Assignment 2 : Question 15 (b)

Abhay Shankar K : cs21btech11001

April 14, 2022

Question:

Find the length of the perpendicular from the origin to the plane

$$\vec{r} \cdot (3i - 4j - 12k) + 39 = 0 \tag{1}$$

Solution: Clearly, the length of the perpendicular from a plane passing through some point is the distance of that point from the plane.

The normal form of a plane is an equation of the form:

$$\vec{A}\vec{x} = D \tag{2}$$

Where :

- $\vec{A} = \begin{pmatrix} a & b & c \end{pmatrix}$
- $\vec{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$, called the point vector
- D is some scalar constant.

We can represent the given plane (*equation* (1)) using normal form from (*equation* (2)) thus :

$$\begin{pmatrix} 3 & -4 & -12 \end{pmatrix} \vec{x} = -39 \tag{3}$$

The formula for the distance of a point from a plane is :

$$Distance = \left| \frac{\begin{pmatrix} a & b & c & D \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}}{\left\| \vec{A} \right\|} \right| \tag{4}$$

Substituting input parameters into equation (4),

- $\begin{pmatrix} a & b & c \end{pmatrix} = \vec{A} = \begin{pmatrix} 3 & -4 & -12 \end{pmatrix}$
- $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \vec{x} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$
- $D = -39$

$$Distance = \left| \frac{\begin{pmatrix} a & b & c & D \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}}{\left\| \vec{A} \right\|} \right| \tag{5}$$

$$= \left| \frac{\begin{pmatrix} 3 & -4 & -12 & -39 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}}{\sqrt{3^2 + (-4)^2 + (-12)^2}} \right| \tag{6}$$

$$= \left| \frac{-39}{\sqrt{169}} \right| \tag{7}$$

$$= |-3| \tag{8}$$

$$= \underline{3} \text{ units} \tag{9}$$

$\therefore$ The length of the perpendicular from the origin to the plane (*equation* (1)) is $\underline{3}$ units.

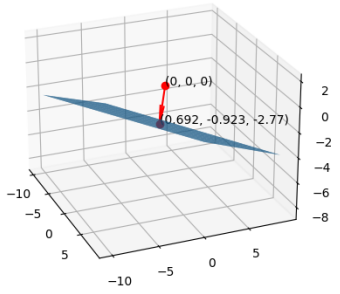

Figure 1: Graph of the given plane