★ Descrete Mathematics

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Reference - Descrete Mathematics & its Appⁿ by Kenneth H. Rosen

- Set theory
- Mathematical inclination
- Recurrence
- Relations
- Permatation & combination
- Trees
- Graphs

(1) Set Theory

- Set is a collection well defined objects

  $A = \{1, 2, 3, 4\}$, $3 \in A$

  $B = \{a, e, i, o, u\}$

- Cardinality of a set :- No. of elements in a set.

  $|A| = 4$ & $|B| = 5$

- ~~Subject~~ Subset :- $[\subseteq]$, $A \subseteq B$, $\phi \in A$

  All element of A ~~(not)~~ in B

  Null set is subset of every set

- Special set :- Set of Natural No → $\mathbb{N}$

  Integers → $\mathbb{Z}$

  Rational No. → $\mathbb{Q}$

  Real No. → $\mathbb{R}$

$\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$

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* Two ways - Set Representation
  1. Listing Method $A = \{1, 2, \ldots\}$
  2. Set builder notation $A = \{2n \mid n \leq 5\}$

- Power set of a set :- It is a set of all subsets of a set.

$A = \{1, 2, 3\}$

$P(A) = \{\phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$

$|P(A)| = 8$

↳ Cardinality of set $\xrightarrow{\text{Always}} 2^{|A|} \Rightarrow 2^3$

- Operations

1. Union :- $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

   $A = \{1, 2, 4\}$
   $B = \{1, 2, 3\}$ $\quad A \cup B = \{1, 2, 3, 4\}$

2. Intersection :- $A \cap B = \{x \mid x \in A \;\&\; x \in B\}$

   $A \cap B = \{1, 2\}$

3. Set Difference :- $A - B = \{x \mid x \in A \;\&\; x \notin B\}$

   $A - B = 4$, $B - A = 3$

4. Symmetric Difference :- $A \oplus B = \{x \mid (x \in A \text{ or } x \in B) \;\&\; x \notin (A \cap B)\}$

   $A \oplus B = (A \cup B) - (A \cap B)$
   $\{1, 2, 3, 4\} - \{1, 2\}$

   $A \oplus B \Rightarrow \{3, 4\}$

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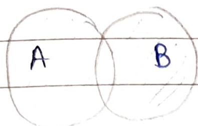
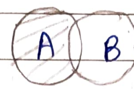
* Universal Set $[U]$:-

→ Complement of a set $\bar{A}$

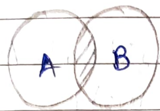
All the elements in U but not in A ; $\bar{A} = U - A$



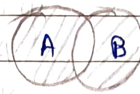
$A \cup B$



$A - B$



$A \cap \bar{B}$



$A \oplus B$

* De-Morgan's Law :-

$\overline{A \cup B} = \bar{A} \cap \bar{B}$

$\overline{A \cap B} = \bar{A} \cup \bar{B}$

* Laws of Sets :-

1) Commutative :- $a+b = b+a$

$A \cup B = B \cup A$

$A \cap B = B \cap A$

$A \oplus B = B \oplus A$

$A - B \neq B - A$

2) Associative Operation :- $(a+b)+c = a+(b+c)$

$(A \cup B) \cup C = A \cup (B \cup C)$

$(A \cap B) \cap C = A \cap (B \cap C)$

$(A \oplus B) \oplus C = A \oplus (B \oplus C)$

3) Distributive Law :-

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

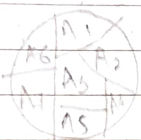
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

4) Identity element :-

$2 + \underline{0} = 2$

$2 \times \underline{\underline{1}} = 2$

5) Partition of a set :-

A 

union

$A = A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5 \cup A_6 \cup A_7$

Intersection is empty

$|A_1|$ or $n(A)$

cardinality

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(\ldots$

no. of element

$+ n(A \cap B \cap C)$

$n(A \cap B \cap C) = n(A \cap B) + n(B \cap C) + n(A \cap C) - n(A) - n(B) - n(C) + n(A \cup B \ldots$

1. Q) P, Q, R are the subsets of universal set U

If $n(U) = 390$, $n(P) = 210$, $n(Q) = 165$

$n(R) = 120$, $n(P \cap Q) = 60$, $n(Q \cap R) = 45$

$n(P \cap R) = 54$, $n(P \cap Q \cap R) = 24$

Illustrate using venn diagram & also find

no. $(P) = 120$

no. $(Q) = 84$

no $(R) = 45$

no $(P \cap Q) = 36$

no $(Q \cap R) = 21$

no. $(P \cap R) = 30$

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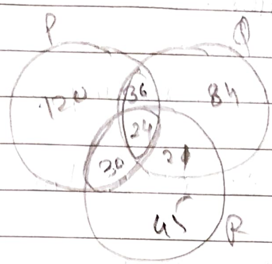
$390 - 210 = 180$ $= no.(P) = 180$



60 − 24 common ele. − 24

330 − 165 = 225

Q − 165 − 36 − 24 − 21 = 84

45 − 24 = 21

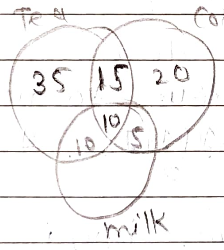
54 − 24 = 30

R − 120 − 30 − 24 − 21 = 45

P − 210 − 210 − 36 − 24 − 30 = 120

(Q2) In a survey of a group of people
60 → like Tea — T
45 → Coffee — C
30 → milk — M
25 → coffee & Tea
20 → Tea & milk
15 → Coffee & milk
10 → all (coffee, tea & milk)

How many people where there



$60 - 25 = 35$

$45 - 25 = 20$

30 −

$n(A\cup B\cup C) = n(A) + n(B) + n(C) - n(A\cap B) - n(B\cap C)$
$- n(A\cap C) + n(A\cap B\cap C)$
$= 60 + 45 + 30 + 10 - 25 - 20 - 15$
$= 145 - 60$
$= 85.$

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★ Mathematical Induction:

$P(n)$:   where n is any positive integer

defn ⇒ It is a technique to prove the formula, theorem, ...

Principle of Mathematical Induction

Two steps :-

1. Basis step :- $P(n)$: given, Show that $P(n)$ is true for $n=0$ or $n=1$

2. Inductive step :- Assume that $P(n)$ is true for $n=k$, where k is any positive number/Integer. then, show that $P(n)$ is also true for $n=k+1$.

Ex: $P(n)$: $1+2+3+4 \cdots + n = \frac{n(n+1)}{2}$

1. Basis step : $P(n)$: is true for $n=1$.

$LHS = 1$, $RHS = \frac{1(1+1)}{2} = 1$

2. Inductive step :- Assume $P(n)$ is true for $n=k$

$1+2+3+4 \cdots + k = \frac{k(k+1)}{2} \Rightarrow$ True — 1

Show that It is also true for $n=k+1$

$1+2+3+4 \cdots + k+1 =$

$\underbrace{1+2+3+4 \cdots + k}_{\text{small}} + k+1$

$$\frac{k(2k-1)(2k+1)+3(2k+1)^2}{3} = \frac{(2k+1)(k(2k-1)+3(2k+1))}{3}$$

$$\frac{(2k+1)\ (2k^2-k+6k+3)}{3} = \frac{(2k+1)(2k^2+5k+3)}{3}$$

$$= \frac{(k+1)(2k+1)(2k+3)}{3}$$

substitute $n = k+1$ in $\frac{n(2n-1)(2n+1)}{3}$

$$\frac{(k+1)\ (2(k+1)-1)\ (2(k+1)+1)}{3} = \frac{(k+1)\ (2k+2-1)(2k+2+1)}{3}$$

$$= \frac{(k+1)\ (2k+1)\ (2k+3)}{3}$$

$\therefore$ LHS = RHS

Hence, $p(n)$ is true

2) $1^3 + 2^3 + 3^3 + \dots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$

→ 1. Basis step: $P(n)$: is true for $n=1$

LHS = 1, RHS = $\left(\frac{1(1+1)}{2}\right)^2 = 1$

2. Inductive step: Assume $P(n)$ is true for for $n = k$

$1^3+2^3+3^3+\dots+k^3 = \left(\frac{k(k+1)}{2}\right)^2$ = True —— (1)

Show that It is also true for $n = k+1$

$1^3+2^3+3^3+\dots+k^3+1 =$

$1^3+2^3+3^3+\dots k^3+(k+1)^3$

from eq^n (1)

LHS: $\left(\frac{k(k+1)}{2}\right)^2 + (k+1)^3$

$\frac{k^2(k+1)^2}{4} + (k+1)^3$

$= \frac{k^2(k+1)}{4} + k^3+1^3+3k^2+3k)$

$= k^2(k+1) + 4k^3 + 4 +$

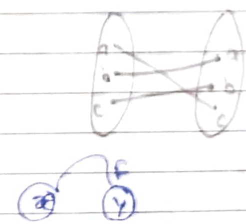
☆ **Functions** :- It is a rule that relates one entity to other. It maps from one set to other set

$F: A \to B$

$f(a) = c$

$f(b) = a$

$f(c) = b$



Domain Codomain

Range

(i) Injective or one to one :- A function is one to one if it follows

(i) for all $a, b \in A$; if $f(a) = f(b)$ then $a = b$

(ii) for all $a, b \in A$, if $a = b$ then $f(a) = f(b)$

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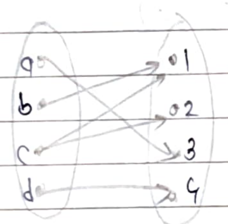
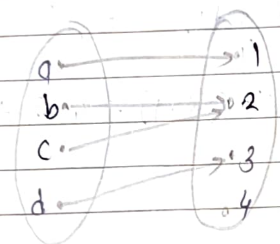
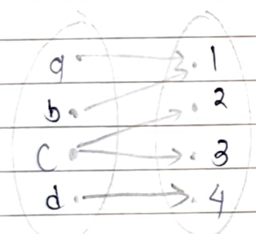
(ii) Onto / Surjective

For all $y \in B$ there ~~exists~~ $x \in A$ $y = f(x)$







Question

(A) Function are one to one / onto, and find the range.

