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Set theory

1) Null set or empty set
2) Singleton Set → 1 element

S.T

1) Null or Empty Set {}

2) Singleton Set {a} → 1 element

3) Finite Set {1, 2 … 10}

4) Infinite set {1, … n}

5) Power Set : Set of Subset

$S = \{1, 2, 3\}$

$P(S) = \{ \{\}, \{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}, \{1,3\}, \{1,2,3\} \}$

2^n = cardinality of power of set

* Properties ~~Cardinality~~ of a Set

1) Union (∪)
2) Intersection (∩)
3) Set Difference (A-B)
4) Set Complement (A^c)
5) Cross product (A×B) / Cartesian product of set

$A = \{1, 2, 3\}$ $B = \{5, 3, 1\}$

$A \times A = \{ (1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3) \}$

$B \times B = \{ (5,5), (5,3), (5,1), (3,5), (3,3), (3,1), (1,5), (1,3), (1,1) \}$

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$A \times B = \{ (1,5), (1,3), (1,1), (2,5), (2,3), (2,1), (3,5), (3,3), (3,1) \}$

* Cardinality of a set
No. of elements in set

Q. What will be the cardinality of power set of $S = \{0, 1, 2, 3, 4, 5\}$

→ Cardinality of power set of $S = 2^6 = 64$

Q. Find the power set of even no. b/w 20 to 30 and also find cardinality of given set

→ ~~S = {20~~

$S = \{20, 22, 24, 26, 28, 30\}$ Cardinality $= 2^6 = 64$

* Cardinality of empty set $= 2^0 = 1$

1 Q. in Mid & End term

* Relation

Relation may exist between objects of the same set or b/w objects of two or more sets

$A = \{1, 2, 3, 4\}$ $B = \{4, 5\}$

$A \times A = \{ (1,1), (1,2), (1,3), (1,4), (2,1), \ldots \}$

a, b

a < b

$A \times B = \{ (1,4), (1,5), (2,4), (2,5), (3,4), (3,5), (4,4), (4,5) \}$

a = b

B

* Definition and Properties

A binary relation R is from a set X to Y (written as xRy or $R(x,y)$) is a subset of Cartesian product)

ex $A=\{1,2,3\}$ $B=\{4,5,6\}$ 11/8/23

$A\times B=\{(1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6)\}$

1) even pairs

$R=\{(2,4),(2,6)\}$

2) Sum is even

$R=\{(1,5),(2,4),(2,6),(3,5)\}$

* Relation:

In a simple terms relation is a set of rules on two sets A and B

- A Relation R from A to B is defined as a subset of $A\times B$.

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$A=\{1,2,5\}$ $B=\{2,4\}$

1) Find relation $x<y$
2) — $x>y$
3) — $x=y$

$R_1=A\times B=\{(1,2),(1,4),(2,2),(2,4),(5,2),(5,4)\}$

$R_1(x<y)=\{(1,2),(1,4),(2,4)\}$

$R(x>y)=\{(5,2),(5,4)\}$

$R(x=y)=\{(2,2)\}$

$A=\{1,2,5\}$
$\{(1,1),(1,2),(1,5),(2,1),(2,2),(2,5),(5,1),(5,2),(5,5)\}$

$R(x<y)=\{(1,2),(1,5),(2,5)\}$

$R(x>y)=\{(2,1),(5,1),(5,2),(5,5)\}$

$R(x=y)\{(1,1),(2,2),(5,5)\}$

** Total number of distinct relation from a set A to set B is 2^{mn} where $n(A)=m, n(B)=n$

ex $n(A)=4$ $n(B)=3 \Rightarrow$ Total no. of Rel $=2^{4\times3}$
$=2^{12}$

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✱ Domain and Range

The set $\{a \in A : (a,b) \in R$ for some $b \in B\}$ is called domain of R and denoted by dom (R)

- The set $\{b \in B : (a,b) \in R$ for some $a \in A\}$ is called range and denoted by Ran (R)

Ex

Let $A = \{2,3,4\}$ & $B = \{3,4,5\}$ list the element of each relation R defined below and also find Domain & Range?

→

(a) $a \in A$ is related to $b \in B$ that is aRb if and only if $a<b$.

