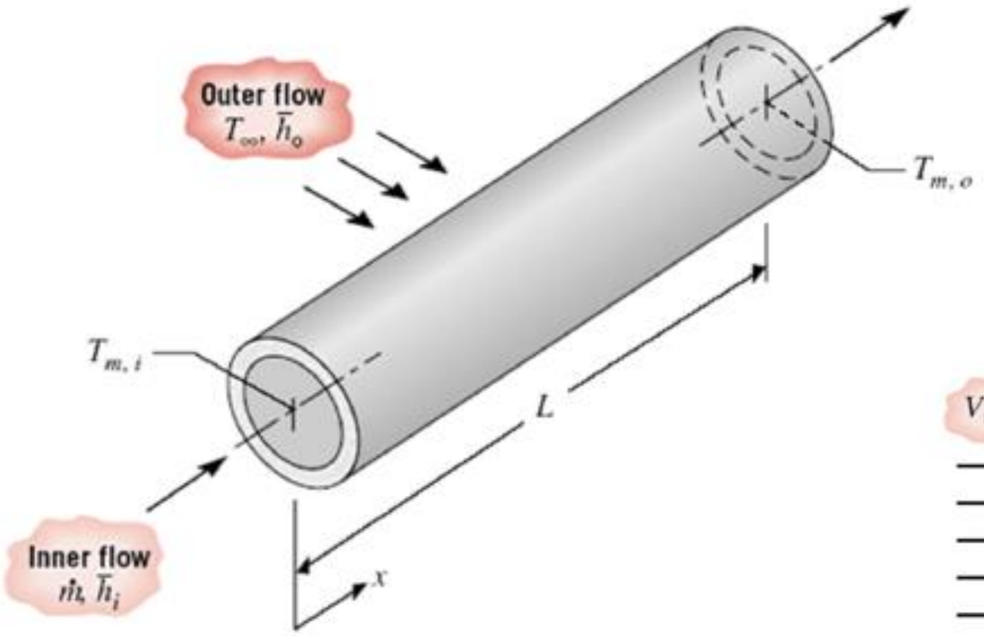
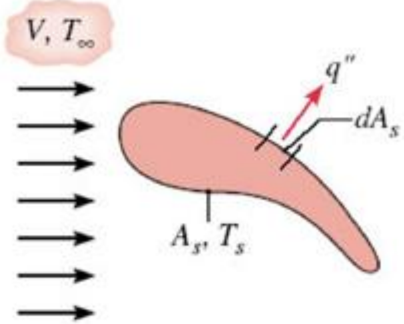
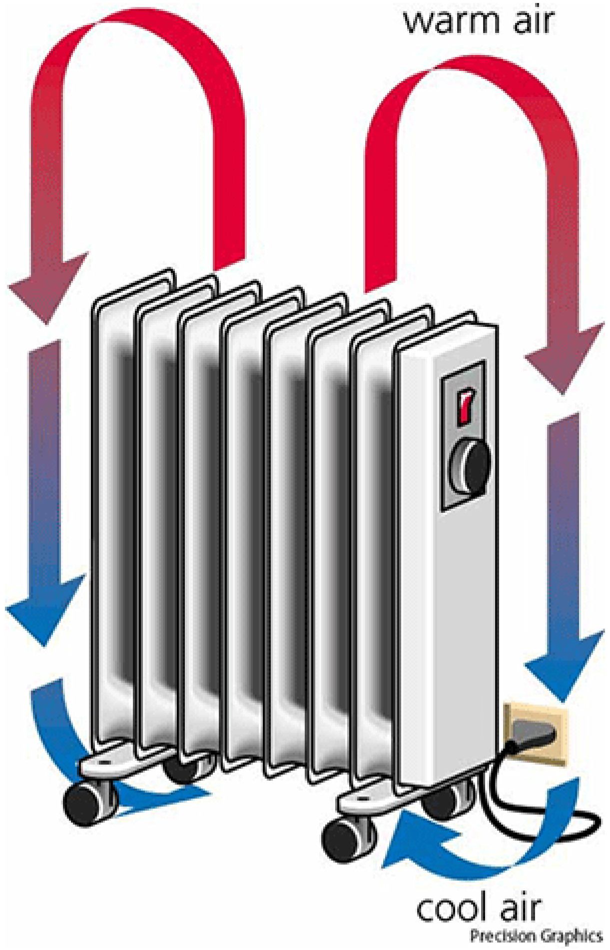
Session-6. Heat Transfer by Convection



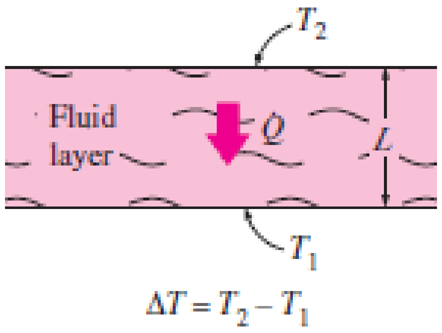




Dr. Jogender Singh (JS)