(1) $F: R \to R$ where $F(x) = x^2$

$\rightarrow$ If $F(a) = f(b)$, then $a = b$

$a^2 = b^2$, $a \neq b$, The function is not one to one

$y = f(x) = a^2$, $a^2 = b$, $a = \pm\sqrt{b} \in R$

So The func^n is onto.

Range $\Rightarrow$

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(2) $F: R \to R$, where $f(x) = 2x+3$

$\rightarrow$ If $f(a) = f(b)$ then $a = b$

$2a+3 = 2b+3$, $2a = 2b$, $a = b$

Thus, the func^n is one to one

$y = f(x)$

$2a+3 = b$, $2a = b-3$,

$a = \left(\frac{b-3}{2}\right) \in R$

So Thus, the func^n is onto.

Range:-

(3) $F: Z \to Z$, where $f(x) = 2x+3$

$\rightarrow$ If $f(a) = f(b)$ then $a = b$

$2a+3 = 2b+3$, $a = b$

So The func^n is one to one

let $y = f(x)$, $2a+3 = b$, $2a = b-3$

$a = \frac{b-3}{3} \notin Z$

Thus function is not onto

Range:-

(4) $F: R \to R$, where $f(x) = x^2+1$

→ If $f(a) = f(b)$ then $a = b$

$a^2 + \cancel{1} = b^2 + \cancel{1}$, $a \neq b$, Thus function is not one to one

let $y = f(x)$

$a^2+1 = b$, $a^2 = b-1$, $a = \pm\sqrt{b-1} \notin R$

Thus $funct^n$ is not onto

Range:-

(5) $F: Z \to Z$ where $f(x) = x^2+1$

→ $f(a) = f(b)$ the $a = b$

$a^2+1 = b^2+1$, $a \neq b$, the function is not one to one

$y = f(x) \Rightarrow a^2+1 = b$, $a = \pm\sqrt{b-1} \in Z$

Thus the function is onto

Range:-

☆ Relation

Cartesian Product

$A \times B = \{ (a,b) \mid a \in A, b \in B \}$

$|A| = m$ $\quad |B| = n$

$cond^n \Rightarrow A \times B \neq B \times A$

$A = \{1, 2\}$

~~B =~~

$B = \{a, b, c\}$

$A \times B = \{ (1,a)\ (1,b)\ (1,c)$
$(2,a)\ (2,b)\ (2,c) \}$

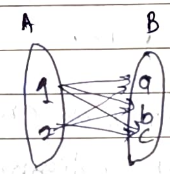
$|A| = 2$

$|B| = 3$

$|A \times B| = 2 \times 3 = 6$ $\xrightarrow{Note}$ $|A \times B| \neq |B \times A|$

$B \times A = \{ (a,1), (a,2) (b,1)$
$(b,2)\ (c,1)\ (c,2) \}$

$func^n$ →



Cartesian Product :-

A Cartesian product is a binary operator bet^n two set's & it is set of ordered pair.

def^n (1)

Relation :- A rel^n from set A to B set is nothing but a any possible subset of a Cartesian product $A \times B$

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(2) A Relaⁿ on a set A is a relation from A to A
in other words relatⁿ on a set A is a subset of A×A

Types of Relation.

(1) Reflexive Relation :-

Example $A = \{a, b, c\}$

$A \times A = \{a, b, c\} \times \{a, b, c\}$

$= \{(a,a)\ (a,b), (a,c), (b,a)\ (b,b)\ (b,c)$
$(c,a), (c,b)\ (c,c)\}$

Matrix :

	a	b	c
a	aa	ab	ac
b	ba	bb	bc
c	ca	cb	cc

— digonal pair.

$\Rightarrow \boxed{\forall a \in A, (a,a) \in R}$ it focus on only digonal pair, digonal pair should be ther = Reflexive Relⁿ

Yes Reflexive relⁿ → $R = \{(a,a)\ (b,b)\ (c,1)\ (c,2)$
$(c,c)\ (a,b)\ (b,a)\}$

$R = \phi$

No Not a Reflexive relⁿ → $R = \{(a,b)\ (b,c)\ (a,a)\ (c,a)\}$

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defⁿ ⟹ A Relation R on A set A is said to be reflexive if $\forall a \in A, (a,a) \in R$

+example

(2) Irreflexive Relation :-

$\Rightarrow \boxed{\forall a \in A, (a,a) \notin R}$

Previous example # digonal element - Not to take.

$R = \{(a,b)\ (b,a)\ (a,a)\ (b,b)\ (a,c)\}$
→ Not a irreflexive relⁿ

$R = \{(a,b)\ (b,a)\ (c,a)\}$
→ yes reflexive relⁿ

defⁿ ⟹ A Relation R on A set A is said to be irreflexive for $\forall a \in A, (a,a) \notin R$

Ex:- $R = A \times A$ ⇒ Not a irreflexive relⁿ.
$R = \phi$ ⇒ Not a irreflexive relⁿ.

(3) Symmetric Relation:-

defⁿ ⟹ A Relation R on A set A is said to be symmetric for $\forall a, b \in A, (a,b) \in R$ then $(b,a) \in R$

It contains all element pair (a,b) ex- digonal pair

$R = A \times A$ is symmetric relⁿ.

$R = \{(a,b)\ (a,c), (a,b)\}$ → Not a symmetric Relⁿ

$R = \{(a,a)\ (b,b)\ (c,c)\}$ → Yes it is a symmetric Relⁿ

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(4) Anti Symmetric Relation

⟹ A relation R on a set A is said to be anti symmetric if $\forall a, b \in A$ $(a,b) \in R$ $(b,a) \in R$ then $a = b$

It allows diagonal pair but (a,b) (b,a) → Not.

(1) R = {(a,a) (b,b) (b,a) (a,c)} ✓

(2) R = {(a,b) (b,c) (a,c)} → is anti symmetric

(3) R = A × A → Not a Anti symmetric

(4) R = {(A,b), (b,c) $\underbrace{(a,c)\ (c,a)}_{\text{Symmetric pair}}$ (a,a) (c,c)} → Not a Anti symmetric

Not allow symmetric pair.

(5) Asymmetric Relation:

⟹ does not allow diagonal pair & Symmetric pair

⟹ $\forall a, b \in A$, $(a,b) \in R$ then $(b,a) \notin R$

⟹ A relation R on a set A is said to be asymmetric if $\forall a, b \in A$, $(a,b) \in R$, $(b,a) \notin R$

R = {(a,a) (b,b) (c,c) (a,b) (b,a)} → ✗

R = A × A → ✗ Not a Asymmetric Relation

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(1). R = {(a,b) (b,c)} → is Asymmetric

(2) R = {(a,b) (b,b) (c,c)} → Not a Asymmetric

(3) R = {(a,b) (b,a) (a,b) (b,b)} → Not a Asymmetric Relⁿ

(6) Transitive Relation

⟹ A relation R on a set A is said to be transitive if $\forall a, b \in A$, $(a,b) \in R$, $(b,c) \in R$ then $(a,c) \in R$

(1) R = {(a,b) (b,c)} → Not a Transitive Relation

(2) R = {(a,b) (b,c) (a,c)} → Yes

a — b — c ✓

(3) R = {(a,b) (b,b) (c,c)} → Yes

(4) R = {(c,a) (b,c)} → Not a Transitive Relⁿ.

b ⇄ c ← a

(1) 1 - 1 element 2 - 1 element same
(2) 1 - 2 element 2 - 2 elem. - Same
} Not Trick

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Ex:- (1) $R = \{(a,b)\ (b,a)\ (a,b)\ (b,b)\} \rightarrow$ Transitive

(2) $R = \{(c,a)\ (b,c)\} \rightarrow$ Not a Transitive.

