Size Reduction

Introduction

- Size means physical dimension and reduction means process of decreasing size.
- Therefore, size reduction means process of changing the object from large physical dimension to the smaller one.
- The other term used for size reduction is comminution.

Objective of Size Reduction

- To increase the surface area
- To produce solid particles of desired shape, size or size ranges, and specific surface,
- To improve the handling (storage and transportation) characteristics, etc.

Mechanism of size reduction

Different mechanisms of size reductions are:

Impact : In this, particle breaks by single rigid force or in general, give coarse, medium, or fine products,

Compression: In this, particle disintegration by two rigid forces or in general used for the coarse reduction of hard solids,

Shear: produced by fluid or by particle – particle interaction.

Attrition: arising from particle scraping or rubbing between two surfaces or gives very fine products

Most of the size-reduction equipment's employ a combination of all these size reduction methods. Another, size reduction method is

Cutting: which gives a particle of definite size and shape

Non-Mechanical introduction of energy : For example, thermal shock, explosive shattering. Ultrasonic grinding, etc.

Impact

- It refers to the sharp, instantaneous collision of one moving object against another.
- Both objects may be moving, such as a cricket bat connecting with a fast moving ball, or one object may be motionless, such as a rock being struck by a hammer blow
- Two types of impact—gravity impact and dynamic impact.
- In gravity impact, the free-falling material is momentarily stopped by the stationary object.
- For example: Coal dropped onto a hard steel surface
- Materials dropping in front of a moving hammer is an example of dynamic impact.

Compression

- Particle is broken by two forces and the size reduction is done between two surfaces, with the work being done by one or both surfaces
- For example, Jaw crushers
- Suitable for reducing extremely hard and abrasive rock.

Attrition

- Attrition is a method of size reduction by rubbing or, scrubbing the materials between two hard surfaces. Hammer mills (discussed later in this chapter) operate with close clearances between the hammers and the screen bars, reduce the size of materials by attrition combined with shear and impact actions
- Attrition consumes more power, it is preferred for crushing the less abrasive materials such as pure limestone and coal

Shear

- Shear consists of a trimming or cleaving action rather than the rubbing action associated with attrition.
- It is usually combined with other size-reduction actions, e.g., single-roll crushers (discussed later in this chapter) employ shear together with impact and compression.

Model Predicting Energy Requirement and Product Size Distribution

- It is almost impossible to find out the accurate amount of energy requirement in order to effect size reduction of a given material, mainly because:
- There is a wide variation in the size and shape of particles both in the feed and product, and
- Some energy is wasted as heat and sound, which can not be determined exactly

- But, a number of empirical laws have been proposed to relate the size reduction with the energy input to the machine. They are Rittinger's Law (1867), Kick's Law (1885), and Bond's Law (1952)

Energy Requirement

The power consumption is determined using crushing or grinding efficiency.

$$\text{Crushing efficiency } \eta_c = \frac{\text{Surface energy created by crushing}}{\text{Total energy absorbed by solid}} \quad (1)$$

if, W_a = Total energy absorbed by a unit mass of solid, J/kg

E_s = Surface energy per unit area, J/m² and

A_{ssf}, A_{ssp} = Areas per unit mass of feed & product, respectively (or specific surfaces), m²/kg

Specific energy created by crushing = $E_s(A_{ssp} - A_{ssf})$ — (2)

$$\therefore \boxed{\eta_c = \frac{E_s(A_{ssp} - A_{ssf})}{W_a}} \longleftarrow (2)$$

But Energy absorbed by unit mass of the solids (W_a) is less than the energy fed (W) because most of portion of total energy input is used to overcome friction in the bearings and other moving parts & rest is available for size reduction, i.e mechanical efficiency

Mechanical efficiency $\eta_m = \frac{\text{Energy absorbed}}{\text{Energy fed to the machine}}$

thus.

$$\eta_m = \frac{W_a}{\text{Energy fed to the system}} \quad \text{——} \quad (4)$$

$$\text{Total energy input} = \frac{W_a}{\eta_m} \quad \text{——} \quad (5)$$

putting value from Eqn (3)

$$= \frac{E_s(A_{ssp} - A_{ssf})}{\eta_m \eta_c} \quad \text{——} \quad (6)$$

If $\dot{m}$ is the flow rate of solids to machine then power required, P, will be -

$$= \dot{m} \times \text{Total energy input} \quad \text{——} \quad (7)$$

$$\boxed{P = \frac{E_s(A_{ssp} - A_{ssf})}{\eta_m \eta_c} \dot{m}} \quad \text{——} \quad (8)$$

Specific surfaces of feed & product materials -

$$A_{ssf} = \frac{6}{\phi_f D_f \rho_{pf}} \quad \& \quad A_{ssp} = \frac{6}{\phi_p D_p \rho_{pp}}$$

Where ϕ_f & ϕ_p = sphericity of the feed & product

D_f & D_p = Sauter mean diameter for the feed & product, meter

ρ_{pf} & ρ_{pp} = Density of the feed & product material kg/m^3

For Homogenous materials $\rho_{pf} = \rho_{pp} = \rho_p$

putting all values in equ^n (8)

$$P = \frac{6 E_s \dot{m}}{\eta_m \eta_c \rho_p} \left\{ \frac{1}{\phi_p D_p} - \frac{1}{\phi_f D_f} \right\}$$

- This relation tells us that the power requirement for crushing will be more for particles having higher surface energy and also for the higher flow rate.
- All the particles (each having certain surface area) in an unit mass of solid particles have a definite amount of surface energy and when their size is reduced, their surface area as well as the surface energy per unit mass increases. And when this occurs, the power requirement becomes more and more for reducing fine particles to still finer ones than for breaking down large pieces of rock.

Rittinger's Law

According to this law, the work required for size reduction is proportional to the new surface area created. Mathematically,

$$W_R = \frac{P}{\dot{m}} = K E_s \left(A_{ssp} - A_{ssf}\right) \quad , \text{ where } k = \frac{1}{\eta_c} = \text{constant}$$

$$W_R = \frac{P}{\dot{m}} = 6 K E_S \left(\frac{1}{\phi_p D_p \rho_{pp}} - \frac{1}{\phi D_f \rho_{pf}}\right)$$

for constant sphericity + density, the work required.

$$W_R = \frac{P}{\dot{m}} = \frac{6KE_s}{\phi \rho_p}\left(\frac{1}{D_p} - \frac{1}{D_f}\right) = K_R\left(\frac{1}{D_p} - \frac{1}{D_f}\right)$$

Where $K_R = \frac{6KE_s}{\phi \rho_p}$ = Rittingers constant

The inverse of rittingers constant is known as Rittingers number.

- Rittinger’s law is applicable mainly to that part of the process, where new surface is being created and holds most accurately for fine grinding where the increase in surface per unit mass of material is predominant,
- Also, this law is applied in cases where the energy input per unit mass of material is not too high.
- This law is applicable for feed size of less than 0.05 mm.

⇒ Kick's law.

This law states that the work required for crushing a given mass of material is constant for a given reduction ratio irrespective of the initial size. Reduction ratio is the ratio of initial particle size to final particle size

$$W_K = \frac{P}{\dot{m}} = K_K \ln\left(\frac{D_f}{D_p}\right)$$

K_K = kicks constant

- For example, if a given quantity material is being crushed from 100 mm to 20 mm or, from 30 mm to 6 mm then in both the cases the energy requirement will be the same as the reduction ratio (100/20 = 30/6 = 5) is same for both the cases.
- This law is more accurate than Rittinger's law for coarse crushing where the surface area produced per unit mass is considerably less.
- This law is applicable for feed size of greater than 50 mm.

Bonds Law

- Neither of the two laws mentioned above (Rittinger's or Kick's) give the accurate energy requirement and both the laws are applicable over a limited range of particle size, and hence, they have limited utility.
- But in the year 1952, F. C. Bond suggested an intermediate law, which states that the work required to form particles of size D_{pp} from a very large particle size is proportional to the square root of the surface to volume ratio s_p/v_p of the product.
- This law is applicable for feed size between 0.05 and 50 mm.

Bond's law -

$$\frac{S_p}{V_p} = \frac{6}{\phi_s D_p}$$

Mathematically,

$$W_B = \frac{P}{\dot{m}} = K\left\{\left(\sqrt{\frac{S_p}{V_p}}\right)_p\right\} = k\sqrt{\frac{6}{\phi D_{pp}}} = k\sqrt{\frac{6}{\phi}} \times \frac{1}{\sqrt{D_{pp}}} = k_b \frac{1}{\sqrt{D_{pp}}}$$

Where k = constant & $k_b = k\sqrt{\frac{6}{\phi}}$ = Bonds constant

or

$$W_B = \frac{P}{\dot{m}} = k_b\left(\frac{1}{\sqrt{D_{pp}}} - \frac{1}{\sqrt{D_{pf}}}\right)$$

⇒ The Bonds constant (K_b) is dependent on the type of machine used and material to be crushed.

