① $$\lim_{x \to a}\left[\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-\sqrt{2x}}\right]$$

$$\lim_{x \to a}\left[\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{2x}} \times \frac{\sqrt{a+2x}+\sqrt{3x}}{\sqrt{a+2x}+\sqrt{3x}} \times \frac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{3a+x}+2\sqrt{x}}\right]$$

$$\lim_{x \to a}\frac{(a+2x-3x)(\sqrt{3a+x}+2\sqrt{x})}{(3a+x-4x)(\sqrt{a+2x}+3\sqrt{x})}$$

$$\lim_{x \to a}\frac{(a-x)(\sqrt{3a+x}+2\sqrt{x})}{(3a-3x(\sqrt{a+2x}+\sqrt{3x})}$$

$$\frac{1}{3}\lim_{x \to a}\frac{(a-x)(\sqrt{3a+x}+2\sqrt{x})}{(a-x)(\sqrt{a+2x}+\sqrt{3x})}$$

$$\frac{1}{3}\;\frac{\sqrt{3a+a}+2\sqrt{a}}{\sqrt{a+2a}+\sqrt{3a}}$$

$$\frac{1}{3}\times\frac{\sqrt{4a}+2\sqrt{a}}{\sqrt{3a}+\sqrt{3a}}$$

$$\frac{1}{3}\;\frac{2\sqrt{a}+2\sqrt{a}}{\sqrt{3a}+\sqrt{3a}}$$

$$\frac{1}{3}\;\frac{\overset{2}{\cancel{4}}\sqrt{a}}{\cancel{2}\sqrt{a}}$$

$$\frac{2}{3\sqrt{3}}$$

(2) $\lim_{y \to 0} \left[\frac{\sqrt{a+y} - \sqrt{a}}{y \cdot \sqrt{a+y}} \right]$

$$\lim_{y \to 0} \left[\frac{\sqrt{a+y} - \sqrt{a}}{y \sqrt{a+y}} \times \frac{\sqrt{a+y} + \sqrt{a}}{\sqrt{a+y} + \sqrt{a}} \right]$$

$$\lim_{y \to 0} \frac{a+y-a}{y \sqrt{a+y} (\sqrt{a+y} + \sqrt{a})}$$

$$\lim_{y \to 0} \frac{\cancel{y}}{\cancel{y} \cdot \sqrt{a+y} (\sqrt{a+y} + \sqrt{a})}$$

$$\frac{1}{\sqrt{a+y} \; (\sqrt{a+y} + \sqrt{a})}$$

$$\frac{1}{\sqrt{a} (\sqrt{a} + \sqrt{a})}$$

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(5) $\lim_{x \to \pi/6} \frac{\cos x - \sqrt{3} \sin x}{\pi - 6x}$

By substituting $x - \pi/6 = h$

$x = h + \pi/6$

where $h \to 0$

$\lim_{h \to 0} \frac{\cos(h + \pi/6) - \sqrt{3} \sin(h + \pi/6)}{\pi - 6(h + \pi/6)}$

using
$\cos(A + B) = \cos A \cos B - \sin A \sin B$
$\sin(A + B) = \sin A \cos B + \cos A \sin B$

$\lim_{h \to 0} \frac{\cos h \cdot \cos \pi/6 - \sin h \sin \pi/6 - \sqrt{3}\,(\sin h \cos \pi/6 + \cos h \sin \pi/6)}{\pi - 6\left(\frac{6h + \pi}{6}\right)}$

$\cos \pi/6 = \cos 30° = \sqrt{3}/2$
$\sin \pi/6 = \sin 30° = 1/2$

$\lim_{h \to 0} \frac{\cos h \cdot \frac{\sqrt{3}}{2} - \sin h \frac{1}{2} - \sqrt{3}\left(\sin h \frac{\sqrt{3}}{2} + \cos h \cdot \frac{1}{2}\right)}{\pi - \cancel{6h + \pi}}$

$\lim_{h \to 0} \frac{\cos \frac{\cancel{\sqrt{3}}}{2} h - \sin \frac{h}{2} - \sin \frac{3h}{2} - \cos \frac{\cancel{\sqrt{3}}}{2} h}{-6h}$

٤.

$$\lim_{h\to 0} \frac{\cancel{x}\ \frac{\sin 4h}{2}}{\cancel{x}\ 6h}$$

$$\lim_{h\to 0} \frac{\sin 4h}{3}$$

$$\frac{1}{3}\lim_{h\to 0} \frac{\sin h}{h} = \frac{1}{3} \times 1 = \frac{1}{3}$$

(4) $\lim_{x\to\infty}\left[\dfrac{\sqrt{x^2+5}-\sqrt{x^2-3}}{\sqrt{x^2+3}-\sqrt{x^2+1}}\right]$

By rationalizing Numerator & Denominator both

$\lim_{x\to\infty}\dfrac{\sqrt{x^2+5}-\sqrt{x^2-3}}{\sqrt{x^2+3}-\sqrt{x^2+1}}\times\dfrac{\sqrt{x^2+5}+\sqrt{x^2-3}}{\sqrt{x^2+5}+\sqrt{x^2-3}}\times\dfrac{\sqrt{x^2+3}+\sqrt{x^2+1}}{\sqrt{x^2+3}+\sqrt{x^2+1}}$

$\lim_{x\to\infty}\left[\dfrac{(x^2+5-x^2+3)}{(x^2+3-x^2+1)}\dfrac{(\sqrt{x^2+3}+\sqrt{x^2+1})}{(\sqrt{x^2+5}+\sqrt{x^2-3})}\right]$

$\lim_{x\to\infty}\dfrac{\cancel{8}^{\,4}}{\cancel{2}}\dfrac{(\sqrt{x^2+3}+\sqrt{x^2+1})}{(\sqrt{x^2+5}+\sqrt{x^2-3})}$

$4\lim_{x\to\infty}\dfrac{\sqrt{x^2(1+3/x^2)}+\sqrt{x^2(1+\frac{1}{x^2})}}{\sqrt{x^2(1+\frac{5}{x^2})}+\sqrt{x^2(1-\frac{3}{x^2})}}$

After applying limit
we get

$=4$

8.

(5) $f(x) = \frac{\sin 2x}{\sqrt{1-\cos^2 x}}$, for $0 < x < \pi/2$, for $\pi/2 < x < \pi$ } at $x = \pi/2$

$f(\pi/2) = \frac{\sin 2(\pi/2)}{\sqrt{1-\cos 2(\pi/2)}}$ $\therefore f(\pi/2) = 0$

f at $x = \pi/2$ define.

$\lim_{x \to \pi/2} f(x) = \lim_{x \to \pi/2} \frac{+\cos x}{\pi - 2x}$

By substituting method
$x - \pi/2 = h$
$x = h + \pi/2$

where $h \to 0$

$\lim_{h \to 0} \frac{\cos(h + \pi/2)}{\pi - 2(h + \pi/2)}$

$\lim_{h \to 0} \frac{\cos(h + \pi/2)}{\pi - 2\left(\frac{2h + \pi}{2}\right)}$

$\lim_{h \to 0} \frac{\cos(h + \pi/2)}{-2h}$

$\lim_{h \to 0} \frac{\cos h \cdot \cos \pi/2 - \sin h \cdot \sin \pi/2}{-2h}$

$\lim_{h \to 0} \frac{\cos h \cdot 0 - \sin h}{-2h}$

$\lim_{h \to 0} \frac{\cancel{-} \sin h}{\cancel{-} 2h}$

$\frac{1}{2} \lim_{h \to 0} \frac{\sin h}{h}$

$= \frac{1}{2}$

h) $\lim_{x \to \pi/2} f(x) = \lim_{x \to \pi/2} - \frac{\sin 2x}{\sqrt{1-\cos 2x}}$ using $\sin 2x = 2\sin x \cdot \cos x$

$\lim_{x \to \pi/2} \frac{2 \cancel{\sin x} \cdot \cos x}{\sqrt{2} \cancel{\sin x}}$

$\lim_{x \to \pi/2} = \frac{2\cos x}{\sqrt{2}}$

$\frac{2}{\sqrt{2}} \lim_{x \to \pi/2} \cancel{\cos x} \cos x$

$\therefore L.H.L = R.H.L$

$\therefore f$ is not continous at $x = \pi/2$

i) $f(x) \frac{x^2-9}{x-3}$ $\quad 0 < x < 3$

$x+3$ $\quad 3 \le x \le 6$

$= \frac{x^2-9}{x+3}$ $\quad 6 \le x < 9$

at $x = 3$

at $x = 3$

i) $f(x) = \frac{x^2-9}{x-3} = 0$

f at $x=3$ define.

ii) $\lim_{x\to 3} f(x) \lim_{x\to 3} x+3$

$f(3) = x+3 = 3+3 = 6$

f is define at $x=3$

$\lim_{x\to 3} f(x) = \lim_{x\to 3} (x+3) = 6$

$\lim_{x\to 3} f(x) = \lim_{x\to 3} \frac{x^2-9}{x-3} = \frac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}}$

$\therefore$ L.H.L = R.H.L

f is continous at $x=3$

for $x=6$

$f(6) = \frac{x^2-9}{x+3} = \frac{36-9}{6+3} = \frac{27}{9} = 3$

2) $\lim_{x\to 6} \frac{x^2-9}{x+3}$

$\lim_{x\to 6} = \frac{(x-3)\cancel{(x+3)}}{\cancel{(x+3)}}$

$\lim_{x\to 6} (x-3) = 6-3 = 3$

$\lim_{x\to 6} x+3 = 3+6 = 9$

function is not continous.

