

3. Factorise

(i) $x^2 + xy + 8x + 8y$

Regrouping method

$$\underbrace{x^2 + \cancel{xy}}_{x^2 + 8x} + \underbrace{8x + 8y}_{8(x+y)}$$

$$x^2 + xy = x(x+y)$$

$$8x + 8y = 8(x+y)$$

$$x^2 + \cancel{xy} + 8x + 8y$$

$$x(x+y) + 8(x+y)$$

$$= (x+y)(x+8)$$

Take common & write remaining

(ii) $15xy - 6x + 5y - 2$

$$15xy - 6x = 3x(5y - 2)$$

$$5y - 2 = 1(5y - 2)$$

$$15xy - 6x + 5y - 2$$

$$= 3x(5y - 2) + 1(5y - 2)$$

$$= (5y - 2)(3x + 1) //$$

(iii) $ax + bx - ay - by$

$$ax + bx - x(a+b)$$

$$-ay - by = -y(a+b)$$

$$ax + bx - ay - by$$

$$x(a+b) - y(a+b)$$

$$= (a+b)(x-y) //$$

(iv) $15pq + 15 + 9q + 25p$

$$15pq + 15 = 15(pq + 1)$$

$$9q + 25 = ?$$

so shuffle the terms,

$$\underbrace{15pq + 9q}_{e} + \underbrace{15 + 25p}_{e}$$

$$15pq + 9q = 3q(5p + 3)$$

$$15 + 25p = 5(3 + 5p)$$

$$15pq + 9q + 15 + 25p$$

$$3q(5p + 3) + 5(3 + 5p)$$

$$= (5p + 3)(3q + 5)$$

(v) $z - 7 + 7xy - xyz^2$

$$z - 7 = 1(z - 7)$$

$$7xy - xyz^2 = xy(7 - z)$$

$$z - 7 + 7xy - xyz^2$$

$$1(z - 7) + xy(7 - z)$$

$$1(z - 7) - xy(z - 7)$$

$$= (z - 7)(1 - xy) //$$

Exercise - 12.2

1. Factorise the following expressions.

$$(i) a^2 + 8a + 16$$

$$\begin{array}{l} \text{use of identities} \\ \text{So, } a^2 = a^2 \quad | \quad b^2 = 16 \\ a = a \quad | \quad b = 4 \end{array}$$

$$2ab = 2 \times a \times 4$$

$$2ab = 8a$$

$$\therefore a^2 + 8a + 16 = (a+4)^2 //$$

$$(i) (a+b)^2 = a^2 + b^2 + 2ab$$

$$(ii) (a-b)^2 = a^2 - 2ab + b^2$$

$$(iii) a^2 - b^2 = (a+b)(a-b)$$

$$(iv) (x+a)(x+b) = x^2 + x(a+b) + ab$$

$$(ii) p^2 - 10p + 25 //$$

$$\begin{array}{l} a^2 = p^2 \quad | \quad b^2 = 25 \\ a = p \quad | \quad b = 5 \end{array}$$

$$\therefore p^2 - 10p + 25 = (p-5)^2 //$$

$$(iii) 25m^2 + 30m + 9$$

$$\begin{array}{l} a^2 + 2ab + b^2 = (a+b)^2 \\ a^2 = 25m^2 \quad | \quad b^2 = 9 \\ a = 5m \quad | \quad b = 3 \end{array}$$

$$\therefore 25m^2 + 30m + 9 = (5m+3)^2 //$$

$$(QV) 4xy^2 + 84y^2 + 36x^2$$

$$\begin{aligned} & \therefore a^2 + 2ab + b^2 = (a+b)^2 \\ & a^2 = 4xy^2 \quad b^2 = 36x^2 \\ & a = 4y \quad b = 6x \\ & \therefore = 84yz. \end{aligned}$$

$$\therefore 49y^2 + 84y^2 + 36x^2 = (7y+6x)^2$$

$$\therefore u^2 - 8x + u = (2x-2)^2$$

$$\text{or } u(x-1)^2 //$$

$$(i) ux^2 - 8x + 4 //$$

2.

