A Project Report on

TO DESIGN OF FRACTIONAL ORDER LOW PASS FILTER

submitted by

Vikas Agarwal (1804331056)

&

Abhay Kushwaha (1904331901)

under the guidance of

DR. ATUL KUMAR DWIVEDI

AND PROF. D. K. SHRIVASTAVA

in partial fulfilment of the requirements
for the award of the degree of

BACHELOR OF TECHNOLOGY

Report Submitted to

DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING

BUNDELKHAND INSTITUTE OF ENGINEERING & TECHNOLOGY, JHANSI, INDIA

2021-2022

Declaration

We hereby proclaim that the work that is being presented in this report entitled **"To design of fractional order low pass filter"** is our own original work and has not been reported by anyone else or submitted in any form for another degree or diploma to any other institution. Information derived from the other sources has been accredited in the text and a list of references has been given. We also declare that I have adhered to the ethical norms and guidelines provided by the Institute.

Vikas Agarwal | **Abhay Kushwaha**
Roll No. 1804331056 | Roll No. 1904331901

Certificate

This is to certify that the thesis entitled "To design of fractional order low pass filter" submitted by Vikas Agarwal and Abhay Kushwaha for the award of the degree of BTech, is carried out by them under my supervision.

Dr. Atul Kumar Dwivedi
Department of ECE, BIET Jhansi

Verification

This is to verify that the project entitled "**To design of fractional order low pass filter**" submitted.

Dr. Atul Kumar Dwivedi
Officer in Charge, B. Tech. Projects
Department of EC

Prof. Deepak Nagaria
Head of EC
& EE Department

DEDICATION

To my beloved Parents and almighty.

Project Outcomes(Ps)

S. N.	Project outcomes After completing this project students will be able to	Bloom's knowledge level
P1	1 To design fractional order butter-worth low pass filter.	KL6
P2	2 To verify the design of filter proteus software is used for simulation .	KL5
P3	3 To obtain fractional order capacitor use of fourth order integer approximation.	KL6
P4	4 To optimize the result using Genetic Algorithm (GA).	KL3
P5	5 To improve the overall performance and stability of Tow Thomas Bi-quad Filter	KL4

KL: Bloom's knowledge level, KL1: remember, KL2: Understand, KL3: Apply,
KL4: Analyse, KL5: Evaluate, KL6: Create/Design

Mapping of project outcomes with Program Outcomes (POs)

Ps	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12
P1	_	3	_	_	2	3	1	3	2	3	3	3
P2	3	_	3	3	3	3	3	3	3	3	3	_
P3	3	3	3	2	3	3	3	3	3	3	3	3
P4	3	3	3	3	3	3	3	3	3	3	3	_
P5	3	3	3	3	_	3	3	3	3	3	3	3

Mapping rules (Rubrics)

1: poor 2: medium 3: best

Acknowledgment

It is my great pleasure to acknowledge the support and encouragements I have received from the people along the way of the research work. First and foremost, I would like to express my heartiest Above all, with all humility, I offer my deepest feelings of gratitude to the blessings and mercy of the Supreme Person for providing me everything I need. We would like to express our sincere thanks and deep sense of gratitude towards Dr. Atul Kumar Dwivedi, Department of Electronics and communication engineering, who has not only helped us in enhancement of my technical knowledge regarding subject topic but also taken us to zenith where we could reset our views and exhibit our talent in front of others. The enthusiastic crystal-clear view on the format and layout and presentation helped us immensely.

Vikas Agarwal	**Abhay Kushwaha**
Roll No. 1804331056	Roll No. 1904331901

ABSTRACT

Keywords - Butterworth; continued fraction expansion; fractional order; RC ladder; simulated annealing; Tow-Thomas bi-quad topology.

In this paper, simulated annealing and suitable scaling optimization techniques are used to design the fractional order low pass filters with Butterworth approximation. The frequency responses of the obtained optimal designs are closer to the ideal one as compared to the other existing designs. The designed filters are further realized using Tow-Thomas bi-quad topology by replacing traditional capacitors with the fractional capacitors, which are approximated by continued fraction expansion and then realized with the RC ladder network. The effectiveness of the proposed circuit realizations is also shown by Proteus simulated results.

TABLE OF CONTENTS

Table of Contents (continued) **Page**

LIST OF TABLES

LIST OF FIGURES

Major Contributions

The main objective of this project is to design fractional order butter-worth low pass filter. The main contributions of the work are

1 To design fractional order butter-worth low pass filter.

2 To verify the design of filter proteus software is used for simulation .

3 To obtain fractional order capacitor use of fourth order integer approximation.

4 To optimize the result using Genetic Algorithm (GA).

5 To improve the overall performance and stability of Tow Thomas Bi-quad Filter.

6 To implement the design filter in hardware.

ABBREVIATIONS

GA	Genetic Algorithm
LPF	Low Pass Filter
FIR	Finite Impulse Response
RB	Resistance Bridge
TT	Tow Thomas

NOTATION

English Symbols

R resistance

C capacitance

Greek Symbols

α order of filter in decimal

Ω unit of resistance

Miscellaneous

$|x|$ Absolute value of x

% Per-cent

Introduction

1.1 Filter Definition

In electronics, a **filter** (signal processing) is a kind of devices or process that removes some unwanted components or features from a signal. Filtering is a class of signal processing, the defining feature of filters being the complete or partial suppression of some aspect of the signal. Most often, this means removing some frequencies or frequency bands. However, filters do not exclusively act in the frequency domain; especially in the field of image processing many other targets for filtering exist. As is known to all, electronic filters remove unwanted frequency components from the applied signal, enhance wanted ones, or both.

1.2 Type of Filters

Filters have different effects on signals of different frequencies. According to this fact, the basic filter types can be classified into four categories: low-pass, high-pass, band-pass, and band-stop. Each of them has a specific application in DSP. One of the objectives may involve digital filters design in applications. Generally, the filter is designed based on the specifications primarily for the passband, stopband, and transition band of the filter frequency response. The filter passband is the frequency range with the amplitude gain of the filter response being approximately unity. The filter stopband refers to the frequency range over which the filter magnitude response is attenuated to eliminate the input signal whose frequency components are within that range. The transition band means the frequency range between the passband and the stopband.

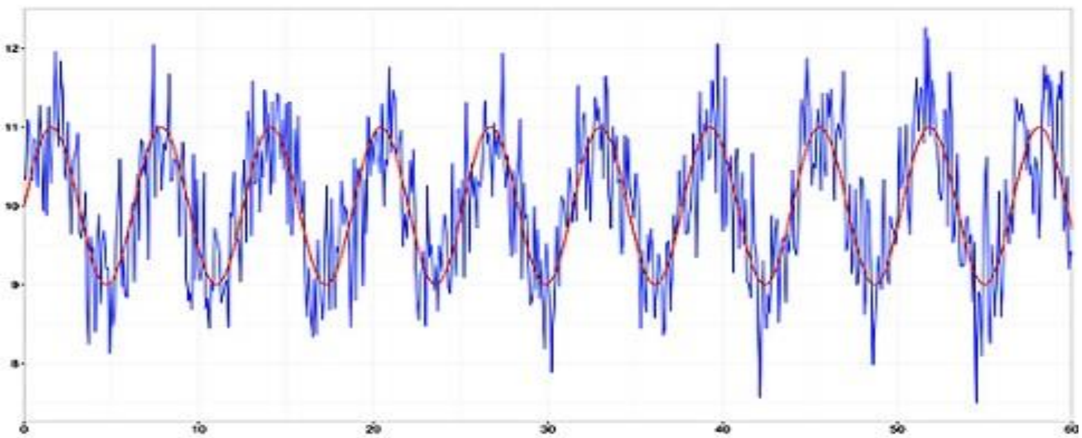

Figure 1.1 Filtering Out the Noise (signal processing)

Because there are many different standards of classifying filters and these overlap in many different ways, there is no clearly distinctive classification. Filters may be:

- non-linear or linear
- analog or digital
- time-variant or time-invariant, also known as shift invariance.
- discrete-time (sampled) or continuous-time
- passive or active type of continuous-time filter
- infinite impulse response (IIR) or finite impulse response (FIR) type

1.2.1 Passive Filter & Active Filter

Passive FilterA passive filter is composed of passive components only. It is based on the principle that the reactance of the capacitive and inductive components changes with frequency. The advantages of this type of filter are: simple circuit, causal power supply, and high reliability. Also there are disadvantages: the signal in the pass-band has energy loss, the load effect is relatively obvious, and electromagnetic induction is easy to cause when using inductive components. When the inductance is large, the size and weight of the filter are relatively large, which is not applicable in the low frequency range.

The passive filter circuit has a simple structure and is easy to design, but its pass-band magnification and cut-off frequency change with the load, so it is not suitable for occasions with large signal processing requirements. Passive filter circuits are usually used in power circuits, such as filtering after DC power rectification, or LC (inductance, capacitor) circuit filtering when high current loads are used.

Active Filter Active filters are composed of passive components and active devices. The advantages of this type of filter are that the signal in the pass-band has no energy loss, even be amplified; the load effect is not obvious, and the mutual influence is small when multi-levels are connected. The simple method of cascading is easy to form high-order filter, and the device is small, lightweight, and does not require magnetic shielding.

