

# **Lecture 8**

## **Database Normalization**

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# Database Anomalies

- In a **Database management system (DBMS)**, **insertion, deletion, and update anomalies** refer to **issues** that can arise when modifying the data in a **relational database**.
- These **anomalies** can **lead to inconsistencies** and **errors** in the **database**.
- **Insertion Anomaly**
- An **insertion anomaly** occurs when we try to **insert a new record** into a database, but due to the way the tables are structured, we are required to provide values for attributes that may not be applicable or available at the time of insertion.
- As a result, we may be **forced to insert incomplete or incorrect data** into the **database**, which can lead to **inconsistencies** and **inaccuracies**.
- **Insertion anomalies** make it **difficult to add new data** without violating the database's **integrity constraints**.

# Database Anomalies

- **Example Insertion Anomaly**
- Consider a **Database** that stores information about **students** and their **courses**:
  - *Students\_Courses(Student\_ID, Name, Course\_ID, Course\_Name)*
- Suppose a student named Alice has not yet enrolled in any course.
- When we try to insert Alice's record into the "Students" table, we are required to provide a Course\_ID since it is a non-null attribute.
- However, since Alice hasn't enrolled in any course yet, we are forced to enter a placeholder or default value for Course\_ID.
- This results in an **insertion anomaly** because we are adding incomplete or irrelevant data to the database.

# Database Anomalies

- **Deletion Anomaly**
- A **deletion anomaly** occurs when we **delete** a record from a database, but unintentionally lose other relevant information that is associated with that record.
- It happens when deleting a record also removes data that is needed by other records or queries in the database.
- **Deletion anomalies** can lead to **data loss** and affect the **overall integrity** of the database.
- **Example** *Students\_Courses(Student\_ID, Name, Course\_ID, Course\_Name)*
- Let us assume that **Alice** is the only student enrolled in a particular course.
- If we delete Alice's record from the "Students" table, we will unintentionally lose information about the course itself.
- Even though the course may still be relevant and other students may want to enroll in it, deleting the single record causes the loss of course information.

# Database Anomalies

- **Update Anomaly**
- An **update anomaly** arises when updating data in a database results in **inconsistencies** or **data duplication**.
- It occurs when modifying an attribute value in one place of the database and failing to update the corresponding values in other places where the same data is stored.
- **Update anomalies** can lead to data inconsistencies and make it challenging to maintain the accuracy and coherence of the database.

# Database Anomalies

- **Example Update Anomaly**
- Imagine a database that tracks inventory for a retail store with table:
- ***Products(Product\_ID, Product\_Name, Quantity, Price)***
- Suppose a product's price changes, and we updated the Price attribute for that product in the table.
- However, if the same product appears in multiple records/tables, we must update the price value in each occurrence.
- If we miss updating any record, it leads to inconsistencies, and the database would contain different prices for the same product, resulting in an **update anomaly**.

# Database Anomalies

- To **mitigate** these **anomalies**, proper **Database Normalization** can be applied.
- By applying **normalization techniques** and structuring the database appropriately, these anomalies can be minimized, ensuring **data integrity** and **consistency** within the DBMS.

# Normalization

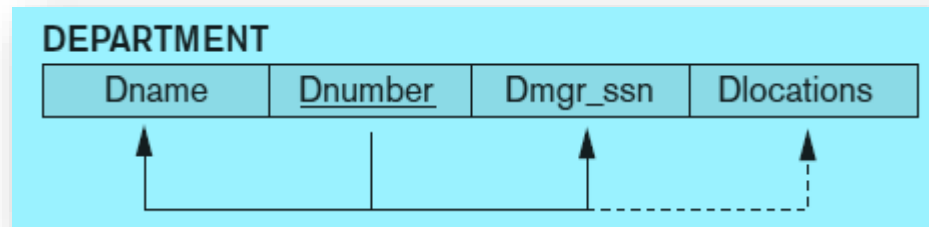
- The **Normalization process**, as first proposed by **Codd (1972)**, takes a **relation schema** through a **series of tests** to certify whether it satisfies a certain **normal form**.
- **Normalization** of data can be considered a process of **analyzing** the given **relation schemas** based on their **FDs** and **primary keys** to achieve the **desirable properties** of:
  - **Minimizing redundancy** and
  - **Minimizing the insertion, deletion, and update anomalies.**
- **Unsatisfactory relation schemas** that do not meet certain constraints are **decomposed** into smaller relation schemas that meet the tests and hence possess the desirable properties.
- Initially, **Codd proposed three normal forms**, which he called **first, second, and third normal form**.
- A **stronger definition of 3NF**—called **Boyce-Codd normal form (BCNF)**—was proposed later by **Boyce and Codd**.



# First Normal Form

- **First normal form (1NF)** was defined to disallow **multivalued attributes** and **composite attributes**.
- It states that the **domain** of an **attribute** must include **only *atomic* (simple, indivisible) values**.
- **Example:**
- Consider the **DEPARTMENT** relation schema.
- We assume that each **department** can have *a number of* **locations**.
- The **DEPARTMENT relation** state is not in **1NF** because **Dlocations** is not an **atomic attribute**.