☆ Transitive closure Using Warshall algorithm :-

Q. Set A = {1, 2, 3, 4}

$R = \{(1,4)\ (2,1)\ (2,3)\ (3,1)\ (3,4)\ (4,3)\}$

Ist step :-

$$M_R = \begin{array}{c|c} & 1\quad 2\quad 3\quad 4 \\ \begin{matrix}1\\2\\3\\4\end{matrix} & \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array} = W_0$$

1st column(C)   Ist Row (R)

{2,3}   {4}

$C \times R = \{(2,4)\ (3,4)\}$ Transitive pair

$$W_1 = \begin{array}{c|c} & 1\quad 2\quad 3\quad 4 \\ \begin{matrix}1\\2\\3\\4\end{matrix} & \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$W_1 = W_2$

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II step :- 2nd Row 2nd column

C   R

ϕ   {1,3,4}

I = ϕ .

IIIrd step :-

C   R

{2,4}   {1,4}

$C \times R = \{(2,1)\ (2,4)\ (4,1)\ (4,4)\}$

1 put then remain W_2 element same in W_3

$$W_3 = \begin{array}{c|c} & 1\quad 2\quad 3\quad 4 \\ \begin{matrix}1\\2\\3\\4\end{matrix} & \begin{bmatrix} 0 & 0 & 0 & \phi \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & \phi \\ 1 & 0 & \phi & 1 \end{bmatrix} \end{array}$$

IVth step :-

C   R

{1,2,3,4}   {1,3,4}

$C \times R = \{(1,1)\ (1,3)\ (1,4)\ (2,1)\ (2,3)\ (2,4)\ (3,1)\ (3,3)\ (3,4)\ (4,1)\ (4,3)\ (4,4)\}$

$R^* = \{C \times R\}$

Q Set $A = \{1, 2, 3, 4\}$

$R = \{(1,2)\ (2,1)\ (2,3)\ (3,4)\}$

→ Ist step :-

$$M_R = \begin{array}{c|c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array} = W_0.$$

1st column C $\{2\}$ 1st row R $\{2\}$

$C \times R = \{(2,2)\}$ Transitive pair

$$W_1 = \begin{array}{c|c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

IInd Step :-

2nd ~~row~~ column C $\{1, \cancel{2}, 2\}$ 2nd Row R $\{1, \cancel{2}, 3\}$

$C \times R = \{(2,1), (2,2), (2,3)\ (1,1)\ (1,2), (1,3)\}$

$$W_2 = \begin{array}{c|c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

IIIrd Step :- 3rd ~~row~~ column $\{1, 2\}$ 3rd row $\{4\}$

$C \times R = \{\cancel{R}(1,4)\ (2,4)\}$

$$W_3 = \begin{array}{c|c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Step 4 :- 4th column $\{1, 2, 3\}$ 4th row ϕ

$\Rightarrow C \times R = \phi$

$R^+ = \{(2,2), (2,1)\ (2,3)\ (1,1)\ (1,2)\ (1,3)\ (1,4)\ (2,4)\}$

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★ Equivalence Relatⁿ :- A relation R on a set A is said to be equivalence Relatⁿ if R is reflexive, symmetric & transitive.

Ex :- $A = \{1, 2, 3\}$

$R_1 = \{(1,1)\ (2,2)\ (3,3)\}$

$R_2 = \{(1,1)\ (2,2)\ (3,3)\ (2,1)\}$

$R_3 = \{(1,1)\ (1,3)\ (2,1)\ (3,1)\}$

→ Check (i) Reflexive
(ii) Symmetric
(iii) Transitive.

$R_1 \Rightarrow$ Equivalence

$R_3 \Rightarrow$ Not Equivalence.

$R_2 \Rightarrow$ Not Equivalence

Ex :- $\phi \Rightarrow$ Not Equivalence

Ex :- $R = \{(a,b) / a-b$ is a int$\}$

* Which of the following are relatⁿ are Equivalence Relⁿ in R

① $R = \{(a,b) / a-b$ is a int$\}$

→ (i) Reflexive → ex → $(2,2) \Rightarrow 2-2=0$ or $(1,1) \to 1-1=0$ int ✓ Reflexive.

(ii) Symmetric → ex → $(1,2)\ (2,1) \to 1-2=-1,\ 2-1=1 \Rightarrow$ int. ✓ Symmetric

(a,b)
(b,a)

(iii) Transitive

ex. $(5,2)\quad (2,1)$

$5-2=3 \qquad 2-3=1 \qquad 5-1=4$

all we get integer Thus, it is Transitive

∴ The Relatⁿ is Equivalence Relⁿ in R.

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② $R_2 = \{(a,b) / a-b$ is divisible by $3\}$

→ (i) Reflexive

ex:- $(2,2) = 2-2=0,\quad 0 \div 3 = 0$

∴ It satisfy Reflexive property.

(ii) Symmetric

ex:- $(1,2)\quad (2,1)$

$1-2 \qquad 2-1$

$=-1 \qquad =1$

$-1 \div 3 \qquad 1 \div 3$

$(6,3)$	$(3,6)$
$6-3$	$3-6$
$\frac{3}{3}=1$	$\frac{-3}{3}=-1$

∴ It satisfy Symmetric property

(iii) Transitive

ex:- $(5,2)\quad (2,1)$

$5-2 \qquad 2-1 \qquad 5-1$

$=3 \qquad =1 \qquad =4$

$3 \div 3 = 1 \qquad 1 \div 3 \qquad 4 \div 3$

$(a,b)\ (b,c) \Rightarrow (a,c)$

$(7,4)\quad (4,1)$

$\frac{3}{3}=1 \qquad \frac{3}{3}=1$

∴ It satisfy Transitive property

Hence, it is Equivalence relation.

③ $R_3 = \{(a,b) / a-b$ is odd$\}$

→ (i) Reflexive

ex :- $(2,2) = 2-2 = 0$

It does not satisfy Reflexive property

Hence, Relation R is not Equivalence relⁿ.

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→ Equivalence classes :- It used to find ~~certian~~ Partiation

Equivalence classes of x is denoted by $[x]$

$[x] = \{ y / y \in A, \ \& \ (x,y) \in R \}$

Ex. :- $A = \{1,2,3,4,5\}$

$R = \{(1,1)\ (2,2)\ (3,3)\ (4,4)\ (5,5)\ (1,2)\ (2,1)\ (4,5)\ (5,4)\}$

1st check it is equivalence relⁿ. → $[1] = \cancel{(1,1)\ (1,2)\ (2,1)\}}\ \{1,2\}$ → किससे Relate करता है

$[2] = \{2,1\} = \{1,2\}$

$\Big\}\ P_1$

$[3] = \{3\} \quad P_2$

$[4] = \{4,5\}$

$[5] = \{4,5\}$

$\Big\}\ P_3$

★ Partiation ⇒

defⁿ → Is a non empty set A, is collection of R if it satisfied $A_1 \cup A_2 = \text{set } A$, $A_1 \cap A_2$ then it is ~~the~~ partiation.

$\{1,2\} = P_1$

$\{3\} = P_2$

$\{4,5\} = P_3$

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ex ⇒ $A = \{1,2,3,4\}$

$R = \{(1,1)\ (2,2)\ (3,3)\ (4,4)\ (3,1)\ (1,3)\ (4,2)\ (2,4)\}$.

→ it is equivalence relⁿ.

$[1] = \{1,3\}$

$[2] = \{2,4\}$

$[3] = \{3,1\}$

$[4] = \{4,2\}$

P_1, P_2

$P_1 = \{1,3\}$

$P_2 = \{2,4\}$.

★ Partial order Relation :-

→ A relation R on a set A is said to be partial order relation if it is reflexive, Antisymmetric & transitive.

ex :→ $A = \{1,2,3\}$

$R_1 = \{(1,1)\ (2,2)\ (3,3)\}$

$R_2 = \{(1,1)\ (2,2)\ (3,3),\ (1,2)\ (2,1)\}$

Antisymmetric Allows digonal pair & it doesn't allow symmetric pair

Ex → $A = \{1,2,3\}$

$R_1 = A \times A$ is not partial order

R_2 = Not a Partial order

$R_3 = \{(1,1)\ (2,2)\ (3,3)\}$: is a partial order

$R_3 = (1,1)\ (2,2)\ (3,3)\ (1,3)\ (2,3)$ – is a partial order.

Q1) $R = \{(a,b) \mid a,b \in Z, \underline{\underline{a < b}}\}$ integer

→ ① Reflexive ex:- $(2,2)$

$2 \not< 2$

condition not satisfied it is not a partial order Rel^n

Q2) $R = \{(a,b) \mid a,b \in Z, a \leq b\}$

→ ① Reflexive

ex:- $(2,2)$

$2 \leq 2$

Condition satisfied

② Anti symmetric

$(1,2)\ (2,1)$

$1 \leq 2$ ✓ $\quad 2 \leq 1$ ✗

$cond^n$ satisfied

③ Transitive

$(1,2)\ (2,1)$

$cond^n$ satisfied

∴ It is partial order Rel^n.

Q3) $R = \{(a,b) \mid a,b \in Z, b/a \in Z\}$

→ ① Reflexive

ex:- $(2,2)$

$2/2 = 1 \in Z$

$Cond^n$ satisfied

② Anti Symmetric

$(2,1)\ (1,2)$

$\left\{\frac{b}{a}\right\}$ $\quad \frac{1}{2} \quad \frac{2}{2}$

$= 0.5 \notin \quad = 1 \in Z$

$cond^n$ satisfied

③ Transitive.

