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Remark

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★ Descrete Mathematics

Reference - Descrete Mathematics & its Appⁿ by Kenneth H. Rosen

- Set theory
- Mathematical inclination
- Recurrence
- Relations
- Permutation & combination
- Trees
- Graphs

(1) Set Theory

- Set is a collection well defined objects
  $A = \{1, 2, 3, 4\}$, $3 \in A$
  $B = \{a, e, i, o, u\}$

- Cardinality of a set :- No. of elements in a set.
  $|A| = 4$ & $|B| = 5$

- ~~Subject~~ Subset :- $[\subseteq]$, $A \subseteq B$, $\phi \in A$
  All element of A pnt in B
  Null set is subset of every set

- Special set :- Set of Natural No → $\mathbb{N}$
  Integers → $\mathbb{Z}$
  Rational No. → $\mathbb{Q}$
  Real No. → $\mathbb{R}$

$\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$

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* Two ways- Set Representation
① Listing Method $A = \{1, 2, \dots\}$
② Set builder notation $A = \{2n \mid n \leq 5\}$

- Power set of a set :- It is a set of all subsets of a set.

$A = \{1, 2, 3\}$

$P(A) = \{ \phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\} \}$

$|P(A)| = 8$

Cardinality of set $\xrightarrow{\text{Always}} 2^{|A|} \Rightarrow 2^3$

- Operations

① Union :- $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

$A = \{1, 2, 4\}$

$B = \{1, 2, 3\}$ $A \cup B = \{1, 2, 3, 4\}$

② Intersection :- $A \cap B = \{x \mid x \in A \ \& \ x \in B\}$

$A \cap B = \{1, 2\}$

③ Set Difference :- $A - B = \{x \mid x \in A \ \& \ x \notin B\}$

$A - B = 4$, $B - A = 3$

④ Symmetric Difference :- $A \oplus B = \{x \mid (x \in A \text{ or } x \in B) \ \& \ x \notin (A \cap B)\}$

$A \oplus B = (A \cup B) - (A \cap B)$

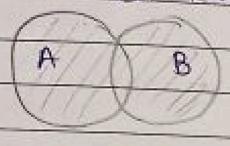
$\{1, 2, 3, 4\} - \{1, 2\}$

$A \oplus B \Rightarrow \{3, 4\}$

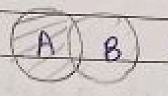
* Universal set [U] :-

→ Complement of a set $\bar{A}$

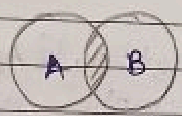
All the elements in U but not in A ; $\bar{A} = U - A$



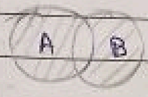
$A \cup B$



$A - B$



$A \cap \bar{B}$



$A \oplus B$

* De-Morgan's law :-

$$\overline{A \cup B} = \bar{A} \cap \bar{B}$$

$$\overline{A \cap B} = \bar{A} \cup \bar{B}$$

* Laws of Sets :-

1) Commutative :- $a+b = b+a$

$$A \cup B = B \cup A$$

$$A \cap B = B \cap A$$

$$A \oplus B = B \oplus A$$

$$A - B \neq B - A$$

2) Associative operation :- $(a+b)+c = a+(b+c)$

$$(A \cup B) \cup C = A \cup (B \cup C)$$

$$(A \cap B) \cap C = A \cap (B \cap C)$$

$$(A \oplus B) \oplus C = A \oplus (B \oplus C)$$

3) Distributive Law :-

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

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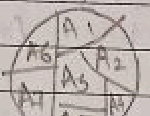
4) Identity element :-

$2 + \underline{\underline{0}} = 2$
$2 \times \underline{\underline{1}} = 2$

5) Partition of a set :-



union
$A = A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5 \cup A_6 \cup A_7$
Intersection is empty
$|A_1|$ or $n(A)$
cardinality

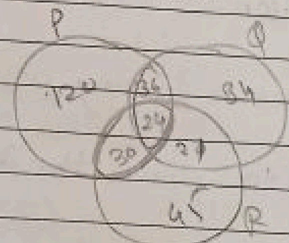
$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$
no. of element

$n(A \cap B \cap C) = n(A \cap B) + n(B \cap C) + n(A \cap C) - n(A) - n(B) - n(C) + n(A \cup B \cup C)$

1.Q) P, Q, R are the subsets of universal set U
If $n(U) = 390$, $n(P) = 210$, $n(Q) = 165$
$n(R) = 120$, $n(P \cap Q) = 60$, $n(Q \cap R) = 45$
$n(P \cap R) = 54$, $n(P \cap Q \cap R) = 24$