Their disadvantages are that the pass-band range is limited by the bandwidth of the active device and requires a DC power supply; the reliability is not as high as that of a passive filter, and it is not suitable for high voltage, high frequency, and high power applications.

The load of the active filter circuit does not affect the filtering characteristics, so it is often used in places with superior signal processing requirements. Active filter circuit is generally composed of an RC network and integrated operational amplifier, so it can only be used under the condition of suitable DC power supply, and it can also be amplified. However, the composition and design of the circuit are also more complicated. Active filter circuits are not suitable for high voltage and high current applications.

1.2.2 Digital Filter & Analog Filters

A digital filter is an algorithm or device consisting of a digital multiplier, an adder, and a delay unit. The function of the digital filter is to perform arithmetic processing on the digital code of the input discrete signal to achieve the purpose of changing the signal spectrum. Digital filters can be made by computer software or large-scale integrated digital hardware. There are active and passive analog filters. Active filters mainly consist of op amps, op amps, resistors, and capacitors. They have problems such as voltage drift, temperature drift, and noise, while digital filters do not get these problems, so they can achieve high stability and accuracy.

Differences between Digital filter & Analog filters

Digital filters are used for discrete systems, analog filters are used in continuous-time systems, and they can also be used in discrete-time systems, such as SC (switched capacitor) filters.

From the point of view of implementation, analog filters are generally built with analog devices such as capacitors and inductors. Digital filters can be implemented by software or digital chips. It is troublesome to replace the capacitor and inductor when the technique parameters of the analog filter are changed. If there is a need for replacement, it is necessary to modify the coefficients (such as when implemented in software).

From the technical view, for example, it is very difficult for analog filters to reach -60dB, and digital filters can easily reach this.

The biggest difference between analog and digital filters is that the digital filter on the $F_s/2$ frequency is flipped, that is, symmetrical, while analog filters are not. Therefore, a large number of interpolation filters are selected in the DAC, and the image frequency is placed at a far frequency point, and then the analog filter regarded as a sound meter is used to filter out the image frequency in the radio frequency band.

The expression of analog filters is different from digital filters: analog filters are represented by H (S), and digital filters are represented by H (Z). Analog filter is based on the approximation of amplitude-frequency characteristics, while digital filters can achieve phase matching.

1.3Characteristics of Filter

1.3.1 Characteristic Frequency

①The pass-band cutoff frequency

$$f_p = w_p / (2p)$$

It is the frequency of the boundary point between the passband and the transition band, at which the signal gain decreases to the specified lower limit.

② Stop-band cut-off frequency

$$f_r = w_r / (2p)$$

It is the frequency of the boundary point between the stopband and the transition band, at which the signal attenuation (reciprocal of the gain) decreases to the specified lower limit.

③The corner frequency

$$f_c = w_c / (2p)$$

It is the frequency when the signal power is attenuated to 1/2 (about 3dB). In many cases, $f_c$is often used as the pass-band or stop-band cutoff frequency.

④ Natural frequency

$$f_o = w_o / (2p)$$

When there is no loss in the circuit, it refers to the resonance frequency of the filter, and complex circuits often have multiple natural frequencies.

1.3.2 Gain and Attenuation

The gain of the filter in the pass-band is not constant.

①For the low-pass filter pass-band gain K_p, for the ordinary filters, it refers to the gain at w = 0; for the high-pass, it refers to the gain at $w \to \infty$; for the band pass, it refers to the gain at the center frequency.

②For the band-stop filter, the stop-band attenuation should be given, and the attenuation is defined as the inverse of the gain.

③The change amount of the pass-band gain ΔK_p, refers to the maximum change amount of the gain at each point in the pass-band. If ΔK_p is in dB, it means the variation of the gain dB value.

1.3.3 Damping coefficient and Quality factor

The damping coefficient is a characterization of a filter's damping effect on a signal with an angular frequency at w_o, and is an indicator of energy loss in the filter.

The reciprocal of the damping coefficient is called quality factor, and is an important indicator of the frequency selection characteristics of the valence band-pass and band-stop filters, $Q = w_o / \Delta w$, where Δw in the formula is the 3dB bandwidth of the band-pass or band-stop filter, w_o is the center frequency, and in many cases the center frequency is equal to the natural frequency.

1.3.4 Sensitivity

The filtering circuit is composed of many components, and changes of parameter values of each component will affect the performance of the filter. The sensitivity of a certain performance index y of the filter to the change of a certain component parameter x is recorded as S_{xy}, which is defined as: $S_{xy} = (d_y / y) / (d_x / x)$.

This sensitivity is not the same concept with the sensitivity of measuring instruments or circuit systems. The smaller the sensitivity, the stronger the fault tolerance of the circuit, and the higher the stability.

1.3.5 Group Delay Function

When the filter's amplitude-frequency characteristics meet the design requirements, in order to ensure that the output signal distortion does not exceed the allowable range, certain requirements should be put forward for its phase-frequency characteristic $\oint(w)$. In filter design, the closer the group delay function $d\oint(w) / dw$ is to a constant, the smaller the signal phase distortion.

1.3.6 Frequency Response

Filter circuits (such as low-pass filters, high-pass filters, band-pass filters, and band-reject filters) shape the frequency content of signals by allowing only certain frequencies to pass through. You can describe these filters based on simple circuits.

You find the sinusoidal steady-state output of the filter by evaluating the transfer function $T(s)$ *at* $s = j\omega$. The transfer function relates the input/output signals in the *s*-domain and assumes zero initial conditions. The radian frequency ω is a variable that stands for the frequency of the sinusoidal input. After you substitute the $s = j\omega$ into $T(s)$, the transfer function becomes a ratio of complex numbers $T(j\omega)$.

Because the function $T(j\omega)$ is a complex number for all frequencies, you can determine the gain $|T(j\omega)|$ and phase $\theta(j\omega)$. Here are the gain and phase relationships:

$$|T(j\omega)| = \frac{\text{Output amplitude}}{\text{Input amplitude}}$$

$$\theta(j\omega) = \angle T(j\omega)$$
$$= \text{Output phase} - \text{Input phase}$$

You can present the gain and phase as a function of frequency ω graphically, as shown in this approximation of a typical filter. In a passband region, the gain function has nearly constant gain for a range of frequencies. In the stopband region, the gain is significantly reduced for a range of frequencies.

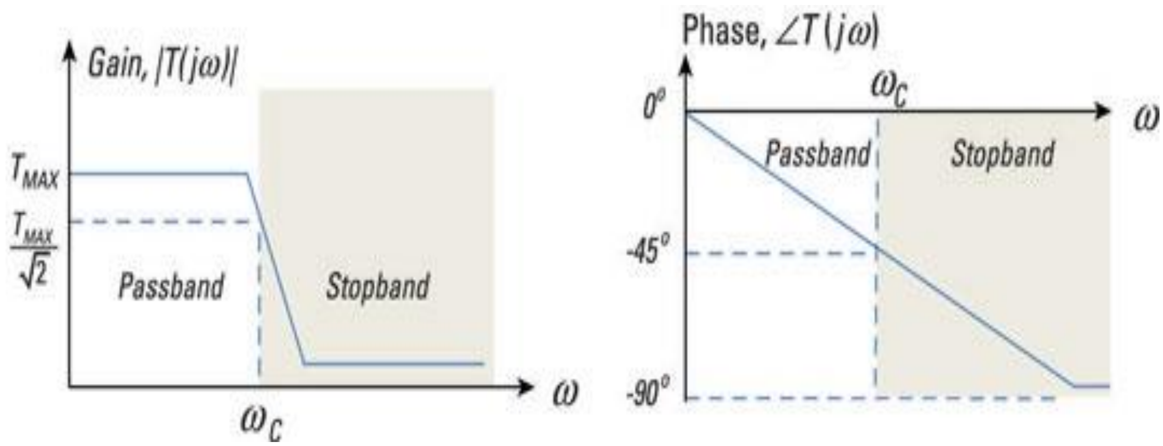

Figure 1.2 **Frequency Response**

For nonideal filters, a transition region occurs between adjacent passband and stopband regions. The cutoff frequency ω_C occurs within the transition region, according to a prescribed definition. One widely used definition says the *cutoff* frequency occurs when the passband gain is decreased by a factor of 0.707 from a maximum value T_{MAX}. The mathematical condition for ω_C is therefore

$$|T(j\omega)| = \frac{1}{\sqrt{2}} T_{MAX} = 0.707 \cdot T_{MAX}$$

At the cutoff frequency, the output power has dropped to one half of its maximum passband value. Here, the passband includes those frequencies where the relative power is greater than the half-power point (0.707 of the maximum value of the transfer function). Frequencies that are less than the half-power point fall in the stopband.