$$\begin{aligned} & (i) 4p^2 - 9a^2 \\ & a^2 - 2ab + b^2 = (a-b)^2 \\ & a^2 = b^2 = (a+b)(a-b) \\ & a = 2p \quad b = 3a \\ & \therefore = 7(a^2 - b^2) \\ & a = 9a^2 \quad b^2 = 16b^2 \\ & a = 3a \quad b = 4b \end{aligned}$$

$$(ii) 63a^2 - 112b^2$$

$$4p^2 - 9a^2$$

$$a^2 - b^2 = (a+b)(a-b)$$

$$a = 9a^2 \quad b^2 = 16b^2$$

$$a = 3a \quad b = 4b$$

$$(iii) 69x^2 - 36$$

$$\begin{aligned} & a^2 - b^2 = (a+b)(a-b) \\ & a^2 = 16ax^2 \quad b^2 = 36 \\ & a = 4x \quad b = 6 \end{aligned}$$

$$16x^3 (x^2 - 9)$$

$$16x^3 (x+3)(x-3)$$

$$(iv) 16x^5 - 144x^3$$

$$\begin{aligned} & (v) (a+mn)^2 - (a-mn)^2 \\ & (a+m)(a-m) \\ & a^2 = a^2 \quad m^2 = m^2 \\ & m = mn \\ & \therefore 16x^2 - 36 = (4x+6)(4x-6) // \\ & = 16x^2 - 144x^2 \\ & = 16x^2 + 24mn + mn^2 - 16x^2 - 24mn - mn^2 \\ & = 24mn + 24mn \\ & = 48mn // \end{aligned}$$

$$(vi) (a+mn)^2 - (a-mn)^2$$

$$\begin{aligned} & (a+m)(a-m) \\ & a^2 = a^2 \quad m^2 = m^2 \\ & m = mn \\ & \therefore 16x^2 - 36 = (4x+6)(4x-6) // \\ & = 16x^2 - 144x^2 \\ & = 16x^2 + 24mn + mn^2 - 16x^2 - 24mn - mn^2 \\ & = 24mn + 24mn \\ & = 48mn // \end{aligned}$$

$$\begin{aligned} & (vii) a^4 + 2a^2b^2 + b^4 \\ & a^2 + 2ab + b^2 = (a+b)^2 \\ & a^2 = a^4 \quad b^2 = b^4 \\ & a = a^2 \quad b = b^2 \\ & \therefore 16x^2 - 36 = (4x+6)(4x-6) // \\ & = 16x^2 - 144x^2 \\ & = 16x^2 + 24mn + mn^2 - 16x^2 - 24mn - mn^2 \\ & = 24mn + 24mn \\ & = 48mn // \end{aligned}$$

$$(viii) 9x^2y^2 - 16$$

$$\begin{aligned} & a^2 - b^2 = (a+b)(a-b) \\ & a^2 = 9x^2y^2 \quad b^2 = 16 \\ & a = 3xy \quad b = 4 \end{aligned}$$

$$\therefore 9x^2y^2 - 16 = (3xy+4)(3xy-4) //$$

$$(vii) (x^2 - 2xy + y^2) - z^2$$

$$(viii) 25a^2 - 14b^2 + 28bc - 49c^2$$

$$(ix) y(y+2) + 9(ay+2)$$

$$(x) 5y^2 - 20y - 82 + 24z$$

$$25a^2 - [a^2 + 2ab + b^2]$$

$$5y^2 - 20y + 2yz - 82 \quad (\text{shuffle})$$

$$a^2 - x^2 \quad | \quad b^2 - y^2 \quad | \quad 2ab - 2xy \\ a^2 - x^2 \quad | \quad b^2 - y^2 \quad | \quad 2ab - 2xy \\ a^2 - x^2 \quad | \quad b^2 - y^2 \quad | \quad 2ab - 2xy$$

$$a^2 - x^2 \quad | \quad b^2 - y^2 \quad | \quad 2ab - 2xy \\ a^2 - x^2 \quad | \quad b^2 - y^2 \quad | \quad 2ab - 2xy \\ a^2 - x^2 \quad | \quad b^2 - y^2 \quad | \quad 2ab - 2xy$$

$$5y^2 - 20y = 5y(y-4)$$

$$2yz - 82 = 2z(y-4)$$

$$a^2 - b^2 = (a+b)(a-b) \quad | \quad a^2 - b^2 = (a+b)(a-b)$$

$$a^2 - (x-y)^2 \quad | \quad b^2 - z^2 \quad | \quad a^2 - 25a^2 \quad | \quad b^2 - (2b-7c)^2 \\ a^2 - (x-y)^2 \quad | \quad b^2 - z^2 \quad | \quad a^2 - 25a^2 \quad | \quad b^2 - (2b-7c)^2 \\ a^2 - (x-y)^2 \quad | \quad b^2 - z^2 \quad | \quad a^2 - 25a^2 \quad | \quad b^2 - (2b-7c)^2$$