# First Normal Form



DEPARTMENT

Dname	<u>Dnumber</u>	Dmgr_ssn	Dlocations
Research	5	333445555	{Bellaire, Sugarland, Houston}
Administration	4	987654321	{Stafford}
Headquarters	1	888665555	{Houston}

# First Normal Form

- There are **three** main techniques to achieve **first normal** form for such a relation:
- **Method 1:**
- **Remove** the attribute **Dlocations** that violates **1NF** and place it in a separate relation **DEPT\_LOCATIONS** along with the **primary key** Dnumber of DEPARTMENT.
- The **primary key** of this relation is the combination {**Dnumber, Dlocation**}.
- A distinct tuple in DEPT\_LOCATIONS exists for *each location* of a department.
- This **decomposes** the non-1NF relation into **two 1NF relations**.

DEPARTMENT		
Dname	<u>Dnumber</u>	Dmgr_ssn
Research	5	333445555
Administration	4	987654321
Headquarters	1	888665555

DEPT_LOCATIONS	
<u>Dnumber</u>	<u>Dlocation</u>
1	Houston
4	Stafford
5	Bellaire
5	Sugarland
5	Houston

# First Normal Form

- **Method 2:**
- **Expand the key** so that there will be a separate tuple in the original DEPARTMENT relation for each location of a DEPARTMENT.
- In this case, the **primary key** becomes the combination {**Dnumber, Dlocation**}.
- This solution has the **disadvantage** of introducing *redundancy* in the relation.

DEPARTMENT			
Dname	<u>Dnumber</u>	Dmgr_ssn	<u>Dlocation</u>
Research	5	333445555	Bellaire
Research	5	333445555	Sugarland
Research	5	333445555	Houston
Administration	4	987654321	Stafford
Headquarters	1	888665555	Houston

# First Normal Form

- **Method 3:**
- If a *maximum number of values* is known for the **attribute**—for example, if it is known that *at most three locations* can exist for a department—replace the **Dlocations** attribute by **three atomic attributes: Dlocation1, Dlocation2, and Dlocation3.**
- This solution has the **disadvantage** of introducing *NULL values* if most departments have fewer than three locations.
- It further introduces spurious semantics about the ordering among the location values that is not originally intended.
- Querying on this attribute becomes more difficult; for example, consider how you would write the **query**: *List the departments that have ‘Bellaire’ as one of their locations* in this design.

# Second Normal Form

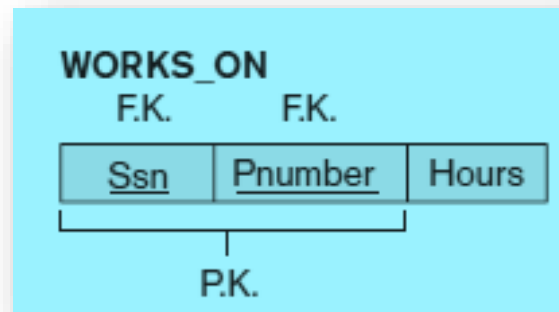
- **Second normal form (2NF)** is based on the concept of *full functional dependency*.
- A functional dependency  $X \rightarrow Y$  is a **full functional dependency** if removal of any attribute  $A$  from  $X$  means that the **dependency does not hold** any more;
- That is, for any attribute  $A \in X$ ,  $(X - \{A\})$  does *not* **functionally determine**  $Y$ .
- A functional dependency  $X \rightarrow Y$  is a **partial dependency** if some attribute  $A \in X$  can be removed from  $X$  and the **dependency still holds**; that is, for some :

$$A \in X, (X - \{A\}) \rightarrow Y.$$

- **Example:**
- $\{\text{Ssn}, \text{Pnumber}\} \rightarrow \text{Hours}$  is a **full dependency** (neither  $\text{Ssn} \rightarrow \text{Hours}$  nor  $\text{Pnumber} \rightarrow \text{Hours}$  holds).
- However, the dependency  $\{\text{Ssn}, \text{Pnumber}\} \rightarrow \text{Ename}$  is **partial** because  $\text{Ssn} \rightarrow \text{Ename}$  holds.

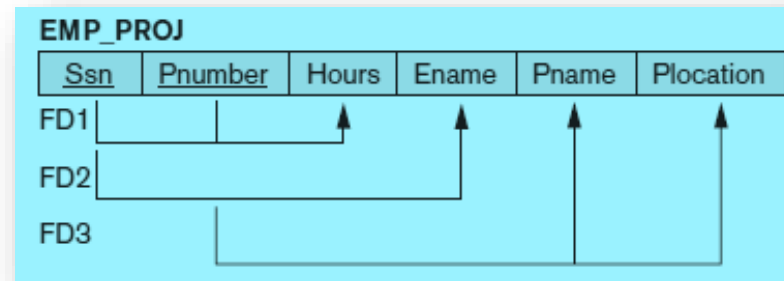
# Second Normal Form

- **Prime attribute**
- An **attribute** of relation schema ***R*** is called a **prime attribute** of ***R*** if it is a **member** of some ***candidate key*** of ***R***.
- An **attribute** is called **nonprime** if it is not a **prime attribute**—that is, if it is not a member of any **candidate key**.
- **Example:** Both **Ssn** and **Pnumber** are **prime attributes** of **WORKS\_ON**, whereas other attribute of **WORKS\_ON** are **nonprime**.