Illustrate using venn diagram & also find
no. (P) = 120
no. (Q) = 84
no (R) = 45
no $(P \cap Q)$ = 36
no $(Q \cap R)$ = 21
no. $(P \cap R)$ = 30

$390 - 210 = 180$



Q2) In a survey of a group of people 60 → like Tea — T

45 → Coffee — C

30 → milk — M

25 → coffee & Tea

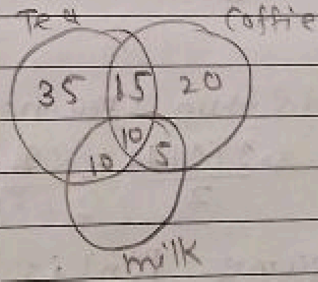
20 → Tea & milk

15 → Coffee & milk

10 → all (coffee, tea & milk)

How many people where there

→



$60 - 25 = 35$

$45 - 25 = 20$

30 -

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$

$$= 60 + 45 + 30 + 10 - 25 - 20 - 15$$

$$= 145 - 60$$

$$= 85.$$

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★ Mathematical Induction:-

$P(n)$: . where n is any positive integer

defn → It is a technique to prove the formula, theorem,...

Principle of Mathematical Induction
Two steps :-

1. Basis step :- $P(n)$: given, show that $P(n)$ is true for $n=0$ or $n=1$

2. Inductive step :- Assume that $P(n)$ is true for $n=k$, where k is any positive number/Integer. then, show that $P(n)$ is also true for $n=k+1$.

Ex :- $P(n) : 1+2+3+4 \cdots +n = \frac{n(n+1)}{2}$

1. Basis step : $P(n)$: is true for $n=1$.
$LHS = 1$, $RHS = \frac{1(1+1)}{2} = 1$

2. Inductive step :- Assume $P(n)$ is true for $n=k$
$1+2+3+4 \cdots +k = \frac{k(k+1)}{2}$ → True — ①

Show that It is also true for $n=k+1$
$1+2+3+4 \cdots + k+1 =$
$\underbrace{1+2+3+4 \cdots +k}+k+1$

From ① $\quad LHS = \frac{K(K+1)}{2} + K+1 = \frac{K(K+1)+2(k+1)}{2}$

$$= \frac{(K+1)(k+2)}{2}$$

$\{n = k+1\}$

$$RHS = \frac{(K+1)(k+2)}{2}$$

Hence, $P(n)$ is true

Ex:- Prove the following formula using M.I (Mathematical Induction)

① $1^2 + 3^2 + 5^2 + \cdots + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$

$\longrightarrow$ 1. Basis step: $P(n)$: is true for $n = 1$

$LHS = 1, \quad RHS = \frac{1(2(1)-1)(2(1)+1)}{3} = \frac{3}{3} = 1$

2. Inductive step: Assume $P(n)$ is true for $n = K$

$1^2 + 3^2 + 5^2 + \cdots + (2K-1)^2 = \frac{K(2K-1)(2K+1)}{3} \rightarrow$ True — ①

Show that It is also true for $n = k+1$

$1^2 + 3^2 + 4^2 + \cdots + (2K+1 \cancel{\mp} 1)^2 =$

$1^2 + 3^2 + 4^2 + \cdots + (2K - 1 \cancel{K} + 1)^2$

From ① $\quad n = k+1$

$LHS = \frac{\cancel{k}(2k-1)(2k+1)}{3} \cancel{(2K+1-1)^2} + (2K+1)^2$

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$$\frac{k(2k-1)(2k+1)+3(2k+1)^2}{3} = \frac{(2k+1)(k(2k-1)+3(2k+1))}{3}$$

$$\frac{(2k+1)(2k^2-k+6k+3)}{3} = \frac{(2k+1)(2k^2+5k+3)}{3}$$

$$= \frac{(k+1)(2k+1)(2k+3)}{3}$$

substitute $n = k+1$ in $\frac{n(2n-1)(2n+1)}{3}$

$$\frac{(k+1)(2(k+1)-1)(2(k+1)+1)}{3} = \frac{(k+1)(2k+2-1)(2k+2+1)}{3}$$

$$= \frac{(k+1)(2k+1)(2k+3)}{3}$$ $\therefore$ LHS = RHS

Hence, $p(n)$ is true

2) $1^3+2^3+3^3+\dots+n^3 = \left(\frac{n(n+1)}{2}\right)^2$

→ 1. Basis step: $P(n)$: is true for $n=1$

LHS $= 1$, RHS $= \left(\frac{1(1+1)}{2}\right)^2 = 1$

2. Inductive step: Assume $P(n)$ is true for for $n=k$

$$1^3+2^3+3^3+\dots+k^3 = \left(\frac{k(k+1)}{2}\right)^2 = \text{True} \quad \text{--- (1)}$$