$$f(x) = \frac{1-\cos 4x}{x^2} \quad x<0 \\ x=0 \Big\} \text{ at } x=0$$

f is continuous at $x = 0$

$\lim_{x\to 0} f(x) = f(0)$

$\lim_{x\to 0} \frac{1-\cos 4x}{x^2} = k$

$\lim_{x\to 0} \frac{2\sin^2 2x}{x^2} = k$

$\Rightarrow \lim_{x\to 0} \frac{\sin^2 2x}{x^2} = k$

$\Rightarrow \lim_{x\to 0} \left(\frac{\sin 2x}{x}\right)^2 = k$

$\Rightarrow (2)^2 = k$

$\underline{k = 8}$

ii) $f(x) = (\sec^2 x)^{\cot^2 x} \quad x \neq 0 \\ x = 0 \Big\}$ at $x = 0$

$= k$

$f(x) = (\sec^2 x)^{\cot^2 x}$

using

$\tan^2 x - \sec^2 x = 1$

$\sec^2 x = 1 + \tan^2 x$

& $\cot^2 x = \frac{1}{\tan^2 x}$

$\lim_{x\to 0} (\sec^2 x)^{\cot^2 x}$

$\lim_{x\to 0} (1+\tan^2 x)^{\frac{1}{\tan^2 x}}$

$$\lim_{h \to 0} \frac{}{\pi - 3(\ldots)}$$

using

$$\tan(A + B) = \frac{\tan A + \tan}{1 - \tan A \tan}$$

$$\lim_{h \to 0} \frac{\dfrac{\sqrt{3} - \tan \pi/3 + \tan h}{1 - \tan \pi/3 \cdot \tan h}}{\pi - \pi - 3h}$$

$$\lim_{h \to 0} \frac{\sqrt{3}\left(\dfrac{1 - \tan \pi/3 \cdot \tan h}{3}\right) - \tan\left(\pi/3 - \tan h\right)}{\dfrac{1 \tan \pi/3 \cdot \tan h}{-3h}}$$

$$\lim_{h\to 0} \frac{\dfrac{(\sqrt{3}-\sqrt{3}\times\sqrt{3}\tan h)-(\sqrt{3}+\tan h)}{1-\tan\pi/3\cdot\tan h}}{-3h}$$

$$\lim_{h\to 0} \frac{\dfrac{(\sqrt{3}-3\tan h)-(\sqrt{3}+\tan h)}{1-\sqrt{3}\cdot\tan h}}{-3h}$$

$$\lim_{h\to 0} \frac{\dfrac{(\sqrt{3}-3\tan h-\sqrt{3}-\tan h)}{1-\sqrt{3}\cdot\tan h}}{-3h}$$

$$\lim_{h\to 0} \frac{-4\tan h}{-3h(1-\sqrt{3}\tan h)}$$

$$\frac{4}{3}\lim_{h\to 0}\frac{\tan h}{h}\lim_{h\to 0}\frac{1}{(1-\sqrt{3}\tan h)}$$

$$\frac{4}{3}\frac{1}{(1-\sqrt{3}\cdot t(0))}$$

$$\frac{4}{3}\left(\frac{1}{1}\right)=\frac{4}{3}$$

7.

① $f(x) = \frac{1-\cos 3x}{x \tan x}$ $x=0$ $x=0$ } at $x=0$

$f(x) = \frac{1-\cos 3x}{x \tan x}$

$\lim_{x \to 0} \frac{2 \sin^2 3/2\, x}{x \tan x}$

$\lim_{x \to 0} \frac{\frac{2 \sin^2 3x/2}{x^2} \times x^2}{\frac{x \cdot \tan x}{x^2} \times x^2}$

$2 \lim_{x \to 0} \frac{(3/2)^2}{1} = \frac{2 \times 9}{4^2} = \frac{9}{2}$

$\lim_{x \to 0} f(x) = \frac{9}{2} \quad g = f(0)$

$\therefore$ f is not continous at $x=0$

Re define function.

$f(x) = \left\{ \frac{1-\cos 3x}{x \tan x} \right.$ $x \neq 0$, $x = 0$ }

Now $\lim_{x \to 0} f(x) = f(0)$

f has removable discontinuity at $x=0$

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(7) $f(x) = \dfrac{(e^{3x}-1)\sin x^\circ}{x^2}$ $\left.\begin{matrix} x \neq 0 \\ x = 0 \end{matrix}\right\}$ at $x = 0$

$\lim_{x\to 0} \left(\dfrac{e^{3x}-1}{x}\right) \lim_{x\to 0} \sin\left(\dfrac{\pi\, x}{180\, x}\right)$

$\lim_{x\to 0} \dfrac{3 \cdot e^{3x}-1}{3x} \lim_{x\to 0} \sin\left(\dfrac{\pi/x}{180\, x}\right)$

$3 \lim_{x\to 0} \dfrac{e^{3x}-1}{3x} \lim_{x\to 0} \sin\left(\dfrac{\pi x}{180\, x}\right)$

$3 \log_e \dfrac{\pi}{180} = \dfrac{\pi}{60} = f(0)$

f is continous at $x = 0$

8. $f(x) = \cancel{\dfrac{e^{x^2} - \cos x}{x^2}}$ $x = 0$

is continous at $x = 0$

$\therefore$ Given

f is continous at $x = 0$

$\lim_{x\to 0} f(x) = f(0)$

$$\lim_{x \to 0} \frac{e^{x^2} - \cos x}{x^2} = f(0)$$

$$\lim_{x \to 0} \frac{e^{x^2} - \cos x - 1 + 1}{x^2}$$

$$\lim_{x \to 0} \frac{(e^{x^2} - 1) + (1 - \cos x)}{x^2}$$

$$\lim_{x \to 0} \frac{e^{x^2} - 1}{x^2} + \lim_{x \to 0} \frac{1 - \cos x}{x^2}$$

$$\log e + \lim_{x \to 0} \frac{2 \sin^2 x/2}{x}$$

$$\log e + 2 \lim_{x \to 0} \left(\frac{\sin x/2}{x} \right)^2$$

multiply with 2 on Num & Denominator

$$1 + \cancel{x} + \frac{1}{\cancel{x}\,2} = \frac{3}{2}\ f(0)$$

7) $f(x) = \frac{\sqrt{2} - \sqrt{1-\sin x}}{\cos^2 x} \quad x = \pi/2$

$f(x)$ is continous at $x = \pi/2$

$$\lim_{x \to \pi/2} \frac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x} \times \frac{\sqrt{2} + \sqrt{1+\sin x}}{\sqrt{2} + \sqrt{1+\sin x}}$$

$$\lim_{x \to \pi/2} \frac{2 - 1 + \sin x}{\cos^2 x (\sqrt{2} + \sqrt{1+\sin x})}$$

$$\lim_{x \to \pi/2} \frac{\cancel{1+\sin x}}{(1-\sin x)\cancel{(1+\sin x)}(\sqrt{2} + \sqrt{1+\sin x})}$$

$$\lim_{x \to \pi/2} \frac{1}{(1-\sin x)(\sqrt{2} + \sqrt{1+\sin x})}$$

$$\cancel{\frac{1}{2\sqrt{}}} \quad \frac{1}{2(\sqrt{2}+\sqrt{2})}$$

$$\frac{1}{2(2\sqrt{2})} = \frac{1}{4\sqrt{2}}$$

$$f(\pi/2) = \frac{1}{4\sqrt{2}} \;//$$

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Practical 2

Show that the following function defined from $\mathbb{R}$ to $\mathbb{R}$ are differentiable.

cot x

$f(x) = \cot x$

$$Df(a) = \lim_{x \to 0} \frac{f(x) - f(a)}{x - a}$$

$$\lim_{x \to 0} \frac{\cot x - \cot a}{x - a}$$

$$\lim_{x \to a} \frac{1/\tan x - 1/\tan a}{x - a}$$

$$\lim_{x \to 0} \frac{\tan a - \tan x}{(x - a) \tan x \tan a}$$

put $x - a = h$

$x = a + h$

as $x \to a$, $h \to a$

$$Df(h) = \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{(a + h - a) \tan(a+h) \tan a}$$

$$= \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{h \times \tan(a+h) \cdot \tan a}$$

formula: $\tan(A - B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

$\tan A - \tan B = \tan(A - B)(1 + \tan A \tan B)$

$$\lim_{h \to 0} \frac{\tan(a - a + h) - [1 + \tan a \tan(a+h)}{h \times \tan(a+h) \tan a}$$

$$\lim_{h\to 0} \frac{\tan h}{h} \times \frac{1+\tan a \tan(a+h)}{\tan(a+h)\tan a}$$

$$= 1 \times \frac{1+\tan^2 a}{\tan^2 a}$$

$$= \frac{\sec^2 a}{\tan^2 a}$$

$$= \frac{1}{\cos^2 a} \cdot \frac{\cos^2 a}{\sin^2 a}$$

$$= -\operatorname{cosec}^2 a$$

$$\therefore Df(a) = -\operatorname{cosec}^2 x$$

ii) $\operatorname{cosec} x$

$$f(x) = \operatorname{cosec} x$$

$$Df(a) = \lim_{x\to 0} \frac{f(x)-f(a)}{x-a}$$

$$Df(h) = \lim_{h \to 0} \frac{\sin a - \sin(a+h)}{(a+h-a)\sin a \cdot \sin(a+h)}$$

formula

$$\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$$

$$= \lim_{h \to 0} \frac{2\cos\left(\frac{a+a+h}{2}\right)\sin\left(\frac{a-a-h}{2}\right)}{h \times \sin a \cdot \sin(a+h)}$$

$$\lim_{h \to 0} -\frac{\sin h/2}{h/2} \times \frac{1}{2} \times \frac{2\cos\left(\frac{2a+h}{2}\right)}{\sin a \sin(a+h)}$$

$$-\frac{1}{2} \times \frac{2\cos\left(\frac{2a+a}{2}\right)}{\sin(a+b)}$$

$$-\frac{\cos a}{\sin^2 a} = -\cot a \operatorname{cosec} a$$

iii) $Sec\, x$

$f(x) = Sec\, x$

$Df(a) = \lim_{x \to a} \dfrac{f(x) - f(a)}{x - a}$

$\lim_{x \to a} \dfrac{Sec\, x - Sec\, a}{x - a}$

$\lim_{x \to a} \dfrac{\frac{1}{\cos x} - \frac{1}{\cos a}}{x - a}$

$\lim_{x \to a} \dfrac{\cos a - \cos x}{(x - a)\cos a \cos x}$

Put $x - a = h$

$x = a + h$

as $x \to a$, $h \to a$

$Df(h) = \lim_{h \to 0} \dfrac{\cos a - \cos(a+h)}{h \times \cos a \cos(a+h)}$

formula $-2\sin\left(\frac{C+D}{2}\right) \sin\left(\frac{C-D}{2}\right)$

$\lim_{h \to 0} \dfrac{-2\sin\left(\frac{a+a+h}{2}\right) \sin\left(\frac{\cancel{a} - \cancel{a} - h}{2}\right)}{h \times \cos a \cos(a+h)}$

$\lim_{h \to 0} \dfrac{-2\sin\left(\frac{2a+h}{2}\right) \sin \frac{h}{2}}{\cos a \cos(a+h) \times \frac{h}{2}} \times \frac{-1}{2}$

$$= \frac{-\frac{1}{2} \times -2 \sin\left(\frac{2a+0}{2}\right)}{\cos a \left(\cos(a+0)\right)}$$

$$= -\frac{1}{2} \times -2 \frac{\sin a}{\cos a \times \cos a}$$

$= \tan a \sec a$

If $f(x) = 4x + 1$, if $x \leq 2$
$x^2 + 5$, $x > 0$ at $x = 2$ then find function is differentiable or not.