# Second Normal Form

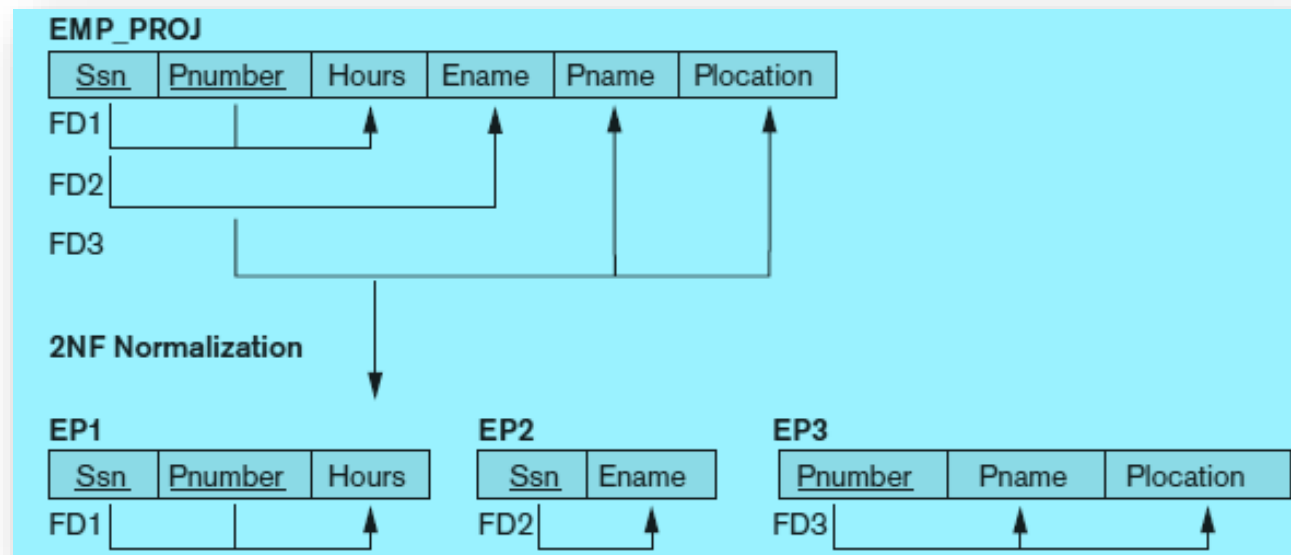
- **2NF Definition**
- A relation schema  $R$  is in **2NF** if every **nonprime attribute**  $A$  in  $R$  is *fully functionally dependent* on the **primary key** of  $R$ .
- **Example:**
- The **EMP\_PROJ** relation is in **1NF** but is not in **2NF**.
- The **nonprime attribute** **Ename** violates **2NF** because of **FD2**, as do the **nonprime attributes** **Pname** and **Plocation** because of **FD3**.
- The **functional dependencies** **FD2** and **FD3** make **Ename**, **Pname**, and **Plocation** **partially dependent** on the **primary key** {**Ssn**, **Pnumber**} of **EMP\_PROJ**, thus violating the **2NF test**.





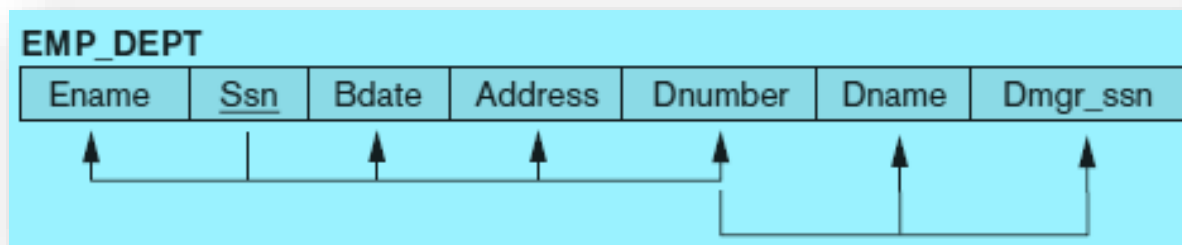
# Second Normal Form

- If a **relation schema** is not in **2NF**, it can be *second normalized* or *2NF normalized* into a number of **2NF relations** in which **nonprime attributes** are associated only with the part of the **primary key** on which they are **fully functionally dependent**.
- Therefore, the **functional dependencies FD1, FD2, and FD3** lead to the **decomposition** of **EMP\_PROJ** into the **three relation** schemas **EP1, EP2, and EP3** shown below, each of which is in **2NF**.



