Assignment 7

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Outline

Question

Exercise 13.3 : Question 2

A bag contains 4 red and 4 black balls, and another bag contains 2 red and 6 black balls. One of the bags is chosen at random and a ball is drawn from it which is found to be red. Find the probability that the ball is drawn from the first bag.

Definitions

X

Let the random variable X represent the bag chosen.

$$X = 0 \leftarrow \text{First bag}$$
$$X = 1 \leftarrow \text{Second bag} \tag{1}$$

Y

Let the random variable Y represent the colour of the ball chosen.

$$Y = 0 \leftarrow \text{Red ball}$$
$$Y = 1 \leftarrow \text{Black ball} \tag{2}$$

Reframing

Required: $P(\text{First Bag} \mid \text{Red ball}) = P(X = 0 \mid Y = 0)$

Bayes' theorem

The problem is a classic application of Bayes' theorem.

Bayes' theorem states :

$$\frac{P(A \mid B)}{P(B \mid A)} = \frac{P(A)}{P(B)} \tag{3}$$

where A and B are any two events.

Probabilities

Event	Formula	Probability
First bag ($P_X(0)$)	None	0.5
Second bag ($P_X(1)$)	None	0.5
Red ball from First bag ($P(Y=0 \mid X=0)$)	$\frac{4}{4+4}$	0.5
Red ball from Second bag ($P(Y=0 \mid X=1)$)	$\frac{2}{2+6}$	0.25
Red ball from either bag ($P_Y(0)$)	$(0.5 \times 0.5) + (0.5 \times 0.25)$	0.375

Using the formula

$$P_Y(0) = P_X(0) \times P(Y=0 \mid X=0) + P_X(1) \times P(Y=0 \mid X=1) \quad (4)$$

Calculation

Substituting the operational variables from the table into equation (3) and subsequently rearranging,

$$\frac{P(X=0 \mid Y=0)}{P(Y=0 \mid X=0)} = \frac{P_X(0)}{P_Y(0)} \tag{5}$$

$$\implies \frac{P(X=0 \mid Y=0)}{0.5} = \frac{0.5}{0.375} \tag{6}$$

$$\implies P(X=0 \mid Y=0) = \frac{2}{3} \tag{7}$$

Therefore, the probability that the ball is drawn from the first bag is $\frac{2}{3}$.