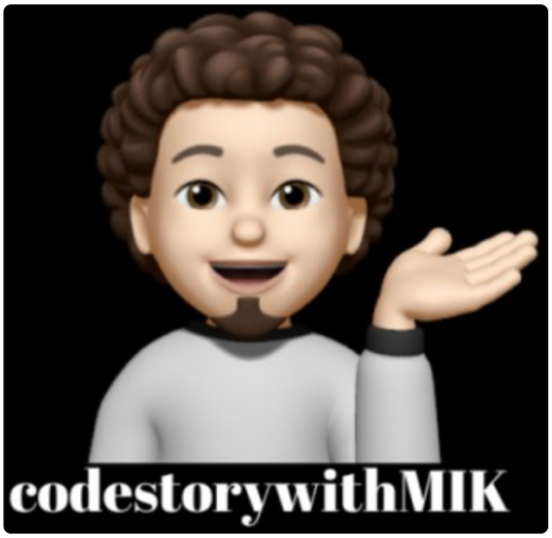
Maths Concepts & Qns

Video-4



Instagram → codestorywithmik

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Motivation :-

If you don't push yourself out of

the comfort zone / limits, you will never grow and learn new things.

Go beyond your limits to be Exceptional.

Modular ^nC_r
Fermat's Little Theorem

$a/b = x$

Basic Arithmetic :- $16/4 = 4$

$10/5 = 2$ etc.

Modular Arithmetic :- we only care about remainder about remainder

$a \% b$ = remainder

when a number is divided by another.

i.e $10 \% 7 = 3$ → remainder

$\% \ 10^9+7$

$10 \overline{) 7} (1$
$\underline{10}$
3

"This is used heavily in problems dealing with large numbers"

$$^nC_r$$

— — — — — n items

$$\boxed{^nC_r = \frac{n!}{r! * (n-r)!}}$$

When dealing with large numbers, value of ^nC_r is very large.

$M = 10^9+7$

So, we are asked to find

$\left({}^{n}C_{r}\right) \%$ Pime no $\equiv$ "Modular ${}^{n}C_{r}$"

$\downarrow$

$M\ (10^{9}+7)$

What's the problem??

$\left(\frac{14}{7}\right) \% 7 = 0 \quad \left(\frac{10}{2}\right) \% 7 = 5 \% 7 = 5$

$\boxed{\left(\frac{3}{2}\right) \% 7} \equiv \boxed{(1.5)} \% 7 \quad \times$

$$3 * \frac{1}{2} \quad \% 7$$

$$3 * (2)^{-1} \quad \% 7$$

"Modular inverse of 2 mod 7" $\equiv$ P

4

$$(2 * P) \% 7 = 1$$

$$(2 * 1) \% 7 \neq 1$$

$$(2 * 2) \% 7 \neq 1$$

$$(2 * 3) \% 7 \neq 1$$

$$\boxed{(2 * 4) \% 7 = 1}$$

Brute Force

$$(3 * 4) \% 7$$

$$= 12 \% 7$$

$= 5 \longrightarrow$ correct answer

Fermat's little theorem

"Modular inverse of a mod M"

$$\Rightarrow \boxed{a^{M-2} \% M}$$

Binary Exponentiation

$a = 2$
$M = 7$

$2^5 \% 7$

$= 32 \% 7$
$= 4$

How this helps to find ^nC_r ?

$$^nC_r = \left(\frac{\overbrace{n!}^{a}}{\underbrace{r! * (n-r)!}_{b}} \right) \% M$$

$$\left(\frac{a}{b}\right) \% M$$

$a \rightarrow n!$

$b \rightarrow r! * (n-r)!$

$$\left(a * (\text{Modular inverse of } b \bmod M) \right) \% M$$

$$b^{M-2} \% M$$

Fermat's theorem.

^nC_r

$r < 0$
$r > n$

Code :- $(^nC_r) \% M$

$M = 10^9 + 7;$

```
int modularnCr( int n, int r) {
    if (r<0 || r>n)
        return 0;

    long long a = fact(n);
```

$$^nC_r = \frac{n!}{r!\,(n-r)!}$$

```
long long b = (fact(r) * fact(n-r)) % M ;

return a * findPower(b, M-2) % M ;
}
```

Fermat's little theorem

Leetcode - 2338

POTD.

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