21AIE201-INTRODUCTION TO ROBOTICS

Lecture 11

DEPARTMENT OF MECHANICAL ENGINEERING

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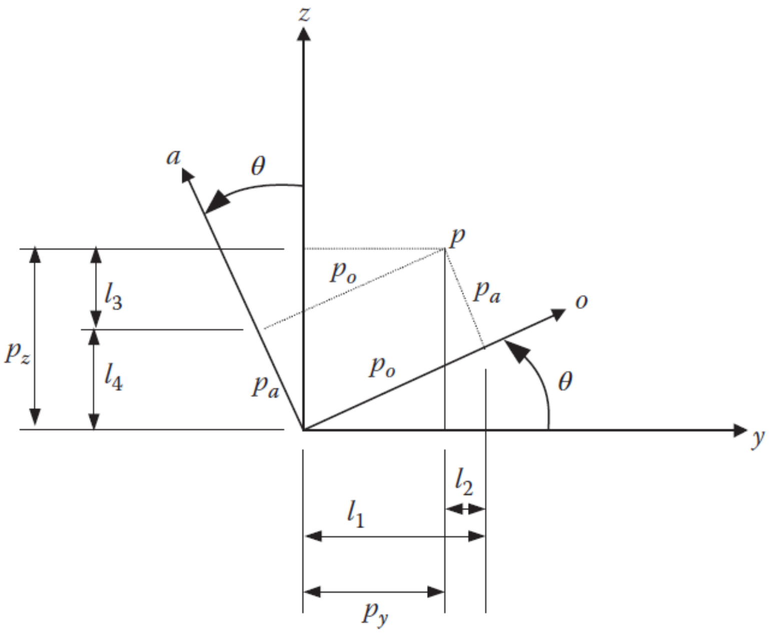
Representation of a Pure Rotation about an Axis

$$Rot(x,\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\theta & -S\theta \\ 0 & S\theta & C\theta \end{bmatrix} \qquad Rot(y,\theta) = \begin{bmatrix} C\theta & 0 & S\theta \\ 0 & 1 & 0 \\ -S\theta & 0 & C\theta \end{bmatrix} \text{ and } Rot(z,\theta) = \begin{bmatrix} C\theta & -S\theta & 0 \\ S\theta & C\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Representation of a Pure Rotation about an Axis



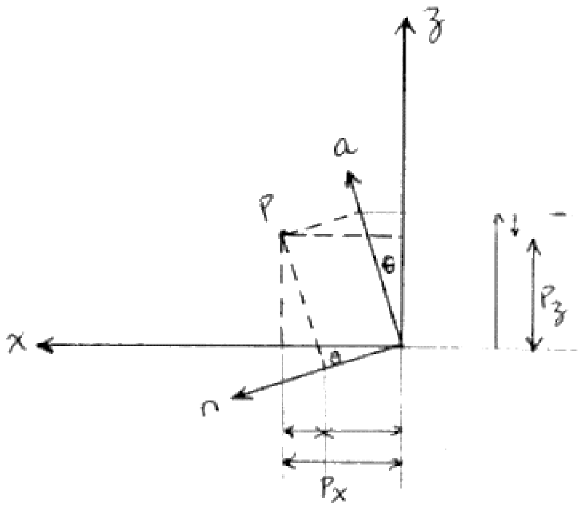
$$p_x = p_n$$

$$p_y = l_1 - l_2 = p_o \cos\theta - p_a \sin\theta$$

$$p_z = l_3 + l_4 = p_o \sin\theta + p_a \cos\theta$$

$$\begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} p_n \\ p_o \\ p_a \end{bmatrix}$$

Representation of a Pure Rotation about an Axis



$$p_x = p_n \cos\theta + p_a \sin\theta$$
$$p_y = p_o$$
$$p_z = -p_n \sin\theta + p_a \cos\theta$$

and

$$\begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} = \begin{bmatrix} C & 0 & S \\ 0 & 1 & 0 \\ -S & 0 & C \end{bmatrix} \begin{bmatrix} p_n \\ p_o \\ p_a \end{bmatrix}$$

