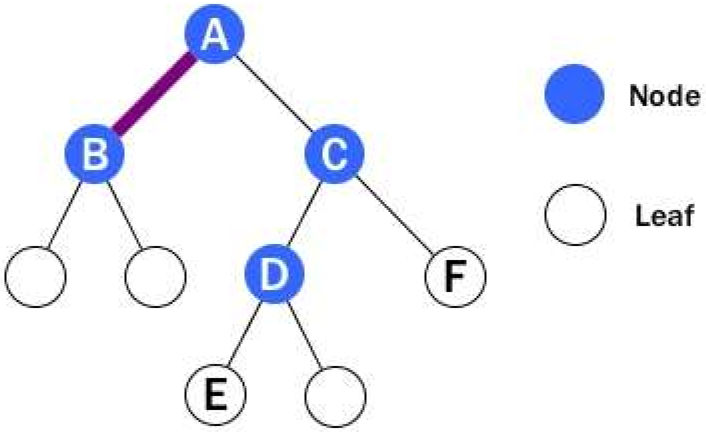
Height, Depth and Level of a Tree

Three important properties of trees: *height*, *depth* and *level*, together with *edge* and *path*.

Edge

Edge – connection between one node to another.

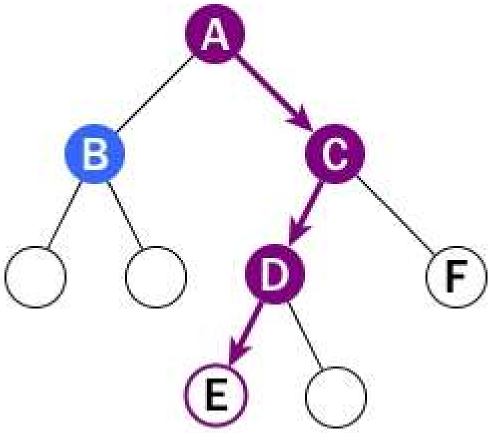


About Edge

An example of edge is shown above between *A* and *B*. Basically, an edge is a line between two nodes, **or a node and a leaf**.

Path

Path – a sequence of nodes and edges connecting a node with a descendant.



About Path

A path starts from a node and ends at another node or a leaf. Please don't look over the following points:

1. When we talk about a path, it includes all nodes and all edges along the path, *not just edges*.

2. The direction of a path is strictly from top to bottom and cannot be changed in middle.

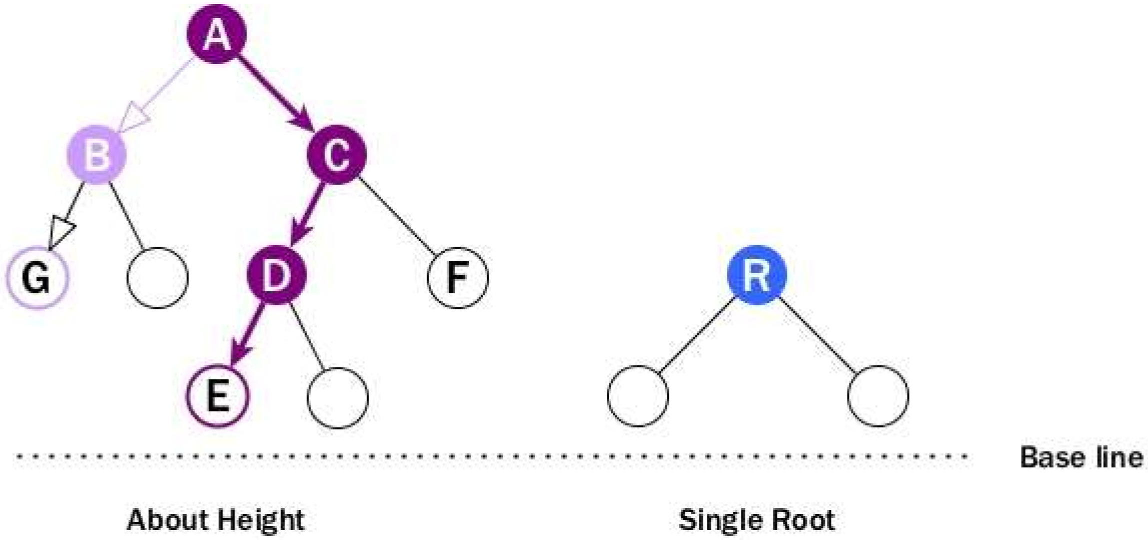
    - In the diagram, we can't really talk about a path from *B* to *F* although *B* is above *F*.
    - Also there will be no path starting from a leaf or from a child node to a parent node.

