Q1. ~~A~~ given in the q.s,

$$C = \frac{1}{n} X X^T.$$

We ~~can~~ see that the new covariance is found after the removal of the first principal eigenvector., ~~we know~~

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We know that the ~~p~~ first principle vector v_1 satisfies the condition

$$Cv_1 = \lambda_1 v_1$$

or

$$C = \lambda_1 v_1 v_1^T.$$

Hence,

$$\boxed{\tilde{C} = \frac{1}{n} XX^T - \lambda_1 v_1 v_1^T}$$

Q2. We can see that the if. we remove any eigen value from the matrix, we would not have any change in the other eigen values as the new diagonal matrix has all the information except the removed value.

$$C = \frac{1}{n} XX^T$$

$$C = B \Lambda B'$$

B → eigen vectors

Λ → eigen values.

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We can say that removing correlation is the goal of PCA and hence the eigen vector can be called the ~~PCA~~ principal components. As the correlation has been removed, we can say that v_j will remain a principal eigen vector of $\check{C}$ with same eigen value λ_j.

Q.3 Now, we got $\check{C}$ by removing the principal component v_1 which is the first principal eigen vector. As proved in Q2, we can say that v_2 of C will have the same value in $\check{C}$. But, now as v_1 has been removed, v_2 is the principal component for $\check{C}$. Hence, (assuming v to be a unit norm) we can say that $v = v_2$.

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Q4. Pseudo code for eigenvectors and eigen values.

```
def first_k_vectors(C, k, f):
    eigenvalist, eigenvallist = [], []
    for x in range(k):
        [vec, val] = f(c)
        eigenveclist.append(vec)
        eigenvallist.append(val)
        c = c - [1 *(val @ val.T)]

    return eigenveclist, eigenvallist.
```

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