2. Electronic Filter Topology

2.1Definition

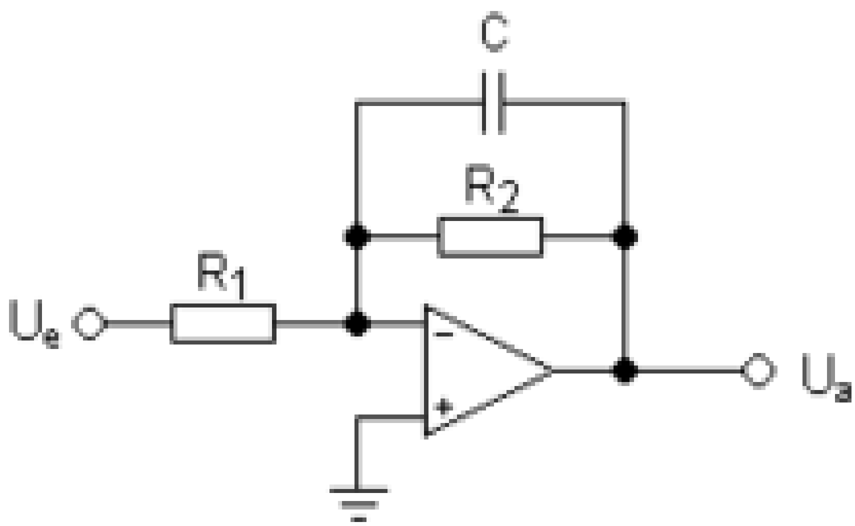

Figure 2.1 An elementary filter topology

An elementary filter topology introduces a capacitor into the feedback path of an op-amp to achieve an unbalanced active implementation of a low-pass transfer function

Electronic filter topology defines electronic filter circuits without taking note of the values of the components used but only the manner in which those components are connected.

Filter design characterises filter circuits primarily by their transfer function rather than their topology. Transfer functions may be linear or nonlinear. Common types of linear filter transfer function are; high-pass, low-pass, bandpass, band-reject or notch and all-pass. Once the transfer function for a filter is chosen, the particular topology to implement such a prototype filter can be selected so that, for example, one might choose to design a Butterworth filter using the Sallen–Key topology.

Filter topologies may be divided into passive and active types. Passive topologies are composed exclusively of passive components: resistors, capacitors, and inductors. Active topologies also include active components (such as transistors, op amps, and other integrated circuits) that require power. Further, topologies may be implemented either in unbalanced form or else in balanced form when employed in balanced circuits. Implementations such as electronic mixers and stereo sound may require arrays of identical circuits.

2.2 Passive Topology

Passive filters have been long in development and use. Most are built from simple two-port networks called "sections". There is no formal definition of a section except that it must have at least one series component and one shunt component. Sections are invariably connected in a "cascade" or "daisy-chain" topology, consisting of additional copies of the same section or of completely different sections. The rules of series and parallel impedance would combine two sections consisting only of series components or shunt components into a single section.

Some passive filters, consisting of only one or two filter sections, are given special names including the L-section, T-section and Π-section, which are unbalanced filters, and the C-section, H-section and box-section, which are balanced. All are built upon a very simple "ladder" topology (see below). The chart at the bottom of the page shows these various topologies in terms of general constant k filters.

Filters designed using network synthesis usually repeat the simplest form of L-section topology though component values may change in each section. Image designed filters, on the other hand, keep the same basic component values from section to section though the topology may vary and tend to make use of more complex sections.

L-sections are never symmetrical but two L-sections back-to-back form a symmetrical topology and many other sections are symmetrical in form.

2.2.1 Ladder Topology

Ladder topology, often called Cauer topology after Wilhelm Cauer (inventor of the elliptic filter), was in fact first used by George Campbell (inventor of the constant k filter). Campbell published in 1922 but had clearly been using the topology for some time before this. Cauer first picked up on ladders (published 1926) inspired by the work of Foster (1924). There are two forms of basic ladder topologies; unbalanced and balanced. Cauer topology is usually thought of as an unbalanced ladder topology.

A ladder network consists of cascaded asymmetrical L-sections (unbalanced) or C-sections (balanced). In low pass form the topology would consist of series inductors and shunt capacitors. Other bandforms would have an equally simple topology transformed from the lowpass topology. The transformed network will have shunt admittances that are dual networks of the series impedances if they were duals in the starting network - which is the case with series inductors and shunt capacitors.

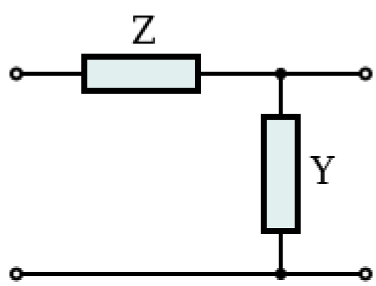

Figure 2.2 L Half section

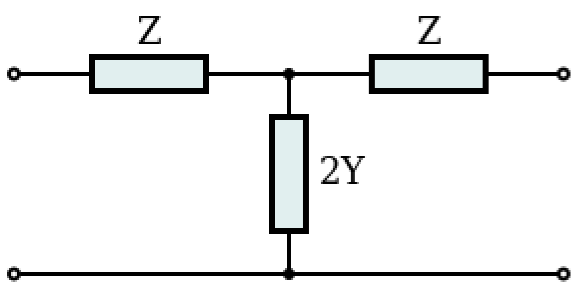

Figure 2.3 T Section

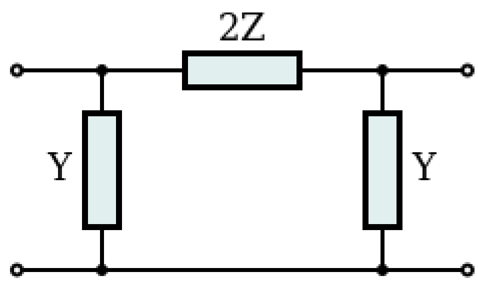

Figure 2.4 Π Section

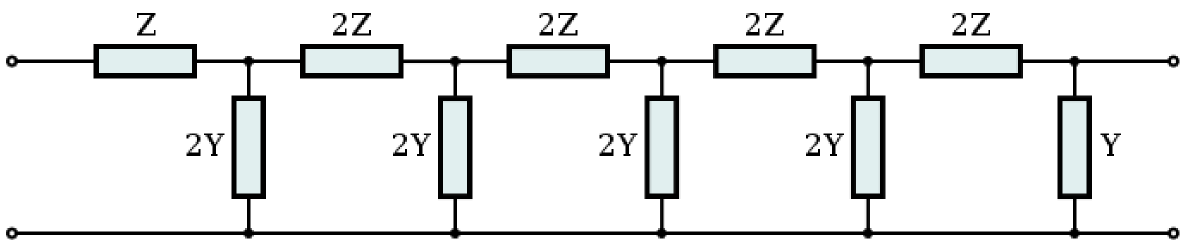

Figure 2.5 Ladder network

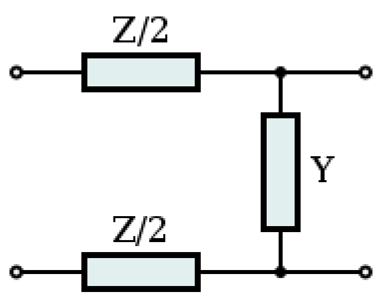

Figure 2.6 C Half-section

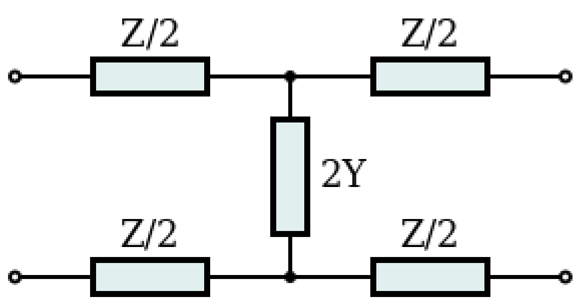

Figure 2.7 H Section

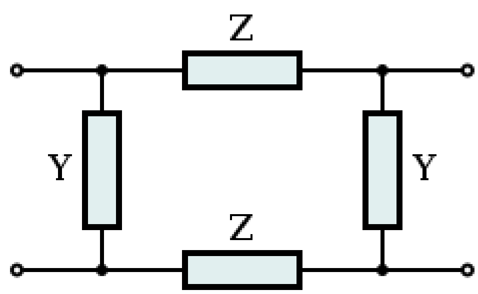

Figure 2.8 Box Section

2.2.2 Modified Ladder Topology

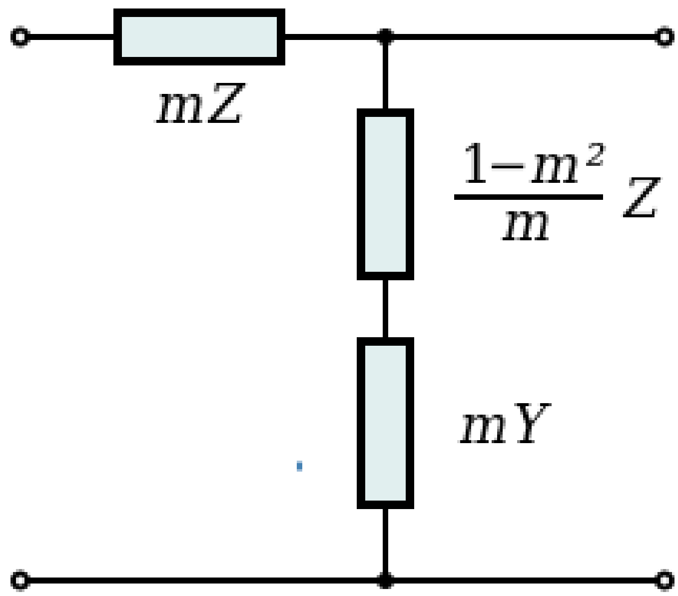

Figure 2.9 series m-derived topology

Image filter design commonly uses modifications of the basic ladder topology. These topologies, invented by Otto Zobel,[1] have the same passbands as the ladder on which they are based but their transfer functions are modified to improve some parameter such as impedance matching, stopband rejection or passband-to-stopband transition steepness. Usually the design applies some transform to a simple ladder topology: the resulting topology is ladder-like but no longer obeys the rule that shunt admittances are the dual network of series impedances: it invariably becomes more complex with higher component count. Such topologies include;

The m-type (m-derived) filter is by far the most commonly used modified image ladder topology. There are two m-type topologies for each of the basic ladder topologies; the series-derived and shunt-derived topologies. These have identical transfer functions to each other but different image impedances. Where a filter is being designed with more than one passband, the m-type topology will result in a filter where each passband has an analogous frequency-domain response. It is possible to generalise the m-type topology for filters with more than one passband using parameters m_1, m_2, m_3 etc., which are not equal to each other resulting in general m_n-type[2] filters which have bandforms that can differ in different parts of the frequency spectrum.