⇒ Work index W_i is defined as the amount of energy in KW-hr per ton of feed material, required to reduce a very large feed to such a size that 80% of the product passes through a 100 μm screen.

So. from bond law $$W_i = K_b \sqrt{\frac{1}{D_{pp}}}$$

$$K_b = W_i \sqrt{D_{pp}}$$

if P is in kW, $\dot{m}$ is in ton per hour, and

(i) D_{pp} is in μm then $K_b = 10\, W_i$, and

(ii) D_{pp} is in mm then $K_b = \sqrt{0.1}\, W_i = 0.3162\, W_i$

Thus, if 80% of feed particles pass through a D_{pf} mm screen and 80% of product particles pass through a D_{pp} mm screen then,

$$\boxed{W_B = \frac{P}{\dot{m}} = 0.3162\, W_i \left(\frac{1}{\sqrt{D_{pp}}} - \frac{1}{\sqrt{D_{pf}}} \right)}$$

Generalised law:

All the 3 laws can be derived from a generalised equ^n relating work required for crushing and the particle size

$$W = \frac{P}{\dot{m}} = -K\left(\frac{1}{D_{vs}^{\,N}}\right) = \frac{dP}{dD_{sn}}$$

for $N = 2$ $\quad K = K_R$ for Rittinger

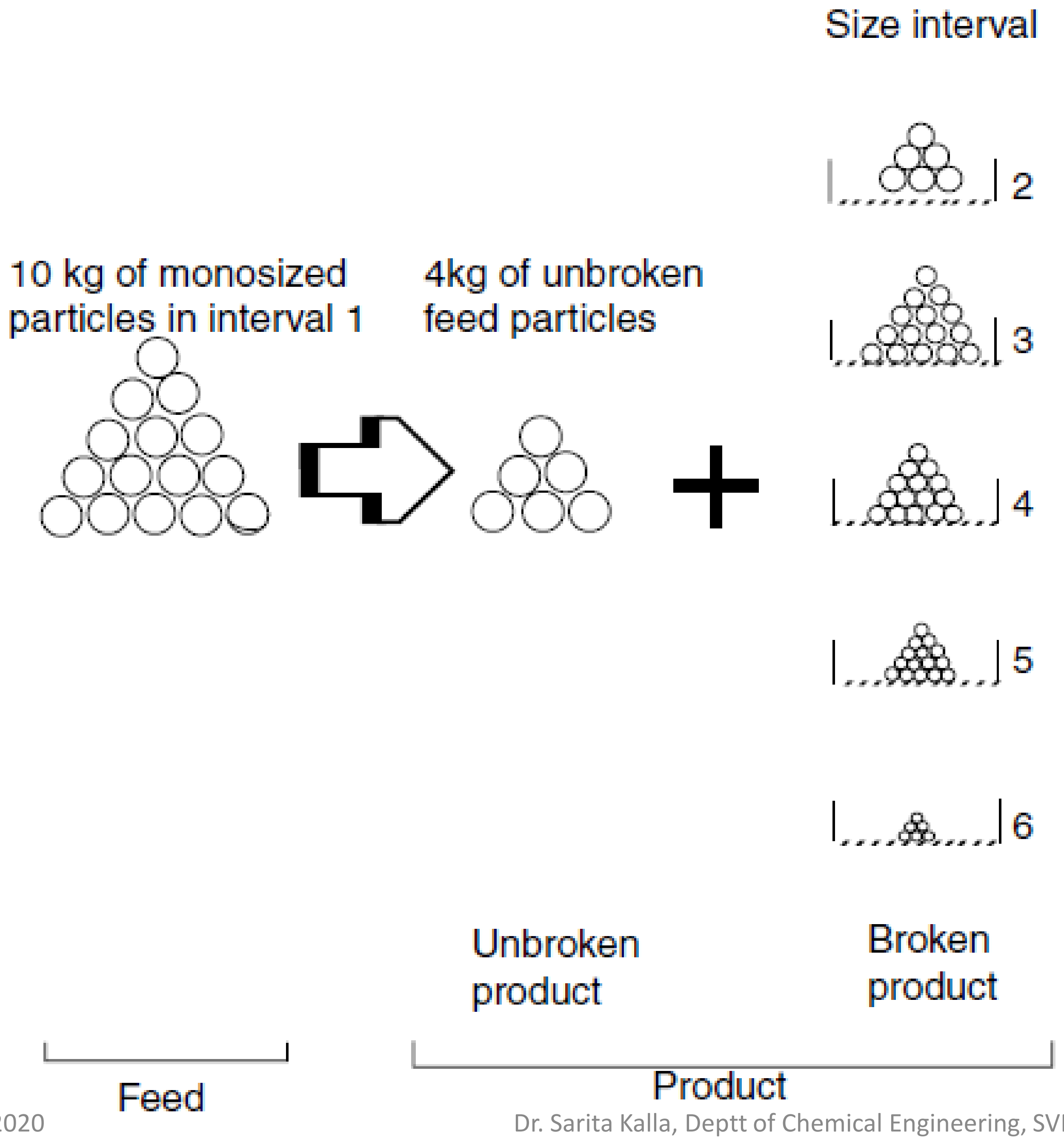
$N = 1$ $\quad K = K_K$ for Kick

$N = 1.5$ $\quad K = K_B$ for Bond

Prediction of the Product Size Distribution

- It is common practice to model the breakage process in comminution equipment on the basis of two functions, the specific rate of breakage and the breakage distribution function.
- The specific rate of breakage S_j is the probability of a particle of size j being broken in unit time (in practice, 'unit time' may mean a certain number of mill revolutions, for example)
- The breakage distribution function b(i, j) describes the size distribution of the product from the breakage of a given size of particle. For example , b(i, j) is the fraction of breakage product from size interval j which falls into size interval i.

- When dealing with 10 kg of monosized particles in size interval 1. If S1 = 0.6 we would expect 4 kg of material to remain in size interval 1 after unit time. The size distribution of the breakage product would be described by the set of b(i, j) values. Thus, for example, if b(4,1)=0.25 we would expect to find 25% by mass from size interval 1 to fall into size interval 4
- The breakage distribution function may also be expressed in cumulative form as B (i,j), the fraction of the breakage product from size interval j which falls into size intervals j to n, where n is the total number of size intervals. [B(i,j) is thus a cumulative undersize distribution.]



Mass (kg)	$b(i, j)$	$B(i, j)$
0.9	0.15	1
1.8	0.3	0.85
1.5	0.25	0.55
1.2	0.2	0.3
0.6	0.1	0.1

S is a *rate* of breakage the rate of change of the mass of particles in size interval i with time:

$$\frac{\mathrm{d}m_i}{\mathrm{d}t} = \sum_{i=1}^{j=i-1} [b(i,j)S_j m_j] - S_i m_i$$

where

$$\sum_{j=1}^{j=i-1} [b(i,j)S_j m_j] = \text{mass broken into interval } i \text{ from all intervals of } j > i$$

$$S_i m_i = \text{mass broken out of interval } i$$

Since $m_i = y_i M$ and $m_j = y_j M$, where M is the total mass of feed material and y_i is the mass fraction in size interval i, then we can write a similar expression for the rate of change of mass fraction of material in size interval, i with time:

Types & classification of size reduction equipment

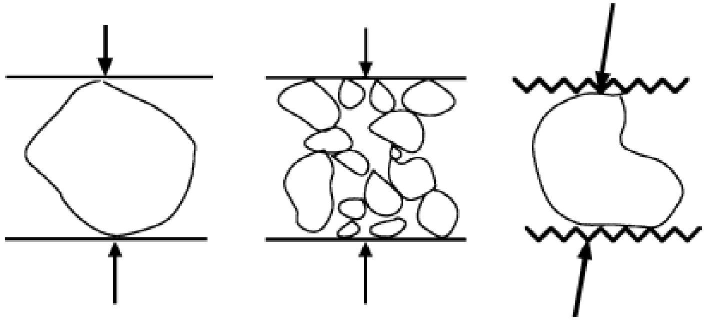
Factors Affecting Choice of Machine

The choice of machine selected for a particular grinding operation will depend on the following variables:

- stressing mechanism;
- size of feed and product;
- material properties;
- carrier medium;
- mode of operation;
- capacity;
- combination with other unit operations

Stressing Mechanism

(1) Stress applied between two surfaces (either surface-particle or particle-particle) at low velocity, 0.01–10 m/s.



- Crushing plus attrition

Contd.

