

Assignment 8

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Question

A box contains n identical balls labelled 1 through n . Suppose k balls are drawn in succession. What is the probability that :

- (i) m is the largest number drawn. (Event A)
- (ii) The largest number drawn is less than or equal to m . (Event B)

Solution

Let M be the event that m is drawn.

Clearly,

$$P(A) = P(MB) \quad (1)$$

Therefore, we shall find $P(B)$ first.

$$\begin{aligned} P(B) &= \frac{\# \text{ Ways of drawing from } [m]}{\# \text{ Ways of drawing from } [n]} \\ &= \frac{{}^m C_k}{{}^n C_k} \end{aligned} \quad (2)$$

We know,

$$P(M) = \frac{k}{n} \quad (3)$$

$$\begin{aligned} \Rightarrow P(B|M) &= \frac{\text{\#Ways of drawing } i \in [m-1] \text{ } k-1 \text{ times}}{\text{\#Ways of drawing } i \in [n] - \{m\} \text{ } k-1 \text{ times}} \\ &= \frac{{}^{m-1}C_{k-1}}{{}^{n-1}C_{k-1}} \end{aligned} \quad (4)$$

We know $P(MB) = P(B|M) \times P(M)$. Thus, substituting (4) in (1),

$$P(A) = \frac{{}^{m-1}C_{k-1}}{{}^{n-1}C_{k-1}} \times \frac{k}{n} \quad (5)$$

Graphs

Taken $n = 10$ and $k = 3$.

Figure: Graph for event A

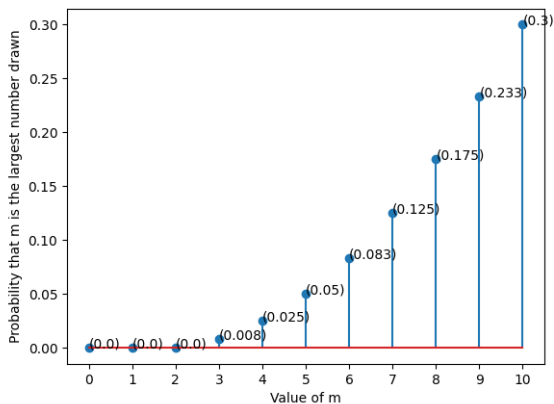


Figure: Graph for event B

