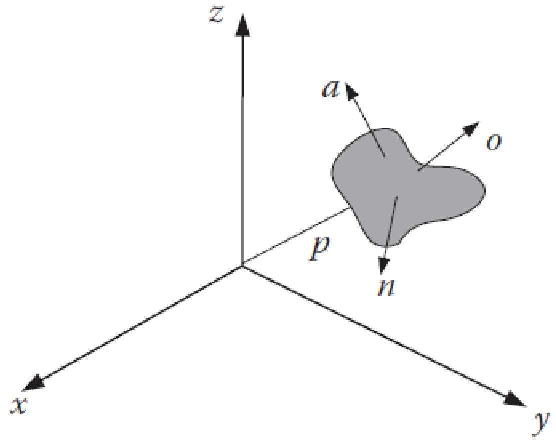
21AIE201-INTRODUCTION TO ROBOTICS

Lecture 9

Representation of a Rigid Body

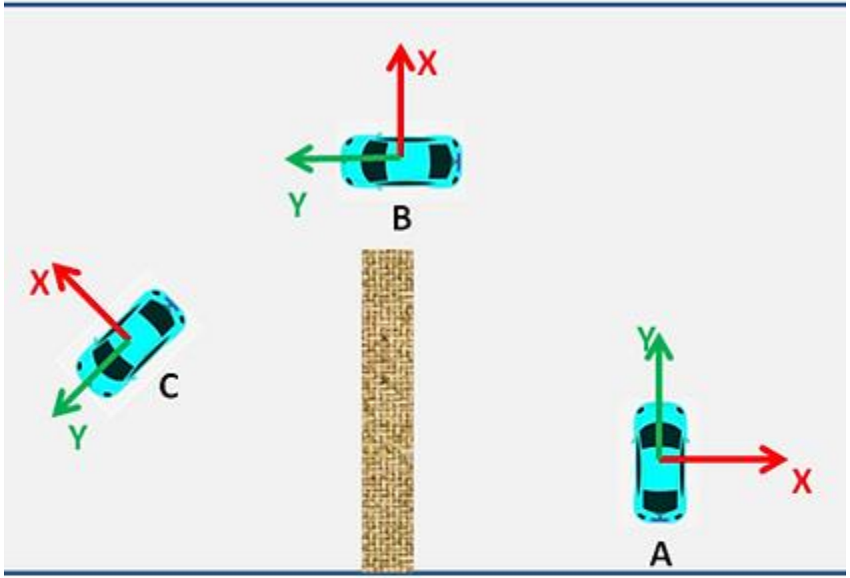


$$F_{object} = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- The three unit vectors **n**, **o**, **a** are mutually perpendicular, and
- Each unit vector representing directional cosines must be equal to 1

These constraints translate into the following six constraint equations:

1) $\mathbf{n} \cdot \mathbf{o} = 0$ (the dot product of $\mathbf{n}$ and $\mathbf{o}$ vectors must be zero)
2) $\mathbf{n} \cdot \mathbf{a} = 0$
3) $\mathbf{a} \cdot \mathbf{o} = 0$
4) $|\mathbf{n}| = 1$ (the magnitude of the lenght of the vector must be 1)
5) $|\mathbf{o}| = 1$
6) $|\mathbf{a}| = 1$



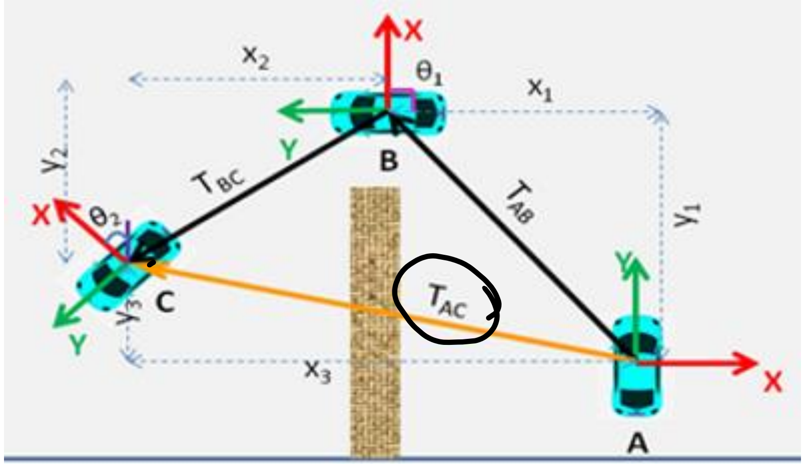
Assume there are three cars in a parking area A, B and C as shown

TRANSFORMATION BETWEEN B TO C

Similarly person in car B has figured out that car C is at $[x_2, y_2]$ units and rotated at an angle θ_2 with respect to his car B.

