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Design & Analysis of Algorithm

Tutorial Sheet 6

Ques 1 What do you mean by minimum spanning tree? What are the applications of MST?

Ans A minimum spanning tree is a subset of the edges of a connected, edge-weighted directed or undirected graph that connect all the vertices together, without any cycles and with the minimum possible total edges weight. It is a spanning tree whose sum of edge weight is as small as possible

Application of MST

I Cluster analysis

II Telecommunication network

III Civil Network Planning

IV Handwriting recognition

V Image segmentation

VI Computer Network Routing Protocol

Ques 2 Please analyse the time & space complexity of Prim, Kruskal, Dijkstra & Bellman ford algorithm.

Ans

Dijkstra's Algorithm

$T.C = \theta(V^2)$

$O(V + E\log V)$ min priority Queue

$SC = O(V^2)$

Prim's Algorithm

$T.C. = O(V^2)$

$= O(E\log V)$ fibonacci heap

$S.C. = O(V+E)$

Kruskal's Algorithm

$S.C. = O(|E| + |V|)$

$T.C. = O(E\log V)$ Disjoint-Set

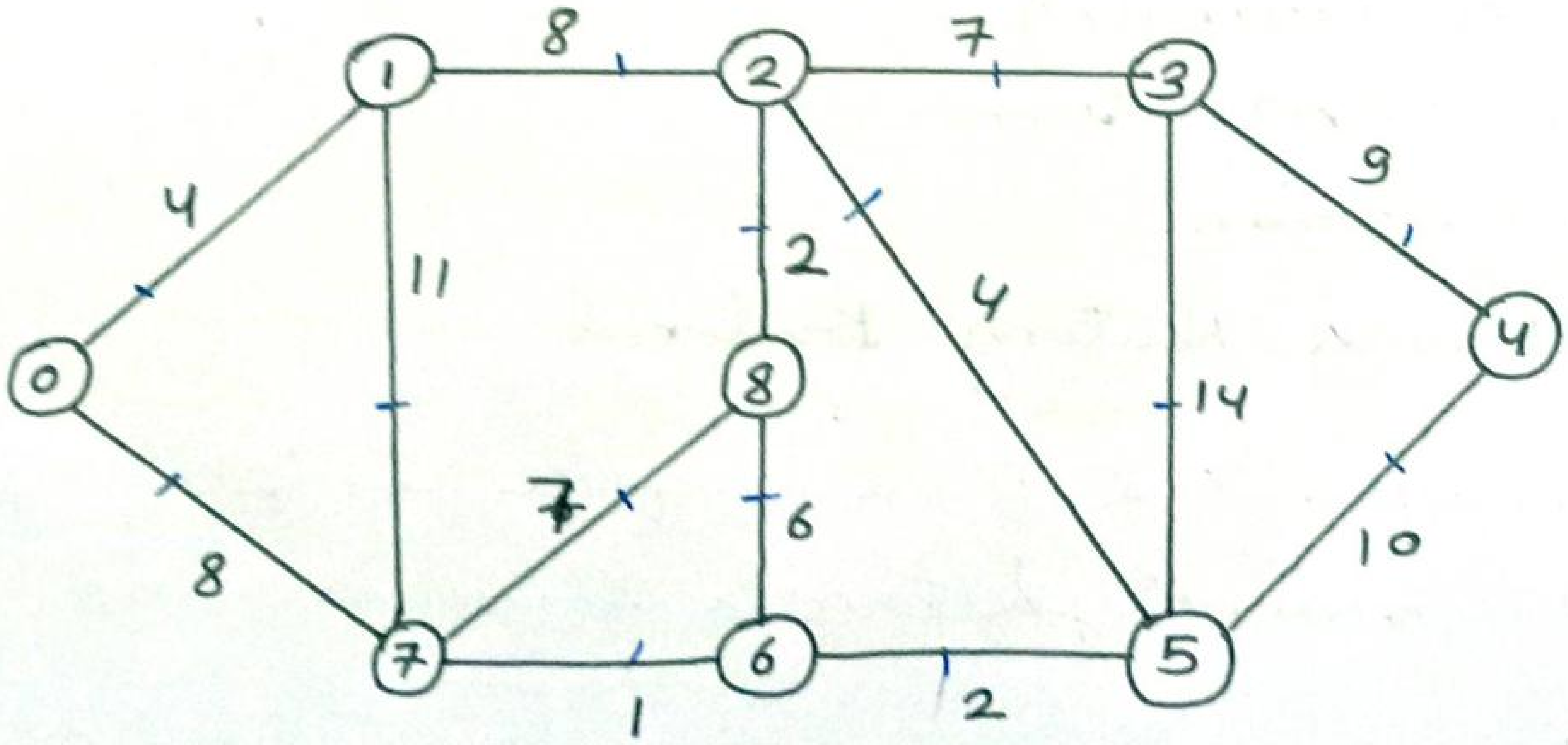
Bellman ford

$T.C. = O(VE)$

$= O(V^3)$ complete graph.

$S.C. = O(E)$

Ques3 Apply Kruskal & Prim's algorithm on graph given on right side to compute MST & its weight?