$(2,4)\ (4,8)$

$\frac{4}{2} = 2 \quad \frac{8}{4} = 2$

$Cond^n$ satisfied

∴ It is partial order ~~Rel~~ Rel^n.

★ Hase Diagram

→ A hase diagram is a graphical rep^n of a partial order rel^n or coset with implied upward orientation.

Steps

① ~~Draw~~ Draw the directed graph as per rel^n

② Directed graph are need to be contain self loop at every vertex because element always grater than or equal to itself.

③ Remove all the self loops because it is reflexive so, don't need to show them

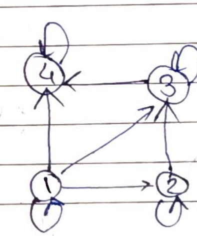
④ Finally remove all ~~the~~ the transitive edges & direction it is partial order.

⑤ Make sure that initial vertex is below the terminal vertex.

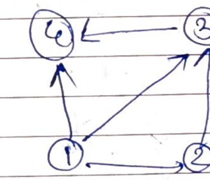
Q. $A = \{1,2,3,4\}$

$R = \{(1,1)\ (1,2)\ (1,3)\ (1,4)\ (2,2)\ (2,3)\ (2,4)\ (3,3)\ (3,4)$ $(4,4)\}$

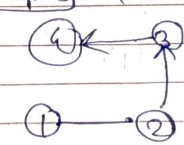
→ Step 1 :-



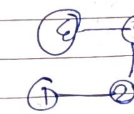
Step 2 :- (Remove self loop)



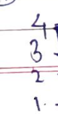
Step 3 :- (Remove transitive pair)



Step 4 - (Remove direction)



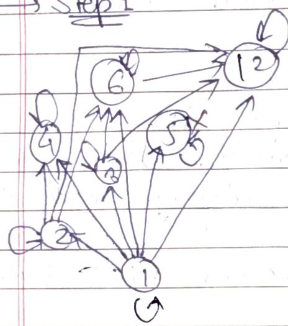
Step 5 Finals -



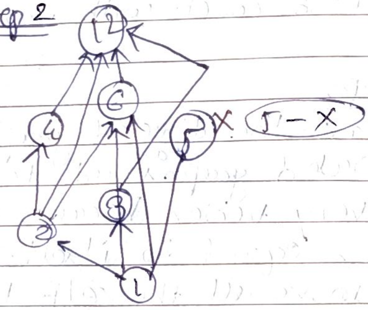
for divisibility

Q1)) $\left[\{1, 2, 3, 4, \cancel{5}, 6, 12\} / \right]$

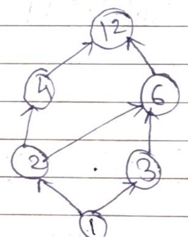
→ Step 1



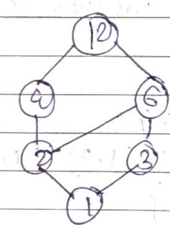
Step 2



Step 3

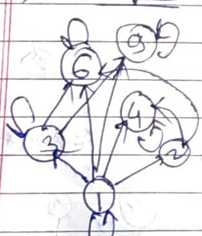


Step 4

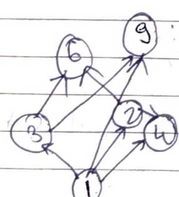


Q2)) $\left[\{1, 2, 3, 4, 6, 9\} / \right]$

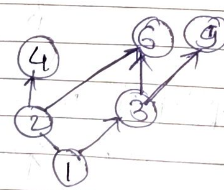
→ Step ①



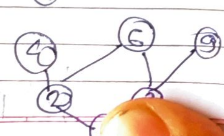
Step ②



Step ③



Step ④



Q3)) Hasse diagram for divisibility.

① $R = \{1, 2, 3, 4, 5, 6\}$

② $R = \{1, 2, 3, 5, 7, 11, 13\}$

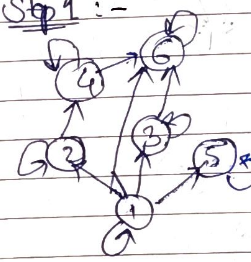
③ $R = \{1, 2, 3, 6, 12, 24, 36, 48\}$

④ $R = \{3, 5, 7, 11, 13, 16, 17\}$

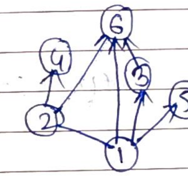
⑤ $\{2, 3, 5, 10, 11, 15, 25\}$

→ ① $R = \left[\{1, 2, 3, 4, 5, 6\} / \right]$:

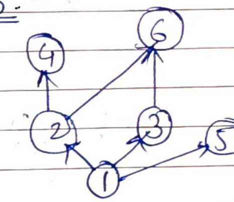
Step 1 :-



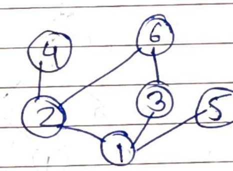
Step 2 :-



Step 3 :-

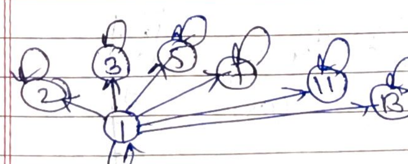


Step 4 :-

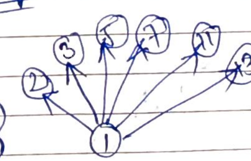


② $R = \left[\{1, 2, 3, 5, 7, 11, 13\} / \right]$

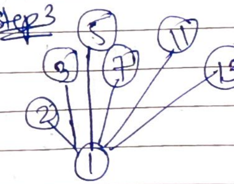
→ Step 1:-



Step 2

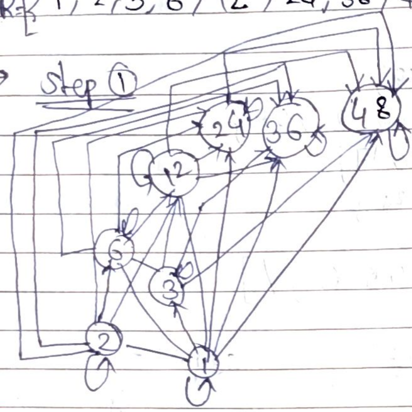


Step 3

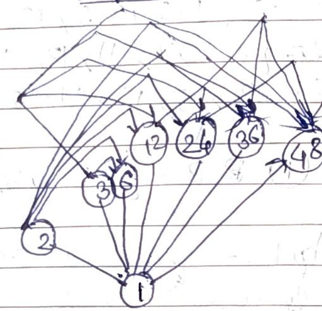


(3) R=[{1, 2, 3, 6, 12, 24, 36, 48}/]

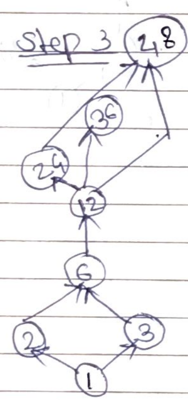
step (1)



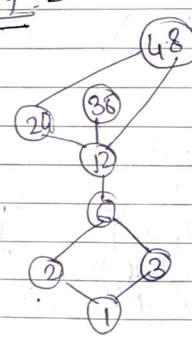
step (2)



step 3

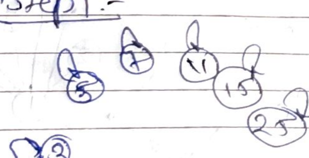


step 4 :-

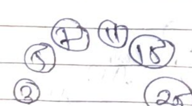


(4) R = [{3, 5, 7, 11, 15, 25}/]

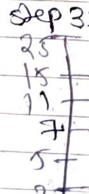
step 1 :-



step 2 :-

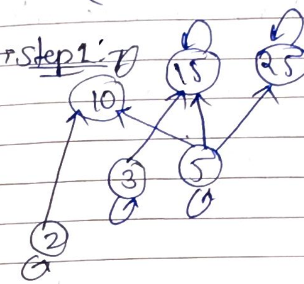


step 3 :-

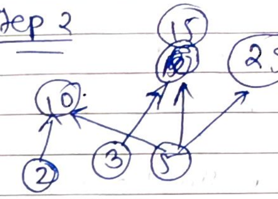


(5) R [{2, 3, 5, 10, 11, 15, 25}/]

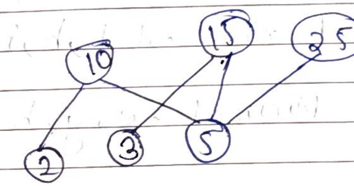
step 1 :-



step 2



step 3

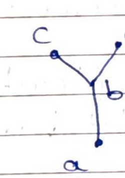


☆ Maximal Element :- If in a poset, an element is not related to any other ~~elet~~ element.

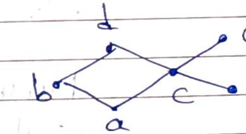
☆ Minimal element :- If in a poset, no element is rotated to an element

Dekhna { maximal → top
minimal → bottom

ex:- ⇒

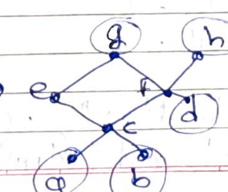


maximal = c, d
Minimal = a.



