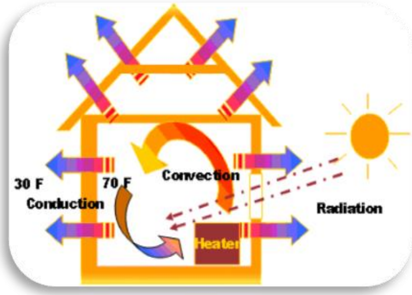
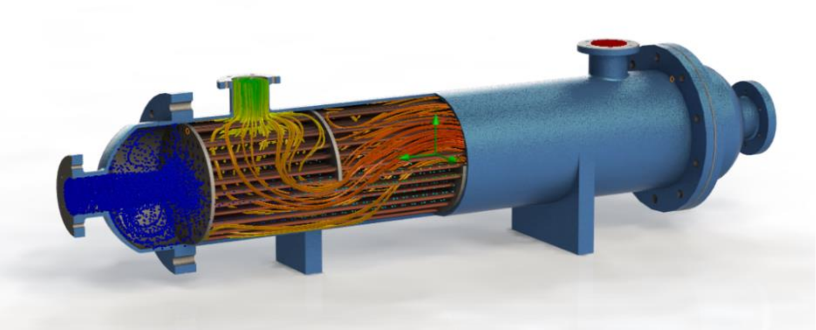
Heat Transfer Operations

Lecture-10. Transient Heat Conduction





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Objectives

- ✓ Consider transient heat conduction when the spatial variation of temperature is negligible, and temperature varies nearly uniformly with time, making the simplified lumped system analysis applicable.
- ✓ Obtain analytical solutions for transient one-dimensional conduction problems in rectangular, cylindrical, and spherical geometries using the method of separation of variables,
- ✓ Understand why a one-term solution is usually a reasonable approximation.
- ✓ Solve the transient conduction problem in large mediums using the similarity variable, and predict the variation of temperature with time and distance from the exposed surface.
- ✓ Construct solutions for multi-dimensional transient conduction problems using the product solution approach.

Introduction

- ✓ We now distinguish that many heat transfer problems are time dependent. Such unsteady, or transient, problems arises typically when the boundary conditions of a system are changed. For example, **a hot metal billet** that is removed from a furnace and exposed to a cool airstream.
  a. Energy is transferred by **convection and radiation** from its surface to the surroundings.
  b. Energy transfer by **conduction** also occurs from the interior of the metal to the surface, and the temperature at each point in the billet decreases until **a steady-state condition** is reached.
  c. The final properties of the metal will depend significantly on the **time-temperature** history that results from heat transfer.
- ✓ Thus, controlling the heat transfer is key to fabricate new materials with enhanced properties.

Learning aim of this chapter is…

To develop procedures for determining

- The time dependence of the temperature distribution within a solid (conduction) during a transient process, and
- Heat transfer between the solid and its surroundings (convection or radiation).
- It depends on the assumption that may be made for the process. If, for example, temperature gradients within the solid may be neglected, a comparatively simple approach, termed the lumped capacitance method, may be used to determine the variation of temperature with time.

A system in which the temperature* of a solid varies with time but remains uniform throughout the solid at any time is called *Lumped Systems.

Lumped System Analysis…

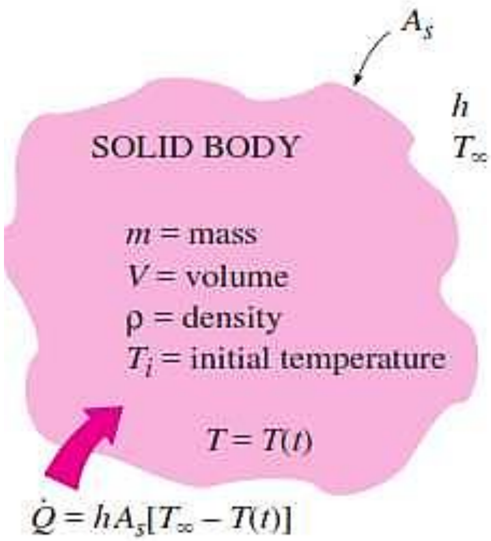
- In the heat transfer operations, some bodies are observed to behave like a "lump" a compact mass.
- Interior temperature of some bodies remains essentially uniform at all times during a heat transfer process.
- The temperature of such bodies can be taken to be a function of time only, $T(t)$.
- Heat transfer analysis that utilizes this idealization is known as lumped system analysis, which provides great simplification in certain classes of heat transfer problems without much sacrifice from accuracy.

