① Find the eigen values and vectors Real and distinct vectors

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ 4 & -4 & 5 \end{bmatrix}$$

$$|A-\lambda I| = \begin{vmatrix} 1-\lambda & 2 & -1 \\ 1 & -\lambda & 1 \\ 4 & -4 & 5-\lambda \end{vmatrix} = 0.$$

Expand a long ROW (or) column using main

$$(1+\lambda)\begin{vmatrix} -\lambda & 1 \\ -4 & 5-\lambda \end{vmatrix} - 2\begin{vmatrix} 1 & 1 \\ 4 & 5-\lambda \end{vmatrix} - 1\begin{vmatrix} 1 & -\lambda \\ 4 & -4 \end{vmatrix}$$

$$(1-\lambda)(-\lambda(5-\lambda)-(-4)] - 2[5-\lambda-4] - 1[-4-(4)(-\lambda)]$$

$$(1-\lambda)[-5\lambda+\lambda^2+4] - 2(1-\lambda) - [4+4\lambda)$$

$$-5\lambda+\lambda^2+4+5\lambda^2-\lambda^3-4\lambda-2+2\lambda+4-4\lambda = -\lambda^3+6\lambda^2$$

$$-\lambda^3+6\lambda^2-11\lambda+6$$

$$\begin{vmatrix} 1-\lambda & 2 & -1 \\ 1 & -\lambda & 1 \\ 4 & -4 & 5-\lambda \end{vmatrix} \rightarrow -\lambda^3+6\lambda^2-11\lambda+6$$

$(-\lambda^3+6\lambda^2-11\lambda+6) \Rightarrow$ factor $(\lambda-3)(\lambda-1)(\lambda-2)=0$

$\lambda = 1, 2, 3$

$\lambda = 1$

Subtract 1 down the main diagonal

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ 4 & -4 & 5 \end{bmatrix}$$

$$(A - 1I)\,x = 0$$

$$\begin{bmatrix} 0 & 2 & -1 \\ 1 & -1 & 1 \\ 4 & -4 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & -5 \\ 0 & 0 & 0 \end{bmatrix}$$

$$v_1 = -\tfrac{1}{2} v_3$$

$$v_2 = \tfrac{1}{2} v_3$$

$$v_3 = \begin{bmatrix} -\tfrac{1}{2} \\ -\tfrac{1}{2} \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix} \rightarrow \text{eigen vector.}$$

$\lambda = 2$

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ 4 & 4 & 5 \end{bmatrix}$$

$(A - 2I)x = \bar{0}$

$$\begin{bmatrix} -1 & 2 & -1 \\ 1 & -2 & 1 \\ 4 & 4 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & -2.5 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} -1/2 \\ 1/4 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1/2 \\ 1/4 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 4 \end{bmatrix}$$ sayan vektör

R=3

$$\begin{bmatrix} -2 & 2 & -1 \\ 1 & -3 & 1 \\ 4 & -4 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -0.25 \\ 0 & 1 & -2.5 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} -1/4 \\ 1/4 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} -1 \\ 1 \\ 4 \end{bmatrix}$$ sayan vektör

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>> A = [1 2 -1;1 0 1;4 -4 5]

A =

     1     2    -1
     1     0     1
     4    -4     5

>> eig(A)

ans =

    3.0000
    2.0000
    1.0000

>> [V,D] = eig(A)

V =

   -0.2357    0.4364    0.4082
    0.2357   -0.2182   -0.4082
    0.9428   -0.8729   -0.8165


D =

    3.0000         0         0
         0    2.0000         0
         0         0    1.0000

>> V = bsxfun(@rdivide, V, abs(min(V,1)))

V =

    -1     1     1
     1    -1    -1
     1    -1    -1
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