Tutorial - 06

1) what do you mean by Minimum spanning tree? what are the applications of MST?

Minimum spanning tree is a subset of edges of a connected edge-weighted undirected graph that connects all the vertices together without any cycles and with minimum possible edge weighted.

Applications -

i) consider n stations are to be linked using a communication network and laying of comm. link between any two stations involves a cost. The ideal solution would be to extract a subgraph termed as minimum cost spanning tree.

ii) Designing LAN

iii) suppose you want to construct highways or railroad spanning several cities, then we can use concept of MST

iv) laying pipelines connecting offshore drilling sites, refineries and consumer markets.

2) Analyze time and space complexity of Prim, Krushal, Dijkstra and Bellman ford algorithm.

Time complexity of Prim's Algo. - O(|E| log |V|)

space complexity of Prim's Algo. - O|V|

Time complexity of Krushal's Algo. - O|E| log |E|

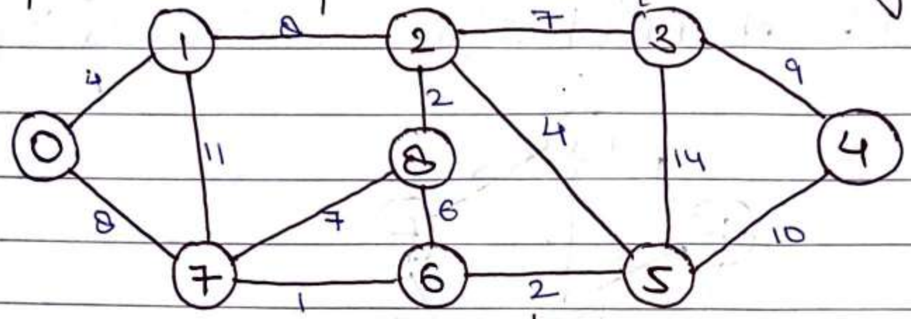
space complexity of Krushal's Algo. - O|V|

Time complexity of Dijkstra's Algo. - $O(V^2)$

space complexity of Dijkstra's Algo. - $O(V^2)$

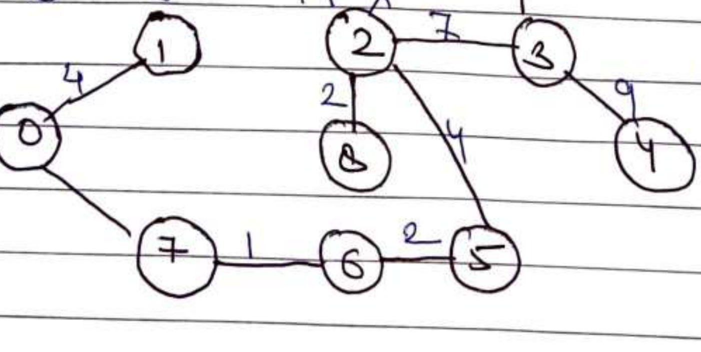
Time complexity of Bellman ford's algo. - O(VE)
space complexity of Bellmon ford's algo - O(E)

3) Apply Krushal and Prim's Algo. on given graph to complete MST & its weight.



Krushal's Algorithm

O	V	W	
6	7	1	✓
5	6	2	✓
2	8	2	✓
0	1	4	✓
2	5	4	✓
6	8	6	X
2	3	7	✓
7	8	7	X
0	7	8	✓
1	2	8	X
4	3	9	✓
4	5	10	X
1	7	11	X
3	5	14	X



weight = 1+2+2+4+4+7+6+9
= 37

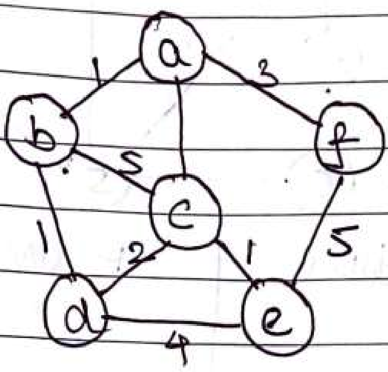
Prim's Algorithm

weight = 4+8+2+4+2+7+9+3
= 37

4) Given a directed weighted graph. you are also given the shortest path from a source vertex 's' to a destination vertex 't'. Does the shortest path remain same in following cases:

i) If weight of every edge is increased by 10 units.

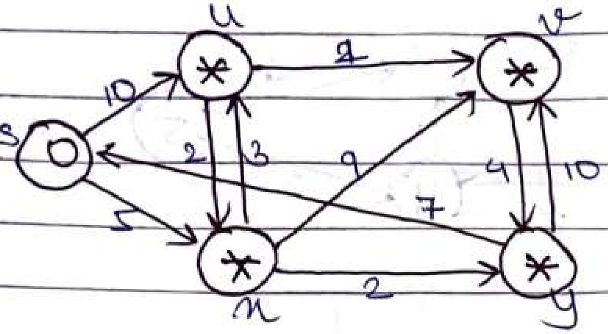
ii) If weight of every edge is multiplied by 10 units.



i) The shortest path may change. The reason is that there may be different no. of edges in different paths from 's' to 't'. for eg- let the shortest path of weight 15 and has edges 5. let there we another path with 2 edges & total weight 25. The weight of shortest path is increased by 5x10 and becomes 15+50. weight of other path is increased by 2x10 & becomes 26+20 so, the shortest path changes to other path with weight as 45.