Consider *a small hot copper ball coming out of an oven. Measurements indicate that the temperature of the copper ball changes with time, but it does not change much with position at any given time.*

Consider *a large roast in an oven. If you cut in half, you will see that the outer parts of the roast are well done while the center part is barely warm.*

So in which case, Lumped system analysis is not applicable..!

Formulation of Lumped System Analysis



i. *Consider a body of arbitrary shape shown fig. At time t = 0, the body is placed into a medium at temperature T_∞, and heat transfer takes place between the body and its environment, with a heat transfer coefficient h.*

ii. *For the sake of discussion, we will assume that $T_\infty > T_i$, but the analysis is equally valid for the opposite case.*

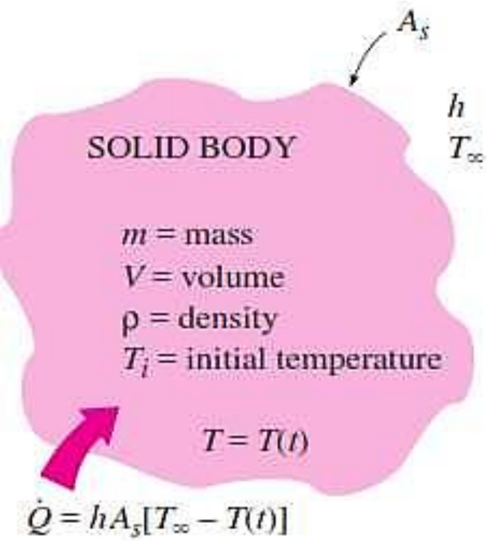
iii. *We assume lumped system analysis to be applicable, so that the temperature remains uniform within the body at all times and changes with time only, T = T(t).*

iv. *During a differential time interval* ***dt****, the temperature of the body rises by a differential amount dT.*

An **energy balance** of the solid for the time interval ***dt*** can be expressed as

$$\begin{bmatrix} Heat\ trasnfer \\ into\ the\ body \\ during\ time\ dt \end{bmatrix} = \begin{bmatrix} Energry\ increase \\ in\ the\ body \\ during\ time\ dt \end{bmatrix} \dots\dots\dots\dots..(10.1)$$

Formulation of Lumped System Analysis



$$hA_s(T_\infty - T)\,dt = mc_p\,dT \quad \ldots\ldots\ldots\ldots..(10.2)$$

$m = \rho V$ and $dT = d(T - T_\infty)$ *since* T_∞ is constant

Can be rearranged as $$\frac{d(T - T_\infty)}{T - T_\infty} = -\frac{hA_s}{\rho V c_p}\,dt \quad \ldots(10.3)$$

Integrating with
$T = T_i$ at $t = 0$
$T = T(t)$ at $t = t$

$$\ln\frac{T(t) - T_\infty}{T_i - T_\infty} = -\frac{hA_s}{\rho V c_p}\,t$$

Taking the exponential of both sides and rearranging, we obtain

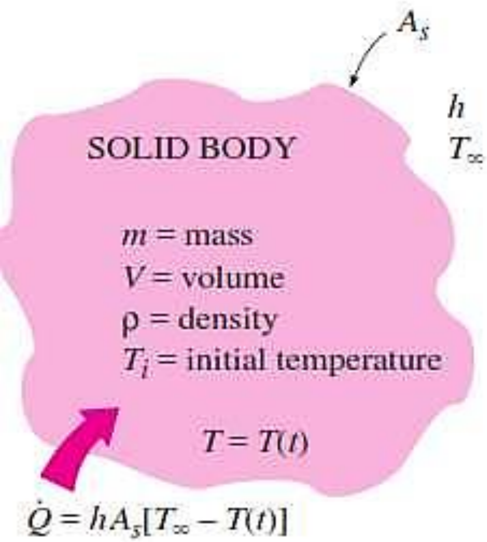
$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \quad \ldots\ldots(10.4), \textit{where,} \quad b = \frac{hA_s}{\rho V c_p}$$

or

$$\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = \exp\left[-\left(\frac{hA_s}{\rho Vc}\right)t\right] \quad \ldots(10.4)$$

