

Tutorial - 04

Ques. 1. $T(n) = 3T(n/2) + n^2$
 $\rightarrow a = 3, b = 2, f(n) = n^2$
 $n^{\log_b a} = n^{\log_2 3}$
 $\therefore n^{\log_b a} < f(n)$
 $\therefore \underline{T(n) = \Theta(n^2)}$

Ques. 2. $T(n) = 4T(n/2) + n^2$
 $\rightarrow a = 4, b = 2, f(n) = n^2$
 $n^{\log_b a} = n^{\log_2 4} = n^{\log_2 2^2}$
 $= n^2$
 $\therefore n^{\log_b a} = f(n)$
 $\therefore \underline{T(n) = \Theta(n^2 \log n)}$

Ques. 3. $T(n) = T(n/2) + 2^n$
 $\rightarrow a = 1, b = 2, f(n) = 2^n$
 $n^{\log_b a} = n^{\log_2 1} = n^0 = 1$
 $n^{\log_b a} < 2^n$
 $\therefore T(n) = \Theta(2^n)$

Ques. 4. $T(n) = 2^n + T(n/2) + n^{2n}$
 $\therefore a$ is a function.
 \Rightarrow Master's theorem is not possible.

Ques. 5. $T(n) = 16T(n/4) + n$
 $\rightarrow a = 16, b = 4, f(n) = n$
 $n^{\log_b a} = n^{\log_4 16} = n^{\log_4 4^2} = n^2$
 $\therefore n^{\log_b a} \geq f(n)$
 $\therefore \underline{T(n) = \Theta(n^2)}$

Ques. 6. $T(n) = 2T(n/2) + n \log n$
 $\rightarrow a=2, b=2, f(n) = n \log n$
 $n \log_b a = n \log_2 2 = n$
 $f(n) > n \log_b a$
 $\therefore T(n) = \Theta(n \log n)$

Ques. 7. $T(n) = 2T(n/2) + n / \log n$
 $\rightarrow a=2, b=2, f(n) = n / \log n$
 $n \log_b a = n \log_2 2 = n$
 $\therefore n \log_b a < f(n)$
 $\therefore T(n) = \Theta(n)$

Ques. 8. $T(n) = 2T(n/4) + n^{0.51}$
 $\rightarrow a=2, b=4, f(n) = n^{0.51}$
 $n \log_b a = n \log_4 2 = n^{0.51}$
 $\therefore n \log_b a < f(n)$
 $\therefore T(n) = \Theta(n^{0.51})$

Ques. 9. $T(n) = 0.5T(n/2) + 1/n$
 $\therefore a < 1$
 \therefore Master's theorem not applicable.

Ques. 10. $T(n) = 16T(n/4) + n!$
 $\rightarrow a=16, b=4, f(n) = n!$
 $n \log_b a = n \log_4 16 = n \log_4 4^2 = n^2$
 $\therefore n^2 < n! \quad (\text{i.e. } n \log_b a < f(n))$
 $\therefore T(n) = \Theta(n!)$

Ques. 11. $T(n) = 4T(n/2) + \log n$
 $\rightarrow a=4, b=2, f(n) = \log n$
 $n \log_b a = n \log_2 4 = n^2$
 $\therefore n \log_b a > f(n)$
 $\therefore T(n) = \Theta(n^2)$

Ques. 12. $T(n) = \text{sqrt}(n) T(n/2) + \log n$
 $\therefore a$ is not constant
 \therefore Master's theorem not applicable.

Ques. 13. $T(n) = 3T(n/2) + n$
 $\rightarrow a=3, b=2, f(n)=n$
 $n \log_b a = n \log_2 3 = n^{1.58}$
 $n \log_b a > f(n)$
 $\therefore T(n) = \Theta(n^{1.58})$

Ques. 14. $T(n) = 3T(n/3) + \sqrt{n}$
 $\rightarrow a=3, b=3, f(n)=\sqrt{n}$
 $n \log_b a = n \log_3 3 = n$
 $\therefore n \log_b a > f(n)$
 $\Rightarrow T(n) = \Theta(n)$

Ques. 15. $T(n) = 4T(n/2) + cn$
 $\rightarrow a=4, b=2, f(n)=c \cdot n$
 $n \log_b a = n \log_2 4 = n \log_2 2^2 = n^2$
 $\therefore n \log_b a > f(n)$
 $\therefore T(n) = \Theta(n^2)$

Ques. 16. $T(n) = 3T(n/4) + n \log n$
 $\rightarrow a=3, b=4, f(n)=n \log n$
 $n \log_b a = n \log_4 3 = n^{0.79}$
 $\therefore n \log_b a < f(n)$
 $\therefore T(n) = \Theta(n \log n)$

Ques. 17. $T(n) = 3T(n/3) + n/2$
 $\rightarrow a=3, b=3, f(n)=n/2$
 $n \log_b a = n \log_3 3 = n$
 $\therefore n \log_b a > f(n)$
 $\therefore T(n) = \Theta(n)$

Ques. 18. $T(n) = 6T(n/3) + n^2 \log n$
 $\rightarrow a=6, b=3, f(n)=n^2 \log n$
 $n \log_b a = n \log_3 6 = n^{1.63}$
 $n \log_b a < f(n) \Rightarrow T(n) = \Theta(n^2 \log n)$

Ques. 19. $T(n) = 4T(n/2) + n/\log n$
 $\rightarrow a=4, b=2, f(n)=n/\log n$
 $n \log_b a = n \log_2 4 = n^2$
 $n \log_b a > f(n)$
 $\Rightarrow T(n) = \Theta(n^2)$

Ques. 20. $T(n) = 64T(n/8) - n^2 \log n$

→ Master's theorem is not applied as $f(n)$ is not increasing function.

Ques. 21. $T(n) = 7T(n/3) + n^2$

→ $a = 7, b = 3, f(n) = n^2$

$$n^{\log_b a} = n^{\log_3 7}$$

$$= n^{1.3}$$

$$\therefore n^{\log_b a} < f(n)$$

$$\Rightarrow \underline{T(n) = \Theta(n^2)}$$

Ques. 22. $T(n) = T(n/2) + n(2 - \log n)$

→ Master's theorem is not applied.

∵ regularity condition is violated is case 3.