Maximal = d, e
minimal = a, f.

⇒



maximal = g, h
minimal = a, b, d.

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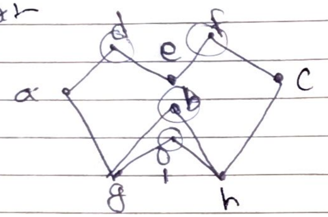
Note → Maximal - more than one possible
Minimal - more than one possible.

ex:-

Maximal = a, b, c
Minimal = a, b, c.

ex:-



Maximal = d, f, b, i

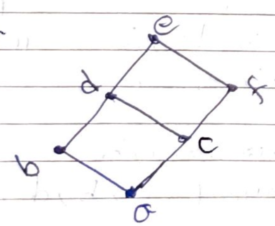
Minimal = e, g, h

☆ Greatest And least Element.
Maximum element — Minimum element.
~~Note~~ → (only one) (~~More than~~ only one)

defn :- Maxm element / Greatest :- If it is Maximal & ~~et~~ every element is related to it

Minimum (least element :- If it is Minimal & it is related in a every element in poset.

ex :-



Maximal = e
Minimal = a

Greatest (Maxm) = e
least element (Mim) = a

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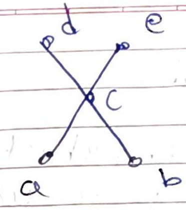
⇒

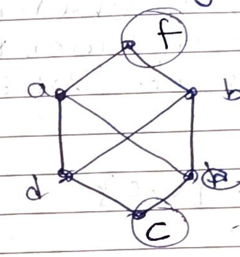


⇒ Maximal ⇒ d, e
Minimal ⇒ a, b.

⇒ Greatest = ϕ
⇒ Least = ϕ

Note ⇒ only single element - Greatest

ex:-

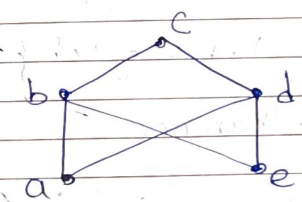


Maximal ⇒ f

Minimal = c

⇒ Greatest ⇒ f
⇒ ~~Minimal~~ least ⇒ c

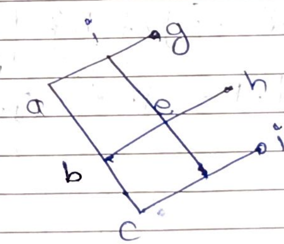
ex:-



Maximal = c
⇒ Greatest = c

Min = a, e
⇒ Least ⇒ ϕ

ex:-



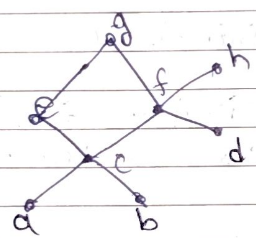
Maximal = g, h, i
⇒ Greatest = ϕ
Minimal = c
⇒ least = c.

10 Mark - least, Great, lower, upper

☆ Upper Bond & lower Bond

Upper Bond ⇒ Let B be a subset of A. an element $x \in A$ is in upper Bond of B if $([y, x] \in$ poset $\forall y \in B)$

Lower Bond ⇒ let B be a subset of A set. An element $x \in A$ is in lower Bound of B if $([x, y] \in$ poset $\forall y \in B)$



B = {e, c}

U.B = {g, e}

L.B = {a, b}

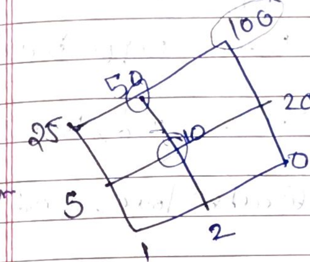
Note ⇒ U.B & L.B - more than one

~~Be~~ B = {c, f, d} ← Relate करो इन element

U.B = {g, f, h}

L.B = φ ← L.B में d का कोई element relate नहीं कर रहा.

ex:- B = {50, 10}



See top ↑ Bottom

U.B = 50, 100

L.B = 1, 5, 2, 10.

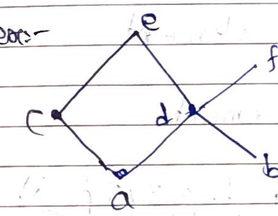
☆ least Upper Bond & Greatest ~~Upper~~ Lower Bond :-

(Minimum) (maxm) or Infimum/meet/∧

Imp Names. (mimum) element in (Supremum)/Join/∨ MCQ Imp

least∧Upper Bond :-

ex:-



B (c, d)

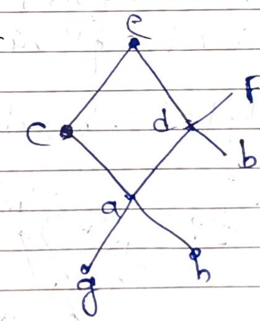
U.B = {e}

⇒ least Upper Bond ⇒ {e}

L.B = {a}

⇒ Greatest lower bond ⇒ {a}

ex:-



B = (c, a)

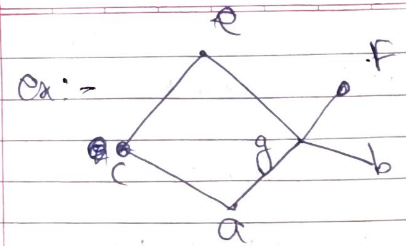
U.B = {c, e}

⇒ least upper Bond ⇒ {c}

L.B = {g, h, a}

⇒ Greatest lower bond ⇒ {a}

Ex:-



$B = \{c, f\}$

$U.B = \phi$

$\Rightarrow$ Lower Upper Bond : ϕ

$L.B = \{a, d, b\}$

$\Rightarrow$ Greatest lower Bond = d.

Q// Poset $[\{3, 5, 9, 15, 24, 45\}, /]$

a) Find the Maximal

b) Find the minimal

c) Is there any greatest element

d) Is there any least element

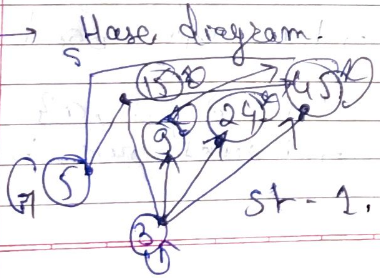
e) Find the upper bond of $\{3, 5\}$ if exist

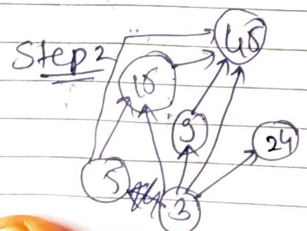
f) ——/— lowest upper bond of $\{3, 5\}$ if it exist.

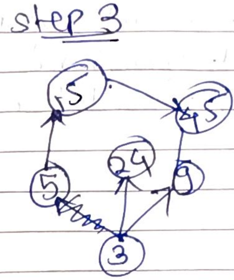
g) find the lower bond of $\{15, 45\}$

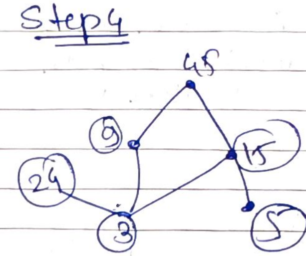
h) Find the greatest lower bond of $\{15, 45\}$ exist

$\rightarrow$ Hase diagram:









a) Find the Maximal

$\Rightarrow$ 45, 24

b) Find the minimal

$\Rightarrow$ 5

c) Is there any greatest element

$\Rightarrow$ Maximal = 45, 24

greatest = ϕ

d) Is there any least element

$\Rightarrow$ Minimal = 5

least element = 5

e) Find the upper bond of $\{3, 5\}$

$\Rightarrow$ 15

f) Find lowest upper bond of $\{3, 5\}$ if it exist

$\Rightarrow$ $L.U.B = \{15, 3, 5\}$

g) Find lower bond of $\{15, 45\}$

$\Rightarrow$

h) Find the greater lower bond of $\{15, 45\}$

$\Rightarrow$

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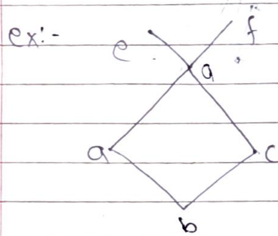
★ Lattice :- A partial order set in which every element has both least upper bond & greatest lower bond is called as Lattice.