Where, $\theta \equiv T - T_\infty$ and $\theta_i \equiv T_i - T_\infty$

Formulation of Lumped System Analysis



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$$\ln\frac{T(t) - T_\infty}{T_i - T_\infty} = -\frac{hA_s}{\rho V c_p}\,t \quad \ldots(10.4)$$

To compute t required to reach T

Taking the exponential of both sides and rearranging, we obtain

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \quad \ldots\ldots(10.5), \textit{where}, \quad b = \frac{hA_s}{\rho V c_p}$$

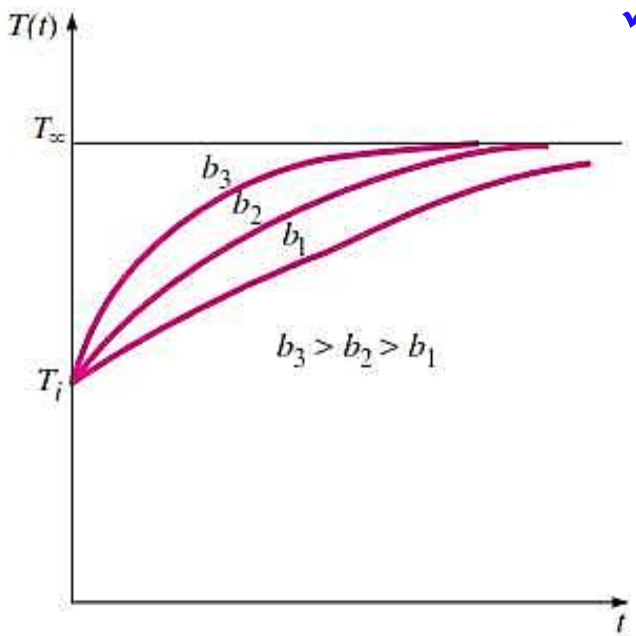
or
$$\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = \exp\left[-\left(\frac{hA_s}{\rho V c}\right)t\right] \quad \ldots(10.6)$$

Where, $\theta \equiv T - T_\infty$ *and* $\theta_i \equiv T_i - T_\infty$

To compute T reached by solid at t

The reciprocal of ***b*** *has* time unit (usually **s**),and is called the **time constant.**

Formulation of Lumped System Analysis



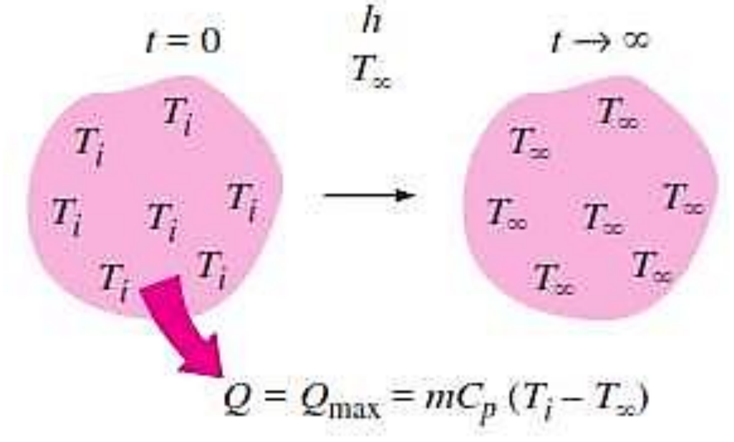
- ✓ Eq. 10.5 enables us to determine the temperature T(t) of a body at time t, or alternatively, the time t required for the temperature to reach a specified value T(t). It can be observed that
  - ✓ The temperature of a body approaches the ambient temperature ***T***∞ exponentially.
  - ✓ The temperature of the body changes rapidly at the beginning, but rather slowly later on.
  - ✓ A large value of b indicates that the body will approach the environment temperature in a short time
  - ✓ The larger the value of the exponent b, the higher the rate of decay in temperature.

