~~Shantanu Mapare~~ Q. write the following stmt in set Builder form.

Cardinality of a set

- Cardinality of a set S, denoted by $|S|$, is the number of elements of the set.
- The number is also refered as the Cardinal number
- If a set has an infinite No. of elements, its cardinality is ∞.

Example:- $|\{1,4,3,5\}| = 4$ $|\{1,2,3,4,5\ldots\}| = \infty$

$|X|=|Y|$, $|X| \leq |Y|$ $|X|<|Y|$ $|X \leq Y|$ & $|X \geq Y|$ then $|X|=|Y|$

* Set operation :- 1) Set Union :-

- 1) The union of set A and B ($A \cup B$) is the set of elements which are in A, in B or in both A and B. Here $A \cup B = \{x / x \in A \text{ OR } x \in B\}$.

2) Intersection ($\cap$) :- which are in both A and B.

$A \cap B = \{x / x \in A \text{ AND } x \in B\}$.

3) Set difference ($A-B$) :- The set difference of set A and B ($A-B$) is the set of elements which are only in A but not in B. Here $A-B = \{x / x \in A \text{ AND } x \notin B\}$.

$A = \{10, 11, 12, 13\}$ and $B = \{13, 14, 15\}$

$A-B = \{10, 11, 12\}$

$B-A = \{14, 15\}$.

③ Complement of set :- (A') :- The complement of a set (denoted by A') is the set of elements which are not in set A. Hence $A' = \{x / x \notin A\}$.

Eg $A = \{x | x$ belongs to set of odd integers, then $A' = \{y | y$ does not belong to set of odd integers.

⑤ Cartesian Product / Cross product :-

if we take two sets $A = \{(a, b)\}$ $B = \{1, 2\}$.

$A \times B = \{(a,1), (a,2) (b,1) (b,2)\}$.

$B \times A = \{(1,a) (1,b)$

⑥ Power Set :- Power set of a set S is the set of all subsets of S including the empty set. The cardinality of a power set of a set S of cardinality n is 2^n. power set is denoted as P(S).

Eg for a set $S = \{a, b, c, d\}$. Let us calculate the subset.

Subsets with 0 elements = $\{\emptyset\}$ (The empty set).

Subset with 1 elem - $\{a\}, \{b\}, \{c\}, \{d\}$.

subset with 2 elem = $\{a,b\}$ $\{a,c\}$ $\{a,d\}$ $\{b,c\}$ $\{b,d\}$ $\{c,d\}$.

Subset with 3 elements = $\{a,b,c\}$ $\{a,b,d\}$ $\{a,c,d\}$ $\{b,c,d\}$.

— " — 4 elem = $\{a,b,c,d\}$.

Hence $P(S) = 2^4 = 16$

Note: The power set of an empty set also an empty set =

$S = \{\emptyset\}$ $P(S) = 2^n = 2^0 = 1$

empty set or Null set :- An empty set contains no elements It denoted by ϕ. As the No. of elements in an empty set is finite, empty set is a finite set.

The cardinality of empty set or null set is zero.

Ex $S = \{x / x \in N \text{ and } 7 < x < 8\} = \phi$

Domain & Range

Relations may exists between objects of the same set or between objects of two or more sets.

Definition & properties :-

A binary relation R from set x to y (written as xRy or R(x,y)) is a subset of cartesian product $x \times y$

Q.1) What will be the cardinality of power set of $\{0, 1, 2, \ldots 5\}$. Ans = 64.

Q.2) Find the power set of $Z = \{2, 7, 9\}$ & total No. of elements. $2^3 = 8$.

~~(x7)(9,2)∈7.~~

Domain & Range

Q1 $A = \{1, 2, 9\}$ & $B = \{1, 3, 7\}$.

Case 1 :- If relation R is "equal to" then

$R = \{(1,1), (3,3)\}$.

Domain (R) $= \{1, 3\}$ Range (R) $= \{1, 3\}$

Case 2 :- If relation R is "less than" then

$R = \{(1,3)\ (1,7)\ (2,3)\ (2,7)\}$.

Dom R $= \{1, 2\}$ Ran (R) $= \{3, 7\}$.