Solution:

L.H.S:

$$Df(2^-) = \lim_{x \to 2} \frac{f(x) - f(2)}{x - 2}$$

$$\lim_{x \to 2} \frac{4x + 1 - (4 \times 2 + 1)}{x - 2}$$

$$\lim_{x \to 2} \frac{4x + 1 - 9}{x - 2}$$

$$\lim_{x \to 2} \frac{4x - 8}{x - 2}$$

$$\lim_{x \to 2} \frac{4(x - 2)}{(x - 2)} = 4$$

$Df(2^-) = 4$

R.H.D:

$$Df(2^+) = \lim_{x \to 2^+} \frac{x^2+5-9}{x-2}$$

$$\lim_{x \to 2^+} \frac{x^2-4}{x-2}$$

$$\lim_{x \to 2^+} \frac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}}$$

$$2+2=4$$

$Df(2^+) = 4$

R.H.D = L.H.D

f is differentiable at $x=2$

3. If $f(x) = 4x+7$, $x<3$

x^2+3x+1, $x \geq 3$ at $x=3$ then

find f is differentiable or not:

Solution:

R.H.D:

$$Df(3^+) = \lim_{x \to 3^+} \frac{f(x)-f(3)}{x-3}$$

$$\lim_{x \to 3^+} \frac{x^2+3x+1-(3^2+3\times3+1)}{x-3}$$

$$\lim_{x \to 3^+} \frac{x^2+3x+1-19}{x-3}$$

$$\lim_{x \to 3^+} \frac{x^2+3x-18}{x-3}$$

$$\lim_{x \to 3^+} \frac{x^2 + 6x - 3x - 18}{x - 3}$$

$$\lim_{x \to 3^+} \frac{x(x+6) - 3(x+6)}{x - 3}$$

$$\lim_{x \to 3^+} \frac{(x+6)(x-3)}{(x-3)} = 3 + 6 = 9$$

$Df(3^+) = 9$

L.H.D. $Df(3^-)$

$$\lim_{x \to 3^-} \frac{f(x) - f(3)}{x - 3}$$

$$\lim_{x \to 3} \frac{4x + 7 - 19}{x - 3}$$

$$\lim_{x \to 3} = \frac{4x - 12}{x - 3}$$

$$\lim_{x \to 3} \frac{4(\cancel{x - 3})}{(\cancel{x - 3})}$$

$Df(3^-) = 4$

R.H.D $\neq$ L.H.D

f is not differentiable at $x = 3$

4. If $f(x) = 8x - 5 \quad , x \le 2$

$= 3x^2 - 4x + 7 \,, x > 2$ at $x = 2$ then.

find f is differentiable or not.

→ $f(2) = 8 \times 2 - 5 = 16 - 15 = 11$

R.H.D :

$$Df(2^+) = \lim_{x \to 2^+} \frac{f(x) - f(2)}{x - 2}$$

$$\lim_{x \to 2^+} \frac{3x^2 - 4x - 4}{x - 2}$$

$$\lim_{x \to 2^+} \frac{3x(x-2) + 2(x-2)}{x - 2}$$

$$\lim_{x \to 2^+} \frac{(3x+2)\cancel{(x-2)}}{\cancel{x-2}}$$

$$3 \times 2 + 2 = 8$$

$$Df(2^+) = 8$$

LHD :

$$Df(2^-) = \lim_{x \to 2} \frac{f(x) - f(2)}{x - 2}$$

$$\lim_{x \to 2} \frac{8x - 5 - 11}{x - 2}$$

$$\lim_{x \to 2} \frac{8x - 16}{x - 2}$$

$$\lim_{x \to 2} \frac{8\cancel{(x-2)}}{\cancel{(x-2)}}$$

$$= 8$$

$$Df(2^-) = 8$$

Practical 3.

$f(x) = x^3 - 5x + 1$

$f'(x) = 3x^2 - 5$

f is increasing iff $f'(x) > 0$

$3x^2 - 5 > 0$

$3x^2 > 5$

$x^2 > 5/3$

$x > \pm\sqrt{\frac{5}{3}}$

$x \in (-\infty, -\sqrt{\frac{5}{3}}) \cup (\sqrt{\frac{5}{3}}, \infty)$

f is decreasing iff $f'(x) < 0$

$3x^2 - 5 < 0$

$3x^2 < 5$

$x^2 < 5/3$

$x < \pm\sqrt{\frac{5}{3}}$

$f(x)\ x^2 - 4x$

$f'(x) = 2x - 4$

f is increasing iff $f'(x) > 0$

$2x - 4 > 0$

$2(x-2) > 0$

$x - 2 > 0$

$\cancel{x} > 0 \quad x > 2$

$x \in (2, \infty)$

$\therefore f$ is decreasing iff $f'(x) < 0$

$2x - 4 < 0$

$2(x - 2) < 0$

$x - 2 < 0$

$x < 2$

$x \in (-\infty, 2)$

iii) $f(x) = 2x^3 + x^2 - 20x + 4$

$f'(x) = 6x^2 + 2x - 20$

$\therefore f$ is increasing iff $f'(x) > 0$

$\Rightarrow 6x^2 + 2x - 20 > 0$

$6x^2 + 12x - 10x - 20 > 0$

$6x(x+2) - 10(x+2) > 0$

$(6x - 10)(x+2) > 0$

$\therefore x \in (-\infty, -2) \cup (10/6, \infty)$

f is decreasing iff $f'(x) < 0$

$6x^2 - 2x - 20 < 0$

$6x^2 + 12x - 10x - 20 < 0$

$6x(x+2) - 10(x-2) < 0$

$(6x - 10)(x+2) < 0$

$\therefore x \in \left(-2, \frac{10}{6}\right)$

$(x) = x^3 - 27x + 5$

$(x) = 3x^2 - 27$

$= 3(x^2 - 9)$

f is increasing iff $f'(x) > 0$

$3(x^2 - 9) > 0$

$x^2 - 9 > 0$

$(x-3)(x+3) = 0$

$\therefore x \in (-\infty, -3) \cup (3, \infty)$

is decreasing iff $f'(x) < 0$

$3(x^2 - 9) < 0$

$x^2 - 9 < 0$

$(x-3)(x+3) < 0$

$\therefore x \in (-3, 3)$

$f(x) = 69 - 24x - 9x^2 + 2x^3$

$f(x) = -24 - 18x + 6x^2$

i.e $6x^2 - 18x - 24$

$6(x^2 - 3x - 4)$

f is increasing iff $f'(x) > 0$

$6(x^2 - 3x - 4) > 0$

$x^2 - 3x - 4 > 0$

$x^2 - 4x + x - 4 > 0$

$x(x-4) + 1(x-4) > 0$

$(x+1)(x-4) > 0$

$x \in (\infty, -1) \cup (4, \infty)$

f is decreasing iff $f'(x) < 0$

$6(x^2 - 3x - 4) < 0$

$x^2 - 3x - 4 < 0$

$x^2 - 4x + x - 4 < 0$

$(x+1)(x-4) < 0$

$x \in (1, 4)$

2.

(i) $y = 3x^2 - 2x^3$

let

$f(x) = y = 3x^2 - 2x^2$

$f'(x) = 6x - 6x^2$

$f''(x) = 6 - 12x$

$6(1 - 2x)$

$f'(x)$ is concave upwards iff,

$f''(x) > 0$

$6(1-2x) > 0$

$1 - 2x > 0$

$-2x > -1$

$2x < -1$

$x < 1/2$

$f(x) = y = x^4 - 6x^3 - 12x^2 + 5x + 7$

$f'(x) = 4x^3 - 18x^2 + 24x + 5$

$f''(x) = 12x^2 - 36x + 24$

$12(x^2 - 3x + 2)$

$f(x)$: concave upward iff

$f''(x) > 0$

$12(x^2 - 3x + 2) > 0$

$x^2 - 3x + 2 > 0$

$x^2 - x - 2x + 2 > 0$

$x(x-1) - 2(x-1) > 0$

$(x-2)(x-1) > 0$

$x \in (-\infty, 1) \cup (2, \infty)$

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$f'(x)$ is concave upward iff,