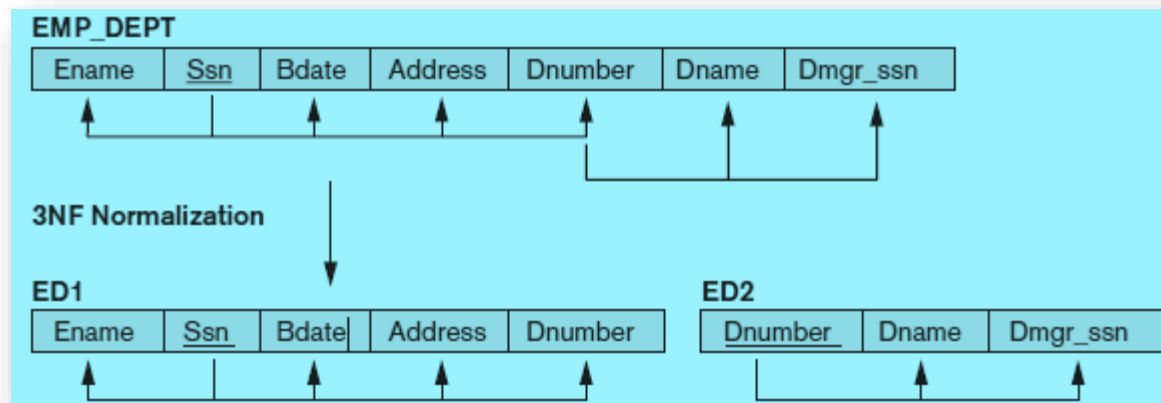
# Third Normal Form

- **Third normal form (3NF)** is based on the concept of *transitive dependency*.
- A functional dependency  $X \rightarrow Y$  in a relation schema  $R$  is a **transitive dependency** if there exists a set of attributes  $Z$  in  $R$  that is **neither a candidate key** nor a **subset of any key of  $R$** , and both  $X \rightarrow Z$  and  $Z \rightarrow Y$  hold.
- The dependency  $Ssn \rightarrow Dmgr\_ssn$  is **transitive** through **Dnumber** in **EMP\_DEPT**, because both the dependencies  $Ssn \rightarrow Dnumber$  and  $Dnumber \rightarrow Dmgr\_ssn$  hold and **Dnumber** is **neither a key** itself **nor a subset of the key** of **EMP\_DEPT**.



# Third Normal Form

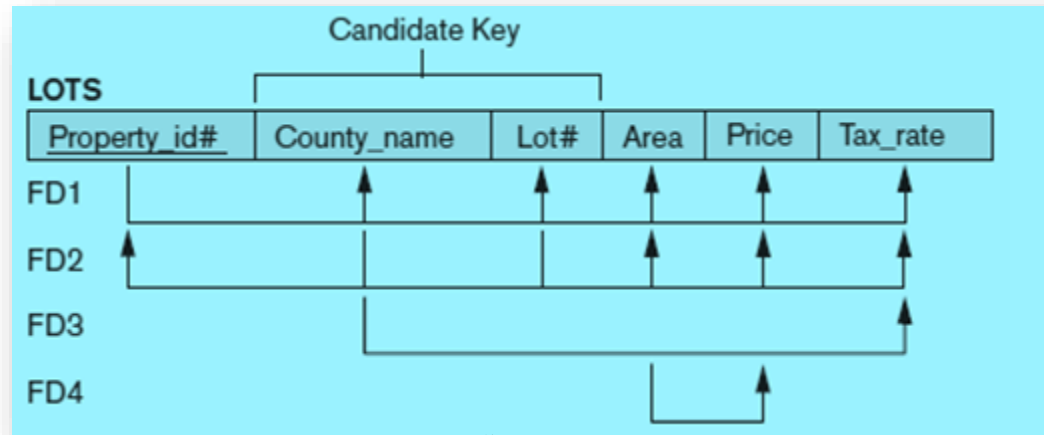
- **Definition 3NF:** A relation schema  $R$  is in 3NF if it satisfies 2NF and no nonprime attribute of  $R$  is transitively dependent on the primary key.
- The relation schema **EMP\_DEPT** is in 2NF, since no partial dependencies on a key exist.
- However, **EMP\_DEPT** is not in 3NF because of the transitive dependency of **Dmgr\_ssn** (and also **Dname**) on **Ssn** via **Dnumber**.
- We can normalize **EMP\_DEPT** by decomposing it into the two 3NF relation schemas **ED1** and **ED2**.



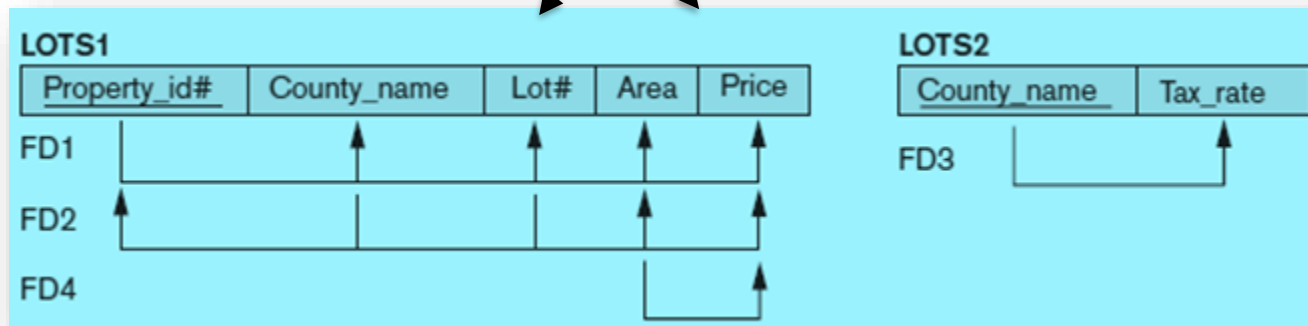
# General Definition of Third Normal Form

- **Definition.** A relation schema  $R$  is in **third normal form (3NF)** if, whenever a *nontrivial* functional dependency  $X \rightarrow A$  holds in  $R$ , **either**
  - (a)  $X$  is a superkey of  $R$ , or
  - (b)  $A$  is a prime attribute of  $R$ .