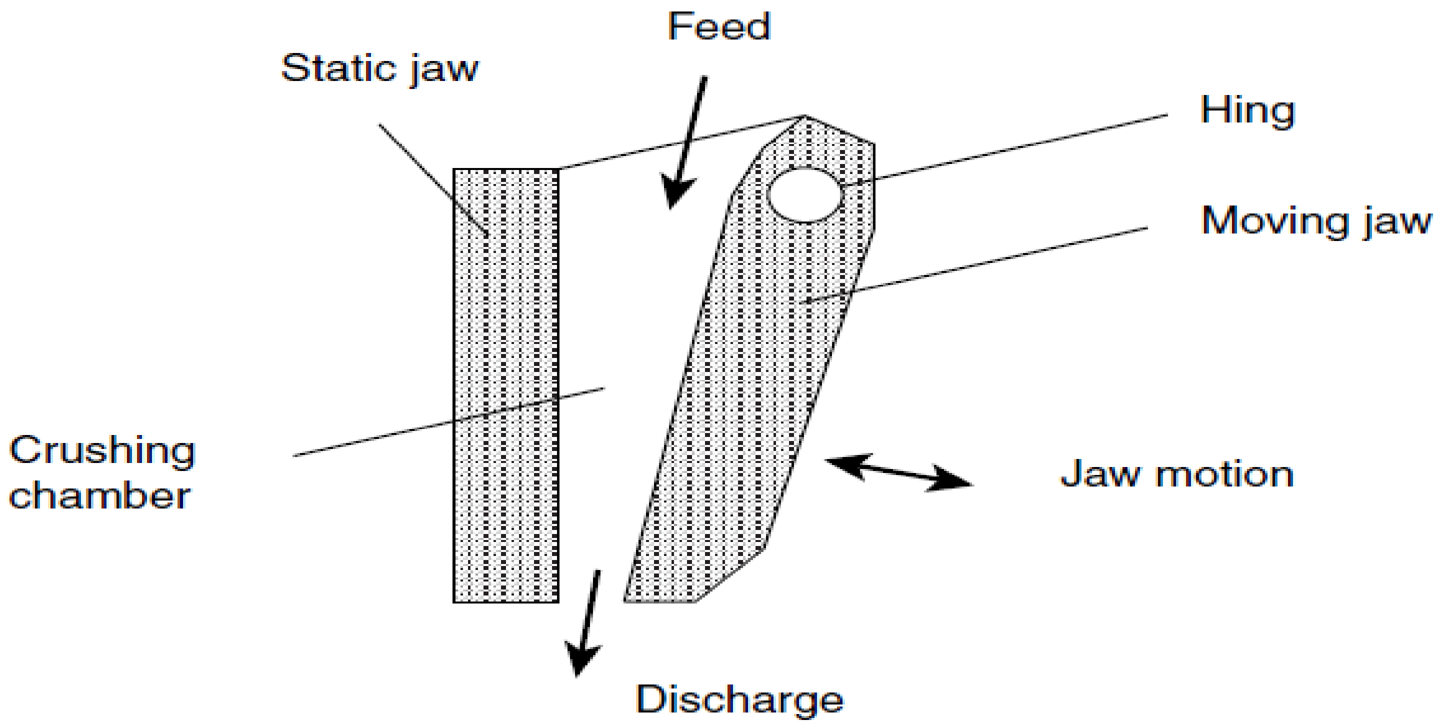
(2) Stress applied at a single solid surface (surface-particle or particle-particle) at high velocity, 10-200 m/s.

- Impact fracture plus attrition

(3) Stress applied by carrier medium-usually in wet grinding to bring about disagglomeration.

Machines using mainly mechanism 1, crushing

The jaw crusher behaves like a pair of giant nutcrackers. One jaw is fixed and the other, which is hinged at its upper end, is moved towards and away from the fixed jaw by means of toggles driven by an eccentric. The lumps of material are crushed between the jaws and leave the crusher when they are able to pass through a grid at the bottom.

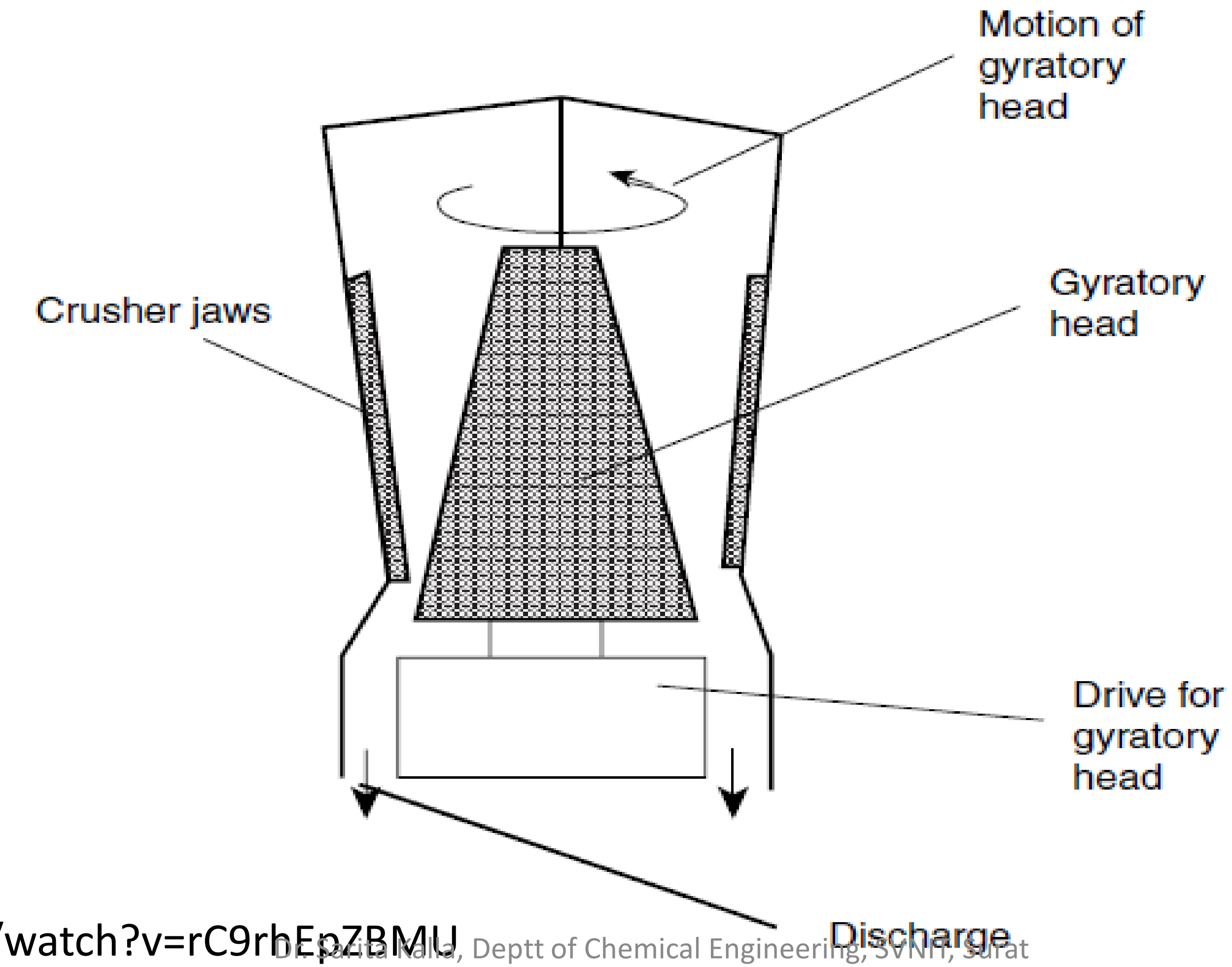


Schematic Diagram of a Jaw Crushers

https://www.youtube.com/watch?v=yTIePG1E-kY

Gyratory Crusher

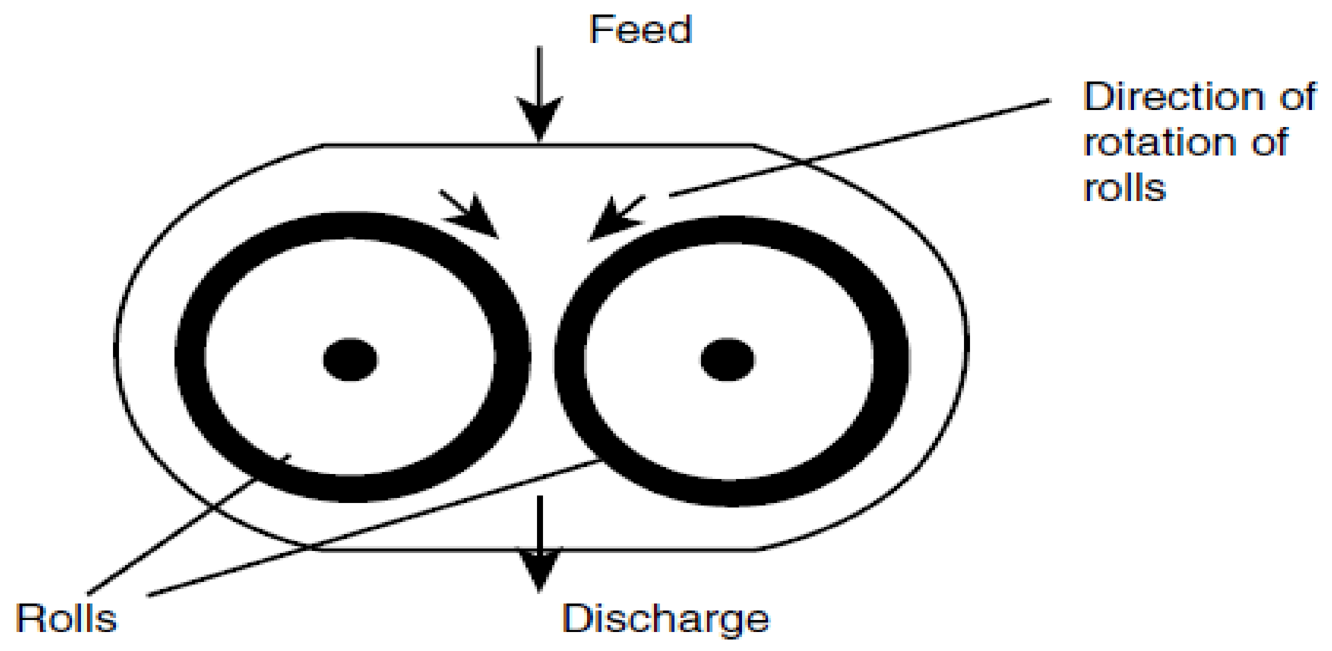
The gyratory crusher has a fixed jaw in the form of a truncated cone. The other jaw is a cone which rotates inside the fixed jaw on an eccentric mounting. Material is discharged when it is small enough to pass through the gap between the jaws.



