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Design & Analysis of Algorithm

Tutorial Sheet 4

For each of the following recurrences, give an expression for the runtime T(n) if the recurrence can be solved with Master Theorem. Otherwise, indicates that the Master Theorem does not apply

Ques! $T(n) = 3T(n/2) + n^2$.

Ans Master equation is

$$T(n) = a\,T(n/b) + f(n)$$

Here

$a = 3 \quad b = 2$

Since $b > 1$ and $a > 0$ Hence here master method is applied.

$c = \log_b a \Rightarrow c = \log_2 3$

$n^c \Rightarrow n^{\log_2 3}$ $\qquad f(n) = n^2$

$n^c \longrightarrow n^c < f(n)$

$$\boxed{T(n) = \Theta(n^2)}$$

Ques 2 $T(n) = 4T(n/2) + n^2$

Ans Master method is

$$T(n) = aT(n/b) + f(n)$$

Here

$a = 4$ & $b = 2$

Since $b > 1$ and $a > 0$ Hence here master method is applied.

$c = \log_b a = \log_2 4 = 2 \log_2 2 = 2$

$n^c = n^2$ $\quad$ $f(n) = n^2$

$$n^c = f(n)$$

~~$T(n) = \Theta(n^2 \log$~~.

$$\boxed{T(n) = \Theta(n^2 \lg n)}$$

Ques 3 $T(n) = T(n/2) + 2^n$

Ans Master Method is

$$T(n) = aT(n/b) + f(n)$$

Here

$a = 1$ & $b = 2$

Since $b > 1$ & $a > 0$ Hence here master method is applied

$c = \log_b a = \log_2 1 = 0$

$n^c = n^0 = 1$ $\quad$ $f(n) = 2^n$

$$n^c < f(n)$$

$$\boxed{T(n) = \Theta(2^n)}$$

Ques 4 $T(n) = 2^n T(n/2) + n^n$

Ans master equation is

$$T(n) = a\,T(n/b) + f(n)$$

Here

$$a = 2^n \quad , b = 2$$

Here a is depending on n since we can't apply master method.

Ques 5 $T(n) = 16\,T(n/4) + n.$

Ans master equation is

$$T(n) = a\,T(n/b) + f(n)$$

Here

$$a = 16 \quad , b = 4$$

Since $b > 1$ and $a > 0$. Hence here master method is applied

$$c = \log_b a \Rightarrow c = \log_4 16 \Rightarrow \log_4 4^2 = 2$$

$$n^c = n^2 \qquad f(n) = n$$

$$n^c > f(n)$$

$$\boxed{T(n) = \Theta(n^2)}$$

Ques6 $T(n) = 2T(n/2) + n \log n$

Ans Master equation is

$$T(n) = aT(n/b) + f(n)$$

Here

$$a = 2, \quad b = 2$$

Since $b \geq 1$ and $a > 0$ Hence here master method is applied.

$$c = \log_b a \Rightarrow c = \log_2 2 = 1$$

$$n^c = n^1 \qquad f(n) = n \log n$$

$$n^c < f(n)$$

$$\boxed{T(n) = \Theta(n \log n)}$$

Ques7 $T(n) = 2T(n/2) + n/\log n$

Ans Master equation is

$$T(n) = aT(n/b) + f(n)$$

Here

$$a = 2, \quad b = 2$$

Since $b > 1$ and $a > 0$. Hence here master method is applied

$$c = \log_b a \Rightarrow c = \log_2 2 = 1$$

$$n^c = n^1 \qquad f(n) = n/\log n$$

$$n^c > f(n)$$

$$\boxed{T(n) = \Theta(n)}$$

Ques 8 $T(n) = 2T(n/4) + n^{0.51}$

Ans Master method equation is

~~$T(n) = 2T(n/4) + n^{0.51}$~~

$T(n) = aT(n/b) + f(n)$

Here

$a = 2$, $b = 4$

Sience $b > 1$ & $a > 0$. Hence here master method is applied

$c = \log_b a \Rightarrow c = \log_4 2 = \log_4 4^{1/2} = 1/2$

$n^c = n^{0.5}$ $\quad f(n) = n^{0.51}$

$n^c < f(n)$

$$\boxed{T(n) = \Theta(n^{0.51})}$$

Ques 9 $T(n) = 0.5\, T(n/2) + 1/n$

Ans master equation is

$T(n) = aT(n/b) + f(n)$

Here

$a = 0.5$, $b = 2$

Since $a > 0$ & $b > 1$. Hence here master method is applied.

$c = \log_b a \Rightarrow \log_2 2^{-1} = -1$

$n^c = n^{-1} \Rightarrow 1/n$ $\quad f(n) = 1/n$.

$n^c = f(n)$

$$T(n) = \Theta((\lg n)/n)$$

Ques 10 $T(n) = 16\,T(n/4) + n!$

Ans Master equation is

$$T(n) = a\,T(n/b) + f(n)$$

$a = 16$, $b = 4$

Since $b > 1$ & $a > 0$. Hence here master method is applied

$c = \log_b a \Rightarrow c = \log_4 16 = 2$

$n^c = n^2$ $\qquad f(n) = n!$

$n^c < f(n)$

$\boxed{T(n) = \Theta(n!)}$

Ques 11 $T(n) = 4T(n/2) + \log n$.

