2nd Semester.

Aim: Continuity

1. $\lim_{x \to a} \left[\dfrac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \right]$

$= \lim_{x \to a} \left[\dfrac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - \sqrt{2x}} \times \dfrac{\sqrt{a+2x} + \sqrt{3x}}{\sqrt{a+2x} + \sqrt{3x}} \times \dfrac{\sqrt{3a+x} + 2\sqrt{x}}{\sqrt{3a+x} + 2\sqrt{x}} \right]$

$= \lim_{x \to a} \left[\dfrac{(a + 2x - 3x)}{(3a + x - 4x)} \dfrac{(\sqrt{3a+x} + 2\sqrt{x})}{(\sqrt{a+2x} + \sqrt{3x})} \right]$

$= \dfrac{1}{3} \lim_{x \to a} \dfrac{(a-x)}{(a-x)} \dfrac{(\sqrt{3a+x} + 2\sqrt{x})}{(\sqrt{a+2x} + \sqrt{3x})}$

$= \dfrac{1}{3} \times \dfrac{\sqrt{4a} + 2\sqrt{a}}{\sqrt{3a} + \sqrt{3a}}$

$= \dfrac{1}{3} \times \dfrac{\cancel{4}\sqrt{a}}{\cancel{2}\sqrt{\cancel{a}}\sqrt{3}}$

$= \dfrac{2}{3\sqrt{3}}$

2. $$\lim_{y\to 0}\left[\frac{\sqrt{a+y}-\sqrt{a}}{y\sqrt{a+y}}\right]$$

$$=\lim_{y\to 0}\left[\frac{\sqrt{a+y}-\sqrt{a}}{y\sqrt{a+y}}\times\frac{\sqrt{a+y}+\sqrt{a}}{\sqrt{a+y}+\sqrt{a}}\right]$$

$$=\lim_{y\to 0}\frac{y}{y\sqrt{a+y}\,(\sqrt{a+y}+\sqrt{a})}$$

$$=\frac{1}{\sqrt{a}\,(\sqrt{a}+\sqrt{a})}$$

$$=\frac{1}{2a}\ //$$

3. $$\lim_{x\to\pi/6}\frac{\cos x-\sqrt{3}\sin x}{\pi-6x}$$

By substituting $\cancel{x}\ x-\frac{\pi}{6}=h$

$$x=h+\frac{\pi}{6}$$

Where $h\to 0$

$$\lim_{h\to 0}\frac{\cos\left(h+\frac{\pi}{6}\right)-\sqrt{3}\cdot\sin\left(h+\frac{\pi}{6}\right)}{\cancel{x}\ \pi-6\left(h+\pi/6\right)}$$

$$\lim_{h\to 0}\frac{\cos h\cdot\cos\frac{\pi}{6}-\sin h\cdot\sin\frac{\pi}{6}-\sqrt{3}\cdot\left(\sin h\cos\frac{\pi}{6}+\cos h\cdot\sin\frac{\pi}{6}\right)}{\pi-6\left(\frac{6h+\pi}{6}\right)}$$

$$\lim_{h\to 0}\frac{\cos h\cdot\frac{\sqrt{3}}{2}-\sin h\,\frac{1}{\sqrt{2}}-\sqrt{3}\left(\sin h\,\frac{\sqrt{3}}{2}+\cos h\cdot\frac{1}{\sqrt{2}}\right)}{\pi-6h+\pi}$$

$$\lim_{h\to 0}\frac{\cos\frac{\sqrt{3}h}{2}-\sin\frac{h}{2}-\sin\frac{3h}{2}-\cancel{\cos\frac{3h}{2}}}{-6h}$$

$$=\lim_{h\to 0}\frac{-\sin\frac{4h}{2}}{-6h}$$

$$=\lim_{h\to 0}\frac{\sin 4h}{3\times 2h}$$

$$=\frac{1}{3}\lim_{h\to 0}\frac{\sin h}{h}=\frac{1}{3}\times 1=\frac{1}{3}$$

4) $\lim_{x\to\infty}\left[\frac{\sqrt{x^2+5}-\sqrt{x^2-3}}{\sqrt{x^2+3}-\sqrt{x^2+1}}\right]$

By rationalising Nr. & Dr.

$$= \lim_{x\to\infty}\left[\frac{\sqrt{x^2+5}-\sqrt{x^2-3}}{\sqrt{x^2+5}-\sqrt{x^2+1}}\right] \times \frac{\sqrt{x^2+5}+\sqrt{x^2-3}}{\sqrt{x^2+5}+\sqrt{x^2-3}}$$

$$= \lim_{x\to\infty}\left[\frac{(x^2+5-x^2+3)(\sqrt{x^2+3}+\sqrt{x^2+1})}{(x^2+3-x^2-1)(\sqrt{x^2+5}+\sqrt{x^2-3})}\right]$$

$$= \lim_{x\to\infty}\frac{8(\sqrt{x^2+3}+\sqrt{x^2+1})}{2(\sqrt{x^2+5}+\sqrt{x^2-3})}$$

$$= 4\lim_{x\to\infty}\frac{\sqrt{x^2(1+\frac{3}{x^2})}+\sqrt{x^2(1+\frac{1}{x^2})}}{\sqrt{x^2(1+\frac{5}{x^2})}+\sqrt{x^2(1-\frac{3}{x^2})}}$$

After applying limit

we get, $= \underline{\underline{4}}$

5) $f(x) = \frac{\sin 2x}{\sqrt{1-\cos 2x}}$, for $0 < x \leq \pi/2$ } at $x =$

for $\frac{\pi}{2} < x < \pi$

$$f(\pi/2) = \frac{\sin 2(\pi/2)}{\sqrt{1-\cos^2(\pi/2)}}$$

f at $x = \pi/2$

$$f(x) = \lim_{x\to\pi/2}\frac{\cos x}{\pi - 2x}$$

By subs method.

$x - \frac{\pi}{2} = h$

$x = \frac{\pi}{2} + h$

where $h \to 0$

$$\lim_{h\to 0}\frac{\cos(h+\frac{\pi}{2})}{\pi - 2(h+\frac{\pi}{2})}$$

$$\lim_{h\to 0}\frac{\cos(h+\frac{\pi}{2})}{-2h}$$

$$\lim_{h\to 0}\frac{\cos h \cdot \cos h \cdot \sin h \cdot \sin\frac{\pi}{2}}{-2h}$$

$$= \lim_{h\to 0}\frac{\cos h \cdot 0 - \sin h}{-2h}$$

$$= \lim_{h\to 0}\frac{-\sin h}{-2h}$$

$$= \frac{1}{2}\lim_{h\to 0}\frac{\sin h}{h}$$

$$= \frac{1}{2}$$

b) $\lim_{x \to \frac{\pi}{2}} f(x) = \lim_{x \to \frac{\pi}{2}^-} \frac{\sin 2x}{\sqrt{1-\cos 2x}}$

$= \lim_{x \to \frac{\pi}{2}^-} \frac{2\sin x \cdot \cos x}{\sqrt{2\sin^2 x}}$

$= \lim_{x \to \frac{\pi}{2}^-} \frac{2\,\cancel{\sin x} \cdot \cos x}{\sqrt{2} \cdot \cancel{\sin x}}$

$= \frac{2}{\sqrt{2}} \lim_{x \to \frac{\pi}{2}^-} \cos x$

L.H.L $\neq$ R.H.L

$\therefore$ f is not cont at $x = \pi/2$

5) ii) $f(x) = \frac{x^2 - 9}{x-3}$ $\quad 0 < x < 3$

$= x+3$ $\quad 3 \leq x \leq 6$

$= \frac{x^2 - 9}{x+3}$ $\quad 6 \leq x < 9$

at $x=3$, $x=6$

at $x=3$

i) $f(3) = \frac{x^2 - 9}{x-3} = 0$

f at $x=3$ define

ii) $\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} x+3$

$f(3) = x+3 = 3+3 = 6$

f is define at $x = 3$

$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} x+3 = 6$

$= \lim_{x \to 3^-} \frac{x^2 - 9}{x-3} = \frac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}}$

L.H.L = R.H.L.

f is cont at $x = 3$

for $x = 6$

$f(6) = \frac{x^2 - 9}{x+3} = \frac{36 - 9}{6+3} = \frac{27}{9} = 3$

$$\lim_{x \to 6^+} \frac{x^2-9}{x+3}$$

$$= \lim_{x \to 6^+} \frac{(x-3)\cancel{(x+3)}}{\cancel{(x+3)}}$$

$$= \lim_{x \to 6^+} (x-3) = 6-3 = 3$$

$$= 3+6 = 9$$

L.H.L ≠ R.H.L

∴ f is not cont.

6) i) $f(x) = \begin{cases} \dfrac{1-\cos 4x}{x^2} & x < 0 \\ = k & x = 0 \end{cases}$ at $x=0$

Solution:- f is cont at $x=0$

$$\lim_{x\to 0} f(x) = f(0)$$

$$= \lim_{x\to 0} \frac{1-\cos 4x}{x^2} = k$$

$$= \lim_{x\to 0} \frac{2\sin^2 2x}{x^2} = k$$

$$= 2\lim_{x\to 0} \left(\frac{\sin 2x}{x}\right)^2 = k$$

$$2(2)^2 = k = \underline{\underline{8}}$$

ii) $f(x) = \begin{cases} (\sec^2 x)^{\cot^2 x} & x \neq 0 \\ = k & x = 0 \end{cases}$ at $x=0$

Solution:- $f(x) = (\sec^2 x)^{\cot^2 x}$

Using,

$\tan^2 x - \sec^2 x = 1$

$\sec^2 x = 1 + \tan^2 x$

&

$\cot^2 x = \dfrac{1}{\tan^2 x}$

$$\therefore \lim_{x\to 0} (\sec^2 x)^{\cot^2 x}$$

w.k.t.

$$\lim_{x\to 0} (1+px)^{1/px} = e$$

$K = e$

$K = e$

ii) $f(x) = \dfrac{\sqrt{3} - \tan x}{\pi - 3x}$ $\quad x \neq \frac{\pi}{3}$ $\Big\}$ at $x = \frac{\pi}{3}$

$\quad = k \qquad x = \frac{\pi}{3}$

$x - \frac{\pi}{3} = h$

$x = h + \frac{\pi}{3}$

$h \to 0$

$$f(\pi/3 + h) = \frac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3(\pi/3 + h)}$$

$$= \lim_{h \to 0} \frac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3(\pi/3 + h)}$$

$$= \lim_{h \to 0} \frac{\sqrt{3} - \dfrac{\tan \pi/3 + \tan h}{1 - \tan\frac{\pi}{3} \cdot \tan h}}{\pi - \pi - 3h}$$

$$= \lim_{h \to 0} \frac{\dfrac{\sqrt{3}\left(1 - \tan\frac{\pi}{3} \cdot \tan h\right) - \left(\tan\frac{\pi}{3} + \tan h\right)}{1 - \tan\frac{\pi}{3} - \tan h}}{-3h}$$

$$= \lim_{h \to 0} \frac{\dfrac{\left[\sqrt{3} - \sqrt{3} \times \sqrt{3} - \tan h\right] - (\sqrt{3} + \tan h)}{1 - \sqrt{3}\tan h}}{-3h}$$

$$= \lim_{h \to 0} \frac{\dfrac{(\sqrt{3} - 3\tan h - \sqrt{3} - \tan h)}{1 - \sqrt{3}\tan h}}{-3h}$$

$$= \lim_{h \to 0} \frac{-4\tan h}{-3h(1 - \sqrt{3}\tan h)}$$

$$= \lim_{h \to 0} \frac{4\tan h}{3h(1 - \sqrt{3}\tan h)}$$

$$= \frac{4}{3} \lim_{h \to 0} \frac{\tan h}{h} \cdot \lim_{h \to 0} \frac{1}{(1 - \sqrt{3}\tan h)}$$

$$= \frac{4}{3}\left(\frac{1}{1}\right) = \frac{4}{3} //$$

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$$\lim_{x \to 0} f(x) = \frac{a}{2} \qquad g = f(0)$$