Height

Height of node – The height of a node is the number of edges on the **longest downward path between that node and a leaf.**

At first, we can see the above definition has a redundant term - *downward* - inside.

As we already know: path can only be downward.



When looking at height:

1. Every node has height. So *B* can have height, so does *A*, *C* and *D*.
2. Leaf cannot have height as there will be no path starting from a leaf.
3. It is the longest path from the node **to a leaf**. So *A*'s height is the number of edges of the path to *E*, NOT to *G*. And its height is 3.

The height of the root is 1.

Height of tree –The height of a tree is the number of edges on the longest downward path between the root and a leaf.

So **the height of a tree is the height of its root**.

Frequently, we may be asked the question:

***What is the max number of nodes a tree can have if the height of the tree is h?*.**

Of course the answer is $2h-1$

When $h=1$, the number of node inside is 1, which is just the root; also when a tree has just root, the height of the root is 1. Hence, the two inductions match.

How about giving a height of 0?

Then it means we don't have any *node* in the tree; but still we may have *leaf* inside (note that in this case we may not call it *root* of the tree as it makes not much sense).

This is why in most languages, the type of a tree can be a leaf alone.

Moreover, when we use $2h-1$

to calculate the max number of nodes, leaves are not taken into account.

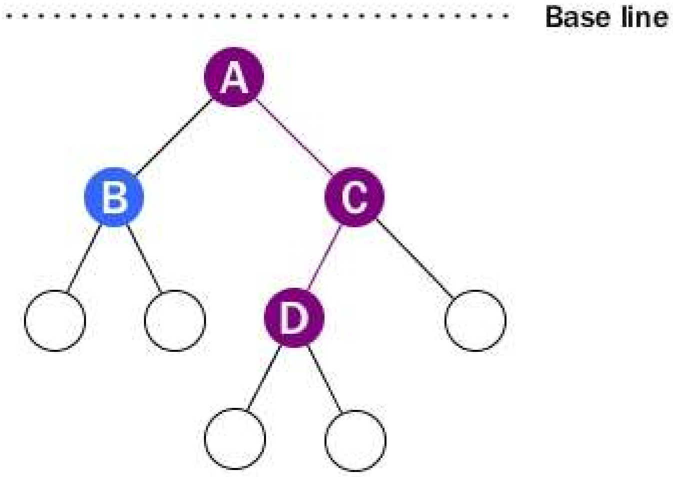
***Leaf* is not *Node*.**

It carries no key or data, and acts only like a STOP sign.

We need to remember this when we deal with properties of trees.

Depth

Depth –The depth of a node is the number of edges from the node to the tree's root node.



About Depth

We don't care about path any more when depth pops in.

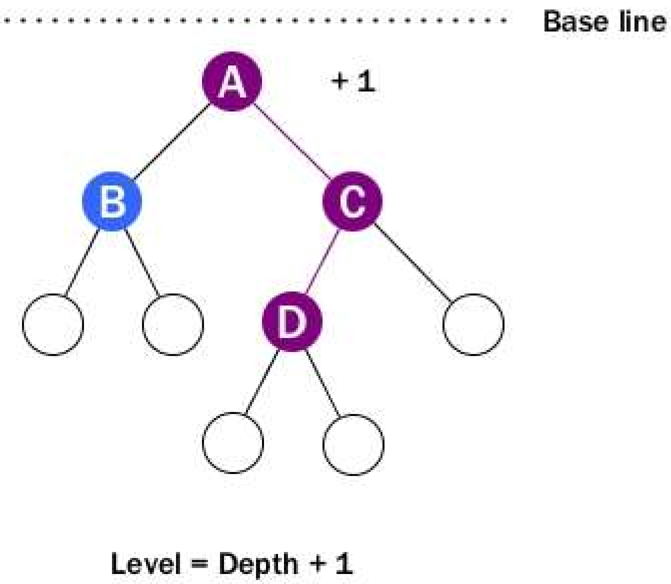
We just count how many edges between the targeting node and the root, ignoring directions. For example, *D*'s depth is 2.

Recall that when talking about height, we actually imply a baseline located at bottom. For depth, the baseline is at top which is root level. That's why we call it depth.

Note that **the depth of the root is 0**.

Level

Level – The level of a node is defined by **1 + the number of connections between the node and the root.**



Simply, **level is depth plus 1.**

The important thing to remember is when talking about level, it **starts from 1** and **the level of the root is 1**.

We need to be careful about this when solving problems related to level.
