Unit 2:

Principle of Inclusion and Exclusion:

Let A and B Finite sets

$|A \cup B| = |A| + |B| - |A \cap B|$

$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$

Q. ① In a survey of group of 80 people it is found that 60 like egg, 30 like fish, fish & egg like?

$|E \cap F| = ?$

60 people like egg

$|E| = 60$

$|F| = 30$

$|E \cup F| = 80$

$|E \cup F| = |E| + |F| - |E \cap F|$ (By principle)

$80 = 60 + 30 - x$

$x = 90 - 80 = 10$

Thus, $|E \cap F| = 10$

② Out of 200 students 50 take maths, 140 of them take economics, 24 take both. How many of them who did not take any course.

$|M| = 50$

$|E| = 140$

$|M \cap E| = 24$

By principle of Inclusion and exclusion

$|M \cup E| = |M| + |E| - |M \cap E|$

$= 50 + 140 - 24$

$= 190 - 24$

$|M \cup E| = 166$

No. of students who did not take any of cours

$= 200 - |M \cup E|$

$= 200 - 166 = 34$

(3) It is known that At the university, 60% professor play tennis, 50% professor play football, 70% play hockey, 20% play tennis and football, 30% tennis and hockey, 40% football and hockey. So if someone claim that 20% professor play all the three games, are you believe the claim.

$|T| = 60$ $|T \cap H| = 30$

$|F| = 50$ $|F \cap H| = 40$

$|H| = 70$

$|T \cap F| = 20$

Let there are 100 professor which play atleast one game

$|T \cup F \cup H| = 100$

$|T \cup F \cup H| = |T| + |F| + |H| - |T \cap F| - |T \cap H| - |F \cap H| + |T \cap F \cap H|$

$100 = 60 + 50 + 70 - 20 - 30 - 40 + |T \cap F \cap H|$

$100 = 180 - 90 + x$

$100 = 90 + x$

$x = 100 - 90$

$x = 10$

$|T \cap F \cap H| = 10$

$\therefore$ 10% professor play all the games

Thus, claim is incor

(4) In a survey of usage, of three toothpastes A, B and C it is found that 60 like A, 55 like B, 40 like C, 20 like A and B, 35 like B and C, 15 like A & C and 10 like all.

$|A| = 60$ $|B \cap C| = 35$

$|B| = 55$ $|A \cap C| = 15$

$|C| = 40$

$|A \cap B| = 20$ $|A \cap B \cap C| = 10$

$\ldots - |A\cap B| - |B\cap C| - |A\cap C| + |A\cap B\cap C|$

$= 60+55+40-20-35-15+10$

$= 115+50-70 = 95$

$|A\cup B\cup C| = 95$

5) A survey on a sample of 25 new cars being sold by local autodealer was conducted to see which of three popular options AC (A), Radio (R), power window (W) are already installed. The survey found that 15 had A, 12 had R, 11 had W, 5 had A & W, 9 had A & R, 4 had R & W, 3 had all options. Find ① cars with atleast one option. ② with no option ③ only A and R not W ④ only A and W not R ⑤ only R and W not A ⑥ only A ⑦ only R ⑧ only W ⑨ only 1.

$|A| = 15$

$|R| = 12$

$|W| = 11$

$|A\cap W| = 5$

$|A\cap R| = 9$

$|R\cap W| = 4$

$|A\cap R\cap W| = 3$

$|A\cup R\cup W| = |A| + |B| + |C| - |A\cap R| - |A\cap W| - |A\cap W| + |A\cap R\cap W|$

$= 15+12+11 - 5 - 9 - 4 + 3$

$= 38 - 18 + 3$

$= 20+3$

i) $|A\cup R\cup W| = 23$

ii) with no option $= 25 - |A\cup R\cup W| = 25-23 = 2$

iii) only A & R not W $= |A\cap R| - |A\cap R\cap W|$

$= 9-3 = 6$

iv) only A & W not R $= |A\cap W| - |A\cap R\cap W|$

$= 5-3 = 2$

v) only R and W not A $= |R\cap W| - |A\cap R\cap W| = 4-3 = 1$

vi) only A $= |A| - |A\cap R| - |A\cap W| + |A\cap R\cap W|$

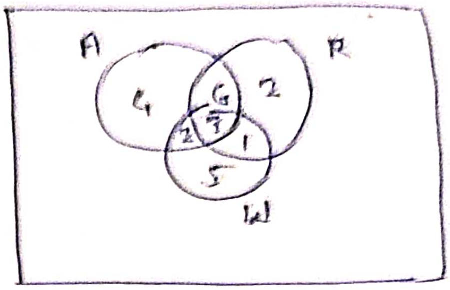
$= 15 - 5 - 9 + 3$

only A $= 4$

vii) only R : $|R| - |A\cap R| - |R\cap W| + |A\cap R\cap W|$
$= 12 - 9 - 4 + 3$
$= 2$

viii) only W : $|W| - |A\cap W| - |R\cap W| + |A\cap R\cap W|$
$= 11 - 5 - 4 + 3$
$= 5$