$$(x-y)^2 - z^2 = (x-y+2)(x-y-2) \quad | \quad a^2 - 25a^2 \quad | \quad b^2 - (2b-7c)^2$$

$$-25a^2 - (2b-7c)^2 \quad | \quad (5a+10b-7c)(5a-2b+7c)$$

$$-25a^2 - (2b-7c)^2 \quad | \quad (5a+10b-7c)(5a-2b+7c) \quad | \quad 10ab + 5b^2 - 5b(2a+1)$$

$$-25a^2 - (2b-7c)^2 \quad | \quad 10ab + 5b^2 - 5b(2a+1) \quad | \quad 10ab + 5b^2 - 5b(2a+1)$$

$$-25a^2 - (2b-7c)^2 \quad | \quad 10ab + 5b^2 - 5b(2a+1) \quad | \quad 10ab + 5b^2 - 5b(2a+1)$$

$$-25a^2 - (2b-7c)^2 \quad | \quad 10ab + 5b^2 - 5b(2a+1) \quad | \quad 10ab + 5b^2 - 5b(2a+1)$$

$$-25a^2 - (2b-7c)^2 \quad | \quad 10ab + 5b^2 - 5b(2a+1) \quad | \quad 10ab + 5b^2 - 5b(2a+1)$$

$$-25a^2 - (2b-7c)^2 \quad | \quad 10ab + 5b^2 - 5b(2a+1) \quad | \quad 10ab + 5b^2 - 5b(2a+1)$$

$$-25a^2 - (2b-7c)^2 \quad | \quad 10ab + 5b^2 - 5b(2a+1) \quad | \quad 10ab + 5b^2 - 5b(2a+1)$$

$$-25a^2 - (2b-7c)^2 \quad | \quad 10ab + 5b^2 - 5b(2a+1) \quad | \quad 10ab + 5b^2 - 5b(2a+1)$$

$$-25a^2 - (2b-7c)^2 \quad | \quad 10ab + 5b^2 - 5b(2a+1) \quad | \quad 10ab + 5b^2 - 5b(2a+1)$$

$$-25a^2 - (2b-7c)^2 \quad | \quad 10ab + 5b^2 - 5b(2a+1) \quad | \quad 10ab + 5b^2 - 5b(2a+1)$$

$$-25a^2 - (2b-7c)^2 \quad | \quad 10ab + 5b^2 - 5b(2a+1) \quad | \quad 10ab + 5b^2 - 5b(2a+1)$$

$$-25a^2 - (2b-7c)^2 \quad | \quad 10ab + 5b^2 - 5b(2a+1) \quad | \quad 10ab + 5b^2 - 5b(2a+1)$$

$$-25a^2 - (2b-7c)^2 \quad | \quad 10ab + 5b^2 - 5b(2a+1) \quad | \quad 10ab + 5b^2 - 5b(2a+1)$$

$$-25a^2 - (2b-7c)^2 \quad | \quad 10ab + 5b^2 - 5b(2a+1) \quad | \quad 10ab + 5b^2 - 5b(2a+1)$$

4. Factorise

$$(i) a^4 - b^4$$

$$a^2 - b^2 = (a+b)(a-b)$$

$$\begin{array}{l|l} a^2 = a^4 & b^2 = b^4 \\ a = a^2 & b = b^2 \end{array}$$

$$a^4 - b^4 = (a^2 + b^2)(a^2 - b^2)$$

$$= (a^2 + b^2)(a+b)(a-b) //$$

$$(ii) p^6 - 81$$

$$a^2 - b^2 = (a+b)(a-b)$$

$$\begin{array}{l|l} a^2 = p^6 & b^2 = 81 \\ a = p^3 & b = 9 \end{array}$$

$$p^6 - 81 = (p^2 + 9)(p^2 - 9)$$

$$= (p^2 + 9)(p+3)(p-3) //$$

$$(iii) x^4 - (y+z)^4$$

$$a^2 - b^2 = (a+b)(a-b)$$

$$\begin{array}{l|l} a^2 = x^4 & b^2 = (y+z)^4 \\ a = x^2 & b = (y+z)^2 \end{array}$$