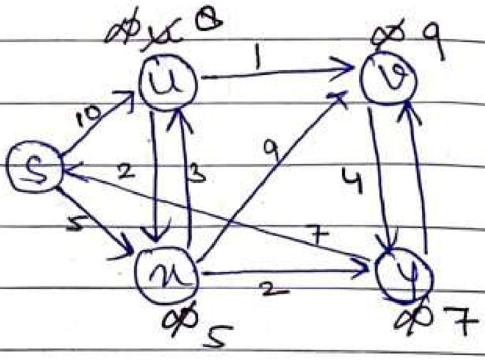
ii) If we multiply all edges weight by 10, the shortest path decannot change. The reason is that weights of all path from 's' to t' gets multiplied by same unit. The no. of edges on path doesn't matter.

5) Apply Dijkstra & Bellman ford algorithm on graph given right side to compute shortest path to all nodes from nodes S.



→ Dijkstra's Algorithm-

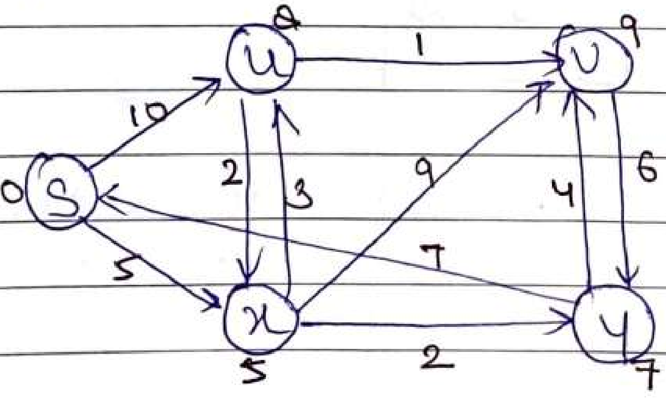
Node	shortest dist. from source node
u	8
x	5
v	9
y	7



→ Bellman ford Algorithm-

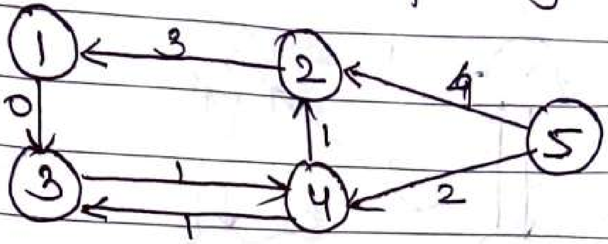
1st →	S^{0}	$u^{10\ \cancel{\infty}}$	$V^{\cancel{\infty}}$	$X^{\cancel{\infty}\ 5}$	Y^{∞}
2nd →	S^{0}	u^{10}	$V^{\cancel{\infty}\ 11}$	X^{5}	Y^{∞}
3rd →	S^{0}	$u^{\cancel{10}\ 8}$	$V^{\cancel{11}\ 9}$	X^{5}	$Y^{\cancel{\infty}\ 7}$
4rd →	S^{0}	u^{8}	V^{9}	X^{5}	Y^{7}

graph does not have negative cycle



→ find Graph

6) Apply all pair shortest path algorithm. Floyd warshall on below mentioned graph. Also analyse space & time complexity of it.



$$\begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & \infty & 6 & 3 & \infty \\ 2 & 2 & 0 & \infty & \infty & \infty \\ 3 & \infty & \infty & 0 & 2 & \infty \\ 4 & \infty & 1 & 1 & 0 & \infty \\ 5 & \infty & 4 & \infty & 2 & 0 \end{array}$$

$$\begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & \infty & 6 & 3 & \infty \\ 2 & 2 & 0 & 8 & 5 & \infty \\ 3 & \infty & \infty & 0 & 2 & \infty \\ 4 & \infty & 1 & 1 & 0 & \infty \\ 5 & \infty & 4 & \infty & 2 & 0 \end{array}$$

$$\begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & \infty & 6 & 3 & \infty \\ 2 & 2 & 0 & 8 & 5 & \infty \\ 3 & \infty & \infty & 0 & 2 & \infty \\ 4 & 3 & 1 & 1 & 0 & \infty \\ 5 & 6 & 4 & 12 & 2 & 0 \end{array}$$

$$\begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & \infty & 6 & 3 & \infty \\ 2 & 2 & 0 & 8 & 5 & \infty \\ 3 & \infty & \infty & 0 & 2 & \infty \\ 4 & 3 & 1 & 1 & 0 & \infty \\ 5 & 6 & 4 & 12 & 2 & 0 \end{array}$$

T.C. → $O(|V|^3)$

S.C. → $O(|V|^2)$