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CSE 512 Fall 2019 – Machine Learning – Homework 2

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Question 1

1.1.1

X follows Poisson distribution this means that function of x is

$$P(X = \cancel{x}k \mid \lambda) = \frac{\lambda^k}{k!} e^{-\lambda}$$

Now, if we have to find the log likelihood, then

$$\log P = -\lambda - \sum_{1}^{k} \log(k!) + k \log(\lambda)$$

Here, $k = \sum_{1}^{k} (x_i)$ or

$$-\lambda - \sum_{1}^{k} \log(x_i!) + \log(\lambda) \sum_{1}^{k} x_i$$

1.1.2

for MLE of this, we will differentiate w.r.t. λ and keep it to λ.

$$\max(x = k \mid \lambda)$$

$$\Rightarrow \frac{\partial}{\partial \lambda}\left(-\lambda - \sum_1^k \log(x_i) + \log(\lambda)\sum_1^k x_i\right)$$

$$= -1 + \frac{1}{\lambda}\sum_1^k x_i$$

keeping this to zero.

$$-1 + \frac{1}{\lambda}\sum_1^k x_i = 0.$$

$$\frac{1}{\lambda}\sum_1^k x_i = 1$$

$$\boxed{\sum_1^k x_i = k\lambda}$$

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1.1.3

1.1.3

for the given problem, the observed X, we will get MLE of λ as $\sum x_i$.

Which means.

$$\sum_{1}^{k} x_i = 4+12+3+5+6+9+10$$

$$= 49.$$

Question 1.2

1.2.1

Here, we are given that the pdf for Gamma is

$$p(\lambda \mid \alpha, \beta) = \frac{\beta^{\alpha}}{\Gamma(\alpha)} \lambda^{\alpha-1} e^{-\beta\lambda}$$

Now, for posterior let us combine the likelihood and prior which means.

$$p(\lambda \mid x) = \left(\frac{\beta^{\alpha}}{\Gamma(\alpha)}\right) \lambda^{\alpha-1} e^{-\beta\lambda}$$

$$\left(\frac{1}{\prod_{i}^{k} x_i}\right) \lambda^{\sum x_i} e^{-\lambda}$$

[$\because$ from 1.1]

If we now ignore the proporti-onality constants i.e. $\frac{\beta^{\alpha}}{\Gamma(\alpha)}$ and $\left(\frac{1}{\sum x_i}\right)$ we get

$$p(\lambda \mid x) \propto \lambda^{\sum x_i + \alpha - 1} e^{-(\beta+1)\lambda}$$

This means that the posterior distribution is

$P(\lambda | X) \propto \text{Gamma}(k+\alpha-1, \beta+1)$

1.2.2

Now, the MAP, we have to take the derivative of the log of the posteriori

$$\log(p(\lambda|X)) \propto \log(\lambda^{\sum x_i+\alpha-1} e^{-(\beta+1)\lambda})$$

$\downarrow$

(L)

$$L = (\sum x_i+\alpha-1)\log\lambda - (\beta+1)\lambda$$

Diff. w.r.t. λ

$$\frac{\sum x_i+\alpha-1}{\lambda} - (\beta+1) = 0$$

$$\frac{\sum x_i+\alpha-1}{\lambda} = \beta+1$$

$$\boxed{\frac{\sum x_i+\alpha-1}{\beta+1} = \lambda}$$