \frac{512}{15} \\ y_3^{(cubic)} &= -\frac{1039}{45} + \frac{789}{5}x^2 - \frac{15809}{45}x + \frac{3872}{15} \end{aligned}$$



Clamped Endpoint Condition

$$\begin{bmatrix} 2 & 1 & 0 & 0 \\ x_2-x_1 & 2(x_3-x_1) & x_3-x_2 & 0 \\ 0 & x_3-x_2 & 2(x_4-x_2) & x_4-x_3 \\ 0 & 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} y_1'' \\ y_2'' \\ y_3'' \\ y_4'' \end{bmatrix} = \begin{bmatrix} 6\left(\frac{y_2-y_1}{x_2-x_1}-c_n\right) \\ 6\left(\frac{y_3-y_2}{x_3-x_2}-\frac{y_2-y_1}{x_2-x_1}\right) \\ 6\left(\frac{y_4-y_3}{x_4-x_3}-\frac{y_3-y_2}{x_3-x_2}\right) \\ 6\left(c_n-\frac{y_4-y_3}{x_4-x_3}\right) \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 & 0 & 0 \\ 1-0 & 2(2-0) & 2-1 & 0 \\ 0 & 2-1 & 2(3-1) & 3-2 \\ 0 & 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} y_1'' \\ y_2'' \\ y_3'' \\ y_4'' \end{bmatrix} = \begin{bmatrix} 6\left(\frac{5-3}{1-0}-c_n\right) \\ 6\left(\frac{2-5}{2-1}-\frac{5-3}{1-0}\right) \\ 6\left(\frac{1-2}{3-2}-\frac{2-5}{2-1}\right) \\ 6\left(c_n-\frac{1-2}{3-2}\right) \end{bmatrix}$$

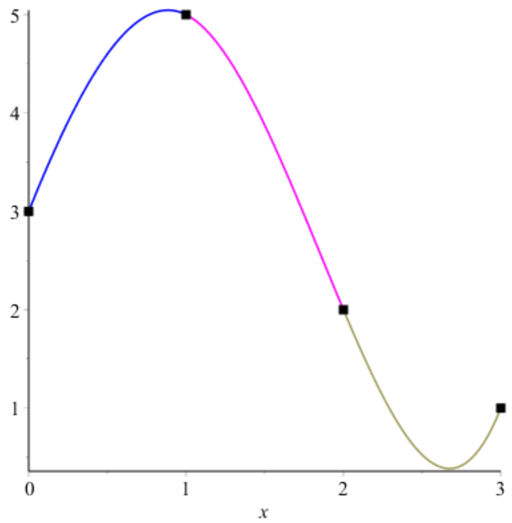
$$\begin{bmatrix} 2 & 1 & 0 & 0 \\ 1 & 4 & 1 & 0 \\ 0 & 1 & 4 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} y_1'' \\ y_2'' \\ y_3'' \\ y_4'' \end{bmatrix} = \begin{bmatrix} 12-6c_n \\ -30 \\ 12 \\ 6(c_n+1) \end{bmatrix}$$

Choosing $c_n = 4$, we have

$$\begin{bmatrix} y_1'' \\ y_2'' \\ y_3'' \\ y_4'' \end{bmatrix} = \frac{1}{45} \begin{bmatrix} 26 & -7 & 2 & -1 \\ -7 & 14 & -4 & 2 \\ 2 & -4 & 14 & -7 \\ -1 & 2 & -7 & 26 \end{bmatrix} \begin{bmatrix} -12 \\ -30 \\ 12 \\ 30 \end{bmatrix}$$

$$\begin{bmatrix} y_1'' \\ y_2'' \\ y_3'' \\ y_4'' \end{bmatrix} = \begin{bmatrix} -\frac{12}{5} \\ -\frac{36}{5} \\ \frac{6}{5} \\ \frac{72}{5} \end{bmatrix}$$

$$\begin{aligned} y_1^{(cubic)} &= -\tfrac{4}{5}x^3 - \tfrac{6}{5}x^2 + 4x + 3 \\ y_2^{(cubic)} &= \tfrac{7}{5}x^3 - \tfrac{39}{5}x^2 + \tfrac{53}{5}x + \tfrac{4}{5} \\ y_3^{(cubic)} &= -\tfrac{11}{5}x^3 - \tfrac{63}{5}x^2 + \tfrac{101}{5}x - \tfrac{28}{5} \end{aligned}$$



