Summer of Science

Quantum Symphony

Assignment 1

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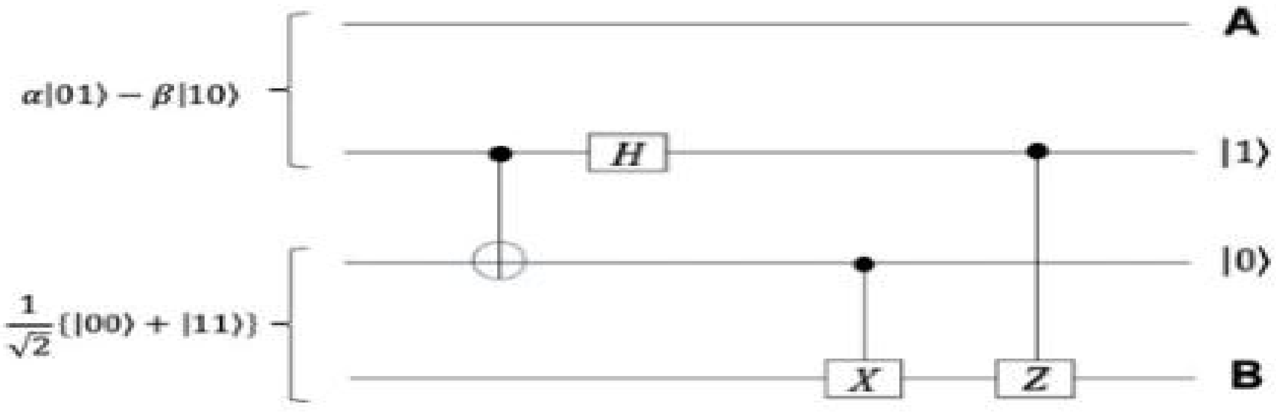
1. For the circuit below, find the final state at A and B



Sol.

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$$(\alpha|01\rangle - \beta|10\rangle)\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$$\frac{1}{\sqrt{2}}\alpha|01\rangle(|10\rangle + |01\rangle) - \frac{1}{\sqrt{2}}\beta|10\rangle(|00\rangle + |11\rangle)$$

$$\frac{1}{2}\alpha|0\rangle(|0\rangle - |1\rangle)(|10\rangle + |01\rangle) - \frac{1}{2}\beta|1\rangle(|0\rangle + |1\rangle)(|00\rangle + |11\rangle)$$

$$\frac{1}{2}\alpha|0\rangle(|0\rangle - |1\rangle)(|11\rangle + |01\rangle) - \frac{1}{2}\beta|1\rangle(|0\rangle + |1\rangle)(|00\rangle + |10\rangle)$$

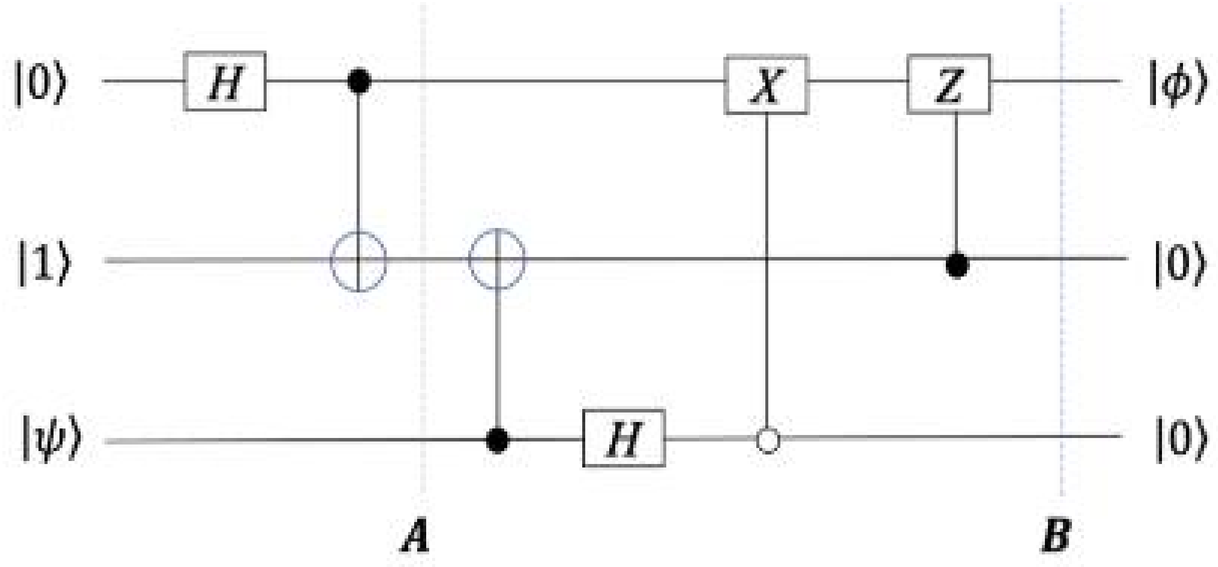
$$\frac{1}{2}\alpha|00\rangle(|11\rangle + |01\rangle) + \frac{1}{2}\alpha|01\rangle(|11\rangle + |01\rangle) - \frac{1}{2}\beta|10\rangle(|00\rangle + |10\rangle) - \frac{1}{2}\beta|11\rangle(|00\rangle + |10\rangle)$$