The mm'-type topology can be thought of as a double m-type design. Like the m-type it has the same bandform but offers further improved transfer characteristics. It is, however, a rarely used design due to increased component count and complexity as well as its normally requiring basic ladder and m-type sections in the same filter for impedance matching reasons. It is normally only found in a composite filter.

2.2.3 Bridged-T topology

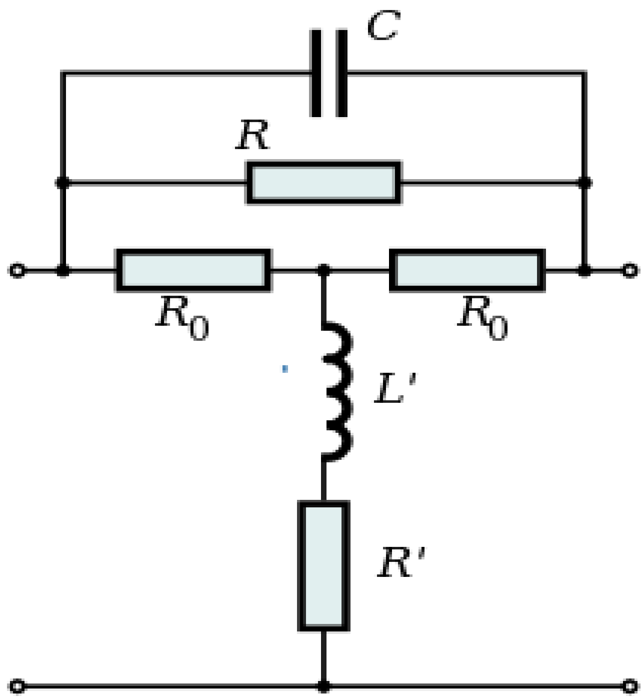

Figure 2.10 Typical bridged-T Zobel network equaliser used to correct high end roll-off

Zobel constant resistance filters[3] use a topology that is somewhat different from other filter types, distinguished by having a constant input resistance at all frequencies and in that they use resistive components in the design of their sections. The higher component and section count of these designs usually limits their use to equalisation applications. Topologies usually associated with constant resistance filters are the bridged-T and its variants, all described in the Zobel network article;

- Bridged-T topology
- Balanced bridged-T topology
- Open-circuit L-section topology
- Short-circuit L-section topology
- Balanced open-circuit C-section topology
- Balanced short-circuit C-section topology

The bridged-T topology is also used in sections intended to produce a signal delay but in this case no resistive components are used in the design.

2.2.4Lattice Topology

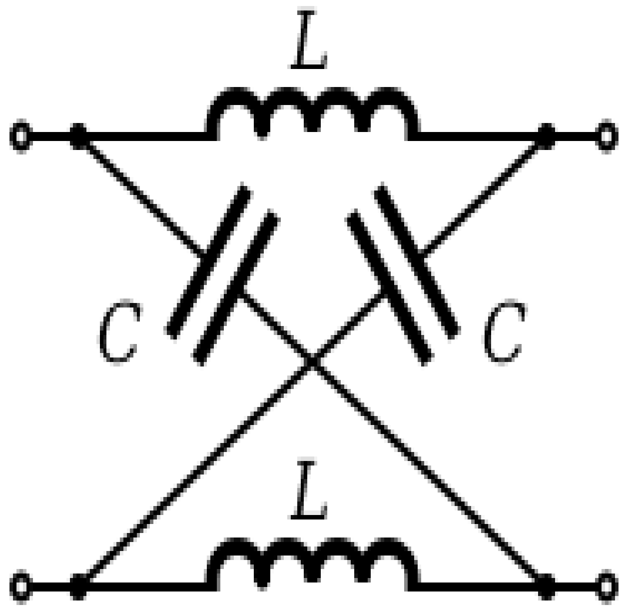

Figure 2.11 Lattice topology X-section phase correction filter

Both the T-section (from ladder topology) and the bridge-T (from Zobel topology) can be transformed into a lattice topology filter section but in both cases this results in high component count and complexity. The most common application of lattice filters (X-sections) is in all-pass filters used for phase equalisation.[4]

Although T and bridged-T sections can always be transformed into X-sections the reverse is not always possible because of the possibility of negative values of inductance and capacitance arising in the transform.

Lattice topology is identical to the more familiar bridge topology, the difference being merely the drawn representation on the page rather than any real difference in topology, circuitry or function.

2.3 Active Topologies

2.3.1Multiple Feedback Topology

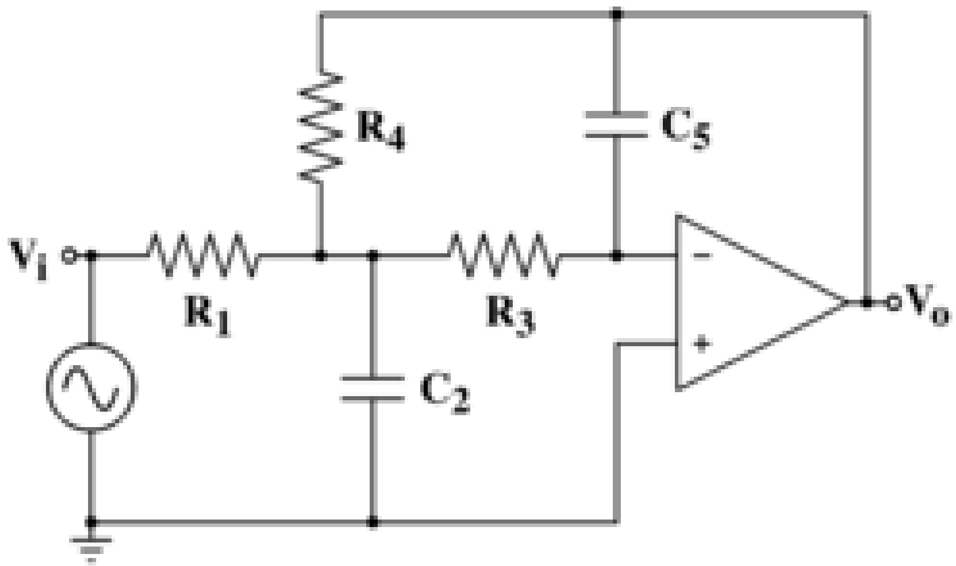

Figure 2.12 Multiple feedback topology circuit.

Multiple feedback topology is an electronic filter topology which is used to implement an electronic filter by adding two poles to the transfer function. A diagram of the circuit topology for a second order low pass filter is shown in the figure on the right.

2.3.2 Bi-Quad Filter Topology

A bi-quad filter is a type of linear filter that implements a transfer function that is the ratio of two quadratic functions. The name *bi-quad* is short for *biquadratic*. Any second-order filter topology can be referred to as a *bi-quad*, such as the MFB or Sallen-Key.However, there is also a specific "bi-quad" topology. It is also sometimes called the 'ring of 3' circuit.

Bi-quad filters are typically active and implemented with a single-amplifier bi-quad (SAB) or two-integrator-loop topology.

The SAB topology uses feedback to generate complex poles and possibly complex zeros. In particular, the feedback moves the real poles of an RC circuit in order to generate the proper filter characteristics.

The two-integrator-loop topology is derived from rearranging a biquadratic transfer function. The rearrangement will equate one signal with the sum of another signal, its integral, and the integral's integral. In other words, the rearrangement reveals a state variable filter structure. By using different states as outputs, any kind of second-order filter can be implemented.

The SAB topology is sensitive to component choice and can be more difficult to adjust. Hence, usually the term bi-quad refers to the two-integrator-loop state variable filter topology.

