▷ Number System

- Number system defines a set of values used to represent quantity.

Name (system)	Base (r)
Binary	2
Octal	8
Decimal	10
Duodecimal	12
Hexadecimal	16

Base (r) → called as Radix
↳ Base is denoted as (r)

Decimal → commonly used

10 → there are 10 diff. digits in decimal number system (0 to 9)

To find number of digits
0 to (r-1)
↳ Base

For,

① Binary → 0 to (2-1) ⇒ 0 & 1 (bits) two numbers
↳ called as binary

② Octal → 0 to (8-1) ⇒ 0 to 7 (8 digits)
↳ 0, 1, 2, 3, 4, 5, 6, 7

③ Duodecimal
0 to (12-1) ⇒ 0 to (11)
↳ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

④ Hexadecimal
0 to (16-1) ⇒ 0 to 15

0
1
2
3
4
5
6
7
8
9
10 → A
11 → B
12 → C
13 → D
14 → E
15 → F

▷ Base 4

↳ 0 to 3

$\boxed{0, 1, 2, 3}$

$7892 = 7000 + 800 + 90 + 2$

$= 7\times10^3 + 8\times10^2 + 9\times10^1 + 2\times10^0$

10^3 10^2 10^1 10^0

The number system with weight on position, called as weightage number system.

& The number system in which there is no weight on the position is called as unweighted number system.

Number System and codes

- Weighted
  - eg. Decimal
  - Binary
  - Octal
  - (BCD), etc — Binary coded decimal
- Unweighted
  - eg. Gray code,
  - Excess-3
  - etc.

Let r_1 & N_1 → Number system 1

r_2 & N_2 → No. system 2.

Base ← r_1 N_1 ← total no. of digits required.

r_2 N_2

Thus $r_1 < r_2$ then $N_1 > N_2$

eg. 1

Decimal → Binary

$(7892)_{10} \rightarrow (\underbrace{1110011100000}_{N_2})_2$ ← Here $r_1 > r_2$, $N_1 < N_2$

N_1 no. of digits, r_1, N_2 no. of bits, r_2

④ $(7992)_{10} \rightarrow (16840)_8$

Decimal $\rightarrow$ octal

$r_1 = 10$ $\quad$ $r_2 = 8$

$N_1 = 4$ $\quad$ $N_2 = 5$

$r_1 > r_2$

$N_1 < N_2$

⑤ $(7992)_{10} \rightarrow (1CE0)_{16}$

$r_1 < r_2$

$N_1 = N_2$

Note: we can say that, N_1 is not strictly greater than N_2 but can be equal.

▷ Binary Number System $\quad$ Base $\Rightarrow 2$ Radix

computer cannot operate on decimal number they do their operation on binary number.

$r = 2$

0 to $(2-1)$

(0, 1)

* Binary digits (0 & 1) are called as 'bits.'

eg. 10101

↳ weight number

thus, base is 2,

$\therefore$ 1 0 1 0 1

$2^4 \quad 2^3 \quad 2^2 \quad 2^1 \quad 2^0$

we can write as,

$(10101)_2 = 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$

$= 16 + 0 + 4 + 0 + 1$

$(10101)_2 = (21)_{10}$

Binary $\rightarrow$ Decimal

eg. ① $(10101.11)_2$ (Has binary point)

$= 1\times2^4+0\times2^3+1\times2^2+0\times2^1+1\times2^0+1\times2^{-1}+1\times2^{-2}$

$= 16+0+4+0+1+0.5+0.25$

$= (21.75)_{10}$

▷ MSB and LSB

MSB → Most significant bit (left most bit)

LSB → Least significant bit (right most bit)

eg. ① 10101 $= (21)_{10}$

MSB LSB

1) changing LSB (1 to 0)

$b_4\ b_3\ b_2\ b_1\ b_0$

1 0 1 0 0 = 20

(not much difference)

↳ 1

2) changing MSB (1 to 0)

0 0 1 0 1 = 5

(the difference is large)

↳ 16

Hence the significance of left most bit is higher than significance of right most bit.

* Bit is smallest unit of data.

1 Nibble = 4 bits

↳ used to represent BCD and Hexadecimal number

1 byte = 8 bits

1 word = 16 bits = 2 bytes

1 Double word = 32 bits = 4 bytes

Decimal to Binary conversion

To convert decimal to any other base 'r', divide integer part by r and multiply fractional part by r.

1) $(13)_{10} \rightarrow (1101)_2$

	1	1	0	1
	b_3	b_2	b_1	b_0
	↑	↑	↑	↑
$\times 2^4$	2^3	2^2	2^1	2^0
↓	↓	↓	↓	↓
16	8	4	2	1
	13			

$13 = 8 + 5$
$= 8 + 4 + 1$

Another method

Here, $r = 2$

$(13)_{10} \rightarrow (1101)_2$

2	13	1 ← lsb
2	6	0
2	3	1
2	1	1 ← msb
	0	

↑ From bottom to top

2) $(25.625)_{10} \rightarrow (11001.101)_2$

2	25	1 ← lsb
2	12	0
2	6	0
2	3	1
2	1	1 ← msb
	0	

11001.101

$0.625 \times 2 = 1.25$ ← consider only integer

$0.25 \times 2 = 0.50$

$0.50 \times 2 = 1.00$

$0.00 \times 2 = 0.00$

↓ top to

H.W.

① $(67)_{10}$

2	67	1
2	33	1
2	16	0
2	8	0
2	4	0
2	2	0
2	1	1
	0	

$(67)_{10} \rightarrow (1000011)_2$

② $(129.75)_{10}$

2	129	1
2	64	0
2	32	0
2	16	0
2	8	0
2	4	0
2	2	0
2	1	1
	0	

10000001.11

$0.75 \times 2 = 1.5$

$0.5 \times 2 = 1.0$

$0.0 \times 2 = 0$

$(129.75)_{10} \rightarrow (10000001.11)_2$

▷ Decimal to Octal conversion

For decimal to octal conversion, divide integer part by 8 and multiply fractional part by 8. (As $r = 8$)

e.g. ① $(112)_{10} \rightarrow$

lsb

8	112	0
8	14	6
8	1	1
	0	

msb

top ↑ bottom

$(112)_{10} \rightarrow (160)_8$

② $(25.625)_{10}$

lsb

8	25	1
8	3	3
	0	

msb

31.5

$0.625 \times 8 = 5.0$

$0.0 \times 8 = 0$

Thus, $(25.625)_{10} \rightarrow (31.50)_8$

H.W.

① $(2048)_{10} \rightarrow$

lsb

8	2048	0
8	256	0
8	32	0
8	4	4
	0	

msb

(4000).

Thus,

$(2048)_{10} \rightarrow (4000)_8$

② $(82.250)_{10}$

8	82	2
8	10	2
8	1	1
	0	

122.2

$0.250 \times 8 = 2$

Thus,

$(82.250)_{10} \rightarrow (122.2)_8$

Decimal to Hexadecimal conversion

For decimal to hexadecimal conversion, divide integer part by 16 and multiply Fractional part by 16.

$r = 16$

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E - 14, F - 15

1) $(254)_{10}$

16	254	14 E → lsb
16	15	15 F → msb
	0	

$(254)_{10} \rightarrow (FE)_{16}$

2) $(25.625)_{10}$

19 . A

$0.625 \times 16 = 10\ (A)$

16	25	9 ↑
16	1	1
	0	

$(25.625)_{10} \rightarrow (19.A)_{16}$

3) $(27.4)_{10} \longrightarrow (\quad)_4$

$r = 4$

4	27	3
4	6	2 ↑
4	1	1
	0	

123 . 1212

$0.4 \times 4 = 1.6$

$0.6 \times 4 = 2.4$

$0.4 \times 4 = 1.6$

$0.6 \times 4 = 2.4$

(1 2 1 2) 12 ....

$(27.4)_{10} \longrightarrow (123.1212)_4$

HW

$(25.625)_{10} \longrightarrow (\quad)_4$

4	25	1
4	6	2
4	1	1
	0	

121 . 22

$0.625 \times 4 = 2.5$

$0.5 \times 4 = 2$

Thus $(25.625)_{10} \longrightarrow (121.22)_4$

Binary to Decimal conversion

others to Decimal:

$(a_3 a_2 a_1 a_0 . a_{-1} a_{-2})_r \longrightarrow (\quad)_{10}$

$$\boxed{a_3 r^3 + a_2 r^2 + a_1 \times r^1 + a_0 \times r^0 + a_{-1} r^{-1} + a_{-2} r^{-2} = (\quad)_{10}}$$

eg. $(\underset{4\,3\,2\,1\,0\;\;-1\,-2}{10101.11})_2 \longrightarrow (\quad)_{10}$

Binary point

$1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 + 1 \times 2^{-1} + 1 \times 2^{-2}$

$= 16 + 0 + 4 + 0 + 1 + 0.5 + 0.25$

$= 21.75$

Thus,

$(10101.11)_2 \longrightarrow (21.75)_{10}$

HW. ① $(\underset{6\,5\,4\,3\,2\,1\,0\;\;-1\,-2\,-3}{1101100.101})_2$

$= 1 \times 2^6 + 1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 0 \times 2^0 + 1 \times 2^{-1} + 0 \times 2^{-2}$