Ans

Ans **Krushal Algorithm**

Step I list all edges

(0,1) = 4   (0,7) = 8   (1,7) = 11   (1,2) = 8

(2,8) = 2   (2,5) = 4   (2,3) = 7   (8,7) = 7

(8,6) = 6   (6,7) = 1   (6,5) = 2   (3,5) = 14

(3,4) = 9   (4,5) = 10

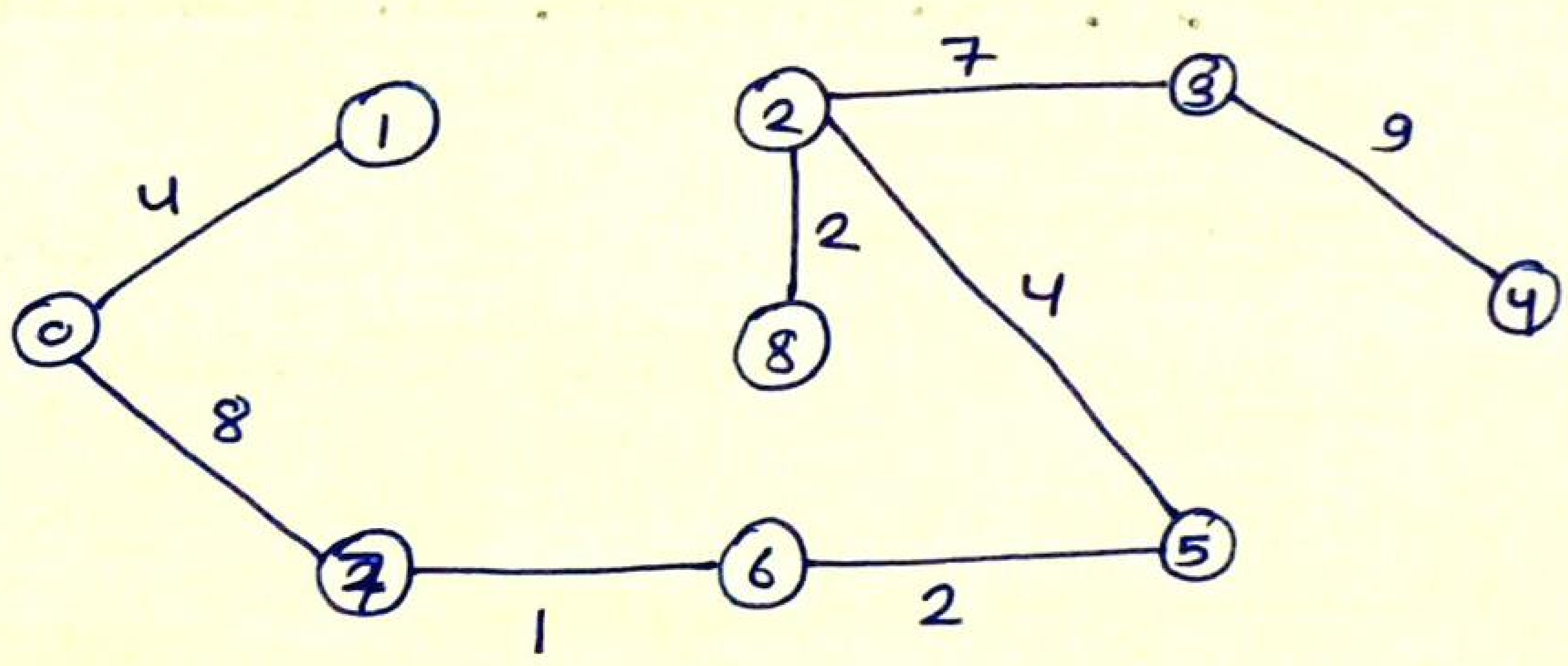
Step II Sorted in assending order

(6,7) = 1   (2,8) = 2   (6,5) = 2   (2,5) = 4

(0,1) = 4   (8,6) = 6   (2,3) = 7   (8,7) = 7

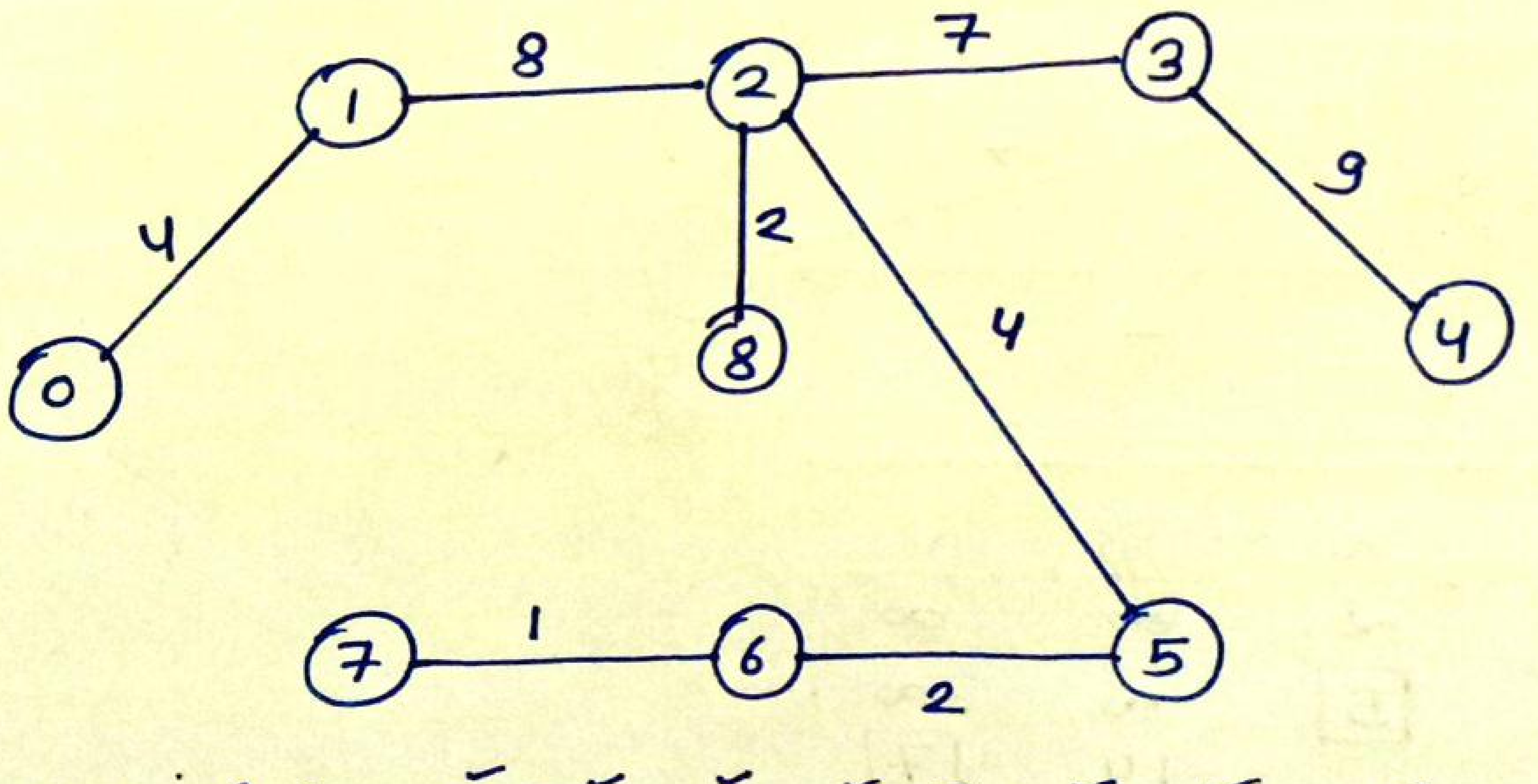
(0,7) = 8   (1,2) = 8   (3,4) = 9   (4,5) = 10