Note that, b is proportional to the surface area, but inversely proportional to the mass and the specific heat of the body. Which means that it takes longer to heat or cool a larger mass, especially when it has a ***large specific heat.***

$$b = \frac{hA_s}{\rho V c_p}$$

Formulation of Lumped System Analysis

- ✓ *Once the temperature T(t) at time t is available from Eq. (10.5) , the rate of convection heat transfer between the body and its environment at that time can be determined from Newton's law of cooling as*



$$\dot{Q}(t) = hA_s(T(t) - T_\infty) \ldots..(10.6a)$$

The *rate* of convection heat transfer (W) between the body and its environment at time *t*.

$$Q(t) = mC_P(T(t) - T_i) \ldots..(10.6b)$$

The *total amount* of heat transfer (kJ) between the body and the surrounding medium over the time interval $t = 0$ to t

$$Q_{max} = mC_P(T(t) - T_i) \ldots..(10.6b)$$

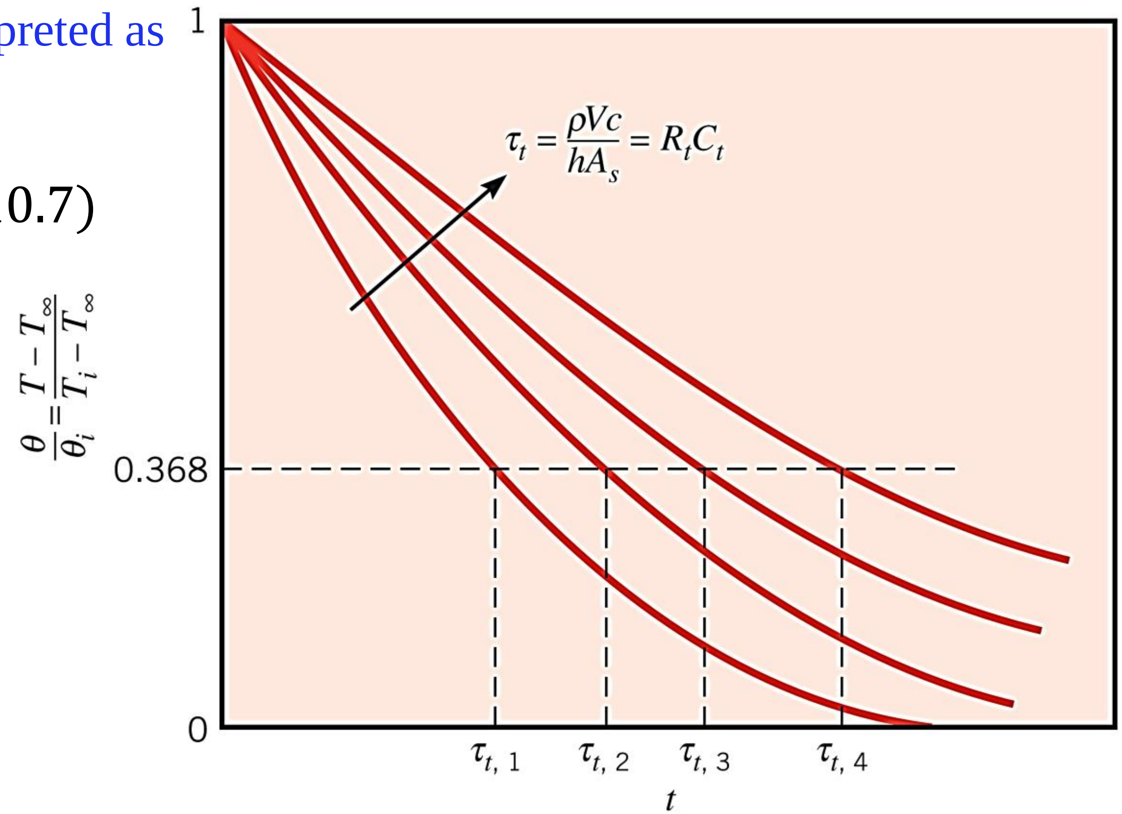
The *maximum* heat transfer between the body and its surroundings when body reaches the surrounding temperature (***T***∞).