(b) $a \in A$ is related to $b \in B$ that is aRb, if a and b are odd numbers

$A \times B = \{(2,3)(2,4)(2,5)(3,3)(3,4)(3,5)(4,3)(4,4)(4,5)\}$

⟶ (a) $R(a<b)$ $R = \{(2,3),(2,4),(2,5),(3,4),(3,5)$
$(4,5)\}$

$Dom(R) = \{2,3,4\}$ $Ran(R) = \{3,4,5\}$

$R(odd) = \{(3,3),(3,5)\}$

$Dom(R) = \{3\}$ $Ran(R) = \{3,5\}$

Graph: Collection of edges and vertices

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$R = \{(2,4)(1,8)(3,5)(2,8)(1,3)(1,5)(4,5)\}$

$Dom(R) = \{1,2,3,4\}$

$Ran(R) = \{3,4,5,8\}$

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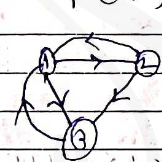
✱ Graphical Representation of Relation.

$R = \{(1,2)\ (1,3)(2,1)\ (2,3)(3,1)\}$



Graphical Rep of Rel^n

✱ Different Types of Relations

① Empty Relation $R = \{\} = \phi$

② Full Relation A set $A = \{1,2,3\}$
$A \times B = \{\ \}$
all elements

③ Inverse Relation
$R = \{(a,b), (a,c)\}$
$R^{-1} = \{(b,a), (c,a)\}$

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* Reflexive Relation $R = \{a, b, c\}$

if there exist $S = \{(a,a), (b,b), (c,c)\}$

A relation R on set A is called reflexive relation if all $a \in A$ is related to (aRa)

eg. $R = \{a, b, c\}$

$S = \{(a,a), (b,b), (c,c)\}$

* Symmetric Relation

$S = \{(a,b), (b,a)\}$ | $S \neq \{(a,b) (b,b)\}$
$S = \{(1,2), (2,1)\}$ | is not symmetric

A Relation R on set A is called symmetric relation if ~~all~~ xRy implies yRx $\forall x \in A$ and $\forall y \in A$

* Transitive Relation

A relation R on set A is called transitive relation if aRb and bRc implies aRc for all $(a, b, c \in A)$

$R = \{a, b, c\}$
$S = \{(a,b) (b,c) (a,c)\}$

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V. Imp. → 5 marks

** Equivalence Relation.

A Relation R on set A is called equivalence Relation if it satisfies following properties

1) Reflexive $\{(a,a), (b,b), (c,c)\}$
2) Symmetric $\{(a,b) (b,a)\}$
3) Transitive $\{(a,b) (b,c), (a,c)\}$

Q. Show that the given relation is equivalence relation or not.

$A = \{1,2,3\}$, $R = A \times A = \{(1,1) (1,2), (1,3)$
$(2,1)\ (2,2)\ (2,3)$
$(3,1)\ (3,2)\ (3,3)\}$

Satisfies all 3 properties
It is equivalence Relation

GBSRC

Q.1. Consider a set
$A = \{ 1, 2, 3, 4, 5 \}$

$R = \{ (5,5), (5,3), (2,2), (2,4), (3,5), (3,3), (4,2), (4,4) \}$

Reflexive — $\{ (5,5), (2,2), (3,3), (4,4) \}$
Symmetric — $\{ (5,3), (3,5), (4,2), (2,4) \}$
Transitive
~~Equivalence~~ $\{$ ~~(5,5)~~ $(5,3)$ $(3,5)$, $(5,5)$, $(2,4)$ $(4,2)$ $(2,2)$ $\}$
$(4,2)$ $(2,4)$ $(4,4)$. $(3,5)$ $(5,3)$ $(3,3)$ $\}$

$\Rightarrow$ Equivalence Rel^n

a, b, ~~c should be distinct~~

Q. Let us assume that R is a relation on the set S of Real numbers defined by xRy iff $(x-y)$ is an integer prove that R is a equivalence set.