Ans Master equation is

$$T(n) = a\,T(n/b) + f(n)$$

$a = 4$, $b = 2$

Since $b > 1$ & $a > 0$. Hence here master method is applied

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$ $\qquad f(n) = \log n$

$n^c > f(n)$

$\boxed{T(n) = \Theta(n^2)}$

Ques 12 $T(n) = sqrt(n)\, T(n/2) + \log n$

Ans Master equation is

$$T(n) = a\,T(n/b) + f(n)$$

Here

$a = \sqrt{n}$, $b = 2$

Here a is depending on n since we can't apply Master method

Ques 13 $T(n) = 3T(n/2) + n$

Ans Master Method equation is

$$T(n) = aT(n/b) + f(n)$$

Here,

$a = 3$, $b = 2$

Since $b > 1$ & $a > 0$. Hence here master method is applied

$c = \log_b a \Rightarrow c = \log_2 3$

$n^c = n^{\log_2 3}$ $\quad f(n) = n$

$n^c > f(n)$

$$\boxed{T(n) = \Theta(n^{\log_2 3})}$$

Ques 14 $T(n) = 3T(n/3) + \sqrt{n}$

Ans Master Method equation is

$$T(n) = aT(n/b) + f(n)$$

Here

$a = 3$, $b = 3$

Since $a > 0$ & $b > 1$. Hence here master method is applied

$c = \log_b a \Rightarrow \log_3 3 = 1$

$n^c = n^1$ $\qquad$ $f(n) = \sqrt{n}$

$n^c > f(n)$

$\boxed{T(n) = \Theta(n)}$

Ques 15 $T(n) = 4T(n/2) + cn$.

Ans Master method equation is

$$T(n) = a\,T(n/b) + f(n)$$

Here,

$a = 4$, $b = 2$

Since $b > 1$ & $a > 0$. Hence here master method is applied

$c = \log_b a \Rightarrow c = \log_2 4 = 2$

$n^c = n^2$ $\qquad$ $f(n) = n$

$n^c > f(n)$

$\boxed{T(n) = \Theta(n^2)}$

Ques 16 $T(n) = 3T(n/4) + n \log n$

Ans Master method equation is

$$T(n) = a\,T(n/b) + f(n)$$

Here

$a = 3,\ b = 4$

Since $b > 1$ & $a > 0$. Hence here master method is applied

$c = \log_b a \Rightarrow c = \log_4 3$

$n^c = n^{\log_4 3}$ $\qquad f(n) = n \log n$

$n^c < f(n)$

$$\boxed{T(n) = \theta(n \log n)}$$

Ques17 $T(n) = 3T(n/3) + n/2$

Ans Master Method equation is

$$T(n) = a\,T(n/b) + f(n)$$

Here,

$a = 3,\ b = 3$

Since $a > 0$ & $b > 1$. Hence here master method is applied

$c = \log_b a \Rightarrow c = \log_3 3 = 1$

$n^c = n^1$ $\qquad f(n) = n/2$

$n^c = f(n)$

$T(n) = \theta(n \log n)$

Ques18 $T(n) = 6\,T(n/3) + n^2 \log n$

Ans Master Method equation is

$$T(n) = a\,T(n/b) + f(n)$$

Here

$a = 6 \quad b = 3$

Since $b > 1$ & $a > 0$. Hence here master method is applied

$c = \log_b a \Rightarrow c = \log_3 6$

$n^c = n^{\log_3 6} \qquad f(n) = n^2 \log n$

$n^c < f(n)$

$$\boxed{T(n) = \Theta(n^2 \log n)}$$

Ques 19 $T(n) = 4T(n/2) + n/\log n$

Ans Master Method equation is

$$T(n) = aT(n/b) + f(n)$$

Here

$a = 4, \quad b = 2$

Since $b > 1$ & $a > 0$. Hence here master method is applied.

$c = \log_b a \Rightarrow c = \log_2 4 = 2$

$n^c = n^2 \qquad f(n) = n/\log n$

$n^2 > f(n)$

$$\boxed{T(n) = \Theta(n^2)}$$

Ques 20 $T(n) = 64T(n/8) - n^2 \log n$

Ans Master Method equation is

$$T(n) = aT(n/b) + f(n)$$

Here

$a = 64 \quad b = 8$

Since $b > 1$ & $a > 0$. Hence here master method is applied.

$c = \log_8 64 = 2$

$n^c = n^2 \qquad f(n) = n^2 \log n$

$n^c < f(n)$

$$\boxed{T(n) = \Theta(n^2 \log n)}$$

Ques21 $T(n) = 7T(n/3) + n^2$

Ans Master method equation is

$$T(n) = aT(n/b) + f(n)$$

Here

$a = 7, b = 3$

Since $b > 1$ & $a > 0$. Hence here master method is applied.

$c = \log_b a = \log_3 7$

$n^c = n^{\log_3 7} \qquad f(n) = n^2$

$n^c < f(n)$

$$\boxed{T(n) = \Theta(n^2)}$$

Ques22 $T(n) = T(n/2) + n(2 - \cos n)$

Ans Master Method equation is

$$T(n) = aT(n/b) + f(n)$$

Here

$a = 1, b = 2$

Since $b > 1$ & $a > 0$. Hence here master method is applied

$c = \log_b a \Rightarrow c = \log_2 1$

$n^c = n^{\log_2 1}$ $\quad f(n) = n(2 - \cos n)$

$n^c < f(n)$

$T(n) = \Theta(n(2 - \cos n))$

$\boxed{T(n) = \Theta(n)}$