Objectives: Convection (Without Phase Change)

- ✓ Energy balance.
- ✓ Heat transfer global coefficient.
- ❑ Logarithmic mean temperature difference (LMTD).
- ✓ Newton's Law of cooling and the local heat transfer coefficient.
- ❑ Forced convection in laminar and turbulent regimes inside tubes.
- ❑ Forced convection in outside tubes
- ❑ Natural convection.
- ❑ Tubular heat exchangers.
- ❑ Plate heat exchangers
- ❑ Extended surfaces (fins)



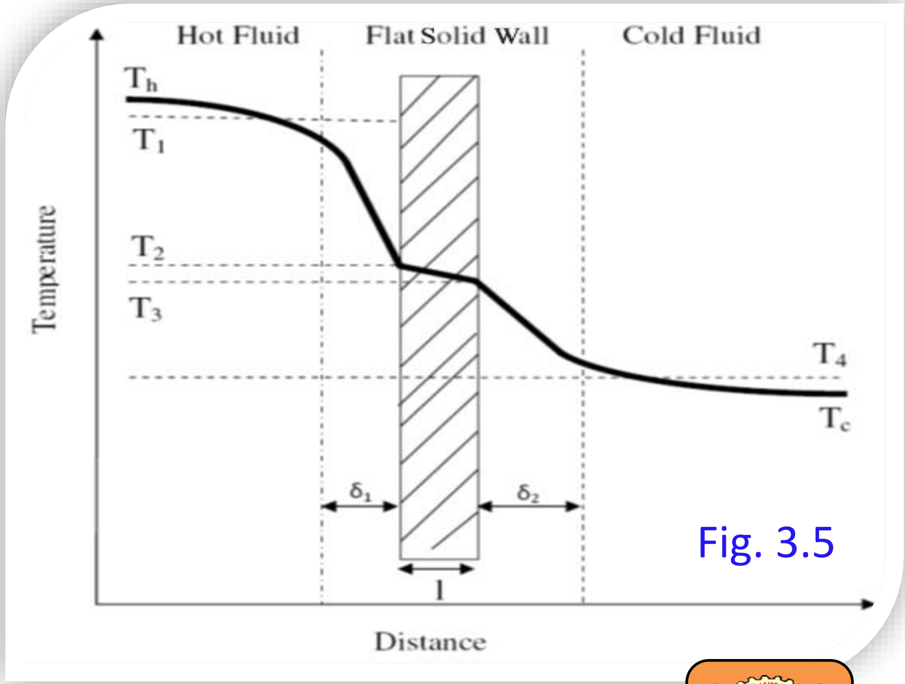


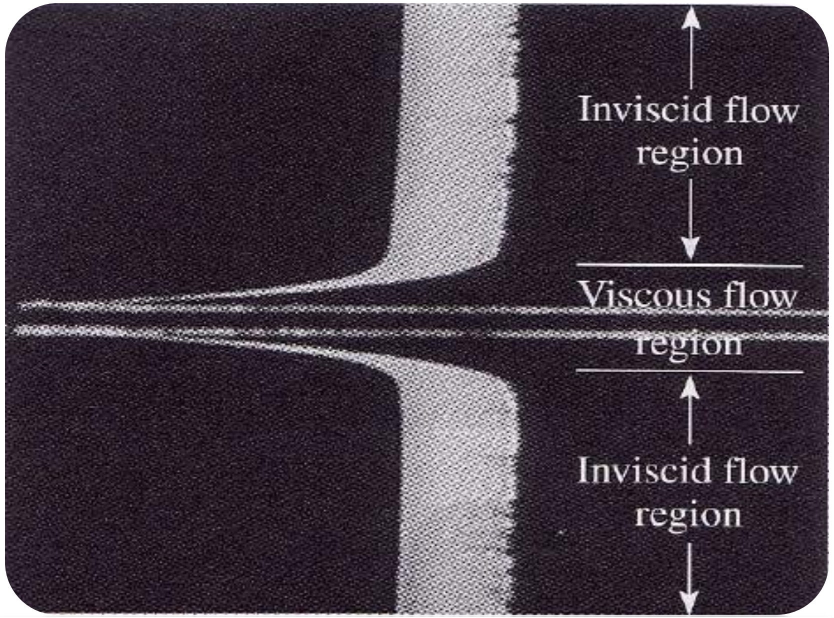
Fig. 3.5

Convection: Classification of Fluid Flows

Convection heat transfer is closely **linked with fluid mechanics**, which is the science that deals with the **behavior of fluids at rest or in motion**, and the interaction of fluids with solids or other fluids at the boundaries.

a. Viscous versus Inviscid Flow

- When two fluid layers move relative to each other, a friction force develops between them and the slower layer tries to slow down the faster layer. This internal resistance to flow is called the **viscosity**. Viscosity is caused by cohesive forces between the molecules in liquids, and by the molecular collisions in gases.
- There is no fluid with zero viscosity, and thus all fluid flows involve viscous effects to some degree. Flows in which the effects of viscosity are significant are called viscous flows.
- The effects of viscosity are very small in some flows, and neglecting those effects greatly simplifies the analysis without much loss in accuracy. Such idealized flows of zero-viscosity fluids are called frictionless or inviscid flows.