https://www.youtube.com/watch?v=rC9rhEpZBMU

Crushing Roll machine

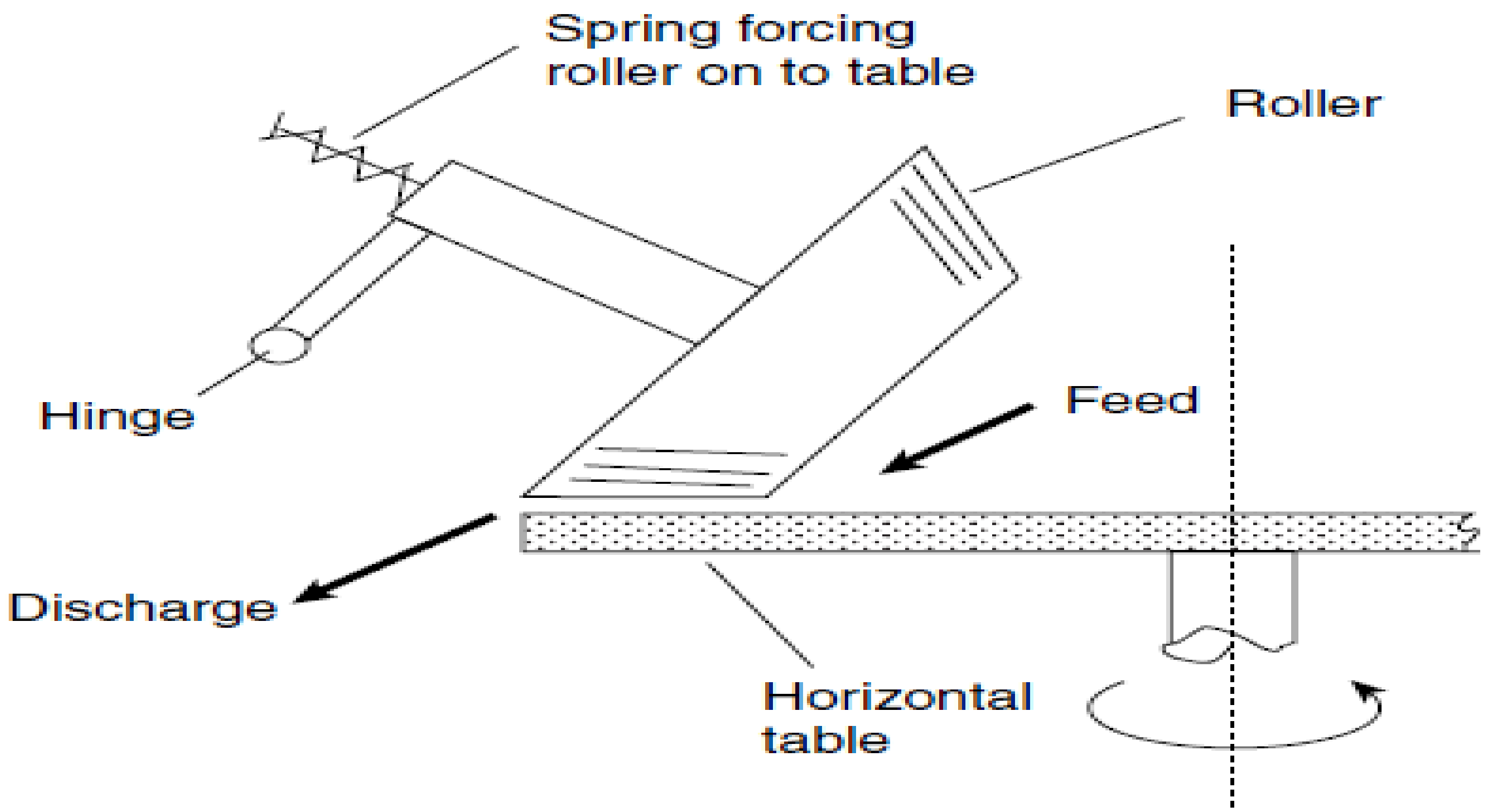
- In the crushing roll machine two cylindrical rolls rotate in opposite directions, horizontally and side by side with an adjustable gap between them. As the rolls rotate, they drag in material which is choke-fed by gravity so that particle fracture occurs as the material passes through the gap between the rolls. The rolls may be ribbed to give improved purchase between material and rolls.



https://www.youtube.com/watch?v=r0XZTIWXQ3M

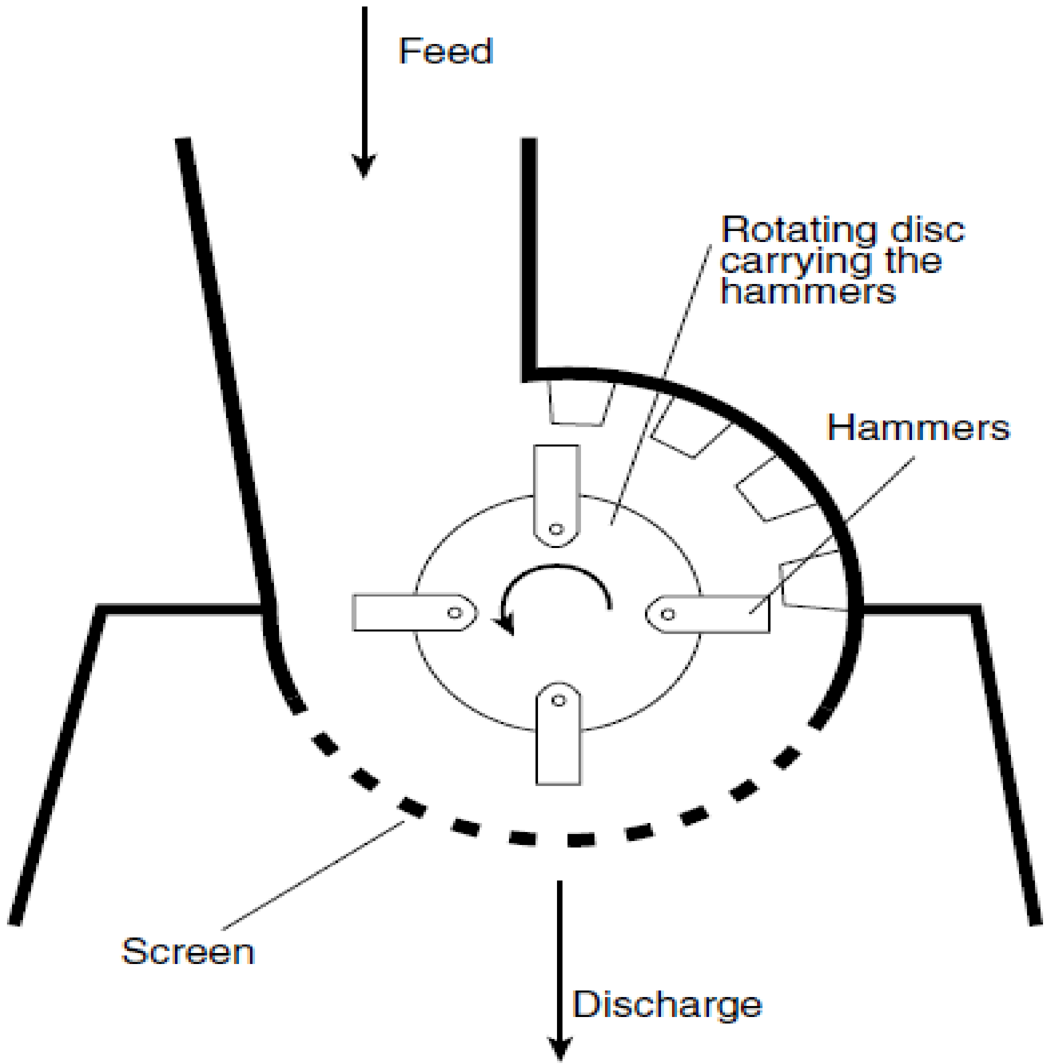
Horizontal table mill

- In the horizontal table mill, shown in Figure, the feed material falls on to the centre of a circular rotating table and is thrown out by centrifugal force. In moving outwards the material passes under a roller and is crushed.



