APPLICATIONS OF EIGEN VALUES AND EIGEN VECTORS

Some applications of Eigen values and eigen vectors are as follows:

1) Used in SVD(singular value decomposition) for Image compression

2) In the derivation of Einstein's second postulate in special relativity

3) In spectral clustering

4) For dimensionality reduction(PCA)

5) Principle axes theorem to find the equation of rotated conic

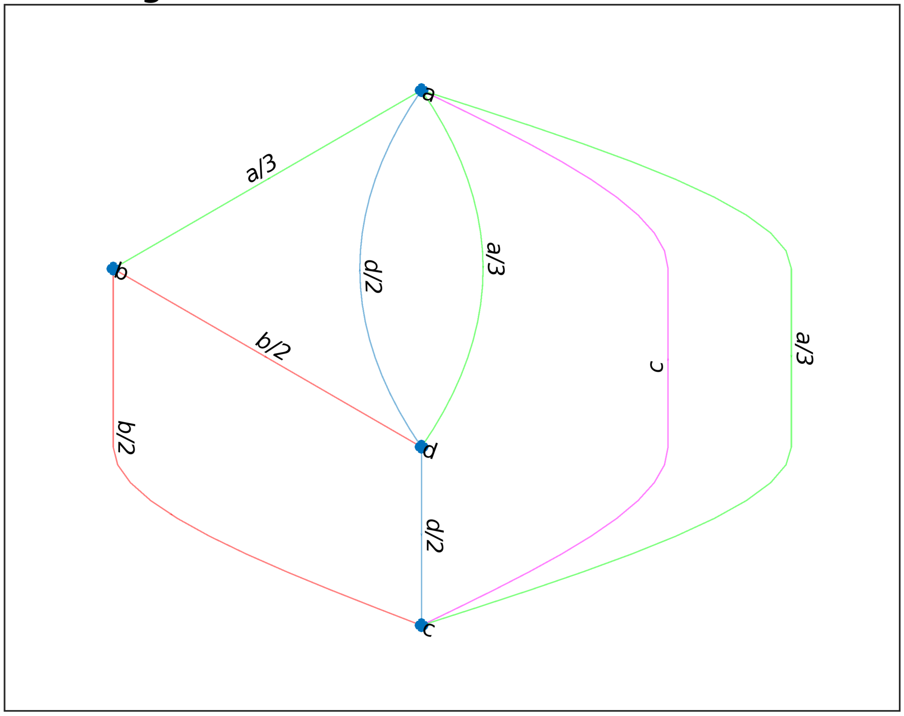
6) Designing bridges and car stereo system

7) By Oil companies to explore land for oil

8) Googel's Page rank algorithm

```
% Page rank algorithm
% For example, the Internet consists of just 4 web sites: a,b,c,d
% linked as follows:
s = {'a', 'a', 'a', 'b', 'b', 'c', 'd', 'd'};
t = {'b', 'c', 'd', 'c', 'd', 'a', 'a', 'c'};
G = digraph(s,t);
labels = {'a/3' 'a/3' 'a/3' 'b/2' 'b/2' 'c' 'd/2' 'd/2'};
p = plot(G,'Layout','layered','EdgeLabel',labels);
highlight(p,[1 1 1],[2 3 4],'EdgeColor','g')
highlight(p,[2 2],[3 4],'EdgeColor','r')
highlight(p,3,1,'EdgeColor','m')
title('PageRank Score Transfer Between Nodes')
```

PageRank Score Transfer Between Nodes




```
% Let A be the transistion matrix of the graph
A = [0 0 1 1/2; 1/3 0 0 0; 1/3 1/2 0 1/2; 1/3 1/2 0 0]
```

```
A = 4×4
         0         0    1.0000    0.5000
    0.3333         0         0         0
    0.3333    0.5000         0    0.5000
    0.3333    0.5000         0         0
```

```
[V, D] = eig(A)
```

```
V = 4×4 complex
   0.7210 + 0.0000i   0.7552 + 0.0000i   0.7552 + 0.0000i   0.5065 + 0.0000i
   0.2403 + 0.0000i  -0.3037 - 0.3461i  -0.3037 + 0.3461i  -0.6057 + 0.0000i
   0.5408 + 0.0000i  -0.0932 + 0.2747i  -0.0932 - 0.2747i  -0.3815 + 0.0000i
   0.3605 + 0.0000i  -0.3584 + 0.0714i  -0.3584 - 0.0714i   0.4807 + 0.0000i
D = 4×4 complex
   1.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i  -0.3606 + 0.4110i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i  -0.3606 - 0.4110i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i  -0.2788 + 0.0000i
```

Initially, the page rank is divided equally among the 4 websites.

Since PageRank should reflect only the relative importance of the nodes, and since the eigenvectors are just scalar multiples of each other, we can choose any of them to be our PageRank vector.

Choose v* to be the unique eigenvector with the sum of all entries equal to 1.The eigen vector for eigen value of 1 is c*[12; 4; 9; 6]

To get sum of all entries as 1, divide by (12+4+9+6=31), which gives c*[0.38; 0.12; 0.29; 0.19]

The first column of V satisfies thiscondition when c=2, hence it is the required page rank vector.

Since rank of page A is high, it is the most relevant website.