Assignment 5 : Miscellaneous Exercise 10

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Question:

The number lock of a suitcase has 4 wheels, each labelled with ten digits i.e. 0 to 9. The lock opens with a sequence of four digits with no repeats. What is the probability of a person getting the right sequence to open the suitcase?

Solution:

Let $U = \{0,1,2,3,4,5,6,7,8,9\}$ be the sample space.

Let the correct sequence $C = c_1c_2c_3c_4$, and let the selected sequence $S = s_1s_2s_3s_4$.

Let the random variables X_1, X_2, X_3, X_4 represent the boolean equality of the digits in the selected sequences, i.e. ,

$$\begin{aligned} &\forall i \in [4]\,, \\ &\quad c_i = s_i \implies X_i = 1 \\ &\quad c_i \neq s_i \implies X_i = 0 \end{aligned} \tag{1}$$

$\therefore$ **Required value** $= P_{X_1}(1) \times P_{X_2}(1) \times P_{X_3}(1) \times P_{X_4}(1)$

Progressing in ascending order,

$$P_{X_1}(1) = \frac{1}{|U|} \tag{2}$$

$$P_{X_2}(1) = \frac{1}{|U - \{c_1\}|} \tag{3}$$

$$P_{X_3}(1) = \frac{1}{|U - \{c_1, c_2\}|} \tag{4}$$

$$P_{X_4}(1) = \frac{1}{|U - \{c_1, c_2, c_3\}|} \tag{5}$$

Substituting the values into equations (2), (3), (4) and (5),

$$P_{X_1}(1) = \frac{1}{10} \tag{6}$$

$$P_{X_2}(1) = \frac{1}{9} \tag{7}$$

$$P_{X_3}(1) = \frac{1}{8} \tag{8}$$

$$P_{X_4}(1) = \frac{1}{7} \tag{9}$$

Multiplying (6), (7), (8) and (9),

$$\begin{aligned} \textbf{Required value} &= \frac{1}{10 \cdot 9 \cdot 8 \cdot 7} \\ &= \underline{1.98 \cdot 10^{-4}} \end{aligned} \tag{10}$$

Alternatively,

We know that the number of sequences of 4 digits formed from among 10 digits without repetitions is

$$N = {}^{10}P_4 = 5040 \tag{11}$$

So, the probability of the selected sequence matching the correct one is

$$\begin{aligned} \frac{1}{N} &= \frac{1}{{}^{10}P_4} \\ &= \underline{1.98 \cdot 10^{-4}} \end{aligned} \tag{12}$$

Therefore, the probability of a person getting the right sequence to open the suitcase is $\underline{1.98 \cdot 10^{-4}}$.

TABLE I
RESULTS OF PYTHON SIMULATION

$\frac{favourable}{total} \times 10^4$
1.84
2.15
1.98
1.81
2.13
2.03
2.11
2.04
1.93
1.97