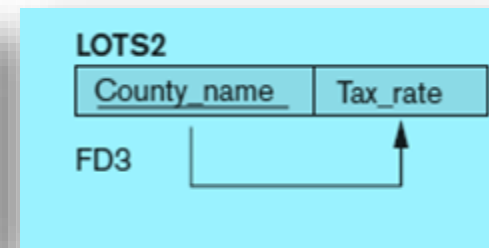
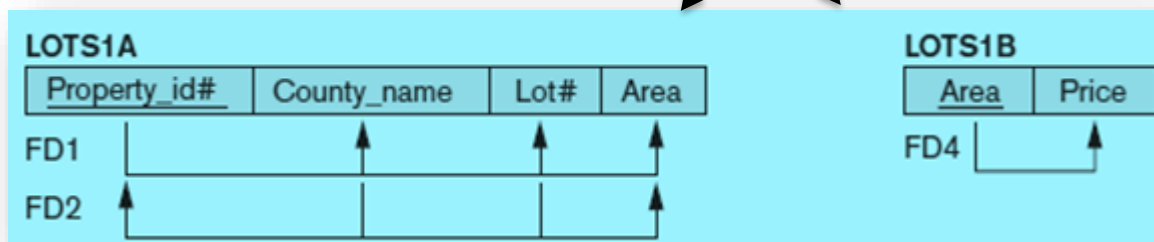
# Example



1NF



2NF



3NF

# Example of 3NF

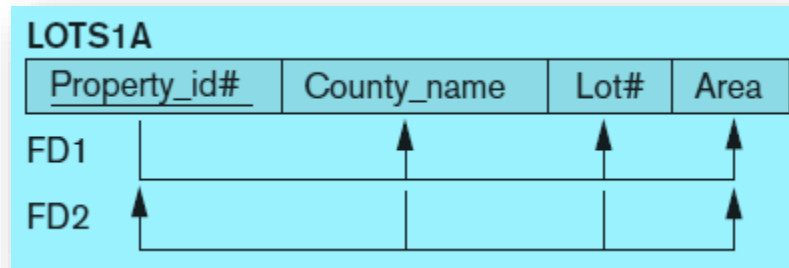
- According to general definition, **LOTS2** is in **3NF**.
- However, **FD4** in **LOTS1** violates **3NF** because **Area** is not a **superkey** and **Price** is **not a prime attribute** in **LOTS1**.
- To normalize **LOTS1** into **3NF**, we decompose it into the relation schemas **LOTS1A** and **LOTS1B**.
- We construct **LOTS1A** by removing the attribute **Price** that violates **3NF** from **LOTS1** and placing it with **Area** (the lefthand side of **FD4** that causes the transitive dependency) into another relation **LOTS1B**.
- Both **LOTS1A** and **LOTS1B** are in **3NF**.

# General Definition of 3NF

- This **general definition** can be **applied *directly*** to test whether a **relation schema** is in **3NF**.
- It **does *not*** have to go through **2NF first**.
- If we apply the above **3NF definition** to **LOTS** with the dependencies **FD1** through **FD4**, we find that ***both* FD3 and FD4 violate 3NF**.
- Therefore, we could decompose **LOTS** into **LOTS1A, LOTS1B, and LOTS2** directly.

# Boyce-Codd Normal Form

- **Boyce-Codd normal form (BCNF)** was proposed as a simpler form of **3NF**, but it was found to be **stricter than 3NF**.
- That is, **every relation in BCNF is also in 3NF**; however, a **relation in 3NF is *not necessarily* in BCNF**.
- **Definition BCNF:** A relation schema ***R*** is in **BCNF** if whenever a ***nontrivial* functional dependency  $X \rightarrow A$**  holds in ***R***, then ***X*** is a **super key of *R***.
- Consider the **LOTS1A relation** schema with its two functional dependencies **FD1** and **FD2**.



- Suppose that we have **thousands of lots** in the relation but the lots are from **only two counties: ABC and XYZ**.

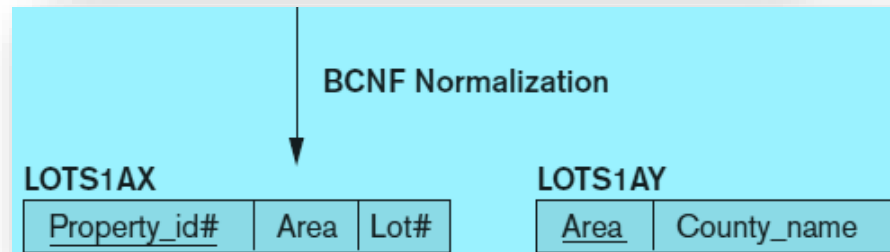
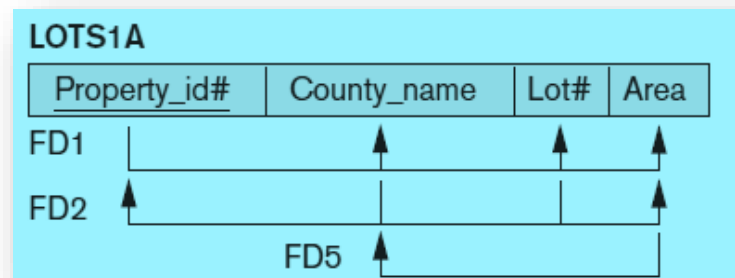