Case 3 :- If relation R is "greater than" then

$R = \{(2,1)\ (9,1)\ (9,3)\ (9,7)\}$.

Dom (R) $= \{2, 9\}$ Range (R) $= \{1, 3, 7\}$.

* Relation :-

- In a simple terms, relation is a set of rule on two sets A and B.

- A relation R from A to B is defined as a subset of A × B.

Eg A = {1, 2, 5} B = {2, 4}

then A × B = {(1,2) (1,4) (2,2) (2,4) (5,2) (5,4)}

i) Relation $x < y$.

R = {(1,2) (1,4) (2,4)}.

ii) Relation $x > y$.

iii) Relation $x = y$. R = {2, 2}.

Note : suppose R is a relation from A to B.

(1) R is a set of order pair (a,b) where a ∈ A, b ∈ A

(2) every such order pair is written as aRb and read as 'a' is related to 'b' by R

(3) R is called Binary relation

(4) Total Number of distinct relation from a set A to a set B is 2^{mn} where n(A) = m and n(B) = n

* Domain & Rang I

* The set $\{a \in A : (a,b) \in R \text{ for some } b \in B\}$ is called domain of R and denoted by dom(R)
* The set $\{b \in B : (a,b) \in R \text{ for some } a \in A\}$ is called rang of R and denoted by Ran(R)

Ex let $A = \{2,3,4\}$ and $B = \{3,4,5\}$. find the elements of each relation R defined below and also find domain & range.

(a) $a \in A$ is related to $b \in B$, that is aRb if and only if $a < b$.

(b) $a \in A$ is related to $b \in B$, that is aRb if a and b are odd numbers.

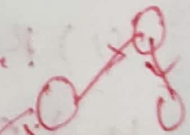
Soln Given $A = \{2,3,4\}$ $B = \{3,4,5\}$.

$A \times B = \{(2,3)(2,4)(2,5)(3,3)(3,4)(3,5)(4,3)(4,4)(4,5)\}$.

$dom(R) = \{2,3,4\}$; $Rang(R) = \{3,4,5\}$.

Representation of Relation using graph:

$R = \{(1,1)(1,2)(3,2)\}$ $S = \{1,2,3\}$.



Types of Relation :-

- The empty relation bet^n x and y is the empty set $\emptyset$
- Full Relation bet^n set x and y is the set $x \times y$
- The identity Relation on set x is the set $\{(x,x) / x \in X\}$.
- The inverse Relation R^{-1} of Relation R is defined as $R^{-1} = \{(b,a) / (a,b) \in R\}$

$R = \{(1,2)(2,3)$ then R' will be $\{(2,1)(3,2)\}$.

* Reflexive : A Relation R on set A is called reflexive if all $a \in A$ is related to a (aRa holds).

ex The Relation $R = \{(a,a),(b,b)\}$ on set $X = \{a,b\}$ is reflexive.

$(a,a) \in R$

* Irreflexive : —"— irreflexive if no $a \in A$ is related to a (aRa does not hold)

ex The relation $R = \{(a,b)(b,a)\}$ on set $X = \{a,b\}$ is irreflexive.

* Symmetric :- —"— symmetric if xRy implies yRx, $\forall x \in A$ and $\forall y \in A$.

$(1,2)(2,1)$

ex : The Relation $R = \{(1,2)(2,1)(3,2)(2,3)\}$ on set $A = \{1,2,3\}$. is symmetric.

* Anti-symmetric :- —"— Anti-symmetric if xRy and yRx implies $x = y$ for all $x \in A$ and $\forall y \in A$.

ex The Relation $R = \{(x,y) \to N / x \leq y\}$ is anti-symmetric since $x \leq y$ and $y \leq x \to x = y$

* ~~The Relation $R = \{(x,y) \to N / x \leq y\}$~~

* Transitive :- A Relation R on set A is called Transitive if xRy and yRz implies xRz for all $x, y, z \in A$.

— A relation is an equivalance Relation if it is Reflexive symmetric & Transitive

$R = \{(1,1)(2,2)(3,3)(1,2)(2,1)(2,3)(3,2)(1,3)(3,1)\}$ on set $A = \{1,2,3\}$ is an equivalance relation since it is reflexive, symmetric & Transitive.

