Assignment 6 : Exercise 13.3 Question 2

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Question:

A bag contains 4 red and 4 black balls, and another bag contains 2 red and 6 black balls. One of the bags is chosen at random and a ball is drawn from it which is found to be red. Find the probability that the ball is drawn from the first bag.

Solution:

Let the random variable X represent the bag chosen.

$$\begin{aligned} X = 0 &\leftarrow \text{First bag} \\ X = 1 &\leftarrow \text{Second bag} \end{aligned} \tag{1}$$

Let the random variable Y represent the colour of the ball chosen.

$$\begin{aligned} Y = 0 &\leftarrow \text{Red ball} \\ Y = 1 &\leftarrow \text{Black ball} \end{aligned} \tag{2}$$

Required:

$P\left(\text{First Bag} \mid \text{Red ball}\right) = P\left(X = 0 \mid Y = 0\right)$

Bayes' theorem states :

$$\frac{P\left(A \mid B\right)}{P\left(B \mid A\right)} = \frac{P\left(A\right)}{P\left(B\right)} \tag{3}$$

The various probabilities required for the computation are tabularized below :

TABLE I
PROBABILITIES

Event	Formula	Probability
First bag $\left(P_X\left(0\right)\right)$	None	0.5
Second bag $\left(P_X\left(1\right)\right)$	None	0.5
Red ball from First bag $\left(P\left(Y = 0 \mid X = 0\right)\right)$	$\frac{4}{4+4}$	0.5
Red ball from Second bag $\left(P\left(Y = 0 \mid X = 1\right)\right)$	$\frac{2}{2+6}$	0.25
Red ball from either bag $\left(P_Y\left(0\right)\right)$	$(0.5 \times 0.5) + (0.5 \times 0.25)$	0.375

Using the formula

$$\begin{aligned} P_Y\left(0\right) = P_X\left(0\right) \times P\left(Y = 0 \mid X = 0\right) \\ + P_X\left(1\right) \times P\left(Y = 0 \mid X = 1\right) \end{aligned} \tag{4}$$

Substituting the operational variables from table I into equation (3) and subsequently rearranging,

$$\frac{P\left(X = 0 \mid Y = 0\right)}{P\left(Y = 0 \mid X = 0\right)} = \frac{P_X\left(0\right)}{P_Y\left(0\right)} \tag{5}$$

$$\implies \frac{P\left(X = 0 \mid Y = 0\right)}{0.5} = \frac{0.5}{0.375} \tag{6}$$

$$\implies P\left(X = 0 \mid Y = 0\right) = \frac{2}{3} \tag{7}$$

Therefore, the probability that the ball is drawn from the first bag is $\frac{2}{3}$.