Typically regions not close to solid surface, where viscous forces are negligible small compared to pressure forces.→ inviscid flow regions

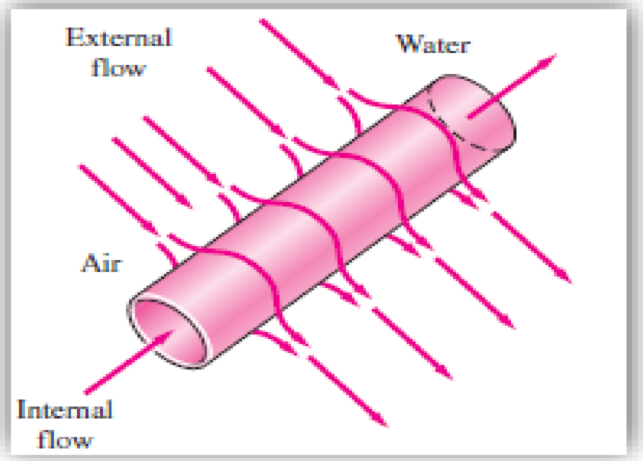
Convection: Classification of Fluid Flows

Convection heat transfer is closely **linked with fluid mechanics**, which is the science that deals with the **behavior of fluids at rest or in motion**, and the interaction of fluids with solids or other fluids at the boundaries.

b. Internal versus External Flow

Depending on whether the fluid is forced to flow.

- In a pipe or duct (Internal Flow): The fluid completely bounded by solid surface. Dominated by the influence of viscosity throughout the flow field.
- Over a surface such as plate or pipe (External Flow): Unbounded fluid. The viscous effects are limited to boundary layers near solid surfaces.



Internal flow of water in a pipe and the external flow of air over the same pipe.

c. Compressible versus Incompressible Flow

Depending on the level variation of density during flow.

- Incompressible: If the flow density remains nearly constant & the volume of energy portion of fluids remain unchanged over the course of its motion. Gas flows as incompressible depends on the Mach number $Ma = v/c$ where c is the speed of sound, c = 346 m/s.

Convection: Classification of Fluid Flows

Convection heat transfer is closely **linked with fluid mechanics**, which is the science that deals with the **behavior of fluids at rest or in motion**, and the interaction of fluids with solids or other fluids at the boundaries.

d. Laminar versus Turbulent Flow

Depending on whether the fluid if forced to flow.

- Fluid motion characterized by smooth layer of the fluid → **Laminar layer.**
- Disordered fluid motion typically occur at high velocities & is characterized by velocity fluctuations → **Turbulent Layer.**
- The flow that alternates between being laminar & turbulent is called → **Transitional .**

e. Natural versus Force Flow

Depending on how the fluid motion is initiated.

- Forced flow: fluid is forced to flow over a surface or in pipe by external means such as pump or fan.
- Natural flow: fluid motion is due to natural means such as the buoyancy effect.

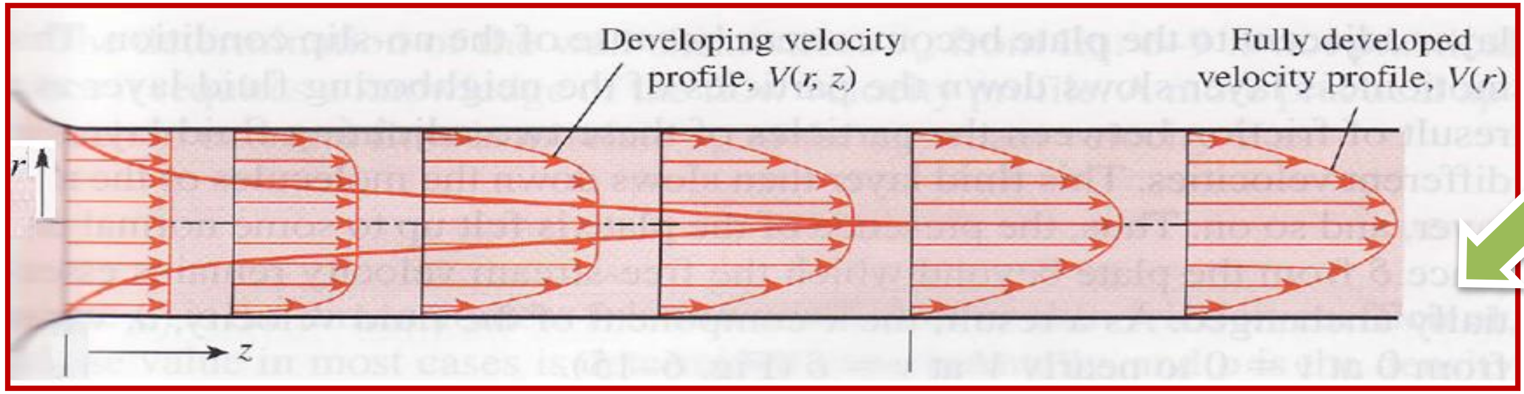
f. Steady versus Unsteady Flow

- Steady: no change at a point with time.
- Unsteady: change at a point with time.
- Uniform: no change with location over a specified region.

