DP on trees

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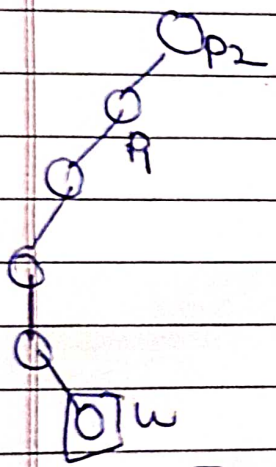
Binary Lifting

* Given a tree, find which node is K level above.

for every node : u
↳ up(u, 2^i)

* And a query (u, k)

⇒ $K = (0|0|1|0)_2 = 2^4 + 2^2 + 2^1$



K = 5

p_1 = up(u, 2) ⇒ 2^2

p_2 = up(p_1, 0) ⇒ 2^0

1. To find up[root][x] = -1
2. up(u, 0) = par(u) [Use DFS]

up(u, x) = up[up[u, x-1], x-1]

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```
int lca(int u, int v)

    if(lvl[u] < lvl[v])
        swap(u,v);

    u = lift_node(u, lvl[u] - lvl[v]);

    if(u==v)
        return u;

    for(int i=19; i>=0; i--){
        if(up[u][i] != up[v][i]){
            u = up[u][i];
            v = up[v][i];
        }
    }

    return lift_node(u,1);
}

int lift_node(int node, int jump_req){
    for(i=19; i>=0; i--){
        if(jump_req==0 || node==-1)
            break;

        if(jump_req >= (1<<i)){
            jump_req -= (1<<i);
            node = up[node][i];
        }
    }
    return node;
}
```

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```
void dfs(int src, int par, int level){

    level[src] = level;
    for(int child : tree[src]){
        if(child != par)
            dfs(child, src, level+1);
        }
    }
}


void binary_lifting(int src, int par){

    up[src][0] = par;
    for(int i=1; i<20; i++){
        if(up[src][i-1] != -1){
            up[src][i] = up[up[src][i-1]][i-1];
        else
            up[src][i] = -1;
    }
    for(int child : tree[src]){
        if(child != par){
            binary_lifting(child, src);
        }
    }
}
```