2.3.3 Tow Thomas Bi-quad Filter

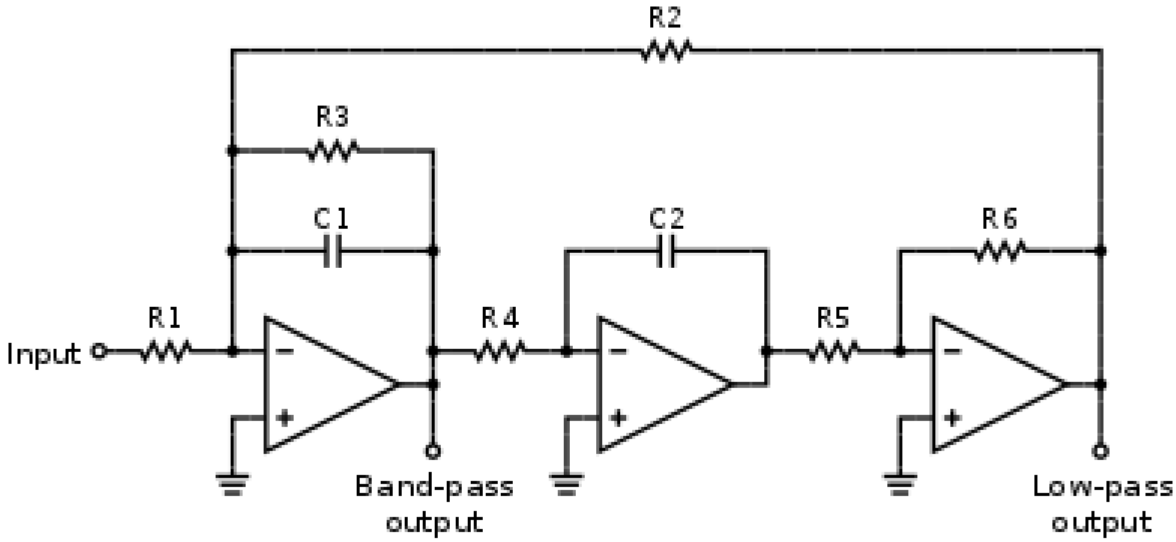

Figure 2.13 The common Tow-Thomas biquad filter topology.

For example, the basic configuration in Figure 1 can be used as either a low-pass or bandpass filter depending on where the output signal is taken from.
If a noninverting low-pass filter is required, the output can be taken at the output of the second operational amplifier, after the order of the second integrator and the inverter has been switched. If a noninverting bandpass filter is required, the order of the second integrator and the inverter can be switched, and the output taken at the output of the inverter's operational amplifier.

2.3.4 Akerberg-Mossberg Filter

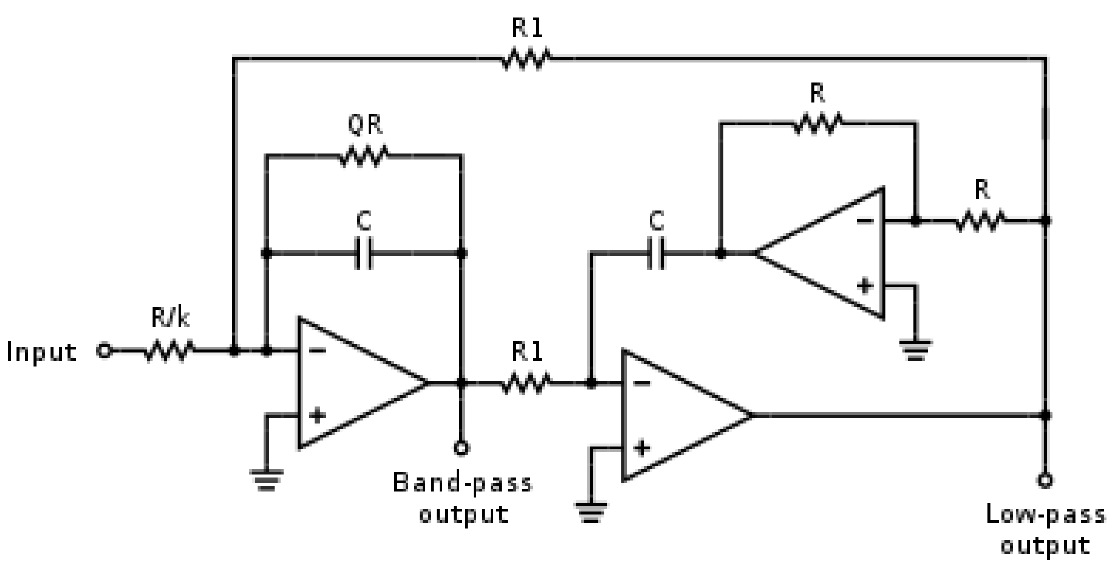

Figure 2.14 TheAkerberg-Mossberg biquad filter topology.

Figure 2 shows a variant of the Tow-Thomas topology, known as Akerberg-Mossberg topology, that uses an actively compensated Miller integrator, which improves filter performance.

2.3.5 Sallen–Key Topology

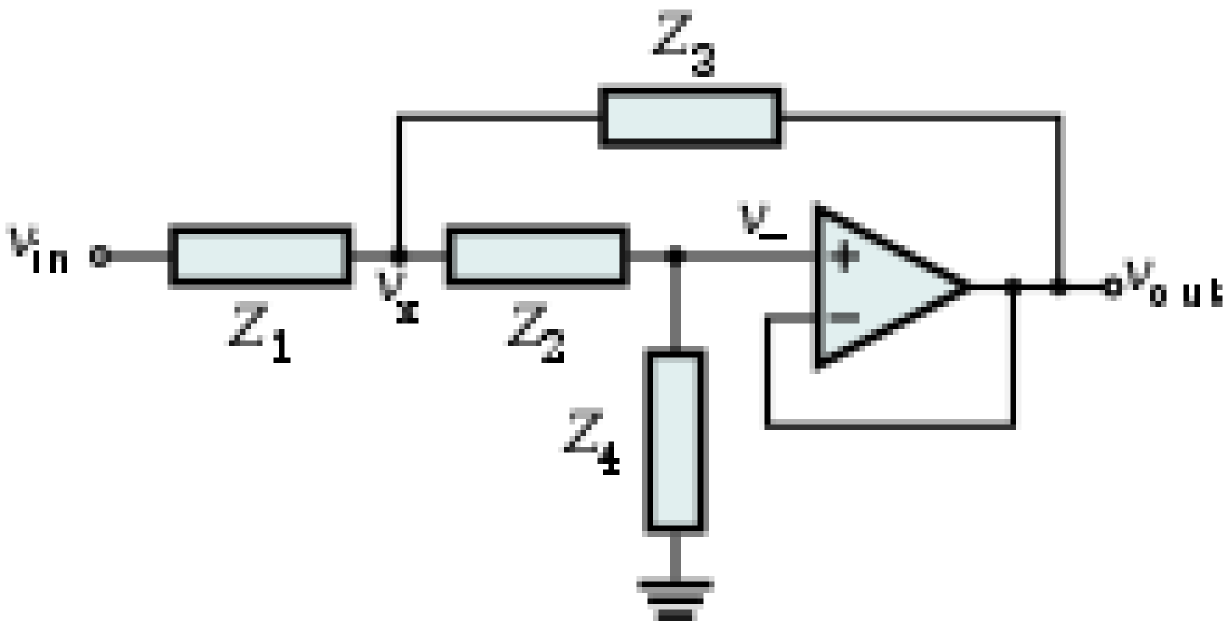

Figure 2.15 The generic Sallen–Key filter topology

The Sallen-Key design is a non-inverting second-order filter with the option of high Q and passband gain.