Convection: Classification of Fluid Flows

g. 1D, 2D and 3D Flows

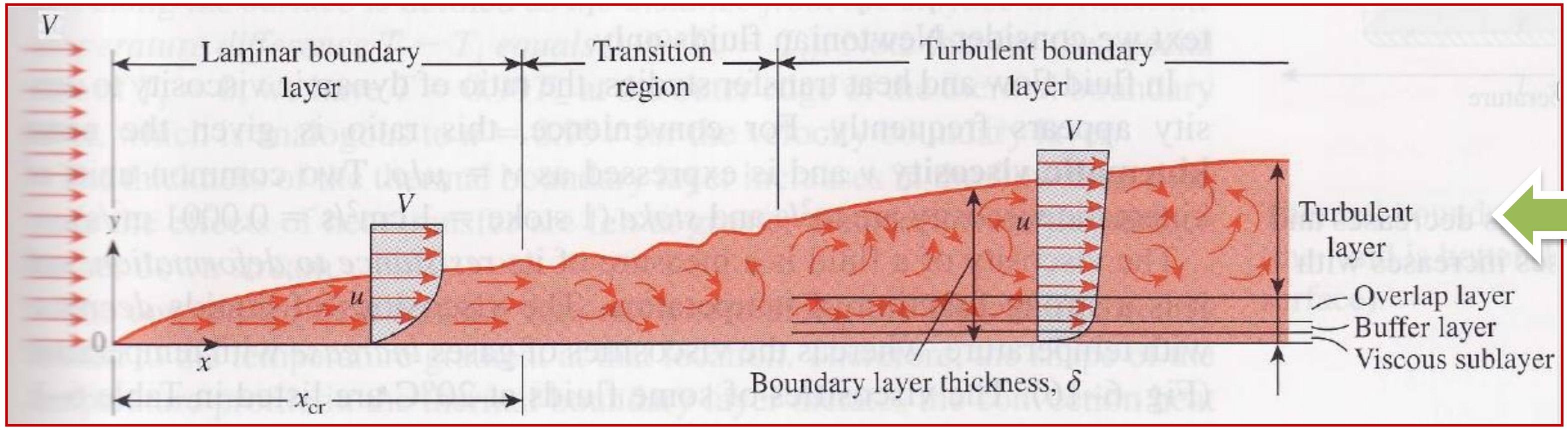
- A flow field is best characterized by the velocity distribution & thus a flow is said to be **1D, 2D or 3D if the flow velocity varies in 1, 2 or 3 primary dimensions**.
- A typical fluid flow involves a 3D geometry & the velocity may vary in all 3 dimensions, V(x,y,z) in rectangular. However, the variation of velocity in certain direction can be small relative to the variation on other directions & can be ignored.
- Consider: the steady flow of a fluid through a circular pipe attached to a large tank. The fluid velocity everywhere on the pipe surface is zero (because no-slip condition).



- The flow is 2D in the entrance region of the pipe since the velocity change in both r and z directions.
- The velocity profile develops fully & remains unchanged after some distance from the inlet and the flow in this region is said to be fully developed (1D since velocity just varies in the radial r-direction).

Convection: Velocity Boundary Layer

Consider the parallel flow of a fluid over a flat plate. The x-coordinate measured along the plate surface from the leading edge of the plate in the direction of the flow. The y-coordinates measured from the surface in the normal direction.



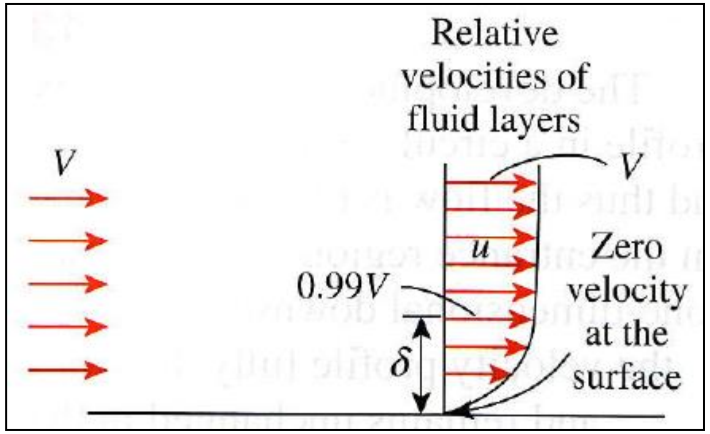
- The fluid approaches the plate in the x-direction with a uniform velocity, V which is practically identical to the free stream velocity over the plate away from the surface.

The development of the boundary layer for flow over a flat plate & the different flow regimes.