Once the second and third bit are measured, out of eight terms we are left with only two terms having second and third bits are $|0\rangle$ and $|1\rangle$ respectively.

So the final state is superposition of the two term,

$$\frac{1}{\sqrt{2}}(\alpha|01\rangle - \beta|10\rangle)$$

2. Consider the quantum state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ and the initial joint state $|\Psi\rangle = |0\rangle \otimes |1\rangle \otimes |\psi\rangle$. What is the joint state $|\Psi\rangle$ of the system at A, and the state $|\phi\rangle$ at B?



Sol.

- Basically this is “Quantum Teleportation”

Question 2

$$|00\rangle \otimes (\alpha|0\rangle + \beta|1\rangle)$$

$$A = \frac{|01\rangle + |10\rangle}{\sqrt{2}} \otimes (\alpha|0\rangle + \beta|1\rangle)$$

$$\alpha\frac{|01\rangle + |10\rangle}{\sqrt{2}} \otimes \frac{(|0\rangle + |1\rangle)}{\sqrt{2}} + \beta\frac{|00\rangle + |11\rangle}{\sqrt{2}} \otimes \frac{(|0\rangle - |1\rangle)}{\sqrt{2}}$$

$$\alpha\frac{|00\rangle + |11\rangle}{2} \otimes |0\rangle + \alpha\frac{|01\rangle + |10\rangle}{2} \otimes |1\rangle + \beta\frac{|10\rangle + |01\rangle}{2} \otimes |0\rangle - \beta\frac{|00\rangle + 11\rangle}{2} \otimes |1\rangle$$

$$\alpha\frac{|00\rangle - |11\rangle}{2} \otimes |0\rangle + \alpha\frac{|01\rangle + |10\rangle}{2} \otimes |1\rangle + \beta\frac{|10\rangle + |01\rangle}{2} \otimes |0\rangle - \beta\frac{|00\rangle - 11\rangle}{2} \otimes |1\rangle$$

When we measure the second and third and get $|00\rangle$, we will certainly get $\alpha|0\rangle + \beta|1\rangle$ as the final state at first qubit.