(1,7) = 11   (3,5) = 14



weight = 4 + 8 + 1 + 2 + 4 + 2 + 7 + 9 = 37

Prim Algorithm



weight = 4 + 8 + 2 + 1 + 2 + 4 + 7 + 9 = 37

Ques 4 Given a directed weight graph. You are also given the shortest path from a source vertex 's' to a destination vertex 't'. Does the shortest path remain same in the modified graph in following cases?

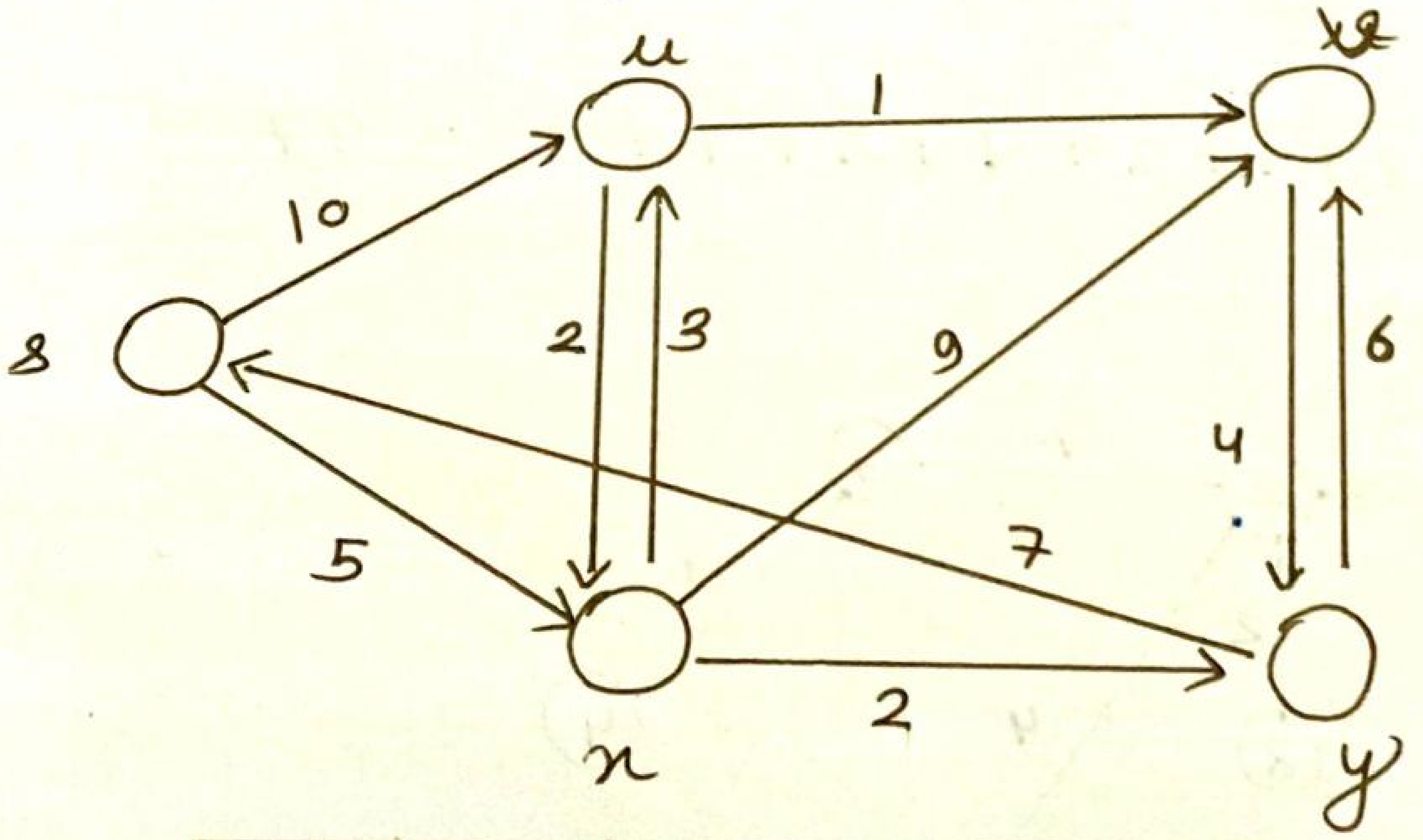
(i) If weight of every edge is increase by 10 unit.

