Assignment 8

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Outline

Question

A box contains n identical balls labelled 1 through n. Suppose k balls are drawn in succession. What is the probability that :

(i) m is the largest number drawn. (Event A)

(ii) The largest number drawn is less than or equal to m. (Event B)

Random Variables

Let the random variable X denote the largest number drawn.

Required :

(i) $P\left(A\right) = P_X\left(m\right)$

(ii) $P\left(B\right) = \Sigma_{i=1}^{m} P_X\left(i\right)$

Solution

Let M be the event that m is drawn. Clearly,

$$P(A) = P_X(m) = P(MB) \tag{1}$$

Therefore, we shall find $P(B)$ first.

$$\begin{aligned} P(B) &= \frac{\#\text{ Ways of drawing from } [m]}{\#\text{ Ways of drawing from } [n]} \\ &= \frac{{}^{m}C_k}{{}^{n}C_k} \end{aligned} \tag{2}$$

We know,

$$P(M) = \frac{k}{n} \tag{3}$$

$$\implies P(B \mid M) = \frac{\#\text{Ways of drawing } i \in [m-1]\ k-1 \text{ times}}{\#\text{Ways of drawing } i \in [n] - \{m\}\ k-1 \text{ times}}$$

$$= \frac{{}^{m-1}C_{k-1}}{{}^{n-1}C_{k-1}} \tag{4}$$

We know $P(MB) = P(B \mid M) \times P(M)$. Thus, substituting (4) in (1),

$$P(A) = \frac{{}^{m-1}C_{k-1}}{{}^{n-1}C_{k-1}} \times \frac{k}{n} \tag{5}$$

Graphs

Figure: Graph for event A

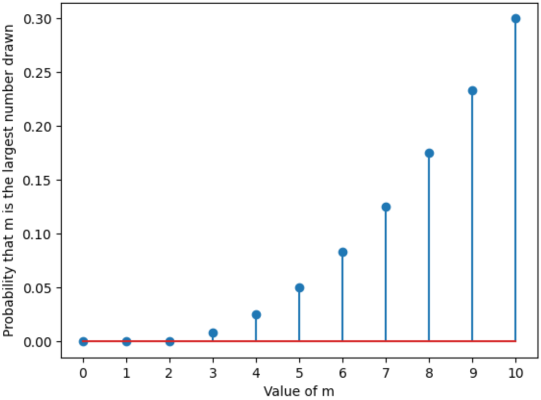

Figure: Graph for event B