$$x^4 - (y+z)^4 = (x^2 + (y+z)^2)(x^2 - (y+z)^2)$$

$$= (x^2 + (y+z)^2)(x + (y+z))(x - (y+z))$$

$$= (x^2 + (y+z)^2)(x+y+z)(x-y-z) //$$

$$(iv) x^4 - (x-z)^4$$

$$a^2 - b^2$$

$$\begin{array}{l|l} a^2 = x^4 & b^2 = (x-z)^4 \\ a = x^2 & b = (x-z)^2 \end{array}$$

$$x^4 - (x-z)^4 = (x^2 + (x-z)^2)(x^2 - (x-z)^2) \rightarrow (a^2 - b^2)$$

$$= (x^2 + x^2 - 2xz + z^2)(x + (x-z))(x - (x-z))$$

$$= (2x^2 - 2xz + z^2)(x+x-z)(x-x+z)$$

$$= (2x^2 - 2xz + z^2)(2x-z)(z)$$

$$= 2(2x-z)(2x^2 - 2xz + z^2) //$$

$$(v) a^4 - 2a^2b^2 + b^4$$

$$a^2 - 2ab + b^2 = (a-b)^2$$

$$\begin{array}{l|l} a^2 = a^4 & b^2 = b^4 \\ a = a^2 & b = b^2 \end{array} \quad \begin{array}{l} 2ab = 2 \times a^2 \times b^2 \\ = 2a^2b^2 \end{array}$$

$$a^4 - 2a^2b^2 + b^4 = (a^2 - b^2)^2$$

$$\begin{aligned} &= [(a+b)(a-b)]^2 \\ &= (a+b)^2(a-b)^2 \end{aligned}$$

5. Factorise the following expressions.

$$(i) p^2 + 6p + 8$$

$$\begin{array}{r} \diagup \\ + 2+4 \\ \diagdown \\ 1 \quad 8 \quad x \end{array}$$

* first write factors of 8

* Match the possible signs to the central expression

* Regroup them & take common expression & remaining expression.

$$p^2 + 6p + 8$$

$$= p^2 + 2p + 4p + 8$$

$$= p(p+2) + 4(p+2)$$

$$= (p+2)(p+4)$$

$$(ii) q^2 - 10q + 21$$

$$\begin{array}{r} \diagup \\ 21 \\ \diagdown \\ -3 \quad -7 \quad x \end{array}$$

$$(iii) p^2 + 6p - 16$$

$$\begin{array}{r} \diagup \\ 16 \\ \diagdown \\ -2 \quad +8 \quad x \end{array}$$

$$q^2 - 3q - 7q + 21$$

$$= q(q-3) - 7(q-3)$$

$$= (q-7)(q-3)$$

$$p^2 - 2p + 8p - 16$$

$$\begin{aligned} &= p(p-2) + 8(p-2) \\ &= (p-2)(p+8) \end{aligned}$$

Exercise - 12.3

1. Carry out the following divisions:

$$(i) 88x^4 \div 56x \quad (ii) -36y^3 \div 9y^2$$

$$\begin{array}{r} 1 \\ \underline{56x} \quad 2 \\ 88x^4 \quad 2x^4 \\ \hline 3x \end{array}$$