Convection: Velocity Boundary Layer

Concept:

- The velocity of the particles in the first fluid layer adjacent to the plate become → zero (because of the no-slip condition).
- This motionless layer slows down the particle of the neighboring fluid layers as a result of **friction between the particles** of these two adjoining fluid layers at different velocity. This fluid layer then slows down the molecules of the next later and so on.
- Thus, the presence of the plate →is felt up to some normal distance from the plate beyond which the free stream velocity remain essentially unchanged.



The development of a boundary layer on a surface is due to the no-slip condition & friction

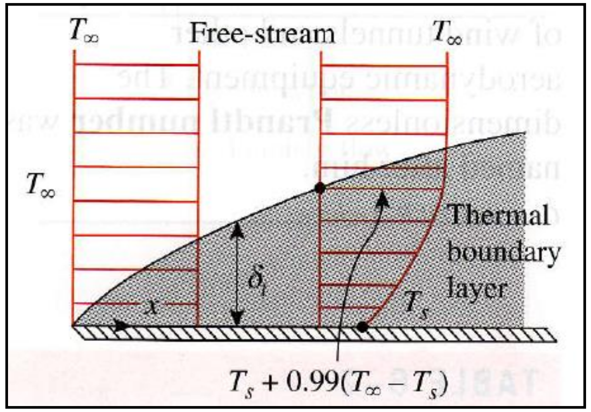
As a result, the x-component of the fluid velocity, u varies from 0 at $Y = 0$ to → V at $Y = \delta$. The region of the flow above the plate bounded by called “***velocity boundary layer*** “. The ***boundary layer thickness***, define as the distance y from the surface at which u = 0.99*V.

Convection: Thermal Boundary Layer

A thermal boundary layer develops → when a fluid at a specified temperature flows over a surface which is at a different temp.

Consider:

- The flow of a fluid at a uniform temp. of T_∞ over an isothermal flat plate at temp. T_s. These fluid particle then exchange energy with the particles in the adjoining fluid layer. The flow region over the surface in which the temp. variation in the direction normal to the surface is thermal boundary layer.
- The thickness of thermal boundary layer δ_t, at any location along the surface is defined as the distance from the surface at which the temp. difference →$T - T_s = 0.99(T_\infty - T_s)$.



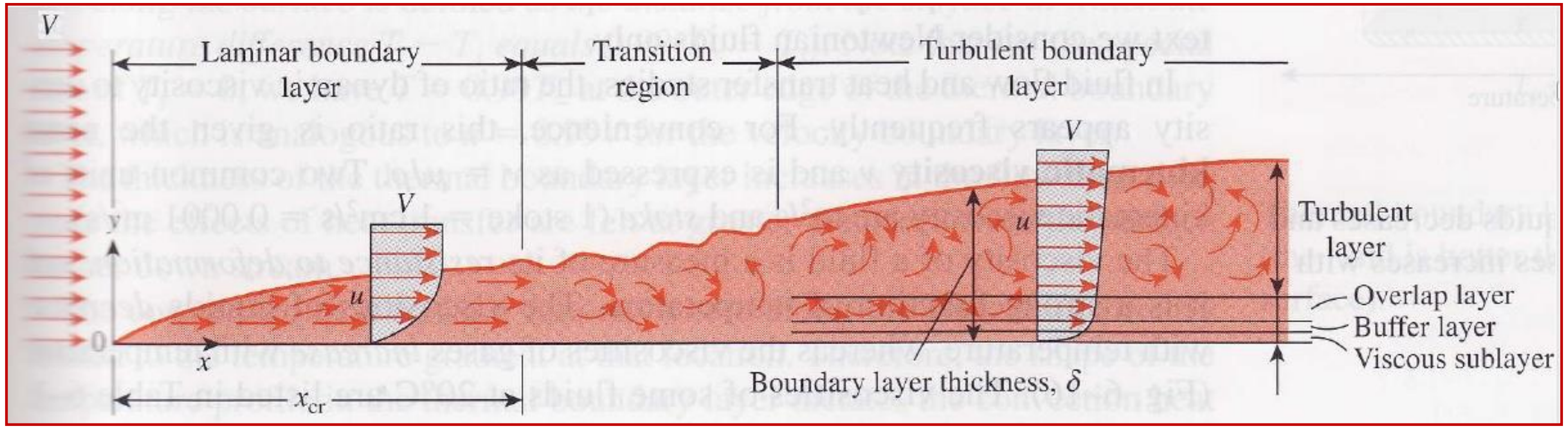
Thermal boundary layer on a flat plate (the fluid is hotter than the plate surface)

Note: the fluid velocity has a strong influence on the temp. profile, the development of the velocity boundary layer relative to the thermal boundary layer will have a **strong effect on the convection heat transfer**.

Convection: Laminar & Turbulent Flow

The flow regimes:

a. Laminar: Characterized by smooth stream-lines & highly-ordered motion.
b. Transition: Flow from laminar to turbulent occur over some region in which the flow fluctuates between laminar & turbulent.
c. Turbulent: characterized by velocity fluctuations & highly-disordered motion.



