1.1.1

Let us assume that $L(i,j)$ be the loss incurred by assigning individual categories.

Probability densities at x are $f_1(x), f_2(x) \cdots f_M(x)$.

Now the prior probability for M categories is

$$\hat{\eta}_i = \frac{\eta_i f_i}{\sum \eta_i f_i}, \quad i = 1, 2 \cdots M.$$

Thus, for random variable x_ posterior probability will be, $\hat{\eta}(x)$. the corresponding loss is

$$r_i(x) = \sum_{i=1}^{M} \eta_i(x) L(i,j)$$

For a given x, the conditional loss is minimum when assigned to category j for which $r_i(x)$ is lowest. The minimizing decision rule δ^* will be know. Now the risk $r^*(x)$ will be different for different values as the difference of α is given.

Hence,

$$r^*(x) = \min_i \left\{ \sum_{i=1}^{M} \hat{\eta}_i(x) \, L(i,j) \right\}$$

or $$r^*(x) = \min \left\{ \eta(x), \alpha(1-\eta(x)) \right\}$$