

Assignment 2 : Question 15 (b)

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Question:

Find the length of the perpendicular from the origin to the plane

$$\mathbf{r} \cdot (3i - 4j - 12k) + 39 = 0 \quad (1)$$

Solution: Clearly, the length of the perpendicular from a plane passing through some point is the distance of that point from the plane.

The normal form of a plane is an equation of the form:

$$\mathbf{A}\mathbf{x} = D \quad (2)$$

Where :

- $\mathbf{A} = (a \ b \ c)$
- $\mathbf{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$, called the point vector
- D is some scalar constant.

We can represent the given plane (equation (1)) using normal form from (equation (2)) thus :

$$(3 \ -4 \ -12) \mathbf{x} = -39 \quad (3)$$

The formula for the distance of a point from a plane is :

$$Distance = \frac{1}{\|\mathbf{A}\|} \left| (a \ b \ c \ D) \cdot \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} \right| \quad (4)$$

The input parameters for equation (4) are:

$$(5)$$

$$\mathbf{A} = (a \ b \ c) = (3 \ -4 \ -12) \quad (6)$$

$$\mathbf{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad (7)$$

$$D = -39 \quad (8)$$

Upon substitution of equations (5), (7), (8) into equation (4),

$$Distance = \frac{1}{\|\mathbf{A}\|} \left| (a \ b \ c \ D) \cdot \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} \right| \quad (9)$$

$$= \frac{\left| (3 \ -4 \ -12 \ -39) \cdot \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right|}{\sqrt{3^2 + (-4)^2 + (-12)^2}} \quad (10)$$

$$= \frac{|-39|}{\sqrt{169}} \quad (11)$$

$$= \underline{3} \text{ units} \quad (12)$$

\therefore The length of the perpendicular from the origin to the plane (equation (1)) is 3 units.

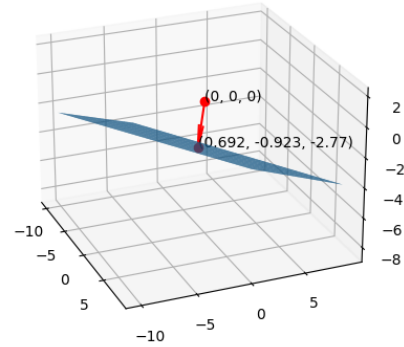


Fig. 1. Graph of the given plane