Show that It is also true for $n = k+1$

$$1^3+2^3+3^3+\dots+k^3+1 =$$

$$1^3+2^3+3^3+\dots k^3+(k+1)^3$$

from eq^n (1)

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LHS: $\left(\frac{K(K+1)}{2}\right)^2 + (K+1)^3$

$\frac{K^2(K+1)^2}{4} + (K+1)^3$

$= \frac{K^2(K+1)}{4} + K^3 + 1^3 + 3K^2 + 3K)$

$= K^2(K+1) + 4K^3 + 4 +$

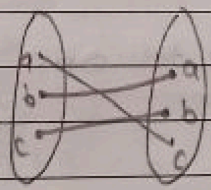
☆ Functions :- It is a rule that relates one entity to other. It maps from one set to other set.

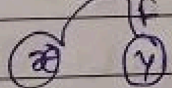
$F: A \to B$

$f(a) = c$

$f(b) = a$

$f(c) = b$





Domain Codomain

Range

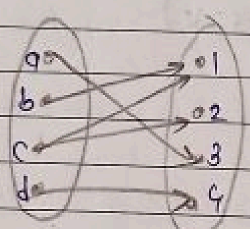
(i) Injective or one to one :- A function is one to one if it follows

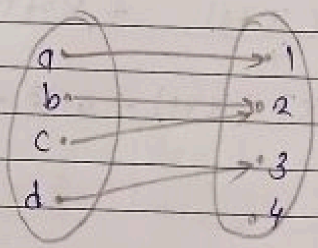
(i) For all $a, b \in A$; If $f(a) = f(b)$ then $a = b$

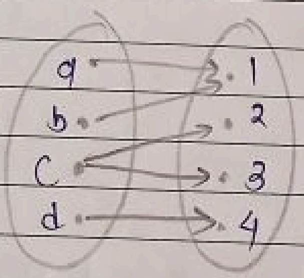
(ii) for all $a, b \in A$, if $a = b$ then $f(a) = f(b)$

(ii) Onto / Surjective

For all $y \in B$ there exists $x \in A$ $y = f(x)$







Question

(A) Function are one to one / onto, and Find the range.

(1) $F : R \to R$ where $F(x) = x^2$

$\Rightarrow$ If $F(a) = f(b)$, then $a = b$

$a^2 = b^2$, $a \neq b$, The function is not one to one

$y = f(x) = a^2$, $a^2 = b$, $a = \pm\sqrt{b} \in R$

$\therefore$ The func^n is onto.

Range $\Rightarrow$

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(2) $F: R \to R$, where $f(x) = 2x+3$

$\longrightarrow$ If $f(a) = f(b)$ then $a=b$

$2a+3 = 2b+3$, $2a = 2b$, $a=b$

Thus, the func^n is one to one

$Y = f(x)$

$2a+3 = b$, $2a = b-3$,

$a = \left(\frac{b-3}{2}\right) \in R$

So Thus, the func^n is onto.

Range:-

(3) $F: Z \to Z$, where $f(x) = 2x+3$

$\longrightarrow$ If $f(a) = f(b)$ then $a=b$

$2a+3 = 2b+3$, $a=b$

So The func^n is one to one

let $Y = f(x)$, $2a+3 = b$, $2a = b-3$

$a = b - 3/_3 \notin Z$

Thus function is not onto

Range:-

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(4) $F: R \to R$, where $f(x) = x^2+1$

→ If $f(a) = f(b)$ then $a = b$

$a^2 + \cancel{1} = b^2 + \cancel{1}$, $a \neq b$, Thus Function is not one to one

let $y = f(x)$

$a^2 + 1 = b$, $a^2 = b-1$, $a = \pm\sqrt{b-1} \notin R$

Thus funct^n is not onto

Range:-

(5) $F: Z \to Z$ where $f(x) = x^2+1$

→ $f(a) = f(b)$ the $a = b$

$a^2 + 1 = b^2 + 1$, $a \neq b$, the function is not one to one

$y = f(x) \Rightarrow a^2+1 = b$, $a = \pm\sqrt{b-1} \in Z$

Thus the function is onto

Range:-