OBE

REFNO044662

B.Sc. (Hons.) SEMESTER V EXAMINATION 2021-22

COMPUTER SCIENCE

CS – 106 : Discrete Mathematical Structures

Time : 4.30 hours **Max. Marks : 70**

Instructions

1. **The Question Paper contains 08 questions out of which you are required to answer any 04 questions. The question paper is of 70 marks with each question carrying 17.5 marks.**

   प्रश्नपत्र में आठ प्रश्न पूँछे गये हैं जिनमें से 4 प्रश्नों का उत्तर देना है। प्रश्नपत्र 70 अंकों का है, जिसमें प्रत्येक प्रश्न 17.5 अंक का है।

2. The total duration of the examination will be **4.30 hours** (Four hours and thirty minutes), which includes the time for downloading the question paper from the Portal, writing the answers by hand and uploading the hand-written answer sheets on the portal.

   परीक्षा का कुल समय 4.30 घंटे का है जिसमें प्रश्नपत्र को पोर्टल से डाउनलोड करके पुनः हस्तलिखित प्रश्नों का उत्तर पोर्टल पर अपलोड करना है।

3. For the students with benchmark disability as per Persons with Disability Act, the total duration of examination shall be **6 hours** (six hours) to complete the examination process, which includes the time for downloading the question paper from the Portal, writing the answers by hand and uploading the hand-written answer sheets on the portal. .

   दिव्यांग छात्रों के लिये परीक्षा का समय 6 घंटे निर्धारित हैं जिसमें प्रश्नपत्र को पोर्टल से डाउनलोड करना एवं हस्तलिखित उत्तर को पोर्टल पर अपलोड करना है।

4. Answers should be hand-written on a plain white A4 size paper using black or blue pen. Each question can be answered in upto 350 words on 3 (Three) plain A4 size paper (only one side is to be used).
   हस्तलिखित प्रश्नों का उत्तर सादे सफेद A4 साइज के पन्ने पर काले अथवा नीले कलम से लिखा होना चाहिये। प्रत्येक प्रश्न का उत्तर 350 शब्दों तक तीन सादे पृष्ठ A4 साइज में होना चाहिये। प्रश्नों के उत्तर के लिए केवल एक तरफ के पृष्ठ का ही उपयोग किया जाना चाहिए।

5. Answers to each question should start from a fresh page. All pages are required to be numbered. You should write your Course Name, Semester, Examination Roll Number, Paper Code, Paper title, Date and Time of Examination on the first sheet used for answers.

   प्रत्येक प्रश्न का उत्तर नये पृष्ठ से शुरू करना है। सभी पृष्ठों को पृष्ठांकित करना है। छात्र को प्रथम पृष्ठ पर प्रश्नपत्र का विषय, सेमेस्टर, परीक्षा अनुक्रमांक, प्रश्नपत्र कोड, प्रश्नपत्र का शीर्षक, दिनांक एवं समय लिखना है।

Questions

1. (a). Define a successor of a set. Give two sets A and B such that $A \in B$ and $A \subseteq B$. **4**

   (b). Define difference and symmetric difference of set. Give in general how these two set operation is achieved when sets are represented using bit strings. **5**

   (c). Define partition and covering of a set. Generate universal set E using $A_1 = \{3,4,5\}$, $A_2 = \{1,2\}$ and then give the partition of generated universal set using sets A_1, A_2. **3**

(d) List the ordered pairs in the equivalence relation R, which generates the partitions $A_1=\{1,2,3\}$, $A_2=\{4,5\}$, and $A_3=\{6\}$ of $S = \{1, 2, 3, 4, 5, 6\}$ and also represent R using matrix and diagraph. **5.5**

2. **(a)** Consider following relation, R, on set $A =\{ 1, 2,3, 4\}$

$R = \{(a,b) \mid a \text{ divides } b\}$

(i) List all ordered pairs of relation R. **1**

(ii) Represent relation R using diagraph. **1**

(iii) Give relation matrix of the relation R. **1**

(iv) Find the matrix of inverse relation relation R. **1**

(b) (i) Define partial order relation and POSET. **2**

(ii) Using only matrix operation show that the relation R defined in question 2(a) is partial order relation and hence (A,R) is POSET. **3**

(iii) Draw the Hasse diagram of the above partial order relation. **1**

(iv) Find the maximal and the minimal element, if exist, of the POSET 2(b)(ii). **1**

(v) Find the greatest and the least element, if exist, of the POSET 2(b)(ii). **1**

(c) Define lattice. Sow that (S_{30}, D) is lattice, where S_{30}: the set of divisor of 30, D: division relation. **5.5**

3. **(a)** Let A be a set having n elements. Answer the following:

(i) Compute total possible reflective relations on A. **2**

(ii) Compute total possible symmetric relations on A. **2**

(iii) Compute total possible symmetric and reflexive relations on A. **2**

(iv) Compute total possible anti-symmetric relations on A. **2**

(b) Define number theoretic function, partial function and primitive recursive function and give at least one example of each. **3**

(c) Show that the total one-one function from A to B is $n(n-1)(n-2)....(n-m+1)$. Where $|A| = m$ and $|B| = n$. **2.5**

(d) Let a relation defined over a finite set. Answer the following considering the matrix representation of the relation: **4**

(i) How to check that the relation is function.

