21AIE201-INTRODUCTION TO ROBOTICS

Lecture 10

Representation of Combined Transformations

To see how combined transformations are handled, let's assume that a frame F_{noa} is subjected to the following three successive transformations relative to the reference frame F_{xyz}:

1) Rotation of α degrees about the x-axis,
2) Followed by a translation of $[l_1, l_2, l_3]$ (relative to the x-, y-, and z-axes respectively),
3) Followed by a rotation of β degrees about the y-axis.

$$(p_{xyz})_1 = Rot(x,\alpha) \times p_{noa}$$

$$(p_{xyz})_2 = Trans(l_1, l_2, l_3) \times (p_{xyz})_1 = Trans(l_1, l_2, l_3) \times Rot(x,\alpha) \times p_{noa}$$

$$p_{xyz} = (p_{xyz})_3 = Rot(y,\beta) \times (p_{xyz})_2 = Rot(y,\beta) \times Trans(l_1, l_2, l_3) \times Rot(x,\alpha) \times p_{noa}$$

Representation of Combined Transformations

Example A point $p[7, 3, 1]^T$ is attached to a frame F_{noa} and is subjected to the following transformations:

1) Rotation of 90° about the z-axis
2) Followed by a rotation of 90° about the y-axis
3) Followed by a translation of [4, –3, 7]

$$p_{xyz} = Trans(4,-3,7)Rot(y,90)Rot(z,90)p_{noa} =$$

$$\begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 7 \\ 3 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \\ 10 \\ 1 \end{bmatrix}$$

Representation of Combined Transformations

Example In this case, assume the same point $p[7, 3, 1]^T$, attached to F_{noa}, is subjected to the same transformations, but the transformations are performed in a different order, as shown:

1) A rotation of 90° about the z-axis
2) Followed by a translation of $[4, -3, 7]$
3) Followed by a rotation of 90° about the y-axis

Find the coordinates of the point relative to the reference frame at the conclusion of transformations.

The matrix equation representing the transformation is:

$$p_{xyz} = Rot(y,90)\,Trans(4,-3,7)\,Rot(z,90)\,p_{noa} =$$

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 7 \\ 3 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 8 \\ 4 \\ -1 \\ 1 \end{bmatrix}$$

Transformations Relative to the Current (Moving) Frame

- To calculate the changes in the coordinates of a point that is attached to the current frame relative to the reference frame, the transformation matrix is post-multiplied instead

In this case, assume the same point $p[7, 3, 1]^T$, attached to F_{noa}, is subjected to the same transformations, but all relative to the current moving frame as:

1) A rotation of 90° about the a-axis
2) Then a translation of $[4, -3, 7]$ along n-, o-, a-axes
3) Followed by a rotation of 90° about the o-axis

$$p_{xyz} = Rot(a,90)\,Trans(4,-3,7)\,Rot(o,90)\,p_{noa} =$$

$$p_{xyz} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 7 \\ 3 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ 0 \\ 1 \end{bmatrix}$$

Mixed Transformations Relative to Rotating and Reference Frames

Example A frame B was rotated about the x-axis 90° followed by a translation about the current o-axis of 2 inches, followed by a rotation about the a-axis of 90° and a translation along the current y–axis of 3 inches.

a) Write an equation that describes the motions.

b) Find the final location of a point ${}^{B}p = [1,3,2]^{T}$ relative to the reference frame.

Mixed Transformations Relative to Rotating and Reference Frames

Example A frame B was rotated about the x-axis 90° followed by a translation about the current o-axis of 2 inches, followed by a rotation about the a-axis of 90° and a translation along the current y–axis of 3 inches.

a) Write an equation that describes the motions.
b) Find the final location of a point $^{B}p = [1,3,2]^{T}$ relative to the reference frame.

Solution:

In this case, transformations alternate relative to the reference and current frames.

a) Starting with frame B and pre- or post-multiplying each motion's matrix accordingly, we get:

$$^{U}T_{B} = Trans(0,3,0)Rot(x,90)[B]Trans(0,2,0)Rot(a,90)$$

Mixed Transformations Relative to Rotating and Reference Frames

Solution:

In this case, transformations alternate relative to the reference and current frames.

a) Starting with frame B and pre- or post-multiplying each motion's matrix accordingly, we get:

b) Substituting the matrices and multiplying them, we get:

$$^{U}p = {}^{U}T_{B} \times {}^{B}p$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \\ 3 \\ 1 \end{bmatrix}$$

Mixed Transformations Relative to Rotating and Reference Frames

Example A frame F was rotated about the o-axis $-90°$, followed by a rotation about the y-axis of $30°$, followed by a translation of 3 units along the a-axis, and finally, a rotation of $90°$ along the x-axis. Find the transformed frame.

$$F_{old} = \begin{bmatrix} 0 & 0 & 1 & 3 \\ 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Solution:

The following set of matrices, written in the proper order to represent transformations relative to the reference frame or the current frame, describes the total transformation and the new frame. Starting with the frame F, we find:

$$[F_{new}] = Rot(x,90)Rot(y,30)[F_{old}]Rot(o,-90)Trans(0,0,3)$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.866 & 0 & 0.5 & 0 \\ 0 & 1 & 0 & 0 \\ -0.5 & 0 & 0.866 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 3 \\ 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.866 & 0.5 & 0 & 5.6 \\ 0.5 & -0.866 & 0 & -3.7 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Time for Discussions

Thank You!