Numerical Analysis Project 2

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Justification of Method

I elected to use cubic splines due to the discrete nature of the data set - because we have no guarantee of consistent, polynomial-like behavior at the data points, it is likely that a single interpolating polynomial would suffer from erratic behavior in between points to accommodate inconsistencies. Although Bezier Curves would likely provide a smooth, appealing interpolation of the data, because they do not necessarily agree with the data points, we would have less fine control over their behavior which is problematic for precise determination of ideal lighting. With cubic splines, we can make adjustments solely through manipulation of the number, spacing, etc of data points without the need to also tweak control points to get expected behavior.

Additionally, to find y values corresponding to a given x value on a Bezier curve, we would need to find the roots of $x = (1-t)^3 * x_0 + 3 * (1-t)^2 * t * x_1 + 3 * (1-t) * t^2 * x_2$, giving us the t corresponding to that x value, and then use that to calculate $y(t)$. Although Newton's method is fast and reliable for cubic equations, it is preferable to not do more work.

Computation of Splines

Raw Output for Data Set 1

```
Coefficients for Data Set 1

0.000000 0.892633 0.923469 1.172533 0.000000

 Values of Cubic Interpolant for Data Set 1
(-0.952381, -0.404201)
(-0.857143, -0.487113)
(-0.761905, -0.567712)
(-0.666667, -0.644455)
(-0.571429, -0.715801)
(-0.476190, -0.780210)
(-0.380952, -0.836615)
(-0.285714, -0.884856)
```

```
(-0.190476, -0.924882)
(-0.095238, -0.956637)
(0.000000, -0.980070)
(0.095238, -0.995064)
(0.190476, -1.001251)
(0.285714, -0.998202)
(0.380952, -0.985485)
(0.476190, -0.962672)
(0.571429, -0.929503)
(0.666667, -0.887225)
(0.761905, -0.837856)
(0.857143, -0.783424)
(0.952381, -0.725952)
```

Raw Output for Data Set 2

```
Coefficients for Data Set 2

0.000000 8.444919 -11.663674 8.724819 0.000000

 Values of Cubic Interpolant for Data Set 2
(-0.952381, -0.433899)
(-0.857143, -0.569682)
(-0.761905, -0.683580)
(-0.666667, -0.761003)
(-0.571429, -0.787360)
(-0.476190, -0.748191)
(-0.380952, -0.644578)
(-0.285714, -0.507793)
(-0.190476, -0.372577)
(-0.095238, -0.273673)
(0.000000, -0.245820)
(0.095238, -0.312099)
(0.190476, -0.448947)
(0.285714, -0.621138)
(0.380952, -0.793448)
(0.476190, -0.930652)
(0.571429, -1.001062)
(0.666667, -1.003772)
(0.761905, -0.953724)
(0.857143, -0.865992)
(0.952381, -0.755650)
```

Plots of Output

Data Set 1

Data Set 2

$1 - x^2$

$\sqrt{5 - x^2}$

RMS Error Table

Interpolated Function	RMS Error
$-\cos(x - 0.2)$	
$1 - x^2$	
$\sqrt{5 - x^2}$	

Error Bounds

Accuracy of Derivative Values