Texture Mapping in Practice

CSE606: Computer Graphics

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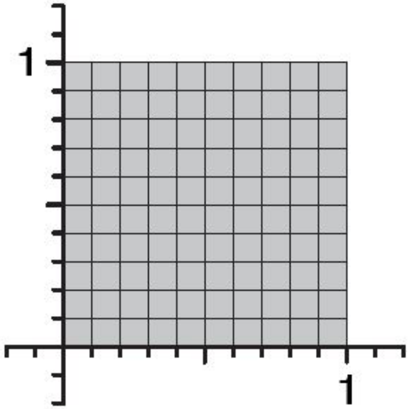
Midterm Rubric Discussion (in-class)

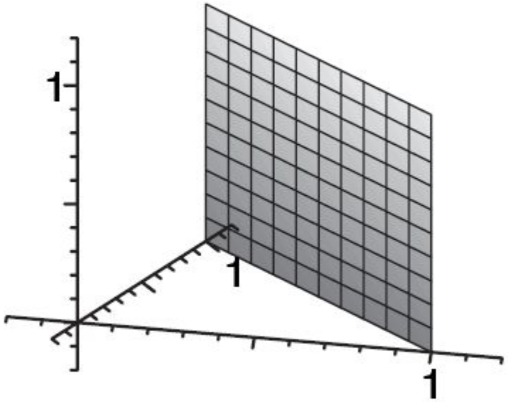
Surface Parametrizations

Surface Parametrizations: Rectangle

1. simple linear function:

$$
\begin{aligned}
\textit{parameter domain:} &\quad \Omega = \{(u,v) \in \mathbb{R}^2 : u, v \in [0,1]\} \\
\textit{surface:} &\quad S = \{(x,y,z) \in \mathbb{R}^3 : x, y, z \in [0,1], x + y = 1\} \\
\textit{parameterization:} &\quad f(u,v) = (u, 1-u, v) \\
\textit{inverse:} &\quad f^{-1}(x,y,z) = (x,z)
\end{aligned}
$$



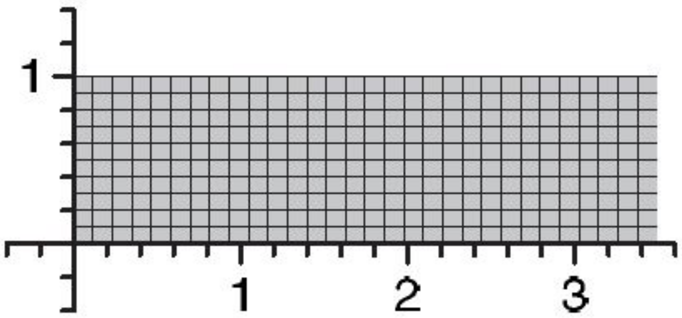


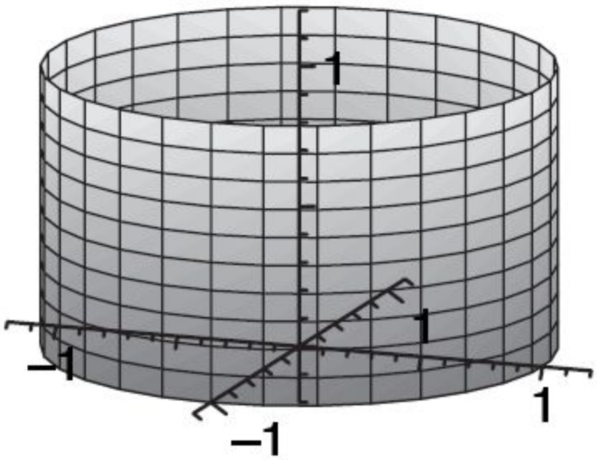
Kai Hormann et al, SIGGRAPH Course Notes

Surface Parametrizations: Cylinder

2. cylinder:

$$
\begin{aligned}
\textit{parameter domain:} \quad & \Omega = \{(u,v) \in \mathbb{R}^2 : u \in [0, 2\pi), v \in [0,1]\} \\
\textit{surface:} \quad & S = \{(x,y,z) \in \mathbb{R}^3 : x^2 + y^2 = 1, z \in [0,1]\} \\
\textit{parameterization:} \quad & f(u,v) = (\cos u, \sin u, v) \\
\textit{inverse:} \quad & f^{-1}(x,y,z) = (\arccos x, z)
\end{aligned}
$$

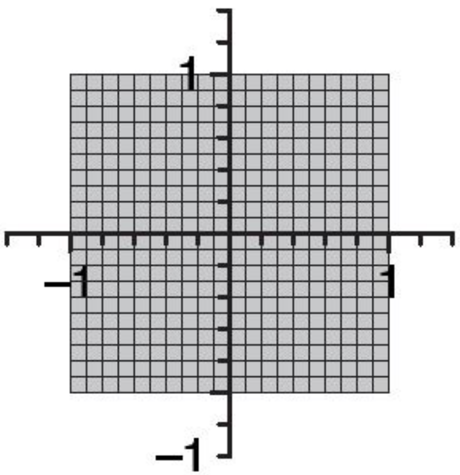


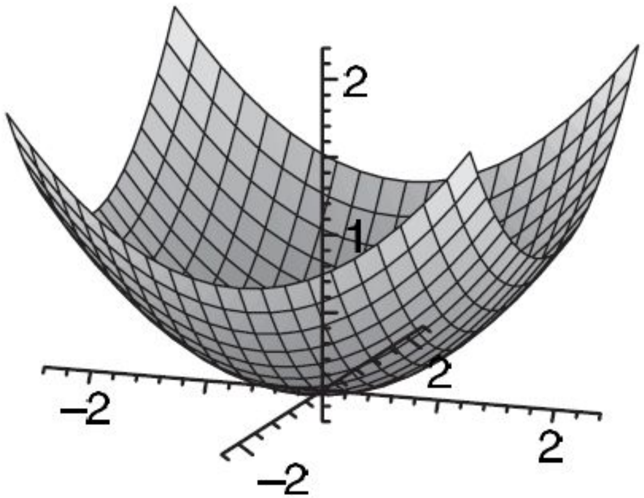


Surface Parametrizations: Paraboloid

3. paraboloid:

$$\begin{aligned} \textit{parameter domain:} \quad & \Omega = \{(u,v) \in \mathbb{R}^2 : u, v \in [-1,1]\} \\ \textit{surface:} \quad & S = \{(x,y,z) \in \mathbb{R}^3 : x, y \in [-2,2], z = \tfrac{1}{4}(x^2+y^2)\} \\ \textit{parameterization:} \quad & f(u,v) = (2u, 2v, u^2+v^2) \\ \textit{inverse:} \quad & f^{-1}(x,y,z) = (\tfrac{x}{2}, \tfrac{y}{2}) \end{aligned}$$





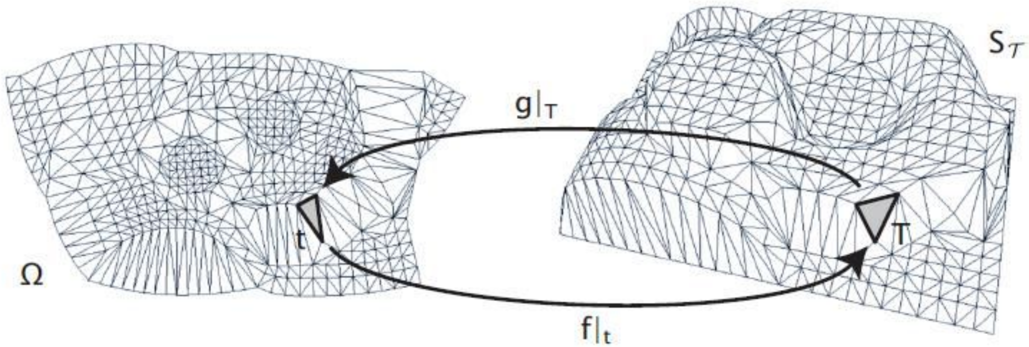
Texture Mapping for Parametrized Surfaces

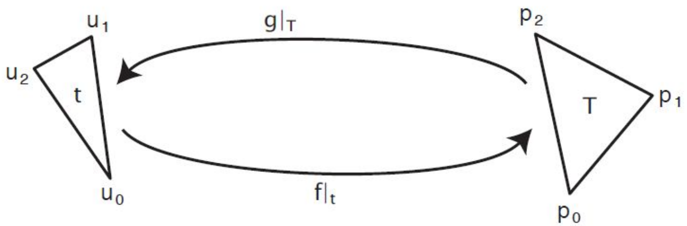
Parameterization of 3D Mesh

Parameterization defines a correspondence between points on a surface mesh in 3D and a 2D domain.

– Should be bijective (one-to-one) or, at least, locally bijective (i.e. triangles are not flipped/inverted in 2D).

– Can be computed using a spring network model with minimizing spring energy; which is done using affine combination of neighbors.