(ii) How to check that the relation is one-to-one function.

(iii) How to check that the relation is onto function.

(iv) How to check that the relation is one-to-one and onto function.

4. **(a)** Prove the following ,here T denotes the Tautology:

i. $((\neg P \wedge (P \vee Q)) \rightarrow Q) \Leftrightarrow T$ **2**

ii. $((P \wedge (P \rightarrow Q)) \rightarrow Q) \Leftrightarrow T$ **2**

(b) Represent following facts using proposition. And find what rules of inference is used in each of these argument: **6**

(i) Alice is mathematics major. Therefore, Alice is either mathematics major or a computer science major.

(ii) Jerry is mathematics major and a computer science major. Therefore, Jerry is mathematics major.

(iii) If, I go swimming, then, I will sky in the sun too long. If, I sky in the sun too long, then I will sun burn. Therefore, if I go swimming, then I will sun burn.

(c) Write the negation of propositions and conclusions given in question 4(b). **4.5**

(d) What is limitation of statement calculus and how it is removed in predicate calculus? **3**

5. **(a)** (i). Suppose the domain of the proposition function $P(x)$ consists of the integers { 1, 2, 3, 4, 5}. Express following statements without using quantifiers: **5**

$$\exists x P(x),\ \forall x P(x),\ \neg\exists x P(x),\ \exists x \neg P(x),\ \forall x \neg P(x)$$

(ii). Explain the concept of consistency and inconsistency of premises. Describe the concept of the proof by contradiction. **6**

(b) Let P(x) denotes the statement "$x \leq 4$". What are these truth values? **3**

i) P(0) ii) P(4) iii) P(6)

(c) Show that $(\exists x)M(x)$ follows logically from the premises $(\forall x)\big(H(x) \rightarrow M(x)\big)$ and $(\exists x)H(x)$. **3.5**

6. **(a)** Model the round-robin tournament using a graph. **3**

(b) What do the in-degree and out-degree of a vertex in graph 6(a) represent? **2**

(c) Draw the graph of following: **4**

i. K_7 ii. C_7 iii. W_7 iv. Q_4

(d) For which value of n are these graphs bipartite? And also find their chromatic number. **4**

i. K_n ii. C_n iii. W_n iv. Q_n

(e) Give the adjacency matrix, adjacency list, and adjacency matrix representation of the graph K_7. **4.5**

7. **(a)** Is five houses can be connected to two utilities without connection crossing? Justify your answer. **2**

(b) Connected planar graph has 30 edges. If planar representation of this graph divides the plane into 20 regions, how many vertices does this graph have? **4**

(c) Find the chromatic number of the Hasse diagram of the POSET (X_{12}, D) as well as maximal, minimal, least and greatest element. Where X_{12}: set of divisor of 12, and D: $\{(x,y) / x \text{ divides } y\}$. **5**

(d) Is the Hasse diagram given in question 7(c) has an Euler circuit? If yes, give at least one Euler circuit present in this graph. **3.5**

(d) Determine whether following statements are true or false or not valid statement: 3

i. $2+2=4$ iff $1+1=2$.

ii. If $1+1=3$, then unicorn exist.

iii. $x+1=2$

8. (a) Define phrase-structured grammar and describe the Chomsky classification of phrase-structured grammar. 3

(b) Let $G = (V, S, T, P)$ be the phrase-structured grammar with $V = \{0, 1, A, B, S\}$, $T = \{0, 1\}$, and set of productions $P = \{S \to 0A, S \to 1A, A \to 0B, B \to 1A, B \to 1\}$. Answer the following:

(i) Show that 10101 belong to the language generated by G. 2

(ii) Show that 10110 does not belong to the language generated by G. 2

(iii) What is the language generated by G? 2

(c) Give production rules in Backus-Naur form that generate all identifiers in the C programming language. In C an identifier starts with a letter or an underscore (_) that is followed by one or more lowercase letters, uppercase letters, underscores, and digits. 3

(d) Give formal definition of finite state machine, which produces an output. Model two input logical AND gate using any finite state machine. 5.5