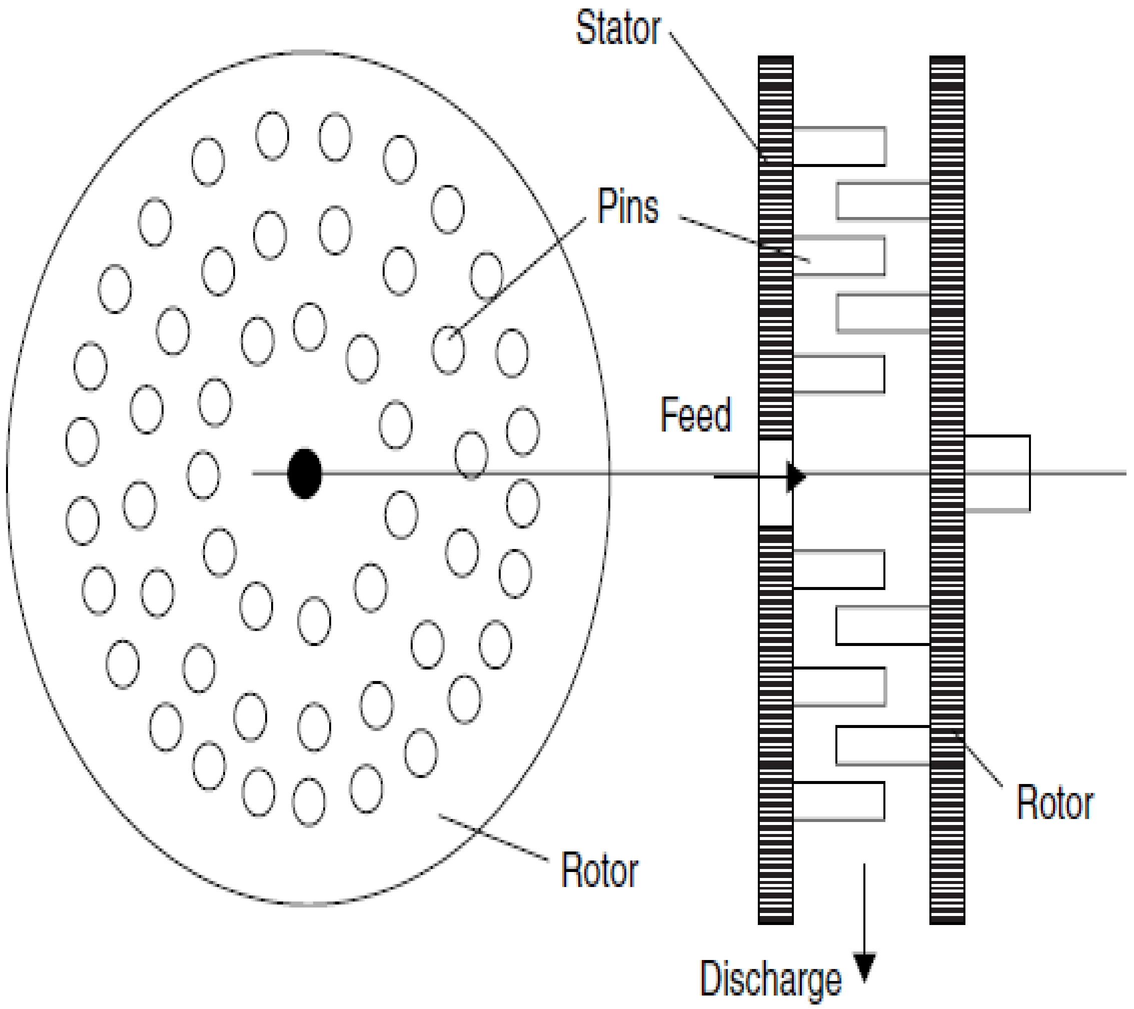
Machines using mainly mechanism 2, high velocity impact

The hammer mill, shown in Figure, consists of a rotating shaft to which are attached fixed or pivoted hammers. This device rotates inside a cylinder. The particles are fed into the cylinder either by gravity or by gas stream. In the gravity-fed version the particles leave the chamber when they are small enough to pass through a grid at the bottom.



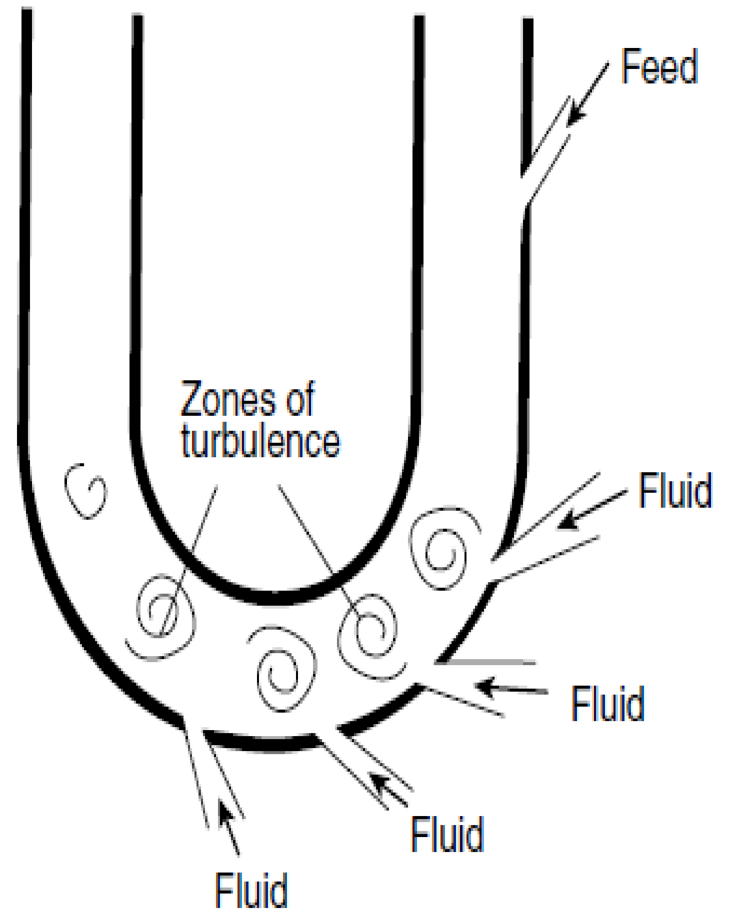
Pin mill

A pin mill consists of two parallel circular discs each carrying a set of projecting pins, as shown in Figure, One disc is fixed and the other rotates at high speed so that its pins pass close to those on the fixed disc. Particles are carried in air into the centre and as they move radially outwards are fractured by impact or by attrition



