ASSIGNMENT 1 : QUESTION 11 (A)

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Question :

Using the Remainder theorem find the remainders obtained when $x^3 + (kx+8)x + k$ is divided by $x+1$ and $x-2$.

Hence find k if the sum of the two remainders is 1.

Given:

- $f(x) = x^3 + (kx+8)x + k$
- Sum of remainders of $f(x)$ after dividing by $(x+1)$ and $(x-2)$ is 1

Find:

- Remainders of $f(x)$ after dividing by $(x+1)$ and $(x-2)$
- The value of k

Solution :

By the remainder theorem,

The remainder after dividing a polynomial $p(x)$ by $(x-r)$ is equal to $p(r)$.

Therefore,

$$f(x) \mod (x+1) = f(-1) \tag{1}$$
$$f(x) \mod (x-2) = f(2) \tag{2}$$

$$f(-1) = (-1)^3 + (k(-1)+8) * (-1) + k \tag{3}$$
$$= -1 + k - 8 + k \tag{4}$$
$$= \underline{2k-9} \tag{5}$$

$$f(2) = 2^3 + (2k+8) \cdot 2 + k \tag{6}$$
$$= 8 + 4k + 16 + k \tag{7}$$
$$= \underline{5k+24} \tag{8}$$

Given that $(2k-9) + (5k+24) = 1$

Rearranging, we get $7k = -14$

Therefore

$$\underline{k = -2} \tag{9}$$

Substituting the value of k, we get

$$2k - 9 = -13 \tag{10}$$
$$5k + 24 = 14 \tag{11}$$

Therefore, the remainders are :

$$f(x) \mod (x+1) = -13 \tag{12}$$
$$f(x) \mod (x-2) = 14 \tag{13}$$