$f'(x) > 0$

$6x > 0$

$x > 0$

$x \in (0, \infty)$

$f''(x)$ is concave downward iff,

$f'(x) < 0$

$6x < 0$

$x < 0$

$x \in (\infty, 0)$

iv) $y = 69 - 24 - 9x^2 + 2x^3$

let,

$f(x)\ y = 69 - 24x - 9x^2 + 2x^3$

$f'(x) - 24 - 18x + 6x^2$

$f''(x) = -18 + 12x$

$f(x)$ is concave upward iff,

$f''(x) > 0$

$-18 + 12x > 0$

$12x = 18$

$x = \frac{18}{12}$

$x \in (\frac{3}{2}, \infty)$

$f''(x)$ is concave downwards iff

$f''(x) < 0$

$-18x + 12x < 0$

$12x < 18$

$x < 18/12$

$x \in (-\infty, 3/2)$

v) $y = 2x^3 + x^2 - 20x + 4$

let,

$f(x) = y = 2x^3 + x^2 - 20x + 4$

$f''(x) = 12x + 2$

$2(6x + 1)$

$f'(x)$ is concave upward:-

$f''(x) > 0$

$2(6x-1) > 0$

$6x > 1$

$x > \frac{1}{6}$

$x \in (\frac{1}{6}, \infty)$

$f''(x)$ is concave downward:-

$f''(x) < 0$

$2(6x-1) < 0$

$6x-1 < 0$

$6x < 1$

$6x < 1/6$

$x \in (-\infty, 1/6)$

x^4

$$f''(2) = 2 + \frac{96}{2^4}$$

$$2 + \frac{96}{16}$$

$$2 + 6$$

$$8 > 0$$

$\therefore f$ has minimum value at $x = 2$

$$f(2) = 2^2 + \frac{16}{2^2}$$

$$15x^4 = 15x^2$$

$$x^2 = 1$$

$$x = \pm 1$$

$$f''(x) = -30x + 60x^3$$

$$f(1) = -30 + 60$$

$$30 > 0$$

f has minimum Value at $x = 1$

$$\therefore f(1) = 3 - 5(1)^3 + 3(1)^5$$

$$6 - 5$$

$$\underline{\underline{1}}$$

$f''(-1) = -30(-1) + 60(-1)^3$
$\quad 30 - 60$
$\quad -30 < 0$

$\therefore f$ has maximum value at $x = -1$

$f(-1) = 3 - 5(-1)^3 + 3(-1)^5$
$\quad 3 + 5 - 3$
$\quad \underline{\underline{5}}$

f has the maximum value 5 at $x = -1$ and has the m[illegible]
[illegible] value 1 & at $x = 1$

7) $f(x) = x^3 - 3x^2 + 1$

$f'(x) = 3x^2 - 6x$

$\rightarrow$ Consider
$f'(x) = 0$

$3x^2 - 6x = 0$

$3x(x-2) = 0$

$3x = 0 \quad x - 2 = 0$

$x = 0$ & $x = 2$

$f''(x) = 6x - 6$

$f''(0)\ 6(0) - 6$
$\quad -6 < 0$

f has maximum value at $x = 0$

5.

$f(0) = (0)^3 - 3(0)^2 + 1$

$\underline{\underline{1}}$

$f''(2) = 6(2) - 6$

$12 - 6$

$6 > 0$

$\therefore f$ has minimum value at $x = 2$

$f(2) = (2)^3 - 3(2)^2 + 1$

$8 - 3(4) + 1$

$9 - 12$

$\underline{\underline{-3}}$

f has ~~mini.~~ maximum 1 at $x = 0$ and
f has minimum value -3 at $x = 2$

$f(x) = 2x^3 - 3x^2 - 12x + 1$
$f'(x) = 6x^2 - 6x - 12$

Consider,
$f'(x) = 0$
$6x^2 - 6x - 12 = 0$
$\therefore 6(x^2 - x - 2) = 0$
$x^2 - x - 2 = 0$
$x(x+1) - 2(x+1) = 0$
$x = 2$ & $x = -1$

$\therefore f''(x) = 12x - 6$
$\therefore f''(2) = 12(2) - 6$
$= 24 - 6$
$18 > 0$

$\therefore f$ has minimum value at $x = 2$

$f(2) = 2(2)^3 - 3(2)^2 - 12(2) + 1$
$2(8) - 3(4) - 24 + 1$
$16 - 12 - 24 + 1$
-19

$f''(-1) = 12(-1) - 6$

$= -12 - 6$

$= -18 < 0$

$\therefore f$ has maximum value at $x = -1$

$\therefore f(-1) = 2(-1)^3 - 3(-1)^2 - 12(1) + 1$

$= -2 - 3 + 12 + 1$

$= \underline{8}$

$\therefore f$ has maximum value 8 at $x = -1$ &
f has minimum value -19 at $x = 2$

2. (i) $f(x) = x^3 - 3x^2 - 55x + 9.5$

$f'(x) = 3x^2 - 6x - 55$

By Newton's method,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 0 + \frac{9.5}{55}$$

$$x_1 = 0.1727$$

$$\therefore f(x_1) = (0.1727)^3 - 3(0.1727)^2 - 55(0.1727) + 9.5$$

$$0.0051 - 0.0895 - 9.4985 + 9.5$$

$$= -0.0829$$

$$f'(x_1) = 3(0.1727)^2 - 6(0.1727) - 55$$

$$0.0895 - 1.0362 - 55$$

$$= -55.9467$$

$$\therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$0.1727 - \frac{0.0829}{55.9467}$$

$$= 0.1712$$

$$f(x_2) = (0.1712)^3 - 3(0.1712)^2 - 55(0.1712) + 9.5$$
$$= 0.0050 - 0.0879 - 9.416 + 9.5$$
$$= 0.0011$$

$$f'(x_2) = 3(0.1712)^2 - 6(0.1712) - 55$$
$$0.0879 - 1.0272 - 55$$
$$= -55.9393$$

$$\therefore x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$0.1712 + \frac{0.0011}{55.9393}$$

$$\underline{\underline{0.1712}}$$

$\therefore$ The root of the equation is 0.1712

ii) $f(x) = x^3 - 4x - 9$
$f'(x) = 3x^2 - 4$

$$f(2) = 2^3 - 4(2) - 9$$
$$8 - 8 - 9$$
$$\underline{\underline{-9}}$$

$f(3) = 3^3 - 4(3) - 9$

$27 - 12 - 9$

$\underline{\underline{6}}$

Let $x_0 = 3$ be the initial approximation,

$\therefore$ By Newton's method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$3 - \frac{6}{23} = 2.7392$$

$f(x_1) = (2.7392)^3 - 4(2.7392) - 9$

$20.5528 - 10.9568 - 9$

0.596

$f'(x_1) = 3(2.7392)^2 - 4$

$22.5096 - 4$

18.5096

$$x_2 = x_1 - \frac{\cancel{f(x_1)}}{f'(x_1)}$$

$2.7392 - 0.596$

$f'(x_2) = 3(2.7071)^2 - 4.$

$21.9851 - 4$

17.9851

$\therefore x_3 = x_2 - \dfrac{f(x_2)}{f'(x_2)}$

$2.7071 - \dfrac{0.0102}{17.9851}$

$2.7071 - 0.0056$

2.7015

$f(x_3) = (2.7015)^3 - 4(2.7015) - 9$

$19.7158 - 10.806 - 9$

-0.0901

$f'(x_3) = 3(2.7015)^2 - 4$

$21.8943 - 4$

17.8943

$x_4 = 2.7015 + \dfrac{0.0901}{17.8943}$

$2.7015 + 0.0050$

2.7065

iii) $f(x) = x^3 - 1.8x^2 - 10x + 17$

$f'(x) = 3x^2 - 3.6x - 10$

$f(1) = (1)^3 - 1.8(1)^2 - 10(1) + 17$

$- 1.8 - 10 + 17$

$\underline{6.2}$

$f(2) = (2)^3 - 1.8(2)^2 - 10(2) + 17$

$\cancel{8 - 7.9}\ 8 - 7.2 - 20 + 17$

$- \underline{2.2}$

Let $x_0 = 2$ be initial approximation

By newton method,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = x_0 - \frac{f(x_2)}{f'(x_2)}$$

$$2 - \frac{2.2}{5.2}$$

$$2 - 0.4230$$

$$1.577$$

$f(x_1) = (1.577)^3 - 1.8(1.577)^2 - 10(1.577) + 17$

$3.9219 - 4.4764 - 15.77 + 17$

0.6755

$f'(x_1) = 3(1.577)^2 - 3.6(1.577) - 10$

$7.4608 - 5.6772 - 10$

$- 8.2164$

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$\therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$1.577 + \frac{0.6755}{8.2164}$

$1.577 + 0.0822$

$\underline{\underline{1.6592}}$

$f(x_2) = (1.6592)^3 - 1.8(1.6592)^2 - 10(1.6592) + 17$

$4.5677 - 4.9553 - 16.592 + 17$

$\underline{\underline{0.0204}}$

$f'(x_2) = 3(1.6592)^2 - 3.6(1.6592) - 10$

$8.2588 - 5.97312 - 10$

-7.7143

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$1.659 + \frac{0.0204}{7.7143}$

$1.6592 + 0.0026$

1.6618

$f(x_3) = (1.6618)^3 - 1.8(1.6618)^2 - 10(1.6618) + 17$

$4.5892 - 4.9708 - 16.618 + 17$

0.0004

$f'(x_3) = (1.6618)^3 - 1.8(1.6618)^2 - 10(1.6618) + 17$
$9.5892 - 4.9708 - 16.618 + 17$
0.0004

$f'(x_3) = 3(1.6618)^2 - 3.6(1.6618) - 10$
$8.2847 - 5.9824 - 10$
-7.6977

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$$1.6618 + \frac{0.0004}{7.6977}$$

1.6618

The root of equation is 1.6618

AB
Oppraksh

Practical = 5

① $\int \frac{dx}{\sqrt{x^2+2x-3}}$

$I = \int \frac{dx}{\sqrt{x^2+2x-3}}$

$\int \frac{dx}{\sqrt{x^2+2x+1-4}}$

$\int \frac{dx}{\sqrt{(x+1)^2-(2)^2}}$

Comparing with $\int \frac{dx}{\sqrt{x^2-a^2}}$; $x^2 = (x+1)^2$
$a^2 = (2)^2$

$I = \log|x + \sqrt{x^2 - a^2}| + c$

$\int(2x^2 - 3\sin x + 5\sqrt{x})\,dx$

(i) $\int(2x^2 - 3\sin x + 5\sqrt{x})\,dx$

$I = \int(2x^2 - 3\sin x + 5\sqrt{x})\,dx$

$2\int x^2\,dx - 3\int \sin x\,dx + 5\int\sqrt{x}\,dx$

$\frac{2}{3}x^3 + 3\cos x + 5 \times \frac{2}{3} x^{3/2} + C$

$\frac{2}{3}x^3 + 3\cos x + \frac{10x^{3/2}}{3} + C$

(ii) $\int \frac{x^3 + 3x + 4}{\sqrt{x}}\,dx$

$\int\left(\frac{x^3}{\sqrt{x}} + \frac{3x}{\sqrt{x}} + \frac{4}{\sqrt{x}}\right)dx$