Formulation of Lumped System Analysis

The results indicate that the difference between the solid and fluid temperatures decays exponentially to zero as t approaches infinity as shown in fig.

The quantity ($\rho Vc/hA_s$) may be interpreted as a *thermal time constant* expressed as

$$\tau_t = \left(\frac{1}{hA_s}\right)(\rho Vc) = R_t C_t \qquad \ldots(10.7)$$



R_t is the resistance to convection heat transfer and C_t is the ***lumped thermal capacitance*** of the solid. Any increase in R_t or C_t will cause the solid to respond more slowly to changes in its thermal environment.

To determine the total energy transfer Q occurring up to some time t, we simply write

$$Q = \int_0^t q\,dt = hA_s\int_0^t \theta\,dt \qquad \ldots(10.8)$$

Formulation of Lumped System Analysis

Substituting for θ from Equation 10.6 and integrating, we obtain

$$Q = (\rho V c)\theta_i \left[1 - \exp\left(-\frac{t}{\tau_t}\right)\right] \qquad \ldots(10.9)$$

The quantity Q is, of course, related to the change in the internal energy of the solid, and from $\Delta E_{\mathrm{st}}^{\mathrm{tot}} = Q - W$

$$-Q = \Delta E_{\mathrm{st}} \qquad \ldots(10.9)$$

For quenching, Q is positive and the solid experiences a decrease in energy.

Validity of the Lumped Capacitance Method

In this section, we determine under what conditions it may be used with reasonable accuracy for solving transient heating and cooling problems.
To develop a suitable criterion consider steady-state conduction through the plane wall of area A

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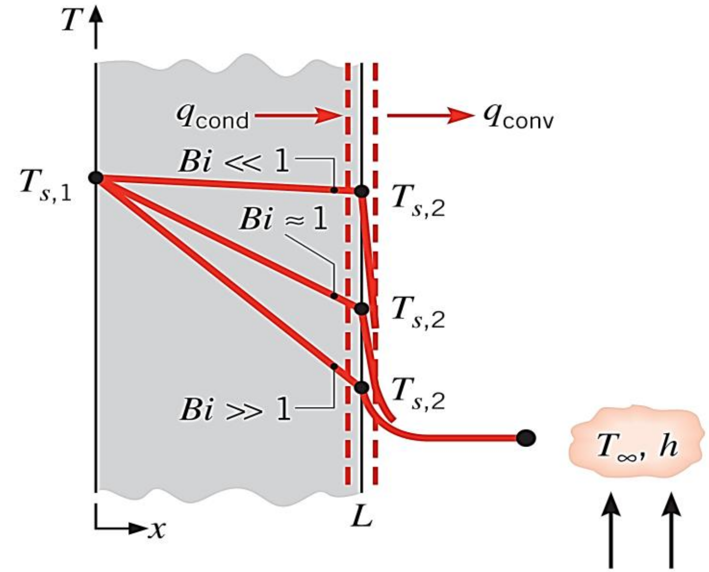


Although we are assuming steady-state conditions, the following criterion is readily extended to transient processes. One surface is maintained at a temperature Ts,1 and the other surface is exposed to a fluid of temperature T∞ < Ts,1. The temperature of this surface will be some intermediate value Ts,2, for which T∞ < Ts,2 < Ts,1. Hence under steady-state conditions the surface energy balance

$$\dot{E}_{in} - \dot{E}_{out} = 0 \qquad \textbf{Or} \qquad \frac{kA}{L}(T_{s,1} - T_{s,2}) = hA(T_{s,2} - T_\infty)$$

where *k* is the thermal conductivity of the solid. Rearranging, we then obtain

Validity of the Lumped Capacitance Method

$$\frac{T_{s,1}-T_{s,2}}{T_{s,2}-T_\infty}=\frac{(L/kA)}{(1/hA)}=\frac{R_{t,\text{cond}}}{R_{t,\text{conv}}}=\frac{hL}{k}\equiv Bi \quad \ldots(10.10)$$