Representation of Combined Transformations

Example In this case, assume the same point $p[7,3,1]^T$, attached to F_{noa}, is subjected to the same transformations, but the transformations are performed in a different order, as shown:

1) A rotation of 90° about the z-axis
2) Followed by a translation of $[4,-3,7]$
3) Followed by a rotation of 90° about the y-axis

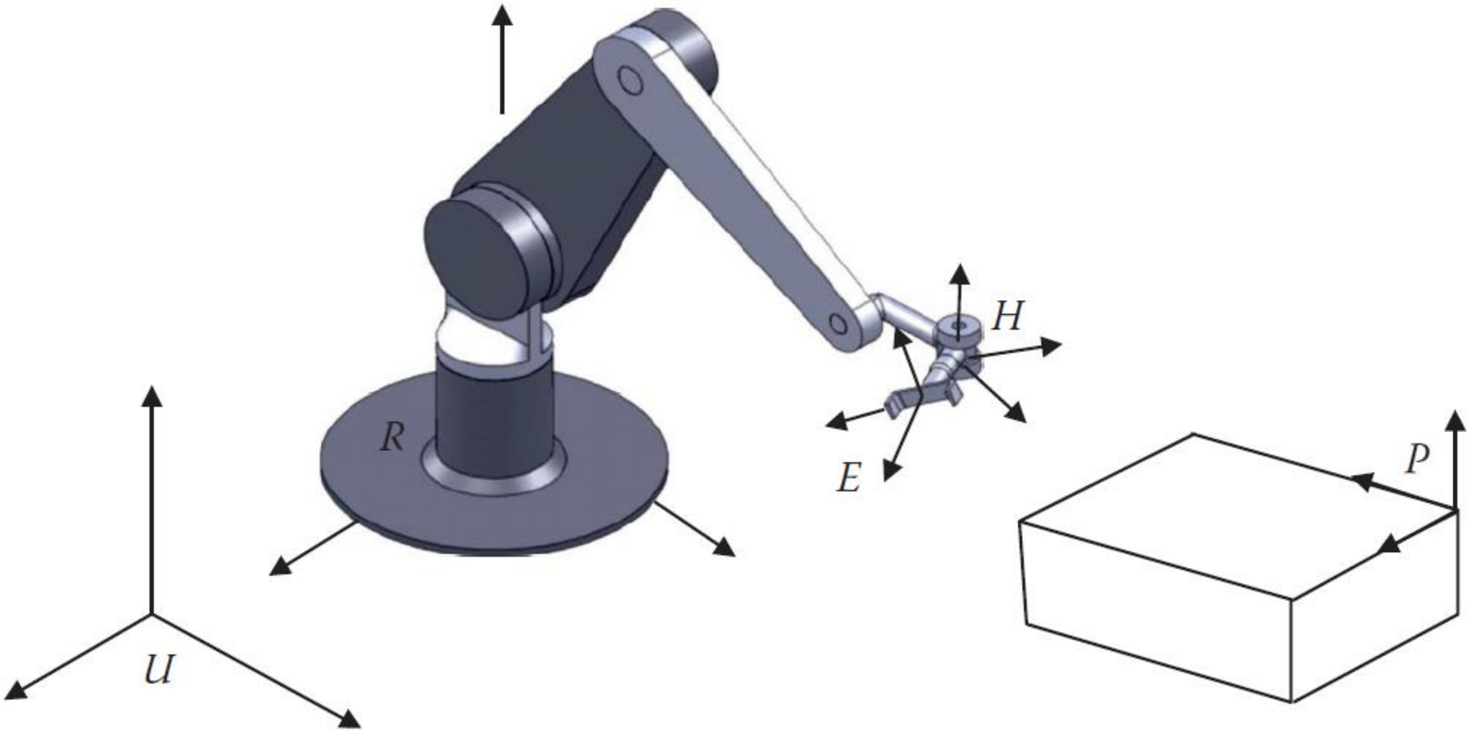
Find the coordinates of the point relative to the reference frame at the conclusion of transformations.