$\int\left(\frac{x^3}{x^{1/2}} + \frac{3x}{x^{1/2}} + \frac{4}{x^{1/2}}\right)dx$

$\int(x^3 \cdot x^{-1/2} + 3x \cdot x^{-1/2} + 4x^{-1/2})\,dx$

$\int x^{5/2}\,dx + 3\int x^{1/2}\,dx + 4\int x^{-1/2}\,dx$

$\frac{2}{7}x^{7/2} + 3 \cdot \frac{2}{3} x^{3/2} + 4x^{1/2} \cdot 2 + C$

$\frac{2}{7}x^{7/2} + 2x^{3/2} + 8\sqrt{x} + C$

68

(5) $\int t^7 \sin(2t^4)\, dt$

$I = \int t^7 \sin(2t^4)\, dt$

let, $t^4 = x$

$4t^3 dt = dx$

$I = \frac{1}{4}\int 4t^3\, t^4 \sin(2t^4)\, dt$

$\frac{1}{4}\int x \cdot \sin(2x)\, dx$

$\frac{1}{4}\left[x \int \sin 2x - \int\left[\int \sin 2x \cdot \frac{d}{dx}(x)\right]\right]$

$\frac{1}{4}\left[-\frac{x\cos 2x}{2} + \frac{1}{2}\int \cos 2x \cdot 1\right]$

$\frac{1}{4}\left[-\frac{x\cos 2x}{2} + \frac{1}{4}\sin 2x\right] + C$

$= -\frac{1}{8} x\cos 2x + \frac{1}{16}\sin 2x + C$

Resubstituting $x = t^4$

$\therefore I = -\frac{1}{8} t^4 \cos(2t^4) + \frac{1}{16}\sin(2t^4) + C$

Q

$\int \sqrt{x}\,(x^2-1)\,dx$

$I = \int \sqrt{x}\,(x^2-1)\,dx$

$\int (\sqrt{x}\cdot x^2 - \sqrt{x})\,dx$

$\int (x^{5/2} - \sqrt{x})\,dx$

$\int x^{5/2}\,dx - \int \sqrt{x}\,dx$

$\frac{2}{7}x^{3/2} - \frac{2}{3}x^{3/2} + C$

6

$\int \frac{1}{x^3}\sin\left(\frac{1}{x^2}\right)dx$

$I = \int \frac{1}{x^3}\sin\left(\frac{1}{x^2}\right)dx$

Let

$\frac{1}{x^2} = t$

$x^{-2} = t$

$-\frac{2}{x^3}\,dx = dt$

$I = \frac{1}{-2}\int -\frac{2}{x^3}\sin\left(\frac{1}{x^2}\right)dx$

$-\frac{1}{2}\int \sin t\,dt$

$-\frac{1}{2}(-\cos t)+C$

$\frac{1}{2}\cos t+C$

Put Resubstituting $t=\frac{1}{x^2}$

$\therefore I=\frac{1}{2}\cos\left(\frac{1}{x^2}\right)+C$

(8) $\int \frac{\cos x}{\sqrt[3]{\sin^2 x}}$

$I=\int \frac{\cos x}{\sqrt[3]{\sin^2 x}}\,dx$

Let $\sin x=t$

$\cos x\,dx=dt$

$\therefore I=\int \frac{dt}{\sqrt[3]{t^2}}$

$I=\int \frac{dt}{t^{2/3}}$

$\int t^{-2/3}\,dt$

$3t^{1/3}+C$

$2(2-x)^{1/2} + c$

$2\sqrt{2-x} + c$

$\int e^{\cos^2 x} \sin 2x \, dx$

$I = \int e^{\cos^2 x} \sin 2x \, dx$

Let $\cos^2 x = t$

$-2\cos x \sin x \, dx = dt$

$-\sin 2x \, dx = dt$

$I = -\int -\sin 2x \, e^{\cos^2 x} dx$

$= -\int e^{t} dt$

$= -e^{t} + c$

Resubstituting $t = \cos^2 x$

$\cancel{I = e}\cos^2$ $I = e^{\cos^2 x} + c$

Practical 6

$x = t - \sin t$

$L = \int_a^b \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}\, dx$

$\frac{dy}{dt} = \sin t \qquad \frac{dx}{dt} = 1 - \cos t$

$L = \int_0^{2\pi} \sqrt{(\sin t)^2 + (1-\cos t)^2}\, dt$

$\int_0^{2\pi} \sqrt{\sin^2 t + 1 + \cos^2 t - 2\cos t}\, dt$

$\int_0^{2\pi} \sqrt{2 - 2\cos}\, dt$

$\sqrt{2} \int_0^{2\pi} \sqrt{1 - \cos t}\, dt$

$\sqrt{2} \int_0^{2\pi} \sqrt{2\sin^2 (t/2)}\, dt$

$2 \times 2 \left[-\cos\left(\frac{t}{2}\right) \right]_0^{2\pi}$

$-4\{-1-1\}$

$= 8$

13

(i) $y = \sqrt{4-x^2}$ $\qquad x \in (-2, 2)$

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$\frac{dy}{dx} = \frac{1 - (-2x)}{2\sqrt{4-x}}$$

$$\frac{-x}{\sqrt{4-x}}$$

$$\therefore \quad L = \int_{-2}^{2} 1 + \left(\frac{-x}{\sqrt{4-x^2}}\right)$$

$$\int_{-2}^{2} \sqrt{\frac{4 - x^2 + x^2}{4 - x^2}}\, dx$$

$$\int_{-}^{2} \sqrt{\frac{4}{4 - x^2}}\, dx$$

$y = x^{3/2}$

$L = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$

$\frac{dy}{dx} = \frac{3}{2}\sqrt{x}$

$L = \int_0^4 \sqrt{1+\frac{9}{4}x}\, dx$

$\frac{1}{2}\int_0^4 \sqrt{4+9x}\, dx$

$\frac{1}{2}\left[\frac{(4+9x)^{3/2}}{3/2} \times \frac{1}{9}\right]_0^4$

$\frac{1}{2} \times \frac{2}{3} \times \frac{1}{9}\left[(4+9x)^{3/2}\right]_0^4$

$\frac{1}{27}\left[40^{3/2} - 8\right]$

$x = 3\sin t \qquad y = 3\cos t \qquad t \in [0, 2\pi]$

$L = \int_a^b \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}\, dt$

$\frac{dy}{dt} = -3\sin t \qquad \frac{dx}{dt} = 3\cos t$

$L = \int_0^{2\pi} \sqrt{(-3\sin^2 t)^2 + (3\cos t)^2}\, dt$

$L = 3\int_0^{2\pi} \sqrt{\sin^2 t + \cos^2 t}\, dx$

$3\int_0^{2\pi} \sqrt{1}\, dt$

$3\left[t\right]_0^{2\pi}$

6π

a

(5) $x = \frac{1}{6}y^3 + \frac{1}{2y}$

$L = \int_a^b \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$

$\frac{dx}{dy} = \frac{\cancel{3}\,1}{\cancel{6}\,2}y^2 - \frac{1}{2y^2}$

$L = \int_1^2 \sqrt{1 + \left(\frac{1}{2}y^2 - \frac{1}{2y^2}\right)^2}\, dy$

$\int_1^2 \sqrt{1 + \left(\frac{y^4 - 1}{2y^2}\right)^2}\, dy$

$= \int_1^2 \sqrt{\frac{(y^4-1)^2 + 4 \times y^4 \times 1}{4y^2}}\, dy$

$\int_1^2 \sqrt{\frac{(y^4+1)^2}{(2y^2)^2}}\, dy$

$\int_1^2 \frac{y^4+1}{2y^2}\, dy$

$\int_1^2 \frac{y^2}{2}\, dy + \int_1^2 \frac{1}{2y^2}\, dy$

$\frac{1}{2}\left[\frac{y^3}{3}\right]_1^2 - \frac{1}{2}\left[\frac{1}{y}\right]_1^2$

$\frac{1}{2}\left[\frac{8}{3} - \frac{1}{3}\right] - \frac{1}{2}\left[\frac{1}{2} - 1\right]$

$\frac{7}{6} - \frac{1}{4}$

$\frac{14-3}{12}$

$11/12$

$\int_0^2 e^{x^2} dx$ with $n=4$

$a=0, b=2, n=4$

$n = \frac{b-a}{n} = \frac{2-0}{4} = 1/2$

x	0	0.5	1	1.5	2
y	1	1.2840	2.7182	9.4877	54.5981

∴ By Simpson's Rule

$\int_0^2 e^{x^2} dx = \frac{0.5}{3}\left[(1+54.5981)+4(1.2840+9.4877)+2(2.\right.$

17.3535

$\int_0^4 x^2 dx$ with $n=4$

$a=0, b=4, n=4$

$h = \frac{b-a}{n} = \frac{4-0}{4} = 1$

x	0	1	2	3	4
y	0	1	4	9	16

By Simpson's Rule.

$\int_0^4 x^2 dx = \frac{1}{3}\left[(0+16)+4(1+9)+2(4)\right]$

$\frac{1}{3}(16+40+8)$

$\frac{64}{3}$

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$\textcircled{3}$ $\int_0^{\pi/3} \sqrt{\sin x}\, dx$ with $n = 6$

$a = 0,\ b = \pi/3,\ n = 6$

$h = \frac{\pi/3 - 0}{6} = \pi/18$

x	0	$\pi/18$	$2\pi/18$	$3\pi/18$	$4\pi/18$	$5\pi/18$	$6\pi/18$
y	0	0.4167	0.5848	0.7071	0.8017	0.8752	0.9306

By Simpson's Rule.