The quantity (hL/k) is a dimensionless parameter. It is termed the ***Biot number****, and it plays a* ***fundamental role*** *in* ***conduction problems that involve surface convection*** *effects. Biot number provides a* ***measure of the temperature drop in the solid relative to the temperature difference between the solid's surface*** *and the fluid.*

It is also evident that the Biot number may be interpreted as **a ratio of thermal resistances**. *In particular, if $Bi \ll 1$, the resistance to conduction within the solid is much less than the resistance to convection across the fluid boundary layer. Hence, the assumption of a uniform temperature distribution within the solid is reasonable if the Biot number is small.*

The Lumped system analysis assumes a ***uniform temperature distribution*** *throughout the body, which will be the case only when the thermal resistance of the body to heat conduction (the* ***conduction resistance****) is* ***zero****.*

Thus, lumped system analysis is exact when Bi = 0 and approximate when Bi > 0.

Of course, the smaller the Bi number, the more accurate the lumped system

Then the question we must answer is, How much accuracy are we willing to sacrifice for the convenience of the lumped system analysis?

Thus the criterion is

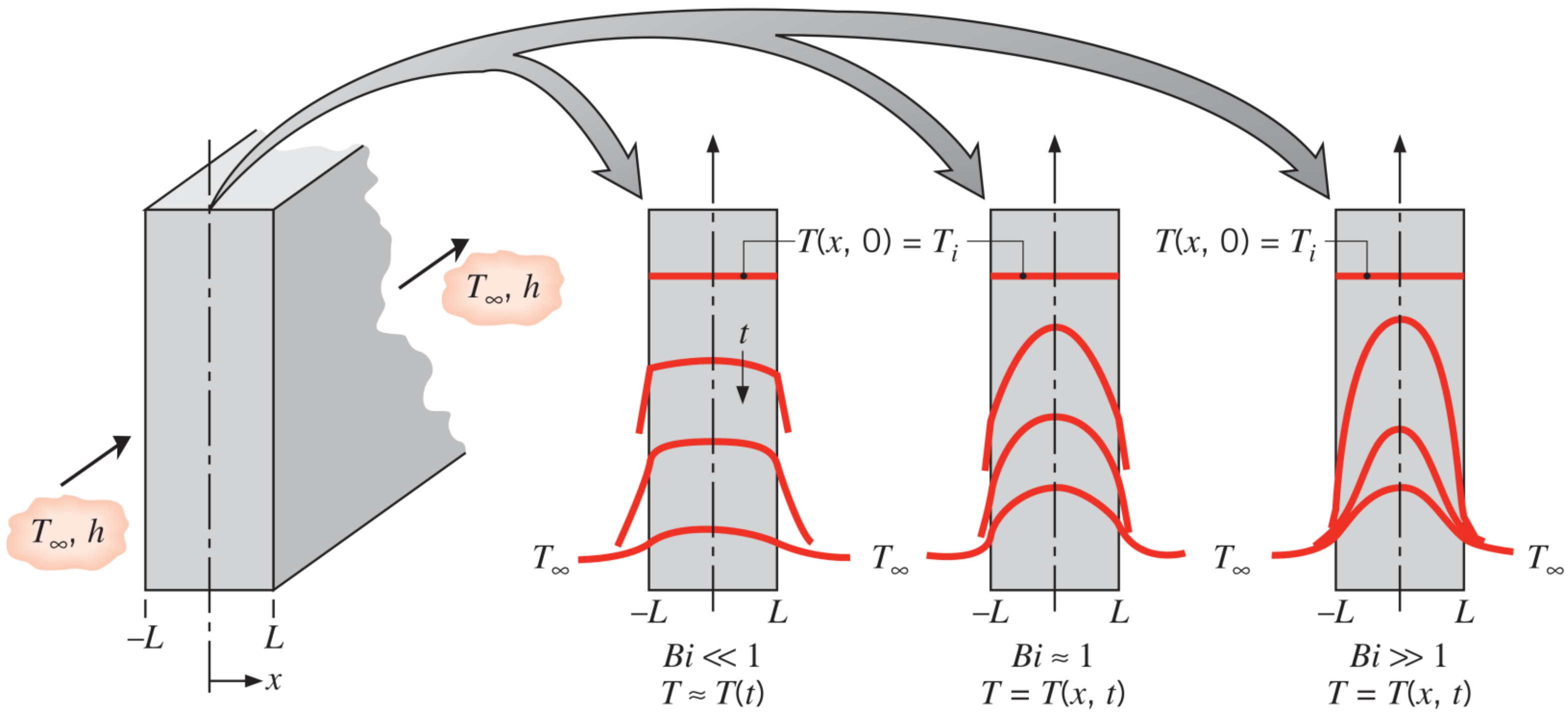
The temperatures within the body relative to the surroundings (i.e., $T - T\infty$) remain within 5 % of each other even for well-rounded geometries such as a spherical ball.