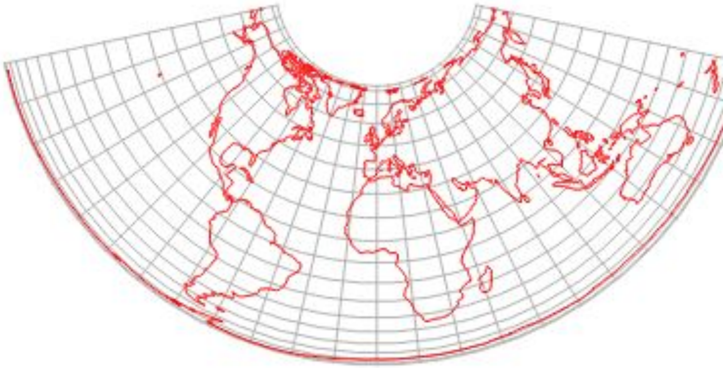
Properties of Parametric Maps

1. Isometric: Length of a curve is preserved
2. Conformal: Angle between intersecting curves is preserved
3. Equiareal: Area of area element is preserved

Isometric ⇔ Conformal + Equiareal

Orthographic map [Wikipedia]

Lee conformal map in a tetrahedron
[Wikipedia]



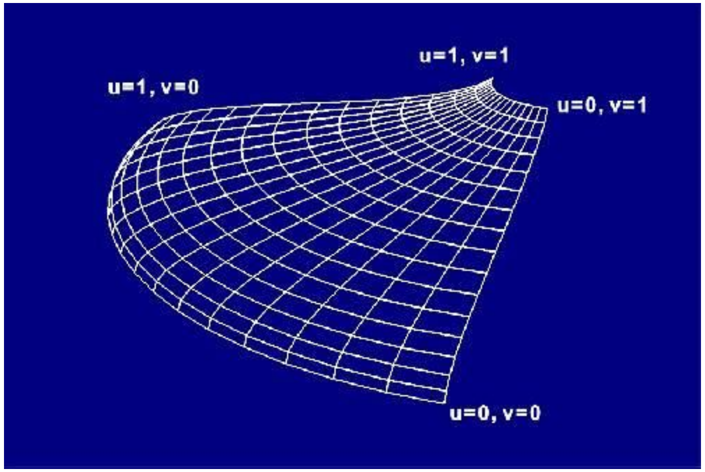
Albers equiareal projection
From [WolframAlpha]

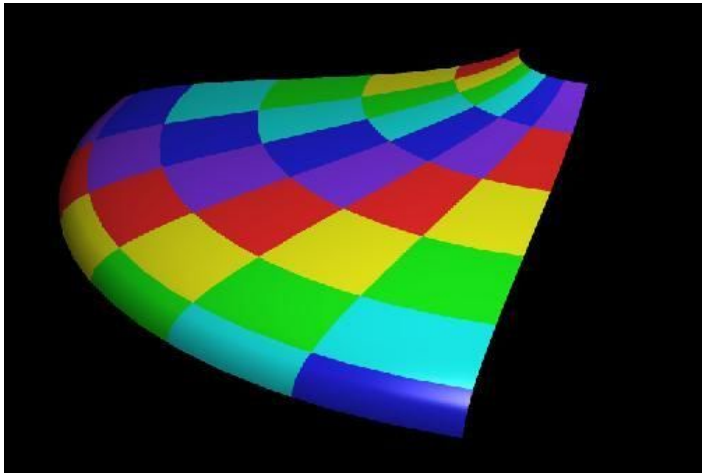
Textures for Parametric Surfaces

Surfaces often modelled as parametric patches:

- Defined as f(u,v)→(x,y,z)
- E.g. planes, cylinders, spheres, Bezier surfaces, b-splines, NURBS

Use the natural u,v parameters as texture parameters



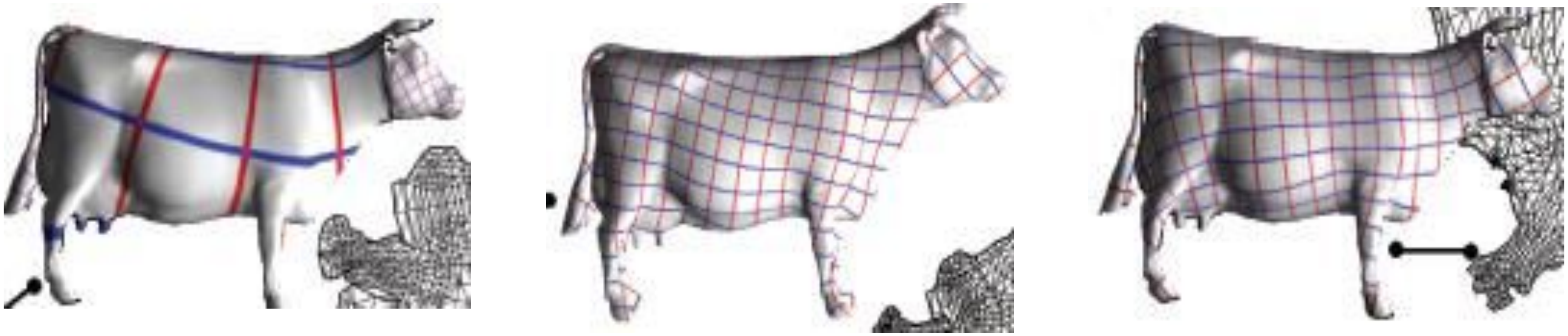


Texture Mapping for Generalized Surfaces – Intermediate Analytic Forms

Parameterization of Complex Surfaces

How do we assign parameters to objects where sizes of quads (or tris) are not uniform ?

- Typically, smaller tris/quads in areas of higher curvature or other detail
- A straightforward texture mapping will produce uneven and unrealistic images
  - e.g. mapping checkerboard onto the teapot: size of checkers will depend on size of quads
  - Unrealistic, since object is made up a single smooth material/surface
- In practice, difficult to work out a general inverse mapping that would correctly handle all kinds of surfaces



ABF++, Fast and Robust Angle Based Flattening A. Sheffer et al, ACM Transactions on Graphics,

Mapping Analytic Forms

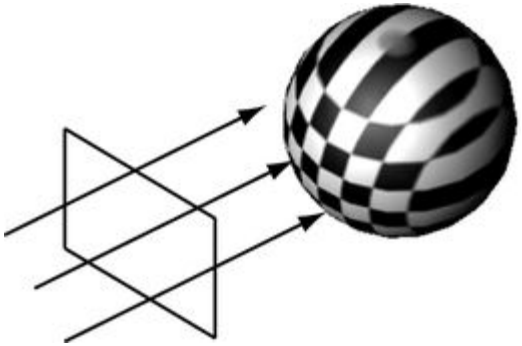
Images: Autodesk Maya

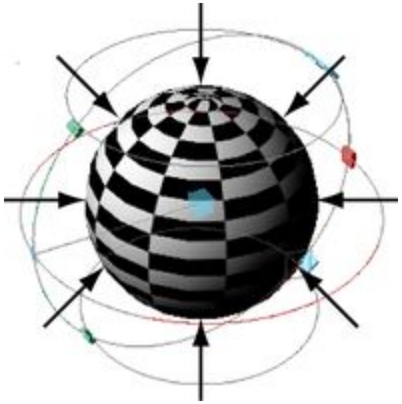
General problem of determining texture coordinates (or parameters) is difficult.

Hence, use an intermediate shape to map to, as a first-cut.

Texturing a sphere using maps that are:

- Planar
- Cylindrical
- Spherical



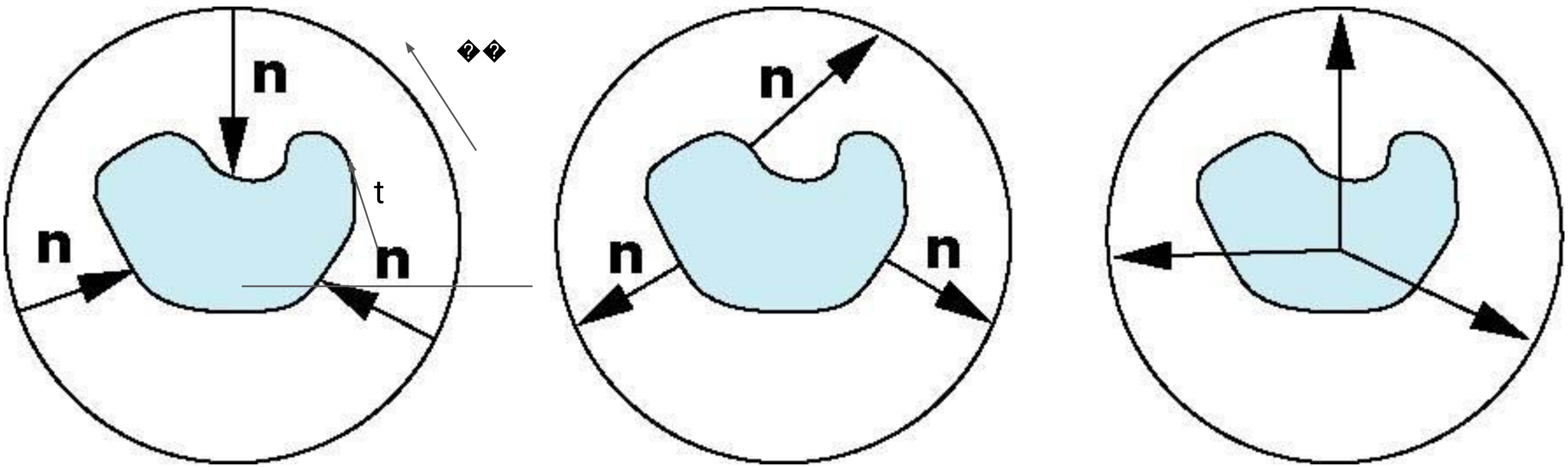


Intermediate Maps

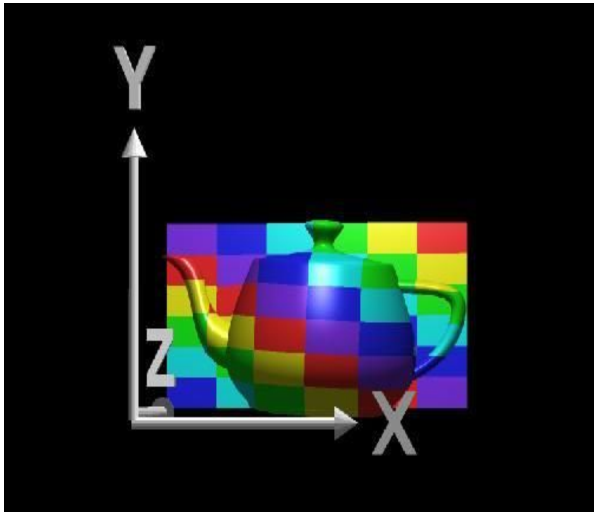
Map object to intermediate object

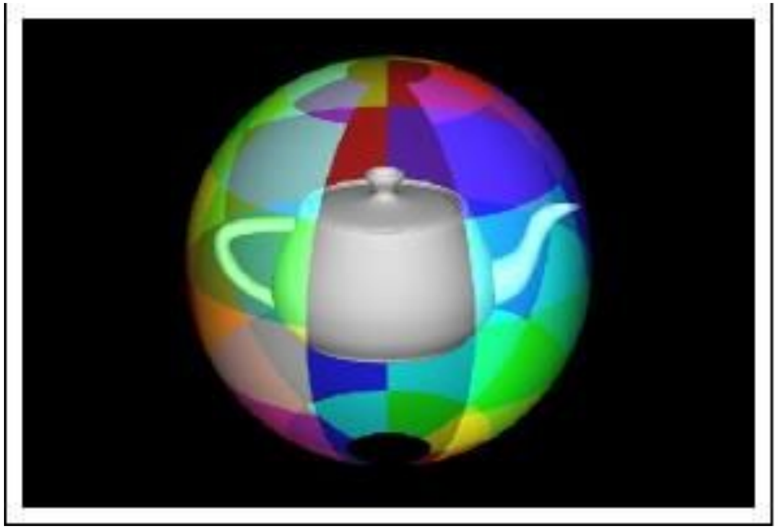
- Normals from intermediate to actual
- Normals from actual to intermediate
- Vectors from center of intermediate

Use an appropriate mapping of point **p** on the surface to ***θ*** of the intermediate map, and hence compute **t**



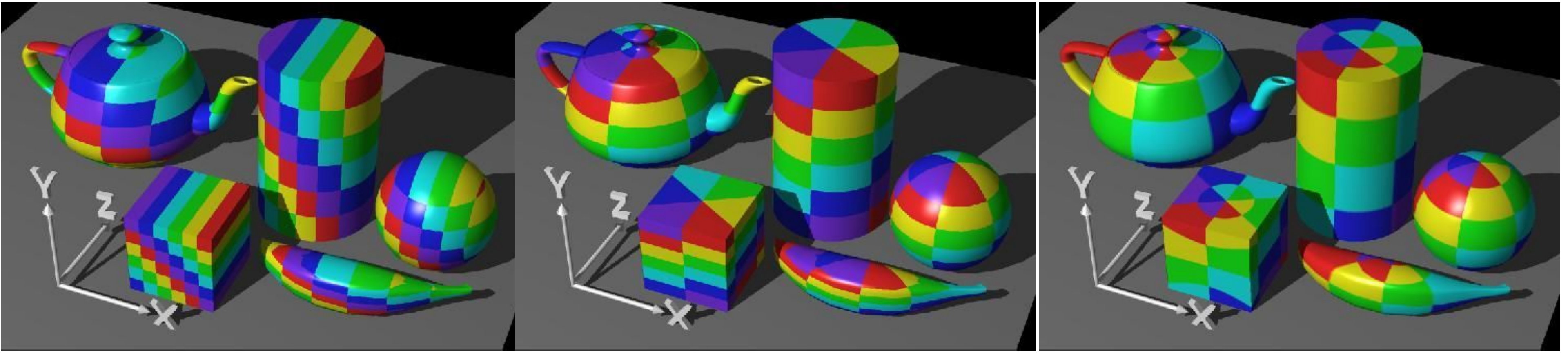
Projections





http://www.siggraph.org/education/materials/HyperGraph/mapping/r_wolfe/r_wolfe_mapping_3.htm

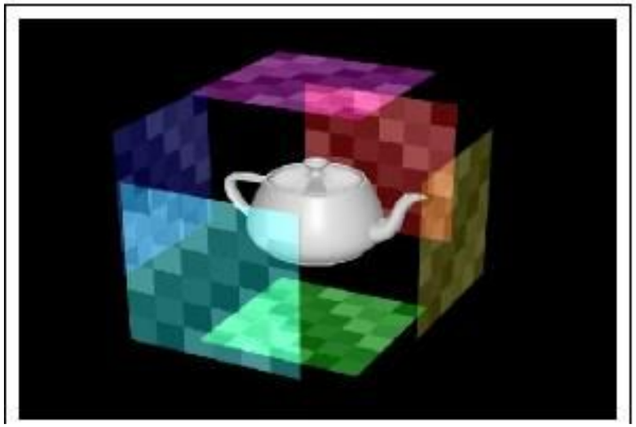
Intermediate Objects

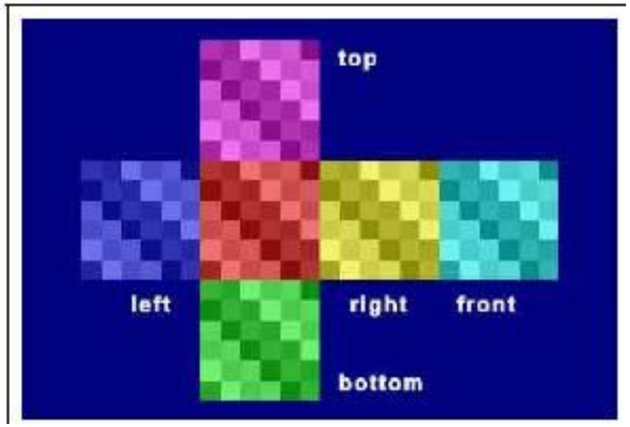


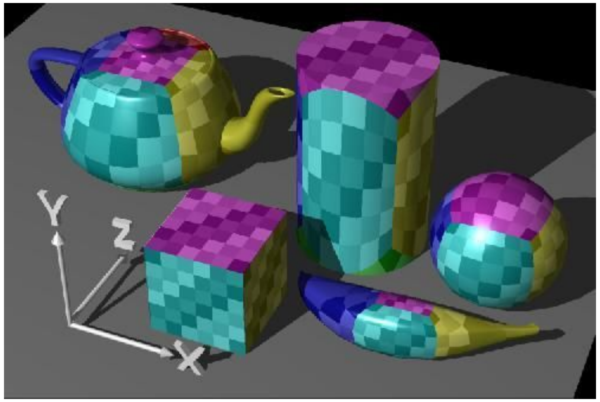
What were the intermediate objects used here?

http://www.siggraph.org/education/materials/HyperGraph/mapping/r_wolfe/r_wolfe_mapping_3.htm