$= 64 + 32 + 0 + 8 + 4 + 0 + 0 + 0.5 + 0 + 0.125$

$= 108.625$

$(1101100.101)_2 \longrightarrow (108.625)_{10}$

Octal to Decimal conversion

$$\underbrace{O_3\ O_2\ O_1\ O_0}_{\text{Integer part}}\ \underset{\text{Octal point}}{.}\ \overbrace{O_{-1}\ O_{-2}}^{\text{fractional part}}$$

$r = 8$

$$O_3 \times 8^3 + O_2 \times 8^2 + O_1 \times 8^1 + O_0 \times 8^0 + O_{-1} \times 8^{-1} + O_{-2} \times 8^{-2} = (\quad)_{10}$$

eg. ① $(5\overset{}{7}.4)_8 \longrightarrow (\quad)_{10}$

$5 \times 8^1 + 7 \times 8^0 + 4 \times 8^{-1}$

$= 40 + 7 + 0.5$

$= 47.5$

Thus, $(57.4)_8 \rightarrow (47.5)_{10}$

② $(\overset{3}{4}\overset{2}{5}\overset{1}{0}\overset{0}{7}.\overset{-1}{4}\overset{-2}{4})_8$

$4 \times 8^3 + 5 \times 8^2 + 0 \times 8^1 + 7 \times 8^0 + 4 \times 8^{-1} + 4 \times 8^{-2}$

$= 4 \times 512 + 5 \times 64 + 0 + 7 + \frac{1}{2} + \frac{1}{16}$

$= 2048 + 320 + 7 + \frac{9}{16}$

$= 2375 + 0.5625$

$= 2375.05625$

$(4507.44)_8 \longrightarrow (2375.5625)_{10}$

③ $(601)_8$

$6 \times 8^2 + 0 \times 8^1 + 1 \times 8^0$

$= 6 \times 64 + 0 + 1$

$= 384 + 0$

$= 385$

$(601)_8 \longrightarrow (385)_{10}$

2) $(1070.02)_8 \longrightarrow$

$1 \times 8^3 + 0 \times 8^2 + 7 \times 8^1 + 0 \times 8^0 + 0 \times 8^{-1} + 2 \times 8^{-2}$

$= 512 + 56 + 0 + \frac{2}{64}$

$= 568 + \frac{8}{64} \times 0 + \frac{2}{64}$

$= 568.03125$

$(1070.02)_8 \longrightarrow (568.03125)_{10}$

Hexadecimal to Decimal conversion

$$h_3\ h_2\ h_1\ h_0\ \underset{}{.}\ h_{-1}\ h_{-2} \qquad r = 16$$

Hexadecimal point

$h_3 \times 16^3 \quad h_2 \times 16^2 \quad h_1 \times 16^1 \quad h_0 \times 16^0 \quad h_{-1} \times 16^{-1} \quad h_{-2} \times 16^{-2}$

$$h_3 \times 16^3 + h_2 \times 16^2 + h_1 \times 16^1 + h_0 \times 16^0 + h_{-1} \times 16^{-1} + h_{-2} \times 16^{-2} = (\quad)_{10}$$

eg. ① $(BAD)_{16} \longrightarrow (\quad)_{10}$

$(11\ 10\ 13)$

$11 \times 16^2 + 10 \times 16^1 + 13 \times 16^0$

$= 2816 + 160 + 13$

$= (2989)_{10}$

② $(57.4)_{16}$

$5 \times 16^1 + 7 \times 16^0 + 4 \times 16^{-1}$

$= 80 + 7 + 0.25$

$= 87.025$

Thus, $(57.4)_{16} \longrightarrow (87.025)_{10}$

HW

$(BFF.C)_{16}$

11 15 15 12

$11 \times 16^2 + 15 \times 16^1 + 15 \times 16^0 + 12 \times 16^{-1}$

$= 2816 + 240 + 15 + 0.75$

$= 3071.75$

$(BFF.C)_{16} \rightarrow (3071.75)_{10}$

Octal to Binary & Binary to Octal conversion

$r = 8$ digits ε 0 to (8-1)
0 to 7

There are eight different digits in octal number system each one of them can be represented by a equivalent 3-bit number.

0 →	0 0 0
1 →	0 0 1
2 →	0 1 0
3 →	0 1 1
4 →	1 0 0
5 →	1 0 1
6 →	1 1 0
7 →	1 1 1

Octal → Binary

① $(37.45)_8 \rightarrow (\quad)_2$

011 111 . 100 101

$(37.45)_8 \rightarrow (011111.100101)_2$

② $(22.07)_8$

$(22.07)_8 \rightarrow (010010.000111)_2$

Binary to Octal

i) $(010110.110)_2$

↑ Binary point

← I.P. F.P. →

$(010110.110)_2 \rightarrow (26.6)_8$

ii) $(01001.1)_2$

$(001001.100) \rightarrow (11.4)_8$

H.W.

① $(672.13)_8 \rightarrow (\quad)_2$

$(672.13)_8 \rightarrow (110111010.001011)_2$

② $(10111011.1001)_2$

$(010111011.100100)_2 \rightarrow (273.44)_8$

▷ Hexadecimal to Binary and Binary to Hexadecimal

- In hexadecimal number system we have 16 different digits.
- Each one of them can be represented by a equivalent 4-bit binary number.

$r = 16$ (0 to 15) → digits.

	8	4	2	1
0 →	0	0	0	0
1 →	0	0	0	1
2 →	0	0	1	0
3 →	0	0	1	1
4 →	0	1	0	0
5 →	0	1	0	1
6 →	0	1	1	0
7 →	0	1	1	1
8 →	1	0	0	0
9 →	1	0	0	1
10 A →	1	0	1	0
11 B →	1	0	1	1
12 C →	1	1	0	0
13 D →	1	1	0	1
14 E →	1	1	1	0
15 F →	1	1	1	1

▷ Hex to Binary

eg ① $(259A)_{16} \rightarrow (\quad)_2$

$(259A)_{16} \rightarrow (0010010110011010)_2$

② $(CAFE.3D)_{16}$

$(CAFE.3D)_{16} \rightarrow (1100101011111110.00111101)_2$

* Binary to Hex

① $(10001001.11)_2$

$(1000\,1001.1100)_2 \rightarrow (89.C)_{16}$

② $(110001001.11)_2$

$(000110001001.1100)_2 \rightarrow (189.C)_{16}$

H.W.

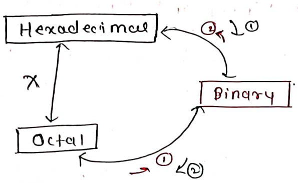
① $(FACE)_{16}$

$(FACE)_{16} \longrightarrow (1111101011001110)_2$

② $(1101111010101111)_2$

$(1101\,1110\,1010\,1111)_2 \rightarrow (DEAF)_{16}$

▷ Hexadecimal to Octal and Octal to Hexadecimal conversion



Hex → Octal

eg. ① $(CAD)_{16} \rightarrow (\ \ )_8$

i) Hex → Binary

$(CAD)_{16} \rightarrow (110010101101)_2$

ii) Binary → octal

$(110010101101)_2 \longrightarrow (6255)_8$

i.e.

$(CAD)_{16} \rightarrow (6255)_8$

2) Octal → Hexadecimal

$(652)_8 \rightarrow (\ )_{16}$

i) $(652)_8 \rightarrow (0110\,1010\,0010)_2$ (octal – binary)

ii) binary – Hexadecimal

$(0110\,1010\,0010)_2 \longrightarrow (1AA)_{16}$

H.W.

1) $(CAFE)_{16} \rightarrow (\ \ )_8$

i) $(CAFE)_{16} \rightarrow (1100101011111110)_2$

ii) $(001100101011111110)_2 \longrightarrow (145376)_8$

2) $(7071)_8 \rightarrow (\ \ )_{16}$

i) $(7071)_8 \rightarrow (111000111001)_2$

ii) $(111000111001)_2 \longrightarrow (E39)_{16}$

correct problem

$(707)_8 \rightarrow (\ )_{16}$

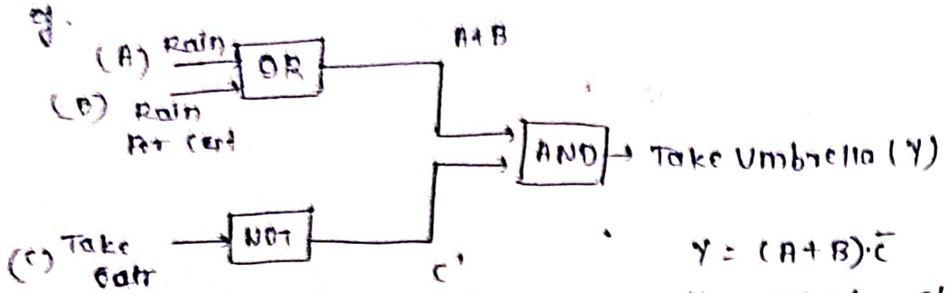
i) $(707)_8 \rightarrow (111000111)_2$

ii) $(000111000111)_2 \longrightarrow (1C7)$

Introduction to Boolean Algebra

- It is the set of rules used to simplify the given logic expression without changing its functionality.
- It is used when number of variables are less.

eg.



