Intro to AI Homework 3

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1 Problem 1

1.1 Probability all variables are True

$$P(A,B,C,D,E) = P(A) * P(B) * P(C) * P(D|A,B) * P(E|B,C)$$

$$P(A,B,C,D,E) = 0.2 * 0.5 * 0.8 * 0.1 * 0.3 = 0.0024$$

The probability all variables are True is 0.0024

1.2 Probability all variables are False

Similarly to the previous question, we use the same expression however we compute and use the false probability for each variable.

$$P(\neg A, \neg B, \neg C, \neg D, \neg E) = P(\neg A)*P(\neg B)*P(\neg C)*P(\neg D|\neg A, \neg B)*P(\neg E|\neg B, \neg C)$$

The False probabilities for the variables are not given but we calculate them by subtracting their True probabilities from 1.

$$P(\neg A, \neg B, \neg C, \neg D, \neg E) = 0.8 * 0.5 * 0.2 * 0.1 * 0.8 = 0.0064$$

The probability all the variables are False is 0.0064

1.3 Probability A is False given all other variables are True

$$P(\neg A|B,C,D,E) = \frac{P(\neg A,B,C,D,E)}{P(B,C,D,E)}$$

To find P(B,C,D,E), we can calculate $P(A,B,C,D,E) + P(\neg A,B,C,D,E)$

$$P(A,B,C,D,E) = 0.2 * 0.5 * 0.8 * 0.1 * 0.3 = 0.0024$$

$$P(\neg A,B,C,D,E) = 0.8 * 0.5 * 0.8 * 0.6 * 0.3 = 0.0576$$

$$P(A,B,C,D,E) + P(\neg A,B,C,D,E) = 0.0024 + 0.0576 = 0.06$$

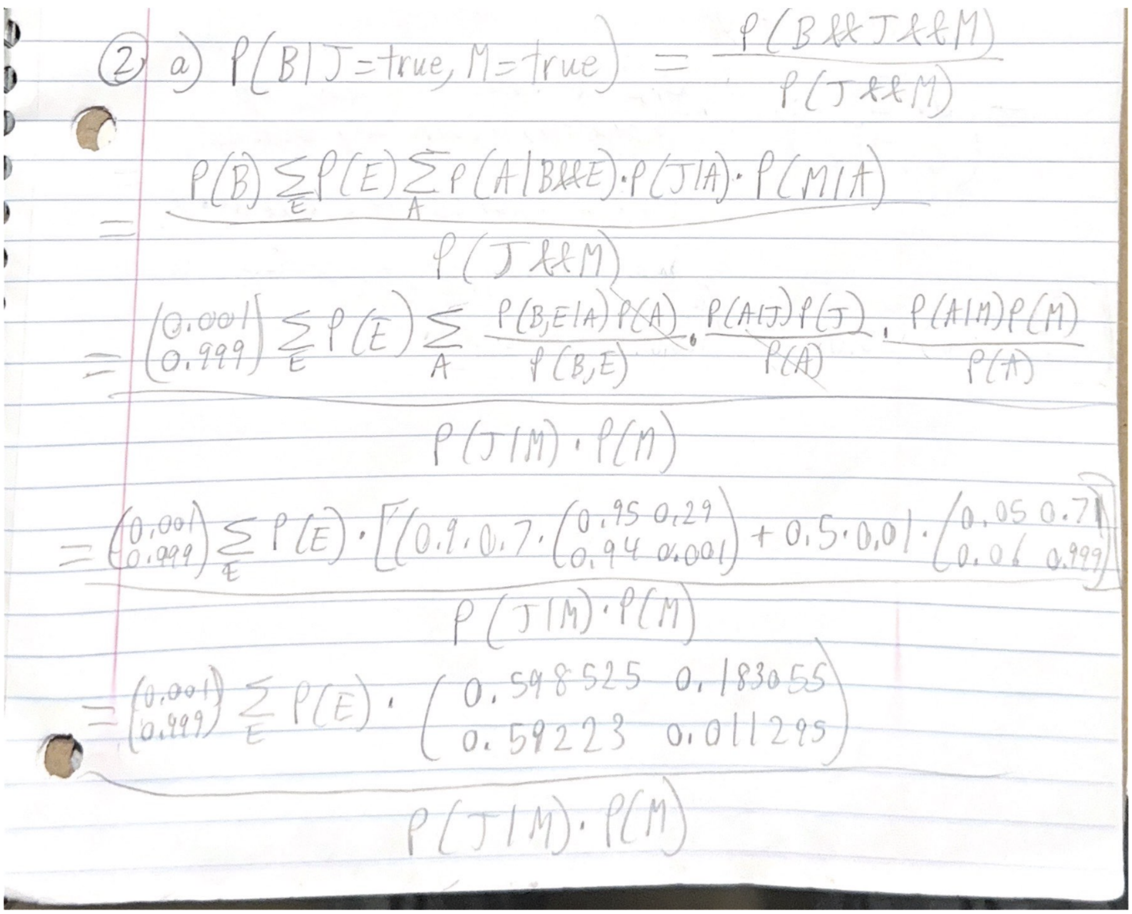
Now we know $P(\neg A,B,C,D,E)$ and $P(B,C,D,E)$, therefore we calculate

$$P(\neg A|B,C,D,E) = \frac{0.0576}{0.06} = 0.96$$

The probability A is True given that all the other variables are True is 0.96

2 Problem 2

2.1 Bayes Network



② a) $P(B|J=true, M=true) = \dfrac{P(B \&\& J \&\& M)}{P(J \&\& M)}$

$$= \frac{P(B)\sum_E P(E) \sum_A P(A|B\&\&E)\cdot P(J|A)\cdot P(M|A)}{P(J\&\&M)}$$

