

Assignment 7 : Exercise 13.3 Question 2

Abhay Shankar K : cs21btech11001

Question:

A bag contains 4 red and 4 black balls, and another bag contains 2 red and 6 black balls. One of the bags is chosen at random and a ball is drawn from it which is found to be red. Find the probability that the ball is drawn from the first bag.

Solution:

Let the random variable X represent the bag chosen.

$X = 0 \leftarrow$ First bag

$X = 1 \leftarrow$ Second bag (1)

Let the random variable Y represent the colour of the ball chosen.

$Y = 0 \leftarrow$ Red ball

$Y = 1 \leftarrow$ Black ball (2)

Required:

$$P(\text{First Bag} \mid \text{Red ball}) = P(X = 0 \mid Y = 0)$$

Bayes' theorem states :

$$\frac{P(A \mid B)}{P(B \mid A)} = \frac{P(A)}{P(B)} \quad (3)$$

The various probabilities required for the computation are tabularized below :

TABLE I
PROBABILITIES

Event	Formula	Probability
First bag ($P_X(0)$)	None	0.5
Second bag ($P_X(1)$)	None	0.5
Red ball from First bag ($P(Y = 0 \mid X = 0)$)	$\frac{4}{4+4}$	0.5
Red ball from Second bag ($P(Y = 0 \mid X = 1)$)	$\frac{2}{2+6}$	0.25
Red ball from either bag ($P_Y(0)$)	$(0.5 \times 0.5) + (0.5 \times 0.25)$	0.375

Using the formula

$$P_Y(0) = P_X(0) \times P(Y = 0 \mid X = 0) + P_X(1) \times P(Y = 0 \mid X = 1) \quad (4)$$

Substituting the operational variables from table I into equation (3) and subsequently rearranging,

$$\frac{P(X = 0 \mid Y = 0)}{P(Y = 0 \mid X = 0)} = \frac{P_X(0)}{P_Y(0)} \quad (5)$$

$$\Rightarrow \frac{P(X = 0 \mid Y = 0)}{0.5} = \frac{0.5}{0.375} \quad (6)$$

$$\Rightarrow P(X = 0 \mid Y = 0) = \frac{2}{3} \quad (7)$$

Therefore, the probability that the ball is drawn from the first bag is $\frac{2}{3}$.