Find eigen values and eigen vectors of the given matrix $A = \begin{bmatrix} 2 & 0 & 4 \\ 0 & 3 & 0 \\ 0 & 1 & 2 \end{bmatrix}$

eigen values can be found from $\det(A - \lambda I) = 0$

$$A - \lambda I = \begin{bmatrix} 2 & 0 & 4 \\ 0 & 3 & 0 \\ 0 & 1 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} 2-\lambda & 0 & 4 \\ 0 & 3-\lambda & 0 \\ 0 & 1 & 2-\lambda \end{bmatrix}$$

$$\det(A - \lambda I) = 0$$

$$(2-\lambda)\left[(3-\lambda)(2-\lambda)\right] - 0 + 4(0-0) = 0$$

$$(2-\lambda)(3-\lambda)(2-\lambda) = 0$$

$$\lambda = 2, 2, 3$$

The eigen values are 2 and 3.

To find eigen vectors :

1) For $\lambda = 2$

$$A - \lambda I = \begin{bmatrix} 2-2 & 0 & 4 \\ 0 & 3-2 & 0 \\ 0 & 1 & 2-2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 4 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$(A - \lambda I)x = 0$$

$$\begin{bmatrix} 0 & 0 & 4 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$4x_3 = 0$$

$$x_2 = 0$$

$$\therefore x = \begin{bmatrix} x_1 \\ 0 \\ 0 \end{bmatrix}$$

If $x_1 = 1$, $x = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$.

ii) For $\lambda = 3$

$$A - \lambda I = \begin{bmatrix} -1 & 0 & 4 \\ 0 & 0 & 0 \\ 0 & 1 & -1 \end{bmatrix}$$

$$(A - \lambda I)x = 0$$

$$\begin{bmatrix} -1 & 0 & 4 \\ 0 & 0 & 0 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$-x_1 + 4x_3 = 0 \Rightarrow x_1 = 4x_3$$

$$x_2 - x_3 = 0 \Rightarrow x_2 = x_3$$

$$x = \begin{bmatrix} 4x_3 \\ x_3 \\ x_3 \end{bmatrix}$$

If $x_3 = 1$, $x = \begin{bmatrix} 4 \\ 1 \\ 1 \end{bmatrix}$.