Assignment 11

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Outline

Question

In an exit poll of 900 voters questioned, 360 responded that they favor a particular proposition. On this basis, it was reported that 40% of the voters favor the proposition.

(**i**) Find the margin of error if the confidence coefficient of the results is 0.95.

(**ii**) Find the confidence coefficient if the margin of error is $\mp 2\%$.

Solution: (i)

Consider the random variables $x_i \forall i \in [900]$, and the random variable $X = \Sigma_{i \in [900]} x_i$. where x_i is the boolean of a voter's opinion. Clearly, each x_i is a Bernoulli variable with parameter p, and X has a binomial distribution. Thus, the variance of X is known, and equals $np(1-p)$, where n $= 900$.

The sample mean, m, is also evident from the question $\left(m = \frac{360}{900} = 0.4\right)$.

Therefore, the margin of error Δ is given by:

$$\Delta = \mp \sqrt{\frac{m(1-m)}{n}} z_{\frac{\gamma+1}{2}} \tag{1}$$

$$= \mp \frac{0.49}{30} \cdot 1.97$$

$$= \mp 3.2\% \tag{2}$$

Solution: (ii)

Given : $\Delta = \mp 2\%$. From (1),

$$z_{\frac{\gamma+1}{2}} = \sqrt{\frac{n}{m(1-m)}} \cdot \Delta$$

$$= \frac{30 \cdot 0.02}{0.49} = 1.22$$

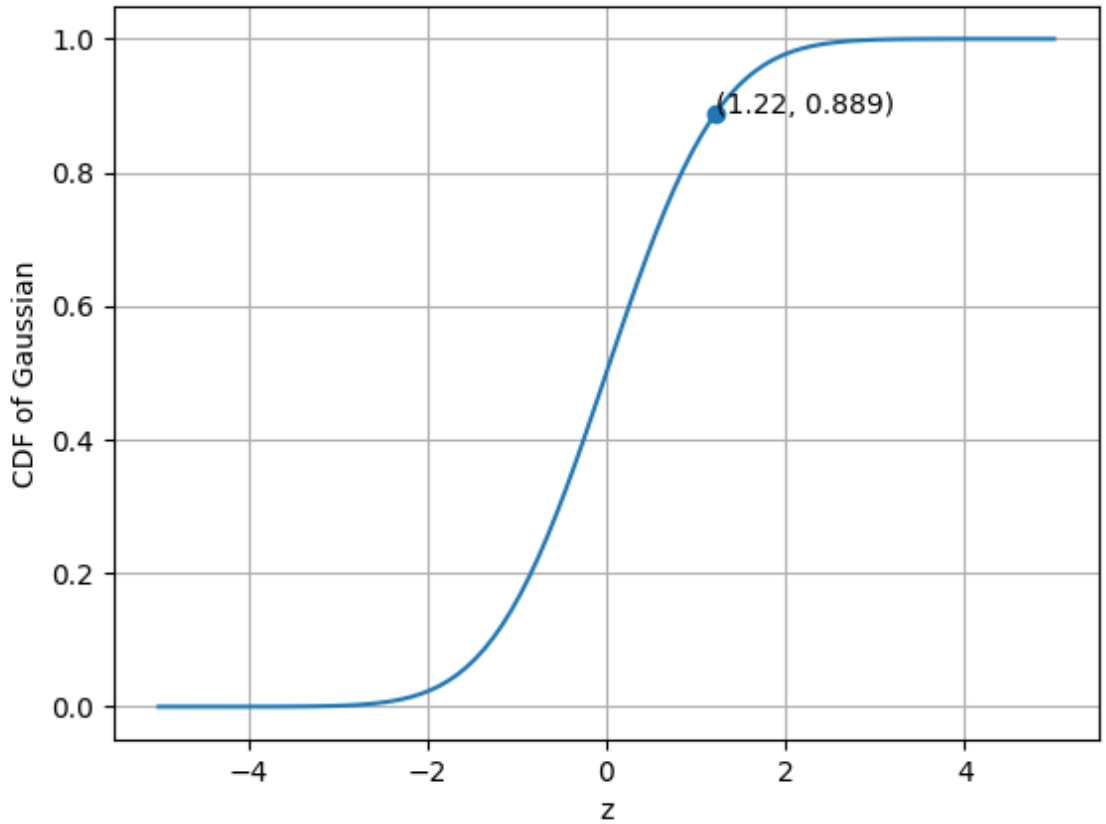
$$\implies \frac{\gamma+1}{2} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{1.22} e^{-\frac{z^2}{2}} dz = 0.89 \quad (3)$$

$$\implies \gamma = 0.78 \quad (4)$$

The value of the integral in (3) was computed in Python.

Graph

Figure: Cumulative function of Gaussian distribution



Result

The results obtained:

(i) $\Delta = \mp 3.2\%$

(ii) $\gamma = 0.78$