$$= \frac{4}{9y^2} y^3$$

$$= \frac{1}{2} x^{4-1}$$

$$= \frac{1}{2} x^3$$

$$= \frac{1}{2} x^3 //$$

$$= -4y^{\frac{3}{2}}$$

$$= -4y^{\frac{3}{2}} y^{\frac{3}{2}-2}$$

$$= -4y^{\frac{3}{2}-2} //$$

$$= -4y^{\frac{3}{2}-2} //$$

$$(iii) 68pqr^2x^3 \div 11qr^2$$

$$(iv) 34x^3y^3z^3 \div 51xy^3z^3$$

$$(v) 8(x^3y^2z^2 + x^2y^3z^2 + x^2y^2z^3) \div 4x^2y^2z^2$$

$$(vi) (x^3 + 2x^2 + 3x) \div 2x$$

$$(vii) 8(p^3q^6 - p^6q^3) \div p^3q^3$$

$$(viii) (10x - 25) \div 5$$

$$(ix) (10x - 25) \div (2x - 5)$$

$$(x) 12a^8b^8 \div (-5a^6b^4)$$

$$(xi) 8(2x - 5) \div 8$$

$$(xii) 8x - 5 //$$

2. Divide the given polynomial by the given monomial.

$$(i) (5x^2 - 6x) \div 3x \quad (ii) 3y^8 - 4y^6 + 5y^4 \div y^4$$

$$= \frac{y^4(5x^2 - 6x)}{3x}$$

$$= y^4(3y^4 - 4y^2 + 5)$$

$$(iii) 8(x^3y^2z^2 + x^2y^3z^2 + x^2y^2z^3) \div 4x^2y^2z^2$$

$$(iv) (x^3 + 2x^2 + 3x) \div 2x$$

$$(v) (p^3q^6 - p^6q^3) \div p^3q^3$$

$$(vi) (10x - 25) \div (2x - 5)$$

$$(vii) 8(p^3q^6 - p^6q^3) \div p^3q^3$$

$$(viii) (10x - 25) \div 5$$

$$(ix) (10x - 25) \div (2x - 5)$$

$$(x) 12a^8b^8 \div (-5a^6b^4)$$

$$(xi) 8(2x - 5) \div 8$$

$$(xii) 8x - 5 //$$

$$(iii) 10y(6y+21) \div 5(ay+7) \quad (iv) 9x^2y^2(3z-2u) \div 27xy(2z)$$

$$\begin{aligned} & 2y(6y+21) \\ & 5(2y+7) \\ & = 2y(ay+7) \\ & = 2y \times 3(ay+7) \\ & = 6y // \end{aligned}$$

$$5apqr(p+q)qr(pr) \div 10a^2pq(q+r)(r+p)$$

$$= \frac{5apqr(p+q)qr(pr)}{10a^2pq(q+r)(r+p)}$$

$$\frac{5(p+q)}{2} //$$

$$(v) 20(y+u)(y^2+sy+3) \div 5(y+u)$$

$$\begin{aligned} & 20abc(3a-12)(5b-30) \div 4u^2(a-u)(b-6) \\ & = \frac{20abc \times 3(a-4) \times 5(b-6)}{4u^2(a-u)(b-6)} \\ & = 10abc // \end{aligned}$$

g. Divide as directed.

$$(v) x(x+1)(x+2)(x+3) \div x(x+1)$$

$$= \frac{x(x+1)(x+2)(x+3)}{x(x+1)}$$

$$(vi) 80ay(x+s)(y-u) \div 16ax(yu)$$

$$= (x+2)(x+3)$$

$$\begin{aligned} & 5(x+1)(3x+s) \\ & = \frac{20ay(x+s)(yu)}{16ax(yu)} \\ & = 5(x+1) // \end{aligned}$$

h. Factorise the expressions & divide them as directed.

$$5(3x+s) //$$

$$= 2y(x+s) //$$

$$= (y+5) //$$

$$\begin{aligned} & (y+5) \\ & = \frac{(y+2)(y+s)}{(y+5)} \\ & = (y+2) // \end{aligned}$$

$$(vii) 5apqr(p+q)qr(pr) \div 10a^2pq(q+r)(r+p)$$

$$(ii) (m^2 + 4wm - 32) \div (m+2)$$

$$= \frac{m^2 - 14m - 32}{m^2 + 2}$$

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$$\begin{aligned} & \frac{(m+2)(m-16)}{(m+2)} = \frac{m^2 + 2m - 16m - 32}{m(m+2) - 16(m+2)} \\ & \quad \vdots \\ & \quad \vdots \end{aligned}$$

$$(800) (5P^2 - 25P + 20) \div (P - 1)$$

$$S^2 = 3CB + 2D$$

$$\begin{aligned} &= 5(p^2 - 5p + 1) \\ &= 5(p-1)(p-4) \end{aligned}$$

$$= 5(CP-4) \cdot 4$$

$$(\text{?}) \log_2 (2^2 + 62 - 16) \stackrel{?}{=} \log(2+8)$$

$$\begin{aligned}
 & \frac{1}{2}y^2(2^2 + 6z - 16) \\
 & = 2y(2+8) \\
 & = 2y(2+8)(2+8) \\
 & = 2y(2+8) \\
 & = 2(2-2)(2+8) \\
 & = (2-2)(2+8)
 \end{aligned}$$

$$\begin{aligned}
 & \frac{3y^3(5y^2 - 98)}{26y^2(5y + 7)} \\
 & = \frac{3y^3(5y^2 - 98)}{26y^2(5y + 7)}
 \end{aligned}$$

$$\frac{sp_{\alpha} (sp_{\alpha}) (p - \alpha)}{2}$$

$$a^2 - b^2 = (a+b)(a-b)$$

$$p^2 - q^2 = (p+q)(p-q)$$