$\{1, 2, 3, 4, 5\}$

$R = \{$ ~~(1,2) (1,3) (1,4) (1,5)~~ $(1,1)$,

let set is
$R = \{ 2, 3, 4, 5 \}$

$R = \{ (2,3)\ (2,2)$

$n(A) = 3$ $A \times A = 9$ $A = \{ 1, 2, 3 \}$

Q. Show that the following Relation is equivalence or not iff ~~(y/x)~~ y is divisible by x $\left(\frac{y}{x}\right)$

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Ans
let's take a set of real numbers as
$A = \{ 2, 3, 4, 5 \}$
$\Rightarrow R = \{ (2,2), (2,3), (2,4), (2,5), (3,2), (3,3), (3,4), (3,5), (4,2), (4,3), (4,4), (4,5), (5,2), (5,3), (5,4), (5,5) \}$
it's Reflexive

Symmetric

+ Transitive also

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Function's

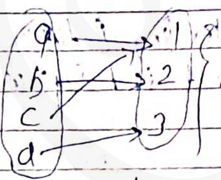
1) One to one (Injection)
2) Onto (Surjection)
3) One to one and onto (Bijection)

Let x, y be two sets with m and n elements and function is defined as $f: x \to y$ then

1) Total no of function $= n^m$
2) Total no of one-to-one function $= {}^nP_m$

$n \geq m$

code Screenshot

Q)  onto

Yes, f is onto since all three elements of the codomain are images of elements in domain.

Q) $f(x): Z \to Z$, Z is set of integers & $f(x) = x^2$ onto?

let $A = \{1, 2, 3\}$

$f(1) = 1$, $f(2) = 4$, $f(3) = 9$

☆ Inverse functions

let f be a bijection from A to B, then the inverse of f, denoted by f^{-1} is the function from B to A defined as

$f^{-1}(y) = x$ iff $f(x) = y$

No inverse exists unless f is a bijection

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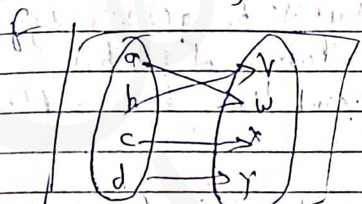
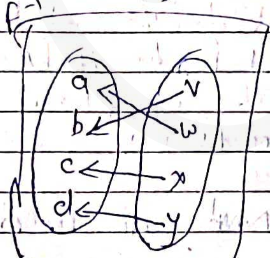
$A = \{a, b, c, d\}$ $B = \{v, w, x, y\}$



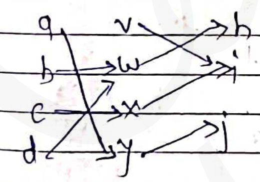
$f: A \to B$

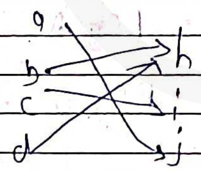


$f^{-1}: B \to A$

☆ Composition

$A \xrightarrow{g} B \xrightarrow{f} C$ $A \xrightarrow{f \circ g} C$





ex: $f(x) = x^2$ & $g(x) = 2x+1$, then

$f(g(x)) = (2x+1)^2$

$g(f(x)) = 2x^2+1$

since $f(g(x)) = f(2x+1) = (2x+1)^2$

$g(f(x)) = 2(x^2)+1$

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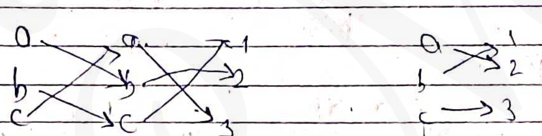
Q) Let g be the function from set $\{a, b, c\}$ to itself such that $g(a) = b$, $g(b) = c$, & $g(c) = a$.
Let f be the function from set $\{a, b, c\}$ to the set $\{1, 2, 3\}$ such that $f(a) = 3$, $f(b) = 2$ and $f(c) = 1$.

What are the compositions of fog & gof?

$\Rightarrow$ fog



$fog(a) = f(g(a)) = f(b) = 2$

$fog(b) = f(g(b)) = f(c) = 1$

$fog(c) = f(g(c)) = f(a) = 3$

Note that gof is not defined, because the codomain of f is not a subset of the domain of g.

