Tutorials-02

Ques1. → find time complexity

```
void func (int n){
    int j=1, i=0;
    while (i<n){
        i=i+j;
        j++;
    }
}
```

j=1
j=2
j=3
⋮
j=k

i=1
i=1+2=3
i=3+3=1+2+3
⋮
i=1+2+3+...+k

sum of k consecutive integers = $\frac{k(k+1)}{2}$

$\frac{k^2+k}{2} < n$

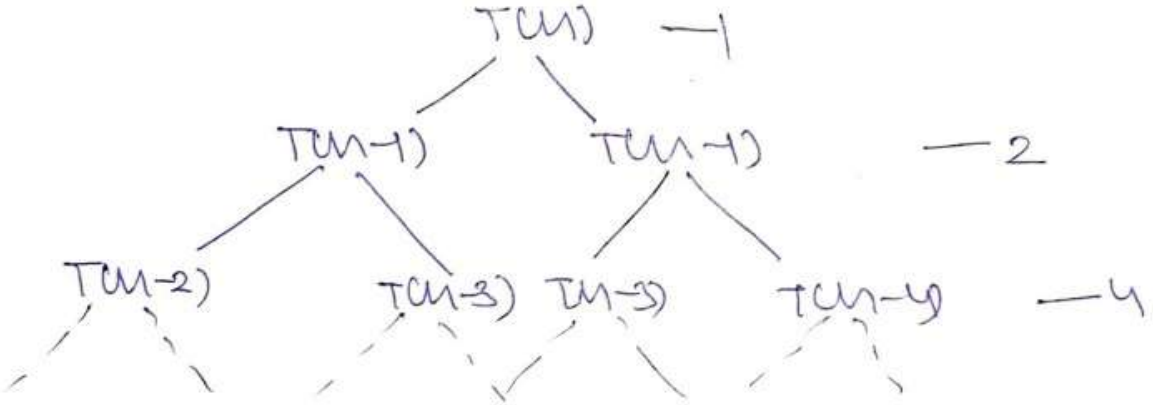
$k^2 < n$ (ignoring constants)

$k < \sqrt{n}$

→ $\boxed{T(n) = O(\sqrt{n})}$

Ques.2. Recursive relation for fibonacci series:

$T(n) = T(n-1) + T(n-2)$



$\Rightarrow 1+2+4+8+\cdots$

Here, $a=1$, $r=2$

So, $\frac{a(r^n-1)}{r-1} = \frac{2^n-1}{1}$

$= 2^n - 1$

$\boxed{T(n) = O(2^n)}$

Ques.3.

(i) $n\log n$

```
void quick_sort (int a[], int lb, int ub)
{
    int i=lb, j=ub;
    int key=a[lb];
    int t=0;
    if (lb>=ub)
      return;
    while (i<j){
      while (key>=a[i] && i<j)
        i++;
      while (key<a[j])
        j--;
      if (i<j){
          t=a[i];
          a[i]=a[j];
          a[j]=t;
      }
    a[lb]=a[j];
    a[j]=key;
    quick_sort(a,0,j-1);
    quick_sort(a,j+1,ub);
}
```

(ii) $O(n^3)$

```
for (int i=0; i<n; i++)
{
    for (int j=0; j<n; j++)
    {
        for (int k=0; k<n; k++)
        {
            sum+=k;
        }
    }
}
```

iii) O(log(log n))

```
int p=0;
for(int i=1; i<n; i=i*2)
{
    p++;
}
for(int j=1; j<p; j=j*2)
{
    // O(1) operation
}
```

Ques.4. $T(n) = T(n/4) + T(n/2) + cn^2$

$= 2T(n/2) + cn^2$

using Master's Method, $T(n) = aT(n/b) + f(n)$

$a \geq 1, b > 1, c = \log_b a$

$c = \log_2 2 = 1$

$\because f(n) > n^c$

$\boxed{T(n) = f(n) = O(n^2)}$

Ques.6.

i	j
1	1, 2, 3, ... n times
2	1, 3, 5, 7, ... n/2 times
3	1, 4, 7, 11, ... n/3 times
4	.
1	.
1	.
1	.
n	j = 1 ... n, n/2, n/3 times

$T(n) = n + n/2 + n/3 + n/4 + \dots + 1$

$= n\left[1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{n}\right]$

$\rightarrow \boxed{T(n) = n\log n}$

Ques.6.

$T(n) = 2 \cdot 2^k \cdot 2^{k^2} \cdot 2^{k^3} \cdots 2^{k \log_k(\log n)}$

2o. $2^{k \log_k(\log n)} = 2^{\log n} = n$

3o. Total time complexity,

$\rightarrow \boxed{T(n) = O(\log k \log n)}$

Ques.7.

i) $100 < \log(\log n) < \log n < \log^2 n < \sqrt{n} < n < n \log n$
$< n^2 < 2^n < 4^n < 2^{2n} < \log(n!) < n!$

ii) $1 < \log(\log n) < \sqrt{\log n} < \log n < \log 2n <$
$2 \log n < n < 2n < 4n < n \log n < n^2 < \log(n!)$
$< n! < 2(2n)$

iii) $96 < \log_2(n) < \log_2 n < 5n < n \log_2 n < n \log_2 n$
$< n! < \log n! < 8^{2n}$