Parabolically Terminated Endpoint Condition

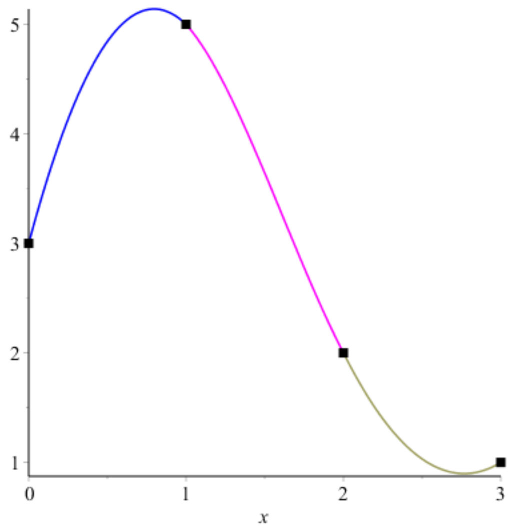
$$\begin{bmatrix} 1 & -1 & 0 & 0 \\ x_2 - x_1 & 2(x_3 - x_1) & x_3 - x_2 & 0 \\ 0 & x_3 - x_2 & 2(x_4 - x_2) & x_4 - x_3 \\ 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} y_1'' \\ y_2'' \\ y_3'' \\ y_4'' \end{bmatrix} = \begin{bmatrix} 0 \\ 6\left(\frac{y_3 - y_2}{x_3 - x_2} - \frac{y_2 - y_1}{x_2 - x_1}\right) \\ 6\left(\frac{y_4 - y_3}{x_4 - x_3} - \frac{y_3 - y_2}{x_3 - x_2}\right) \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & 4 & 1 & 0 \\ 0 & 1 & 4 & 1 \\ 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} y_1'' \\ y_2'' \\ y_3'' \\ y_4'' \end{bmatrix} = \begin{bmatrix} 0 \\ -30 \\ 12 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} y_1'' \\ y_2'' \\ y_3'' \\ y_4'' \end{bmatrix} = \frac{1}{24} \begin{bmatrix} 19 & 5 & -1 & 1 \\ -5 & 5 & -1 & 1 \\ 1 & -1 & 5 & -5 \\ 1 & -1 & 5 & 19 \end{bmatrix} \begin{bmatrix} 0 \\ -30 \\ 12 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} y_1'' \\ y_2'' \\ y_3'' \\ y_4'' \end{bmatrix} = \begin{bmatrix} -\frac{27}{4} \\ -\frac{27}{4} \\ \frac{15}{4} \\ \frac{15}{4} \end{bmatrix}$$

$$\begin{aligned} y_1^{(cubic)} &= -\tfrac{27}{8}x^2 + \tfrac{43}{8}x + 3 \\ y_2^{(cubic)} &= \tfrac{13}{8}x^3 - \tfrac{63}{8}x^2 + \tfrac{37}{4}x + 2 \\ y_3^{(cubic)} &= \tfrac{15}{8}x^2 - \tfrac{83}{8}x - \tfrac{61}{4} \end{aligned}$$



Not-a-Knot Endpoint Condition

$$\begin{bmatrix} x_3-x_2 & -(x_3-x_1) & x_2-x_1 & 0 \\ x_2-x_1 & 2(x_3-x_1) & x_3-x_2 & 0 \\ 0 & x_3-x_2 & 2(x_4-x_2) & x_4-x_3 \\ 0 & x_4-x_3 & -(x_4-x_2) & x_3-x_2 \end{bmatrix} \begin{bmatrix} y_1'' \\ y_2'' \\ y_3'' \\ y_4'' \end{bmatrix} = \begin{bmatrix} 0 \\ 6\left(\frac{y_3-y_2}{x_3-x_2}-\frac{y_2-y_1}{x_2-x_1}\right) \\ 6\left(\frac{y_4-y_3}{x_4-x_3}-\frac{y_3-y_2}{x_3-x_2}\right) \\ 0 \end{bmatrix}$$