# Boyce-Codd Normal Form

- Suppose also that **lot sizes** in **ABC County** are only **0.5, 0.6, 0.7, 0.8, 0.9, and 1.0 acres**, whereas **lot sizes** in **XYZ County** are restricted to **1.1, 1.2, ..., 1.9, and 2.0 acres**.
- In such a situation we would have the **additional functional dependency**

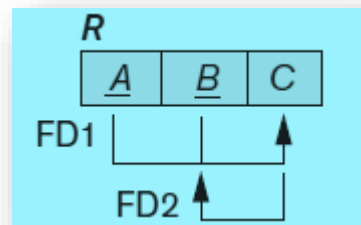
**FD5:  $Area \rightarrow County\_name$**

- If we add this to the other dependencies, the relation schema **LOTS1A** still is in **3NF** because **County\_name** is a **prime attribute** but it is **not in BCNF** as **area** is **not a superkey**.



# Boyce-Codd Normal Form

- We can decompose **LOTS1A** into **two BCNF relations LOTS1AX and LOTS1AY**.
- But **this decomposition loses the functional dependency FD2** because its attributes no longer coexist in the same relation after **decomposition**.
- **Example:** Consider a relation schema **R**



- The relation schema **R** is in **3NF**, but not in **BCNF**.

# Desirable properties of Decomposition

- There are **two desirable properties** of a **decomposition** of a **relation schema**:
  1. **Loss-less Join(Non-additive join)**
  2. **Dependency preservation**

# Lossy Decomposition

- Suppose we have a relation  $R(ABC)$  with  $F=(A \rightarrow B, C \rightarrow B)$
- $R$  is decomposed into **two relations**  $R1(A,B)$  and  $R2(B,C)$

$R(ABC)$

A	B	C
A1	B1	C1
A3	B1	C2
A2	B2	C3
A4	B2	C4

$R1$

A	B
A1	B1
A3	B1
A2	B2
A4	B2

$R2$

B	C
B1	C1
B1	C2
B2	C3
B2	C4

$R1 \bowtie R2$

A	B	C
A1	B1	C1
A1	B1	C2
A3	B1	C1
A3	B1	C2
A2	B2	C3
A2	B2	C4
A4	B2	C3
A4	B2	C4

- There are **spurious tuples** in joined relation.
- Spurious tuples** results in **loss of information (not loss of tuples)**.
- Thus  $R \rightarrow R1 R2$  is a **lossy decomposition**.

# Lossless Decomposition

- Suppose we have a relation **R(XYZ)** with **F=(XY→X, X→Y, X→Z)**
- R is decomposed into two relations R1(X,Y) and R2(X,Z)**

R(ABC)		
X	Y	Z
X1	Y1	Z1
X2	Y2	Z2
X3	Y2	Z1
X4	Y1	Z2

R1	
X	Y
X1	Y1
X2	Y2
X3	Y2
X4	Y1

R2	
X	Z
X1	Z1
X2	Z2
X3	Z1
X4	Z2

R1 ⋈ R2		
X	Y	Z
X1	Y1	Z1
X2	Y2	Z2
X3	Y2	Z1
X4	Y1	Z2

- There is **no spurious tuples** in joined relation.
- Thus **R → R1 R2** is a **loss-less decomposition**.

# Lossless Decomposition

**Definition:** A decomposition  $D=\{R_1, R_2, \dots, R_m\}$  of  $R$  has the **lossless join property** w.r.t. the set of dependencies  $F$  on  $R$  if, for every relation instance  $r$  of  $R$  that satisfies  $F$ , the following holds:  $\bowtie (\pi_{R_1}(r) \dots \pi_{R_m}(r)) = r$

## Check lossless Decomposition:

- Let  $R$  be a relation schema, and let  $F$  be a set of functional dependencies on  $R$ .
- Let  $R_1$  and  $R_2$  form a **decomposition** of  $R$ .
- This **decomposition** is a **lossless-join decomposition** of  $R$  if at least one of the following functional dependencies is in  $F^+$ :
  - $R_1 \cap R_2 \rightarrow R_1$
  - $R_1 \cap R_2 \rightarrow R_2$
- In other words, if  $R_1 \cap R_2$  forms a **super key** of either  $R_1$  or  $R_2$ , the decomposition of  $R$  is a **lossless-join decomposition**.