Ans If we add, 10 to each edge, the sorted array of path will be on same order. Therefore, there will be no change on shortest path.

(ii) If weight of every edge is multiplied by 10 unit.

Ans If we multiply, array will be on same order, there will be no change on shortest path

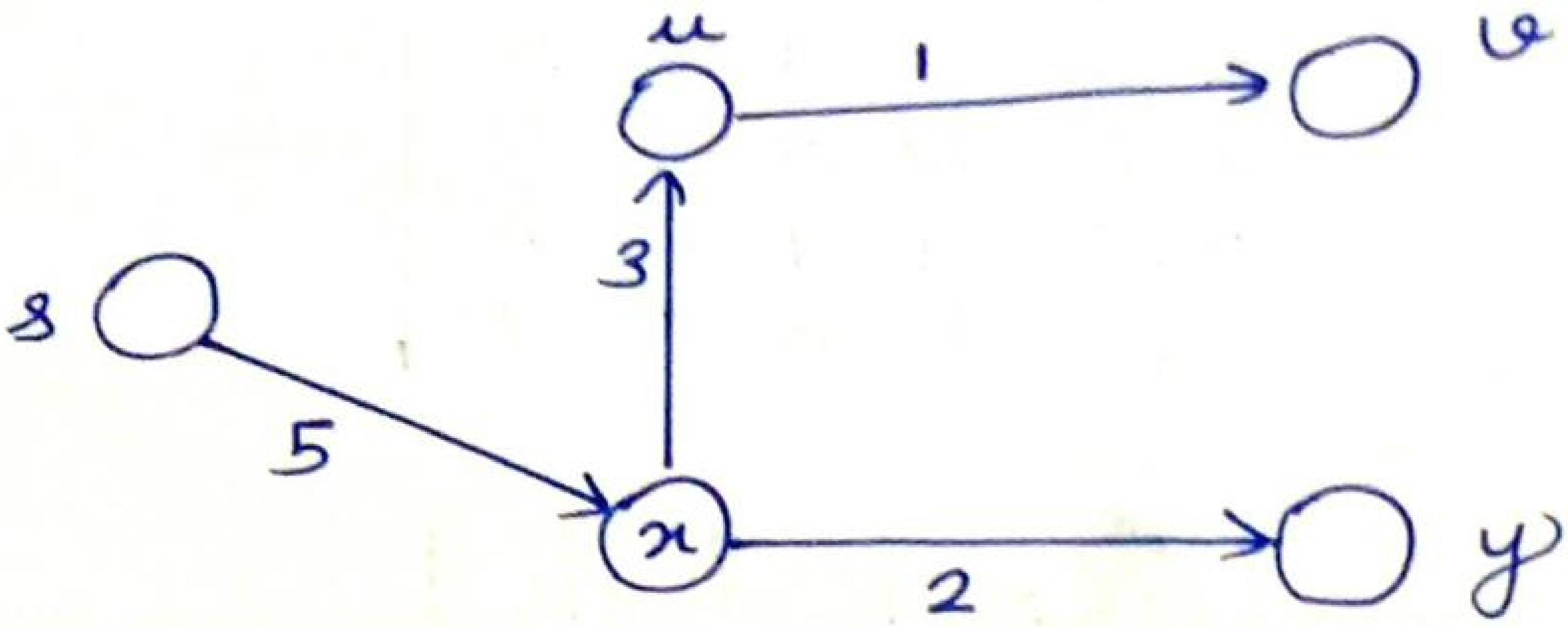
Ques 5 Apply Dijkstra & Bellman algo. on graph given on right side to compute shortest path of all nodes from node S



s	u	x	y	v
$\boxed{0}$	∞	∞	∞	∞
—	10	$\boxed{5}$	∞	∞
—	8	—	14	$\boxed{7}$
—	$\boxed{18}$	—	13	—
—	—	—	[illegible]	—

