Problem Set 11 (For Week of 11/28)

Graphs: Representation, Traversal

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1. Suppose a weighted undirected graph has *n* vertices and *e* edges. The weights are all integers. Assume that the space needed to store an integer is the same as the space needed to store an object reference, both equal to one unit. *What is the minimum value of e* for which the adjacency matrix representation would require less space than the adjacency linked lists representation? Ignore the space needed to store vertex labels.

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2. The complement of an **undirected** graph, **G**, is a graph **GC** such that:
   - **GC** has the same set of vertices as **G**
   - For every edge *(i,j)* in **G**, there is no edge *(i,j)* in **GC**
   - For every pair of vertices *p* and *q* in **G** for which there is no edge *(p,q)*, there is an edge *(p,q)* in **GC**.

   Implement a method that would return the complement of the **undirected** graph on which this method is applied.

```
class Edge {
  int vnum;
  Edge next;
}

public class Graph {
  Edge[] adjlists;  // adjacency linked lists
  ...
  public Graph complement() {
    // FILL IN THIS METHOD
    ...
  }
}
```

   What would be the worst case running time (big O) of an implementation for a graph with *n* vertices and *e* edges?

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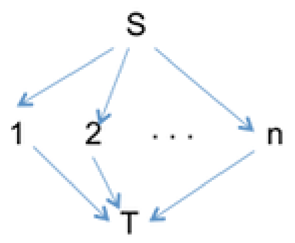
3. **WORK OUT THE SOLUTION TO THIS PROBLEM AND TURN IT IN AT RECITATION**

   Consider this graph:



   This graph has *n*+2 vertices and 2*n* edges. For every vertex labeled *i*, *1* <= *i* <= *n*, there is an edge from *S* to *i*, and an edge from *i* to *T*.

   1. How many different depth-first search sequences are possible if the start vertex is *S*?
   2. How many different breadth-first search sequences are possible if the start vertex is *S*?

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4. * You can use DFS to check if there is a path from one vertex to another in a directed graph.

Implement the method **hasPath** in the following. Use additional class fields/helper methods as needed:

```
public class Neighbor {
  public int vertex;
  public Neighbor next;
  ...
}

public class Graph {
  Neighbor[] adjLists;  // adjacency linked lists for all vertices

  // returns true if there is a path from v to w, false otherwise
  public boolean hasPath(int v, int w) {
   // FILL IN THIS METHOD
   ...
  }
}
```

5. An *undirected* graph may be disconnected, if there are certain vertices that are unreachable from other vertices. In a disconnected graph, each island of vertices is called a *connected component* - in each island, every vertex can reach all other vertices.

   You are given the same Graph class as in problem #4, but this time it represents an undirected graph, and it does not have the hasPath method.

   Implement a method in this class that will use dfs to number all connected components (0..), and return an array that holds, for each vertex, the number of the connected component to which it belongs. Implement helper methods as necessary. What is the big O running time of your implementation?

```
public class Graph {

  Neighbor[] adjLists; // adjacency linked lists for all vertices

  // returns an array of connected component membership of vertices,
  // i.e. return[i] is the number of the connected number to which a vertex belongs
  // connected components are numbered 0,1,2,...
  public int[] connectedComponents() {
    // FILL IN THIS IMPLEMENTATION
  }

  ...
}
```