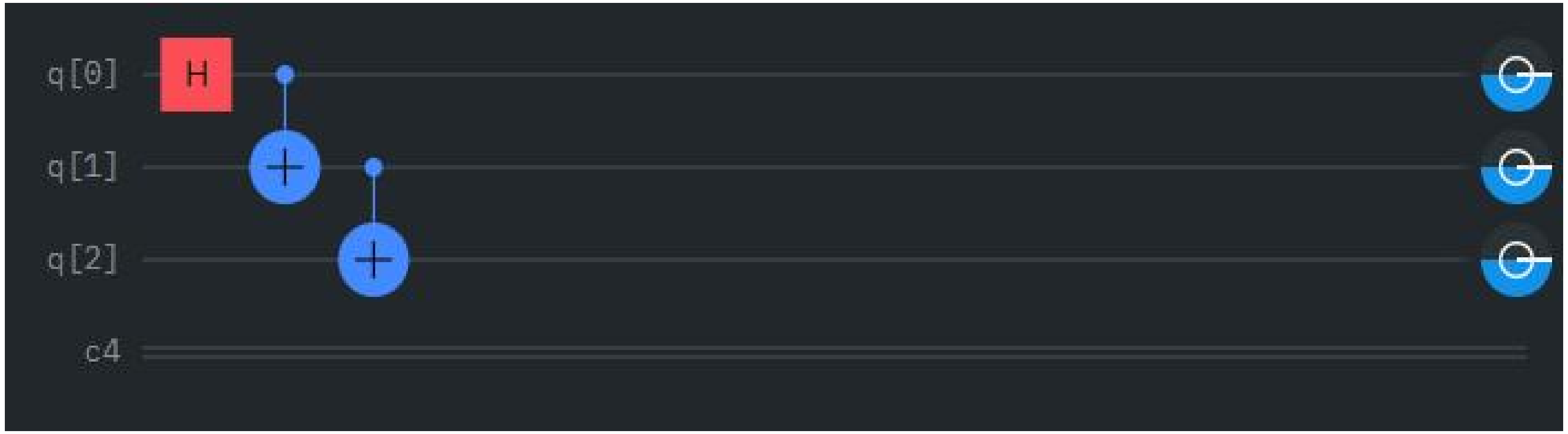
3. The quintessential three-qubit multiparty entangled states are the GHZ and W states:

$$|\psi\rangle_{\mathrm{GHZ}} = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$$

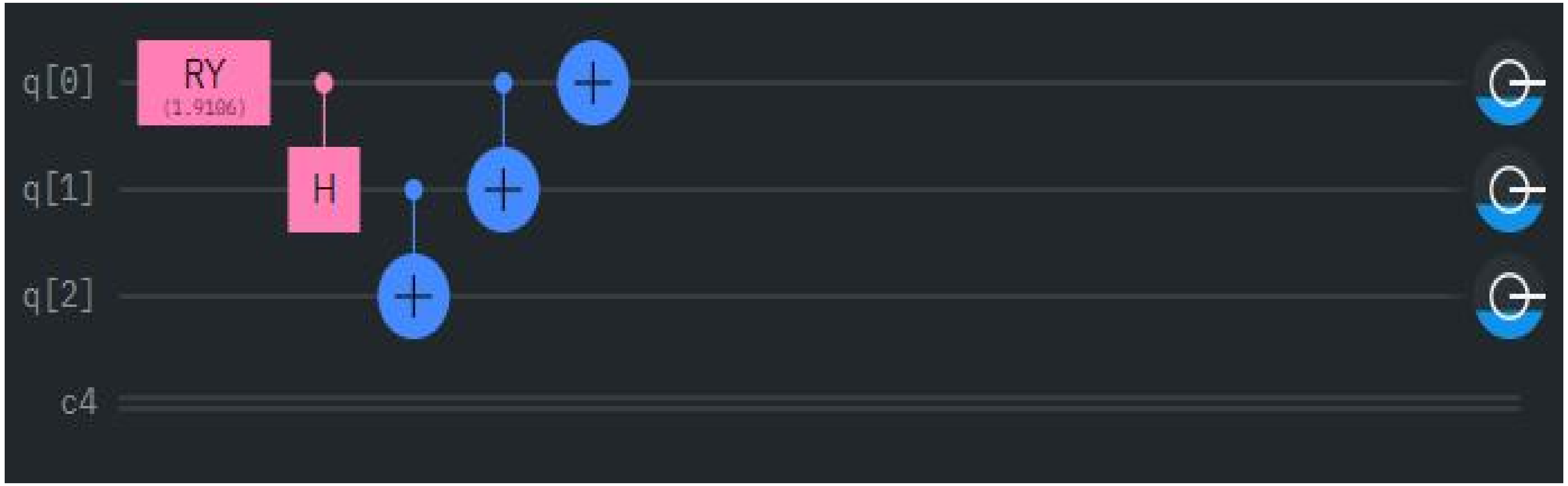
$$|\psi\rangle_{\mathrm{W}} = \frac{1}{\sqrt{3}}(|001\rangle + |010\rangle + |100\rangle).$$

Draw circuits that create the GHZ and W states.