Most flows encountered in practice are turbulent. Laminar flows is encountered when highly viscous fluids such as oils flow in small pipes.

The turbulent boundary layer can consists of 4 regions

i. ***Viscous sublayer:*** The very thin layer next to the wall where viscous effects are dominant . The velocity profile nearly linear & the flow is streamlined.

ii. ***Buffer layer:*** The turbulent effects are becoming significant but the flow still dominated by viscous effects.

iii. ***Overlap layer:*** The turbulent effects are much more significant but still not dominating.

iv. ***Turbulent layer:*** The turbulent effects dominate over viscous effects.

The intense mixing of the fluid in turbulent flow as result of rapid fluctuations enhances → ***heat and momentum transfer*** between fluid particles, which ***increase*** the friction force on the surface & the convection heat transfer rate.

Convection: Reynolds & Prandtl numbers

Reynolds numbers

The flow regime depends on the ratio of the inertia forces to viscous forces in the fluid. This ratio is called → Reynolds no.

$$\mathrm{Re} = \frac{\text{Inertia forces}}{\text{Viscous forces}} = \frac{VL_c}{\nu} = \frac{\rho V L_c}{\mu}$$

Where.
V = Upstream velocity (equivalent to the free-stream velocity for a flat plate)
L_c = Characteristic length of the geometries
υ = kinematic viscosity of the fluid = $^{\mu}/_{\rho}$ (units: m^2/s)

- The critical Reynolds no. → The Reynolds no. at which the flow become turbulent.
- The value of the critical Reynolds no. is different for different geometries and flow conditions. For flow over a flat plate, the general value of the critical Reynolds no is

$$\boxed{\mathrm{Re}_{cr} = V x_{cr} / \nu = 5 \times 10^5}$$

Where x_{cr} is the distance from the leading edge of the plate at which transition from laminar to turbulent flow occurs

Prandtl numbers

The relative thickness of the velocity and the thermal boundary layers is best described by the *dimensionless* parameter **Prandtl number,** defined as

$$\mathrm{Pr} = \frac{\text{Molecular diffusivity of momentum}}{\text{Molecular diffusivity of heat}}$$

$$= \frac{\nu}{\alpha} = \frac{\mu C_p}{k}$$

Some important ***dimensionless numbers*** used in forced heat transfer convection

Dimensionless no	Physical significance	Expression
Nusselt	$\dfrac{\text{Wall temperature gradient}}{\text{Temperature gradient across the fluid in the pipe}}$	$Nu = \dfrac{hl}{k}$
Reynolds	$\dfrac{\text{Inertial force}}{\text{Viscous force}}$	$\dfrac{l\nu\rho}{\mu}$
Prandtl	$\dfrac{\text{Momentum diffusivity}}{\text{Thermal diffusivity}}$	$Pr = \dfrac{c_p\mu}{k} = \dfrac{\nu}{\alpha}$ where, $\nu = \frac{\mu}{\rho}, \alpha = \frac{k}{\rho c_p}$
Stanton	$\dfrac{\text{Heat flux by convection}}{\text{Heat flux by bulk flow}}$	$St = \dfrac{h}{u\rho c_p} = \dfrac{Nu}{Re\,Pr}$
Peclet	$\dfrac{\text{Heat flux by bulk flow}}{\text{Heat flux by conduction}}$	$Pe = \dfrac{lu\rho}{\mu}\cdot\dfrac{c_p\mu}{k}$ $= Re\,Pr$
Graetz	Same as Peclet number, however, used in connection with analysis of heat transfer in laminar flow in pipes of length L at distance d	$Gz = Pe\cdot\dfrac{d}{L}$

Notations:

h = heat transfer coefficient

l = characteristic length

k = thermal conductivity

ν = Momentum diffusivity

u = velocity

ρ = density

μ = viscosity

c_p = specific heat at constant pressure

L = length of the pipe (in Graetz number)

d = diameter of the pipe (in Graetz number)

Flow through a pipe or tube

Turbulent flow: A classical expression for calculating heat transfer in fully developed turbulent flow in smooth tubes/pipes of diameter (d) and length (L) is given by ***Dittus and Boelter***

$$Nu = 0.023Re^{0.8}Pr^{n} \;\ldots\ldots\ldots\ldots (7.1)$$

where,

n = 0.4, for heating of the fluid

n = 0.3, for cooling of the fluid

The properties in this equation are evaluated at the average fluid bulk temperature. Therefore, the temperature difference between bulk fluid and the wall should not be significantly high.