The matrix equation representing the transformation is:

$$p_{xyz} = Rot(y,90)\,Trans(4,-3,7)\,Rot(z,90)\,p_{noa} =$$

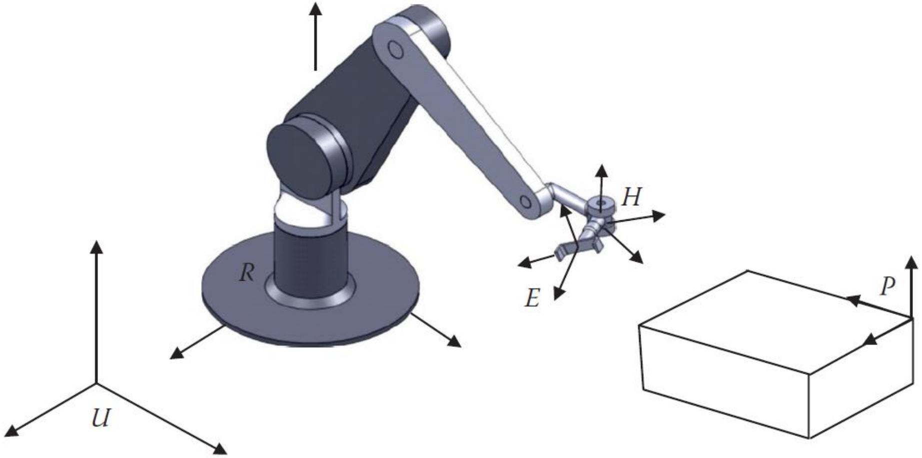
$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 7 \\ 3 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 8 \\ 4 \\ -1 \\ 1 \end{bmatrix}$$

Inverse of Transformation Matrices



The Universe, robot, hand, part, and end effector frames.

Inverse of Transformation Matrices



The Universe, robot, hand, part, and end effector frames.

The location of the point where the hole will be drilled can be related to the reference frame U through two independent paths:

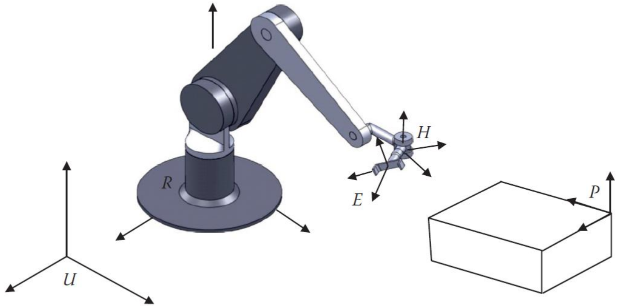
1. **One through the part,**
2. **one through the robot.**

Therefore, the following equation can be written:

$$^{U}T_{E} = {}^{U}T_{R}\,{}^{R}T_{H}\,{}^{H}T_{E} = {}^{U}T_{P}\,{}^{P}T_{E}$$

The location of point E on the part can be achieved by moving from U to P and from P to E, or by a transformation from U to R, from R to H, and from H to E.

Inverse of Transformation Matrices



The Universe, robot, hand, part, and end effector frames.

$$[{}^{U}T_{R}]^{-1}[{}^{U}T_{R}{}^{R}T_{H}{}^{H}T_{E}][{}^{H}T_{E}]^{-1} = [{}^{U}T_{R}]^{-1}[{}^{U}T_{P}{}^{P}T_{E}][{}^{H}T_{E}]^{-1}$$

or since $({}^{U}T_{R})^{-1}({}^{U}T_{R}) = I$ and $({}^{H}T_{E})({}^{H}T_{E})^{-1} = I$, the left side of Eq. simplifies to ${}^{R}T_{H}$, and we get:

$${}^{R}T_{H} = {}^{U}T_{R}{}^{-1}\,{}^{U}T_{P}{}^{P}T_{E}{}^{H}T_{E}{}^{-1}$$

We can check the accuracy of this equation by realizing that ${}^{H}T_{E}{}^{-1}$ is the same as ${}^{E}T_{H}$. Therefore, the equation can be rewritten as:

$${}^{R}T_{H} = {}^{U}T_{R}{}^{-1}\,{}^{U}T_{P}{}^{P}T_{E}{}^{H}T_{E}{}^{-1} = {}^{R}T_{U}{}^{U}T_{P}{}^{P}T_{E}{}^{E}T_{H} = {}^{R}T_{H}$$