# Dependency Preservation

- Let  $F$  be a set of **functional dependencies** on a schema  $R$ , and let  $R_1, R_2, \dots, R_n$  be a **decomposition** of  $R$ .
- The **restriction** of  $F$  to  $R_i$  is the set  $F_i$  of all functional dependencies in  $F^+$  that include only attributes of  $R_i$ .
- Note that the definition of **restriction** uses **all dependencies** in  $F^+$ , not just those in  $F$ .
- For instance, suppose a relation schema  $R(ABC)$ ,  $F = \{A \rightarrow B, B \rightarrow C\}$ , and we have a **decomposition** into  $AC$  and  $AB$ .
- The **restriction** of  $F$  to  $AC$  is then  $F_1 = A \rightarrow C$ , since  $A \rightarrow C$  is in  $F^+$ , even though it is **not** in  $F$ , restriction of  $F$  to  $AB$  is  $F_2 = A \rightarrow B$ .
- **Let  $F' = F_1 \cup F_2 \cup \dots \cup F_n$**
- Then this **decomposition** is dependency preserving if  **$F'^+ = F^+$**

# Dependency Preservation

- **Example 1:**

- Relation schema  $R(ABC)$ ,  $F = \{A \rightarrow B, B \rightarrow C\}$
- The two decompositions are  $R1(AC)$ ,  $F1 = \{A \rightarrow C\}$  and  $R2(AB)$ ,  $F2 = \{A \rightarrow B\}$
- Is not a dependency preserving decomposition as  $F'^+$  is not equal to  $F^+$ .

$$F' = F1 \cup F2 = \{A \rightarrow C, A \rightarrow B\}$$

- **Example 2:**

- Relation schema  $R(ABCD)$ ,  $F = \{A \rightarrow B, A \rightarrow C, A \rightarrow D\}$
- The two decompositions are  $R1(ABD)$ ,  $F1 = \{A \rightarrow B, A \rightarrow D\}$  and  $R2(BC)$ ,  $F2 = \{ \}$
- Thus  $F' = F1 \cup F2 = \{A \rightarrow B, A \rightarrow D\}$
- It is **not a dependency preserving** decomposition as  $F'^+$  will not be equal to  $F^+$ .



# Summary: BCNF

- **Lossless-join** and **Dependency preserving** decomposition into **BCNF impossible**?
- It is **not possible** to have **all three** of the following:
  - (1) **Guaranteed lossless design**
  - (2) **Guaranteed dependency preservation**
  - (3) **All relations in BCNF.**
- The **first condition** is a **must** and **cannot be compromised**.
- The **second condition** is **desirable**, but not a must, and may have to be relaxed if we insist on achieving **BCNF**.

# Exercise 1

- Suppose that we decompose the schema  $R = (A, B, C, D, E)$  into  $R_1(A, B, C)$  and  $R_2(A, D, E)$ . Show that this decomposition is a **lossless-join decomposition** if the following set  $F$  of functional dependencies holds:  $F = \{ A \rightarrow BC, CD \rightarrow E, B \rightarrow D, E \rightarrow A \}$
- **Solution:**
- As  $R_1 \cap R_2 = A$
- Restriction of  $R_1$  over  $F$  is  $F_1' = A \rightarrow BC$  thus  $A$  is a *candidate key* of  $R_1$ .
- Restriction of  $R_2$  over  $F$  is  $F_2' = E \rightarrow A$  thus  $E$  is *not a candidate key* of  $R_2$ .
- As  $R_1 \cap R_2 = A$  is a **superkey** of  $R_1$  thus this **decomposition** is a **lossless-join decomposition**.

# Exercise 2

- Show that the following decomposition of the schema  $R = (A, B, C, D, E)$  is not a lossless-join decomposition:  $R1(A, B, C)$ ,  $R2(C, D, E)$  if the following set  $F$  of functional dependencies holds:  $F = \{A \rightarrow BC, CD \rightarrow E, B \rightarrow D, E \rightarrow A\}$
- **Solution:**
- As  $R1 \cap R2 = C$
- Restriction of  $R1$  over  $F$  is  $F1' = A \rightarrow BC$  thus  $A$  is a *candidate key* of  $R1$ .
- Restriction of  $R2$  over  $F$  is  $F2' = CD \rightarrow E$  thus  $CD$  is a *candidate key* of  $R2$ .
- As  $R1 \cap R2 = C$  is **not a superkey** of either  $R1$  or  $R2$  thus this **decomposition** is not a **lossless-join decomposition** .