Thus, when Bi < 0.1, the variation of temperature with location within the body will be small and can reasonably be approximated as being uniform.

Validity of the Lumped Capacitance Method

Hence, when confronted with such a problem, *the very first thing that one should do is calculate the Biot number.* If the following condition is satisfied

$$Bi = \frac{hL_c}{k} < 0.1 \quad \ldots(10.11),$$ *Where,* L_c *is defined as characteristic length as* $L_c \equiv V/A_s$



Transient temperature distributions for different Biot numbers in a plane wall symmetrically cooled by convection.

Validity of the Lumped Capacitance Method

Hence, the definition of facilitates calculation of L_C for solids of complicated shape and reduces to the half-thickness L for a plane wall of thickness $2L$ (see the fig. above), to $r_o/2$ for a long cylinder, and to $r_o/3$ for a sphere.

However, if one wishes to implement the criterion in a conservative fashion, L_C should be associated with the length scale corresponding to the maximum spatial temperature difference. Accordingly, for a symmetrically heated (or cooled) plane wall of thickness $2L$, Lc would remain equal to the half-thickness L. However, for a long cylinder or sphere, Lc would equal the actual radius r_o, rather than $r_o/2$ or $r_o/3$.

Finally, we note that, with $L_c \equiv V/A_s$, the exponent of Eq. 10.6 may be expressed as

$$\frac{hA_s t}{\rho V c} = \frac{ht}{\rho c L_c} = \frac{hL_c}{k}\frac{k}{\rho c}\frac{t}{L_c^2} = \frac{hL_c}{k}\frac{\alpha t}{L_c^2}$$

Or

$$\frac{hA_s t}{\rho V c} = Bi \cdot Fo \qquad \ldots(10.12)$$

Where,

$$Fo \equiv \frac{\alpha t}{L_c^2} \qquad \ldots(10.13)$$

It is a *dimensionless time*, which, with the Biot number, characterizes transient conduction problems. Substituting Eq. 10.12 into 10.6, we obtain

$$\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = \exp(-Bi \cdot Fo) \qquad \ldots(10.14)$$

Validity of the Lumped Capacitance Method

Q 10.1 A thermocouple junction, which may be approximated as a sphere, is to be used for temperature measurement in a gas stream. The convection coefficient between the junction surface and the gas is $h = 400\,W/m^2 \cdot K$, and the junction thermophysical properties are $k = 20\,W/m \cdot K, c = 400\,J/kg.K,$ and $\rho = 8500\,kg/m^3$. Determine the junction diameter needed for the thermocouple to have a time constant of 1 s. If the junction is at 25 °C and is placed in a gas stream that is at 200 °C, how long will it take for the junction to reach 199 °C?

Validity of the Lumped Capacitance Method