Application of eq. 7.1, lies in the following limits

$$0.7 \leq Pr \leq 160; \qquad \frac{d}{L} < 0.1; \qquad Re \geq 10{,}000$$

Gnielinski suggested that better results for turbulent flow in smooth pipe may be obtained from the following relations

$$Nu = 0.0214\left(Re^{0.8} - 100\right)Pr^{0.4} \;\ldots\ldots\ldots (7.2)$$

$$0.5 \leq Pr \geq 1.5; \qquad 10^{4} \leq Re \geq 5\times 10^{6}$$

$$Nu = 0.012\left(Re^{0.87} - 280\right)Pr^{0.4} \;\ldots\ldots\ldots (7.3)$$

$$1.5 \leq Pr \geq 500; \qquad 3000 \leq Re \geq 10^{6}$$

When the temperature difference between bulk fluid and wall is very high, the viscosity of the fluid and thus the fluid properties changes substantially. Therefore, the viscosity correction must be accounted using **Sieder–Tate equation** given below

$$Nu = 0.027Re^{0.8}Pr^{0.4}\left({}^{\mu}/_{\mu_w}\right)^{0.14} \ldots\ldots\ldots (7.4)$$

$$0.7 \leq Pr \geq 16{,}700; \qquad Re \geq 10{,}000; \frac{d}{} \leq 0.1$$

The fluid properties have to be evaluated at the mean bulk temperature of the fluid except μ_w which should be evaluated at the wall temperature.

The earlier relations were applicable for fully developed flow when entrance length was negligible. Nusselt recommended the following relation for the entrance region when the flow is not fully developed.

$$Nu = 0.036 Re^{0.8} Pr^{0.33} (\mu/\mu_w)^{0.055} \ldots\ldots\ldots (6.4)$$

$$0.7 \leq Pr \geq 16{,}700; \quad Re \geq 10{,}000; \quad 10 \leq \frac{d}{L} \leq 400$$

where, L is the tube length and d is the tube diameter.

As different temperature terms will appear in the course therefore to understand these terms see the following details.

i. **Bulk temperature/mixing cup temperature**: Average temperature in a cross-section.
ii. **Average bulk temperature**: Arithmetic average temperature of inlet and outlet bulk temperatures.
iii. **Wall temperature**: Temperature of the wall.
iv. **Film temperature**: Arithmetic average temperature of the wall and free stream temperature.
v. **Free stream temperature**: Temperature free from the effect of wall.
vi. **Log mean temperature difference**: It will be discussed in due course of time

Q. 6.1 *Pressurized air is to be heated by flowing into a pipe of 2.54 cm diameter. The air at 200°C and 2 atm pressure enters in the pipe at 10 m/s. The temperature of the entire pipe is maintained at 220°C. Evaluate the heat transfer coefficient for a unit length of a tube considering the constant heat flux conditions are maintained at the pipe wall. What will be the bulk temperature of the air at the end of 3 m length of the tube?*

Pr number	0.681
Viscosity	2.57×10^{-5} kg/m s
Thermal conductivity	0.015 W/m °C
Density	1.493 kg/m³
c_p	1.025 kJ/kg °C

$$Re = \frac{d\,v\,\rho}{\mu} = 14756$$

$$Nu = 0.023 Re^{0.8} Pr^{n}$$

n = 0.4 as the air is being heated,

$$\frac{h\,d}{k} = 0.023\,(14756)^{0.8}\,(0.681)^{0.4} = 42.67$$

$$h = 25.20\ W/m^2\ ^{\circ}C$$

Energy balance is required to evaluate the increase in bulk temperature in a 3 m length of the tube,

$$h(\pi d\, dL)(220 - T) = \dot{m} c_p\, dT$$

$$\int_{200}^{T} \frac{dT}{220 - T} = \frac{h \pi d}{\dot{m} c_p} \int_0^L dL$$

$$= \frac{h\,\pi\,d\,L}{\rho v \frac{\pi d^2}{4} c_p}$$

$$= \frac{4hL}{\rho c_p v d}$$

$$= \frac{(4)(25.2)(3)}{(1.493)(1025)(10)(0.0254)}$$

$$= 0.778$$

Or

$$[ln\,(220 - T)]_T^{200} = 0.778$$

$$ln 20 - \ln(220 - T) = 0.778$$

$$T = 210.81\ ^{\circ}C$$

Convection: Derivation of Differential Convection Equations

Consider the parallel flow of a fluid over a surface. For analysis, take the flow direction along the surface to be x & the direction normal to the surface to be y and a differential volume element of length dx, height dy and unit depth in z-direction.

The **continuity, momentum and energy equations** for steady 2D incompressible flow with constant properties are determined from mass. Momentum and energy balance to be:

Continuity → $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Momentum → $$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu\frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x}$$