Intermediate Objects

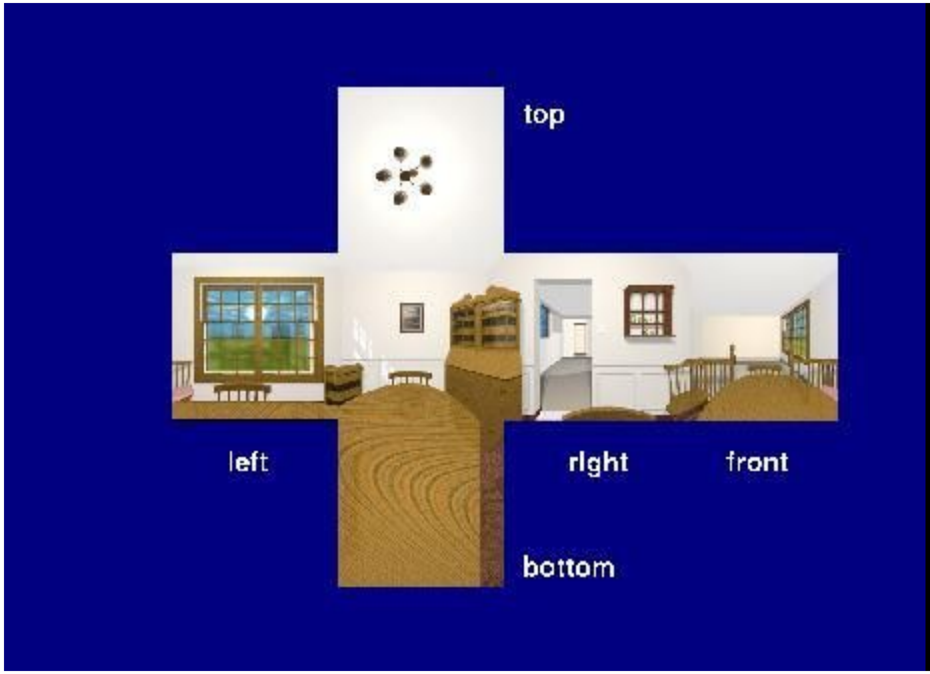






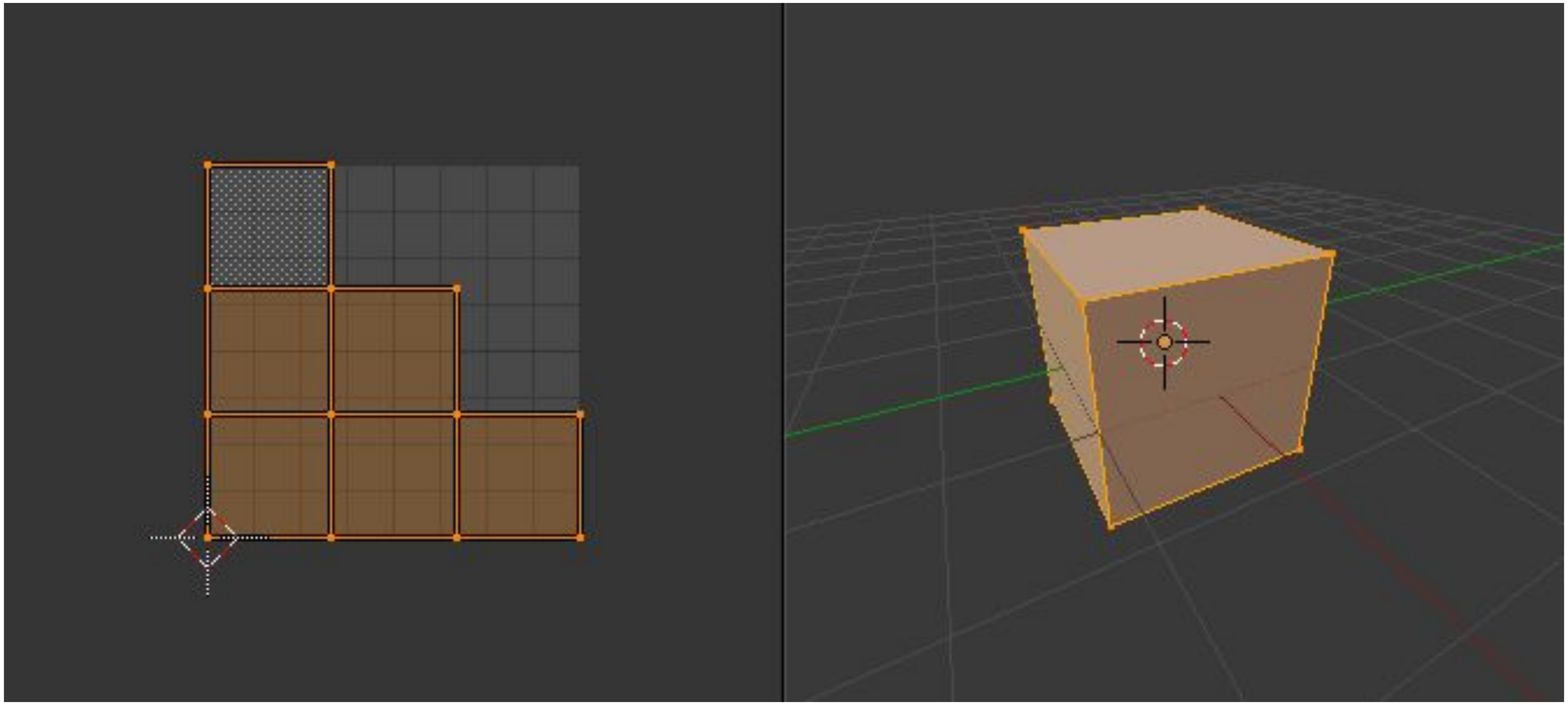
http://www.siggraph.org/education/materials/HyperGraph/mapping/r_wolfe/r_wolfe_mapping_3.htm

Cube Maps



Cube as an intermediate map.
http://www.siggraph.org/education/materials/HyperGraph/mapping/r_wolfe/r_wolfe_mapping_3.htm

Parametric maps of mesh surfaces



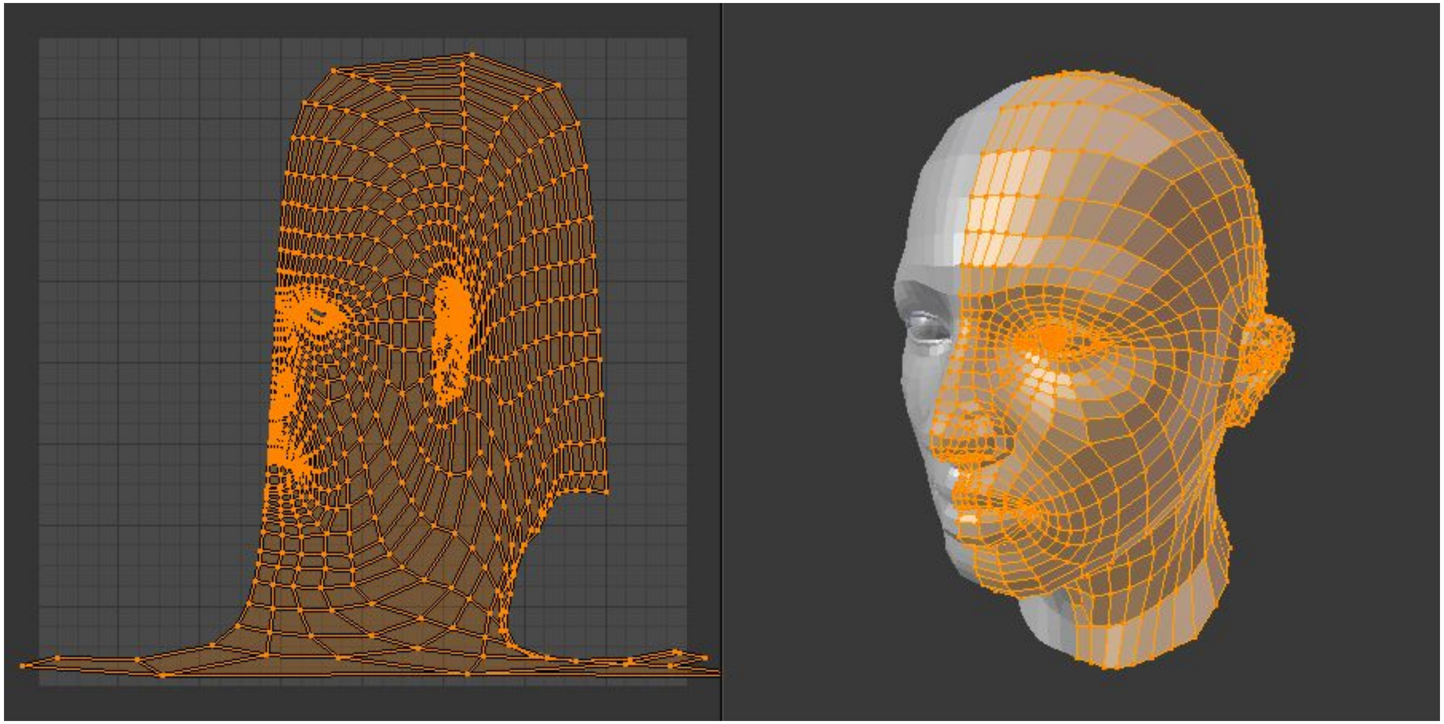
Blender.org

Break surface into multiple patches that are each reasonably flat.

