Assignment 2 : Question 15 (b)

Abhay Shankar K : cs21btech11001

Question:

Find the length of the perpendicular from the origin to the plane

$$\mathbf{r} \cdot (3i - 4j - 12k) + 39 = 0 \tag{1}$$

Solution: Clearly, the length of the perpendicular from a plane passing through some point is the distance of that point from the plane.

The normal form of a plane is an equation of the form:

$$\mathbf{A}\mathbf{x} = D \tag{2}$$

Where :

- $\mathbf{A} = \begin{pmatrix} a & b & c \end{pmatrix}$
- $\mathbf{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$, called the point vector
- D is some scalar constant.

We can represent the given plane $(equation\ (1))$ using normal form from $(equation\ (2))$ thus :

$$\begin{pmatrix} 3 & -4 & -12 \end{pmatrix} \mathbf{x} = -39 \tag{3}$$

The formula for the distance of a point from a plane is :

$$Distance = \frac{1}{\|\mathbf{A}\|} \left| \begin{pmatrix} a & b & c & D \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} \right| \tag{4}$$

The input parameters for equation (4) are:

$$\mathbf{A} = \begin{pmatrix} a & b & c \end{pmatrix} \tag{5}$$
$$= \begin{pmatrix} 3 & -4 & -12 \end{pmatrix} \tag{6}$$
$$\mathbf{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{7}$$
$$D = -39 \tag{8}$$

Upon substitution of equations (5), (7), (8) into equation (4),

$$Distance = \frac{1}{\|\mathbf{A}\|} \left| \begin{pmatrix} a & b & c & D \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} \right| \tag{9}$$

$$= \frac{\left| \begin{pmatrix} 3 & -4 & -12 & -39 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right|}{\sqrt{3^2 + (-4)^2 + (-12)^2}} \tag{10}$$

$$= \frac{|-39|}{\sqrt{169}} \tag{11}$$

$$= \underline{3} \text{ units} \tag{12}$$

$\therefore$ The length of the perpendicular from the origin to the plane $(equation\ (1))$ is $\underline{3}$ units.

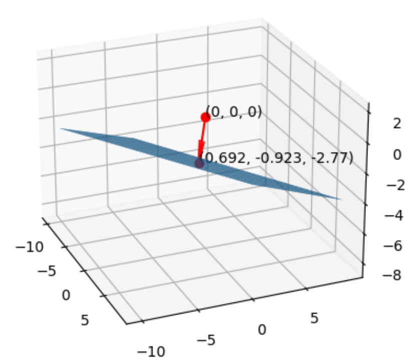

Fig. 1. Graph of the given plane