$Y = (A+B)\cdot\bar{C}$

$Y = BAC' + B'AC' + BC'$

Rules:

1) complement

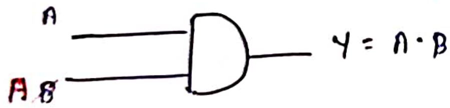
A complement = $\bar{A}$ or A' or (not A) $0' = 1$, $1' = 0$

• $(A')' = A$

2) AND

$2^2 = 4$ combinations

I/P A	I/P B	O/P Y
0	0	0
0	1	0
1	0	0
1	1	1



$A = 0 \rightarrow 0\cdot0 = 0 = Y$

$A = 1 \rightarrow 1\cdot1 = 1 = Y$

ules:

$A\cdot A = A$

$A\cdot 0 = 0$

$A\cdot 1 = A$

$A\cdot A' = 0$

R

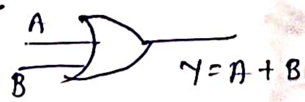
$+A = A$

$+0 = A$

$\boxed{1 = 1}$

$' = 1$

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1



HW.

$Y = A\cdot B + A'\cdot C + B\cdot C$

$= \cancel{A}(\cancel{A+C})\cancel{B} + A'C$

$= AB + A'C + (A+A')BC$

$= AB + A'C + ACB + A'CB$

$\cancel{= AB + AC + (A'C + A'C)}$

$\cancel{= A + BC + A'C}$

$= AB(1+C) + A'C(1+B)$

$= AB\cdot 1 + A'C\cdot 1$

$Y = AB + A'C$

3) $F = (A+B+C)(A+B'+C)(A+B+C')$

$= (A+B+C)\cdot(A+B+C')(A+B'+C)$

$= (A+B+C\cdot C')(A+B'+C)$

$= [(A+B)+0](A+B'+C)$

$= (A+B)\cdot(A+B'+C)$

$= A + B\cdot(B'+C)$

$= A + B\cdot B' + B\cdot C$

$= A + 0 + B\cdot C$

$F = A + B\cdot C$

4) $G = (A+B)(A+B')(A'+B)(A'+B')$

$= (A + B\cdot B')(A' + B\cdot B')$

$= (A+0)(A'+0)$

$= A\cdot A'$

$G = 0$

Redundancy Theorem / consensus theorem

i) Three variables
ii) Each variable is represented twice → may be in normal or complimented form
iii) one variable is complimented
iv) Take the complemented variable. (term)

eg. $Y = AB + A'C + BC$ → redundant term

$= AB + A'C$ (as A is complimented consider only terms containing A)

eg. ① $F = AB + B\bar{C} + AC$

$= B\bar{C} + A\bar{C}$

② $F = A\bar{B} + BC + A\bar{C}$

$F = A\bar{B} + BC$

③ $(A+B)\cdot(\bar{A}+C)\cdot(B+C)$

$F = (A+B)\cdot(\bar{A}+C)$

④ $G = (A+B)\cdot(\bar{B}+C)\cdot(A+C)$

$= (A+B)\cdot(\bar{B}+C)$

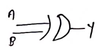
⑤ $F = \bar{A}\bar{B} + A\bar{C} + \bar{B}\bar{C}$

→ It is the only thing ie. non-complimented

→ ommited

$F = \bar{A}\bar{B} + A\bar{C}$

EX-OR

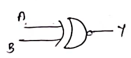


A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

$Y = A \oplus B$

$Y = \bar{A}B + A\bar{B}$

EX-NOR

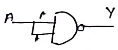


A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

$Y = \overline{A \oplus B}$

$= \bar{A}\bar{B} + AB$

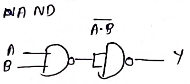
NOT → shorted input NAND gate act as NOT gate.



$Y = \overline{AA} = \bar{A}$

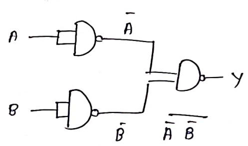
Rule $\overline{AB} = \bar{A} + \bar{B}$

NAND



$Y = \overline{\overline{AB}} = AB$

OR



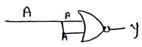
$Y = \overline{\bar{A}\,\bar{B}}$

$= \bar{\bar{A}} + \bar{\bar{B}}$

$Y = A + B$

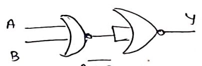
using NOR gate

NOT

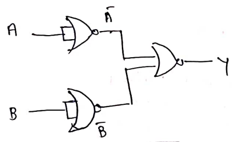


$Y = \overline{A + A} = \bar{A}$

OR



$Y = \overline{\overline{A+B}} = A + B$



$\overline{A+B} = \bar{A} \cdot \bar{B}$

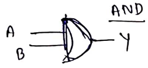
$Y = \overline{\bar{A} + \bar{B}}$

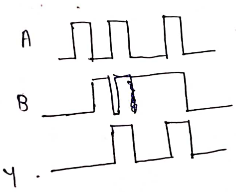
$= \bar{\bar{A}} \cdot \bar{\bar{B}}$

$Y = A \cdot B$

AND

eg.



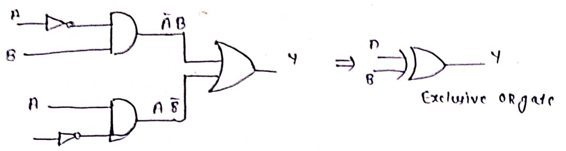


1.10 Theorem $A + BC = (A + B)(A + C)$

$x \cdot y$	A	B	C	BC	$A+B$ (x)	$A+C$ (y)	$A+BC$
0	0	0	0	0	0	0	0
0	0	0	1	0	0	1	0
0	0	1	0	0	1	0	0
1	0	1	1	1	1	1	1
1	1	0	0	0	1	1	1
1	1	0	1	0	1	1	1
1	1	1	0	0	1	1	1
1	1	1	1	1	1	1	1

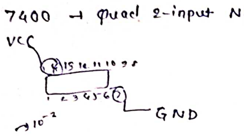
Thus $A + BC = (A + B)(A + C)$

$y = \bar{A}B + A\bar{B} = A \oplus B$



Exclusive OR gate

7400 → Quad 2-input NAND Gate IC



$\rightarrow 10^{-2}$

```
   101          101
   110        × 110
 (1)111       -----
                000
               101
              101
             -----
             11110
```

Decimal	Binary	BCD code
0	0000	
1	0001	
2	0010	
3	0011	
4		
5		
6		
7		
8		
9		
10		0001·0000
11		0001 0001
12		0001 0010
13		0001 0011
14		0001 0100
15		0001 0101

$495)_{10} \rightarrow (0100\ 1001\ 0101)_{BCD}$

↓

$(0111\ 1100\ 1000)_{Excess3}$

Excess 3

0	0011
1	0100
2	0101
3	0110
4	
5	
6	
7	
8	
9	1100
10	
11	
12	
13	
14	
15	

- Binary to gray code

	Binary	Gray
0	000	000
1	001	001
2	010	011
3	011	010
4	100	110
5	101	111
6	110	101
7	111	100

only one bit is changed

using 4 bit

0000	0000
0001	0001
0010	0011
0011	0010
0100	0110
0101	0111
0110	0101
0111	0100
1111	1100
1110	1101
1101	1111
1100	1110
1011	1010
1010	1011
1001	1001
1000	1000

$B_3 \quad B_2 \quad B_1 \quad B_0$ (with $\oplus$ between each)

$G_3 \quad G_2 \quad G_1 \quad G_0$

$G_3 = B_3$

$G_2 = B_3 \oplus B_2$

$G_1 = B_2 \oplus B_1$

$G_0 = B_1 \oplus B_0$

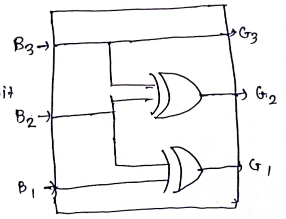
$G_3 G_2 G_1$ 111 – Gray

101 $B_3 B_2 B_1$ Binary

$B_3 = G_3$

$B_2 = B_3 \oplus G_2 = G_3$

$B_1 = B_2 + G_1 = G_3 \oplus$



3-bit Binary input

3-bit Gray code output

1) Binary addition

$$\begin{array}{r} 547 \\ +\ 25 \\ \hline 572 \end{array} \quad \begin{array}{l} \rightarrow 5\times10^2 + 4\times10^1 + 7\times10^0 \\ \rightarrow 2\times10^1 + 5\times10^0 \\ \rightarrow 5 \mid 7 \mid 2 \end{array}$$

$7+5 = 12 > 9$

12×10^0

$12 = 10+2$

$(10+2)\times10^0$

$10\times10^0 + 2\times10^0$

$1\times10^1 + 2\times10^0$

carry

i)

$$\begin{array}{r} 110 \\ +\ 101 \\ \hline 1011 \end{array} \quad \begin{array}{l} \rightarrow 1\times2^2 + 1\times2^1 + 0\times2^0 \\ \rightarrow 1\times2^2 + 0\times2^1 + 1\times2^0 \\ 1\times2^3 + 0\times2^2 + 1\times2^1 + 1\times2^0 \end{array}$$

