Assignment 9

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Outline

1. Question
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5. Graph

Question

Express the density $f_y(y)$ of the random variable $y = g(x)$ in terms of $f_x(x)$ if :

(i) $g(x) = |x|$

(ii) $g(x) = e^{-x}U(x)$ where U (x) is the Uniform Distribution, say over the interval $[-1, 1]$.

Solution : (i)

Clearly, $f_y(0) = f_x(0)$. For nonzero x :

We know

$$\begin{aligned} F_y(y_1) &= P\left(g(x) \leq g(x_1)\right) \\ &= P\left(|x| \leq |x_1|\right) \\ &= P\left(-|x_1| \leq x \leq |x_1|\right) \\ &= \left(F_x(x_1) - F_x(-x_1)\right) \times sgn(x_1) \end{aligned}$$

$$\therefore F_y(y) = \left(F_x(x) - F_x(-x)\right) sgn(x) \tag{1}$$

Assuming $f_x(x)$ is a normal distribution, clearly the normalization factor is 0.5. Differentiating and normalizing (1),

$$f_y(y) = \frac{1}{\sigma\sqrt{2\pi}} \times e^{\frac{x^2}{2\sigma^2}} \tag{2}$$

Solution : (ii)

We know

$$\begin{aligned} F_y(y_1) &= P(g(x) \leq g(x_1)) \\ &= P\left(e^{-x} \leq e^{-x_1}\right) \\ &= P(x \geq x_1) \\ &= 1 - P(x \leq x_1) + P_x(x_1) \\ &= 1 - F_x(x_1) \end{aligned}$$

$$\therefore F_y(y) = 1 - F_x\left(\frac{-\ln y}{U(x)}\right) \tag{3}$$

Assuming $f_x(x)$ is a normal distribution and $\sigma = 0.3$, the normalization constant is 0.1618. Differentiating and normalizing (3),

$$f_y(y) = \left(1 + \frac{4}{\sigma\sqrt{2\pi}} e^{x - \frac{x^2}{2\sigma^2}}\right) \times 0.1618 \tag{4}$$

Result

(i) $y = |x| \implies f_y(y) = f_y(y) = \frac{1}{\sigma\sqrt{2\pi}} \times e^{\frac{x^2}{2\sigma^2}} \rightarrow \ (2)$

(ii) $y = e^{-x}U(x) \implies f_y(y) = \left(1 + \frac{4}{\sigma\sqrt{2\pi}} e^{x - \frac{x^2}{2\sigma^2}}\right) \times 0.1618 \rightarrow \ (4)$

Graph

Assuming : $\sigma = 0.3$