ex:-



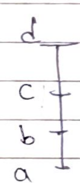
$B = \{a, d\}$

$U.B = \{e, d, f\}$

$L.B = \{a, b\}$

$L.UB = \{d\}$

$G.L.B = \{a\}$

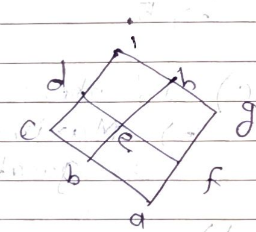


$B = \{B, C\}$

$LUB = \{C\}$

$LB = b, a$

$G.LB = b$



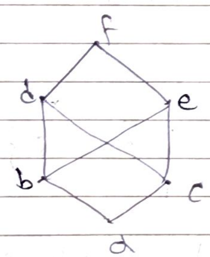
$B = \{c, f\}$

$U.B = d, i$

$L.B = a$

$G.L.B = a$

$L.U.B = d$



$B = \{d, e\}$

$U.B = f$

$L.U.B = f$

$L.B = b, c, a$

$G.L.B = \phi$

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★ Primest :-

exam Fundamental theorem of Arithmatic :- Every integer is greater than 1 can be ~~uniquely~~ uniquely as a prime or as a product of two or more primes where the prime factor are written as order of non-decreasing size

$$n = P_1^{\alpha_1} \cdot P_2^{\alpha_2} \cdot P_3^{\alpha_3} \cdots P_k^{\alpha_k}$$

where,

P_1, P_2, P_3, P_k distinct primes

$\alpha_1, \alpha_2, \alpha_3, \ldots \alpha_k > 0$ are integers.

eg. $100 = 2.5.2.5.$

2	100
5	50
2	10
~~2~~	~~10~~
5	5
	1

$999 = 3^3 \times 37$

3	999
3	333
3	111
	37

$10! = 2 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10$

$= 2 \times 3 \times 2 \times 2 \times 5 \times 2 \times 3 \times 7 \times 2 \times 2 \times 2 \times 3 \times 3 \times 2 \times 5$

$= 2^8 \times 3^4 \times 5^2 \times 7$

exam def^n prime :- An integer p is greater than one is called as prime if the only the factor of p are 1 & 3

ex:- 7, 3, 17, etc.

* Greatest common Divisor :-

gcd (24, 36)

24 = ~~1×2×3×~~ 1, 2, 3, 6, 12, 8, 4

36 = 1, 2, 3, 6, 12, 9, 4, 18

GCD ⇒ (24, 36 = 12)

defn let a & b be a integer not both zero, the largest integer such that d/a, d/b is greatest common divisor of a & b and it is denoted by gcd (a, b)

→ The largest integer that divide both the integer is called the gcd

→ The integer a & b are relatively prime if their greatest common divisor is one

MCQ $\boxed{gcd(17, 21) = 1}$

gcd (a, b) = 1

→ The integer $a_1, a_2, \ldots a_n$ are pairwise relatively prime gcd of a_i & a_j is 1

$gcd(a_i, a_j) = 1$ where $1 \le i \le j \le n$

ex:- 10, 17, 21

(10, 17) (17, 21) (10, 21)

gcd (10, 17) = 1

gcd (17, 21) = 1

gcd (10, 21

10 = 1×2×5

17 = 1×17

21 = 1×3×7

1m

* The Euclidean Algorithm :-

① gcd (25, 60)

grater → a = bq + r

60 = 25 q + r

60 = 25 · 2 + 10

$$25 \overline{)60}^{\,2} \quad 50 \quad 10$$

b become a = 25, r become b = 10

a = 25, b = 10

25 = 10 · 2 + 5

$$10 \overline{)25}^{\,2} \quad 20 \quad 5$$

a = 10, b = 5

$\boxed{10 = 5 \cdot 2 + 0}$

$$5 \overline{)10}^{\,2} \quad 10 \quad 0$$

gcd (25, 60) = 5

② gcd (100, 101)

→ a = bq + r

101 = 100 q + r

101 = 100 · 1 + 1

$$100 \overline{)101}^{\,1} \quad 100 \quad 1$$

a = 100, 1 = b

100 = 1 · 100 + 0

$$1 \overline{)100}^{\,100} \quad 100 \quad 0$$

gcd (100, 101) = 1.

(3) gcd (24,36)

$36 = 24 \cdot q + r$

$36 = 24 \cdot 1 + 12$

$24 \overline{)36}$ quotient 1; -24; remainder 12

$a = 24,\ b = 12$

$24 = 12 \cdot 2 + 0$

$12 \overline{)24}$ quotient 2; -24; remainder 00

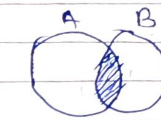
gcd (12, 24) = 12.

Unit 1 - end → some part - hex - 8 page

Unit 2 :-

Principle of Inclusion & Exclusion:-

Let A & B finite sets



$|A \cup B| = |A| + |B| - |A \cap B|$

$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$

Q1) In a servey of group of 80 people it is found that 60 like egg 30 like fish. Find the ~~peple~~ like both egg & fish.

→ Given, 80 people

By principle of Inclusion & Exclusion

Egg = 60 = $|E|$

Fish = 30 = $|F|$

To find, $|E \cap F| = ?$

Solution, $|E \cup F| = |E| + |F| - |E \cap F|$

$80 = 60 + 30 - |E \cap F|$

$|E \cap F| = 10.$

Q2) Out of 200 students, 50 take maths, 140 take economics, 24 take both, how many of both not take any of cources.

→ 200 students

$|M| = 50$

$|E| = 140$

$|M \cap E| = 24$

By Inclusion & Exclusion

$|M \cup E| = |M| + |E| - |M \cap E|$

$= 50 + 140 - 24$

$|M \cup E| = 166$

$\Rightarrow 200 - |M \cup E|$

$200 - 166$

$= 34$

∴ 34 student not take any of cources.

Imp

Q3 It is known that at the university of 60% professor play tennis, 50% professor play Football, 70% Play ~~take~~ hockey, 20% play tennis & Football, 30% tennis & hockey, 40% Football & hockey.
If someone clamed that 20% professor play all the three game.

→ $|T| = 60$

$|F| = 50$

$|H| = 70$

$|T \cap F| = 20$

$|T \cap H| = 30$

$|F \cap H| = 40$

Let assume 100 professor which at least one gam

by ~~assu~~ principle of Inclusion & Exclusion

$|T \cup F \cup H| = |T| + |F| + |H| - |T \cap F| - |T \cap H| - |F \cap H| + |T \cap F \cap H|$

$100 = 60 + 50 + 70 - 20 - 30 - 40 + |T \cap F \cap H|$

$100 - 90 = |T \cap F \cap H|$

$\Rightarrow |T \cap F \cap H| = 10$

∴ 10% professor play all the three games

∴ Clamed is incorrect.

Q4) In a servey wage of three tooth paste A, B, C is found that 60 like A, 55 like B, 40 like C 20 like A & B, 35 like B & C 15 like A & C, 10 like All, find the no. of person including in a servey?

→ $|A| = 60$ $|B \cap C| = 35$
$|B| = 55$ $|A \cap C| = 15$
$|C| = 40$ $|A \cap B \cap C| = 10$
$|A \cap B| = 20$

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$$

$$|A \cup B \cup C| = 60 + 55 + 40 - 20 - 35 - 15 + 10$$
$$\therefore |A \cup B \cup C| = 95$$

∴ 95 person including in a servey.

Q5) A servey of sample on new car, 25 car sold to local autodelar was conducted to see which of three popular options AC (A), Radio (R), power window (W), are alldready installed, The servey found that 15 had A, 12 had R, 11 had W, 5 had A & W, 9 had A & R, 4 had R & W, 3 had all. Find (i) car with atleast 1 one option
(ii) No option
(iii) Only A & R not W
(iv) Only A & W not R
(v) only R & W not A.
(vi) only A
(vii) only R
(viii) only W
(x) Only 1

→ $|A| = 15$ $|A \cap W| = 5$
$|R| = 12$ $|A \cap R| = 9$ $|A \cap R \cap W| = 3$
$|W| = 11$ $|R \cap W| = 4$

(i) $$|A \cup R \cup W| = |A| + |R| + |W| - |A \cap R| - |R \cap W| - |A \cap W| + |A \cap R \cap W|$$

$$3 = 15 + 12 + 11 - 5 - 9 - \underline{|R \cap W|} - 3$$

(ii) No. option.
$25 - 23 = 2$

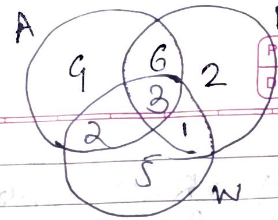
(iii) Only A & R not W.
$= |A \cap R| - |A \cap R \cap W|$
$= 9 - 3$
$= 6$

(iv) only A & W not R
$= |A \cap W| - |A \cap R \cap W|$
$= 5 - 3$
$= 2$

(v) only R & W not A
$|R \cap W| - |A \cap R \cap W|$
$= 4 - 3 = 1$

(vi) Only A
$|A| - |A \cap R| - |A \cap W| + |A \cap R \cap W|$
$= 15 - 9 - 5 + 3$
$= 4$

(vii) only R
$|R| - |A \cap R| - |R \cap W| + |R \cap W \cap A|$
$= 12 - 9 - 4 + 3 = 2$



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(viii) only W

$= |W| - |W \cap R| - |W \cap A| + |A \cap R \cap W|$

$= 11 - 4 - 5 + 3$

$= 5$

(ix) only one option

= only A + only R + only W

$= 4 + 2 + 5 = 11$

Q6) A survey of household U.S. reveals 96% have at least one television set, 98% have telephone service & 95% have telephone service & at least 1 television set. What is the % of household have neither telephone service nor television set

→ $|T| = 96$

$P \leftarrow |T\ \text{service}| = 98$

telephone $|T \cap P| = 95$

$|T \cup P| = |T| + |P| - |T \cap P|$

$= 96 + 98 - 95$

$= 96 + 3$

$= 99$

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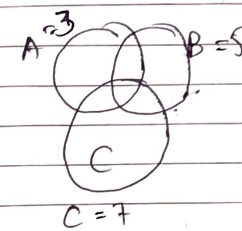
Q7) Among the integer 1 to 300 fill the venn diagram w.r.t divisibility by 3, 5, 7.