$\int_0^{\pi/3} \sqrt{\sin x} = \frac{h}{3}\left[(y_0 + y_6) + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)\right]$

$\frac{\pi}{54}\left[(0 + 0.9306) + 4(0.4167 + 0.707 + 0.8752) + 2(0.5848 + 0.8017)\right]$

$= 0.6806$

$\boxed{\int_0^{\pi/3} \sqrt{\sin x} = 0.6806}$

Practical 7

Solve ~~the~~ the following differential eqn

1) $x\frac{dy}{dx} + y = e^x$

$$\frac{dy}{dx} + \frac{1}{x}y = \cancel{e^x}\ \frac{e^x}{x}$$

Comparing with $\frac{dy}{dx} + P(x)y = Q(x)$

$P(x) = \frac{1}{x}$; $Q(x) = \frac{e^x}{x}$

$$I.F = e^{\int 1/x\, dx}$$

$$e^{\log x}$$

$$I.F = x$$

$$y(I.F) = \int Q(x)(I.F)dx$$

$$y(x) = \int \frac{e^x}{x} \cdot x\, dx$$

$$yx = \int e^x dx$$

$$xy = e^x + c$$

2) $e^x \frac{dy}{dx} + 2e^x y + 1$

$$e^x\left(dy/dx + 2y\right) = 1$$

$$\frac{dy}{dx} + 2y = \frac{1}{e^x}$$

Comparing with $\frac{dy}{dx} + P(x)y = Q(x)$

i)

$\therefore P(x) = 2 \;,\; Q(x) = 1/e^{x}$

$I.F = e^{\int 2 dx}$

e^{2x}

$y\,(I.F) = \int Q(x)\,(I.F)\,dx$

$y e^{2x} = \int \frac{1}{e^{x}} e^{2x}\,dx$

$y e^{2x} = \int e^{-x} \cdot e^{2x}\,dx$

$y e^{2x} = \int e^{x}\,dx$

$y e^{2x} = e^{x} + C$

ii)

$x \frac{dy}{dx} = \frac{\cos x}{x} - 2y$

$x \frac{dy}{dx} + 2y = \frac{\cos x}{x}$

$\frac{dy}{dx} + \left(\frac{2}{x}\right) y = \frac{\cos x}{x^2}$

Comparing with $\frac{dy}{dx} + P(x) y = Q(x)$

$\therefore P(x) = \frac{2}{x} \;,\; Q(x) = \frac{\cos x}{x^2}$

$I.F = e^{\int 2/x\, dx}$

$e^{2\log x}$

$I.F = x^2$

$y\,(I.F) = \int Q(x)\,(I.F)\,dx$

$y\,(x^2) = \int \frac{\cos x}{x^2} (x^2)\,dx$

$x^2 y = \sin x + C$

(4) $x\frac{dy}{dx} + 3y = \frac{x \sin x}{x^2}$

$\frac{dy}{dx} + \left(\frac{3}{x}\right)y = \frac{\sin x}{x^3}$

Comparing with $\frac{dy}{dx} + P(x)y = Q(x)$

$P(x) = 3x^{-1}$ $\quad Q(x)$ $\frac{\sin x}{x^3}$

$I.F = e^{\int 3/x \, dx}$

$e^{3\log x}$

$e^{\log x^3}$

x^3

$\therefore y(I.F) = \int Q(x)(I.F)\,dx$

$y(x^3) = \int \frac{\sin x}{\cancel{x^3}} \cancel{(x^3)}\,dx$

$x^3 y = -\cos x + C$

(5) $e^{2x}\frac{dy}{dx} + 2e^{2x}y = 2x$

$e^{2x}\left(\frac{dy}{dx} + 2y\right) = 2x$

$\frac{dy}{dx} + 2y = \frac{2x}{e^{2x}}$

Comparing with $\frac{dy}{dx} + P(x)y = Q(x)$

$\therefore P(x) = 2$ $\qquad Q(x) = \frac{2x}{e^{2x}}$

$I.F = e^{\int 2\,dx}$

e^{2x}

$\therefore y(I.F) = \int Q(x)(I.F)\,dx$

$$y\cdot(e^{2x}) = \int \frac{2x}{e^{2x}}(e^{2x})\,dx$$

$$ye^{2x} = \frac{2x^2}{2} + C$$

$$ye^{2x} = x^2 + C$$

(6) $\sec^2 x \tan y\,dx + \sec^2 y \tan x\,dy = 0$

$\rightarrow$ $\sec^2 x \tan y\,dx = -\sec^2 y \tan x\,dy$

$$\frac{\sec^2 x}{\tan x}\,dx = \frac{-\sec^2 y}{\tan y}\,dy$$

$$\therefore \int \frac{\sec^2}{\tan x}\,dx = -\int \frac{\sec^2 y}{\tan y}\,dy$$

$$\therefore \log(\tan x) = -\log|\tan y| + C$$

$$\log(\tan x) + \log(\tan y) = C$$

$$\log|\tan x - \tan y| = C$$

$$\tan x \cdot \tan y = e^{x}$$

$\frac{dy}{dx} = \sin^2(x-y+1)$

Put $x-y+1 = v$

$1 - \frac{dy}{dx} = \frac{dv}{dx}$

$\frac{dy}{dx} = 1 - \frac{dy}{dx}$

$1 - \cancel{\frac{dy}{}}\frac{dv}{dx} = \sin^2 v$

$1 - \sin^2 v = \frac{dv}{dx}$

$dx = \frac{dv}{1-\sin^2 v}$

$\int dx = \int \sec^2 v \, dv$

$x = \tan v + C$

But $v = x+y-1$

$x = \tan(x+y-1) + C$

$\frac{dy}{dx} = \frac{2x+3y-1}{6x+9y+C}$

$\frac{dy}{dx} = \frac{2x+3y-1}{3(2x+3y+2)}$

Put $2x+3y = v$

$2 + \frac{3\,dy}{dx} = \frac{dv}{dx}$

$\frac{dy}{dx} = \frac{1}{3}\left(\frac{dv}{dx} - 2\right)$

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$$\frac{1}{3}\left(\frac{dv}{dx}-2\right)=\frac{v-1}{3(v+2)}$$

$$\frac{dv}{dx}=\frac{v-1}{v+2}+2$$

$$\frac{dv}{dx}=\frac{v-1+2v+4}{v+2}$$

$$\frac{dv}{dx}=\frac{3v+3}{v+2}$$

$$\frac{v+2\,dv}{3(v+1)}=dx$$

$$\frac{1}{3}\int\left(\frac{v+1+1}{v+1}\right)dv=\int dx$$

$$\frac{1}{3}\int 1+\frac{1}{v+1}\,dv=\int dx$$

$$\frac{1}{3}\left(v+\log(v+1)\right)=x+c$$

But $v=2x+3y$

$\therefore\ 2x+3y+\log|2x+3y+1|=3x+c$

$\therefore\ 3y=x-\log|2x+3y+1|+c$

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Practical no 8

Topic : Euler's method

$$\frac{dy}{dx} = y + e^x - 2$$

$f(x, y) = y + e^x - 2$ $y_0 = 2$, $x_0 = 0$ $h = 0.5$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	2	1	2.5
1	0.5	2.5	2.487	3.7435
2	1	3.5743	4.2925	5.3615

$y_{n+1} = y_n + h f(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
3	1.5	5.3615	7.8431	9.28315
4	2	9.2831		

$\therefore$ By Euler's formula.

$y(2) = 9.2831$

2. $\frac{dy}{dx} = 1 + y^2$

$f(x,y) = 1 + y^2$, $y_0 = 0$ $x_0 = 0$ $h = 0.2$

Using Euler's iteration formula,

$y_{n+1} = y_n + h f(x_n, y_n)$

n	x_n	y_n	1
0	0.0	0	1.04
1	0.2	0.2	1.1665
2	0.4	0.408	1.4113
3	0.86	0.6413	1.
4	0.8	0.9236	
5	1	1.2942	

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	0	1	0.2
1	0.2	0.2	1.04	0.408
2	0.4	0.408	1.1665	0.6413
3	0.6	0.6413	1.4113	0.9236
4	0.8	0.9236	1.8530	1.2942
5	1	1.2942		

$\therefore$ By Euler's formula,

$y(1) = 1.2942$

$\frac{dy}{dx} = \sqrt{\frac{x}{y}}$ $y(0) = 1$ $x_0 = 0$, $h = 0.2$

using Euler's iteration formula.

$y_{n+1} = y_n + h\, f(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	1	0	1
1	0.2	1	0.4472	1.0895
2	0.4	1.0895	0.6059	1.1211
3	0.6	1.1211	0.7316	1.1463
4	0.8	1.1463	0.8354	1.16708
5	1	1.1671		

$\therefore$ By Euler's for.

$y(1) = 1.1671$

$\frac{dy}{dx} = 3x^2 + 1$ $y_0 = 2$, $x_0 = 1$, $h = 0$

for $h = 0.5$

Using Euler's iteration formula,

$y_{n+1} = y_n + h\, f(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	4
1	1.5	4	49	28.5
2	2	28.5		

$\therefore$ By Euler's formula,

$y(2) = 28.5$

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$\therefore$

for $h = 0.25$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	3
1	1.25	3	5.6875	4.4219
2	1.5	4.4219	7.75	6.3594
3	1.75	6.3594	10.1815	8.9048
4	2	8.9048		

By Euler's formula

$y(2) = 8.9048$

(5) $\frac{dy}{dx} = \sqrt{xy} + 2 \quad y_0 = 1, \ x_0 = 1, \ h = 0.2$

Using Euler's iteration formula,

$y_{n+1} = y_n + h f(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	1	3	1.6
1	1.2	1.6		

$\therefore$ By Euler's formula,

$y(1.2) = 1.6$

9/10

three

$(x,y)\to(4,-1)$ $\quad 2y^2+5$

At $(4,-1)$, denominator $\neq 0$

$\therefore$ By applying limit

$$\frac{(4)-3(-1)+(-1)^2-1}{-4(-1)+5}$$

$$\frac{-64+3+1-1}{4+5} = \frac{-61}{9}$$

$$\frac{(y+1)(x^2+y^2-4x)}{x+3y}$$

$(x,y)\to(2,0)$

At $(2,0)$ denominator $\neq 0$

$\therefore$ By applying limit

$$\frac{(0+1)((2)^2+0-4(2))}{2+0}$$

$$\frac{1(4+(-8))}{2}$$

$$= -2$$

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(5) $\lim_{(x,y,z)\to(1,1,1)} \dfrac{x^2-y^2z^2}{x^3-x^2yz}$

