

1.1.4.

Assuming  $R$  = Asymptotic risk for 1-NN and  $R^*$  is the Bayes risk. As shown in 1.1.3, the correctly classified point will have probability  $(1+\alpha)r^*(n)(1-r^*(n))$ . Now, the NN risk  $R$  is

$$R = \lim_n E[r(n, x_n)].$$

Now, by the dominated convergence theorem

$$R = E\left[\lim_n r(n, x_n)\right]$$

and we know

$$R = E[r(n)]$$

$$\Rightarrow R = E[(1+\alpha)\hat{y}_L(n)\hat{y}_R(n)]$$

$$= E[(1+\alpha)r^*(n)(1-r^*(n))]$$

Since, Bayes risk  $R^*$  is the expectation of  $r^*$ ;

$$R = (1+\alpha)R^*(1-R^*).$$

$$\text{or } R \leq (1+\alpha)R^*(1-R^*).$$