Assignment 2 : Question 15 (b)

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Question:

Find the length of the perpendicular from the origin to the plane

$$\mathbf{r} \cdot (3i - 4j - 12k) + 39 = 0 \tag{1}$$

Solution:

Clearly, the length of the perpendicular from a plane passing through some point is the distance of that point from the plane. The normal form of a plane is an equation of the form:

$$\begin{pmatrix} \mathbf{A} & D \end{pmatrix} \begin{pmatrix} \mathbf{x} \\ 1 \end{pmatrix} = 0 \tag{2}$$

Where :

$$\mathbf{A} = \begin{pmatrix} a & b & c \end{pmatrix}, \mathbf{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \tag{3}$$

We can represent the given plane $(equation\ (1))$ using normal form $(equation\ (2))$ thus :

$$\begin{pmatrix} 3 & -4 & -12 & 39 \end{pmatrix} \begin{pmatrix} \mathbf{x} \\ 1 \end{pmatrix} = 0 \tag{4}$$

The formula for the distance of a point from a plane is :

$$Distance = \frac{\left| \begin{pmatrix} \mathbf{A} & D \end{pmatrix} \begin{pmatrix} \mathbf{x} \\ 1 \end{pmatrix} \right|}{\|\mathbf{A}\|} \tag{5}$$

The input parameters are :

$$\begin{pmatrix} \mathbf{A} & D \end{pmatrix} = \begin{pmatrix} 3 & -4 & -12 & 39 \end{pmatrix}, \mathbf{x} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{6}$$

Substituting (6) in (5),

$$Distance = \frac{\left| \begin{pmatrix} 3 & -4 & -12 & 39 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right|}{\left\| \begin{pmatrix} 3 & -4 & -12 \end{pmatrix} \right\|} \tag{7}$$

$$= \frac{39}{13} \tag{8}$$

$$= 3 \tag{9}$$

$\therefore$ The length of the perpendicular from the origin to the plane $(equation\ (1))$ is $\underline{3}$ units.

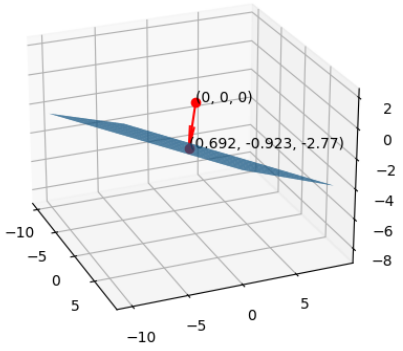

Fig. 1. Graph of the given plane