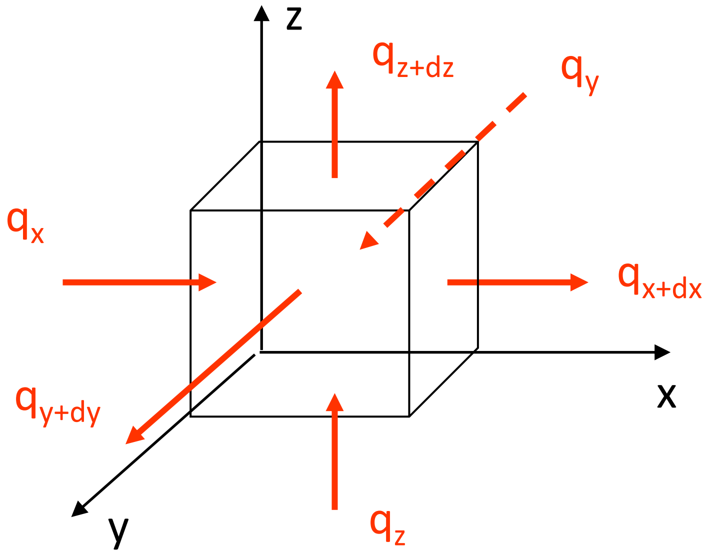
Energy → $$\rho\, c_p\left[u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right] = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \mu\Phi$$

Where the viscous dissipation function, (ϕ)

$$\Phi = 2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right] + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2$$

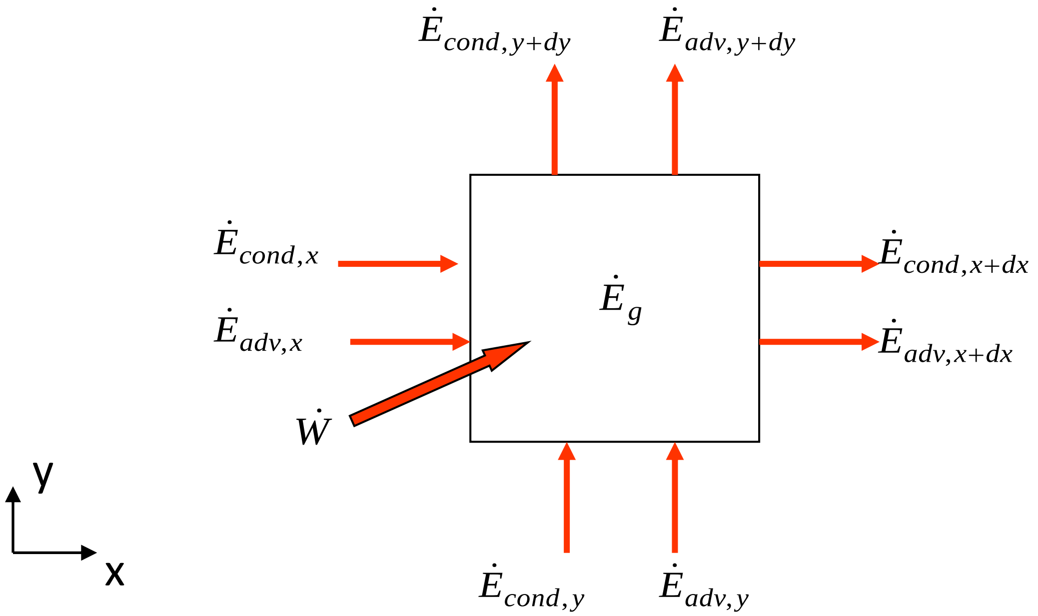
Convection: Conservation of Energy

Reminder: Previously we considered only heat transfer due to conduction and derived the “heat equation”



Energy Conservation Equation

$$\dot{E}_{in} + \dot{E}_g - \dot{E}_{out} = \frac{dE_{st}}{dt} = \dot{E}_{st}$$



- Must consider that energy is also transferred due to bulk fluid motion *(advection)*
  - Kinetic and potential energy
  - Work due to pressure forces

Convection: Thermal Energy Equation

For steady-state, two dimensional flow of an incompressible fluid with constant properties:

$$\underbrace{\rho C_p\left(u\frac{\partial T}{\partial x}+\upsilon\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}\right)}_{\substack{\text{Net outflow of heat due to}\\ \text{bulk fluid motion (advection)}}}=\underbrace{k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)}_{\substack{\text{Net inflow of heat due to}\\ \text{conduction}}}+\mu\Phi+\underbrace{\dot{q}}_{\substack{\text{rate of energy}\\ \text{generation per}\\ \text{unit volume}}}$$

where $$\mu\Phi=\mu\left\{\left(\frac{\partial u}{\partial y}+\frac{\partial \upsilon}{\partial x}+\frac{\partial w}{\partial x}\right)^2+2\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial \upsilon}{\partial y}\right)^2+\left(\frac{\partial w}{\partial z}\right)^2\right]\right\}$$

represents the **viscous dissipation**: *Net rate at which mechanical work is irreversibly converted to thermal energy, due to viscous effects in the fluid*

Take away from todays session

- ✓ Energy balance.
- ✓ Heat transfer global coefficient.
- ❑ Logarithmic mean temperature difference (LMTD).
- ✓ Newton's Law of cooling and the local heat transfer coefficient.
- ✓ Forced convection in laminar and turbulent regimes inside tubes.
- ❑ Forced convection in outside tubes
- ❑ Natural convection.
- ❑ Tubular heat exchangers.
- ❑ Plate heat exchangers
- ❑ Extended surfaces (fins)

Next Session…

Heat Exchangers: Flow arrangement and Design.