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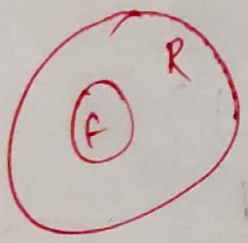
Function: A Relation 'f' from a set A to a set B is called function if to each element $a \in A$, we can assign unique element.

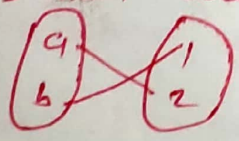
$f: A \to B$



Ex: $A = \{a, b\}$ $B = \{(1, 2)\}$

$A \times B = \{(a,1)(a,2)(b,1)(b,2)\}$.

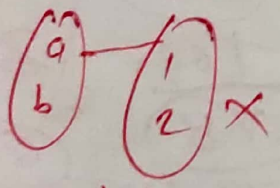
Domain Codomain / Range.



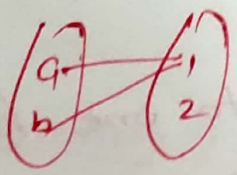
$(a,2)$
$(b,1)$.

$(a,1)$
$(b,2)$

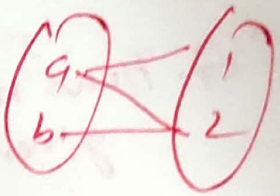
A B.



Not function



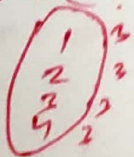
$(a,1)$
$(b,1)$



$(a,1)(a,2)(b,2)$

- How many functions possible?

n^m ; $3^4 = 81$ $\quad 2^{mn} - n^m$



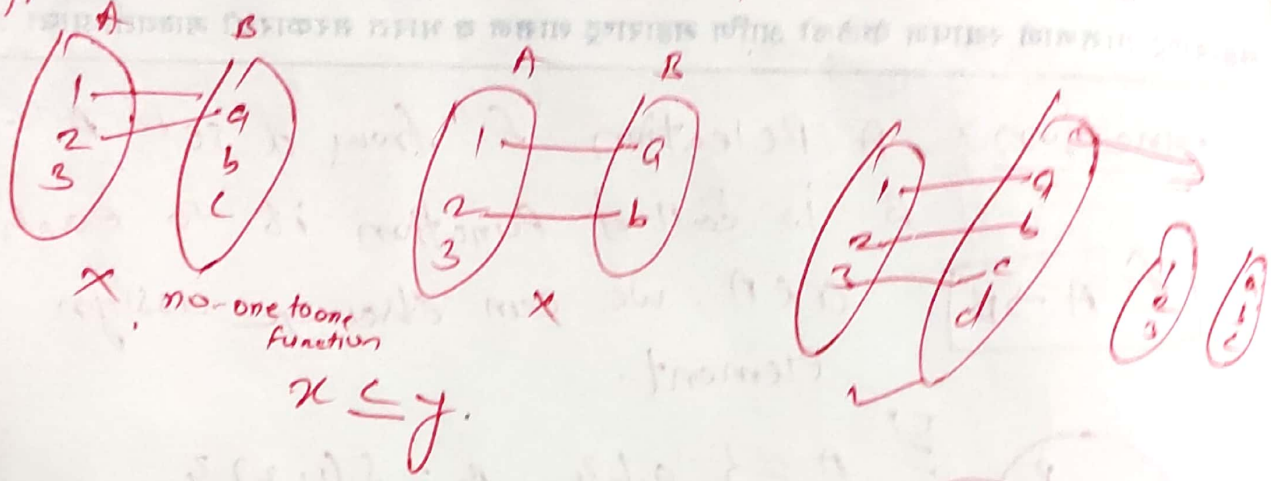
- How many relations which are not function.