Fluid Energy Mill

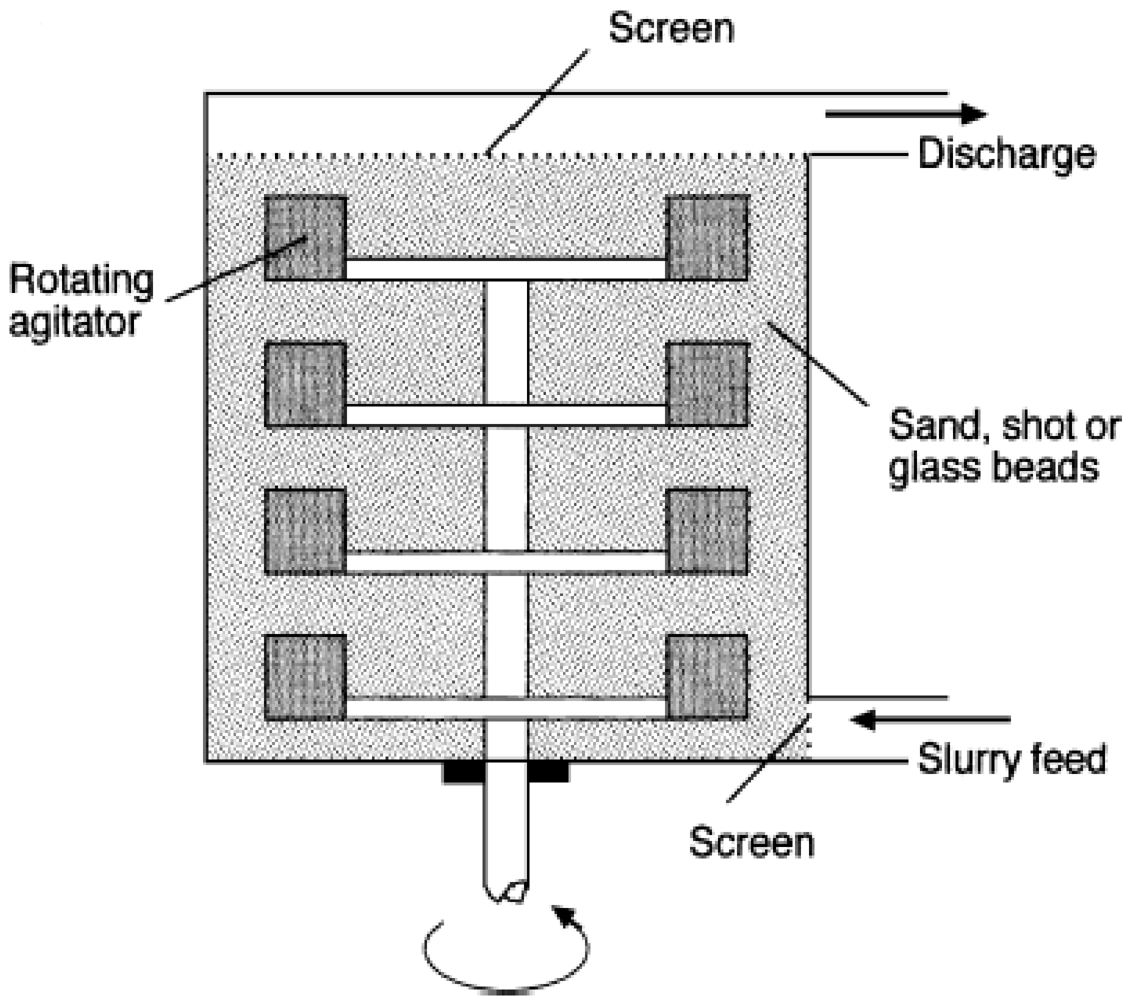
The fluid energy mill relies on the turbulence created in high velocity jets of air or steam in order to produce conditions for interparticle collisions which bring about particle fracture. A common form of fluid energy mill is the loop or oval jet mill shown in Figure. Material is conveyed from the grinding area near the jets at the base of the loop to the classifier and exit situated at the top of the loop. These mills have a very high specific energy consumption and are subject to extreme wear when handling abrasive materials. These problems have been overcome to a certain extent in the fluidized bed jet mill in which the bed is used to absorb the energy from the high-speed particles ejected from the grinding zone.



Schematic diagram of Fluid Energy Mill

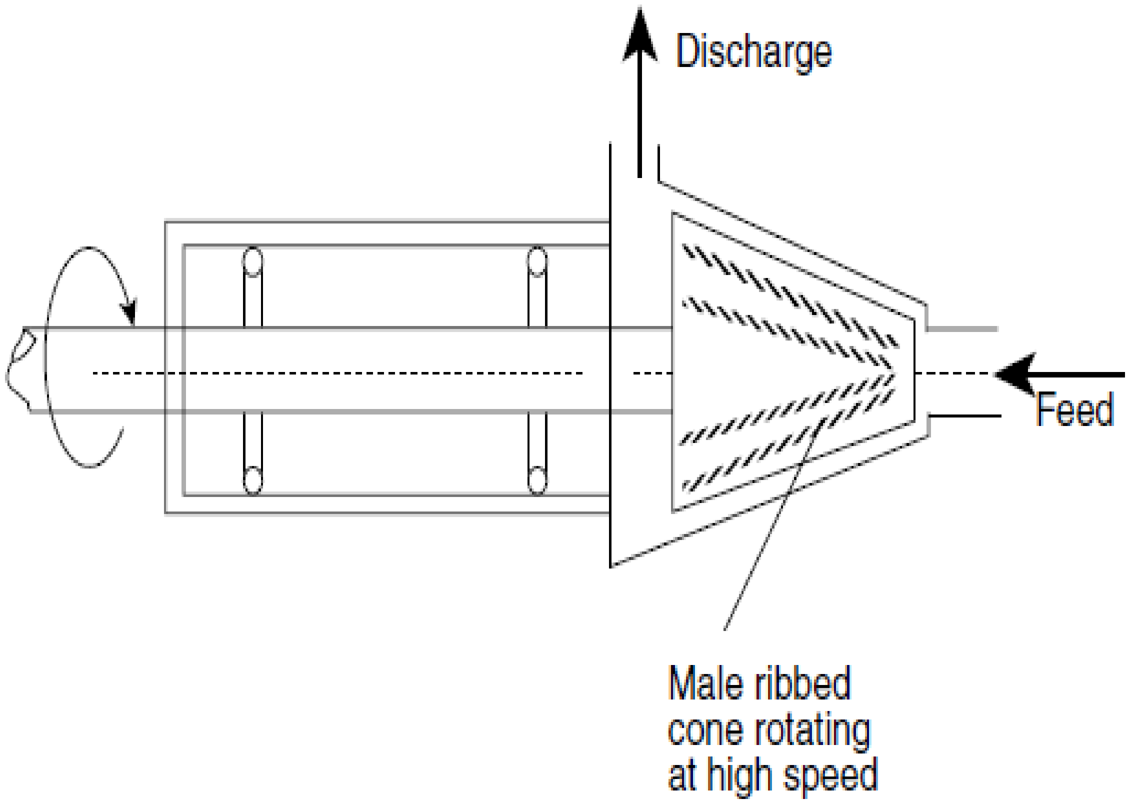
Machines using a combination of mechanisms 1 and 2, crushing and impact with attrition

The sand mill, shown in Figure, is a vertical cylinder containing a stirred bed of sand, glass beads or shot. The feed, in the form of a slurry, is pumped into the bottom of the bed and the product passes out at the top through a screen which retains the bed material.



Colloid Mill

In the colloid mill, the feed in the form of a slurry passes through the gap between a male, ribbed cone rotating at high speed and a female static cone, as shown in the Figure.



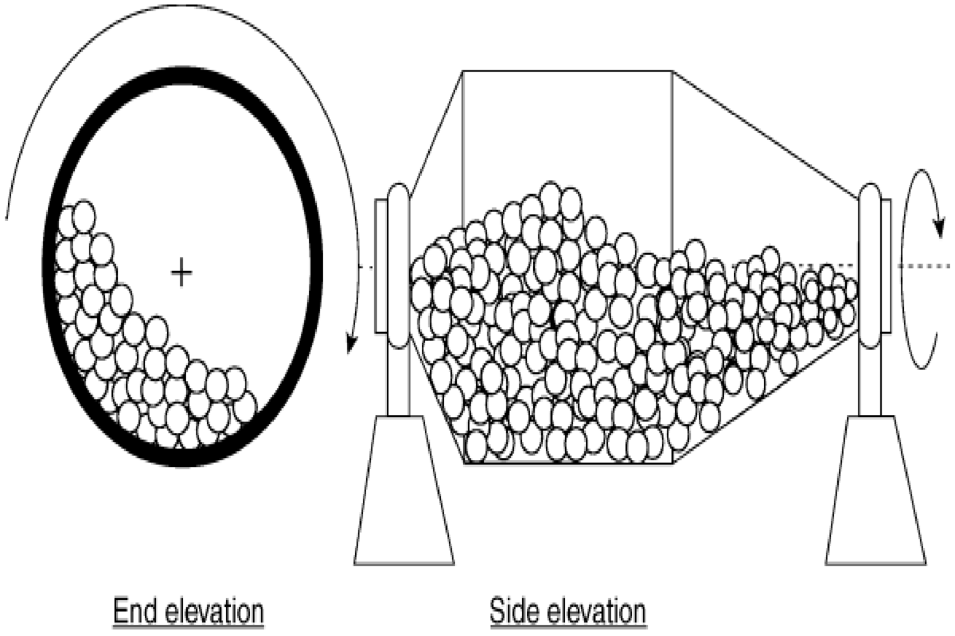
Ball Mill

The ball mill, shown in Figure, is a rotating cylindrical or cylindrical- conical shell about half filled with balls of steel or ceramic. The speed of rotation of the cylinder is such that the balls are caused to tumble over one another without causing cascading.

This speed is usually less than 80% of the critical speed which would just cause the charge of balls and feed material to be centrifuged. In continuous milling the carrier medium is air, which may be heated to avoid moisture which tends to cause clogging.

Ball mills may also be used for wet grinding with water being used as the carrier medium. The size of balls is chosen to suit the desired product size.

The conical section of the mill, shown in Figure, causes the smaller balls to move towards the discharge end and accomplish the fine grinding.



Particle Size

- How the product size determines the type of mill to be used shown in the table

Terminology used in comminution

Size range of product	Term used
1–0.1 m	Coarse crushing
0.1 m	Crushing
1 cm	Fine crushing, coarse grinding
1 mm	Intermediate grinding, milling
100 μm	Fine grinding
10 μm	Ultrafine grinding

Comminution equipment classification according to product size

Down to 3 mm	3 mm–50 μm	<50 μm
Crushers	Ball mills	Ball mills
Table mills	Rod mills	Vibration mills
Edge runner mills	Pin mills	Sand mills
	Tube mills	Perl mills
	Vibration mills	Colloid mills
		Fluid energy mills

Material Properties

- *Hardness* Hardness is usually measured on the Mohs' scale of hardness where graphite is ranked 1 and diamond is ranked 10. The property hardness is a measure of the resistance to abrasion.
- *Abrasiveness* This is linked closely to hardness and is considered by some to be the most important factor in selection of commercial mills. Very abrasive materials must generally be ground in mills operating at low speeds to reduce wear of machine parts in contact with the material

Contd.