Inverse of Transformation Matrices (3× 3)

$$Rot(x,\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\theta & -S\theta \\ 0 & S\theta & C\theta \end{bmatrix}$$

1) Calculate the determinant of the matrix.
2) Transpose the matrix.
3) Replace each element of the transposed matrix by its own minor (adjoint matrix).
4) Divide the adjoint matrix by the determinant.

$$\det[Rot(x,\theta)] = 1\left(C^2\theta + S^2\theta\right) + 0 = 1$$

$$\mathrm{Adj}[Rot(x,\theta)] = Rot(x,\theta)^T_{\mathrm{minor}} = Rot(x,\theta)^T$$

$$Rot(x,\theta)^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\theta & S\theta \\ 0 & -S\theta & C\theta \end{bmatrix}$$

$$Rot(x,\theta)^{-1} = Rot(x,\theta)^T$$

Inverse of Transformation Matrices (3× 3)

EXAMPLE: Calculate the matrix representing $Rot(x,40^\circ)^{-1}$.

Solution:

The matrix representing a 40° rotation about the x-axis is:

$$Rot(x,40^\circ) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.766 & -0.643 & 0 \\ 0 & 0.643 & 0.766 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The inverse of this matrix is:

$$Rot(x,40^\circ)^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.766 & 0.643 & 0 \\ 0 & -0.643 & 0.766 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Inverse of Transformation Matrices (4× 4)

$$T = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad T^{-1} = \begin{bmatrix} n_x & n_y & n_z & -\mathbf{p}\cdot\mathbf{n} \\ o_x & o_y & o_z & -\mathbf{p}\cdot\mathbf{o} \\ a_x & a_y & a_z & -\mathbf{p}\cdot\mathbf{a} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Inverse of Transformation Matrices (4× 4)

EXAMPLE Calculate the inverse of the given transformation matrix:

$$T = \begin{bmatrix} 0.5 & 0 & 0.866 & 3 \\ 0.866 & 0 & -0.5 & 2 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

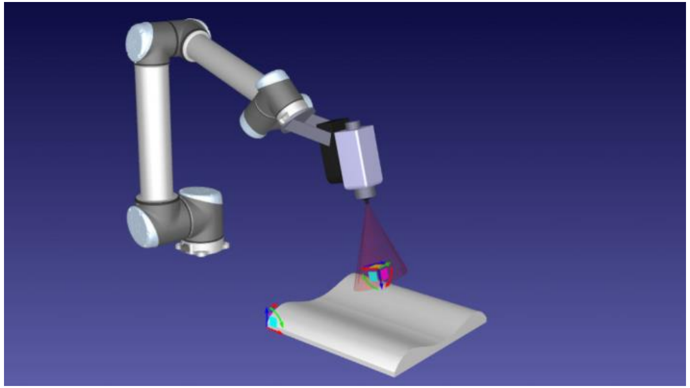
Solution:

Based on the previous discussion, the inverse of the transformation is:

$$T^{-1} = \begin{bmatrix} 0.5 & 0.866 & 0 & -(3\times 0.5 + 2\times 0.866 + 5\times 0) \\ 0 & 0 & 1 & -(3\times 0 + 2\times 0 + 5\times 1) \\ 0.866 & -0.5 & 0 & -(3\times 0.866 + 2\times -0.5 + 5\times 0) \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.5 & 0.866 & 0 & -3.23 \\ 0 & 0 & 1 & -5 \\ 0.866 & -0.5 & 0 & -1.598 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

You may want to verify that TT^{-1} is an identity matrix.

Inverse of Transformation Matrices (4× 4)



In a robotic setup, a camera is attached to the fifth link of a 6-DOF robot. It observes an object and determines its frame relative to the camera's frame. Using the following information, determine the necessary motion the end effector must make to get to the object:

$$^{5}T_{cam} = \begin{bmatrix} 0 & 0 & -1 & 3 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad ^{5}T_{H} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad ^{cam}T_{obj} = \begin{bmatrix} 0 & 0 & 1 & 2 \\ 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad ^{H}T_{E} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Time for Discussions

Thank You!