$\therefore$ f is cont. at $x = 0$

$$f(x) = \begin{cases} \dfrac{1 - \cos 3x}{x \tan x} & ; x \neq 0 \\ a/2 & ; x = 0 \end{cases}$$

Now $\lim_{x \to 0} f(x) = f(0)$

f has removable discontinuity at $x = 0$

ii) $$f(x) = \left.\begin{cases} \dfrac{(e^{3x} - 1)\sin x}{x^2} & x \neq 0 \\ = \pi/6 & x = 0 \end{cases}\right\}$$ at $x = 0$

$$= \lim_{x \to 0} \frac{(e^{3x} - 1)\sin\left(\frac{\pi}{180}\right)}{x^2}$$

$$= \lim_{x \to 0} \frac{(e^{3x} - 1)}{x} \cdot \lim_{x \to 0} \frac{\sin\left(\frac{\pi}{180}\right)}{x}$$

$$= \lim_{x \to 0} 3 \frac{\cancel{e^{3x}} - 1}{3x} \cdot \lim_{x \to 0} \frac{\sin\left(\frac{\pi x}{180}\right)}{x}$$

$$= 3 \lim_{x \to 0} \frac{e^{3x} - 1}{3x} \cdot \lim_{x \to 0} \frac{\sin\left(\frac{\pi x}{180}\right)}{x}$$

$= 3 \log_e \frac{\pi}{180} = \frac{\pi}{60} = f(0)$

f is cont at $x = 0$

a) $f(x) = \dfrac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x}$ $x \neq \pi/2$

$f(0)$ is cont at $x = \pi/2$

$= \lim_{x \to \pi/2} \dfrac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x} \times \dfrac{\sqrt{2} + \sqrt{1+\sin x}}{\sqrt{2} + \sqrt{1+\sin x}}$

$= \lim_{x \to \pi/2} \dfrac{1 + \sin x}{1 - \sin^2 x(\sqrt{2} + \sqrt{1+\sin x})}$

$= \lim_{x \to \pi/2} \dfrac{1 + \sin x}{(1-\sin x)(1+\sin x)(\sqrt{2} + \sqrt{1+\sin x})}$

$= \lim_{x \to \pi/2} \dfrac{1}{(1-\sin x)(\sqrt{2} + \sqrt{1+\sin x})}$

$= \dfrac{1}{2(\sqrt{2} + \sqrt{2})} = \dfrac{1}{2(2\sqrt{2})} = \dfrac{1}{4\sqrt{2}}$

$f(\pi/2) = \dfrac{1}{4\sqrt{2}}$

AV 05/08/2020

formula:

$$= \lim_{h \to 0} \frac{\tan(a-a+h) - (1+\tan a \tan(a+h))}{h \times \tan(a+h)\tan a}$$

$$= \lim_{h \to 0} -\frac{\tan h}{h} \times \frac{1+\tan a \tan(a+h)}{\tan(a+h) \cdot \tan a}$$

$$= -1 \times \frac{1+\tan^2 a}{\tan^2 a}$$

$$= -\frac{\sec^2 a}{\tan^2 a}$$

$$= \frac{-1}{\cos^2 a} \times \frac{\cos^2 a}{\sin^2 a}$$

$$= -\operatorname{cosec}^2 a$$

$\therefore$ ~~$f(a) = -\cos^2 a$~~

$\therefore$ f is differentiable $\forall a \in R$

$\operatorname{cosec} x$

$f(x) = \operatorname{cosec} x$

$$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x-a}$$

$$= \lim_{x \to a} \frac{\operatorname{cosec} x - \operatorname{cosec} a}{x-a}$$

$$= \lim_{x \to a} \frac{\frac{1}{\sin x} - \frac{1}{\sin a}}{x-a}$$

$$= \lim_{x \to a} \frac{\sin a - \sin x}{(x-a)\sin a \cdot \sin x}$$

subs: $x - a = h$

$x = h + a$

as $x \to a$, $h \to 0$

$$Df(h) = \lim_{h \to 0} \frac{\sin a - \sin(a+h)}{(a+h-a)\sin a \cdot \sin(a+h)}$$

$$= \lim_{h \to 0} \frac{2\cos\left(\frac{a+a+h}{2}\right)\sin\left(\frac{a-a-h}{2}\right)}{h \times \sin a \cdot \sin(a+h)}$$

$$= \lim_{h \to 0} \frac{-\sin\frac{h}{2}}{\frac{h}{2}} \times \frac{1}{2} \times \frac{2\cos\left(\frac{2a+h}{2}\right)}{\sin a \cdot \sin(a+h)}$$

$$= -\frac{\cos a}{\sin^2 a} = -\cot a \cdot \operatorname{cosec} a$$

iii) $\sec x$

$f(x) = \sec x$

$$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to a} \frac{\sec x - \sec a}{x - a}$$

$$= \lim_{x \to a} \frac{\frac{1}{\cos x} - \frac{1}{\cos a}}{x - a}$$

$$= \lim_{x \to a} \frac{\cos a - \cos x}{(x-a)\cos a \cdot \cos x}$$

Subs $x - a = h$

$x = h + a$

as $x \to a$ $\therefore h \to 0$

$$Df(x) = \lim_{h \to 0} \frac{\cos a - \cos(a+h)}{h \times \cos a \cdot \cos(a+h)}$$

$$= \lim_{h \to 0} \frac{-2 \sin\left(\frac{a + a + h}{2}\right) \cdot \sin\left(\frac{a-a-h}{2}\right)}{h \times \cos a \cdot \cos(a+h)}$$

$$= -\frac{1}{2} \times -2 \frac{\sin\left(\frac{2a+0}{2}\right)}{\cos a \cdot \cos(a+0)}$$

$$= \tan a \cdot \sec a$$

$Df(2^+) = 4$

$R.H.D = L.H.D$

Q3) If $f(x) = 4x + 7, \quad x < 3$
$= x^2 + 3x + 1, \quad x \geq 3$ at $x = 3$

Find f is differentiable or not

Soln :-

R.H.D

$$Df(3^+) = \lim_{x \to 3^+} \frac{f(x) - f(3)}{x - 3}$$

$$= \lim_{x \to 3^+} \frac{x^2 + 3x + 1 - 19}{x - 3}$$

$$= \lim_{x \to 3^+} \frac{x^2 + 6x - 3x - 18}{x - 3}$$

$$= \lim_{x \to 3^+} \frac{x(x+6) - 3(x+6)}{x - 3}$$

$$= \lim_{x \to 3^+} \frac{(x+6)(x-3)}{(x-3)}$$

$$= 3 + 6 = 9$$

$Df(3^+) = 9$

L.H.D $= Df(3^-)$

$$= \lim_{x \to 3^-} \frac{f(x) - f(3)}{x - 3}$$

$$= \lim_{x \to 3^-} \frac{4x - 12}{x - 3}$$

$$= \lim_{x \to 3} \frac{4\cancel{(x-3)}}{\cancel{(x-3)}}$$

$Df(3^-) = 4$

$R.H.D \neq L.H.D$

$\therefore$ f is not differentiable at $x = 3$

4] If $f(x) = 8x - 5, \quad x \leq 2$
$= 3x^2 - 4x + 7, \quad x > 2$ at $x = 2$

Find f is not differentiable

Soln :- $f(2) = 8 \times 2 - 5 = 16 - 5 = 11$

$$R.H.D = \lim_{x \to 2^+} \frac{f(x) - f(2)}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{3x^2 - 4x + 7 - 11}{x - 2}$$

$$= \lim_{x \to 2^+} 3x^2 - 6x + 2x - 4$$

$$= \lim_{x \to 2^+} \frac{3x(x-2) + 2(x-2)}{x - 2}$$

L.H.D :

$$Df(2^-) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x - 2}$$

$$= \lim_{x \to 2^-} \frac{8x - 5 - 11}{x - 2}$$

$$= \lim_{x \to 2^-} \frac{8\cancel{(x-2)}}{\cancel{(x-2)}}$$

$$Df(2^-) = 8$$

L.H.D = R.H.D

f is differentiable at $x = 3$

11/2/19

Practical NO.3 :

Title: Application of derivatives

Q1. Find intervals in which function is increasing or decreasing:

a] $f(x) = x^3 - 5x - 11$

Solution: f is increasing if & only if

$f'(x) > 0$

$f(x) = x^3 - 5x - 11$

$f'x = 3x^2 - 5$

$3x^2 - 5 > 0$

$x = \pm\sqrt{\frac{5}{3}}$

$\therefore \quad x \in \left(-\infty, -\sqrt{\frac{5}{3}}\right) \cup \left(\sqrt{\frac{5}{3}}, \infty\right)$

Now, f is decreasing if & only if

$f'(x) < 0$

$3x^2 - 5 < 0$

$x = \pm\sqrt{\frac{5}{3}}$

$\therefore \quad x \in \left(-\sqrt{\frac{5}{3}}, \sqrt{\frac{5}{3}}\right)$

b] $f(x) = x^2 - 4x$

Solution:- f is increasing if & only if

$f'(x) > 0$

$f(x) = x^2 - 4x$

$f'(x) = 2x - 4$

$2x - 4 > 0$

$2(x-2) > 0$

$x - 2 > 0$

$x = 2$; $x \in (2, \infty)$

Now f is decreasing if & only if

$f'(x) < 0$

$2x - 4 < 0$

$2(x-2) < 0$

$x - 2 < 0$

$x = 2$

$\therefore x \in (-\infty, 2)$

c] $f(x) = 2x^3 + x^2 - 20x + 4$

Solution: f is ~~decreasing~~ increasing if & only if $f'(x)$

$f'(x) = 2x^3 + x^2 - 20x$

$f'(x) = 6x^2 + 2x - 20$

$\therefore x \in (-\infty, -2) \cup (\frac{5}{3}, \infty)$

Now f is decreasing if & only if

$f'(x) < 0$

$6x^2 + 2x - 20 < 0$

$(x+2)(6x-10) < 0$

$x = -2, 5/3$

$x \in (-2, 5/3)$

] $f(x) = x^3 - 27x + 5$

Solution:- f is increasing if and only if

$f'(x) > 0$

$f(x) = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

$3x^2 - 27 > 0$

$3(x^2 - 9) > 0$

$x^2 - 9 > 0$

$\therefore x \in (-\infty, -3) \cup (3, \infty)$

f is decreasing if & only if, $f'(x) < 0$;

$f(x) = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

$3x^2 - 27 < 0$

$x^2 - 9 < 0$

$x = -3, 3$

QII contd...

d) $y = 69 - 24x - 9x^2 + 2x^3$

Solution:- $\therefore y = f(x)$

$$f(x) = 69 - 24x - 9x^2 + 2x^3$$
$$f'(x) = -24 - 18x + 6x^2$$
$$f''(x) = -18 + 12x$$

$\therefore$ f is concave upward if & only if

$$f''(x) > 0$$
$$-18 + 12x > 0$$
$$6(2x - 3) > 0$$
$$2x - 3 > 0$$
$$x = \frac{3}{2}$$
$$x \in \left(\frac{3}{2}, \infty\right)$$

$\therefore$ f is concave downwards if & only if

$$f''(x) < 0$$
$$-18 + 12x < 0$$
$$2x - 3 < 0$$
$$x = 3/2$$
$$x \in (-\infty, 3/2)$$

e) $y = 2x^3 + x^2 - 20x + 4$

Solution:- $\therefore y = f(x)$

$$f(x) = 2x^3 + x^2 - 20x + 4$$
$$f'(x) = 6x^2 + 2x - 20$$
$$f''(x) = 12x + 2$$

$\therefore$ f is concave upwards if & only if

$$f''(x) > 0$$
$$12x + 2 > 0$$
$$2(6x + 1) > 0$$
$$x = -1/6$$
$$x \in \left(-\frac{1}{6}, \infty\right)$$

$\therefore$ f is concave downwards if & only if

$$f''(x) < 0$$
$$12x + 2 < 0$$
$$2(6x + 1) < 0$$
$$\cancel{6x + 1 < 0}$$
$$\cancel{x = -1/6}$$
$$\cancel{x \in (-\infty, -1/6)}$$
$$6x + 1 < 0$$
$$x + \frac{1}{6} < 0$$
$$x < -\frac{1}{6}$$
$$\cancel{x \in \left(-\frac{1}{6}, \infty\right)}$$
$$x \in (-\infty, -1/6)$$

Q I contd...

e] $f(x) = 69 - 24x - 9x^2 + 2x^3$

Solution:- f is increasing if & only if $f'(x) > 0$

$\therefore f(x) = 69 - 24x - 9x^2 - 12x^3$

$f'(x) = -24 - 18x + 6x^2$

$\cancel{-(3(x+4)(x-1)) > 0}$

$-24 - 18x + 6x^2 > 0$

$-6(-4 - 3x + x^2) > 0$

$x^2 - 3x - 4 > 0$

$x = 4, -1$

$x \in (-\infty, -1) \cup (4, \infty)$

Now f is decreasing if & only if

$f'(x) < 0$

$-24 - 18x + 6x^2 < 0$

$x = -4, -1$

$x\ (-1, 4)$

2] contd...