Define UV map for each patch

Surface can be “cut” along edges above a certain angle threshold, for instance

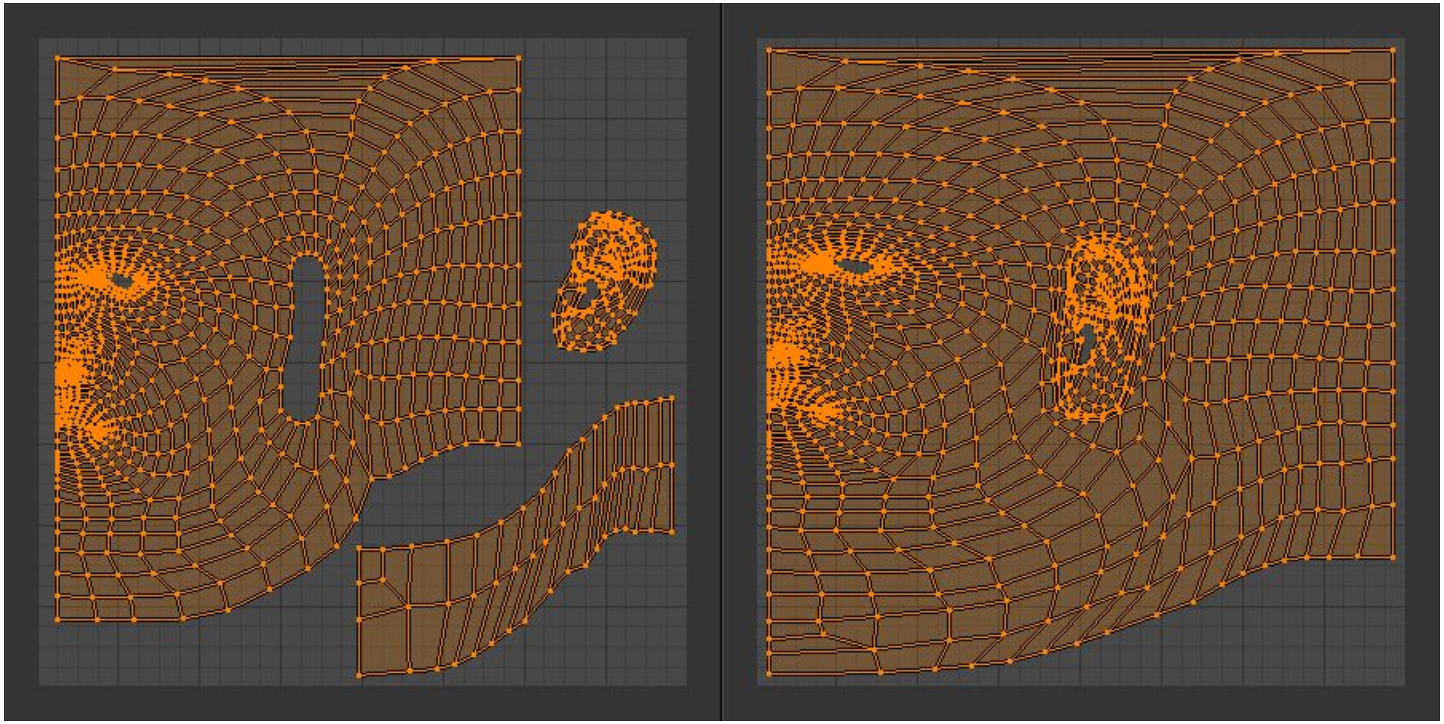
Parameterization for Complex Surfaces



“Flattening” a surface onto a standard shape, e.g. a sphere.
High distortion at ear, eye, neck etc

Blender.org

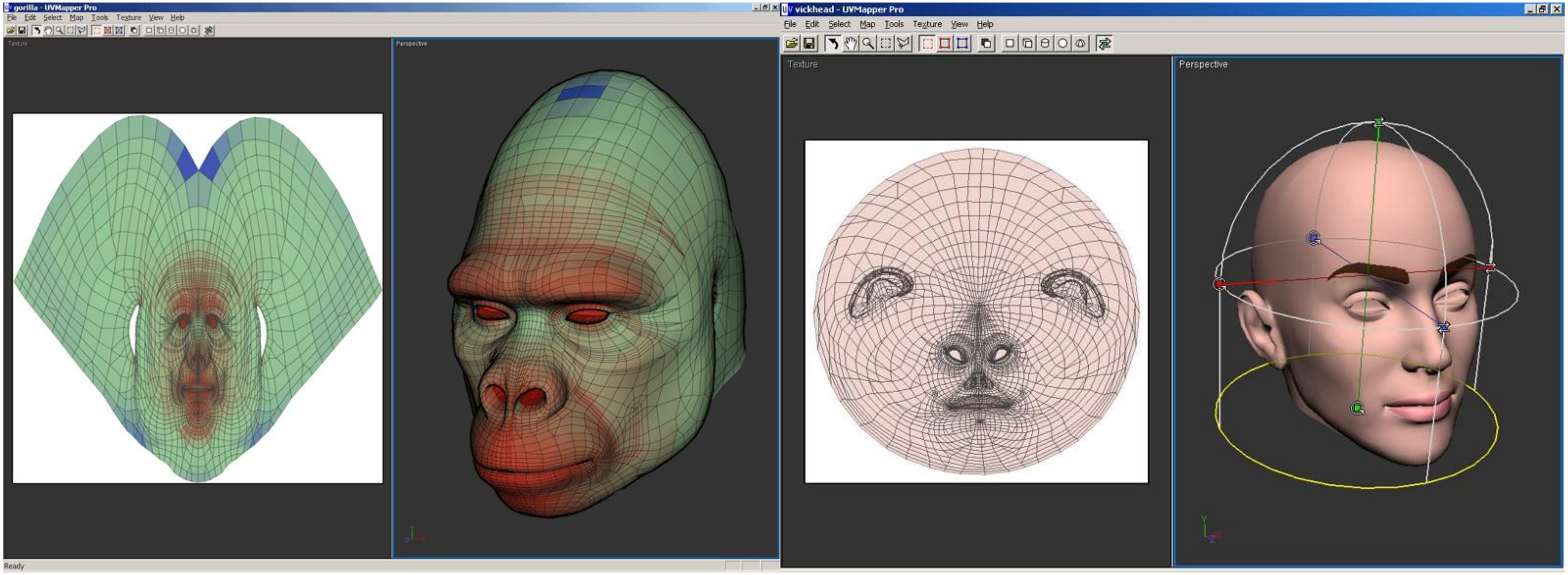
Parameterization for Complex Surfaces



Cut out ear, neck; parameterize them independently, "stitch" back the pieces in uv-space

Blender.org

Examples: Parametric Maps of Complex Surfaces

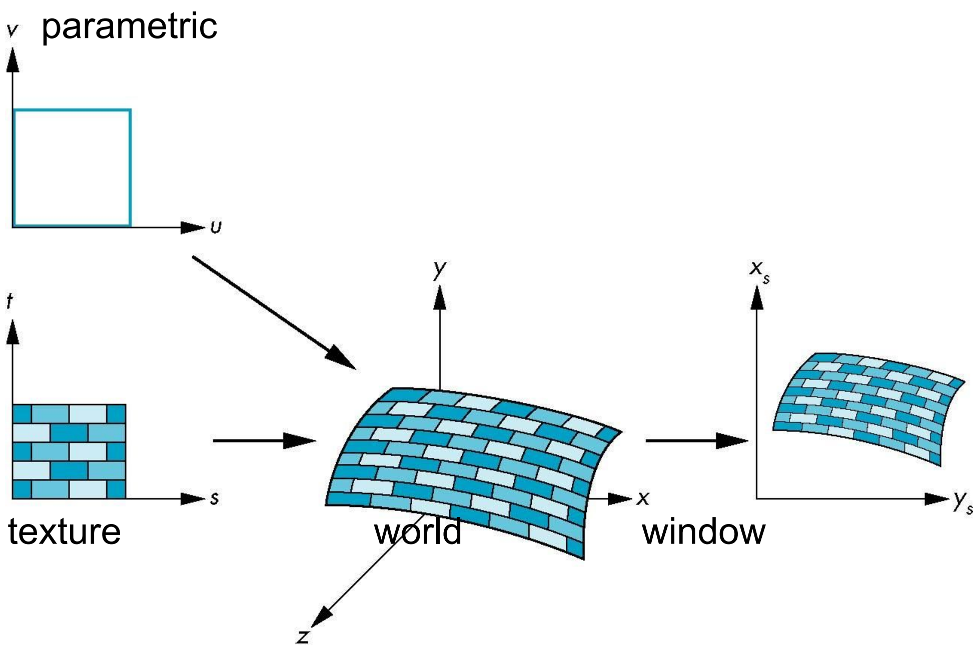


uvmapper.com

For a detailed explanation, see:
[Teaching Texture Mapping Visually, Rosalee Wolfe, SIGGRAPH 97 Education Set]

Texture Operations

Texture Mapping Process



- Parametric coordinates may be used to model curves and surfaces.
- Texture coordinates used to identify points in the image to be mapped.
- Object or World Coordinates, conceptually, where the mapping takes place.
- Window Coordinates, where the final image is really produced