At $(1,1,1)$, Denominator $= 0$

$$\lim_{(x,y,z)\to(1,1,1)} \frac{x^2-y^2z^2}{x^3-x^2yz}$$

$$\lim_{(x,y,z)\to(1,1,1)} \frac{\cancel{(x-yz)}(x+yz)}{x^2\cancel{(x-yz)}}$$

$$\lim_{(x,y,z)\to(1,1,1)} \frac{x+yz}{x^2}$$

on Applying limit

$$\frac{1+1(1)}{(1)^2} = \underline{\underline{2}}$$

$f(x,y) = xy\,e^{x^2+y^2}$

$f_x = \frac{\partial}{\partial x}(f(x,y))$

$\frac{\partial}{\partial x}(xye^{x^2+y^2})$

$ye^{x^2+y^2}(2x)$

$\therefore f_x = \cancel{2x}\ 2xye^{x^2+y^2}$

$f_y = \frac{\partial}{\partial y}(f(x,y))$

$\frac{\partial}{\partial y}(xye^{x^2+y^2})$

$xe^{x^2+y^2}(2y) \quad \therefore f_y = 2ye^{x^2+y^2}$

$f(x,y) = e^x \cos y$

$f_x = \frac{\partial}{\partial x}(f(x,y))$

$\frac{\partial}{\partial x}(e^x \cos y)$

$\therefore f_x = e^x \cos y$

$f_y = \frac{\partial}{\partial y}(f(x,y))$

$\frac{\partial}{\partial y}(e^x \cos y)$

$f_y = -e^x \sin y$

③ $f(x,y) = x^3y^2 - 3x^2y + y^3 + 1$

$f_x = \frac{\partial}{\partial x}(f(x,y))$

$\frac{\partial}{\partial x}(x^3y^2 - 3x^2y + y^3 + 1)$

$f_x = 3x^2y^2 - 6xy$

$f_y = \frac{\partial}{\partial y}(f(x,y))$

$\frac{\partial}{\partial y}(x^3y^2 - 3x^2y + y^3 + 1)$

$f_y = 2x^3y - 3x^2 + 3y^2$

3.

① $f(x,y) = \frac{2x}{1+y^2}$

$f_x = \frac{\partial}{\partial x}\left(\frac{2x}{1+y^2}\right)$

$$\frac{1+y^2 \frac{\partial}{\partial x}(2x) - 2x \frac{\partial}{\partial x}(1+y^2)}{(1+y^2)^2}$$

$$\frac{2+2y^2 - 0}{(1+y^2)^2}$$

$$\frac{2\cancel{(1+y^2)}}{\cancel{(1+y^2)}(1+y^2)} = \frac{2}{1+y^2}$$

at $(0,0) = \frac{2}{1+0}$

$\underline{\underline{2}}$

$$f_y = \frac{\partial}{\partial y}\left(\frac{2x}{1+y^2}\right)$$

$$\frac{1+y^2 \frac{\partial}{\partial x}(2x) - 2x \frac{\partial}{\partial x}(1+y^2)}{(1+y^2)^2}$$

$$\frac{1+y^2(0) - 2x(2y)}{(1+y^2)^2}$$

$$\frac{-4xy}{(1+y^2)^2}$$

at $(0,0)$

$$= \frac{-4(0)(0)}{(1+0)^2} = 0$$

$$f(x,y) = \frac{y^2 - xy}{x^2}$$

$$f_x = \frac{x^2 \frac{d}{\partial x}(y^2 - xy) - (y^2 - xy)\frac{\partial}{\partial x}(x^2)}{(x^2)^2}$$

$$\frac{x^2(-y) - (y^2 - xy)(2x)}{x^4}$$

$$\frac{-x^2y - 2x(y^2 - xy)}{x^4}$$

$$f_y = \frac{2y - x}{x^2}$$

$$f_{xx} = \frac{\partial}{\partial x}\left(-x^2y - 2x(y^2 - xy)\right)$$

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$$\frac{x^4\left(\frac{\partial}{\partial x}(-x^2y-2xy+2x^2y)\right)-(-x^2y-2xy+2x^2y)\frac{\partial}{\partial x}(x^4)}{(x^4)^2}$$

$$\frac{x^4(-2xy-2y^2+4xy)-4x^3(-x^2y-2xy+2x^2y)}{x^6} \quad — (1)$$

$$f_{yy}=\frac{\partial}{\partial y}\left(\frac{2y-x}{x^2}\right)$$

$$\frac{2-0}{x^2}=\frac{2}{x^2} \quad — (2)$$

$$f_{xy}=\frac{\partial}{\partial y}\left(\frac{-x^2y-2xy^2+2x^2y}{x^4}\right)$$

$$\frac{-x^2-4xy+2x^2}{x^4} \quad — (3)$$

$$f_{yx}=\frac{\partial}{\partial x}\left(\frac{2y-x}{x^2}\right)$$

$$\frac{x^2\frac{\partial}{\partial x}(2y-x)-(2y-x)\frac{\partial}{\partial x}(x^2)}{(x^2)^2}$$

$$\frac{-x^2-4xy+2x^2}{x^4} \quad — (4)$$

from 3 & 4

$f_{xy}=f_{yx}$

$f(x,y) = x^3 + 3x^2y^2 - \log(x^2+1)$

$(x^3+3x^2y^2 - \log(x^2+1))$

$f_x = 3x^2 + 6xy^2 - \frac{2x}{x^2+1}$

$f_y = \frac{\partial}{\partial y}(x^3+3x^2y^2-\log(x^2+1))$

$0 + 6x^2y - 0$

$6x^2y$

$f_{xx} = 6x+6y^2 - \frac{(x^2+1)\frac{\partial(2x)}{\partial x} - 2x\frac{\partial(x^2+1)}{\partial x}}{(x^2+1)^2}$

$6x+6y^2 - \left(\frac{2(x^2+1)-4x^2}{(x^2+1)^2}\right)$ —— (1)

$f_{yy} = \frac{\partial}{\partial y}(6x^2y)$

$6x^2$ —— (2)

$f_{xy} = \frac{\partial}{\partial y}\left(3x^2+6xy^2-\frac{2x}{x^2+1}\right)$

$0+12xy-0$

$12xy$ —— (3)

$f_{yx} = \frac{\partial}{\partial x}(6x^2y)$

$12xy$ —— (4)

from 3 & 4

$f_{xy} = f_{yx}$

5.

(1) $f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$

$\rightarrow f(1,1) = \sqrt{(1)^2+(1)^2} = \sqrt{2}$

$f_x = \frac{1}{2\sqrt{x^2+y^2}}(2x)$ $\qquad f_y = \frac{1}{2\sqrt{x^2+y^2}}(2y)$

$\frac{x}{\sqrt{x^2+y^2}}$ $\qquad = \frac{y}{\sqrt{x^2+y^2}}$

f_x at $(1,1) = \frac{1}{\sqrt{2}}$ $\qquad f_y$ at $(1,1) = \frac{1}{\sqrt{2}}$

$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$\sqrt{2} + \frac{1}{\sqrt{2}}(x-1) + \frac{1}{\sqrt{2}}(y-1)$

$\sqrt{2} + \frac{1}{\sqrt{2}}(x-1+y-1)$

$\sqrt{2} + \frac{1}{\sqrt{2}}(x+y-2)$

$\frac{x+y}{\sqrt{2}}$

(3) $f(x,y) = \log x + \log y$

$f(1,1) = \log(1) + \log(1) = 0$

$f_x = 1/x + 0$ $\qquad$ $f_y = 0 + 1/y$

f_x at $(1,1) = 1$ $\qquad$ f_y at $(1,1) = 1$

$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-$

$0 + 1(x-1) + 1(y-1)$

$x - 1 + y - 1$

$x + y - 2$

Practical 10

1. $f(x,y) = x + 2y - 3$ $\quad a = (1,-1)$, $u = 3i - j$

→ Here

$u = 3i - j$ not a unit vector

$\bar{u} = 3\bar{i} - \bar{j}$

$|u| = \sqrt{10}$

unit vector along u is $\frac{\bar{u}}{|u|} = \frac{1}{\sqrt{10}}(3\bar{i} - \bar{j})$

$\frac{1}{\sqrt{10}}(3, -1)$

$\left(\frac{3}{\sqrt{10}} + \frac{-1}{\sqrt{10}}\right)$

Now

$f(a+hu) = \left(f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)\right)$

$f\left(1 + \frac{3h}{\sqrt{10}}, -1 - \frac{h}{\sqrt{10}}\right)$

$1 + \frac{3h}{\sqrt{10}} + 2\left(-1 - \frac{h}{\sqrt{10}}\right) - 3$

$1 - 2 - 3 + \frac{3h}{\sqrt{10}} - \frac{2h}{\sqrt{10}}$

$-4 + \frac{h}{\sqrt{10}}$

$\therefore D_u f(a) = \lim_{h\to 0} \frac{f(a+hu) - f(a)}{h}$

$\lim_{h\to 0} \frac{-4 + \frac{h}{\sqrt{10}} - (-4)}{h}$

$\lim_{h\to 0} \frac{h}{h\sqrt{10}} = \frac{1}{\sqrt{10}}$

$f(x,y) = y^2 - 4x + 1$, $a = (3,4)$, $u = i + 5j$

$\Rightarrow$ Here,

$u = i + 5j$ is not unit vector

$\bar{u} = \bar{i} + 5\bar{j}$

$|\bar{u}| = \sqrt{26}$

$\therefore$ unit vector along u is $\frac{\bar{u}}{|\bar{u}|} = \frac{1}{\sqrt{26}}(\bar{i} + 5\bar{j})$

$\frac{1}{\sqrt{26}}(1,5)$

$\left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$

Now

$f(a+hu) = f\left((3,4) + h\left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)\right)$

$f\left(3 + \frac{h}{\sqrt{26}}, 4 + \frac{5h}{\sqrt{26}}\right)$

$\left(4 + \frac{5h}{\sqrt{26}}\right)^2 - 4\left(3 + \frac{h}{\sqrt{26}}\right) + 1$

$= 16 + \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - 12 - \frac{4h}{\sqrt{26}} + 1$

