1.1.3

We know that the conditional risk $r(x, x_n')$ is given by the conditional independence of θ and θ_n'.

$$r(x, x_n') = E(L(\theta, \theta_n'))$$
$$= \Pr\{\theta \neq \theta_n' | x, x_n'\}$$
$$= \Pr\{\theta = 1 | x\} \Pr\{\theta_n' = 2 | x_n'\}$$
$$+ \Pr\{\theta = 2 | x\} \Pr\{\theta_n' = 1 | x_n'\}$$

So, we get

$$r(x, x_n') = \hat{\eta}_1(x)\hat{\eta}_2(x_n') + \hat{\eta}_2(x)\hat{\eta}_1(x_n')$$

We see that f_1, f_2 are not continuous at x as they are trivially taken and hence they converge with probability one.

$$r(x, x_n') = 2\hat{\eta}_1(x_0)\hat{\eta}_2(x_0)$$

Conditional Bayes risk as show in 1.1.1 is

$$r^*(x) = \min\{\eta(x), \alpha(1-\eta(x))\}$$

We get by symmetry

$$r(x) \leq (1+\alpha) r^*(x) (1 - r^*(x)).$$