Sol. The states are tri-partite entangled state and their circuits are given by

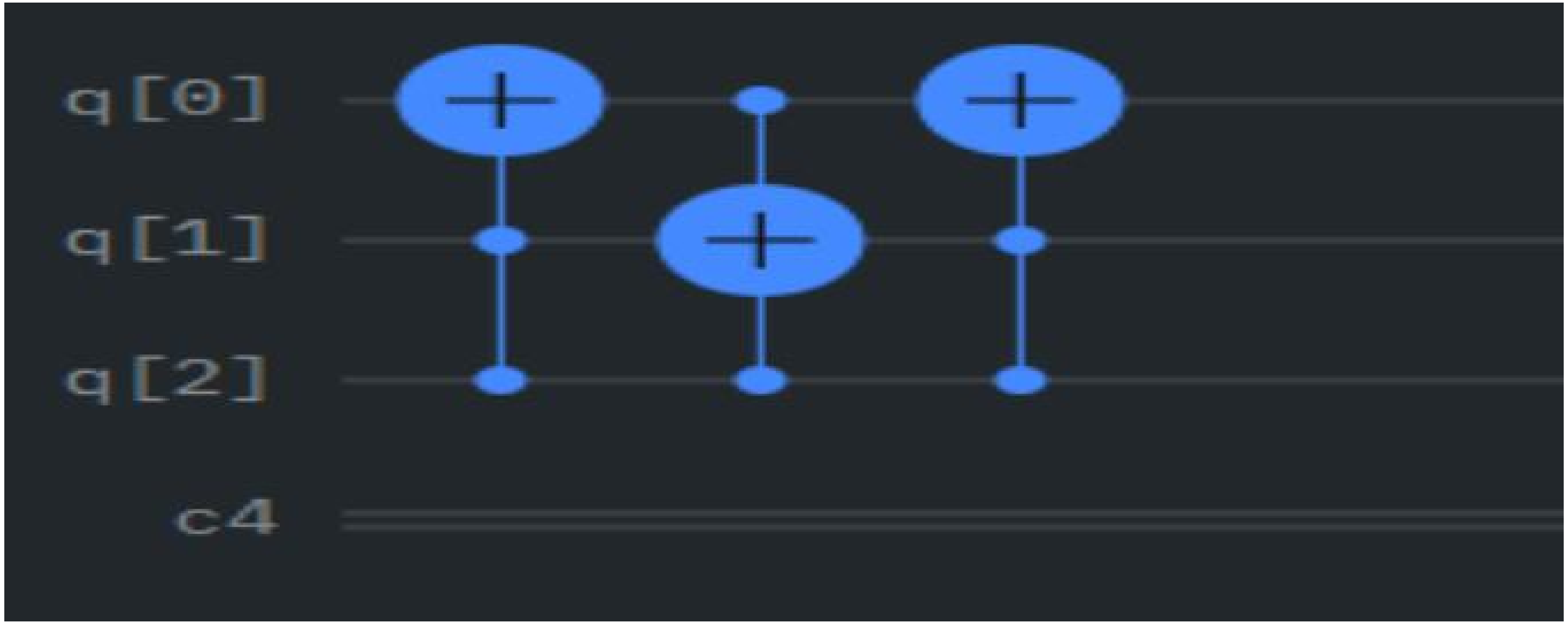


The angle mentioned in rotation about y operator (RY) is $2\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ $(= 1.9106)$



4. Draw a 3-qubit gate, using only Toffoli gates, that implements a controlled swap operation, where the swap is applied to the two target qubits depending on a single control qubit and show the truth table when control is on.

Sol. In the following circuit the control has been given to third qubit



Input(q[2]q[1]q[0])	Output
000	000
001	001
010	010
011	011
100	100
101	110
110	101
111	111

5. Code up the following circuits :

(a) A circuit that swaps the states of two qubits

(b) A circuit that takes the computational basis to Hadamard basis

(c) Circuits drawn in questions 3 and 4

(d) A circuit that decrements a three bit number by 1 and stores the result in the same qubits that are used for the input