~~find~~

$y = 3x^2 - 2x^3$

Solution:- $f(x) = 3x^2 - 2x^3$

$f'(x) = 6x - 6x^2$

$f''(x) = 6 - 12x$

$\therefore$ f is concave upward if & only if,

$f''(x) > 0$

$6 - 12x > 0$

$6(1 - 2x) > 0$

$-(2x - 1) > 0$

$x \in (-\infty, \frac{1}{2})$

$\therefore$ f is concave downward if & only if

$f''(x) < 0$

$6(1 - 2x) < 0$

$-(2x - 1) < 0$

$x \in (\frac{1}{2}, \infty)$

18/12/19

Practical 4 :-

Topic :- Application of Derivative & Newton's Method

Q1) $f(x) = x^2 + \frac{16}{x^2}$

Solu^n :- $f'(x) = 2x + \frac{-32}{x^3}$

For maxima / minima

$f'(x) = 0$

$\therefore 2x - \frac{32}{x^3} = 0$

$2x = \frac{32}{x^3}$

$x^4 = 16$

$x = \pm 2$

$f''(x) = 2 + \frac{96}{x^4}$

$f''(2) = f''(-2)$

$2 + \frac{96}{(\pm 2)^4} = 2 + \frac{96}{16} = 8 > 0$

$\therefore$ $f(x)$ is minimum at $x = \pm 2$

$f(2) = 8$ is the minimum value

ii) $f(x) = 3 - 5x^3 + 3x^5$

Solution:-

$f(x) = 3 - 5x^3 + 3x^5$

$f'(x) = - 15x^4 - 15x^2$

For maxima / minima

$f'(x) = 15x^4 - 15x^2 = 0$

$x^4 - x^2 = 0$

$x^2(x^2 - 1) = 0$

$x = 0 ; -1 ; 1$

$f''(x) = 60x^3 - 30x$

$f''(0) = 0$

$f''(-1) = -60 + 30 = -30 < 0$

$f''(1) = 60 - 30 = 30 > 0$

$f(x)$ is maximum at -1 &

minimum at 1

$f(-1) = 3 + 5 - 3 = 5$

$f(1) = 3 - 5 + 3 = 1$

iv] $f(x) = 2x^3 - 3x^2 - 12x + 1$

Solution :-

$f'(x) = 6x^2 - 6x - 12$

For maxima / minima

$f'(x) = 0$

$6x^2 - 6x - 12 = 0$

$x^2 - x - 2 = 0$

$x^2 - 2x + x - 2 = 0$

$x(x-2) + 1(x-2) = 0$

$(x+1)(x-2) = 0$

$x = -1, 2$

$f''(x) = 12x - 6$

$f''(-1) = -12 - 6 = -18 < 0$

$f''(2) = 24 - 6 = 18 > 0$

$f(x)$ is maximum at $x = -1$ &

minima at $x = 2$

$f(-1) = 8$

$f(2) = -19$

Q2]

i) $f(x) = x^3 - 3x^2 - 55x + 9.5$

$f'(x) = 3x^2 - 6x - 55$

$x_0 = 0$

$f(x_0) = 9.5$

$f'(x_0) = -55$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 0 - \frac{9.5}{-55}$$

$= 0.1727$

$f(x_1) = -0.0828$

$f'(x_1) = -55.9467$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 0.1727 - \frac{0.0828}{-55.9467}$$

$x_2 = 0.1712$

$f(x_2) = 0.0011$

$x_3 = 0.1712$ is the root of the eqn.

] $f(x) = x^3 - 4x - 9$

$f'(x) = 3x^2 - 4$

$f(2) = -9$

$f(3) = -9$

$f(3) = 6$

6 is closer to 0 in number line

$x_0 = 3$

$f(x_0) = 23$ $\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_1)}$

$x_1 = 2.7391$

$f(x_1) = 0.5942$ $f'(x_1) = 18.508$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = \frac{0.5942}{18.508} = 2.707$$

~~$x_2 = x_1 - $~~

$f(x_2) = 0.0085$

$f'(x_2) = 17.9835$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 2.707 - \frac{0.0085}{17.9835}$$

$x_3 = 2.7065$

$f(x_3) = -0.0085$

$f'(x_3) = 17.9757$

$\therefore x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$

$= 2.7065 - \frac{0.0005}{17.9757}$

$x_4 = 2.7065$

$\therefore$ 2.7065 is the root of the given function

iii] $f(x) = x^3 - 1.8x^2 - 10x + 17$

$f'(x) = 3x^2 - 3.6x - 10$

$f(1) = 1 - 1.8 - 10 + 17 = 6.2$

$f(2) = -2.2$

$\therefore$ -2.2 is closer to 0 on the number line

$\therefore x = 2$

$f(x_0) = -2.2$

$f'(x_0) = -5.2$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$= 2 - \frac{-2.2}{-5.2}$

$x_1 = 1.5769$

$f(x_1) = (1.5769)^3 - 1.8(1.5769)^2 - 10(1.5769) + 17$

$= 0.6762$

$f'(x_1) = -8.217$

$\therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$= 1.5769 - \frac{0.6762}{-8.217}$