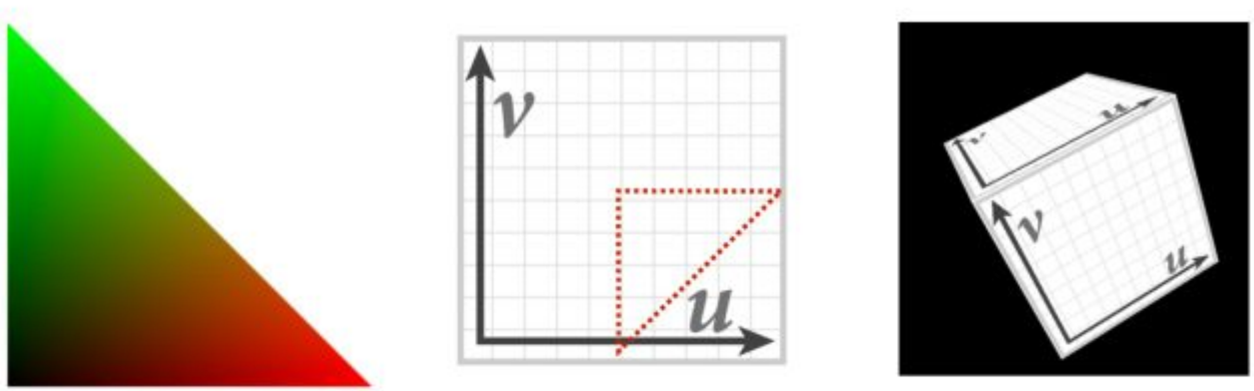
Texture Wrapping

- Wrapping parameters determine what happens if s and t are outside the (0,1) range

Texture Sampling

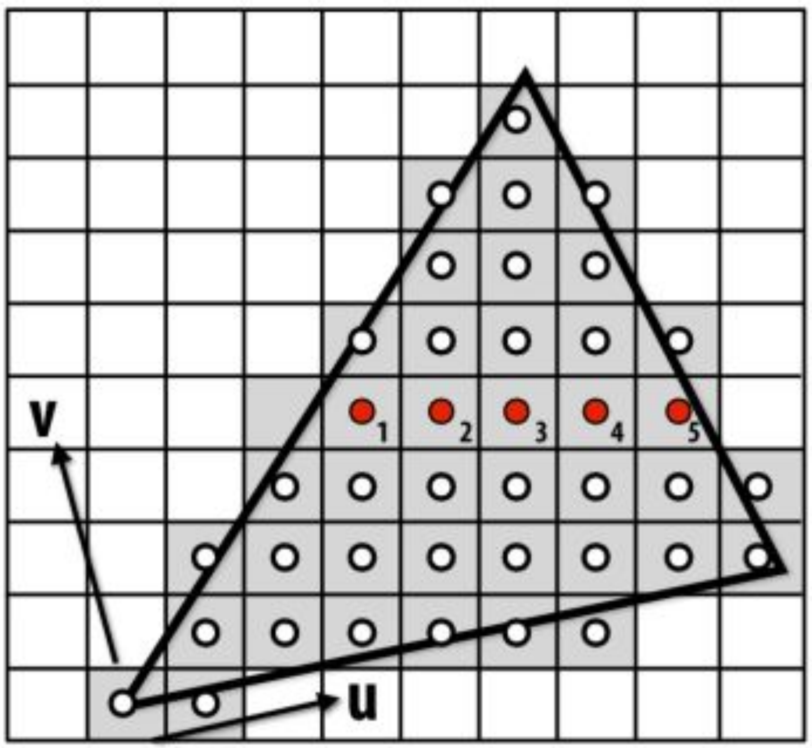
Basic algorithm for mapping texture to surface:

- Interpolate u and v coordinates across triangle.
- For each fragment:
  - Sample (evaluate) texture at (u,v)
  - Set color of fragment to sampled texture value

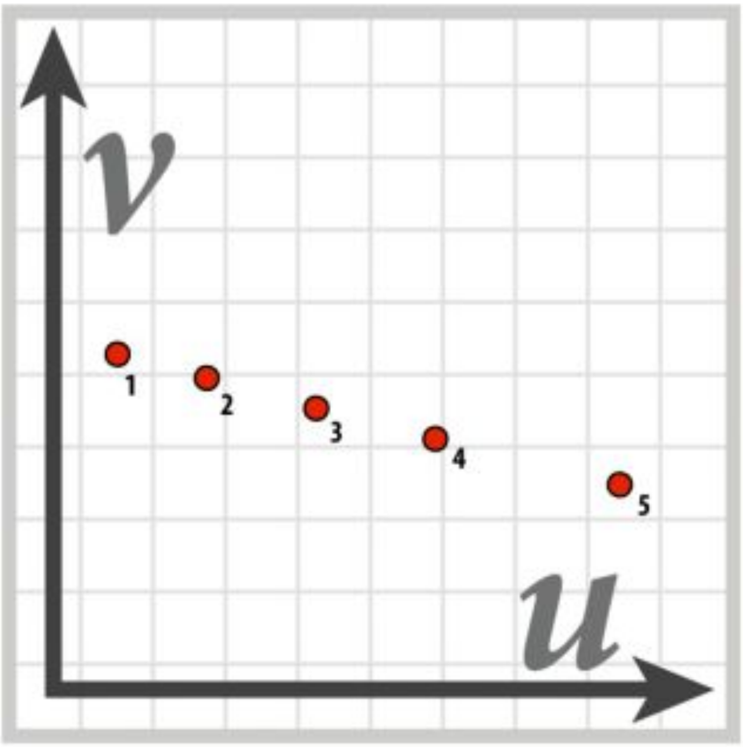


...sadly not this easy in general!

Texture Space Samples



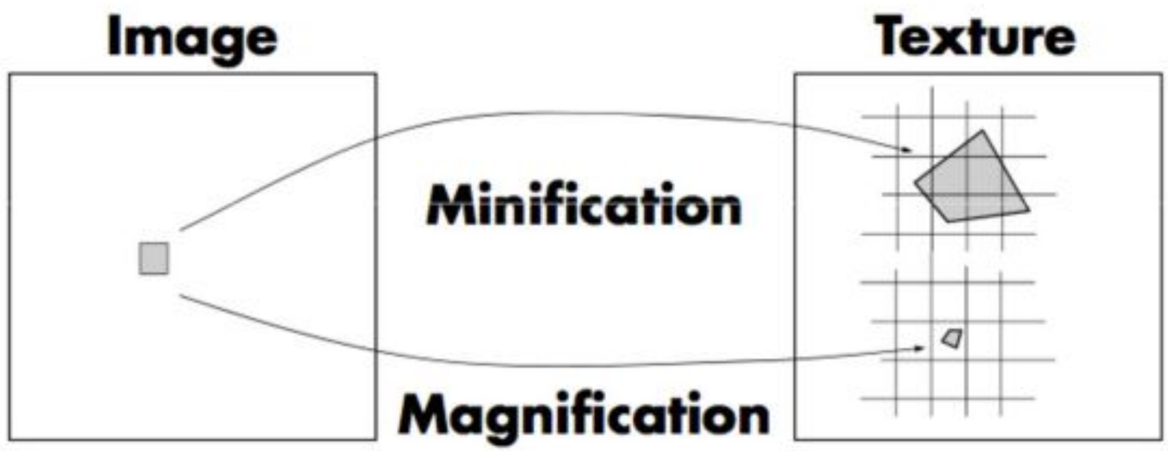
Sample positions are uniformly distributed in screen space (rasterizer samples triangle's appearance at these locations)



Texture sample positions in texture space (texture function is sampled at these locations)

https://15462.courses.cs.cmu.edu/fall2019/lecture/texture/slide_051

Texture Filtering - Minification and Magnification



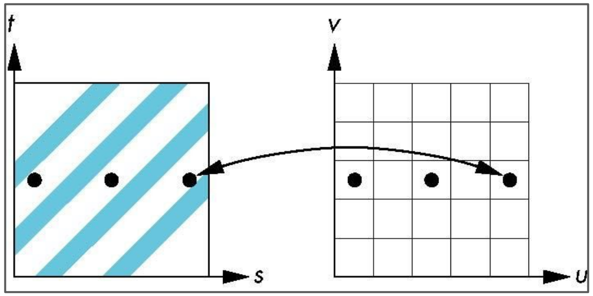
- **Minification:**
  - Area of screen pixel maps to large region of texture (filtering required -- averaging)
  - One texel corresponds to far less than a pixel on screen
  - Example: when scene object is very far away
- **Magnification:**
  - Area of screen pixel maps to tiny region of texture (interpolation required)
  - One texel maps to many screen pixels
  - Example: when camera is very close to scene object (need higher resolution texture map)

https://15462.courses.cs.cmu.edu/fall2019/lecture/texture/slide_055

Issue-1: Aliasing

Point sampling of textures can lead to aliasing errors, especially in inverse mapping:

- Can miss parts of texture pattern



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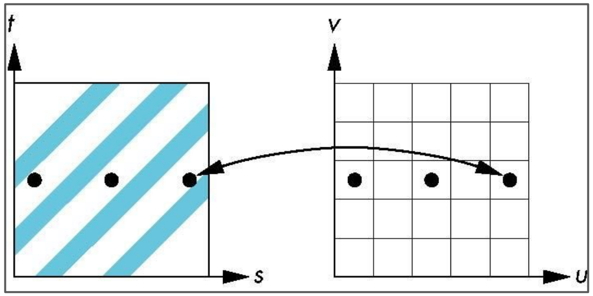


Solution: Texture Sampling

1. Filter modes allow us to use area averaging instead of point samples
   - Mapping (s,t) directly to texel coords (integers) can cause aliasing issues
   - Can apply filters:
     - Nearest
     - Linear (Use 4 texels around the point to compute a weighted average
       - Produces smoother variations
2. **Mipmapping**

Mipmapping

Mipmaps are smaller, pre-filtered versions of a texture image, representing different levels of detail (LOD) of the texture.