Sum: (10) 1 1

carry sum

$2\times2^2 \quad 2>1$

$2 = 2+0$

$(2+0)\times2^2$

$2^3 + 0\times2^2$

Summary of Binary addition

		sum	carry
0+0	=	0	0
1+0 or 0+1	=	1	0
1+1	=	0	1

HW

$$\begin{array}{r} 11011 \\ +\ 1111 \\ \hline 101010 \end{array}$$

16 8 4 2 1

27

15

42

2) Binary subtraction

1)

$$\begin{array}{r} 11011 \\ -\ 10110 \\ \hline 00101 \end{array}$$

borrow

2×2^2

$$\begin{array}{l} 1\times2^4 + 1\times2^3 + 0\times2^2 + 1\times2^1 + 1\times2^0 \\ -\ 1\times2^4 + 0\times2^3 + 1\times2^2 + 1\times2^1 + 0\times2^0 \\ \hline 0\times2^4 + 0\times2^3 + 1\times2^2 + 0\times2^1 + 1\times2^0 \end{array}$$

borrow → 2

ii)

$$\begin{array}{r} 1110 \\ -\ 111 \\ \hline 0111 \end{array} \quad \begin{array}{r} 14 \\ -\ 7 \\ \hline 7 \end{array}$$

HW

16 8 4 2 1

$$\begin{array}{r} 10101 \\ -\ 1111 \\ \hline 00110 \end{array} \quad \begin{array}{r} 21 \\ -\ 15 \\ \hline 6 \end{array}$$

3) Binary multiplication

Addition

		sum	carry
0+0	=	0	0
1+0 / 0+1	=	1	0
1+1	=	0	1

Product

$0\times0 \rightarrow 0$

$1\times0 / 0\times1 \rightarrow 0$

$1\times1 \rightarrow 1$

$$\begin{array}{r} 1010 \\ \times\ 101 \\ \hline 110010 \end{array}$$

		1×2^3 +	0×2^2 +	1×2^1 +	0×2^0
			1×2^2 +	0×2^1 +	1×2^0
	×	1×2^3	0×2^2	1×2^1	0×2^0
	0×2^4	0×2^3	0×2^2	0×2^1	×
1×2^5	0×2^4	1×2^3	0×2^2	×	
1×2^5	1×2^4	0×2^3	0×2^2	1×2^1	0×2^0

$0\times2^0\times2^0$

$0\times1\times1$

0×1

0×2^0

HW

$$\begin{array}{r} 1101 \\ \times\ 11 \\ \hline 1101 \\ +\ 1101\times \\ \hline 100111 \end{array} \quad \begin{array}{l} \rightarrow 13 \\ \rightarrow 3 \\ \\ \\ \\ \rightarrow 39 \end{array}$$

32 16 8 4 2 1

$$\begin{array}{r} 1010 \\ \times\ 11 \\ \hline 1010 \\ +\ 1010\times \\ \hline 11110 \end{array} \quad \begin{array}{l} \rightarrow 10 \\ \rightarrow 3 \\ \\ \\ \\ \rightarrow 30 \end{array}$$

16 8 4 2 1

1) Octal addition

$$\begin{array}{ccc} & \swarrow 8^1 & \swarrow 8^0 \\ & A_1 & A_0 \\ + & B_1 & B_0 \\ \hline \end{array}$$

base(r) = 8 ⇒ 0 to 7

$C_0 \le 7$	$C_0 > 7$
sum = C_0	sum
carry = 0	$C_0 = 1 \times 8 + 5$ ← sum

carry ↑ base ↑

1)
$$\begin{array}{cccc} & 2 & 4 & 3 \\ + & 2 & 1 & 2 \\ \hline & 4 & 5 & 5 \end{array}$$

2)
$$\begin{array}{ccccc} & & 1 & 1 & \\ & & 5 & 6 & 7 \\ + & & 2 & 4 & 3 \\ \hline & 1 & 0\cancel{8} & 3 & 2 \end{array}$$

10 > 7

10 = 1×8 + 2

11 > 7

11 = 1×8 + 3

8 > 7

1×8 + 0

HW

$$\begin{array}{ccccc} & 1 & 1 & 1 & \\ & 1 & 7 & 7 & 6 \\ + & & 3 & 4 & 5 \\ \hline & 2 & 3 & 4 & 3 \end{array}$$

① 11 > 7

1×8' + 3

② 12 > 7

1×8' + 4

2) Octal subtraction:

Borrow - 8

eg. ①

$$\begin{array}{r} 7\;4\;3 \\ -\;5\;6\;4 \\ \hline \text{Ans.}\;1\;5\;7 \end{array}$$

②

$$\begin{array}{r} 6\;9\;4 \\ -\;2\;6\;5 \\ \hline \text{Ans.}\;3\;3\;7 \end{array}$$

HW

①

$$\begin{array}{r} 5\;1\;1 \\ -\;9\;5 \\ \hline 4\;5\;6 \end{array}$$

$$\begin{array}{r} 6\;0\;0\;0 \\ -\;7\;7\;7 \\ \hline 5\;0\;0\;1 \end{array}$$

3) Octal multiplication

1)

$$\begin{array}{r} 7\;2 \\ \times\;1\;2 \\ \hline 1\;6\;4 \\ +\;7\;2\;\times \\ \hline 1\;1\;0\;4 \end{array}$$

$7\times 2 = 14$
$= 1\times 8+6$
$8 = 1\times 8+0$
$9 = 1\times 8+1$

2)

$$\begin{array}{r} 3\;5\;7 \\ 2\;3 \\ \hline 1\;3\;1\;5 \\ +\;7\;3\;6\;\times \\ \hline 1\;0\;6\;7\;5 \end{array}$$

$21 = 7\times 3 = 2\times 8+5$
$17 = \cancel{2\times}\,8\times 2+1$
$11 = 1\times 8+3$
$14 = 1\times 8+6$

HW

1)

$$\begin{array}{r} 6\;3 \\ \times\;2\;4 \\ \hline 3\;1\;4 \\ +\;1\;4\;6\;\times \\ \hline 1\;7\;7\;4 \end{array}$$

$12 = 1\times 8+4$
$25 = 3\times 8+1$
12

2)

$$\begin{array}{r} 6\;3\;5 \\ \times\;5\;5 \\ \hline 4\;0\;2\;1\;\cancel{5} \\ 4\;0\;2\;1\;\times \\ \hline 4\;4\;2\;3\;1 \end{array}$$

$25 = 3\times 8+1$
$18 = 2\times 8+2$
$32 = 4\times 8$

1) Hexadecimal Addition

↳ Base (16)
0 to (16-1)

10 = A
11 = B
12 = C
13 = D
14 = E
15 = F

1)

$$\begin{array}{r} 5\;6\;8\;9 \\ +\;4\;5\;7\;4 \\ \hline 9\;B\;F\;D \end{array}$$

2)

$$\begin{array}{r} A\;D\;D \\ +\;D\;A\;D \\ \hline 1\;8\;8\;A \end{array}$$

$26 = 1\times 16+10 \rightarrow A$
$24 = 1\times 16+8$

HW.

①

$$\begin{array}{r} 8\;9\;9 \\ +\;1\;8\;9 \\ \hline A\;2\;2 \end{array}$$

$18 = 1\times 16+2$

②

$$\begin{array}{r} D\;A\;F \\ +\;B\;A\;F \\ \hline 1\;9\;5\;E \end{array}$$

$30 = 1\times 16+14$
$21 = 1\times 16+5$
$25 = 1\times 16+9$

2) Hexadecimal subtraction

1)

$$\begin{array}{r} 9\;6\;5\;4 \\ -\;5\;3\;2\;1 \\ \hline 4\;3\;3\;3 \end{array}$$

2)

$$\begin{array}{r} 9\;7\;4\;B \\ -\;5\;8\;7\;C \\ \hline 3\;E\;C\;F \end{array}$$

HW

1)

$$\begin{array}{r} F\;A\;B \\ -\;A\;E\;F \\ \hline 4\;B\;C \end{array}$$

2)

$$\begin{array}{r} 7\;7\;8\;8 \\ -\;D\;E\;F \\ \hline 6\;9\;9\;9 \end{array}$$

3) Hexadecimal multiplication

1)
```
     9 4
   × 1 2
   -----
   1 2 8
 + 9 4 ×
   -----
   A 6 8
```
$18 = 1 \times 16 + 2$

```
   1   1
  10  11  12
   A   B   C
 ×     2   3
 -----------
   2   0   3   4
+  1   5   7   8   ×
 -----------------
   1   7   7   B   4
```
$36 = 2 \times 16 + 4$

$35 = 2 \times 16 + 3$

$32 = 2 \times 16 + 0$

$24 = 1 \times 16 + 8$

$23 = 1 \times 16 + 7$

$21 = 1 \times 16 + 5$

HW

①
```
     8 1 8
   ×   2 1
   -------
     8 1 8
+ 1 0 3 0 ×
  ---------
  1 0 8 1 8
```

②
```
    13 10 15
     D  A  F
   ×    5  4
   ---------
  3  6  B  C
+ 4  4  6  3  ×
  -------------
  4  7  D  6  C
```
$60 = 3 \times 16 + 12$

$43 = 2 \times 16 + 11$

$54 = 3 \times 16 + 6$

$75 = 4 \times 16 + 11$

54

$68 = 4 \times 16 + 4$

$21 = 1 \times 16 + 5$

1) Sum of product

SOP form

total no. of combinations $= 2^n \rightarrow$ no. of variables. eg. $2^3 = 8$

	A	B	C	F
0 m_0	0	0	0	0
1 m_1	0	0	1	0
2 m_2	0	1	0	1
3 m_3	0	1	1	0
4 m_4	1	0	0	1
5 m_5	1	0	1	1
6 m_6	1	1	0	1
7 m_7	1	1	1	1

F → SOP form
→ POS

In SOP form → the output must be high i.e. 1.