$x_2 = 1.6592$

$\therefore f(x_2) = 0.0204$

$f'(x_2) = -7.7143$

$\therefore x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

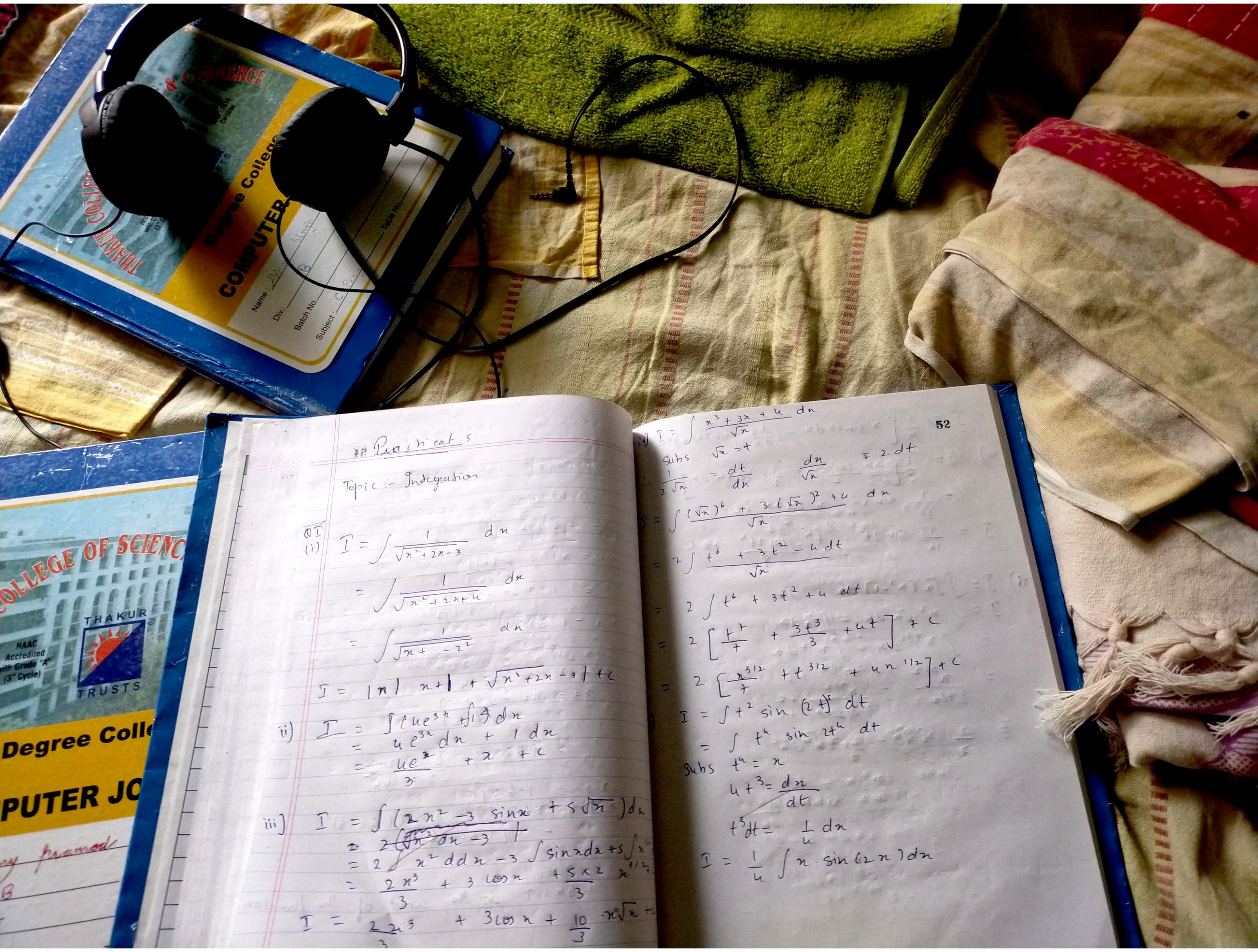
$= 1.6592 - \frac{0.0204}{-7.7144}$

$x_3 = 1.6618$

$f(x_3) = (1.6618)^3 - 1.8(1.6618)^2 - 16(1.6018) + 17$

$\therefore f(x_3) = 0$

$\therefore$ 1.6618 is the root of the function



Practicals
Topic :- Integration
Q I
(i) $I = \int \frac{1}{\sqrt{x^2+2x-3}} dx$
$= \int \frac{1}{\sqrt{x^2+2x+4}} dx$
$= \int \frac{1}{\sqrt{x+\ -2^2}} dx$
$I = |n| \ x+| + \sqrt{x^2+2x-3}| + c$
ii) $I = \int (4e^{3x} + 1) dx$
$= 4e^{3x} dx + 1 dx$
$= \frac{4e^{x}}{3} + x + c$
iii) $I = \int (2x^2 - 3 \sin x + 5\sqrt{x}) dx$
$= 2 \int x^2 dx - 3 \int \sin x dx + 5 \int x^{1/2}$
$= \frac{2x^3}{3} + 3 \cos x + \frac{5 \times 2}{3} x^{3/2}$
$I = \frac{2x^3}{3} + 3\cos x + \frac{10}{3} x\sqrt{x} +$
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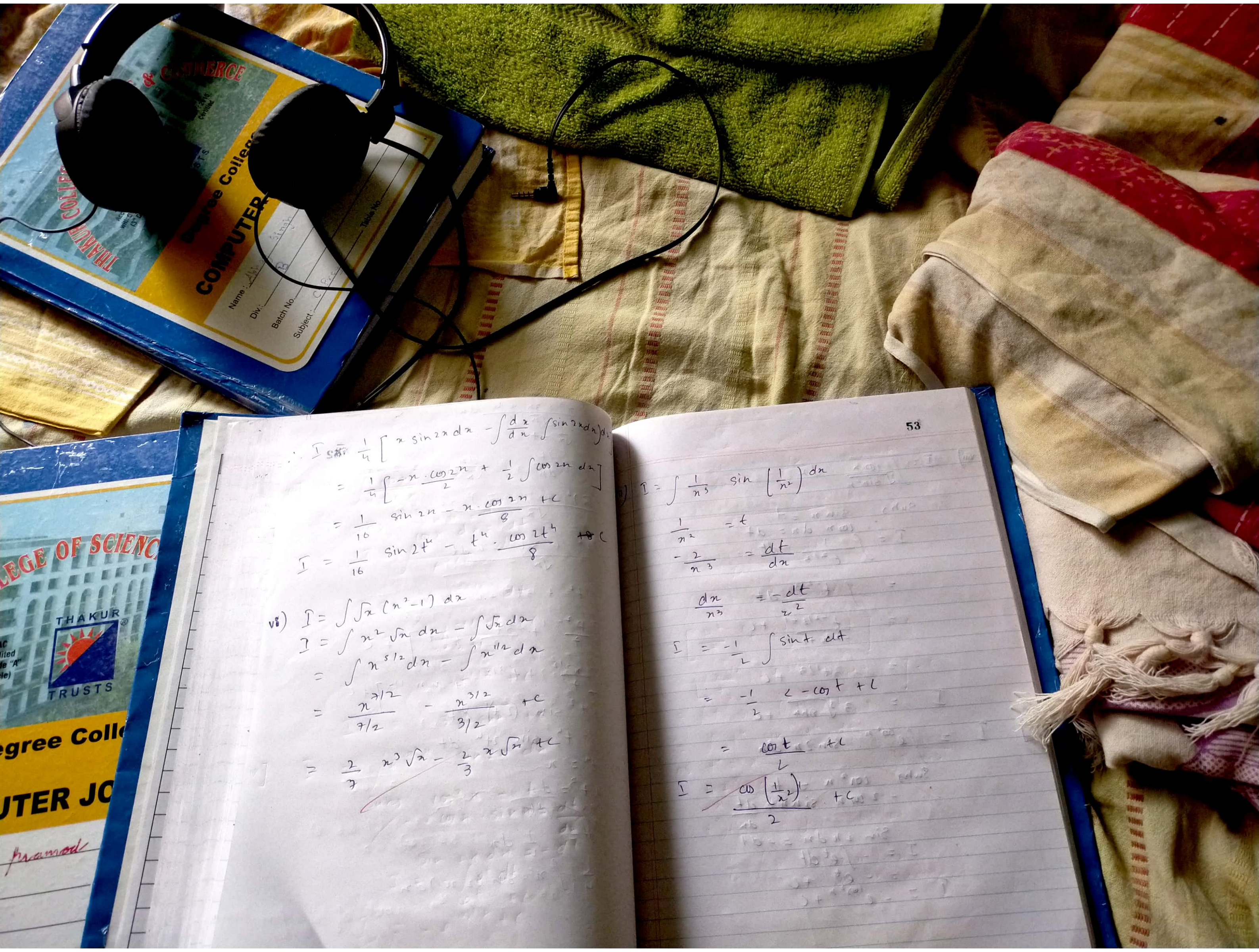
iv) $I = \int \frac{x^3 + 3x + 4}{\sqrt{x}} dx$
Subs $\sqrt{x} = t$
$\frac{1}{2\sqrt{x}} = \frac{dt}{dx}$ $\frac{dx}{\sqrt{x}} = 2dt$
$I = \int \frac{(\sqrt{x})^6 + 3(\sqrt{x})^2 + 4}{\sqrt{x}} dx$
$= 2 \int \frac{t^6 + 3t^2 - 4 dt}{\sqrt{x}}$
$= 2 \int t^6 + 3t^2 + 4 \ dt$
$= 2 \left[\frac{t^7}{7} + \frac{3t^3}{3} + 4t \right] + c$
$= 2 \left[\frac{x^{3/2}}{7} + t^{3/2} + 4x^{1/2} \right] + c$
$I = \int t^2 \sin (2t)^5 dt$
$= \int t^4 \sin 2t^4 \ dt$
Subs $t^4 = x$
$4t^3 = \frac{dx}{dt}$
$t^3 dt = \frac{1}{4} dx$
$I = \frac{1}{4} \int x \cdot \sin (2x) dx$



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$\therefore I = \frac{1}{4}\left[x \sin 2x \, dx - \int \frac{dx}{dx} \left(\int \sin 2x \, dx \right) dx \right.$
$= \frac{1}{4}\left[-x \cdot \frac{\cos 2x}{2} + \frac{1}{2}\int \cos 2x \, dx \right]$
$= \frac{1}{16} \sin 2x - x \cdot \frac{\cos 2x}{8} + c$
$I = \frac{1}{16} \sin 2t^4 - t^4 \cdot \frac{\cos 2t^4}{8} + c$
vii) $I = \int \sqrt{x}\,(x^2 - 1)\, dx$
$I = \int x^2 \sqrt{x}\, dx - \int \sqrt{x}\, dx$
$= \int x^{5/2} dx - \int x^{1/2} dx$
$= \frac{x^{7/2}}{7/2} - \frac{x^{3/2}}{3/2} + c$
$= \frac{2}{7} x^3 \sqrt{x} - \frac{2}{3} x \sqrt{x} + c$
$I = \int \frac{1}{x^3} \sin\left(\frac{1}{x^2}\right) dx$
$\frac{1}{x^2} = t$
$-\frac{2}{x^3} = \frac{dt}{dx}$
$\frac{dx}{x^3} = -\frac{dt}{2}$
$I = -\frac{1}{2}\int \sin t \, dt$
$= -\frac{1}{2} (-\cos t) + c$
$= \frac{\cos t}{2} + c$
$I = \frac{\cos\left(\frac{1}{x^2}\right)}{2} + c$

viii] $I = \int \frac{\cos x}{\sqrt[3]{\sin^2 x}} \, dx$

Subs $\sin x = t$

$\therefore \cos x \, dx = dt$

$I = \int \frac{1}{t^{2/3}} \, dt$

$= \int t^{-2/3} \, dt$

$= \frac{t^{-2/3+1}}{-2/3+1} + c$

$= 3t^{1/3} + c$

$I = 3\sqrt[3]{\sin x} + c$

ix] $I = \int e^{\cos^2 x} \cdot \sin 2x \, dx$

Subs $\cos^2 x = t$

$\therefore -2\cos x \cdot \sin x = \frac{dt}{dx}$

$\sin 2x \, dx = -dt$

$\therefore I = -\int e^t \, dt$

$= -e^t + c$

$I = -e^{\cos^2 x} + c$

x] $I = \int \left(\frac{x^2 - 2x}{x^3 - 3x^2 + 1}\right) dx$

$x^3 - 3x^2 + 1 = t$

$3x^2 - 6x = \frac{dt}{dx}$

$(x^2 - 2x) \, dx = \frac{dt}{3}$

$I = \frac{1}{3} \int \frac{1}{t} \, dt$

$= \frac{1}{3} \log |t| + c$

$I = \frac{1}{3} \log |x^3 - 3x^2 + 1| + c$

Practical 6

Topic:- App. of integration & Numerical Integration.

1] Find the length of the foll:-

$x = t - \sin t$; $y = 1 - \cos t$ $t \in [0, 2\pi]$

Solution:-

$$\text{arc length} = \int_0^{4} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^3}\, dt = \int_0^{2\pi} \sqrt{(1-\cos t)^2 + (\sin t)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{1 - 2\cos t + \cos^2 t + \sin^2 t}\, dt = \sqrt{1 - 2\cos t + 1}\, dt$$

$$= \int_0^{2\pi} \sqrt{2 - 2\cos t}\, dt$$

$$= \int_0^{2\pi} 2\,|\sin \pi/2|\, dt \quad \left(\because \sin^2\frac{t}{2} = \frac{1 - \cos - 1}{2}\right)$$

$$= \int_0^{2\pi} 2 \sin \frac{1}{2}\, dt \quad \left(\because \sin \frac{1}{2} \geq 0 \text{ when } 0 < t \leq 2\pi\right)$$

$$= \left[-4\cos\left(\frac{1}{2}\right)\right]_0^{2\pi} = (-4\cos\pi) + 4\cos 0 = 4 + 4 = 8$$

= 8 units

2] $y = \sqrt{4 - x^2} \quad x \in [-2, 2]$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{4 - x^2}} \times (-2x)$$

$$= \frac{-x}{\sqrt{4 - x^2}}$$

$$L = \int_{-2}^{2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$= \int_{-2}^{2} \sqrt{\frac{4 - x^2 + x^2}{4 - x^2}}\, dx$$

$$= \int_{-2}^{2} \sqrt{\frac{4}{4 - x^2}}\, dx$$

$$= 2 \int_{-2}^{2} \frac{1}{\sqrt{2^2 - x^2}}\, dx$$

$$= 2\left[\sin^{-1}(1) - \sin^{-1}(-1)\right]$$

$$= 2\left[\frac{\pi}{2} + \frac{\pi}{2}\right] = 2\pi$$

3] $y = x^{3/2}$ in $[0, 4]$

$$\frac{dy}{dx} = \frac{d}{dx} x = \frac{3}{2} x^{1/2}$$

$$L = \int_0^4 \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$= \int_0^4 \sqrt{1 + \frac{9}{4} x}\, dx$$

$$= \frac{1}{2} \int_0^4 \sqrt{4 + 9x}\, dx$$

$$= \frac{1}{2} \left[\frac{(4 + 9x)^{3/2}}{3/2} \times \frac{1}{9}\right]_0^4$$

$$= \frac{1}{27} \left[(4 + 9x)^{3/2}\right]_0^4$$

$$= -\frac{1}{27}\left[(4 + 0)^{3/2} - (4 + 36)^{3/2}\right]$$

$$L = \frac{1}{27}\left(40^{3/2} - 8\right) \text{ units}$$

(4) $x = 3\sin t$, $y = 3\cos t$ $t \in (0, 2\pi)$

$$\frac{dx}{dt} = 3\cos t \; ; \; \frac{dy}{dt} = -3\sin t$$

$$L = \int_0^{2\pi} \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{(3\cos t)^2 + (-3\sin t)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{9\sin^2 t + 9\cos^2 t}\, dt$$

$$= \int_0^{2\pi} 3\sqrt{x}\, dt$$

$$= 3\int_0^{2\pi} 3\, dt$$

$$= 3\left[x\right]_0^{2\pi}$$

$$= 3(2\pi - 0)$$

$$L = 6\pi \text{ units}$$

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(5) $x = \frac{1}{6}y^3 + \frac{1}{2y}$ on $y \in (1, 2)$

$$\frac{dx}{dy} = \frac{y^2}{2} - \frac{1}{2y^2}$$

$$\frac{dx}{dy} = \frac{y^4 - 1}{2y^2}$$

$$L = \int_1^2 \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$= \int_1^2 \sqrt{1 + \frac{(y^4-1)^2}{4y^4}}\, dy$$