3. Proposed Methodology

3.1 Second Order Tow Thomas Bi-quad Filter

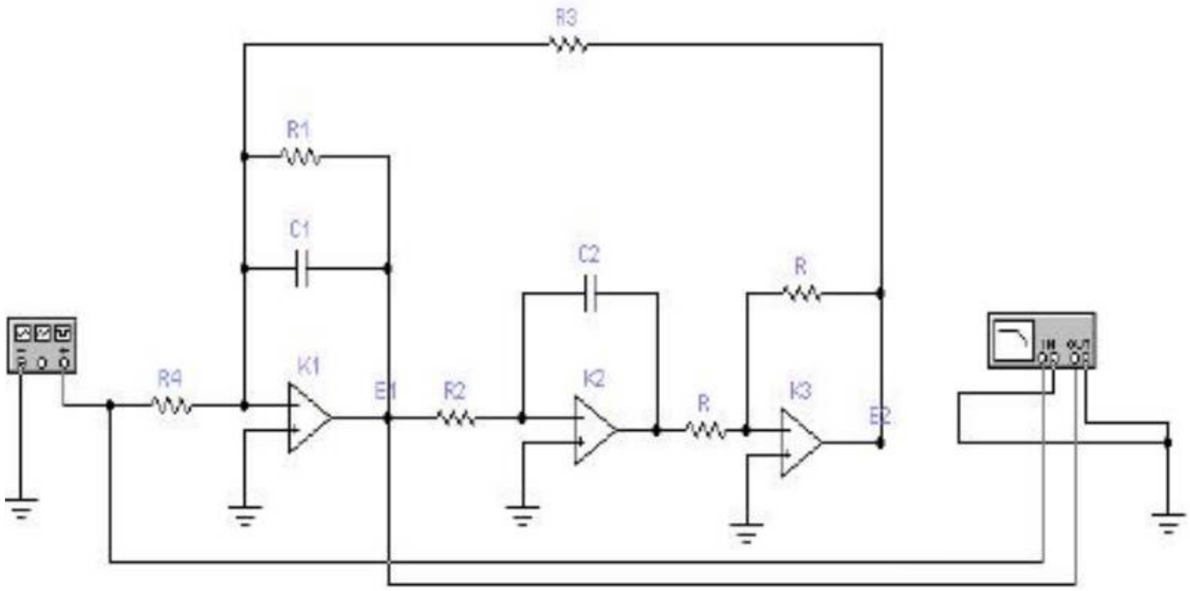

Figure 3.1 Second Order Tow Thomas Bi-quad Filter

A Tow-Thomas Bi-quad Circuit which shall be referred to as theBi-quad in the rest ofthe document. The Bi-quad exhibitsa second-order transfer function that can be used to buildhigher order circuits. The Bi-quad is composed of threeimportant parts: alossy integrator, an inverting integrator and an inverting amplifier. The open-loop operationalamplifier K_1 has a feedback loop $R_1 C_1$ with a R_4 resistorthat defines a lossy-integrator structure. This structure isused to achieve the low-pass and band-pass frequency response states. The second operation amplifier K_2 has afeedback capacitor C_2 and input resistor R_2 is realized asinverting integrator. Thethird operational amplifier

$K_3$is simply configured as an inverting amplifier with resistances R_5 and R_6 as both equal. This stage provides low noise pre-amplificationof the entire circuit. The main feedback loop is composedof a resistor R_3. The inverting amplifier can be removedto reduce production costs. Doing this would mean thereplacement of the second stage inverting integrator circuit with a balanced time-constant (BTC) integrator ora resistance-bridge (RB) integrator. In addition, this structure provides us with ahigh-impedance purely resistive input. An essential aspect of the Bi-quad circuit is its flexibility.The transfer function in is related directly tothe passive RC elements which interconnect the operationalamplifiers. This is the general transfer function for theentire circuit:

$$\frac{V_0(s)}{V_{in}(s)} = \frac{R_3 R_5 / R_4 R_6}{S^{1+\alpha} R_2 R_3 C_1 C_2 + S^{\alpha}(R_2 R_3 C_2 / R_1) + 1}$$

where $\alpha = 1$

3.2 Genetic Algorithm

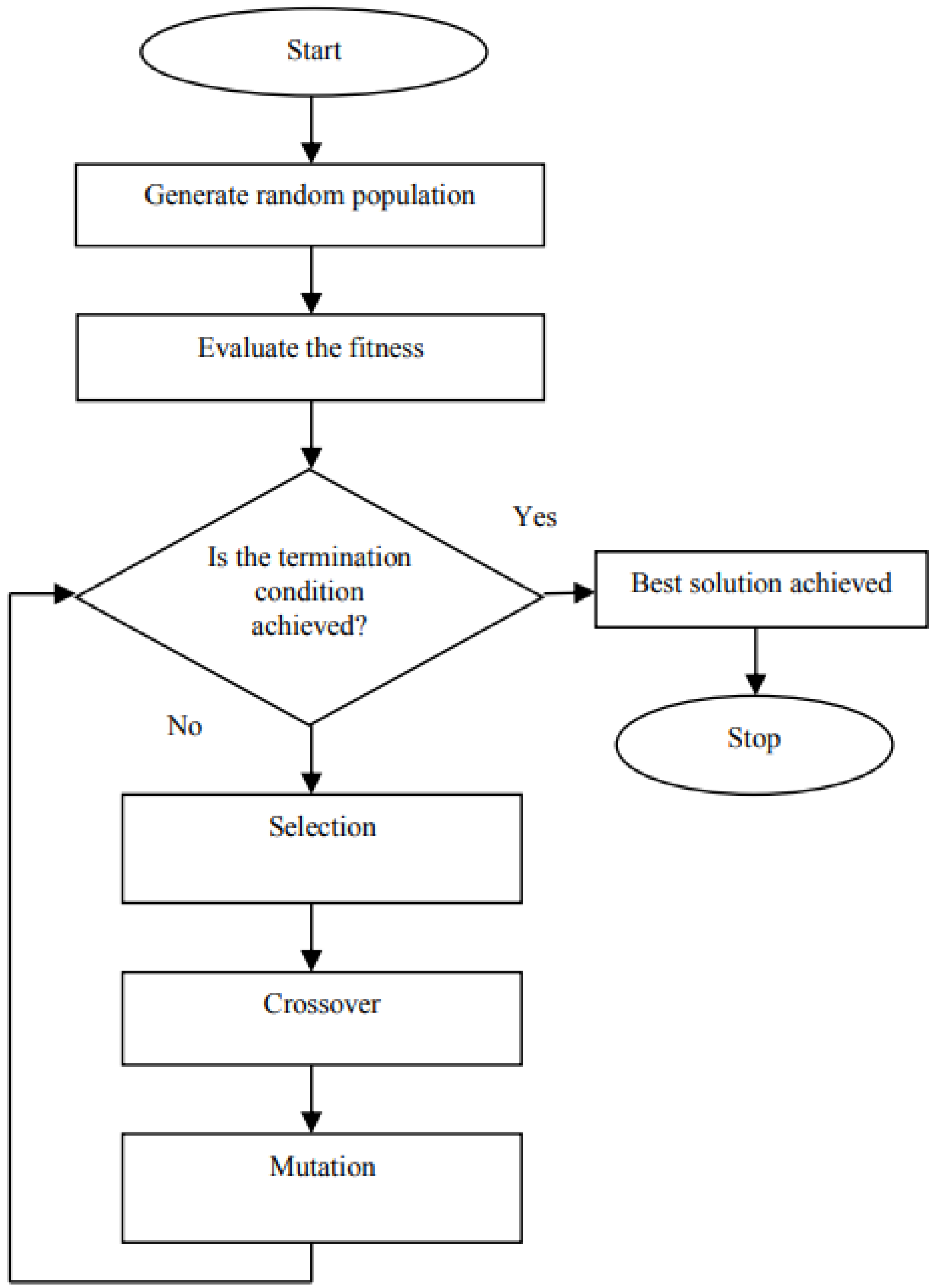

Figure 3.2 Flow chart of Genetic algorithm

GA generates the solutions of problems by using methods motivated by inheritance, mutation, selection and crossover. It is a robust search which based on natural selection and natural genetics. It is not certain in nature, sometimes initial parent population gives best result that would not be achieved after many generation. GA works on iterations and starts from the initial solution. Each iteration step is carefully chosen an operation and used for the current solution. If the new solution is appropriate e.g. if it is better than the current solution then it becomes the current solution for the next iteration, otherwise, it is rejected. This process stops when the requested solution or the number of iterations is achieved.

3.3 OPTIMIZED FRACTIONAL ORDER BUTTERWORTH FILTER

3.3.1 Applying GA on Tow Thomas Transfer FunctionwithStandard Second Order Butterworth LPF

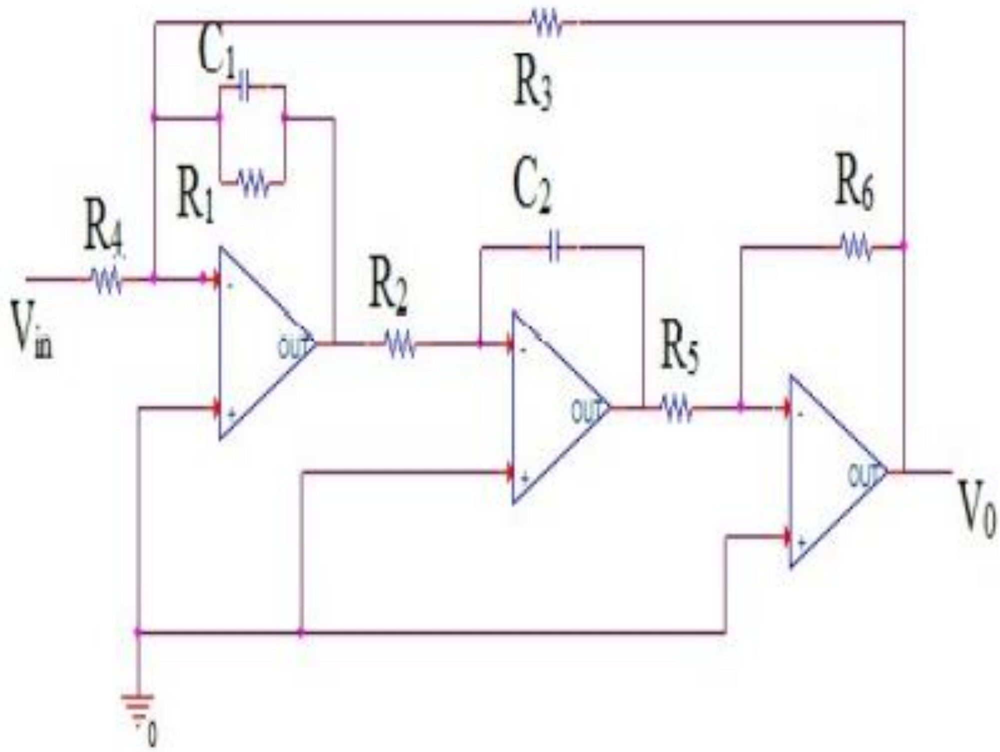

Figure 3.3 Tow Thomas biquad topology

In proposed work, GA has been chosen to approximate the characteristics of fractional order low pass Butterworth filter. Further, GA optimized filter coefficients of proposed filter are utilized to design fractional order low pass Butterworth filter using Tow Thomas bi-quad. The (1+α) order low pass transfer function is known by following equation

$$T(s) = \frac{c}{as^{1+\alpha} + bs^{\alpha} + 1}$$

where c, a and b are filter coefficients.

