

Assignment 2 : Question 15 (b)

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Question:

Find the length of the perpendicular from the origin to the plane

$$\mathbf{r} \cdot (3i - 4j - 12k) + 39 = 0 \quad (1)$$

Solution:

Clearly, the length of the perpendicular from a plane passing through some point is the distance of that point from the plane. The normal form of a plane is an equation of the form:

$$(\mathbf{A} \ D) \begin{pmatrix} \mathbf{x} \\ 1 \end{pmatrix} = 0 \quad (2)$$

Where :

$$\mathbf{A} = (a \ b \ c), \mathbf{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad (3)$$

We can represent the given plane (equation (1)) using normal form from equation (2) thus :

$$(3 \ -4 \ -12 \ 39) \begin{pmatrix} \mathbf{x} \\ 1 \end{pmatrix} = 0 \quad (4)$$

The formula for the distance of a point from a plane is :

$$Distance = \frac{\left| (\mathbf{A} \ D) \begin{pmatrix} \mathbf{x} \\ 1 \end{pmatrix} \right|}{\|\mathbf{A}\|} \quad (5)$$

The input parameters are :

$$(\mathbf{A} \ D) = (3 \ -4 \ -12 \ 39), \mathbf{x} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad (6)$$

Substituting (6) in (5),

$$Distance = \frac{\left| (3 \ -4 \ -12 \ 39) \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right|}{\|(3 \ -4 \ -12)\|} \quad (7)$$

$$= \frac{39}{13} \quad (8)$$

$$= 3 \quad (9)$$

\therefore The length of the perpendicular from the origin to the plane (equation (1)) is 3 units.

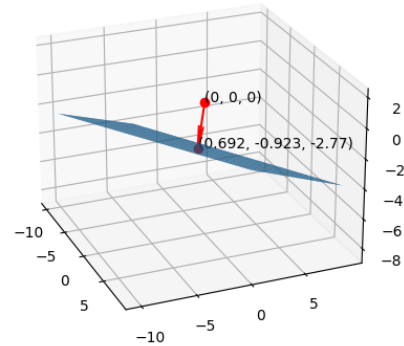


Fig. 1. Graph of the given plane