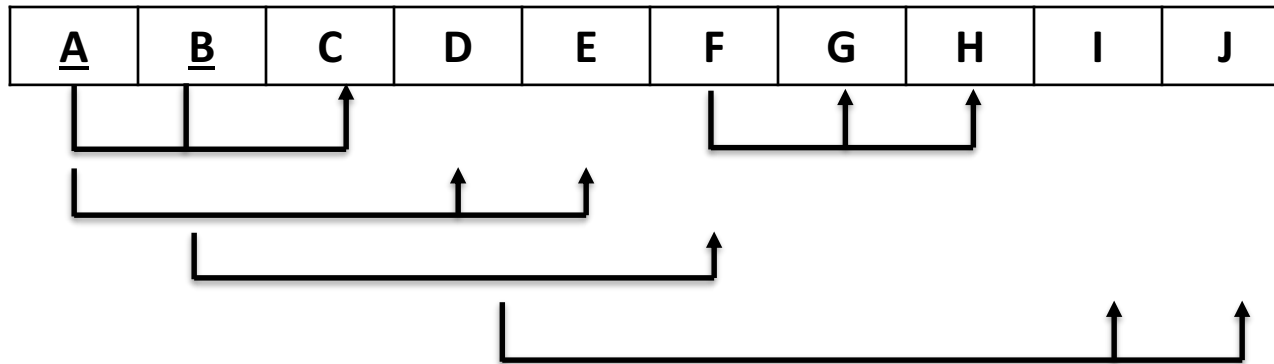
# Exercise 3

- Consider the universal relational schema  $R(A, B, C, D, E, F, G, H, I, J)$  and a set of following functional dependencies  $F = \{AB \rightarrow C, A \rightarrow DE, B \rightarrow F, F \rightarrow GH, D \rightarrow IJ\}$ . Determine the **keys** for  $R$ ? Decompose  $R$  into **2NF** and then in **3NF**.
- Solution:** Here, **AB** are not present in the **RHS**, so Let us find **AB<sup>+</sup>** w.r.t. the set of functional dependencies **F**:
  - AB<sup>+</sup> = AB**
    - = ABC using  $AB \rightarrow C$
    - = ABCDE using  $A \rightarrow DE$
    - = ABCDEF using  $B \rightarrow F$
    - = ABCDEFGH using  $F \rightarrow GH$
    - = **ABCDEFGHIJ** using  $D \rightarrow IJ$

As **AB<sup>+</sup>** contains **all the attributes** of  $R$  thus **AB** is a **key attribute** of  $R$ .

# Exercise 3

- Given a relational schema  $R(A, B, C, D, E, F, G, H, I, J)$  and a set of following functional dependencies  $F = \{AB \rightarrow C, A \rightarrow DE, B \rightarrow F, F \rightarrow GH, D \rightarrow IJ\}$ .



**2NF**

$R1(\underline{A}, B, C)$

$R2(\underline{A}, D, E, I, J)$

$R3(\underline{B}, F, G, H)$



**3NF**

$R1(\underline{A}, \underline{B}, C)$

$R21(\underline{A}, D, E) \quad R22(\underline{D}, I, J)$

$R31(\underline{B}, F) \quad R32(\underline{F}, G, H)$

# Exercise 4

- Consider the universal relation  $R=\{A, B, C, D, E, F, G, H, I, J\}$  and the set of functional dependencies  $F=\{ AB \rightarrow C, BD \rightarrow EF, AD \rightarrow GH, A \rightarrow I, H \rightarrow J\}$ . What is the key for R? Normalize the relation R upto **3NF**, justify your answer.

- Solution:** To find the key check the RHS of the given functional dependencies, and find the closure of the attributes not present at the RHS.

- Here, **ABD** are not present in the **RHS**, so

$$(ABD)^+ = ABD = ABCD \text{ (since } AB \rightarrow C \text{)}$$

$$= ABCDEF \text{ (since } BD \rightarrow EF \text{)}$$

$$= ABCDEFGH \text{ (since } AD \rightarrow GH \text{)}$$

$$= ABECDFGHI \text{ (since } A \rightarrow I \text{)}$$

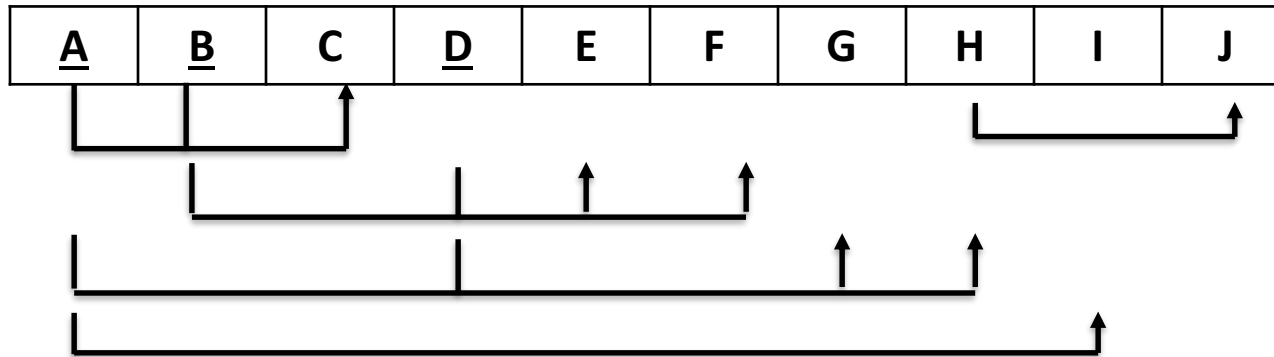
$$= ABECDFGHIJ \text{ (since } H \rightarrow J \text{)}$$

$$= \mathbf{ABCDEFGHI}$$

As  $ABD^+$  contains **all the attributes** of R thus **ABD** is a **key attribute** of R.

# Exercise 4

- Given a relational schema  $R(A, B, C, D, E, F, G, H, I, J)$  and a set of following functional dependencies  $F = \{AB \rightarrow C, BD \rightarrow EF, AD \rightarrow GH, A \rightarrow I, H \rightarrow J\}$ .



## 2NF

$R1(\underline{A}, \underline{B}, C)$

$R2(\underline{B}, \underline{D}, E, F)$

$R3(\underline{A}, \underline{D}, G, H, J)$

$R4(\underline{A}, I)$



## 3NF

$R1(\underline{A}, \underline{B}, C)$

$R2(\underline{B}, \underline{D}, E, F)$

$R31(\underline{A}, \underline{D}, G, H), R31(\underline{H}, J)$

$R4(\underline{A}, I)$