Mathematical Induction 6/9/23

* Principal of MI

Let $S(n)$ be a statement involving an integer n.
Suppose that, for some fixed integer n_0,

a) $S(n_0)$ is true (i.e, $S(n)$ is true if $n = n_0$)

b) When integer $k >= n_0$ & $S(k)$ is true, then $S(k+1)$ is true.

eg: $S(n)$: $1+2+3+ \dots n = \frac{n(n+1)}{2}$

$n = 4 \rightarrow \frac{4 \times 5}{2} = 10$

$n = 100 \rightarrow \frac{(100 \times 101)}{2} = 5050$

a) Show that $S(n)$ is true for some (small) integer k
 a) That means, it is true at least sometimes!

b) Since $S(n)$ is true for some integer k, show it is also true for $k+1$.
 a) That means it is true for all integer $>= k$

→ Lets first show $S(n)$ is true for some small integer

let take 1

$S(1) = 1$

This means that 1 works in formula for n

Now assume that n works, so giving that assumption lets see if $S(n+1)$ works

$S(n+1) = 1+2+3+ \dots + n + n+1 = (n(n+1))/2 + n+1$

$= (n(n+1))/2 + 2(n+1)/2$

$= (n+2)(n+1)/2$

- $F_7 = ?$
- $F_n = 6$
- $F_{n+1} = 5$

→ $n=6$: $F_6 = 6$, $F_{6+1} = 5$, $F_7 = 5$

$F_2 = ?$

$F_n = 6$

$F_{n-1} = 5$

→ $n = 3$

$F_{n+1} = F_n + F_{n-1}$

$F_7 = F_6 + F_5$

$F_7 = 60$, $F_6 = 37$

$F_5 = F_7 - F_6 = 60 - 37 = 23$

$F_6 = F_5 + F_4$

$F_4 = 37 - 23$

$= 14$

$F_5 = F_4 + F_3$

$23 = 14 + F_3$

$\boxed{F_3 = 9}$

$F_4 = F_3 + F_2$

$14 = 9 + F_2$

$\boxed{F_2 = 5}$

$F_3 = F_2 + F_1$

$9 = 5 + F_1$

$\boxed{F_1 = 4}$

25-27 Sep → 2 paper each
30 min each sub.
Attendance 75% (out of 5 marks)

14/9/23

Syllabus for Isc-I
DM : 1st 3 Units
1) Descriptive type 2) (fill in the blanks)
2)

* Propositional logic

Logic - used to distinguish between valid and invalid mathematical arguments

· Proposition - is a declarative sentence either true or false, but not both

eg: 1) 1+1=2 - True
2) What time is it? → Not proposition
3) x+1=2 Not proposition

· letters are used to denote propositions p, q, r, s, ..

* Compound Statements

- combining one or more propositions

eg. John is smart or he studies every night

· One or more propositions can be combining & form a single compound proposition using connectives (logical operators).

Token: smallest individual part
printf ("hi") ;
1 2 3 4 5

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Connectives	Symbol	Name
And	$\wedge$	Conjunction
Or	$\vee$	Disjunction
Not	$\neg$, $\sim$	Negation

* Truth Table:

- can be used to show how logical operators can be combine propositions to compound proposition

- possible value $= 2^n$

- Truth values for proposition could be True or false (T) (F)

* formal Proposition

1] Not (Negation): $\sim$ / $\neg$
- p = "not p"

Truth table

p	$\sim p$
T	F
F	T

2) AND (conjunction): $\wedge$
→ $p \wedge q$ is TRUE when Both p & q are true otherwise is FALSE

p	q	p ∧ q
T	T	T
T	F	F
F	T	F
F	F	F

eg. Student who have taken calculus and computer sciences can take this class

3) OR (disjunction) : ∨
"p or q"

$p \vee q$ = False when both p & q are False

p	q	p ∨ q
T	T	T
T	F	T
F	T	T
F	F	F

ex:
Determine true or false
1. Ice floats in water and 2+2=4 → True
2) China is in Europe and 2+2=4 → False
3) 5-3=1 or 2+2=4 → True

p - Anwar is a good lecturer
q : Hidayah is a good teacher
r : Anwar's student hate mathematics
s Hidayah's student hate mathematics

1) $p \wedge \sim q$
2) $q \vee \sim q$
3) $r \wedge \sim r$
4) $p \vee r \wedge \sim q$
5) $p \wedge r$
6)

MCQ. or Fill in the blanks

18/9/23

Rules of Inference

A theorem is a statement that can be shown to be true

- A proof is the means of doing so

Rules:

1. Modus Ponens
2. Modus Tollens
3. Addition
4. Simplification

Tautology:	Inference Rule
$(p \land (p \to q)) \to q$	Modus Ponens
~~$(\neg p \land (p \to q)) \to \neg q$~~	
$(\neg q \land (p \to q)) \to \neg p$	Modus Tollens
$p \to (p \lor q)$	Addition
$(p \land q) \to p$	Simplification
$((p \lor q) \land \neg q) \to p$	Disjunctive Syllogism
$(p \to q) \land (q \to r) \to (p \to r)$	Hypothetical Syllogism

$p \to q = \sim p \lor q$

- Ellen is a math major or CS major
  1. $M \lor C$
- If Ellen does not like discrete math, she is not a CS major
  2. $\neg D \to \neg C$
- If Ellen likes discrete math, she is smart
  3. $D \to S$
- Ellen is not a math major
  4. $\neg M$

5. C	$(M \lor C) \land \neg M$ DS (1, 4)
6. D	MT (2, 5)
7. S	MP (3, 6)

1. Construct the truth table for the compound proposition

$$(p \to q) \leftrightarrow (\neg p \to \neg q).$$

3. What are the contrapositive, the converse and inverse of conditional statement "If you work hard then you will be rewarded"

4. Find truth table for statement $p \to \neg q$.

9. Show that propositions $p \to q$ & $\neg p \vee q$ are logically equivalent

3. (b) Write the symbolic form and negate following statements:

1.

p	$\neg p$	q	$\neg q$	$p \to q$	$p \leftrightarrow q$	$\neg p \to \neg q$	$(p \to q) \leftrightarrow (\neg p \to \neg q)$
T	F	T	F	T	T	T	T
T	F	F	T	F	F	T	F
F	T	T	F	T	F	F	F
F	T	F	T	T	T	T	T

3. If you work hard then you will be rewarded

$\to$ $p \to q$

Contrapositive: If you will not be rewarded then you don't work hard.

Converse: If q then p.

If you will be rewarded then you work hard

Inverse: If not p then not q

If you don't work hard then you will not be rewarded

4. $p \to \neg q$

p	q	$\neg q$	$p \to \neg q$
T	T	F	F
T	F	T	T
F	T	F	T
F	F	T	T

p ↔ p			p → q		T
T	T	T	T	F	F
F	P	T			

9.

¬p	p	q	p → q	¬p ∨ q
F	T	T	T	T
F	T	F	F	F
T	F	T	T	T
T	F	F	T	T

from this we can see that

$p \to q$ & $\neg p \vee q$ are logically equivalent.

3(b)

13 $p \vee \neg(p \wedge q)$ is a tautology

p	q	p ∧ q	¬(p∧q)	p ∨ ¬(p∧q)
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

$\Rightarrow p \vee \neg(p \wedge q)$ is a tautology

12 Is $\neg p \wedge (p \vee q)) \to q$ a tautology?

→

p	q	¬p	p ∨ q	¬p ∧ (p ∨ q)	¬p ∧ (p∨q) → q
T	T	F	T	F	F
T	F	F	T	F	F
F	T	T	T	T	T
F	F	T	F	F	T

Not a tautology.

19. Indirect proof: If $3n+2$ is odd then n is odd

→

Assume that n is even, then $n = 2k$, for some integer k

$\Rightarrow 3(2k)+2 = 6k+2$, we see that $3n+2$ is even.

But we were given that $3n+2$ is odd, so this is a contradiction

$\Rightarrow$ Our assumption that n is even must be false $\Rightarrow$ n must be odd

$\Rightarrow$ If $3n+2$ is odd then n is odd.

Om Deshmukh

$p \to q = \neg p \vee \neg q$ | $\forall$ - for all

$p \leftrightarrow q = (p \to q)$, | $\exists$ - there exists

4-Q) PDNF of $(\neg p \to r) \wedge (q \leftrightarrow r)$

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~~* Universal Quantifier~~

* Predicates & Quantifiers

All dogs are animals

- All — quantifier
- dogs — subject term
- are — copula
- animals — predicate term

Limitations of propositional logic

· Proposition logic cannot adequately express the meaning of statements

· Suppose we know
"Every computer connected to the university network is functioning properly"

· No rules of propositional logic allow us to conclude
"MATH3 is functioning properly"
where MATH3 is one of computers connected to university network

5/9/23

* Predicate & Quantifiers
- Can be used to express the meaning of a wide range of statements