- *Toughness* This is the property whereby the material resists the propagation of cracks. In tough materials excess strain energy brings about plastic deformation rather than propagation of new cracks. Brittleness is the opposite of toughness. Tough materials present problems in grinding, although in some cases it is possible to reduce the temperature of the material, thereby reducing the propensity to plastic flow and rendering the material more brittle.

Contd.

- Cohesivity/adhesivity The properties whereby particles of material stick together and to other surfaces. Cohesivity and adhesivity are related to moisture content and particle size. Decrease of particle size or increasing moisture content increases the cohesivity and adhesivity of the material. Problems caused by cohesivity/adhesivity due to particle size may be overcome by wet grinding.

Contd.

- Fibrous nature Materials of a fibrous nature are a special case and must be comminuted in shredders or cutters which are based on the hammer mill design.
- Low melting point The heat generated in a mill may be sufficient to cause melting of such materials causing problems of increased toughness and increased cohesivity and adhesivity. In some case

Contd.

- *Other special properties* Materials which are thermally sensitive and have a tendency to spontaneous combustion or high inflammability must be ground using an inert carrier medium (e.g. nitrogen). Toxic or radioactive materials must be ground using a carrier medium operating on a closed circuit.

Carrier Medium

- The carrier medium may be a gas or a liquid. Although the most common gas used is air, inert gases may be used in some cases as indicated above. The most common liquid used in wet grinding is water although oils are sometimes used.
- The carrier medium not only serves to transport the material through the mill but, in general, transmits forces to the particles, influences friction and hence abrasion, affects crack formation and cohesivity/adhesivity. The carrier medium can also influence the electrostatic charging and the flammability of the material

Mode of Operation

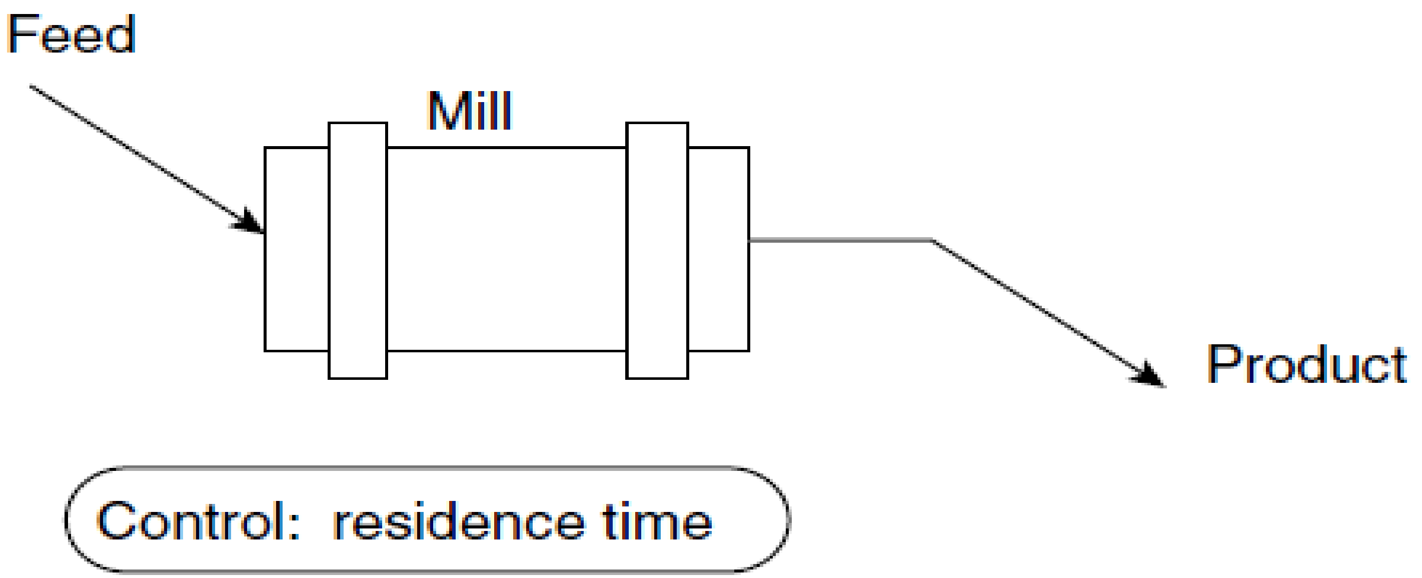
- Mills operate in either batch or continuous mode. Choice between modes will be based on throughput, the process and economics. The capacity of batch mills varies from a few grammes on the laboratory scale to a few tonnes on a commercial scale. The throughput of continuous milling systems may vary from several hundred grammes per hour at laboratory scale to several thousand tonnes per hour at industrial scale.

Combination with other Operations

- Some mills have a dual purpose and thus may bring about drying, mixing or classification of the material in addition to its size reduction.

Types of Milling Circuit

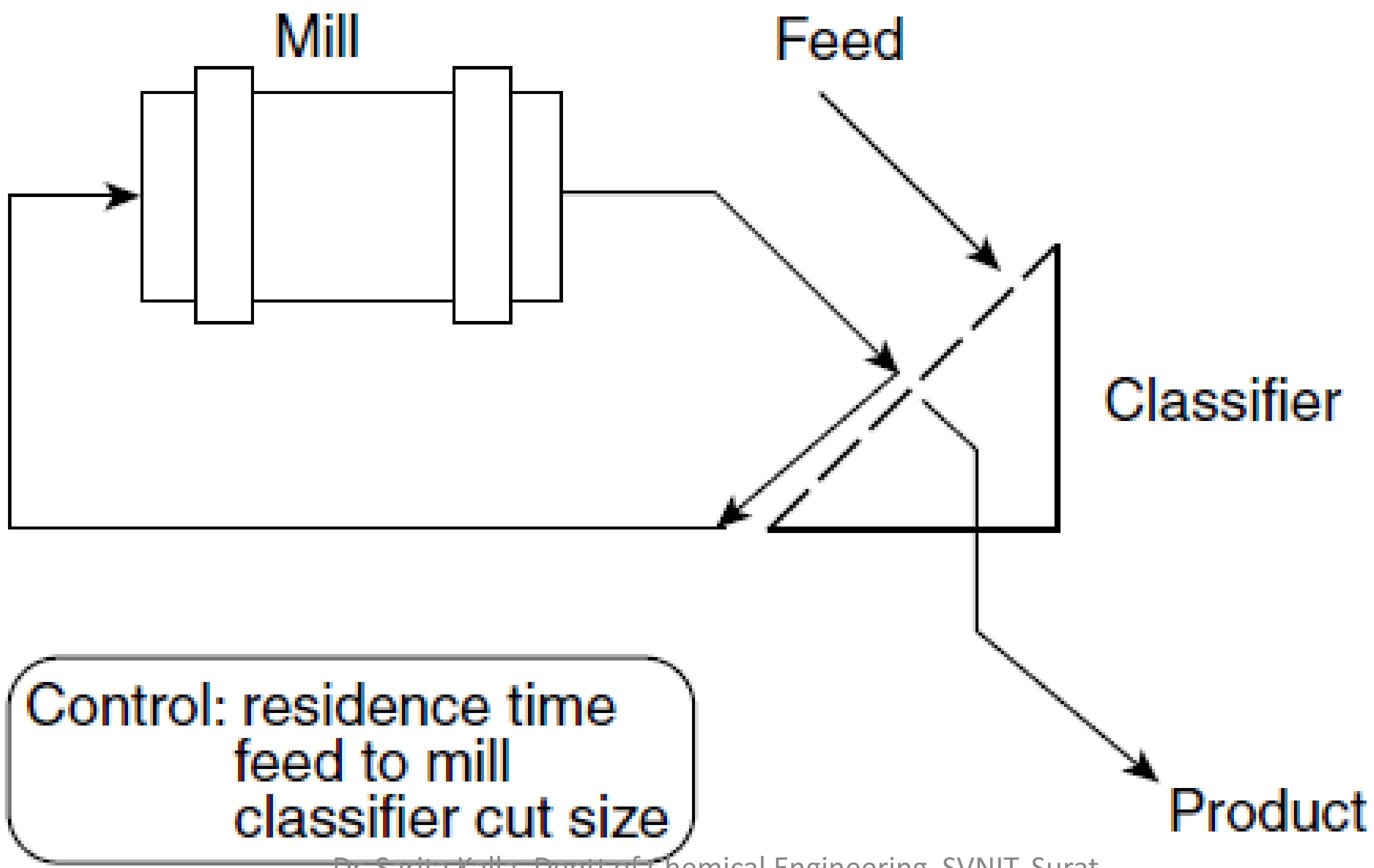
- Milling circuits are either 'open circuit' or 'closed circuit'. In open circuit milling the material passes only once through the mill, and so the only controllable variable is the residence time of the material in the mill. Thus the product size and distribution may be controlled over a certain range by varying the material residence time (thoughput), i.e. feed rate governs product size and so the system is inflexible.