→ 

$|A| = \left\lfloor \frac{300}{3} \right\rfloor$ floor funcn.

$\lceil\ \rceil$ ceil funcn $= 42.8 = 43$

$\lfloor\ \rfloor = 42.8 = 42$ floor funcn

$|A| = \left\lfloor \frac{300}{3} \right\rfloor = 100$

$|B| = \left\lfloor \frac{300}{5} \right\rfloor = 60$ $\quad C = \left\lfloor \frac{300}{7} \right\rfloor = 42.8 = 42$

$|A \cap B| = \left\lfloor \frac{300}{3 \times 5} \right\rfloor = 20$

$A \cap C = \left\lfloor \frac{300}{3 \times 7} \right\rfloor = 14$

$|B \cap C| = \left\lfloor \frac{300}{5 \times 7} \right\rfloor = 8.$

$|A \cap B \cap C| = \left\lfloor \frac{300}{5 \times 3 \times 7} \right\rfloor = 2.85 = 2.$

① $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$

$|A \cup B \cup C| = 100 + 60 + 42 - 20 - 8 - 14 + 2$

$|A \cup B \cup C| = 162$

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Sub quan ① divisible non ② only A & B not C
③ only B & C not a
④ only A & C not B
⑤ only A
⑥ only B
⑦ only C
⑧ only 1

① 300 - 162 = 138.

② only A & B not C

= A ∩ B - A ∩ B ∩ C
= 20 - 2 ⇒ 18

③ only A & C not B
= A ∩ C - A ∩ B ∩ C
= 14 - 2 ⇒ 12

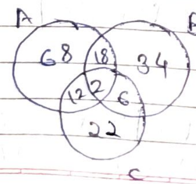
④ only ~~B&C~~ not a
= B ∩ C - A ∩ B ∩ C
= 8 - 2 ⇒ 6

⑤ Only A
= |A| - |A ∩ B| - |A ∩ C|
+ |A ∩ B ∩ C|
= 100 - 20 - 14 + 2
= 68

⑥ only B
= |B| - |B ∩ A| - |B ∩ C|
+ |A ∩ B ∩ C|
= 60 - 20 - 8 + 2
= 34

⑦ only C
= |C| - |C ∩ A| - |C ∩ B|
+ |A ∩ B ∩ C|
= 42 - 14 - 8 + 2
= 22

⑧ only 1
⇒ only A + only B + only C
⇒ 124.



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* 1 unit part (Remaining).
* Recursive Defination :-
— Recursively defined Func^n :-

Q// Suppose that f is defined recursively by $f(0) = 3$,
$f(n+1) = 2f(n) + 3$
Find $f(1), f(2), f(3), f(4)$
Ist step :- put f(0) in place of n
$f(n+1) = 2f(n) + 3$
$f(0+1) = 2 \times 3 + 3 = 9$
$f(1) = 9$
$n=1 \rightarrow f(2) = 2f(1) + 3 = 2 \times 9 + 3 = 21$
$n=2 \rightarrow f(3) = 2f(2) + 3 = 2 \times 21 + 3 = 45$
$n=3 \rightarrow f(4) = 2f(3) + 3 = 2 \times 45 + 3 = 93$

Q// find $f(1), f(2), f(3)$ & $f(4)$ if $F(n)$ defined recursively by $f(0) = 1$ & for $n = 0, 1, 2, \ldots$

ⓐ $f(n+1) = f(n) + 2$

$n = 0$
$f(1) = f(0) + 2 = 1 + 2 = 3$
$n = 1$
$f(2) = f(1) + 2 = 3 + 2 = 5$
$n = 2$
$f(3) = f(2) + 2 = 5 + 2 = 7$
$f = 3$
$f(4) = f(3) + 2 = 7 + 2 = 9.$

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→ steps
We use two steps to define the func^n with set of non-negative integer as its domain

step 1:- Basis step:-
Specify the value of the func^n at zero

Step 2:- Recursive Step :-
Give a rule for finding its value at an integer from at value at smaller integer.

Q- part b). = $f(n+1) = 3f(n)$

$n=0$
$f(1) = 3f(0) = 3 \times 1 = 3$

$n=1$
$f(2) = 3f(1) = 3 \times 3 = 9$

$n=2$
$f(3) = 3f(2) = 3 \times 9 = 27$

$n=3$
$f(4) = 3f(3) = 3 \times 27 = 81$.

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c) $f(n+1) = 2^{f(n)}$

$n=0$
$f(1) = 2^{f(0)} = 2^1 = 2$

$n=1$
$f(2) = 2^{f(1)} = 2^2 = 4$

$n=2$
$f(3) = 2^{f(2)} = 2^4 = 2 \times 2 \times 2 \times 2 = 16$

$n=3$
$f(4) = 2^{f(3)} = 2^{16} = 65536$.

d) = $f(n+1) = f(n^2) + f(n) + 1$

⇒ $n=0$
$f(1) = f(0^2) + f(0) + 1 = 1 + 1 + 1 = 3$

$n=1$
$f(2) = f(1^2) + f(1) + 1 = 3^2 + 3 + 1 = 13$

$n=2$
$f(3) = f(2^2) + f(2) + 1 = 13^2 + 13 + 1 = 13^2 + 14 = 183$

$n=3$
$f(4) = f(3^2) + f(3) + 1) = 183^2 + 183 + 1 = 33673$.

Q/ $f(0)=3$

ⓐ $f(n+1) = -2f(n)$

ⓑ $f(n+1) = 3f(n)+7$.

→ ⓐ $f(n+1) = -2f(n)$

$n=0$

$f(1) = -2f(0) = -2(3) = -6$

$n=1$

$f(2) = -2f(1) = -2\times -6 = 12$

$n=2$

$f(3) = -2f(2) = -2\times 12 = -26$

$n=3$

$f(4) = -2f(3) = -2\times -26 = 52$

ⓑ $f(n+1) = 3f(n)+7$

$n=0$

$f(1) = 3f(0)+7 = 3\times 3+7 = 9+7 = 16$

$n=1$

$f(2) = 3f(1)+7 = 3\times 16+7 = 48+7 = 55$

$n=2$

$f(3) = 3f(2)+7 = 3\times 55+7 = 135+7 = 172$

$n=3$

$f(4) = 3f(3)+7 = 3\times 172+7 = 523$.

Q/ ⓒ $f(n+1) = f(n)^2 - 2f(n) - 2$

$f(0)=3 \quad f(5)=?$

→ $n=0$

$f(1) = f(0)^2 - 2f(0) - 2 = 3^2 - 2\times 3 - 2 = 9-6-2$

$=1$

$n=1$

$f(2) = f(1)^2 - 2f(1) - 2 = 1^2 - 2\times 1 - 2 = -3$

$n=2$

$f(3) = f(2)^2 - 2f(2) - 2 = -3^2 - 2\times -3 - 2 = 9-6$

$=13$

$n=3$

$f(4) = f(3)^2 - 2f(3) - 2 = 13^2 - 2\times 13 - 2 = 141$

$n=4$

$f(5) = f(4)^2 - 2f(4) - 2 = 141^2 - 2\times 141 - 2$

$= 19597$.

ⓓ $f(n+1) = 3^{f(n)/3}$

$n=0$

→ $f(1) = 3^{f(0)/3} = 3$

$n=1$

$f(2) = 3^{f(1)/3} = 3$

$n=2$

$f(3) = 3^{f(2)/3} = 3$

$n=3$

$f(4) = 3^{f(3)/3} = 3$

$n=4$

$f(5) = 3^{f(4)/3} = 3$

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Q) $f(0) = 3$

(a) $f(n+1) = -2f(n)$
(b) $f(n+1) = 3f(n) + 7$

→ (a) $f(n+1) = -2f(n)$

$n = 0$
$f(1) = -2f(0) = -2(3) = -6$

$n = 1$
$f(2) = -2f(1) = -2 \times -6 = 12$

$n = 2$
$f(3) = -2f(2) = -2 \times 12 = -26$

$n = 3$
$f(4) = -2f(3) = -2 \times -26 = 52$

(b) $f(n+1) = 3f(n) + 7$

$n = 0$
$f(1) = 3f(0) + 7 = 3 \times 3 + 7 = 9 + 7 = 16$

$n = 1$
$f(2) = 3f(1) + 7 = 3 \times 16 + 7 = 48 + 7 = 55$

$n = 2$
$f(3) = 3f(2) + 7 = 3 \times 55 + 7 = 135 + 7 = 142$

$n = 3$
$f(4) = 3f(3) + 7 = 3 \times 142 + 7 = 523.$

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Q) (c) $f(n+1) = f(n)^2 - 2f(n) - 2$
$f(0) = 3$ $f(5) = ?$

