Assignment 6 : Example 12

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Question:

Three coins are tossed simultaneously. Consider the events :

- E : Three heads or three tails
- F : At least two heads
- G : At most two heads

Which pairs of events are independent, and which are dependent?

Solution:

Let the random variable X represent the number of heads among the three tossed coins.

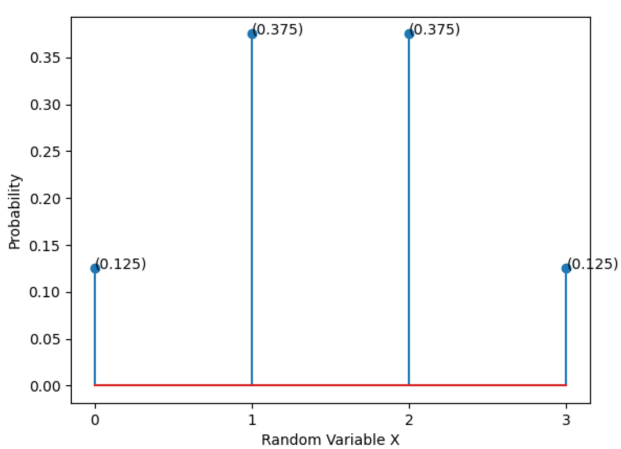
$\therefore X \in \{0, 1, 2, 3\}$

Reframing the events in terms of X,

- E : $X \in \{0, 3\}$
- F : $X \in \{2, 3\}$
- G : $X \in \{0, 1, 2\}$

The probability mass function of X is as follows.

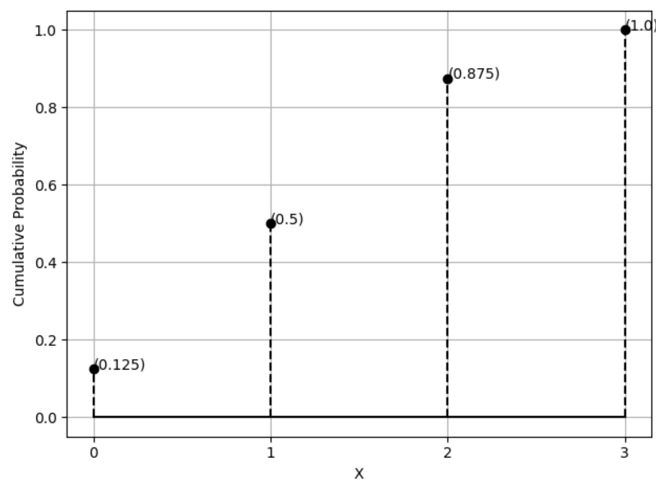
Fig. 1. Probability mass function for X



The corresponding Cumulative distribution can be obtained for a given value of X by adding up the probabilities of all values of X less than the given value.

Thus :

Fig. 2. Cumulative Probability Function for X



From 1, we get

$$P(E) = P_X(0) + P_X(3) \quad = 0.25 \tag{1}$$
$$P(F) = P_X(2) + P_X(3) \quad = 0.5 \tag{2}$$
$$P(G) = P_X(0) + P_X(1) + P_X(2) \quad = 0.875 \tag{3}$$

Therefore, the probabilities of the events are :

TABLE I
PROBABILITIES

Event	Probability
E	0.25
F	0.5
G	0.875

Now, representing pairs of events with X,

- EF : $X = 3$
- FG : $X = 2$
- GE : $X = 0$

And their probabilities,

$$P(EF) = P_X(3) = 0.125 \tag{4}$$
$$P(FG) = P_X(2) = 0.375 \tag{5}$$
$$P(GE) = P_X(0) = 0.125 \tag{6}$$

Which are tabularised below.

TABLE II
PROBABILITIES

Event	Probability
EF	0.125
FG	0.375
GE	0.125

Checking for independence,

$$P(E) \times P(F) = 0.125 = P(EF) \tag{7}$$
$$P(F) \times P(G) = 0.4375 \neq P(FG) \tag{8}$$
$$P(G) \times P(E) = 0.21875 \neq P(EG) \tag{9}$$

Hence, the events (E and F) are independent, whereas the events (F and G) and the events (G and E) are dependent.