Thus, standard / canonical SOP form

$F = \bar{A}\cdot B\cdot\bar{C} + A\cdot\bar{B}\cdot\bar{C} + A\cdot\bar{B}\cdot C + A\cdot B\cdot\bar{C} + A\cdot B\cdot C$

Here all terms are minterms

minterm (it looks like)
↳ $\bar{A}\cdot B\cdot C$
$A\cdot B C$
AB
AC
$\bar{A}C$, etc.

$m_0 \rightarrow$ minterm

$M_0 \rightarrow$ Maxterm

$F(A, B, C) = m_2 + m_4 + m_5 + m_6 + m_7$

$= \Sigma m(2, 4, 5, 6, 7)$

Here,

$F = \bar{A}\cdot B\cdot\bar{C} + A\cdot\bar{B}\cdot\bar{C} + A\bar{B}C + AB\bar{C} + ABC$

$= \bar{A}\cdot B\cdot\bar{C} + A\cdot\bar{B}(\bar{C}+C) + AB(\bar{C}+C)$

$= \bar{A}B\bar{C} + A\cdot\bar{B}\cdot 1 + AB\cdot 1$

$= \bar{A}\cdot B\bar{C} + A\cdot\bar{B} + AB$

$= \bar{A}\cdot B\bar{C} + A(\bar{B}+B)$

$= \bar{A}\cdot B\bar{C} + A\cdot 1$

$= \bar{A}\cdot B\cdot\bar{C} + A$

$y = \bar{A}\cdot\underbrace{(B\cdot\bar{C})}_{x} + A$

$= \bar{A}x + A$

$\boxed{y = x + A}$

SOP form (types) → canonical / standard
→ minimal

$F = A + B\cdot\bar{C}$ (SOP form)

↳ minimal SOP form

1) canonical / standard SOP form: Each minterm is having all the variables in normal or complimented form.

eg. $F = \bar{A}B + AB + \bar{A}\bar{B}$

2) minimal SOP form: Each minterm does not have all the variables in normal or complimented form.

eg. $G = A + \bar{B}C$

Q. For the given truth table, minimize the SOP expression

A	B	Y
0	0	0
0	1	1
1	0	0
1	1	1

Sol

$\boxed{Y = \bar{A}\cdot B + A\cdot B}$ canonical — Y directly written from truth table

$= (\bar{A}+A)B$

$= 1\cdot B$

$\boxed{Y = B} \rightarrow$ minimal

Q. Simplify the expression for $Y(A,B) = \Sigma m(0,2,3)$

Soln

$Y(A,B) = \Sigma m(0,2,3)$

A	B	Y
0	0	1
0	1	0
1	0	1
1	1	1

(canonical) $Y = \bar{A}\cdot\bar{B} + A\bar{B} + AB$

$= (\bar{A}+A)\bar{B} + AB$

$= 1\cdot\bar{B} + AB$

$= \bar{B} + AB$ — neglect $\Rightarrow \bar{A} + AB \Rightarrow \bar{A} + B$

$Y = \bar{B} + A$

(minimal SOP)

Product of sum (POS form)

A	B	C	Y	
0	0	0	0	M_0
0	0	1	0	M_1
0	1	0	1	
0	1	1	0	M_3
1	0	0	1	
1	0	1	1	
1	1	0	1	
1	1	1	1	

POS form is used when the output is low or "0"

0 → A
1 → $\bar{A}$

maxterms

POS $Y = (A+B+C)\cdot(A+B+\bar{C})\cdot(A+\bar{B}+\bar{C})$

canonical pos form

SOP $\bar{Y} = \bar{A}\cdot\bar{B}\cdot\bar{C} + \bar{A}\cdot\bar{B}\cdot C + \bar{A}\cdot B\cdot C$

$\bar{\bar{Y}} = \overline{[\bar{A}\cdot\bar{B}\cdot\bar{C} + \bar{A}\cdot\bar{B}\cdot C + \bar{A}\cdot B\cdot C]}$

$Y = (\bar{\bar{A}}+\bar{\bar{B}}+\bar{\bar{C}})\cdot(\bar{\bar{A}}+\bar{\bar{B}}+\bar{C})\cdot(\bar{\bar{A}}+\bar{B}+\bar{C})$

D'Morgan's theorem

POS $Y = (A+B+C)\cdot(A+B+\bar{C})\cdot(A+\bar{B}+\bar{C})$

$Y = \underline{(A+B+C)}\cdot\underline{(A+B+\bar{C})}\cdot(A+\bar{B}+\bar{C})$

$= (A+B+C\bar{C})\cdot(A+\bar{B}+\bar{C})$

$= (A+B+0)\cdot(A+\bar{B}+\bar{C})$

$= (A+B)\cdot(A+\bar{B}+\bar{C})$

$= A + B(\bar{B}+\bar{C})$

$= A + B\cdot\bar{B} + B\cdot\bar{C}$

$= A + 0 + B\cdot\bar{C}$

$Y = A + B\cdot\bar{C} = (A+B)\cdot(A+\bar{C})$

(minimal POS form)

* $(A+B)\cdot(A+C) = A + BC$

Q. For the given truth table minimize the POS expression

A	B	Y	
0	0	1	M_0
0	1	0	M_1
1	0	1	M_2
1	1	0	M_3

POS (canonical) ⇒

0 → A
1 → $\bar{A}$

$Y = (A+\bar{B})\cdot(\bar{A}+\bar{B})$

$Y = M_1 + M_3$

$Y = (A+\bar{B})\cdot(\bar{A}+\bar{B}) \rightarrow$ canonical

$= (A\cdot\bar{A} + \bar{B})$

$= 0 + \bar{B}$

$Y = \bar{B} \rightarrow$ minimal form

** In minimal POS form each maxterm will not have all variables (in normal or complemented form)

eg. $Y = \Pi(M_1, M_3)$, $Y = \Pi M(1,3)$ } maxterm

representation of above question

or $Y = \Sigma m(0,2) \rightarrow$ minterm

Note

SOP form
no. of AND gate >> no. of OR gate

POS form
no. of OR gate >> no. of AND ga

▷ SOP & POS Form examples

	A	B	C	Y	
	0	0	0	1	m_0
M_1	0	0	1	0	
	0	1	0	1	m_2
	0	1	1	1	m_3
M_4	1	0	0	0	
M_5	1	0	1	0	
	1	1	0	1	m_6
	1	1	1	1	m_7

minterms →

$Y(A,B,C) = \Sigma m(0,2,3,6,7)$ → output is 1

maxterms →

$Y(A,B,C) = \Pi M(1,4,5)$ → output is 0.

SOP Form →

$0 \to \bar{A}$

$1 \to A$

$Y(A,B,C) = \Sigma m(0,2,3,6,7)$

canonical SOP form

$Y = \bar{A}\cdot\bar{B}\cdot\bar{C} + \underline{\bar{A}\cdot B\cdot\bar{C}} + \underline{\bar{A}\cdot B\cdot C} + \underline{AB\bar{C}} + \underline{ABC}$

$= \bar{A}\cdot\bar{B}\cdot\bar{C} + \bar{A}\cdot B(\bar{C}+C) + AB(\bar{C}+C)$

$= \bar{A}\cdot\bar{B}\cdot\bar{C} + \bar{A}\cdot B\cdot 1 + AB\cdot 1$

$= \bar{A}\cdot\bar{B}\cdot\bar{C} + (\bar{A}+A)B$

$= \bar{A}\cdot\bar{B}\cdot\bar{C} + 1\cdot B$

$= \bar{A}\cdot\bar{B}\cdot\bar{C} + B = B + \bar{B}\cdot\bar{A}\cdot\bar{C}$

$\boxed{(A+\bar{A}B) = A+B}$

$Y = B + \bar{A}\cdot\bar{C}$ → minimal SOP form

POS Form →

$0 \to A \;/\; 1 \to \bar{A}$

$Y = \Pi M(1,4,5)$

$= M_1 + M_4 + M_5$

$Y = (A+B+\bar{C})\cdot(\bar{A}+B+C)\cdot(\bar{A}+B+\bar{C})$ → canonical POS Form

$= (A+B+\bar{C})\cdot(\bar{A}+B+C\cdot\bar{C})$

$= (A+B+\bar{C})\cdot(\bar{A}+B+0)$

$= (A+B+\bar{C})\cdot(\bar{A}+B)$

$= B + \bar{A}(A+\bar{C})$

$= B + \bar{A}\cdot A + \bar{A}\cdot C$

$= B + 0 + \bar{A}\cdot C$

$Y = B + \bar{A}\cdot C$

$= (\bar{A}+B)\cdot(B+\bar{C})$ → minimal POS form

▷ Minimal to canonical Form conversion

① $Y = B'C + A$

$= A + B'C$

$= \cancel{(A+B')\cdot(A+C)}$

$= A\cdot 1\cdot 1 + B'C\cdot 1$

$Y = A(B+B')\cdot(C+C') + B'C(A+A')$

$Y = (AB+AB')\cdot(C+C') + B'CA + B'CA'$

$Y = ABC + ABC' + \underline{AB'C} + AB'C' + AB'C + A'B'C$

Redundant (remove)

(canonical)

$Y = ABC + ABC' + AB'C + AB'C' + A'B'C$

Steps

① → Find out no. of variables & what are they

Here, 3 var → A, B & C

② → Find out the absent variable in each term

$m_1 =$ A ✓, B ✗, C ✗ $m_2 =$ A ✗, B ✓, C ✓

③ →

② $F = (A+B+C')(A'+C)$ → minimal POS

$= (A+B+C')(A'+C+0)$

$= (A+B+C')(\underbrace{A'+C}_{x} + B\cdot B')$

$= (A+B+C')(A'+C+B)(A'+C+B')$

canonical POS

① 3 var A, B, C

② $M_1 =$ A ✓, B ✓, C ✓ $M_2 =$ A ✓, B ✗, C ✓

$\boxed{(x+A\cdot A') = (x+A)\cdot(x+A')}$

Q. ① In a minimal SOP form, number of minterms in the logic expression $A+B'C$ are: 5 terms

$Y = A + B'C$

$= A\cdot 1\cdot 1 + 1\cdot B'C$

$= A(B+B')(C+C') + (A+A')B'C$

$= (AB+AB')(C+C') + AB'C + A'B'C$

$= ABC + ABC' + \underline{AB'C} + AB'C' + \underline{AB'C} + A'B'C$

$Y = ABC + ABC' + AB'C' + AB'C + A'B'C$

② With 2 variables maximum possible minterms and maxterms are

(A, B) → 2 variables

$2^2 = 4$ possible combinations

maximum possible minterms & maxterms are 4.