→ $n = 0$
$f(1) = f(0)^2 - 2f(0) - 2 = 3^2 - 2 \times 3 - 2 = 9 - 6 - 2$
$= 1$

$n = 1$
$f(2) = f(1)^2 - 2f(1) - 2 = 1^2 - 2 \times 1 - 2 = -3$

$n = 2$
$f(3) = f(2)^2 - 2f(2) - 2 = -3^2 - 2 \times -3 - 2 = 9 - 6 -$
$= 13$

$n = 3$
$f(4) = f(3)^2 - 2f(3) - 2 = 13^2 - 2 \times 13 - 2 = 141$

$n = 4$
$f(5) = f(4)^2 - 2f(4) - 2 = 141^2 - 2 \times 141 - 2 =$
$= 19597.$

(d) $f(n+1) = 3^{f(n)/3}$
$n = 0$
→ $f(1) = 3^{f(0)/3} = 3$

$n = 1$
$f(2) = 3^{f(1)/3} = 3$

$n = 2$
$f(3) = 3^{f(2)/3} = 3$

$n = 3$
$f(4) = 3^{f(3)/3} = 3$

$n = 4$
$f(5) = 3^{f(4)/3} = 3$

Q/ $f(0) = 3$

(a) $f(n+1) = -2f(n)$
(b) $f(n+1) = 3f(n) + 7$

→ (a) $f(n+1) = -2f(n)$

$n = 0$
$f(1) = -2f(0) = -2(3) = -6$

$n = 1$
$f(2) = -2f(1) = -2 \times -6 = 12$

$n = 2$
$f(3) = -2f(2) = -2 \times 12 = -26$

$n = 3$
$f(4) = -2f(3) = -2 \times -26 = 52$

(b) $f(n+1) = 3f(n) + 7$

$n = 0$
$f(1) = 3f(0) + 7 = 3 \times 3 + 7 = 9 + 7 = 16$

$n = 1$
$f(2) = 3f(1) + 7 = 3 \times 16 + 7 = 48 + 7 = 55$

$n = 2$
$f(3) = 3f(2) + 7 = 3 \times 55 + 7 = 135 + 7 = 172$

$n = 3$
$f(4) = 3f(3) + 7 = 3 \times 172 + 7 = 523$.

Q/ (c) $f(n+1) = f(n)^2 - 2f(n) - 2$

$f(0) = 3 \qquad f(5) = ?$

→ $n = 0$
$f(1) = f(0)^2 - 2f(0) - 2 = 3^2 - 2 \times 3 - 2 = 9 - 6 - 2$
$= 1$

$n = 1$
$f(2) = f(1)^2 - 2f(1) - 2 = 1^2 - 2 \times 1 - 2 = -3$

$n = 2$
$f(3) = f(2)^2 - 2f(2) - 2 = -3^2 - 2 \times -3 - 2 = 9 - 6 - 2$
$= 13$

$n = 3$
$f(4) = f(3)^2 - 2f(3) - 2 = 13^2 - 2 \times 13 - 2 = 141$

$n = 4$
$f(5) = f(4)^2 - 2f(4) - 2 = 141^2 - 2 \times 141 - 2 = $
$= 19597$.

(d) $f(n+1) = 3^{f(n)/3}$

$n = 0$
→ $f(1) = 3^{f(0)/3} = 3$

$n = 1$
$f(2) = 3^{f(1)/3} = 3$

$n = 2$
$f(3) = 3^{f(2)/3} = 3$

$n = 3$
$f(4) = 3^{f(3)/3} = 3$

$n = 4$
$f(5) = 3^{f(4)/3} = 3$

Q. $f(0) = 3$

(a) $f(n+1) = -2f(n)$

(b) $f(n+1) = 3f(n) + 7$

→ (a) $f(n+1) = -2f(n)$

$n = 0$

$f(1) = -2f(0) = -2(3) = -6$

$n = 1$

$f(2) = -2f(1) = -2 \times -6 = 12$

$n = 2$

$f(3) = -2f(2) = -2 \times 12 = -26$

$n = 3$

$f(4) = -2f(3) = -2 \times -26 = 52$

(b) $f(n+1) = 3f(n) + 7$

$n = 0$

$f(1) = 3f(0) + 7 = 3 \times 3 + 7 = 9 + 7 = 16$

$n = 1$

$f(2) = 3f(1) + 7 = 3 \times 16 + 7 = 48 + 7 = 55$

$n = 2$

$f(3) = 3f(2) + 7 = 3 \times 45 + 7 = 135 + 7 = 142$

$n = 3$

$f(4) = 3f(3) + 7 = 3 \times 142 + 7 = 523.$

Q. (c) $f(n+1) = f(n)^2 - 2f(n) - 2$

$f(0) = 3 \qquad f(5) = ?$

→ $n = 0$

$f(1) = f(0)^2 - 2f(0) - 2 = 3^2 - 2 \times 3 - 2 = 9 - 6 - 2$
$= 1$

$n = 1$

$f(2) = f(1)^2 - 2f(1) - 2 = 1^2 - 2 \times 1 - 2 = -3$

$n = 2$

$f(3) = f(2)^2 - 2f(2) - 2 = -3^2 - 2 \times -3 - 2 = 9 - 6 -$
$= 13$

$n = 3$

$f(4) = f(3)^2 - 2f(3) - 2 = 13^2 - 2 \times 13 - 2 = 141$

$n = 4$

$f(5) = f(4)^2 - 2f(4) - 2 = 141^2 - 2 \times 141 - 2 =$
$= 19597.$

(d) $f(n+1) = 3^{f(n)/3}$

$n = 0$

→ $f(1) = 3^{f(0)/3} = 3$

$n = 1$

$f(2) = 3^{f(1)/3} = 3$

$n = 2$

$f(3) = 3^{f(2)/3} = 3$

$n = 3$

$f(4) = 3^{f(3)/3} = 3$

$n = 4$

$f(5) = 3^{f(4)/3} = 3$

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Q/ $f(0) = 3$

(a) $f(n+1) = -2f(n)$
(b) $f(n+1) = 3f(n) + 7$

→ (a) $f(n+1) = -2f(n)$

$n = 0$
$f(1) = -2f(0) = -2(3) = -6$

$n = 1$
$f(2) = -2f(1) = -2 \times -6 = 12$

$n = 2$
$f(3) = -2f(2) = -2 \times 12 = -26$

$n = 3$
$f(4) = -2f(3) = -2 \times -26 = 52$

(b) $f(n+1) = 3f(n) + 7$

$n = 0$
$f(1) = 3f(0) + 7 = 3 \times 3 + 7 = 9 + 7 = 16$

$n = 1$
$f(2) = 3f(1) + 7 = 3 \times 16 + 7 = 48 + 7 = 55$

$n = 2$
$f(3) = 3f(2) + 7 = 3 \times 55 + 7 = 135 + 7 = 172$

$n = 3$
$f(4) = 3f(3) + 7 = 3 \times 172 + 7 = 523.$

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Q1/ (c) $f(n+1) = f(n)^2 - 2f(n) - 2$
$f(0) = 3 \quad f(5) = ?$

→ $n = 0$
$f(1) = f(0)^2 - 2f(0) - 2 = 3^2 - 2 \times 3 - 2 = 9 - 6 - 2$
$= 1$

$n = 1$
$f(2) = f(1)^2 - 2f(1) - 2 = 1^2 - 2 \times 1 - 2 = -3$

$n = 2$
$f(3) = f(2)^2 - 2f(2) - 2 = -3^2 - 2 \times -3 - 2 = 9 - 6 - 2$
$= 13$

$n = 3$
$f(4) = f(3)^2 - 2f(3) - 2 = 13^2 - 2 \times 13 - 2 = 141$

$n = 4$
$f(5) = f(4)^2 - 2f(4) - 2 = 141^2 - 2 \times 141 - 2 =$
$= 19597.$

(d) $f(n+1) = 3^{f(n)/3}$

$n = 0$
→ $f(1) = 3^{f(0)/3} = 3$

$n = 1$
$f(2) = 3^{f(1)/3} = 3$

$n = 2$
$f(3) = 3^{f(2)/3} = 3$

$n = 3$
$f(4) = 3^{f(3)/3} = 3$

$n = 4$
$f(5) = 3^{f(4)/3} = 3$

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★ **Pigenhole principle** :- The principle state that if there are more pigen then the pigenholes then their must be one pigenhole with at least two two pigenhole in it.

defn → If k is +ve integer & k+1 or more object are placed in k boxes then their is at least one box contain two or more objects.

always
⇒ $\boxed{n > k}$

Suppose

P	P
P	PP

4 pigenholes (P)
5 pigeon/ objects

→ $\underline{n > k}$

⇒ Generalized pigeonhole principle :- If n objects (pigeon) are placed into k boxes (pigeon holes) then their is at least one box containing atleast $\lceil n/k \rceil$ objects ; n is greater than k

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