(8) ix) only 1 : only A + only R + only W
$= 4 + 2 + 5$
$= 11$



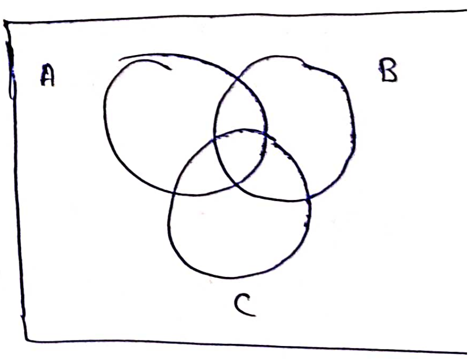
Q(6) A survey of household in US
96% have one television set
98% have telephone service
95% have telephone service and atleast one television set.
What is the % of household in US have neither telephone service nor television set

$|T| = 96$
$|P| = 98$
$|T\cap P| = 95$

$|T\cup P| = |T| + |P| - |T\cap P|$
$= 96 + 98 - 95$
$= 194 - 95$
$|T\cup P| = 99$ (for both)

% of household who have neither telephone service nor television set $= 100 - 99 = 1\%$.

(7) Among the integers 1 to 300 feel the venn diagram w.r.t divisibility by 3, 5 & 7.



$|A| = \left\lfloor \frac{300}{3} \right\rfloor = 100$

$|B| = \left\lfloor \frac{300}{5} \right\rfloor = 60$

$|C| = \left\lfloor \frac{300}{7} \right\rfloor = 42.8 \approx 42$

$|A\cap B| = \left\lfloor \frac{300}{3\times 5} \right\rfloor = 20$

Floor Fun
$\lfloor \ \rfloor$
Ceil Fun
$\lceil \ \rceil$

$|B \cap C| : \left\lfloor \frac{300}{5 \times 7} \right\rfloor = 8.5 \approx 8$

$|A \cap C| = \left\lfloor \frac{300}{3 \times 7} \right\rfloor = 14. \approx 14$ (Floor Function)

$|A \cap B \cap C| = \left\lfloor \frac{300}{3 \times 5 \times 7} \right\rfloor = 2. \approx 2$

① Divisibly by none
② only A & B not C
③ only B & C not A
④ only A & C not B
⑤ only A
⑥ only B
⑦ only C
⑧ only 1.

$|A \cup B \cup C| = |A| + |B| + |C|$
$- |A \cap B| - |A \cap C| - |B \cap C|$
$+ |A \cap B \cap C|$
$= 100 + 60 + 42 - 20 - 8 - 14$
$+ 2$
$= 202 - 40$

$|A \cup B \cup C| = 162$

① Divisible by none $= 300 - 162 = 148$

② only A & B not C $= |A \cap B| - |A \cap B \cap C|$
$= 20 - 2 = 18$

③ only B & C not A $= |A \cap B| - |A \cap B \cap C| = 8 - 2 = 6$

④ only A & C not B $= |A \cap C| - |A \cap B \cap C| = 14 - 2 = 12$

⑤ only A $= |A| - |A \cap B| - |A \cap C| + |A \cap B \cap C|$
$= 100 - 14 - 20 + 2$
$= 68$

⑥ only B $= |B| - |A \cap B| - |B \cap C| + |A \cap B \cap C|$.
$= 60 - 20 - 8 + 2$
$= 34$

⑦ only C $= |C| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$
$= 42 - 14 - 8 + 2$
$= 22$

⑧ only 1 = only A + only B + only C
$= 68 + 34 + 22$

only 1 = 124

▷ Recursive Definition

Recursively defined function-

Q.① Suppose that F is defined recursively by $F(0)=3$, $F(n+1)=2F(n)+3$

Find $F(1)$, $F(2)$, $F(3)$, $F(4)$.

Here, $F(0)=3$

$F(n+1) = 2F(n)+3$

For $n=0$

$F(1) = 2F(0)+3$

$= 2\times3+3$

$F(1) = 9$

For $n=1$

$F(2) = 2F(1)+3$

$= 2\times9+3$

$F(2) = 18+3 = 21$

For $n=2$

$F(3) = 2F(2)+3$

$= 2\times21+3$

$F(3) = 45$

For $n=4$

$F(4) = 2F(3)+3$

$= 2\times45+3$

$F(4) = 93$

② Q. Find $F(1)$, $F(2)$, $F(3)$, $F(4)$ if $F(n)$ defined recursively by $F(0)=1$, and for $n=0,1,2,\ldots$

a) $F(n+1) = F(n)+2$

For $n=0$

$F(1) = F(0)+2$

$= 1+2 = 3$

For $n=1$

$F(2) = F(1)+2$

$= 3+2$

$= 5$

For $n=2$

$F(3) = F(2)+2$

$= 5+2$

$= 7$

For $n=3$

$F(4) = F(3)+2$

$= \cancel{9}7+2$

$= 9$

- We we two steps to define funct with the set of non-negative int as its domain:

① Basis

Specify the value of the functi

② Recursive step

Give a rule for finding its va at an integer from its v smaller integers.