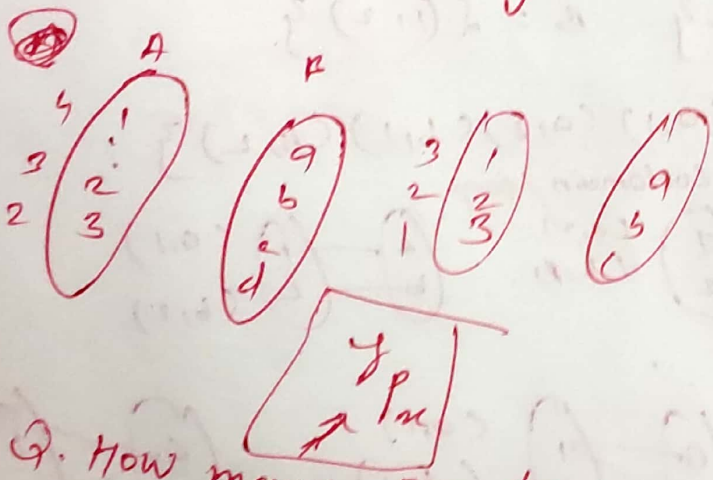
$A = \{1, 2, 3, 4\}$ $B = \{a, b, c\}$. $m = 4$ $n = 3$

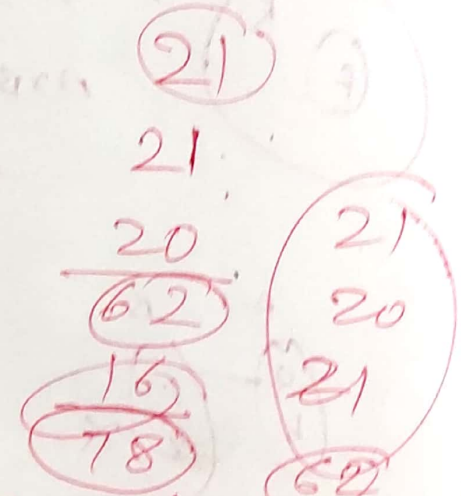
No. of element in cartesian product

$2^{mn} = 2^{4 \times 3} = 2^{12} = 2^{12} - 3^4$

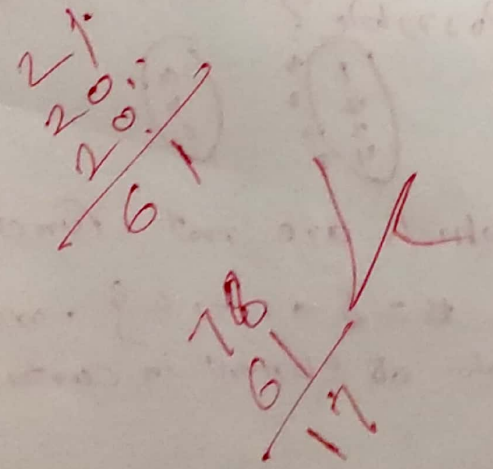
* One to one function :- A function f from set 'A' to set 'B' is one to one if no two elements in 'A' are mapped to same element in B.







Q. How many one to one function possible for the given set $A = 5$ $B = 8$.



Functions:- A function assigns exactly one element of one set to each element of other set.

** **Note:-** Let x and y be two sets, with m and n elements and a function is defined as $f: x \to y$ then,

1) Total Number of functions $= n^m$.
2) Total Number of one-one function $= {}^nP_m$
3) Total No. of onto function $= n^m - {}^nC_1 (n-1)^m + {}^nC_2 (n-2)^m - \dots +$

Composition of function:-

- $fog \neq gof$.
- If f and g both are one-one function then fog is also one-one.
- If f and g both are onto. Functn then fog is also onto.
- If f and fog both are one-one function then g is also one-one.
- If f and fog both are onto function then it is not necessary that g is also onto

$\Rightarrow (fog)^{-1} = g^{-1} \circ f^{-1}$

$f^{-1} \circ f = f^{-1}(f(a)) = f^{-1}(b) = a.$

$f \circ f^{-1} = f(f^{-1}(b)) = f(a) = b.$

Q. Consider $A = \{2,3,4,5\}$ and $R = \{(5,5)(5,3)(2,2)(2,4)(3,5)(3,3)(4,2)(4,4)\}$.

Q: Let us assume that R is a Relation on the set of real numbers defined by xRy if and only if $x-y$ is an integer. P.T. R is an equivalence relation R.

Q.3: y/x check the reft. symmetric & transitive property of the Relation xRy if and only if y is divisible by x when $x, y \in N$.