Butterworth second order transfer function with normalized frequency value of 1 rad/sec is presented by the following equation

$$B_2(s) = \frac{0.5012}{s^2 + 1.414s + 1}$$

In MATLAB, GA is used within the frequency range from $\omega = 10^{-5}$ rad/s to 1 rad/s to get the coefficients of c, a and b. Following equation is used to reduce the square of the error among the low pass (1+α) order transfer function and Butterworth second order transfer function.

$$\min_a \sum_{i=1}^{k} (abs(T(x,\omega_i)) - abs(B_2(\omega_i)))^2$$

The above three equation are used to get the GA optimized filter coefficient values. Optimized filter coefficients of (1+α) order low pass Butterworth filter are shown below in Table for α ranging from 0.1-0.9 in the steps of 0.1.

α	c	a	b
0.1	0.9107	2.3404	1.8097
0.2	0.4324	5.3838	0.6436
0.3	0.9376	68.4905	9.0643
0.4	0.5312	1.6539	0.3513
0.5	0.5169	2.1057	0.3782
0.6	0.5312	34.9333	4.2446
0.7	0.5581	1.4824	1.3613
0.8	0.4852	80.1854	8.4364
0.9	0.5312	0.4487	2.9385

Table 3.1 Simulated filter coefficients using GA

The GA simulated magnitude responses are plotted using filter coefficients c , a and b from Table I for α equals to 0.1 to 0.9 is shown in Fig.2.

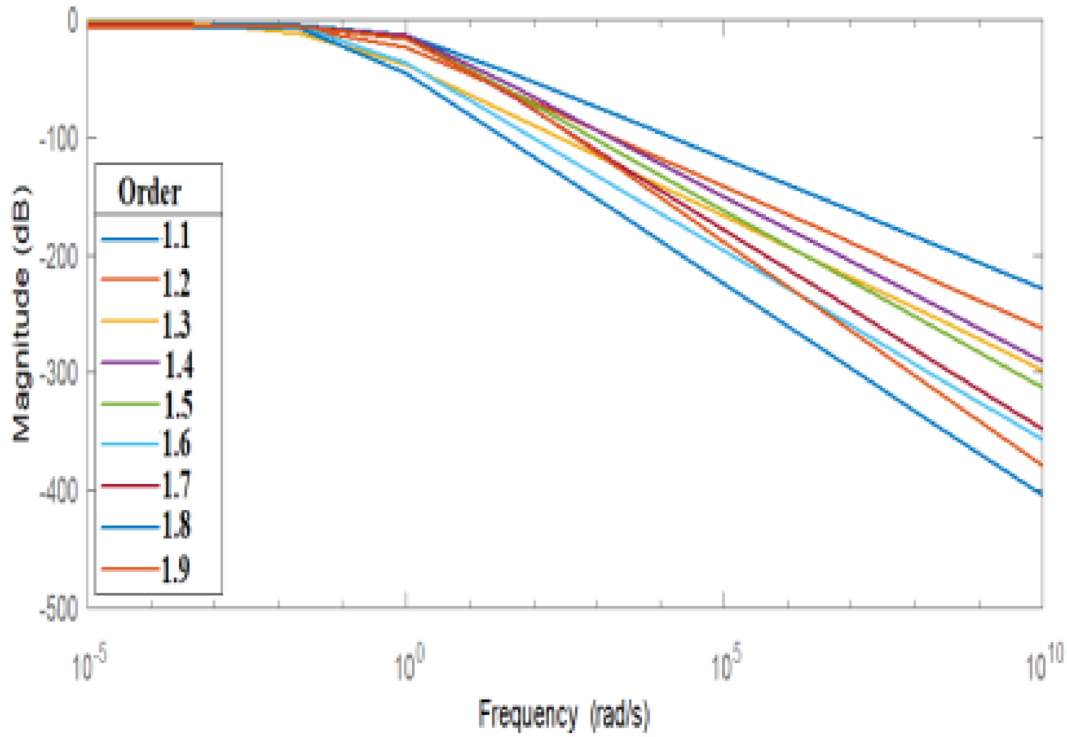

Figure 3.4 GA simulated magnitude response of fractional order low pass Butterworth filter

3.3.2 Comparison of Tow Thomas Transfer Function with Standard Second Order Butterworth LPF

Comparing the coefficients of

$$\frac{V_0(s)}{V_{in}(s)} = \frac{R_3R_5 / R_4R_6}{S^{1+\alpha}R_2R_3C_1C_2 + S^{\alpha}(R_2R_3C_2 / R_1) + 1}$$

AND

$$T(s) = \frac{c}{as^{1+\alpha} + bs^{\alpha} + 1}$$

to get the component values of R1, R2 and R3 of bi-quad. We get,

$$R_1 = a / b$$

$$R_2 = a / c$$

$$R_3 = c$$

by keeping R4 = R5 = R6 =1 KΩ, C1 =C2 =1 F. The frequency of proposed (1+α) order low pass Butterworth filter is shifted to 1 K-Hz and magnitude is scaled by a factor of 1000.

Component	Order		
	1.1	**1.5**	**1.9**
C_1 (μF)	0.159	0.159	0.159
C_2(μF)	415	12.6	0.381
R_1 (Ω)	1293	559	152
R_2 (Ω)	2569.8	4073.7	844.6
R_3 (Ω)	910.7	516.9	531.2
R_4,R_5,R_6(KΩ)	1	1	1

Table3.2 Component values for (1) α + order low pass Butterworth filter

3.4 Fractional order capacitor using fourth order integer approximation

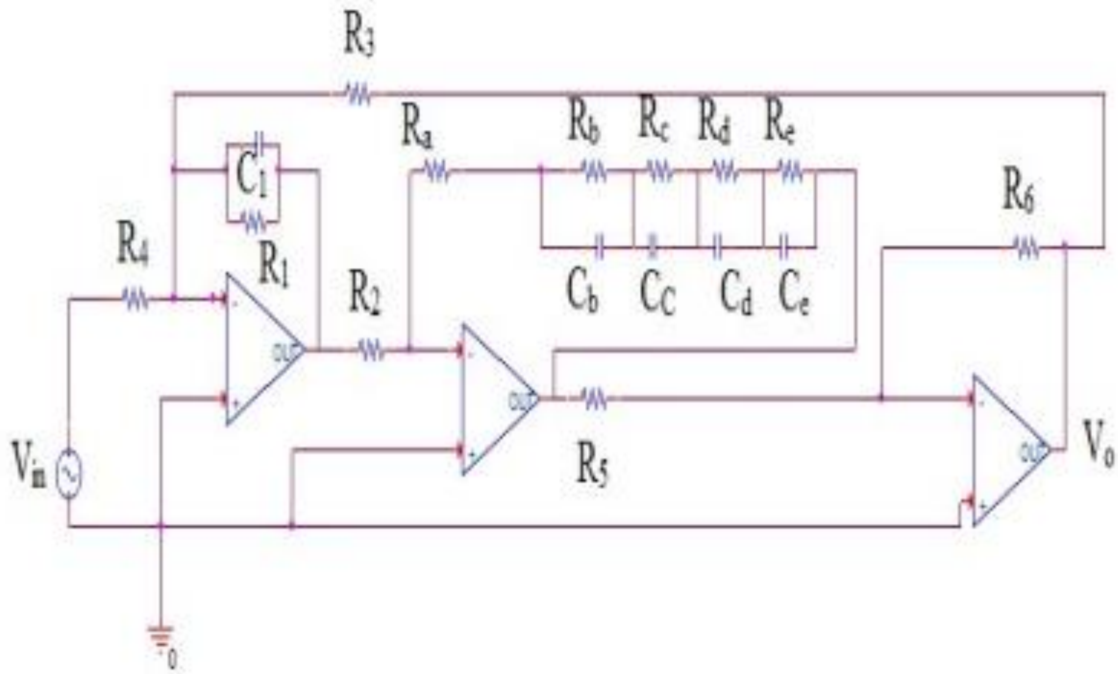

Figure 3.5 Fractional order capacitor using fourth order integer approximation

In order to realize filters that make use of Laplacian operators$^{\alpha}$, integer order approximation needs to be used as commercial fractance devices are not available. The integer order approximations can be done by continued fraction expansion (CFE) and rational approximation methods. In this paper CFE method is used for the approximation of the fractional capacitor. Fourth order approximation by collecting eight terms from the CFE is selected for the fractional order low pass filter realization. The fractional capacitor is then realized by the RC ladder network.

