Assignment 4 : Example 13

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Question:

A committee of two persons is selected from two men and two women. Find the probability that the committee will have :

(**i**) no men

(**ii**) one man

(**iii**) two men

Solution:

Let the random variable X represent the number of men in the committee.

TABLE I
RANGE OF X

Value of X	Represents
0	No men
1	1 man
2	2 men

Let the set of possible candidates for the committee, i.e. the sample space, be denoted as S.

$$S = \{m_1, m_2, w_1, w_2\} \tag{1}$$

where $[m_i]$ are men and $[w_i]$ are women.

Define the relation R on the set S to be :

$$R = \{(a, b) \mid \text{a and b are both in the committee}\} \tag{2}$$

R is also clearly the set of all subsets of S $(equation\ (1))$ of cardinality 2.

In roster form, we can represent the relation $(equation\ (2))$ as follows.

$$\begin{aligned} R = \{&(w_1, w_2), (w_1, m_2), \\ &(m_1, w_2), (w_1, m_1), \\ &(m_2, w_2), (m_1, m_2)\} \end{aligned} \tag{3}$$

Therefore, upon reviewing (3), the frequency distribution of the number of men in the committee (X) is as given in table II.

TABLE II
FREQUENCY OF X

X	Frequency
0	1
1	4
2	1

The probabilities are then :

(**i**) $P_X(0) = \frac{1}{6}$

(**ii**) $P_X(1) = \frac{4}{6} = \frac{2}{3}$

(**iii**) $P_X(2) = \frac{1}{6}$

Alternatively :

$$P_X(r) = \frac{\left({}^2C_r\right)^2}{{}^4C_2} \tag{4}$$

Upon substitution of $r \in \{0, 1, 2\}$ into equation (4), we get the same result as above.

(**i**) $P_X(0) = \frac{1}{6}$

(**ii**) $P_X(1) = \frac{4}{6} = \frac{2}{3}$

(**iii**) $P_X(2) = \frac{1}{6}$