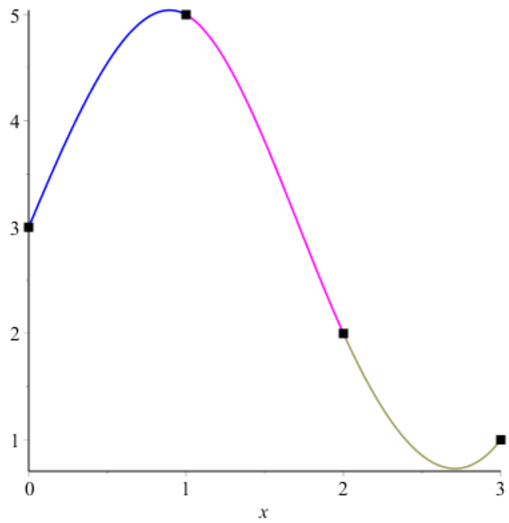
$$\begin{bmatrix} -3 & 1 & 2 & 0 \\ 1-0 & 2(2-0) & 2-1 & 0 \\ 0 & 2-1 & 2(3-1) & 3-2 \\ 0 & -1 & 4 & -3 \end{bmatrix} \begin{bmatrix} y_1'' \\ y_2'' \\ y_3'' \\ y_4'' \end{bmatrix} = \begin{bmatrix} 0 \\ 6\left(\frac{2-5}{2-1}-\frac{5-3}{1-0}\right) \\ 6\left(\frac{1-2}{3-2}-\frac{2-5}{2-1}\right) \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -3 & 1 & 2 & 0 \\ 1 & 4 & 1 & 0 \\ 0 & 1 & 4 & 1 \\ 0 & -1 & 4 & -3 \end{bmatrix} \begin{bmatrix} y_1'' \\ y_2'' \\ y_3'' \\ y_4'' \end{bmatrix} = \begin{bmatrix} 0 \\ -30 \\ 12 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} y_1'' \\ y_2'' \\ y_3'' \\ y_4'' \end{bmatrix} = \frac{1}{198} \begin{bmatrix} -62 & 12 & 21 & 7 \\ 16 & 48 & -15 & -5 \\ -2 & -6 & 39 & 13 \\ -8 & -24 & 57 & -47 \end{bmatrix} \begin{bmatrix} 0 \\ -30 \\ 12 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} y_1'' \\ y_2'' \\ y_3'' \\ y_4'' \end{bmatrix} = \begin{bmatrix} -\frac{6}{11} \\ -\frac{90}{11} \\ \frac{36}{11} \\ \frac{78}{11} \end{bmatrix}$$

$$
\begin{aligned}
y_1^{(cubic)} &= -\tfrac{14}{11}x^3 - \tfrac{3}{11}x^2 + \tfrac{39}{11}x + 3 \\
y_2^{(cubic)} &= \tfrac{21}{11}x^3 - \tfrac{108}{11}x^2 + \tfrac{144}{11}x - \tfrac{2}{11} \\
y_3^{(cubic)} &= \tfrac{7}{11}x^3 - \tfrac{24}{11}x^2 - \tfrac{24}{11}x + 10
\end{aligned}
$$



Problem 5

i.)

ii.)

iii.)

Problem 6

i.)

ii.)

iii.)

Problem 7

$T_{10}(x)$

n	$a_{T'_{10}}$	$a_{T''_{10}}$	$a_{\int T_{10}}$
0	0	500	0
1	20	0	0
2	0	960	0
3	20	0	0
4	0	840	0
5	20	0	0
6	0	640	0
7	20	0	0
8	0	360	0
9	20	0	$\frac{1}{18}$
10	0	0	0
11	0	0	$\frac{1}{22}$

$T_{15}(x)$

n	$a_{T'_{15}}$	$a_{T''_{15}}$	$a_{\int T_{15}}$
0	15	0	0
1	0	3360	0
2	30	0	0
3	0	3240	0
4	30	0	0
5	0	3000	0
6	30	0	0
7	0	2640	0
8	30	0	0
9	0	2160	0
10	30	0	0
11	0	1560	0
12	30	0	0
13	0	840	0
14	30	0	$\frac{1}{28}$
15	0	0	0
16	0	0	$\frac{1}{32}$

$T_{20}(x)$

n	$a_{T'_{10}}$	$a_{T''_{10}}$	$a_{\int T_{10}}$
0	0	4000	0
1	40	0	0
2	0	7920	0
3	40	0	0
4	0	7680	0
5	40	0	0
6	0	7280	0
7	40	0	0
8	0	6720	0
9	40	0	0
10	0	6000	0
11	40	0	0
12	0	5120	0
13	40	0	0
14	0	4080	0
15	40	0	0
16	0	2880	0
17	40	0	0
18	0	1520	0
19	40	0	$\frac{1}{38}$
20	0	0	0
21	0	0	$\frac{1}{42}$