They are often stored in sequences of progressively smaller textures called mipmap chains with each level half as small as the previous one.

Advantages:

- Improved image quality
- Increased performance

https://blog.imaginationtech.com/why-you-really-should-be-using-mipmapping-in-your-graphics-applications/

Mipmapping - Application

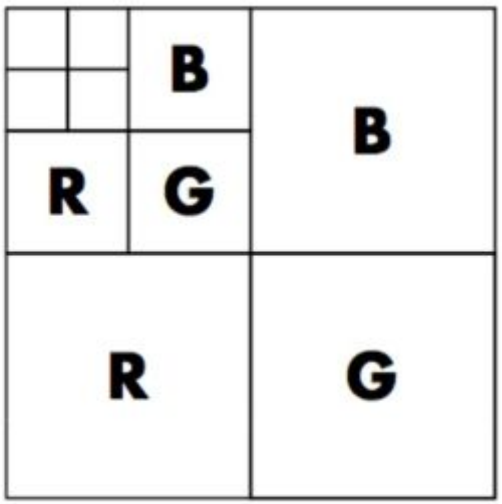
As the object gets further from the camera, the object's texture will eventually appear smaller on-screen than its actual resolution, i.e., there will be more than one texel (pixels of a texture map) per screen pixel.

The texture will have to be scaled down in a process called **minification filtering**, which often requires the application to sample multiple texels to decide on the colour of a pixel. This is a problem when the entire texture needs to be sampled at runtime for an object that could only be a single pixel wide.

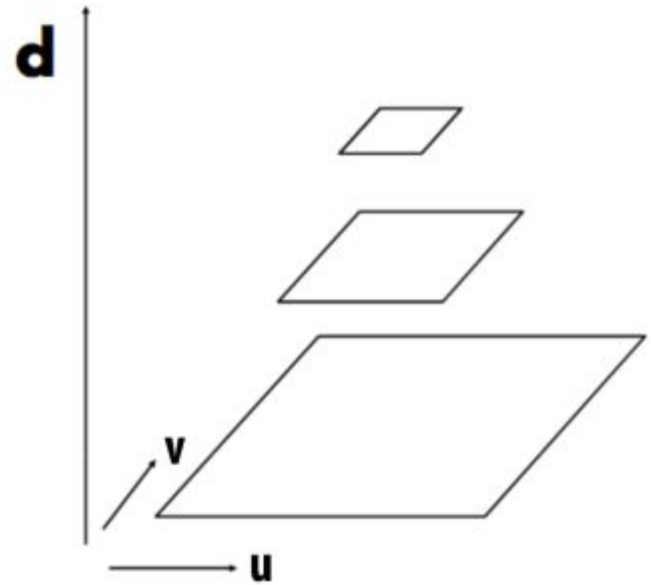
Instead of sampling a single texture, the application can switch between any of the lower resolution mipmaps in the chain depending on the distance from the camera. Thus, mipmaps is a solution to texture aliasing issue.

https://blog.imaginationtech.com/why-you-really-should-be-using-mipmapping-in-your-graphics-applications/

MipMapping - Storage



Williams' original proposed
mip-map layout

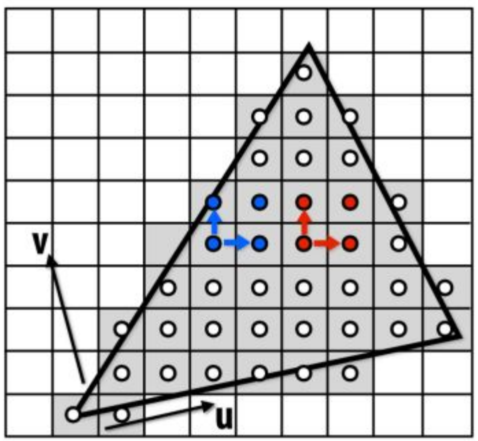


"Mip hierarchy"
level = *d*

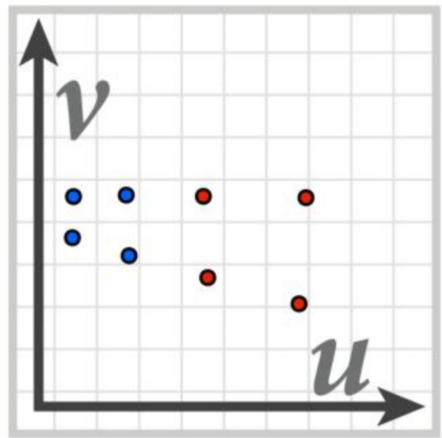
What is the storage overhead of a mipmap?

Williams, L., 1983, July. Pyramidal parametrics. In *Proceedings of the 10th annual conference on Computer graphics and interactive techniques* (pp. 1-11).
https://15462.courses.cs.cmu.edu/fall2019/lecture/texture/slide_058

MipMap Level Determination



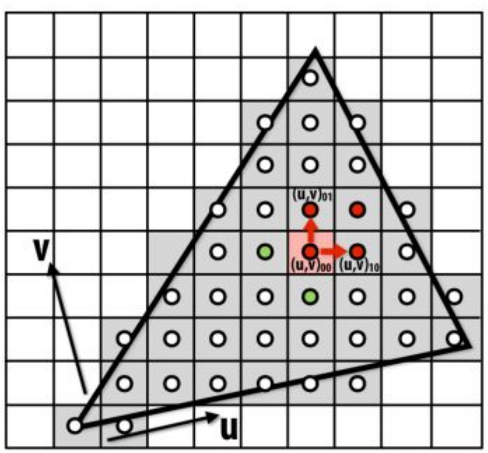
Screen space

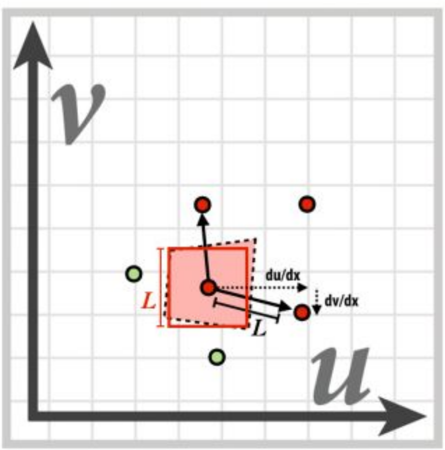


Texture space

Compute differences between texture coordinates of neighboring screen samples / fragments

$$du/dx = u_{10} - u_{00} \qquad dv/dx = v_{10} - v_{00}$$
$$du/dy = u_{01} - u_{00} \qquad dv/dy = v_{01} - v_{00}$$





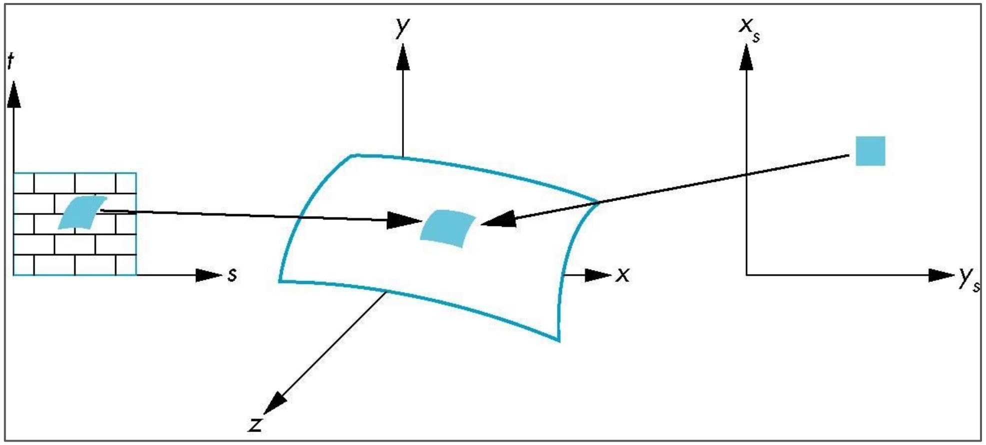
$$L = \max\left(\sqrt{\left(\frac{du}{dx}\right)^2 + \left(\frac{dv}{dx}\right)^2}, \sqrt{\left(\frac{du}{dy}\right)^2 + \left(\frac{dv}{dy}\right)^2}\right)$$