Q ③ $F(n+1) = 3F(n)$

$F(0) = 1$

Soln For $n = 0$

$F(1) = 3F(0)$
$= 3 \times 1 = 3$

For $n=1$, $F(2) = 3F(1)$
$= 3 \times 3$
$= 9$

For $n = 2$

$F(3) = 3F(2)$
$= 3 \times 9$
$= 27$

For $n = 3$

$F(4) = 3F(3)$
$= 3 \times 27$
$= 81$

Q ④ $F(n+1) = 2^{F(n)}$

$F(0) = 1$

For $n = 0$,
$F(1) = 2^{F(0)}$
$= 2^1 = 2$

For $n = 1$
$F(2) = 2^{F(1)}$
$= 2^2 = 4$

For $n = 2$
$F(3) = 2^{F(2)}$
$= 2^4$
$= 16$

For $n = 3$
$F(4) = 2^{F(3)}$
$= 2^{16}$
$F(4) = 65536$

Q ⑤ $F(n+1) = F(n)^2 + F(n) + 1$

$F(0) = 1$

For $n = 0 \Rightarrow F(1) = F(0)^2 + F(0) + 1$
$= 1+1+1 = 3$

For $n = 1 \Rightarrow F(2) = F(1)^2 + F(1) + 1$
$= 9 + 3 + 1 = 13$

For $n = 2 \Rightarrow F(3) = F(2)^2 + F(2) + 1$
$= 169 + 13 + 1 = 183$

For $n = 3 \Rightarrow F(4) = F(3)^2 + F(3) + 1$
$= 183^2 + 183 + 1$

$F(4) = 33673$

Q. ⑥ $F(0) = 3$

ⓐ $F(n+1) = -2F(n)$

ⓑ $F(n+1) = 3F(n) + 7$

ⓐ For $n = 0 \Rightarrow F(1) = -2F(0)$
$= -6$

For $n = 1 \Rightarrow F(2) = -2F(1)$
$= 12$

For $n = 2 \Rightarrow F(3) = -2F(2)$
$= -24$

For $n = 3 \Rightarrow F(4) = -2F(3)$
$= 48$

ⓑ For $n = 0 \Rightarrow F(1) = 3F(0) + 7$
$= 9 + 7 = 16$

For $n = 1 \Rightarrow F(2) = 3F(1) + 7 = 48 + 7 = 55$

For $n = 2 \Rightarrow F(3) = 3F(2) + 7 = 3 \times 55 + 7$
$= 165 + 7 = 172$

For $n = 3 \Rightarrow F(4) = 3F(3) + 7 = 3 \times 172 + 7$
$= 516 + 7$
$= 523$

Q.7 $F(n+1) = F(n)^2 - 2F(n) - 2$, $F(0) = 3$

For $n=0$, $F(1) = F(0)^2 - 2F(0) - 2$

$= 9 - 6 - 2 = 1$

For $n=1$, $F(2) = F(1)^2 - 2F(1) - 2$

$= 1 - 2 - 2$

$= -3$

For $n=2$, $F(3) = F(2)^2 - 2F(2) - 2$

$= 9 + 6 - 2$

$= 13$

For $n=3$, $F(4) = 169 - 26 - 2$

$= 141$

For $n=4$, $F(5) = F(4)^2 - 2F(4) - 2$

$= 141^2 - 282 - 2$

$= 19597$

Q.8 $F(n+1) = 3^{F(n)/3}$ $F(0) = 3$

For $n=0$, $F(1) = 3^{F(0)/3} = 3^{3/3} = 3$

For $n=1$, $F(2) = 3^{F(1)/3} = 3^{3/3} = 3$

For $n=2$, $F(3) = 3^{F(2)/3} = 3^{3/3} = 3$

For $n=3$, $F(4) = 3^{F(3)/3} = 3^{3/3} = 3$

For $n=4$ $F(5) = 3$

▷ Pigeonhole principle

The principle states that if there are more pigeons than there must be atleast one pigeon hole with atleast two ~~pigeons~~ pigeons in it.

Defn :- If k is positive integer and k+1 or more objects are ~~placed~~ placed in k boxes , then there is atleast one box containing two or more of the objects.

$n > k$ (always)

n → no. of pigeons

k → no. of boxes