$$= \int_1^2 \sqrt{\frac{(y^4-1) + 4 \times y^4}{4y^4}}\, dy$$

$$= \int_1^2 \frac{y^4 + 1}{2y^2}\, dy$$

$$= \frac{1}{2}\int_1^2 y^2\, dy + \frac{1}{2}\int_1^2 y^{-2}\, dy$$

$$= \frac{1}{2}\left[\frac{y^3}{3} - \frac{y^{-1}}{1}\right]_1^2$$

$$= \frac{1}{2}\left[\frac{8}{3} - \frac{1}{2} - \frac{1}{3} + 1\right]$$

$$= \frac{1}{2}\left[\frac{7}{3} + \frac{1}{2}\right]$$

$$= \frac{1}{2}\left[\frac{17}{6}\right]$$

$$l = \frac{17}{12} \text{ units}$$

Q II i]

$$\int_0^2 e^{x^2}\,dx \quad \text{with } n = 4$$

$$h = \frac{b-a}{n} = \frac{2-0}{4} = 0.5$$

x	0	0.5	1	1.5	2
y	1	1.284	2.783	9.487	54.598

$$\int_0^2 e^{x^2}\,dx = \frac{h}{3}\left[(y_0 - y_2) + 4(y_1 + y_3) + \ldots\right.$$

$$= \frac{0.5}{3}\left[(1 + 54.5482) + 5(1.284 + \ldots\right.$$

$$= \frac{0.5}{3}\left[55.5982 + 43.0682 + 5.468\right]$$

$$\int_0^2 e^{x^2}\,dx = 17.3535$$

ii]

$$\int_0^4 x^2\,dx \qquad n = 4$$

$$h = \frac{4-0}{4} = 1$$

x	0	1	2	3	4
y	0	1	4	9	16

$$\int x^2\,dx = \frac{h}{3}\left[(y_0 + y_4) + 4(y_1 + y_3)\ 2(y_2)\right]$$

$$= \frac{1}{3}\left[16 + 4(10) + 8\right]$$

$$= \frac{64}{3}$$

$$\int x^2\,dx = 21.333$$

$$\int_0^{\pi/3} \sqrt{\sin x}\,dx \quad \text{with } n = 6$$

$$h = \frac{\pi/3 - 0}{6} = \frac{\pi}{18}$$

x	0	$\frac{\pi}{18}$	$\frac{2\pi}{18}$	$\frac{3\pi}{18}$	$\frac{4\pi}{18}$	$\frac{5\pi}{18}$	$\frac{6\pi}{18}$
y	0	0.4167	0.58	0.70	0.80	0.87	0.93

$$\int_0^{\pi/3} \sqrt{\sin x}\, dx = \frac{h}{3}\left[y_0 + y_6 + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)\right]$$

$$= \frac{\pi}{18}\left[0.4167 + 0.9306 + 2(0.4167 \ldots\ 0.707 + 0.8752) + 4(0.58 \ldots\ 0.997)\right]$$

$$= \frac{\pi}{54}\left[1.3473 + 4(1.999) + 2(1.3805)\right]$$

$$= \frac{\pi}{54} \times 12.1163$$

$$\int_0^{\pi/3} \sqrt{\sin x}\, dx = 0.7049$$

08/01/2020

Practical 7:

Solve the foll differential equation

i) $x\frac{dy}{dx} + y = e^x$

ii) $e^x\frac{dy}{dx} + 2e^x y = 1$

iii] $x\frac{dy}{dx} = \frac{\cos x}{x} - 2y$

iv] $x\frac{dy}{dx} + 3y = \frac{\sin x}{x^2}$

v] $e^{2x}\frac{dy}{dx} + 2e^{2x}y = 2x$

vi] $\sec^2 x \tan y\, dx + \sec^2 \tan x\, dy = 0$

vii] $\frac{dy}{dx} = \sin^2(x - y + 1)$

viii] $\frac{dy}{dx} = \frac{2x + 3y - 1}{6x + 9y + 6}$

Solutions:-

i) $x \dfrac{dy}{dx} + y = e^x$

$\dfrac{dy}{dx} + \dfrac{1}{x} y = \dfrac{e^x}{x}$

$P(x) = 1/x \qquad g(x) = e^{2/x}$

$I.F. = e^{\int P(x)\,dx}$

$= e^{\int 1/x\,dx}$

$= e$

$I.F. = x$

$y.(I.F.) = \int I.F\, dx + c$

$y x = \int \dfrac{e^x}{x} \times x \, dx + c$

$= \int e^x \, dx + c$

$xy = e^x + c$

ii) $e^x \dfrac{dy}{dx} + 2e^x y = 1$

$\dfrac{dy}{dx} + 2y = e^{-x}$

$P(x) = 2 \qquad Q(x) = e^{-x}$

$I.F. = e^{\int P(x)\,dx}$

$= e^{\int 2\,dx}$

$= e^{2x}$

$y(I.F.) = \int Q(x)\,(I.F.)\,dx + c$

$= \int e^{-x} \times e^{2x/x} + c$

$= \int e^{-x+2x}\,dx + c$

$= \int e^x \,dx + c$

$y.e^{2x} = e^x + c$ //

iii] $x\frac{dy}{dx} = \frac{\cos x}{x} - 2y$

$\frac{dy}{dx} + \frac{2}{x}y = \frac{\cos x}{x^2}$

$P(x) = 2/x \quad \therefore \quad Q(x) = \cos x/x^2$

$I.F. = \cancel{\int P(x)\,dx}\; e^{\int P(x)\,dx}$

$= e^{\int 2/x\,dx}$

$= e^{2\ln x}$

$= e^{\ln x^2}$

$= x^2$

$y(I.F.) = \int Q(x)\,(I.F.)\,dx + c$

$x^2 y = \int \frac{\cos x}{\cancel{x^2}} \times \cancel{x^2}\,dx + c$

$x^2 y = \sin x + c$

iv] $x\frac{dy}{dx} + 3y = \frac{\sin x}{x^2}$

$\therefore \frac{dy}{dx} + \frac{3}{x}y = \frac{\sin x}{x^3}$

$P(x) = 3/x \quad Q(x) = \sin x/x^3$

$I.F. = e^{\int P(x)\,dx}$

$= e^{\int 3/x\,dx}$

$= e^{3\ln x}$

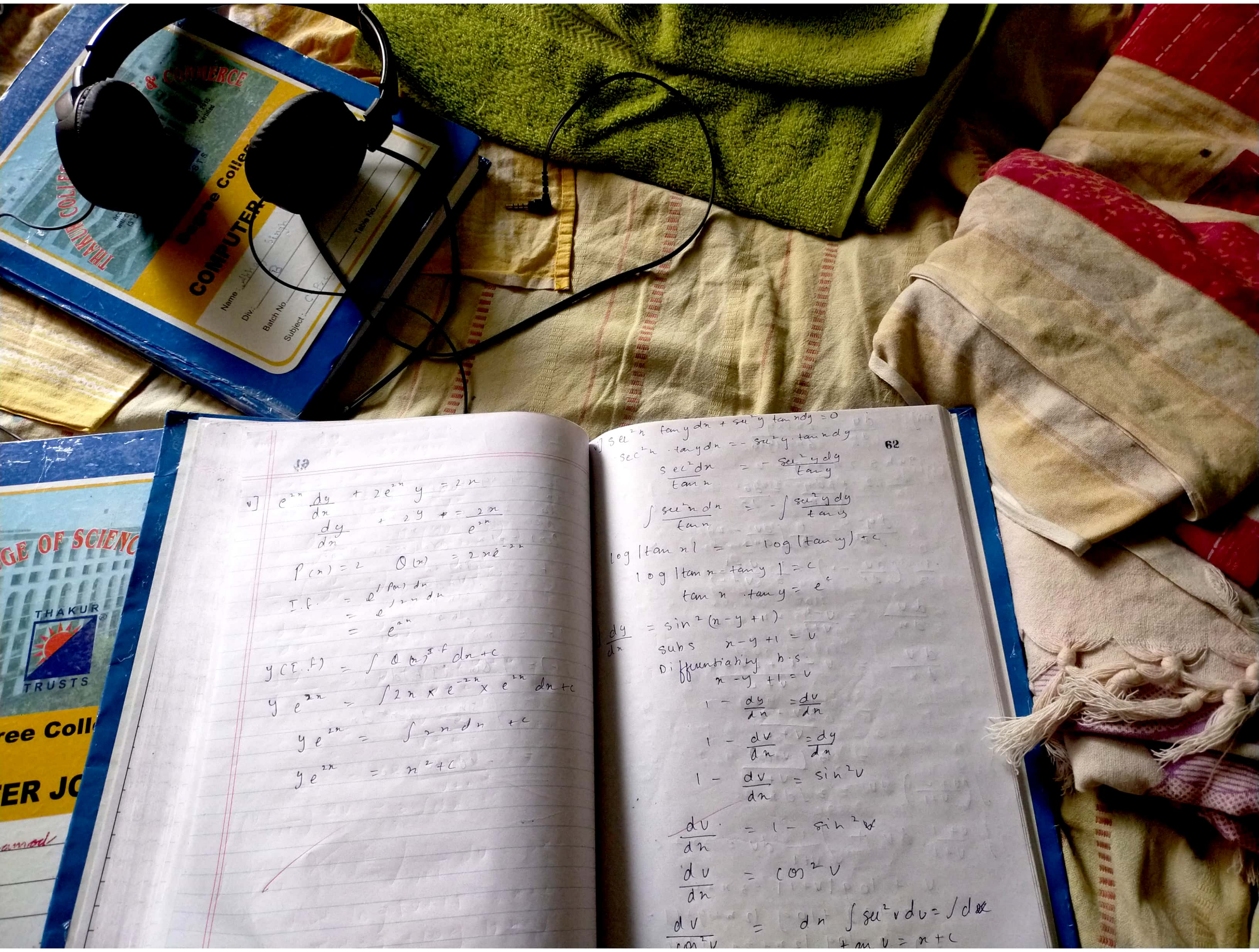
$= e^{\ln x^3}$

$I.F. = x^3$

$y(I.F.) = \int Q(x)\,(I.F.)\,dx + c$

$= \int \frac{\sin x}{\cancel{x^3}} \times \cancel{x^3}\,dx + c$

$x^3 y = -\cos x + c$



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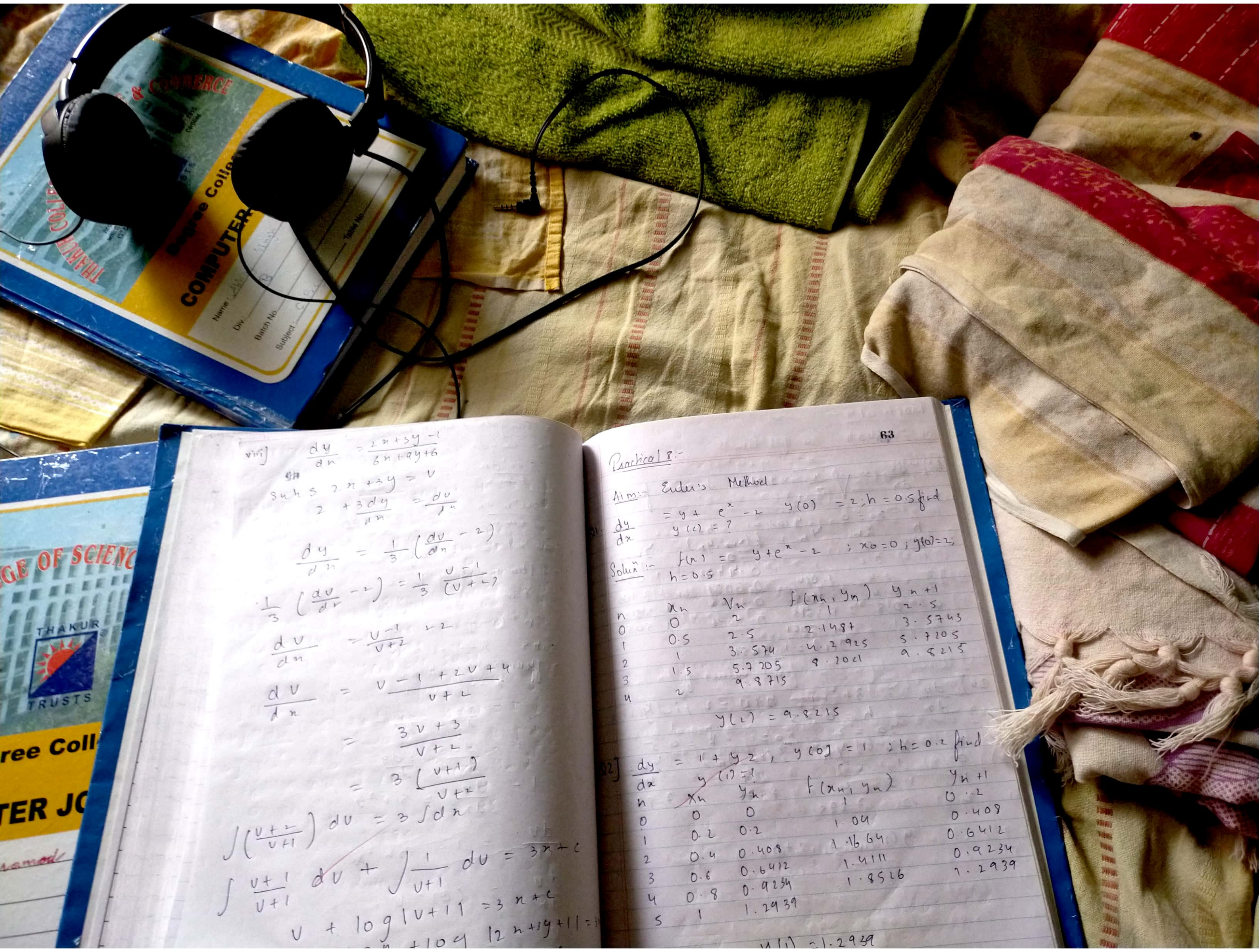
v] $e^{2x}\frac{dy}{dx} + 2e^{2x}y = 2x$
$\frac{dy}{dx} + 2y = \frac{2x}{e^{2x}}$
$P(x) = 2 \quad Q(x) = 2xe^{-2x}$
$I.F. = e^{\int P(x)\,dx} = e^{\int 2\,dx} = e^{2x}$
$y(I.F) = \int Q(x)^{I.F}\,dx + c$
$ye^{2x} = \int 2x \times e^{-2x} \times e^{2x}\,dx + c$
$ye^{2x} = \int 2x\,dx + c$
$ye^{2x} = x^2 + C$
vi] $\sec^2 x \tan y\,dx + \sec^2 y \tan x\,dy = 0$
$\sec^2 x \cdot \tan y\,dx = -\sec^2 y \cdot \tan x\,dy$
$\frac{\sec^2 dx}{\tan x} = -\frac{\sec^2 y\,dy}{\tan y}$
$\int \frac{\sec^2 x\,dx}{\tan x} = -\int \frac{\sec^2 y\,dy}{\tan y}$
$\log|\tan x| = -\log(\tan y) + c$
$\log|\tan x \cdot \tan y| = c$
$\tan x \cdot \tan y = e^c$
$\frac{dy}{dx} = \sin^2(x-y+1)$
subs $x-y+1 = v$
Differentiating b.s
$x - y + 1 = v$
$1 - \frac{dy}{dx} = \frac{dv}{dx}$
$1 - \frac{dv}{dx} = \frac{dy}{dx}$
$1 - \frac{dv}{dx} = \sin^2 v$
$\frac{dv}{dx} = 1 - \sin^2 v$
$\frac{dv}{dx} = \cos^2 v$
$\frac{dv}{\cos^2 v} = dx \quad \int \sec^2 v\,dv = \int dx$
$\tan v = x + c$



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Practical 8:-
Aim:- Euler's Method
dy/dx = y + e^x - 2   y(0) = 2; h = 0.5 find
y(2) = ?
Solun:- f(x) = y + e^x - 2 ; x0 = 0 ; y(0) = 2;
h = 0.5
n   xn   yn   f(xn, yn)   yn+1
0   0   2   1   2.5
1   0.5   2.5   2.1487   3.5743
2   1   3.574   4.2925   5.7205
3   1.5   5.7205   8.2021   9.8215
4   2   9.8715
y(2) = 9.8215
Q2] dy/dx = 1 + y2, y(0) = 1 ; h = 0.2 find
y(1) = ?
n   xn   yn   f(xn, yn)   yn+1
0   0   0   1   0.2
1   0.2   0.2   1.04   0.408
2   0.4   0.408   1.1664   0.6412
3   0.6   0.6412   1.4111   0.9234
4   0.8   0.9234   1.8526   1.2939
5   1   1.2939
y(1) = 1.2939

Q3] $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$ $y(0) = 1$ $h = 0.2$ find $y(1)$

$x_0 = 0$; $y(0) = 1$; $h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	1	0	1
1	0.2	1	0.4472	1.0894
2	0.4	1.084	0.6059	1.2105
3	0.6	1.2105	0.7040	1.3513
4	0.8	1.3513	0.7696	1.5051
5	1	1.5051		

$y(1) = 1.5051$

Q4] $\frac{dy}{dx} = 3x^2 + 1$ $y(1) = 2$ find $y(2)$; $h = 0.25$

$y_0 = 2$; $x_0 = 1$; $h = 0.5$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	4
1	1.5	4	7.75	7.875
2	2	7.895		

$y(2) = 7.875$

Q5] $y_0 = 2$; $x_0 = 1$ $h = 0.25$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	3
1	1.25	3	5.6875	4.4218
2	1.5	4.4218	59.6569	19.3360
3	1.75	19.3360	1122.6426	299.9960
4	2	299.9960		

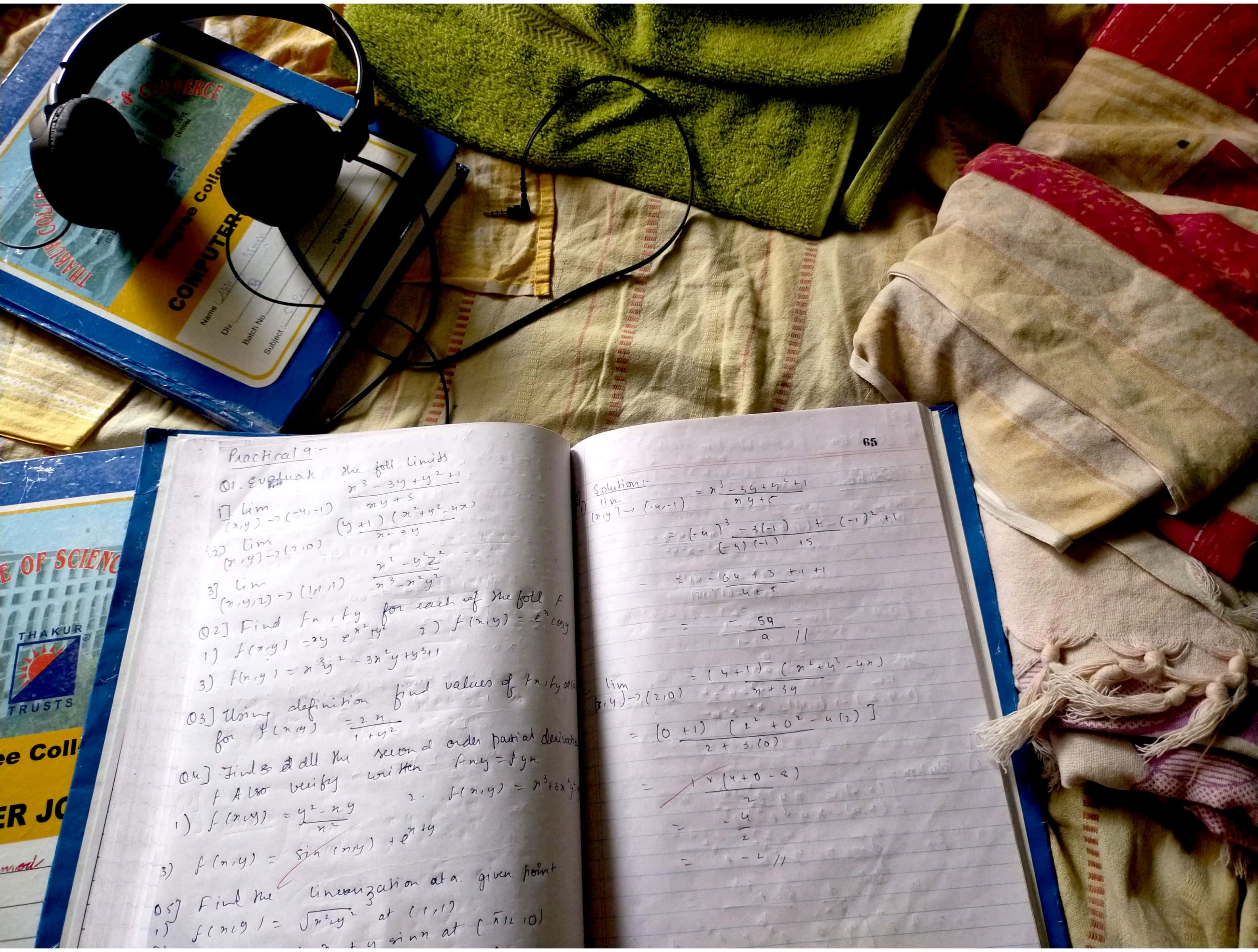
$y(2) = 299.9960$

$\frac{dy}{dx} = \sqrt{xy} + 2$; $y(1) = 1$ $h = 0.2$

$x_0 = 1$ $y_0 = 1$ $h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	1	3	3.6
1	1.2	3.6		

$y(1.2) = 3.6$



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Solution

3 **Solution :-**

$$\lim_{(x,y,z) \to (1,1,1)} = \frac{x^2 - y^2 z^2}{x^3 - x^2 y^2}$$

$$= \frac{1^2 - (1)^2 (1)^2}{(1)^3 - (1)^2 (1)(1)}$$

$$= \frac{1 - 1}{1 - 1}$$

$$= \frac{0}{0}$$

$\therefore \lim_{(x,y,z) \to (1,1,1)} \frac{x^2 - y^2 z^2}{x^4 - x^2 y^2}$ is not de

Q 2]

i] **Solution:-** $f(x,y) = xy\, e^{x^2 + y^2}$

$\therefore f(x,y) = x \cdot y \cdot e^{x^2} \cdot e^{y^2}$

$$f_x = \frac{d}{dx}\, x \cdot y \cdot e^{x^2} \cdot e^{y^2}$$

$$y \cdot e^{y^2} \cdot \frac{d}{dx}\, x \cdot e^{x^2}$$

$f(x) = (2x^2 + 1)\, y\, e^{x^2 + y^2}$

$$f_y = x e^{x^2} \frac{d}{dy}\, y \cdot e^{y^2}$$

$$= x e^{x^2} \left[y \frac{d}{dy} e^{y^2} + e^{y^2} \frac{d}{dy} y \right]$$

$$= x e^{x^2} \left[2y^2 + e^{y^2} \right]$$

$$f_y = (2y^2 + 1)\, x\, e^{x^2 + y^2}$$

$f(x,y) = e^x \cdot \cos y$

Solution :-

$$f_x = \frac{d}{dx}\, e^x \cdot \cos y$$

$$f_x = e^x \cdot \cos y$$

$$f_y = \frac{d}{dy}\, e^x \cdot \cos y$$

$$f_y = -e^x \cdot \sin y$$

$f(x,y) = x^3 y^2 - 3x^2 y + y^3 + 1$

solution :-

$$f_x = \frac{d}{dx}\,(x^3 y^2 - 3x^2 y + y^3 + 1)$$

$$= 3x^2 y^2 - 6xy$$

$$f_y = \frac{d}{dy}\,(x^3 y^2 - 3x^2 y + y^3 + 1)$$

$$= 2x^3 y - 3x^2 + 3y^2$$

22/04/2020

Q3]