· Allow us to reason and explore relationship between objects

· Predicates: statements involving variables,

eg. "$x>3$", "$x=y+3$", "$x+y=z$", "computer x is under attack by an intruder", "computer x is functioning properly"

ex: $x>3$
↳ subject of stmt

· Predicate "is greater than 3" refers to a property that the ~~subject of stmt~~ can have

· $P(x)$, P denotes predicate "is greater than 3"

· $P(x)$ aka propositional function P at x

· Once a value is assigned to variable x, $P(x)$ becomes a proposition and has truth value

- $P(4)$: true
- $P(2)$: false

all, some, many, none, few

5/9/23

Quantifiers

- Express the extent to which a predicate is true
- In English, all, some, many, none, few

- focus on two types:
  - Universal: a predicate is true for every element under consideration
  - Existential: a predicate is true for there is one or more elements under consideration
  - Predicate calculus: the area of logic that deals with predicates & quantifiers

Universal Quantifier ($\forall$)

- " $p(x)$ for all values of x in domain".
  - $\forall x \; p(x)$
- Read it as "for all x $p(x)$" or "for every x $p(x)$".
- A statement $\forall x \; p(x)$ is false if and only if $p(x)$ is not always true

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- An element for which $p(x)$ is false is called a counterexample of $\forall x \; p(x)$
- A single counterexample is all we need to establish that $\forall x \; p(x)$ is not true

Q "$x+1 > x$" what is the truth value of $\forall x \; p(x)$?

- Let $q(x)$ be statement "$x<2$" what is the truth value of $\forall x \; q(x)$ where is the domain consists of all real numbers?

Q What is the truth value of $\forall x \; p(x)$ when $p(x)$ is the statement "$x^2 < 10$" and the domain consists of positive integers not exceeding 4?

→ $\forall x \; p(x)$ is same as $p(1) \wedge p(2) \wedge p(3) \wedge p(4)$
T T T F
T ∧ T ∧ T ∧ F = F

11/10/23

* Existential quantification $\exists$ → There exists
"for some", "for at least one", or "there is"
$\exists x\, p(x)$
Read as "There is x such that $p(x)$", "There is at least one x such that $p(x)$", or "for some x, $p(x)$".

When all elements of domain can be listed, e.g. $x_1, x_2, \ldots, x_n$ it follows that the existential quantification is same as disjunction $p(x_1) \vee p(x_2) \vee \ldots \vee p(x_n)$

- $\exists x\, p(x)$ "$x^2 > 10$" domain consists positive integer not exceeding 4?
→ $\exists P(x)$ is same as $p(1) \vee p(2) \vee p(3) \vee p(4)$

at most 4 — 1, 2, 3, 4
at least 4 — 4, 5, ... n
exactly 4 — 4

* Uniqueness quantifier $\exists !$
- There exists a unique x such that $p(x)$ is true
$\exists !\, p(x)$.
"There is exactly one", "There is one & only one".

* Quantifiers with restricted domain
$\forall x < 0,\ x^2 > 0$ same as $\forall x\, (x < 0 \rightarrow x^2 > 0)$
$\forall y \neq 0,\ y^3 \neq 0$ same as $\forall y\, (y \neq 0 \rightarrow y^3 \neq 0)$
$\exists z > 0,\ z^2 = 2$ same as $\exists z\, (z > 0 \wedge z^2 = 2)$

* Precedence of quantifiers
- $\forall$ & $\exists$ have higher precedence than all logical operators from propositional calculus
$\forall x\, p(x) \vee q(x) \equiv (\forall x\, p(x)) \vee q(x)$ rather than $\forall x (p(x) \vee q(x))$.

* Binding Variables
- Scope
$\exists x\, (x + y = 1)$
$\exists x\, (p(x) \wedge q(x)) \vee \forall x\, R(x)$
$\exists x\, (p(x) \wedge q(x)) \vee \forall y\, R(y)$

* Negating quantification
To negate
$\neg \forall x\, P(x) = \exists x\, \neg P(x)$
$\neg \exists x\, P(x) = \forall x\, \neg P(x)$

- Consider:
- "Some students have visited Mexico" $\Rightarrow \exists x\, M(x)$
- "Every student in this class has visited Canada or Mexico"

Let:
- $S(x)$ be "x is a student in this class"
- $M(x)$ be "x has visited Mexico"
- $C(x)$ be "x has visited Canada":

- So $\exists x\, (S(x) \wedge M(x))$