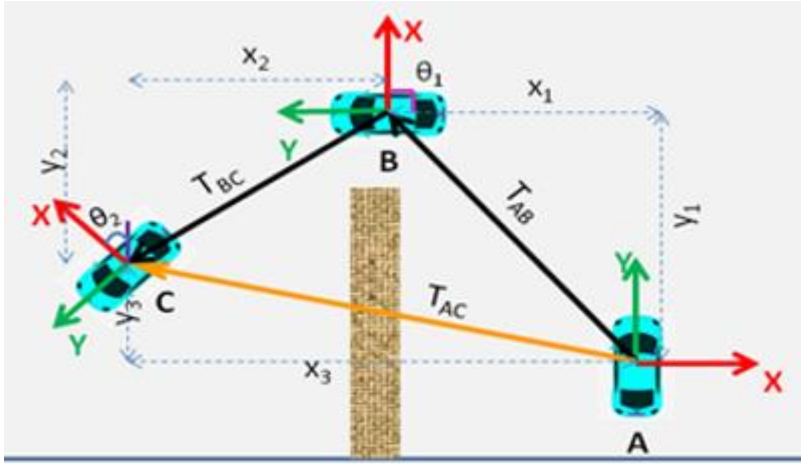
$$T_{BC} = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & x_2 \\ \sin\theta_2 & \cos\theta_2 & y_2 \\ 0 & 0 & 1 \end{bmatrix}$$

Now our task is to find out the position and orientation of car C without seeing it directly. Let's say car C is at $[x_3, y_3]$ units and rotated at an angle θ_3 with respect to my car, car A.



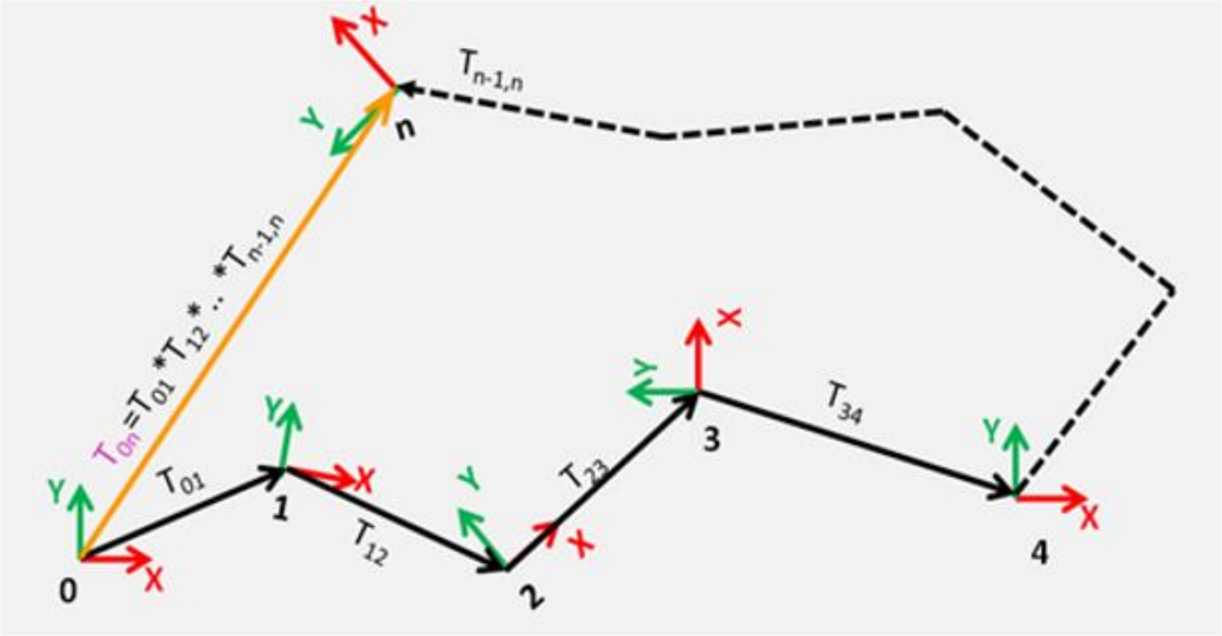
To find our x_3, y_3 and θ_3, we will take the help of person in car B to get T_{BC} matrix. Therefore I know T_{AB} and TBC. That's it I have all required information to calculate x_3, y_3 and θ_3.

Transformation of Car C w.rt Car A with the help of Car B



$$T_{AC} = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & x_1 \\ \sin\theta_1 & \cos\theta_1 & y_1 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & x_2 \\ \sin\theta_2 & \cos\theta_2 & y_2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & x_3 \\ \sin\theta_3 & \cos\theta_3 & y_3 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta_1{*}\cos\theta_2 - \sin\theta_1{*}\sin\theta_2 & -\cos\theta_1{*}\sin\theta_2 - \sin\theta_1{*}\cos\theta_2 & \cos\theta_1{*}x_2 - \sin\theta_1{*}y_2 + x_1 \\ \sin\theta 1\,{*}\cos\theta_2 + \cos\theta_1{*}\sin\theta_2 & -\sin\theta_1{*}\sin\theta_2 + \cos\theta_1{*}\cos\theta_2 & \sin\theta_1{*}x_2 + \cos\theta_1{*}y_2 + y_1 \\ 0 & 0 & 1 \end{bmatrix}$$



$$T_{0n}=T_{01}*T_{12}*T_{23}*\ldots\ldots\ldots T_{n-2,n-1}*T_{n-1,n}$$

NOTE

2D transformations are mostly used in planar manipulators and autonomous mobile robots, self driving cars.