Scanned by TapScanner

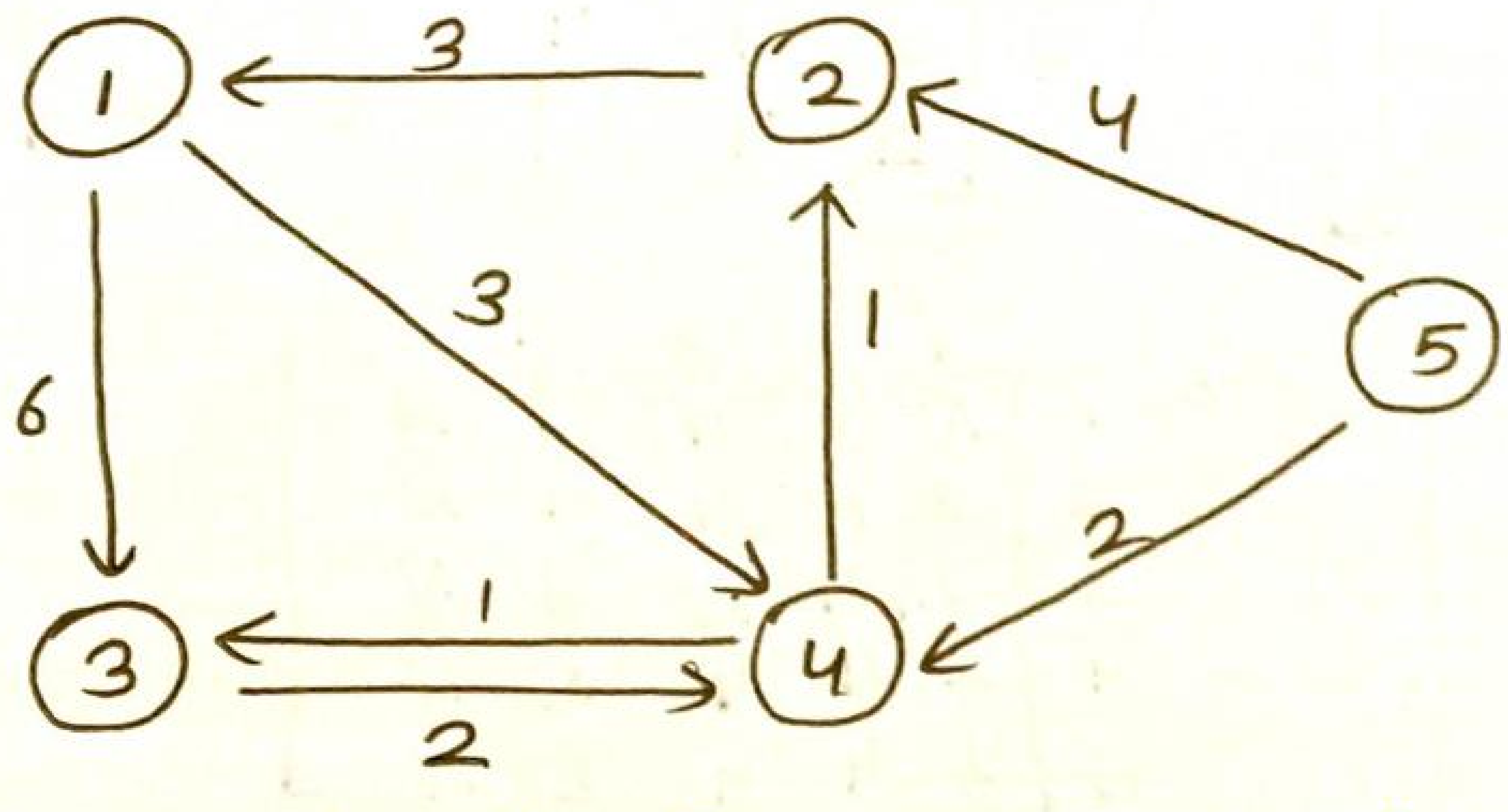
s → x → v → u → y



Bellman ford

s	u	v	x	y
0	∞	∞	∞	∞
0	10	11	5	17
0	8	9	5	7
0	8	9	5	7
0	8	9	5	7

Ques 6 Apply all pair shortest path algo - Floyd warshall on below mentioned graph & also analyse the time & space complexity of algo.