For n variable → 2^n (max/min-terms)

③ For n=4, what is the total number of logical expressions.

n = no. of variables = 4

For n = 2 (A, B)

1	A	$A \cdot \bar{B}$	$AB = BA$
0	$\bar{A}$	$\bar{A} \cdot B$	$A + B = B + A$
$\bar{A}B + A\bar{B}$	B	$A + \bar{B}$	$\bar{A}\bar{B}$
$AB + \bar{A}\bar{B}$	$\bar{B}$	$\bar{A} + B$	$\bar{A} + \bar{B}$

} 16 logical expression

Thus

$16 = 2^4$

$16 = 2^{2^2}$ → variable

2^{2^n} = no. of logic expressions

For n = 4,

$2^{2^4} = 2^{16} = 65536$ logical expressions.

Positive logic and Negative Logic

Positive logic

- Higher voltage corresponds to logic 1.
- "5V" ⇒ logic '1'

"0V" ⇒ logic '0'

eg. logic 0 → −5V

logic 1 → 0V

0V > −5V

1V 0 → $\boxed{\text{+ve logic}}$

This is positive logic

For negative logic

0 1

Negative Logic

- Higher voltage corresponds to logic 0.

→ "5V" → logic "0"

→ "0V" ⇒ logic "1"

logic 0 → −1.7V

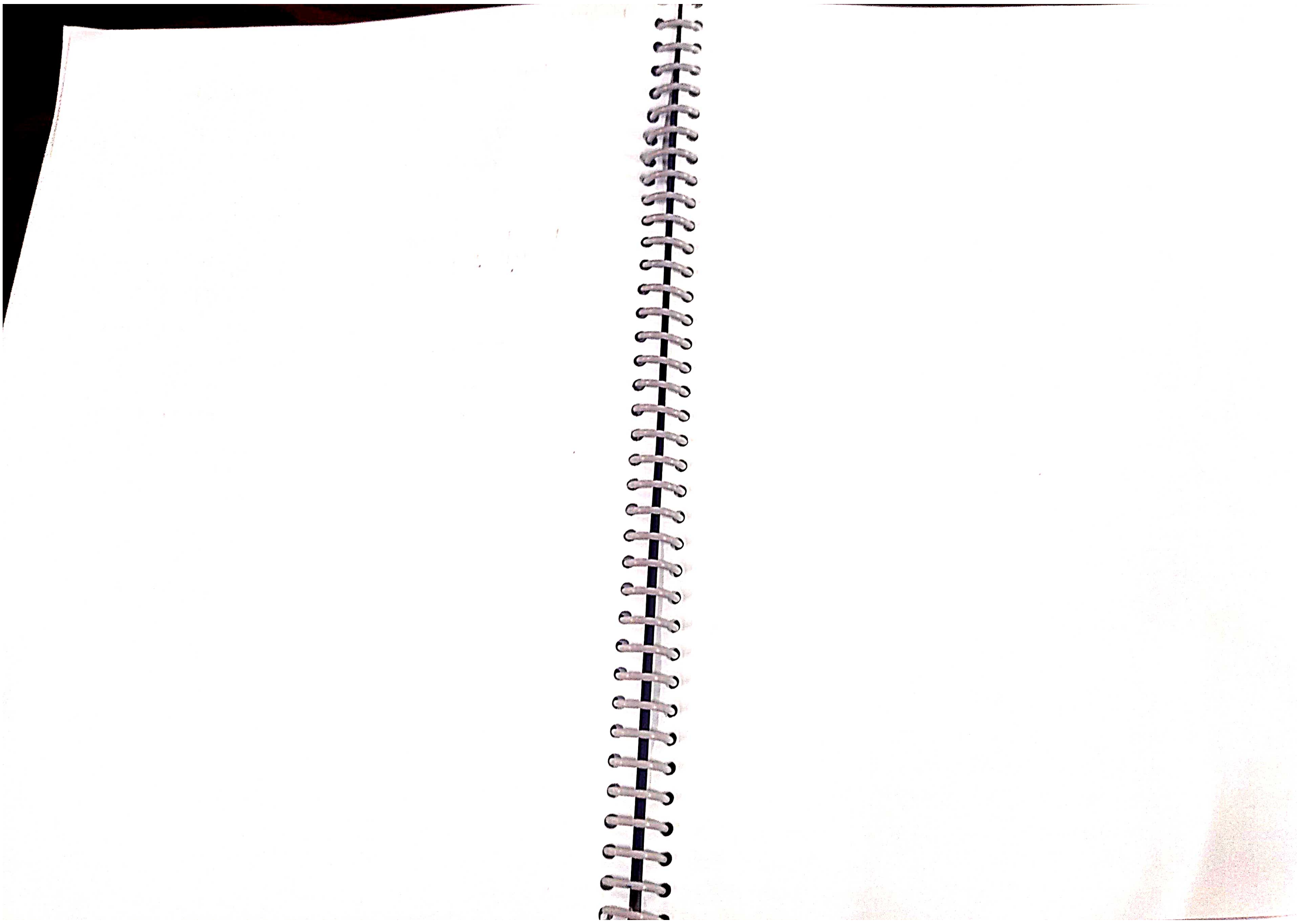
logic 1 → −0.8V

−0.8 > −1.7

+ve 1 0

−ve 0 1

AN → positive logic



1) combination circuits
2) sequential circuits

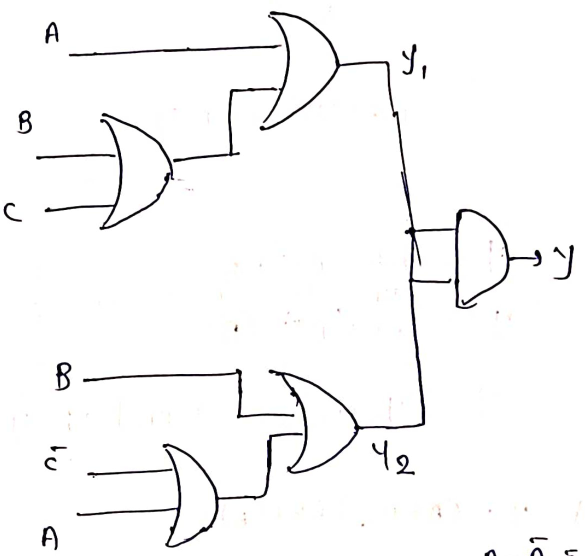
$A \cdot \bar{A} = 1$
$A + 1 = A$

eg. 1

$$y = \underbrace{(A + BC)}_{y_1}\underbrace{(B + \bar{C}A)}_{y_2}$$

=

Design a circuit →



Q. Is the circuit form by using only one type of gate? If yes, design a circuit.

$y = (A + BC)(B + \bar{C}A)$

$= A(B + \bar{C}A) + BC(B + \bar{C}A)$

$= A \cdot B + A \cdot \bar{C} \cdot A + BCB + BC \cdot \bar{C}A$

$= A \cdot B + A \cdot A \cdot \bar{C} + B \cdot BC + BAC \cdot \bar{C}$

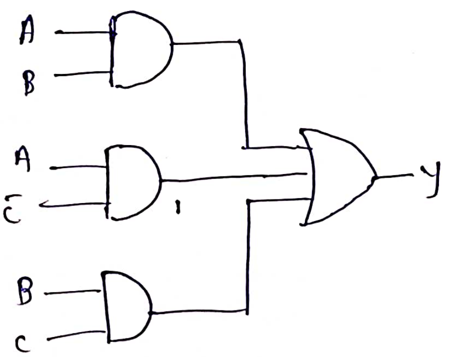
$= A \cdot B + A \cdot \bar{C} + BC + BA \cdot 0$

$= A \cdot B + A \cdot \bar{C} + BC + 0$

$y = AB + A\bar{C} + BC$ (S.O.P Form)

$A \cdot \bar{A} = 0$
$A + \bar{A} = 1$

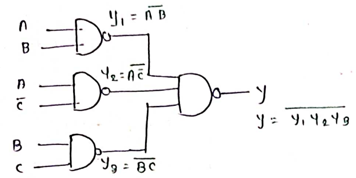
↓
implement two levels
1) AND gates
2) OR



AND - OR

POS Form implementation
1) OR
2) AND

Replacing all gates by NAND gate



NAND - NAND

$$\bar{y} = \overline{AB + A\bar{C} + BC}$$
$$\bar{y} = (\underbrace{\overline{AB}}_{y_1})(\underbrace{\overline{A\bar{C}}}_{y_2})(\underbrace{\overline{BC}}_{y_3}) \leftarrow \text{let}$$
} (∵ D'Morgan's theorem)

$y = \overline{y_1 . y_2 y_3}$ ⇒ And of $y_1 y_2 y_3$ → its complement ⇒ NAND

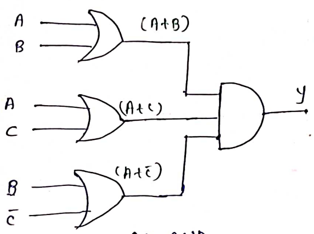
2) $y = (A+BC)(B+\bar{C}A)$.