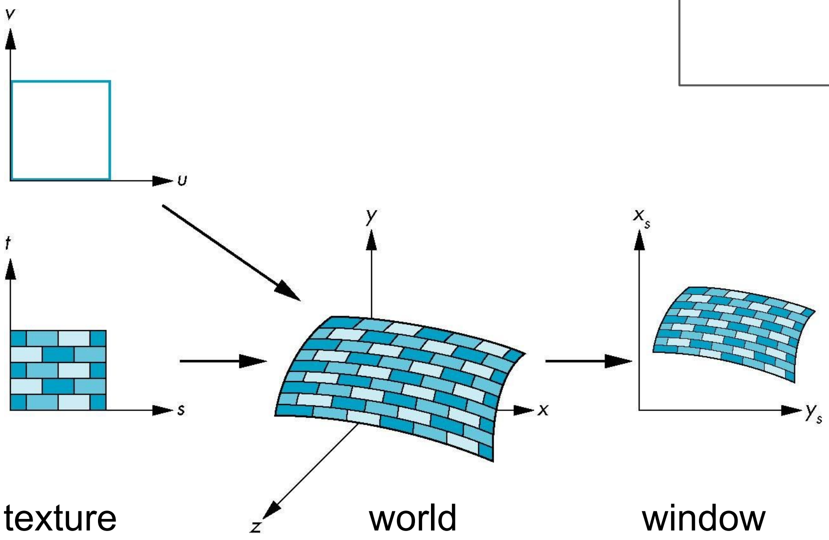
$$\textit{mip-map } d = \log_2 L$$

https://15462.courses.cs.cmu.edu/fall2019/lecture/texture/slide_059
https://15462.courses.cs.cmu.edu/fall2019/lecture/texture/slide_060

Issue-2: Texture Pre-image

Pre-image of (rectangular) pixel may be curved.





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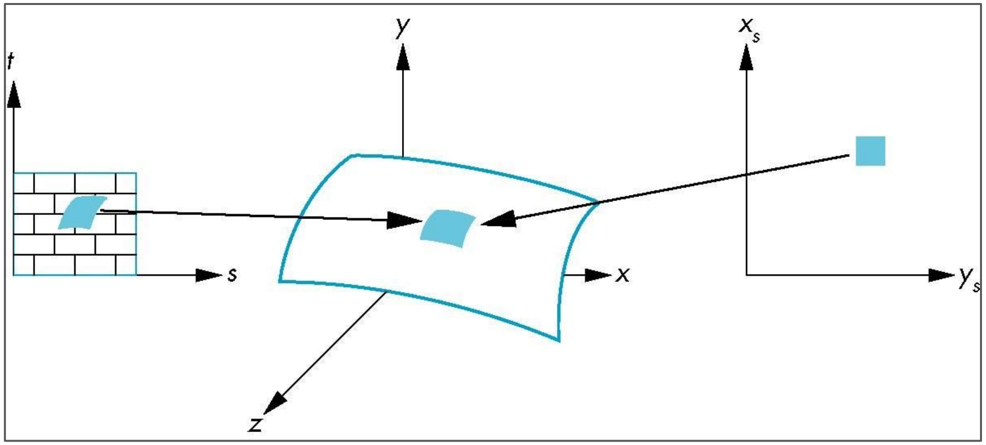
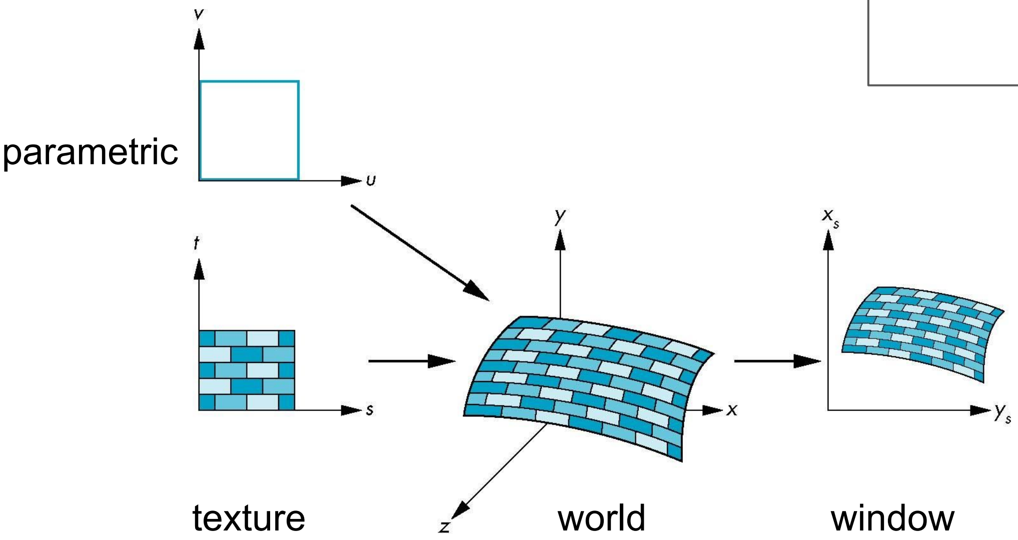


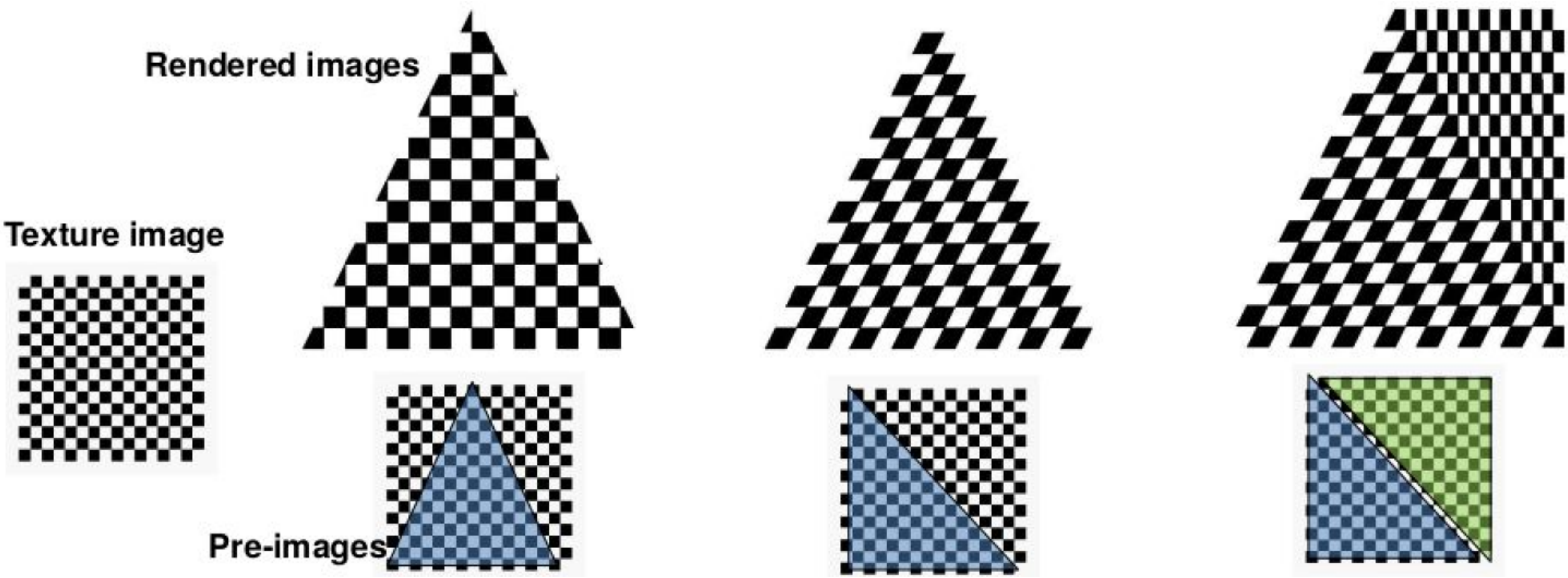


Solution: Use seams

Angel: Interactive Computer Graphics 5E © Addison-Wesley 2009
https://blender.stackexchange.com/questions/268214/how-can-i-fix-the-image-texture-problem-on-this-curved-wall

Texture Interpolation

- Interpolation of texture coordinates (i.e. map to texels)
- Can result in distortions



Texture Mapping Application

https://www.youtube.com/watch?v=Yx2JNbv8Kpg

Summary

- Surface parametrization - analytical forms
- Textures for 3D parametric surfaces
- Textures for general mesh surfaces
    - Intermediate maps
- Texture Operations
    - Wrapping
    - Sampling
    - Mipmapping
    - Curved elements