Analysis

The tables above list the coefficients a_n of T_n for all relevant n for the first two derivatives and integral of each T_n. The patterns in the data can be explained by

$$T_n(x) = \frac{1}{2}\left(\frac{1}{n+1}T'_{n+1}(x) - \frac{1}{n-1}T'_{n-1}(x)\right)$$

which gives us

$$\int T_n(x) = \frac{1}{2}\left(\frac{1}{n+1}T_{n+1}(x) - \frac{1}{n-1}T_{n-1}(x)\right)$$

and

$$T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x)$$

which gives

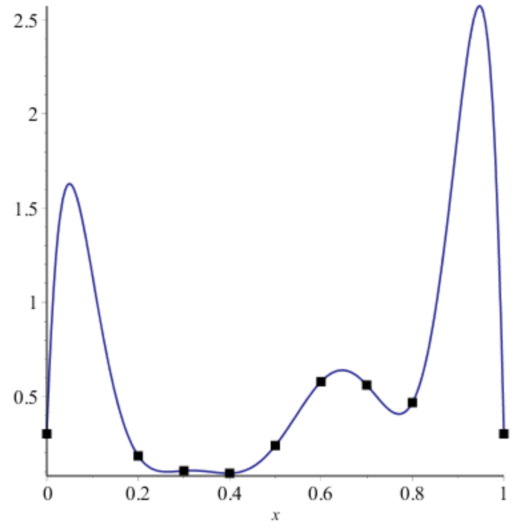
$$T'_n(x) = \frac{1}{1-x^2}\left(-nxT_n(x) + nT_{n-1}(x)\right)$$

which we can of course take the derivative of to obtain higher order derivatives of T_n.

Problem 8

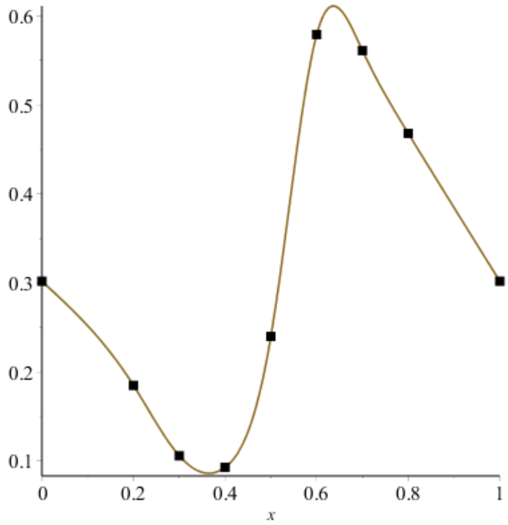
Lagrange Interpolation

$L_8(x) = -10717.01390x^8 + 41523.43743x^7 - 66130.60764x^6 + 55973.20311x^5 - 27150.02639x^4 + 7547.542702x^3 - 1112.916584x^2 + 66.38116670x + .302$



Cubic Spline Interpolation

$$f(x) = \begin{cases} .302 - .473x - 2.802x^3 & x < 0.2 \\ .12974 + 2.11093x - 12.91926x^2 + 18.72996x^3 & x < 0.3 \\ .47410379 - 1.3326681x - 1.44057x^2 + 5.975866x^3 & x < 0.4 \\ -2.430901 + 20.45487x - 55.90942x^2 + 51.36657x^3 & x < 0.5 \\ 26.420189 - 152.651673x + 290.3036659x^2 - 179.442153x^3 & x < 0.6 \\ -37.6981566 + 167.940056x - 244.015883x^2 + 117.40204x^3 & x < 0.7 \\ 5.37168 - 16.644978x + 19.67702x^2 - 8.1660098x^3 & x < 0.8 \\ 1.257759 - 1.2177579x + .39299789x^2 - .130999x^3 & x \geq .8 \end{cases}$$



Analysis

The cubic spline interpolation produces a much more convincing interpolation between the data points - the Lagrange polynomial suffers from very erratic behavior at the endpoints (similar to Runge's phenomenon), and indeed $f(.1) = 0.251906420711722$ is a much more consistent value with the data than $L_8(.1) = 1.14113749$.