$(A+BC) = (A+B)(A+C)$

$= (A+B)(A+C)(B+\bar{C})(B+A)$

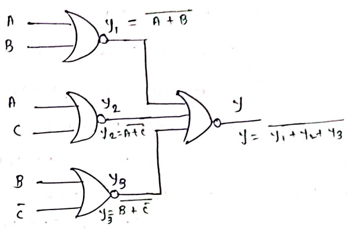
$= (A+B)(A+B)(A+C)(B+\bar{C})$

$= (A+B)(A+C)(B+\bar{C})$ (POS Form) → ① OR ② AND



OR - AND

Replacing by NOR gates



NOR-NOR

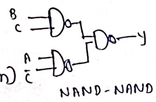
$$\bar{y} = \overline{\underbrace{(A+B)}_{y_1}\underbrace{(A+C)}_{y_2}\underbrace{(B+\bar{C})}_{y_3}}$$
$$= \overline{(A+B)} + \overline{(A+C)} + \overline{(B+\bar{C})}$$
} (D'Morgan's theorem)

$\bar{y} = \bar{y}_1 + \bar{y}_2 + \bar{y}_3$

$y = \overline{\bar{y}_1 + \bar{y}_2 + \bar{y}_3}$

D $y = AB + A\bar{C} + BC$ (SOP Form)

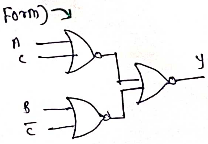
$y = \cancel{AB +} BC + A\bar{C}$ - theorem ~~(POS Form)~~



NAND - NAND

D $y = (A+B)(A+C)(B+\bar{C})$ (POS Form)

$y = (A+C)(B+\bar{C})$



NOR-NOR

▷ canonical SOP and POS forms

eg. three variables
(A, B, C)

$Y = AB\bar{C} + A\bar{C}\bar{B} + \bar{A}BC$ (canonical SOP form)

$Y = (A+B+\bar{C})(A+\bar{C}+\bar{B})(\bar{A}+B+C)$ (canonical POS form)

▷ canonical SOP form

$Y = AB + BC$
$= AB(C+\bar{C}) + BC(A+\bar{A})$
$= ABC + AB\bar{C} + BCA + BC\bar{A}$

▷ canonical POS form

$Y = (A+B)(B+C)$
$= (A+B+C\bar{C})(B+C+A\bar{A})$
$Y = (A+B+\bar{C})(A+B+\bar{C})$
$(B+C+A)(B+C+\bar{A})$

ABC	minterms m_i (output 1)	maxterms M_i (output 0)	y
000	m_0 $\bar{A}\bar{B}\bar{C}$	M_0 $A+B+C$	1
001	m_1 $\bar{A}\bar{B}C$	M_1 $A+B+\bar{C}$	0
010	m_2 $\bar{A}B\bar{C}$	M_2 $A+\bar{B}+C$	0
011	m_3 $\bar{A}BC$	M_3 $A+\bar{B}+\bar{C}$	0
100	m_4 $A\bar{B}\bar{C}$	M_4 $\bar{A}+B+C$	0
101	m_5 $A\bar{B}C$	M_5 $\bar{A}+B+\bar{C}$	0
110	m_6 $AB\bar{C}$	M_6 $\bar{A}+\bar{B}+C$	0
111	m_7 ABC	M_7 $\bar{A}+\bar{B}+\bar{C}$	1

$\left.\begin{matrix}\bar{A}\\ \bar{B}\\ \bar{C}\end{matrix}\right\}0$ $\left.\begin{matrix}A\\ B\\ C\end{matrix}\right\}1$

$Y = \bar{A}\bar{B}\bar{C} + ABC$
↳ SOP

POS

$Y = \Sigma M(1,2,3,4,5,6)$

$Y = (A+B+\bar{C}) \cdot (A+\bar{B}+C) \cdot (A+\bar{B}+\bar{C}) \cdot (\bar{A}+B+C) \cdot (\bar{A}+B+\bar{C}) \cdot (\bar{A}+\bar{B}+C)$

0 0 1 → input
0 0 0 → output
zero
0

$Y = \bar{A}\bar{B}\bar{C} + ABC = \Sigma m(m_0, m_7) = \Sigma m(0,7)$

inputs ABC	o/p y
000	1
001	0
010	0
011	0
100	0
101	0
110	0
111	1

Truth table

Here $Y = \bar{A}\bar{B}\bar{C} + ABC$

SOP
$Y = \bar{A}\bar{B}\bar{C} + ABC$
$= \Sigma m(0,7)$
↳ minterms

POS
$Y = (A+B+\bar{C}) \ldots (\bar{A}+\bar{B}+\bar{C})$
$= \Sigma M(1,2,3,4,5,6)$
↳ maxterms

▷ K-map for 3 variables

$Y = AB + A\bar{C} + BC$ (SOP)

truth table

A	B	C	y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

↳ in minterms
$Y = \Sigma m(3,4,6,7)$
↳ $Y = \Sigma M(0,1,2,5)$
(in maxterms)

K-map

▷ considered any element only once.

according to gray code

A \ BC	00	01	11	10
0	0 (0)	0 (1)	1 (3)	0 (2)
1	1 (4)	0 (5)	1 (7)	1 (6)

Starts from msb → A

0 & 2
4 & 6
are neighbours after rolling K-map

y ▷ changing bit is thrown
Here A is changing
so $Y = \bar{A}BC + ABC$
$= BC(\bar{A}+A)$ ↳ 1
$= BC$

eqn for y →
$\boxed{Y = BC + A\bar{C}}$
simplest SOP form

AND-OR NAND-NAND

POS Form

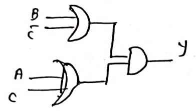
y \ BC	00	01	11	10
A = 0	0	0	1	0
A = 1	1	0	1	1

eqⁿ for POS form

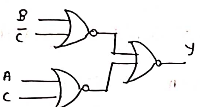
$$y = (B+\bar{C})\cdot(A+C)$$

① as B is 0 then don't take its complement

② For value 1 take complement.

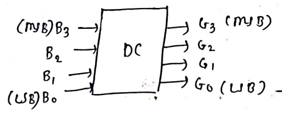


OR-AND



NOR-NOR

eg. Design 4 bit binary to gray code converter. Use k-map for simplification and draw neat stage of required hardware.



Input B_3 A	B_2 B	B_1 C	B_0 D	Output G_3	G_2	G_1	G_0
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	0	0	0	1	1
0	0	1	1	0	0	1	0
0	1	0	0	0	1	1	0
0	1	0	1	0	1	1	1
0	1	1	0	0	1	0	1
0	1	1	1	0	1	0	0
1	0	0	0	1	1	0	0
1	0	0	1	1	1	0	1
1	0	1	0	1	1	1	1
1	0	1	1	1	1	1	0
1	1	0	0	1	0	1	0
1	1	0	1	1	0	1	1
1	1	1	0	1	0	0	1
1	1	1	1	1	0	0	0
8	4	2	1				

K-map

G_3

B_3B_2 \ B_1B_0	00	01	11	10
00	0 (0)	0 (1)	0 (3)	0 (2)
01	0 (4)	0 (5)	0 (7)	0 (6)
11	1 (12)	1 (13)	1 (15)	1 (14)
10	1 (8)	1 (9)	1 (11)	1 (10)

← octave

$G_3 = B_3$

Vertically → B_2 is changing ✗
& B_3 is constant ✓

Horizontally → B_1 is changing ✗
& B_0 is also changing ✗

G_2

B_3B_2 \ B_1B_0	00	01	11	10
00	0	0	0	0
01	1	1	1	1
11	0	0	0	0
10	1	1	1	1

constant

$$G_2 = \bar{B}_3 B_2 + B_3 \bar{B}_2 = B_3 \oplus B_2$$

↖ exclusively OR

$G_1 = B_2 \oplus B_1$

$G_0 = B_1 \oplus B_0$

G_1

B_3B_2 \ B_1B_0	00	01	11	10
00	0	0	1	1
01	1	1	0	0
11	1	1	0	0
10	0	0	1	1

$$G_1 = B_2\bar{B}_1 + B_1\bar{B}_2$$

$$G_1 = B_2 \oplus B_1$$

G_0

B_3B_2 \ B_1B_0	00	01	11	10
00	0	1	0	1
01	0	1	0	1
11	0	1	0	1
10	0	1	0	1

$G_0 = \bar{B_1} B_0 + B_1 \bar{B_0}$

$G_0 = B_1 \oplus B_0$

In the IC we have to use 3 X-OR gate to get required output.

r's complement

- r's complement (radix)
- (r-1)'s complement (Diminized Radix complement)

eg. comp of $(7)_{10}$

Ans. 3

$10-7=3$

$10-9=1$ (comp 9)

$10-6=4$

Let N be the given number

r = base

$n=1$

$\boxed{r^n - N}$ r's complement

eg. ① $N = 5690$

determine 10's complement

$\boxed{n = \text{no. of digits}}$

TO $: 10^4 - 5690$

$= 10000 - 5690$

10's comp. = 4310

② N = 1101. Find 2's complement.