Solution : $f_x(a,b) = \lim_{h \to 0} \frac{f(a+h,b) - f(a,b)}{h}$

$f_y(a,b) = \lim_{h \to 0} \frac{f(a,b+h) - f(a,b)}{h}$

Acc to given $(a,b) = (0,0)$

$f(0,0) = \lim_{h \to 0} \frac{f(h,0) - f(0,0)}{h}$

$= \lim_{h \to 0} \frac{2h - 0}{h} = 2$

$f_y(0,0) = \lim_{h \to 0} \frac{f(0,h) - f(0,0)}{h}$

$= \lim_{h \to 0} \frac{0 - 0}{h} = 0$

$\therefore f_x = 2 , f_y = 0$

$f_x(0,0) = \lim_{h \to 0} \frac{f(h,0) - f(0,0)}{h}$

$= \lim_{h \to 0} \frac{2h - 0}{h} = 2$

$f_y(0,0) = \lim_{h \to 0} \frac{f(0,h) - f(0,0)}{h}$

$= \lim_{h \to 0} \frac{0 - 0}{h} = 0$

ii) $f(x,y) = \frac{y^2 - xy}{x^2}$

$\therefore f_{xx} = \frac{d^2 f}{dx^2} \qquad f_{yy} = \frac{d^2 f}{dy^2}$

Applying $\frac{u}{v}$ rule

$f_x = \frac{x^2(0-y) - (y^2-xy)2x}{x^4}$

$= \frac{-x^2 y - 2xy^2 + 2x^2 y}{x^4}$

$f_x = \frac{x^2 y - 2xy^2}{x^4}$

$f_{xx} = \frac{x^4(2xy - 2y^2) - (x^2 y - 2xy^2)(4x^3)}{x^8}$

$= \frac{2x^5 y - 2x^4 y^2 - (4x^5 y - 8x^4 y^2)}{x^8}$

$= \frac{-2x^5 y + 6x^4 y^2}{x^8}$

$= \frac{6x^4 y^2 - 2x^5 y}{x^8}$

$f_{xx} = \frac{6y^2 - 2xy}{x^4}$

$$f_y = \frac{1}{x^2}(2y - x) \quad \therefore f_{yy} = \frac{1}{x^2}2 = \frac{2}{x^2}$$

$$f_{xy} = \frac{2y - x}{x^2}$$

$$= \frac{x^2(-1) - (2y - x)(2x)}{x^4}$$

$$= \frac{-x^2 - 4xy + 2x^2}{x^4}$$

$$= \frac{x - 4y}{x^3}$$

$$f_{xy} = \frac{x - 4y}{x^3}$$

$$f_{yx} = \frac{x^2 y - 2xy}{x^4}$$

$$= \frac{x^2 - 4xy}{x^4}$$

$$= \frac{x - 4y}{x^3}$$

$$= f_{xy}$$

$$f_{yx} = f_{xy}$$

Hence verified

Q5] i) $f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$

ii) $f(x,y) = 1 - x + y\sin x$ at $(\pi/2, 0)$

iii) $f(x,y) = \log x + \log y$ at $(1,1)$

Soln :-

$$f(x,y) = \sqrt{x^2+y^2} \qquad (a,b) = (1,1)$$

$$f_x = \frac{1}{2\sqrt{x^2+y^2}} \times 2x \qquad f_y = \frac{2y}{2\sqrt{x^2+y^2}}$$

$$f_x = \frac{x}{\sqrt{x^2+y^2}} \qquad \therefore f_y = \frac{y}{\sqrt{x^2+y^2}}$$

$$f_x(1,1) = \frac{1}{\sqrt{2}} \qquad \therefore f_y(1,1) = \frac{1}{\sqrt{2}}$$

$$\therefore L(x,y) \; f(1,1) + f_x(1,1)(x-1) \; f_y(1,1)(y-1)$$

$$= \sqrt{2} + \frac{x}{\sqrt{2}} \cdot 1 + \frac{y}{\sqrt{2}} \cdot 1$$

$$= \frac{\sqrt{2} + \frac{x + y \cdot 2}{\sqrt{2}}}{\sqrt{2}}$$

$$L(x,y) = \frac{x+y}{\sqrt{2}}$$

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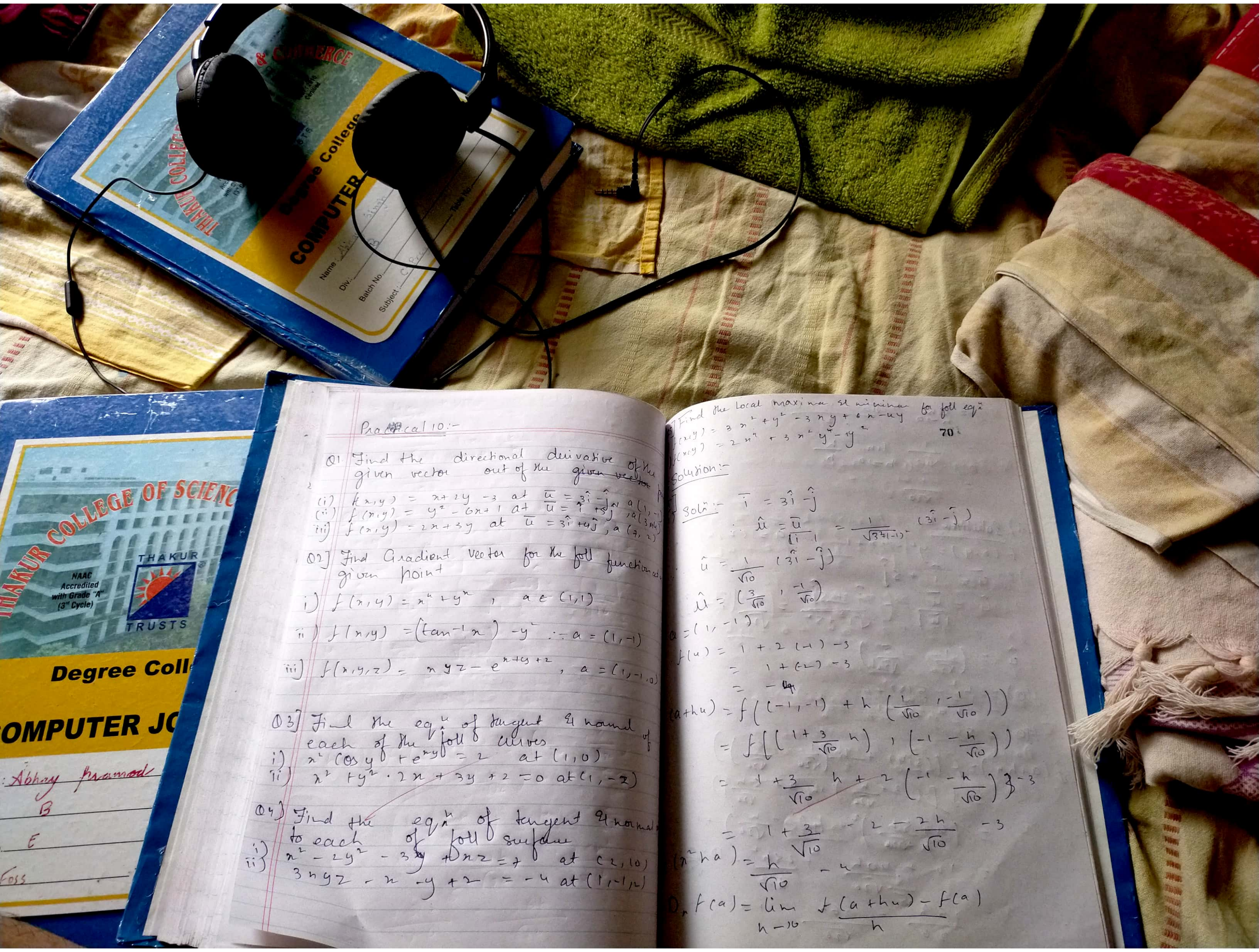
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Practical 10:-
Q1 Find the directional derivative of the given vector out of the given vector
(i) f(x,y) = x+2y -3 at u = 3i - j, a(1,-1)
(ii) f(x,y) = y² - 6x+1 at u = i +5j, a(3,4)
iii) f(x,y) = 2x+3y at u = 3i +4j, a(7,2)
Q2] Find Gradient vector for the foll function at given point
i) f(x,y) = x⁴ + yⁿ, a = (1,1)
ii) f(x,y) = (tan⁻¹x) - y², a = (1,-1)
iii) f(x,y,z) = xyz - e^(x+y+z), a = (1,-1,0)
Q3] Find the eqⁿ of tangent & normal of each of the foll curves
i) x² cos y + e^(xy) = 2 at (1,0)
ii) x² + y² - 2x + 3y + 2 = 0 at (1,-2)
Q4] Find the eqⁿ of tangent & normal to each of foll surface
i) x² - 2y² - 3y + xz = 7 at (2,1,0)
ii) 3xyz - x - y + z = -4 at (1,-1,2)
Find the Local maxima & minima for foll eqⁿ
f(x,y) = 3x² + y² - 3xy + 6x - 4y
f(x,y) = 2x⁴ + 3x²y⁴ - y²
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Solution:-
Solⁿ:- u = 3i - j
û = u/|u| = 1/√(3²+(-1)²) (3i - j)
û = 1/√10 (3i - j)
û = (3/√10, -1/√10)
a = (1,-1)
f(a) = 1 + 2(-1) - 3
= 1 + (-2) - 3
= -4
f(a+hu) = f((1,-1) + h(3/√10, -1/√10))
= f((1 + 3h/√10), (-1 - h/√10))
= 1 + 3h/√10 + 2(-1 - h/√10) - 3
= 1 + 3/√10 - 2 - 2h/√10 - 3
f(a+hu) = h/√10 - 4
Dᵤf(a) = lim h→0 [f(a+hu) - f(a)]/h

$$= \lim_{h \to 0} \frac{h/\sqrt{10} - 4 - (-4)}{2}$$

$$= \frac{1}{\sqrt{10}} \lim_{h \to 0} \frac{h}{h}$$

$$= \frac{1}{\sqrt{10}}$$

ii) Solution:-

$a = (2,4)$

$f(x,y) = y^2 - 4x + 1$

$\bar{u} = \hat{i} + 5\hat{j}$

$$\hat{u} = \frac{\bar{u}}{|\bar{u}|} = \frac{\hat{i} + 5\hat{j}}{\sqrt{1^2 + 5^2}} = \frac{1}{\sqrt{26}}(\hat{i} + 5\hat{j})$$

$$\hat{u} = \left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$$

$f(0) = 4^2 - 4(3) + 1$

$= 16 - 12 + 1$

$f(0) = 5$

$$f(a+hu) = f\left((3,4) + h\left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)\right)$$

$$= f\left(\left(3 + \frac{h}{\sqrt{26}}\right), \left(4 + \frac{5h}{\sqrt{26}}\right)\right)$$

$$= \left(4 + \frac{5h}{\sqrt{26}}\right)^2 - 4\left(3 + \frac{h}{\sqrt{26}}\right) + 1$$

