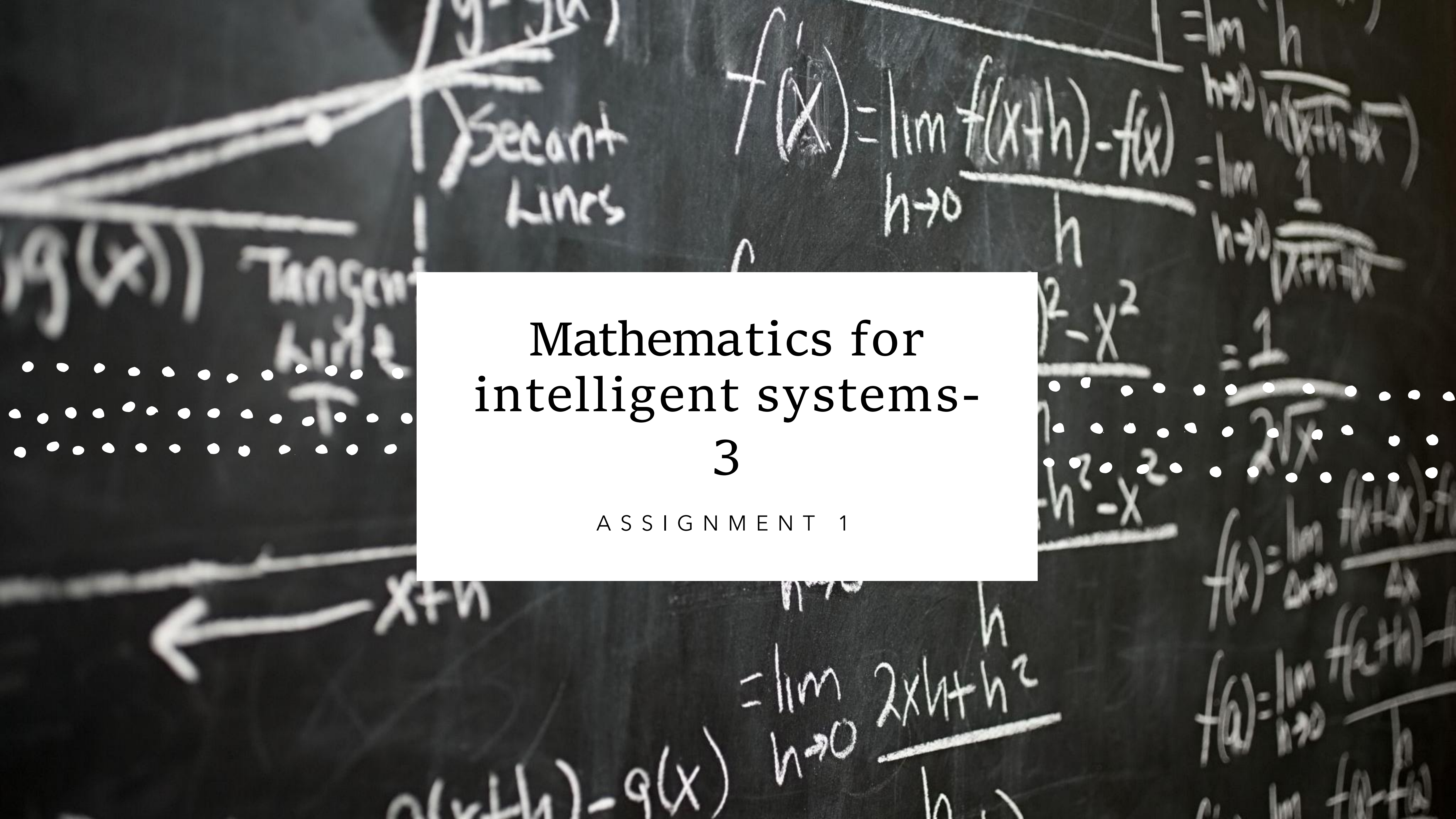

Mathematics for intelligent systems-3

ASSIGNMENT 1

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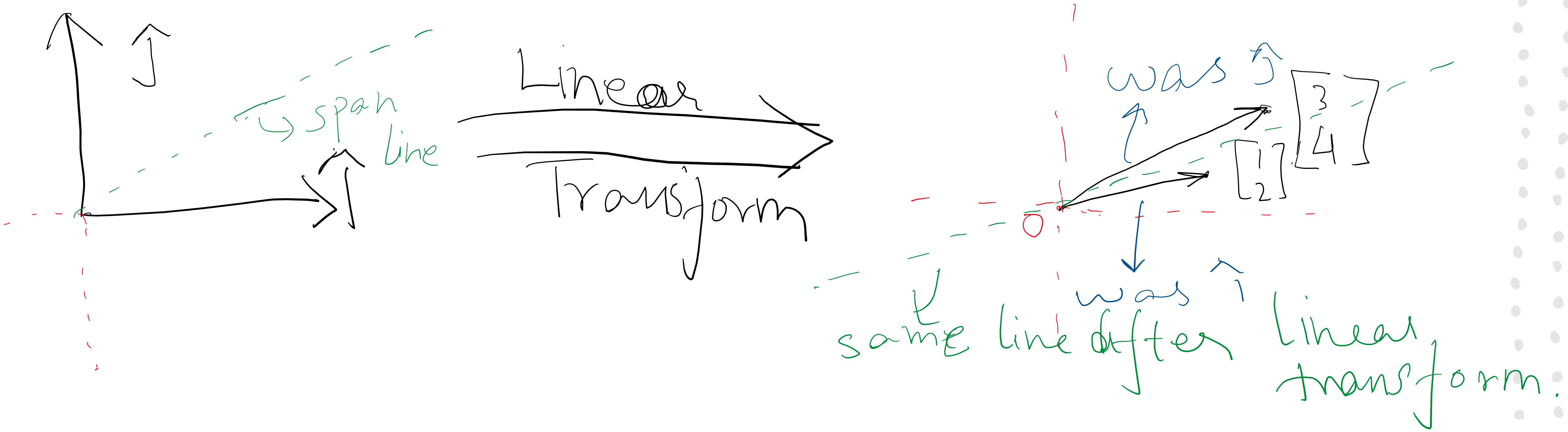
Eigen values and Eigen Vectors

- Geometric Definition
- Eigen Equation
- Arithmetic Multiplicity and Geometric Multiplicity



$A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$; Let 'x' be a vector on the line

$$x = \begin{bmatrix} a \\ b \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 1 \\ 2 \end{bmatrix} a + \begin{bmatrix} 3 \\ 4 \end{bmatrix} b = \lambda \begin{bmatrix} a \\ b \end{bmatrix}$$

$$\begin{bmatrix} a + 3b \\ 2a + 4b \end{bmatrix} = \begin{bmatrix} \lambda a \\ \lambda b \end{bmatrix}$$

$a + 3b = \lambda a$ — (1)

$2a + 4b = \lambda b$ — (2)

From (1);

$\Rightarrow \quad b = \dfrac{(\lambda - 1)a}{3}$

From (2);

$2\cancel{a} = (\lambda - 4)\dfrac{(\lambda - 1)\cancel{a}}{3}$

$\Rightarrow \quad 6 = (\lambda^2 - 5\lambda + 4)$

$$\Rightarrow \lambda^2 - 5\lambda - 2 = 0$$

$$\lambda = \frac{5 \pm \sqrt{25 - 4(1)(-2)}}{2} = \frac{5 \pm \sqrt{17}}{2}$$

These are the eigen values of $\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

This is not a conventional method to do this but its important to get the concept.

The Eigen Equation

It is given by $Ax = \lambda x$, where

A = matrix given.
x = eigen vector.
λ = eigen values.

$$Ax - \lambda x = 0$$

$$(A - \lambda I)x = 0$$

$\because |x| \neq 0$, $|(A - \lambda I)| = 0$

Let's try if we get the same answer as the geometric method.

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} 1-\lambda & 3 \\ 2 & 4-\lambda \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$\therefore \begin{vmatrix} 1-\lambda & 3 \\ 2 & 4-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (\lambda - 1)(\lambda - 4) - 6 = 0$$

$$\Rightarrow \lambda^2 - 5\lambda + 4 - 6 = 0$$

$$\Rightarrow \boxed{\lambda^2 - 5\lambda - 2 = 0}$$

We get the same equation as before

This equation is unique for every square matrix and this is called its "Characteristic Equation".

The characteristic eqn of a 2×2 matrix is given as follows;

$$\lambda^2 - tr(A)\lambda + det(A) = 0$$

In general, $p(\lambda) = det(\lambda I - A)$