$N = 1101$, $n=4$, $r=2$

$= 10_2^4 - 1101$

$= \cancel{10000}^{16} - 1101$

$= (16)_{10} - 1101$

$= 10000 - 1101$

$= 11$

$$\begin{array}{r} 10000 \\ -\ 1101 \\ \hline 00011 \end{array}$$

2	16	0
2	8	0
2	4	0
2	2	0
	1	

HW

① N = 76895

10's complement

$= r^n - N$

$= 10^5 - 76895$

$= 100000 - 76895$

10's com. = 23105

② 16 8 4 2 1
11011

2's complement

$= r^n - 11011$

$= 2^5 - 11011$

$= 32 - 11011$

$= 100000 - 11011$

2's com = 101

$\boxed{32-27=5}$

2	32	0
2	16	0
2	8	0
2	4	0
2	2	0
	1	

$$\begin{array}{r} 100000 \\ -\ 11011 \\ \hline 000101 \end{array}$$

① $F(A,B,C,D) = \Sigma m(0,1,2,3,5,7,8,9,11,14)$

Four variable logical function. (minimize)

$y = F(A,B,C,D) = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C}D + \ldots$

Truth table

A	B	C	D	y
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	0
1	1	1	0	1
1	1	1	1	0

ⓘ SOP Form

y

CD \ AB	00	01	11	10
00	1	4	12	1 8
01	1	1 5	13	1 9
11	1	1 7	15	1 11
10	1 2	6	1 14	10

$y = \bar{A}\bar{B} + \bar{A}\cdot D + \bar{B}D + B\bar{C}D + ABC\bar{D}$

ⅱ) POS Form $y = \Sigma\pi(4,6,10,12,13,15)$

$y = (A+\bar{B}+C+D)\cdot(A+\bar{B}+\bar{C}+D)\cdot(\bar{A}+B+\bar{C}+D)\cdot(\bar{A}+\bar{B}+C+D)\cdot$
$(\bar{A}+\bar{B}+C+\bar{D})\cdot(\bar{A}+\bar{B}+\bar{C}+\bar{D})$

y

CD \ AB	00	01	11	10
00		0	0	
01			0	
11			0	
10		0		0

$y = (C+D+\bar{B})\cdot(\bar{A}+\bar{B}+\bar{D})\cdot$
$(A+\bar{B}+D)\cdot(\bar{A}+B+\bar{C}+D)$

② $y = F(A,B,C,D) = AB\bar{C}D + \bar{A}BCD + \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}\bar{D} + A\bar{C} + A\bar{B}C + \bar{B}$

1101 0111 000 000 10 101 0

A B C D	y
0 0 0 0	1
0 0 0 1	1
0 0 1 0	1
0 0 1 1	1
0 1 0 0	0
0 1 0 1	0
0 1 1 0	0
0 1 1 1	1
1 0 0 0	1
1 0 0 1	1
1 0 1 0	1
1 0 1 1	1
1 1 0 0	0
1 1 0 1	1
1 1 1 0	0
1 1 1 1	0

y

AB \ CD	00	01	11	10
00	1	1	1	1
01	0	0	1	0
11	1	1	0	0
10	1	1	1	1

$y = \bar{B} + A\cdot\bar{C} + \bar{A}\cdot CD$

③ $y = F(A,B,C,D) = \Sigma m(1,3,7,11,15) + d(0,2,5)$ — don't care

↳ SOP form

A	B	C	D	y
0	0	0	0	x
0	0	0	1	1
0	0	1	0	x
0	0	1	1	1
0	1	0	0	0
0	1	0	1	x
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0

y

AB \ CD	00	01	11	10
00	x	1	1	x
01	0	x	1	0
11	0	0	1	0
10	0	0	1	0

let x = 1 use of don't care

$y = CD + \bar{A}\bar{B}$

In SOP form consider (x) as 1

In POS form consider (x) as 0.

POS Form

Y: AB \ CD	00	01	11	10
00	X	1	1	X
01	0	X	1	0
11	0	0	1	0
10	0	0	1	0

consider
$X = 0$

$y = D \cdot (\bar{B} + C) \cdot (\bar{A} + C)$ POS equation.

$y = F(A, B, C, D, E) = \Sigma m(0, 1, 2, 3)$

2^5 comb = 32

ABCDE	y
00000	1
00001	1
00010	1
00011	1
	0

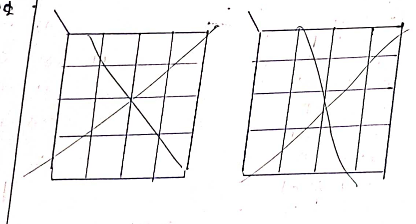
16+16 = 32 cells

A = 0

BC \ DE (A = 0)	00	01	11	10
00	1	1	1	1
01				
11				
10				

A = 1

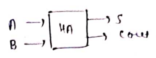
BC \ DE (A = 1)	00	01	11	10
00				
01				
11				
10				



$\boxed{y = \bar{A}\bar{B}\bar{C}}$

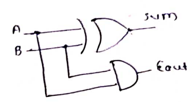
Half Adder

A	B	S	carry
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1



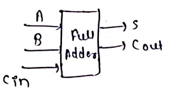
(SOP Form) $S = \bar{A}B + A\bar{B} = A \oplus B$ (minterms)

$C_{out} = AB$



Full Adder

↳ Add two bits along with possible carry



Truth table

A	B	Cin	S	Cout
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

K-map sum

A \ B Cin	00	01	11	10
0	0	1	0	1
1	1	0	1	0

$S = A\bar{B}\bar{C}_{in} + \bar{A}\bar{B}C_{in} + ABC_{in} + \bar{A}B\bar{C}_{in}$

$= A(\bar{B}\bar{C}_{in} + BC_{in}) + \bar{A}(\bar{B}C_{in} + B\bar{C}_{in})$

$= A\bar{B}\bar{C}_{in} + ABC_{in} + \bar{A}(B \oplus C_{in})$

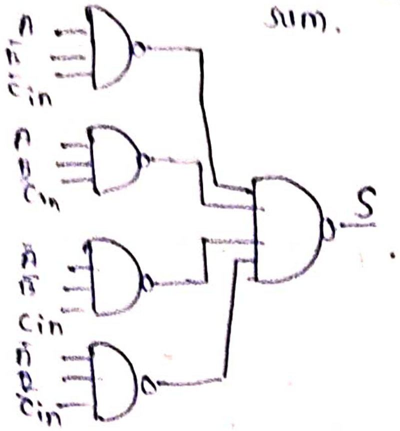
↳ X-OR

Cout

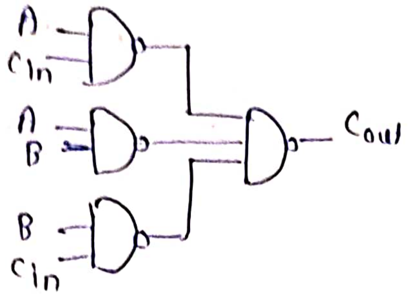
A \ B Cin	00	01	11	10
0	0	0	1	0
1	0	1	1	1

$C_{out} = BC_{in} + AC_{in} + AB$

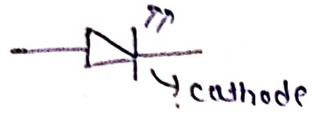
NAND - NAND realization for sum.



carry (cout)

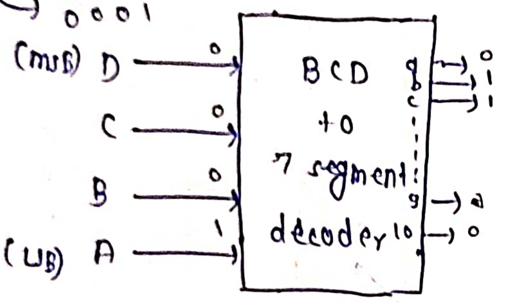


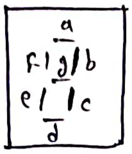
D BCD to 7 segment Decoder
- ① common anode
- ② common cathode type display
  - all cathodes are connected to each other



common cathode type

to display: DCBA
① 0001





displays 1.

Decimal no.	BCD
0	0000
1	0001
2	
⋮	
9	1001
10	0001 0000

DCBA	a	b	c	d	e	f	g
0000	1	1	1	1	1	1	0
0001	0	1	1	0	0	0	0
0010	1	1	0	1	1	0	1
0011	1	1	1	1	0	0	1
0100	0	1	1	0	0	1	1
0101	1	0	1	1	0	1	1
0110	0	0	1	1	1	1	1
0111	1						
1000	1						
1001	1						

K-map for A

a: DC \ BA	00	01	11	10
00	1	0	1	1
01	0	1	1	0
11	X	X	X	X
10	1	1	X	X

$$a = D\bar{C} + BA + CA + \bar{D}\bar{C}\bar{A}$$