Ans

$$D_0 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & \infty & 6 & 3 & \infty \\ 3 & 0 & \infty & \infty & \infty \\ \infty & \infty & 0 & 2 & \infty \\ \infty & 1 & 1 & \infty & \infty \\ \infty & 4 & \infty & 2 & \infty \end{bmatrix} \end{array}$$

$$\pi_0 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} N & N & 1 & 1 & N \\ 2 & N & N & N & N \\ N & N & N & 3 & N \\ N & 4 & 4 & N & N \\ N & 5 & N & 5 & N \end{bmatrix} \end{array}$$

$$D_1 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & \infty & 6 & 3 & \infty \\ 3 & 0 & 9 & 6 & \infty \\ \infty & \infty & 0 & 2 & \infty \\ \infty & 1 & 1 & 0 & \infty \\ \infty & 4 & \infty & 2 & 0 \end{bmatrix} \end{array} \qquad \pi_1 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} N & N & 1 & 1 & N \\ 2 & N & 1 & 1 & N \\ N & N & N & 3 & N \\ N & 4 & 4 & N & N \\ N & 5 & N & 5 & N \end{bmatrix} \end{array}$$

$$D_2 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & \infty & 6 & 3 & \infty \\ 3 & 0 & 9 & 6 & \infty \\ \infty & \infty & 0 & 2 & \infty \\ 4 & 1 & 1 & 0 & \infty \\ 7 & 4 & 13 & 2 & 0 \end{bmatrix} \end{array} \qquad \pi_2 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} N & N & 1 & 1 & N \\ 2 & N & 1 & 1 & N \\ N & N & N & 3 & N \\ 2 & 4 & 4 & N & N \\ 2 & 5 & 2 & 5 & N \end{bmatrix} \end{array}$$

$$D_3 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & \infty & 6 & 3 & \infty \\ 3 & 0 & 9 & 6 & \infty \\ \infty & \infty & 0 & 2 & \infty \\ 4 & 1 & 1 & 0 & \infty \\ 7 & 4 & 13 & 2 & 0 \end{bmatrix} \end{array} \qquad \pi_3 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} N & N & 1 & 1 & N \\ 2 & N & 1 & 1 & N \\ N & N & N & 3 & N \\ 2 & 4 & 4 & N & N \\ 2 & 5 & 2 & 5 & N \end{bmatrix} \end{array}$$

$$D_4 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & 4 & 4 & 3 & \infty \\ 3 & 0 & 7 & 6 & \infty \\ 6 & 3 & 0 & 2 & \infty \\ 4 & 1 & 1 & 0 & \infty \\ 6 & 3 & 3 & 2 & 0 \end{bmatrix} \end{array} \qquad \pi_4 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} N & 4 & 4 & 1 & N \\ 2 & N & 4 & 1 & N \\ 2 & 4 & N & 3 & N \\ 2 & 4 & 4 & N & N \\ 2 & 4 & 4 & 5 & N \end{bmatrix} \end{array}$$

$$D_5 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & 4 & 4 & 3 & \infty \\ 3 & 0 & 7 & 6 & \infty \\ 6 & 3 & 0 & 2 & \infty \\ 4 & 1 & 1 & 0 & \infty \\ 6 & 3 & 3 & 2 & 0 \end{bmatrix} \end{array} \qquad \pi_5 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} N & 4 & 4 & 1 & N \\ 2 & N & 4 & 1 & N \\ 2 & 4 & N & 3 & N \\ 2 & 4 & 4 & N & N \\ 2 & 4 & 4 & 5 & N \end{bmatrix} \end{array}$$

$$\boxed{\begin{array}{l} T.C = O(n^3) \\ S.C = O(n^2) \end{array}}$$