Open Circuit Milling

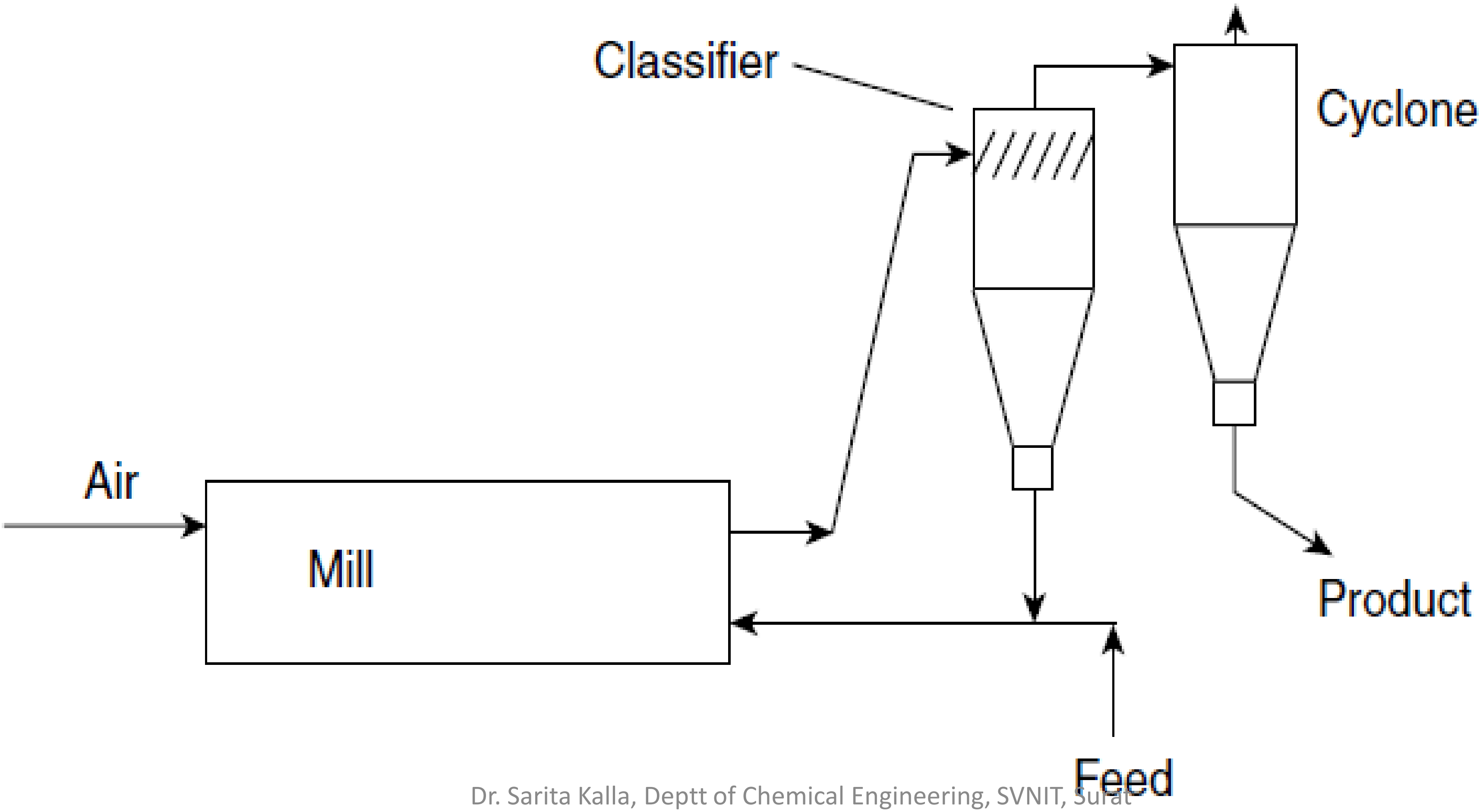
Closed circuit milling

In closed circuit milling the material leaving the mill is subjected to some form of classification (separation according to particle size) with the oversize being returned to the mill with the feed material. Such a system is far more flexible since both product mean size and size distribution may be controlled.



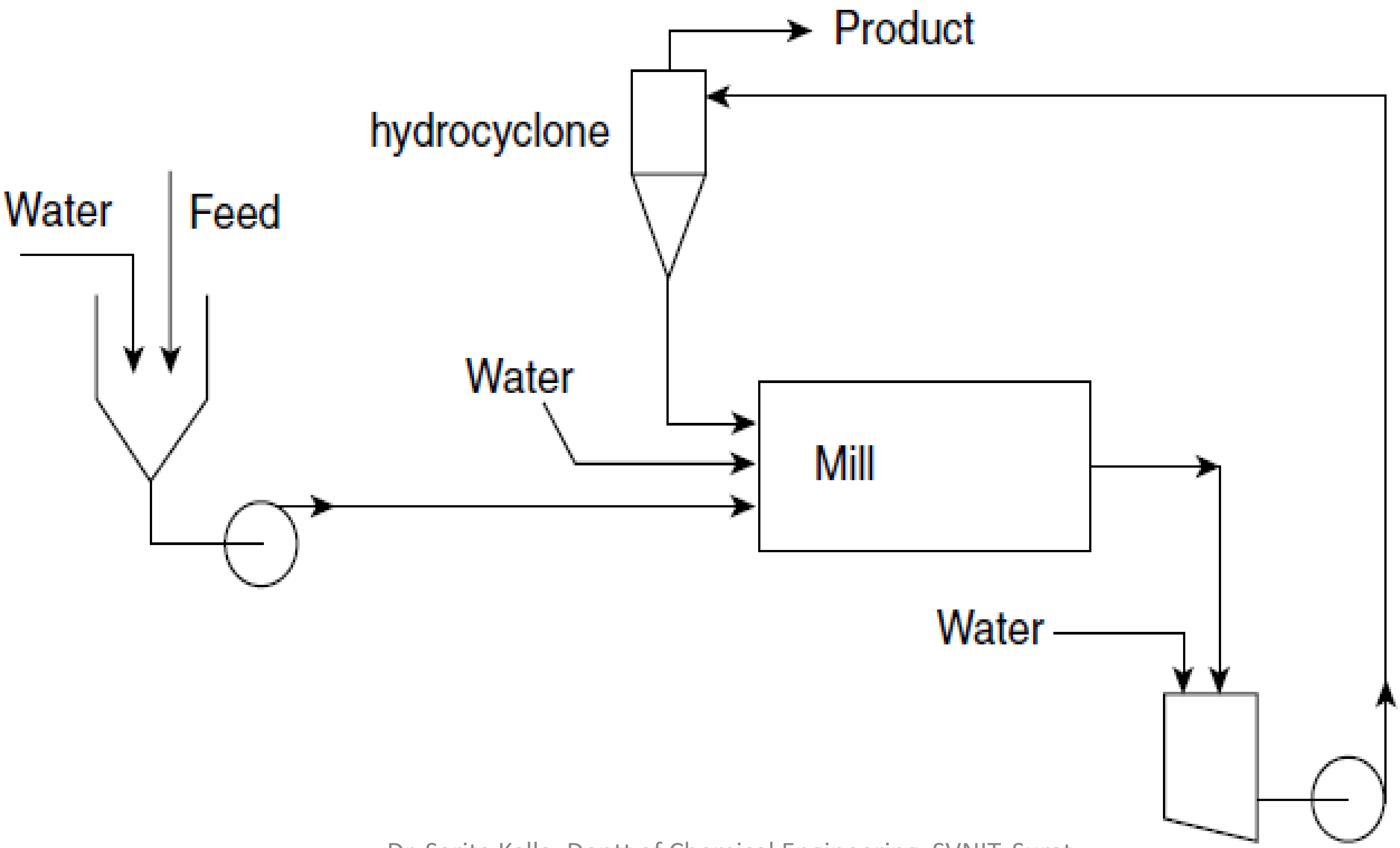
Dry milling: Closed circuit operation

The equipment necessary for feeding material into the mill, removing material from the mill, classifying, recycling oversize material and removing product in the case of dry closed milling circuits, respectively.



Wet milling: closed circuit operation

The equipment necessary for feeding material into the mill, removing material from the mill, classifying, recycling oversize material and removing product in the case of wet closed milling circuits, respectively.



References

- Martin Rhodes, "Introduction to Particle Technology", 2nd Edition, John Wiley & Sons, 2008.

- *Mechanical Operations*. Authors: A.K. *Swain*, H. Patra and G.K. Roy. Publisher: Tata McGraw Hill Education Pvt. Ltd., New Delhi