$= \frac{26h^2}{26} + \frac{36h}{\sqrt{26}} + 5$

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(a) $\lim_{h\to 0} \frac{f(a+hu)-f(a)}{h}$

$$\lim_{h\to 0} \frac{\frac{25h^2}{26}+\frac{36h}{\sqrt{26}}+5-5}{h}$$

$$\lim_{h\to 0} \frac{\frac{25h^2}{26}+\frac{36h}{\sqrt{26}}}{h}$$

$$\lim_{h\to 0} h\left(\frac{25h}{26}+\frac{36}{\sqrt{26}}\right)$$

$$\frac{25(0)}{26}+\frac{36}{\sqrt{26}} = \frac{36}{\sqrt{26}}$$

$f(x,y) = 2x+3y \quad a=(1,2), \; u = 3i+4j$

Here,

$u = 3i+4j$ is not a unit vector

$\bar{u} = 3\bar{i}+4\bar{j}$

$|\bar{u}| = \sqrt{25} = 5$

$\therefore$ Unit vector along $u = \frac{\bar{u}}{|\bar{u}|} = \frac{1}{5}(3i+4j)$

$\frac{1}{5}(3,4)$

$(3/5, 4/5)$

Now,

$f(a+hu) = f\left((1,2) + h\left(\frac{3}{5} + \frac{4}{5}\right)\right)$

$f\left(1 + \frac{3h}{5},\ 2 + \frac{2h}{5}\right)$

$2\left(1 + \frac{3h}{5}\right) + 3\left(2 + \frac{4h}{5}\right)$

$2 + \frac{6h}{5} + 6 + \frac{12h}{5}$

$8 + \frac{18h}{5}$

$D_u f = \lim_{h \to 0} \frac{f(a+hu) - f(a)}{h}$

$\lim_{h \to 0} \frac{\frac{18h}{5} - 8}{h}$

$\lim_{h \to 0} \frac{18h}{5h} = \frac{18}{5}$

$f(x,y) = x^y + y^x \qquad a = (1,1)$

$f_x = y(x^{y-1}) + y^x \log y$

$f_y = x(y^{x-1}) + \cancel{x^y} \log x$

$\nabla f(x,y) = (f_x, f_y)$

$= (y x^{y-1} + y^x \log y,\ x y^{x-1} + \cancel{30}\, x^y \log x)$

$\nabla f(x,y)$ at $(1,1)$

$(1(1)^0 + 1^1(\log(1)),\ 1(1)^{(1-1)} + 1^1 \log 1)$

$(1,1)$

(1) $f(x,y) = (\tan^{-1}x) \cdot y^2 \quad a = (1,-1)$

$f_x = y^2 \left(\frac{1}{1+x^2}\right) = \frac{y^2}{1+x^2}$

$f_y = 2y \tan^{-1}x$

$\nabla f(x,y) = (f_x, f_y)$

$\left(\frac{y^2}{1+x^2}, 2y \tan^{-1}x\right)$

$\nabla f(x,y)$ at $(1,-1)$

$\left(\frac{(-1)^2}{1+(1)^2}, 2(-1)\tan^{-1}(1)\right)$

$\left(\frac{1}{2}, -\frac{2\pi}{4}\right)$

$\left(\frac{1}{2}, -\pi/2\right)$

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$f_x(x - x_0) + f_y(y - y_0) = 0$

$2(x-1) + 1(y-0) = 0$

$2x - 2 + y = 0$

$2x + y - 2 = 0$ → Equation of Tangent.

Now,

for equation of Normal.

$bx + ay + d = 0$

$x + 2y + d = 0$

$(1) + 2(0) + d = 0$ At $(1,0)$

$1 + d = 0$

$d = -1$

$\therefore x + 2y - 1 = 0$ → Equation of normal

(2) $x^2 + y^2 - 2x + 3y + 2 = 0$ at $(2,-2)$

$f(x,y) = x^2 + y^2 - 2x + 3y + 2$

$f_x = 2x + 0 - 2 + 0 + 0$

$2x - 2$

f_x at $(2,-2) = 2(2) - 2$

$= \underline{\underline{2}}$

$f_y = 0 + 2y - 0 + 3 + 0$

$2y + 3$

f_y at $(2,-2) = 2(-2) + 3$

$= \underline{\underline{-1}}$

Equation of tangent

$f_x(x - x_0) + f_y(y - y_0) = 0$

$2(x-2) + (-1)(y+2) = 0$

$2x - 4 - y - 2 = 0$

$2x - y - 6 = 0$ — Equation of Tangent

$bx + ay + d = 0$

$-x + 2y + d = 0$

$-(2) + 2(-2) + d = 0 \quad$ at $(2, -2)$

$-2 - 4 + d = 0$

$d = +6$

$-x + 2y - 6 = 0 \longrightarrow$ Equation of Normal.

4.

(i) $x^2 - 2yz + 3y + xz = 7 \quad$ at $(2, 1, 0)$

$f(x, y, z) = x^2 - 2yz + 3y + xz - 7$

$f_x = 2x - 0 + 0 + z = 0 \quad \therefore f_x$ at $(2,1,0) = 2(2) + 0$

$2x + z \qquad \underline{\underline{4}}$

$f_y = -2z + 3 + 0 - 0 \qquad f_y$ at $(2,1,0) = -2$

$= -2z + 3$

$f_z = 0 - 2y + 0 + x = 0 \qquad f_z$ at $(2,1,0) = -2($

$-2y + x \qquad \underline{\underline{0}}$

Equation of tangent.

$f_x(x - x_0) + f_y(y - y_0) + f_z(z - z_0) = 0$

$4(x - 2) + 3(y - 1) + 0(z - 0) = 0$

$4x - 8 + 3y - 3 = 0$

$\therefore 4x + 3y - 11 = 0 \quad$ — Equation of tangent.

$$\frac{x - x_0}{f_x} = \frac{y - y_0}{f_y} = \frac{z - z_0}{f_z}$$

$$\frac{x - 2}{4} = \frac{y - 1}{3} = \frac{z - 0}{0}$$ — Equation of normal

$3xyz - x - y + z = -4$ at $(1, -1, 2)$

$f(x, y, z) = 3xyz - x - y + z + 4$

$f_x = 3yz - 1 - 0 + 0 + 0$ $\quad f_x$ at $(1, -1, 2) = 3(1)(2$

$= -7$

$f_y = 3xz - 0 - 1 + 0 + 0$ $\quad f_y$ at $(1, -1, 2) = 3(1)(2$

$3xz - 1$ $\quad = 5$

$f_z = 3xy - 0 - 0 + 1 - 0$ $\quad f_z$ at $(1, -1, 2) = 3(1)$

$3xy + 1$ $\quad =$

Equation of tangent

$f_x(x - x_0) + f_y(y - y_0) + f_z(z - z_0) = 0$

$-7(x - 1) + 5(y + 1) + (-2)(z - 2) = 0$

Scanned with CamScanner

5.

(1) $f(x,y) = 3x^2 + y^2 - 3xy + 6x - 4y$

$\therefore f_x = 6x + 0 - 3y + 6 - 0$ — (1)

$= 6x - 3y + 6$

$f_y = 2y - 3x + 0 - 4$ — (2)

$2y - 3x - 4$

$f_x = 0$

$6x - 3y + 6 = 0$

$\cancel{6x - 3y + 6 = 0}$ $3(2x - y + 2) = 0$

$2x - y + 2 = 0$

$2x - y = -2$ — (3)

$f_y = 0$

$2y - 3x - 4 = 0$

$2y - 3x = 4$

Multiply 3 by 2 & subtracting 4 from (3)

$4x - 2y = -4$ $4x - 2y = -4$

$4x - 2y = -4$

$- \; 2y - 3x = 4$

$7x = 0$

$x = 0$

Substituting value of x in (3)

$x(0) - y = -2$

$-y = -2$

$y = 2$

$\therefore$ Critical Points are $(0,2)$

$r = f_{xx} = 6$

$t = f_{yy} = 2$

$s = f_{xy} = -3$

$rt - s^2 = 12 - 9$

$3 > 0$

here $r > 0$ & $rt - s^2 > 0$

f has minimum at $(0, 2)$

$f(0,2) = 3(0)^2 + (2)^2 - 3(0)(2) + 6(0) - 4(2)$

$0 + 4 - 0 - 0 - 8 = 4$

$f(x,y) = 2x^4 + 3x^2y - y^2$

$f_x = 8x^3 + 6xy - 0$

$8x^3 + 6xy$

$f_y = 0 + 3x^2 - 2y$

$3x^2 - 2y$

$f_x = 0$

$8x^3 + 6xy = 0$

$x(4x^2 + 6y) = 0$

$4x^2 + 6y = 0$ — 1

$f_y = 0$

$3x^2 - 2y = 0$

$3x^2 - 2y = 0$

$3x^2 - 2y = 0$ — 2

multiply in 1 by 3 & 2 by 4 & subtract 2 from 1

$12x^2 + 18y = 0$

$-12x^2 - 8y = 0$

$24y = 0$

$y = 0$ — 3

$x^2 - 2(0) = 0$

$3x^2 = 0$

$x^2 = 0$

$x = 0$ — 4

08

Critical point are $(0,0)$

Now

$r = f_{xx} = 24x^2 + 8y$

$t = f_{yy} = -2$

$s = f_{xy} = 6x$

$rt - s^2 = (24x^2 + 6y)(-2) - (6x)^2$

$= -48x^2 - 12y - 36x^2$

$= -84x^2 - 12y$

At $(0,0)$

$r = 24(0)^2 + 6(0)$

$= 0$

$s = 6(0) = 0$

$rt - s^2 = -84(0)^2 - 12(0) = 0$

$r = 0$

& $rt - s^2 = 0$

Nothing can be s

$f(x,y) = x^2 - y^2 + 2x + 8y - 70$

$f_x = 2x - 0 + 2 + 0 - 0$

$f_y = -2y + 0 + 8$
$= -2y + 8$

$f_x = 0$
$2x + 2 = 0$
$2(x+1) = 0$
$x + 1 = 0$
$x = -1$

$f_y = 0$
$-2y + 8 = 0$
$-2(y-4) = 0$
$y = +4$

$\therefore$ Critical Points are $(-1, 4)$

$\therefore r = f_{xx} = 2$

$t = f_{yy} = -2$

$s = f_{xy} = 0$

$rt - s^2 = 2(-2) - 0^2$
$= -4 < 0$

Here $r > 0$ & $rt - s^2 < 0$

Nothing can be said

AK
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