EXAMPLE

For the following frame, find the values of the missing elements and complete the matrix representation of the frame:

$$F = \begin{bmatrix} ? & 0 & ? & 5 \\ 0.707 & ? & ? & 3 \\ ? & ? & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

1 0

−0.707 0

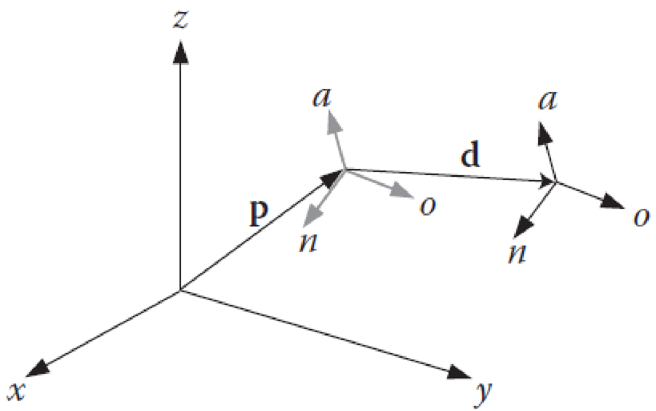
al matrix is:

$$F_1 = \begin{bmatrix} 0.707 & 0 & 0.707 & 5 \\ 0.707 & 0 & -0.707 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ or } F_2 = \begin{bmatrix} -0.707 & 0 & -0.707 & 5 \\ 0.707 & 0 & -0.707 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Find the missing elements of the following frame representation:

$$F = \begin{bmatrix} ? & 0 & ? & 3 \\ 0.5 & ? & ? & 9 \\ 0 & ? & ? & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Representation of a Pure Translation



Representation of a pure translation in space.

$$T = Trans(d_x, d_y, d_z) = \begin{bmatrix} 1 & 0 & 0 & d_x \\ 0 & 1 & 0 & d_y \\ 0 & 0 & 1 & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$F_{new} = \begin{bmatrix} 1 & 0 & 0 & d_x \\ 0 & 1 & 0 & d_y \\ 0 & 0 & 1 & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} n_x & o_x & a_x & p_x + d_x \\ n_y & o_y & a_y & p_y + d_y \\ n_z & o_z & a_z & p_z + d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$F_{new} = Trans\left(d_x, d_y, d_z\right) \times F_{old}$$

A frame F is moved 3 units along the x-axis and 2 units along the z-axis of the reference frame. Find the new location of the frame.

$$F = \begin{bmatrix} 0.527 & -0.574 & 0.628 & 8 \\ 0.369 & 0.819 & 0.439 & 10 \\ -0.766 & 0 & 0.643 & 6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Representation of a Pure Rotation about an Axis

$$Rot(x,\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\theta & -S\theta \\ 0 & S\theta & C\theta \end{bmatrix} \qquad Rot(y,\theta) = \begin{bmatrix} C\theta & 0 & S\theta \\ 0 & 1 & 0 \\ -S\theta & 0 & C\theta \end{bmatrix} \quad \text{and} \quad Rot(z,\theta) = \begin{bmatrix} C\theta & -S\theta & 0 \\ S\theta & C\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

A point p[2, 3, 4]T is attached to a rotating frame. The frame rotates 90∘ about the x-axis of the reference frame. Find the coordinates of the point relative to the reference frame after the rotation, and verify the result graphically.

1 0 0 0

0

Representation of Combined Transformations

To see how combined transformations are handled, let's assume that a frame F_{noa} is subjected to the following three successive transformations relative to the reference frame F_{xyz}:

1) Rotation of α degrees about the x-axis,
2) Followed by a translation of $[l_1, l_2, l_3]$ (relative to the x-, y-, and z-axes respectively),
3) Followed by a rotation of β degrees about the y-axis.

$$(p_{xyz})_1 = Rot(x,\alpha) \times p_{noa}$$

$$(p_{xyz})_2 = Trans(l_1, l_2, l_3) \times (p_{xyz})_1 = Trans(l_1, l_2, l_3) \times Rot(x,\alpha) \times p_{noa}$$

$$p_{xyz} = (p_{xyz})_3 = Rot(y,\beta) \times (p_{xyz})_2 = Rot(y,\beta) \times Trans(l_1, l_2, l_3) \times Rot(x,\alpha) \times p_{noa}$$

Representation of Combined Transformations

Example A point $p[7, 3, 1]^T$ is attached to a frame F_{noa} and is subjected to the following transformations:

1) Rotation of 90° about the z-axis
2) Followed by a rotation of 90° about the y-axis
3) Followed by a translation of [4, −3, 7]

Time for Discussions

Thank You!