$$= 16 + \frac{40h}{\sqrt{26}} + \frac{25h^2}{26} - 12 - \frac{4h}{\sqrt{26}}$$

$$= \frac{25h^2}{26} - \frac{36h}{\sqrt{26}} + 5$$

$$f(a) = 2(1) + 3(2)$$
$$= 2 + 6$$
$$= 8$$

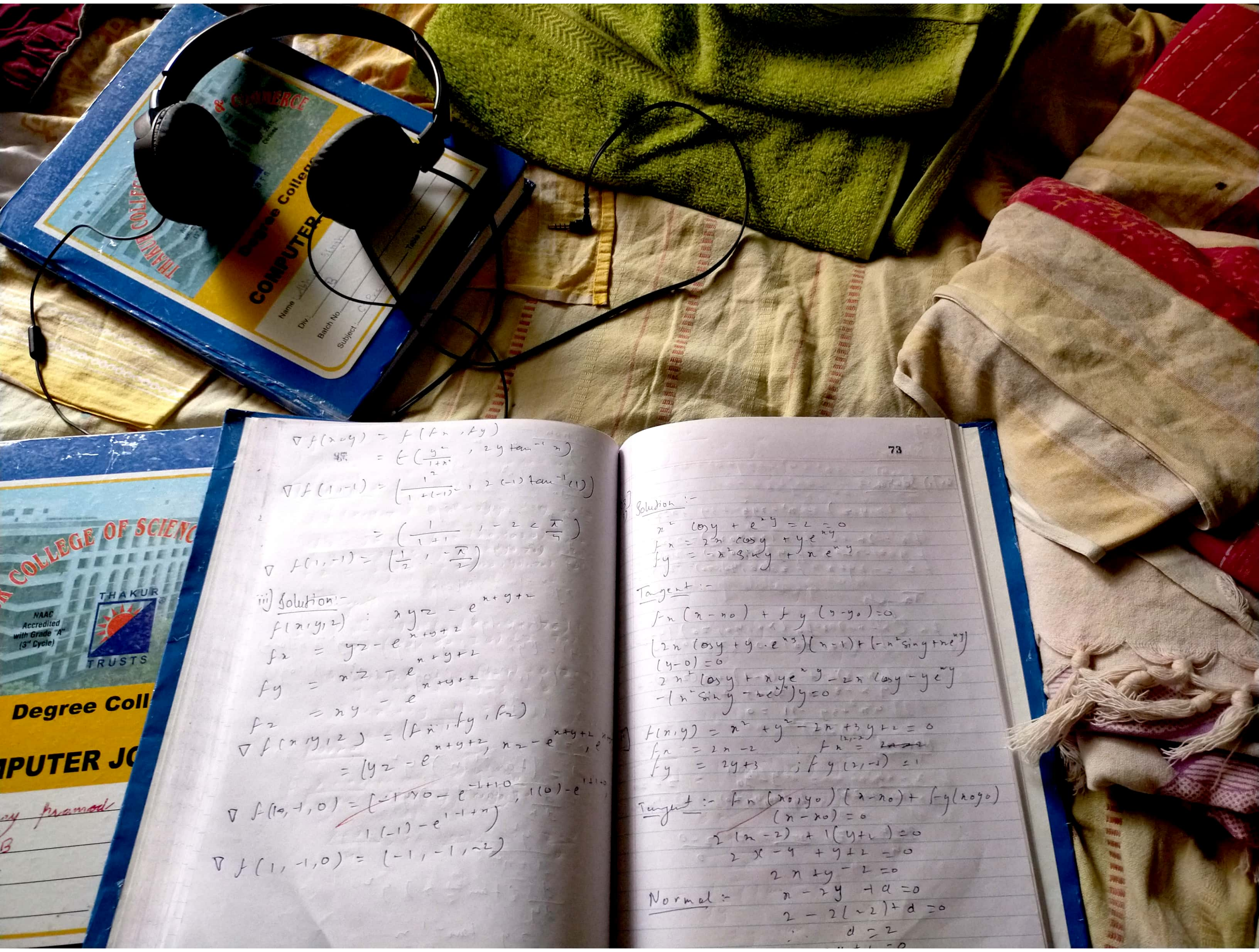
$$f(a+hu) = f\left((1,2) + h\left(\tfrac{3}{5}, \tfrac{4}{5}\right)\right)$$
$$= f\left(\left(1+\frac{3h}{5}\right), \left(2+\frac{4h}{5}\right)\right)$$
$$= 2\left(1+\frac{3h}{5}\right) + 3\left(2+\frac{4h}{5}\right)$$
$$= 2 + \frac{6h}{5} + 6 + \frac{12h}{5}$$
$$= \frac{18h}{5} + 8$$

$$D_u f(a) = \lim_{h \to 0} \frac{f(a+hu) - f(a)}{h}$$

$$D_u f(a) = \lim_{h \to 0} \frac{\frac{18h}{5} + 8 - 8}{h}$$

$$= \lim_{h \to 0} \frac{18h/5}{h}$$

$$= \frac{18}{5}$$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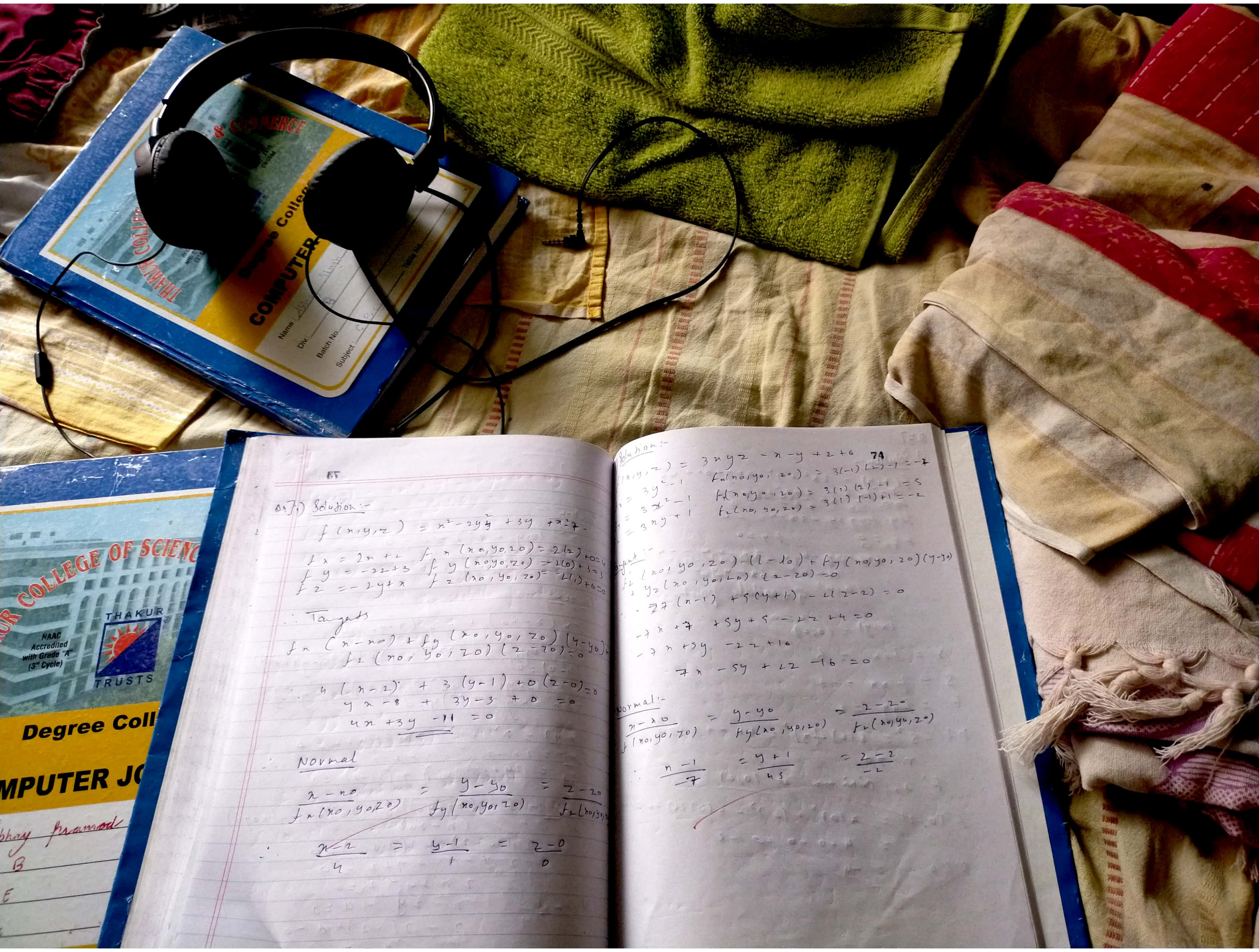

Solution:-
f (x,y,z) = x² - 2yz + 3y + xz - 7
fx = 2x + z   fx (x0,y0,z0) = 2(2) + 0 = 4
fy = -2z + 3   fy (x0,y0,z0) = -2(0) + 3 = 3
fz = -2y + x   fz (x0,y0,z0) = -2(1) + 0 = 0
Tangents
fx (x - x0) + fy (x0, y0, z0) (y - y0) + fz (x0, y0, z0) (z - z0) = 0
4 (x - 2) + 3 (y - 1) + 0 (z - 0) = 0
4x - 8 + 3y - 3 + 0 = 0
4x + 3y - 11 = 0
Normal
(x - x0) / fx (x0, y0, z0) = (y - y0) / fy (x0, y0, z0) = (z - z0) / fz (x0, y0, z0)
(x - 2) / 4 = (y - 1) / 1 = (z - 0) / 0
Solution:-
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f (x,y,z) = 3xyz - x - y + z + 6
fx (x0, y0, z0) = 3(-1)(2) - 1 = -7
fy (x0, y0, z0) = 3(1)(2) - 1 = 5
fz (x0, y0, z0) = 3(1)(-1) + 1 = -2
-7 (x - 1) + 5 (y + 1) - 2 (z - 2) = 0
-7x + 7 + 5y + 5 - 2z + 4 = 0
-7x + 5y - 2z + 16
7x - 5y + 2z - 16 = 0
Normal:-
(x - x0) / fx (x0, y0, z0) = (y - y0) / fy (x0, y0, z0) = (z - z0) / fz (x0, y0, z0)
(x - 1) / -7 = (y + 1) / 5 = (z - 2) / -2
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Q 5]

i) Solution:-

$f(x,y) = 3x^2 + y^2 - 3xy + 6x + 4y$

$f_x = 6x - 3y + 6$

$f_y = 2y - 3x - 4$

$f_x = 0 \quad ; \quad f_y = 0$

$6x - 3y - 6 \qquad 3x - 2y = -4$

$6x - 4y = -8$

$\therefore y = 2$

$x = 0$

$\therefore (x,y) = (0,2)$

$r = f_{xx} = 6$

$s = f_{xy} = -3$

$t = f_{yy} = 2$

$rt - s^2 = 6(2) - (-3)^2 = 12 - 9 = 3 > 0$

$r > 0$

$\therefore f$ is minima at $(0,2)$

$f(0,2) = 3(0)^2 + (2)^2 - 3(0)(2) + 6(0) + 4(2)$

$= 4 + 0 - 0 + 0 - 8$

$f(0,2) = -4$

Solution:-

$f(x,y,z) = 2x^4 + 3x^2y - y^2$

$f_x = 8x^3 + 6xy$

$f_y = -2y + 3x^2$

$f_x = 0$

$8x^3 + 6xy = 0 \qquad f_y = 0$

$\therefore x(8x^2 + 6y) = 0 \qquad -2y + 3(0)^2 = 0$

$\therefore x = 0 \qquad \therefore y = 0$

$(x,y) = (0,0)$ is a root

$8x^2 + 6y = 0$

$x^2 = -\frac{2}{3}y$

$f_y = -2y - \frac{2}{3} \times 3y = 0$

$-6y = 0$

$y = 0$

$x^2 = 0$

$x = 0$

$\therefore (x,y) = (0,0)$ is the only root

$r = f_{xx} = 24x^2 + 6y = 0$
$s = f_{xy} = 6x = 0$
$t = f_{yy} = -2 = -2$

$r = 0$

$rt - t^2 = 0(0) - (2)^2$
$rt - 4^2 = -6 < 0$

$(0,0)$ is saddle point.

AK
05/02/2022

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