$$= \frac{\begin{pmatrix}0.001\\0.999\end{pmatrix}\sum_E P(E)\sum_A \frac{P(B,E|A)P(A)}{P(B,E)}\cdot\frac{P(A|J)P(J)}{P(A)}\cdot\frac{P(A|M)P(M)}{P(A)}}{P(J|M)\cdot P(M)}$$

$$= \frac{\begin{pmatrix}0.001\\0.999\end{pmatrix}\sum_E P(E)\cdot\left[0.9\cdot 0.7\cdot\begin{pmatrix}0.95 & 0.29\\0.94 & 0.001\end{pmatrix} + 0.5\cdot 0.01\cdot\begin{pmatrix}0.05 & 0.71\\0.06 & 0.999\end{pmatrix}\right]}{P(J|M)\cdot P(M)}$$

$$= \frac{\begin{pmatrix}0.001\\0.999\end{pmatrix}\sum_E P(E)\cdot\begin{pmatrix}0.598525 & 0.183055\\0.59223 & 0.011295\end{pmatrix}}{P(J|M)\cdot P(M)}$$

$$= \begin{pmatrix} 0.001 \\ 0.999 \end{pmatrix} \cdot \left[0.002 \cdot \begin{pmatrix} 0.598525 \\ 0.183055 \end{pmatrix} + 0.998 \cdot \begin{pmatrix} 0.59223 \\ 0.011295 \end{pmatrix} \right]$$

$$P(J|M) \cdot P(M)$$

$$= \begin{pmatrix} 0.001 \\ 0.999 \end{pmatrix} \cdot \left[\begin{pmatrix} 0.00119705 \\ 0.00036611 \end{pmatrix} + \begin{pmatrix} 0.59104554 \\ 0.011272241 \end{pmatrix} \right]$$

$$P(J|M) \cdot P(M)$$

$$= \frac{\begin{pmatrix} 0.001 \\ 0.999 \end{pmatrix} \begin{pmatrix} 0.59224259 \\ 0.001493351 \end{pmatrix}}{P(J,M)} = \frac{\begin{pmatrix} 0.00059224259 \\ 0.0014918576 \end{pmatrix}}{0.0020853609}$$

^ from calcs above

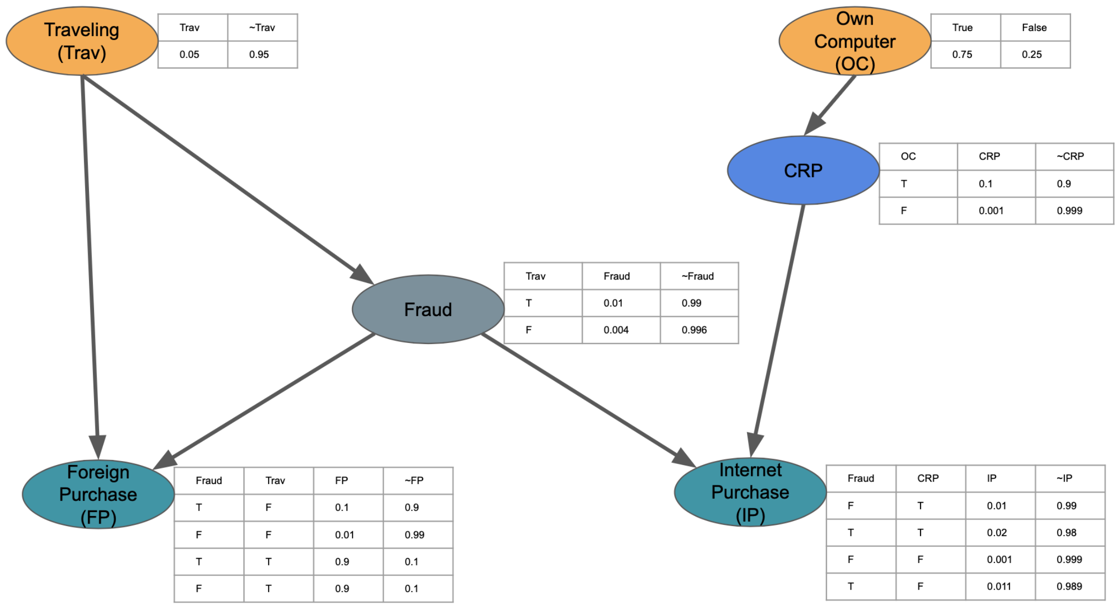
$$= \begin{pmatrix} 0.284 \\ 0.716 \end{pmatrix}$$

0.284 chance a burglary happens, given they both call

0.716 chance the burglary won't happen, given they both call

3 Problem 3

3.1 Bayes Network



Trav	~Trav
0.05	0.95

True	False
0.75	0.25

OC	CRP	~CRP
T	0.1	0.9
F	0.001	0.999

Trav	Fraud	~Fraud
T	0.01	0.99
F	0.004	0.996

Fraud	Trav	FP	~FP
T	F	0.1	0.9
F	F	0.01	0.99
T	T	0.9	0.1
F	T	0.9	0.1

Fraud	CRP	IP	~IP
F	T	0.01	0.99
T	T	0.02	0.98
F	F	0.001	0.999
T	F	0.011	0.989

3.2 Probabilities

3.2.1 Prior Probability

$$P(\text{Fraud}) = P(\text{Fraud}=T \mid \text{Trav}=T) \cdot P(\text{Trav}=T)$$

$$0.01 \times 0.05$$

$$+ P(\text{Fraud}=T \mid \text{Trav}=F) \cdot P(\text{Trav}=F)$$

$$0.004 \times 0.95$$

$$(0.01 \times 0.05) + (0.004 \times 0.95) = 0.0043$$