# Assignment 10

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# Outline

# Question

Let $X$ and $Y$ be two independent random variables with common pmf :

$$P_X(k) = pq^k \text{ where } q = p - 1$$
$$\forall (k \geq 0) \in \mathbb{Z} \tag{1}$$

Show that the following pairs of RV's are independent.

**(i)** $min(X, Y)$ and $X - Y$.

**(ii)** $Z = min(X, Y)$ and $W = max(X, Y) - min(X, Y)$

# Solving for $p$

Due to unitarity,

$$\begin{aligned} \Sigma_{k=0}^{\infty} P_X(k) &= 1 \\ \implies \Sigma_{k=0}^{\infty} p(p-1)^k &= 1 \\ \implies \frac{p}{2-p} &= 1 \\ \implies p &= 1 \end{aligned} \tag{2}$$

Therefore, the PMF becomes a Kronecker Delta function, $P_X(k) = \delta(k)$. We shall be leveraging several of the properties of the delta function for the solution.

# Solution:

**(i)**

Clearly,

$$P(Z = k \; \& \; (X - Y) = k) = \delta(k) \tag{3}$$

And also,

$$\begin{aligned} P_Z(k) \times P((X - Y) = k) &= \delta(k) \times \delta(k) \\ &= \delta(k) \end{aligned} \tag{4}$$

$\therefore Z = min(X, Y)$ and $X - Y$ are independent.

**(ii)**

Clearly,

$$P\left(Z = k \,\&\, W = k\right) = \delta\left(k\right) \tag{5}$$

$$\tag{6}$$

And also,

$$\begin{aligned} P_Z\left(k\right) \times P_W\left(k\right) &= \delta\left(k\right) \times \delta\left(k\right) \\ &= \delta\left(k\right) \end{aligned} \tag{7}$$

$\therefore Z = min\left(X, Y\right)$ and $W = max\left(X, Y\right) - min\left(X, Y\right)$ are independent.

# Graph