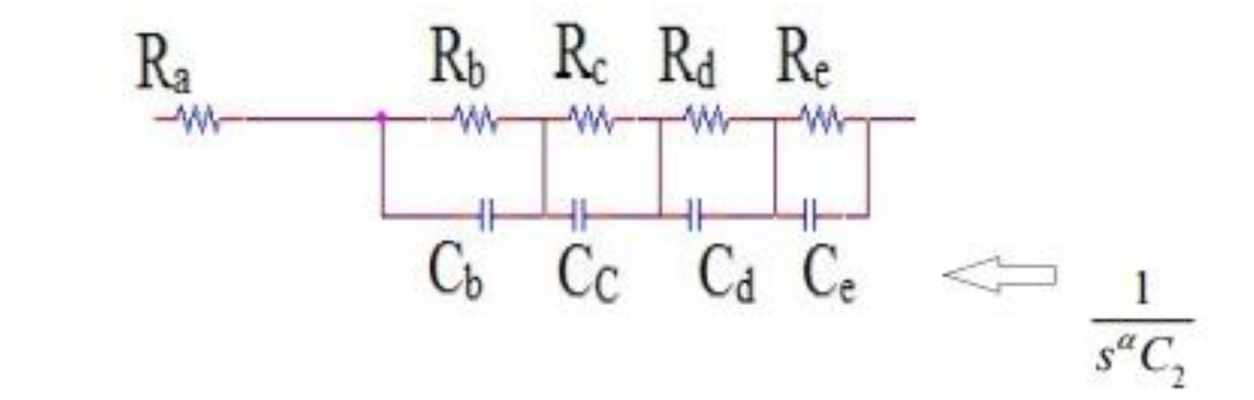

Figure 3.6 Fractional order capacitor using fourth order integer approximation

The values of the resistances and the capacitors required to realize the fractional capacitance C1 for the low pass filters of the order 1.1, 1.5 and 1.9 with the magnitude scaled to 1000 and the cut off frequency shifted to 1 kHz are given in Table below and the resultant circuit, on replacing the traditional capacitor by the fractional capacitor as shown above.

Component	Order		
	1.1	**1.5**	**1.9**
$R_a(\Omega)$	659	111	6.8
$R_b(\Omega)$	195.8	252	43
$R_c(\Omega)$	135	378.8	130.8
$R_d(\Omega)$	158.5	889	670
$R_e(\Omega)$	370	7.4k	146k
$C_b(nF)$	69	84	704
$C_c(\mu F)$	0.627	0.29	1.1
$C_d(\mu F)$	2.1	0.54	1
$C_e(\mu F)$	6.6	0.69	0.20

Table3.3 Component values for fractional capacitor

4.Results and Discussion

In fig. 4.1, we are showing the comparison graph of frequency response between the Tow Thomas Bi-quad Topology and Modified Tow Thomas Bi-quad Topology for the fraction order of 1.1. Frequency is expressed in log and Voltage is expressed in dB. The Tow Thomas Bi-quad Topology is shown by orange colour solid line while Modified Tow Thomas Bi-quad Topology is shown by blue colour solid line.

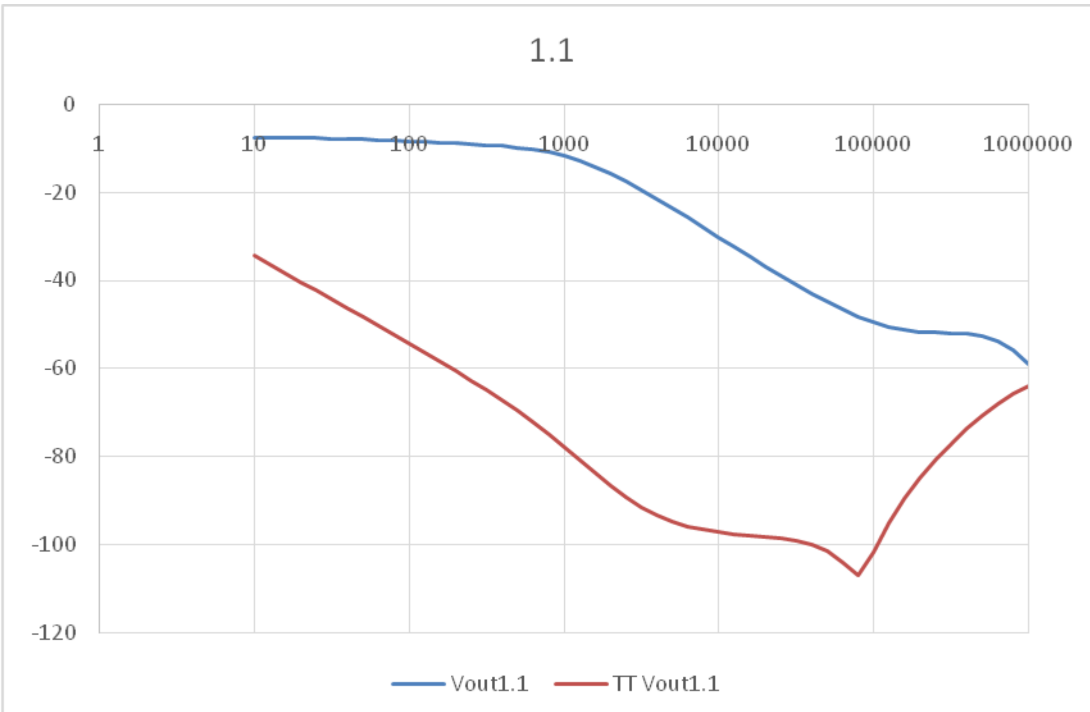

Figure 4.1 Frequnecy vs Voltage Graph for order 1.1

In fig.4.2, we are showing the comparison graph of frequency response between the Tow Thomas Bi-quad Topology and Modified Tow Thomas Bi-quad Topology for the fraction order of 1.5. Frequency is expressed in log and Voltage is expressed in dB. The Tow Thomas

Bi-quad Topology is shown by orange colour solid line while Modified Tow Thomas Bi-quad Topology is shown by blue colour solid line.

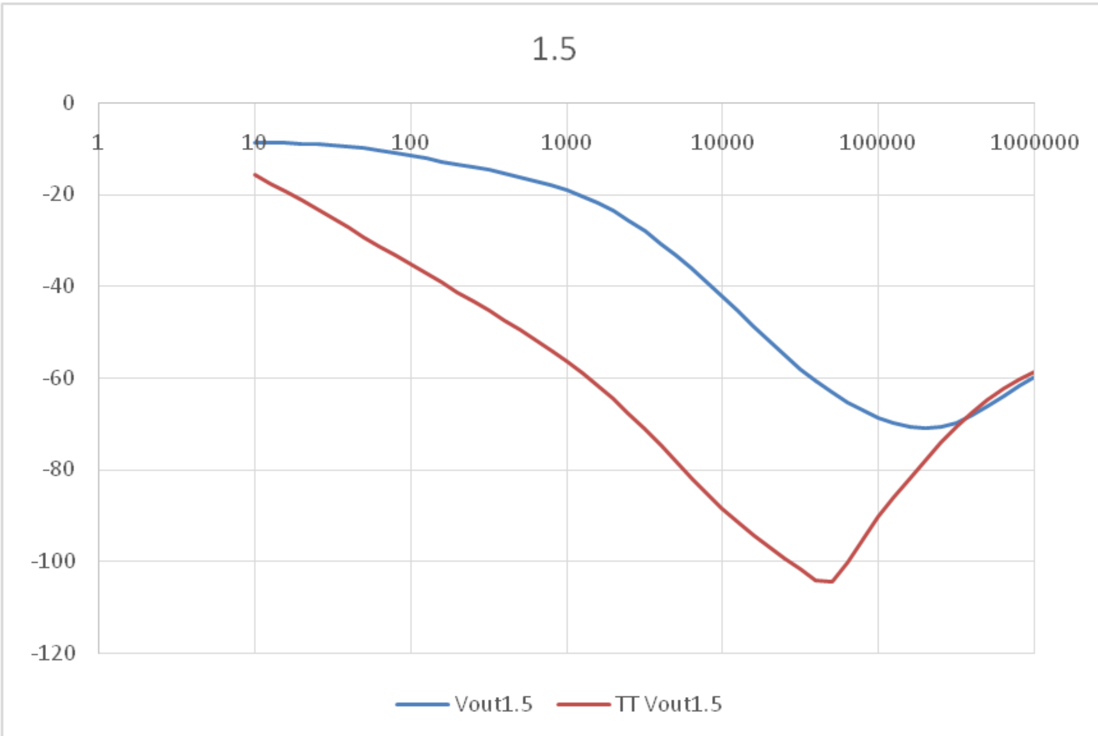

Figure 4.2 Frequnecy vs Voltage Graph for order 1.5

In fig.4.3, we are showing the comparison graph of frequency response between the Tow Thomas Bi-quad Topology and Modified Tow Thomas Bi-quad Topology for the fraction order of 1.9. Frequency is expressed in log and Voltage is expressed in dB. The Tow Thomas Bi-quad Topology is shown by orange colour solid line while Modified Tow Thomas Bi-quad Topology is shown by blue colour solid line.

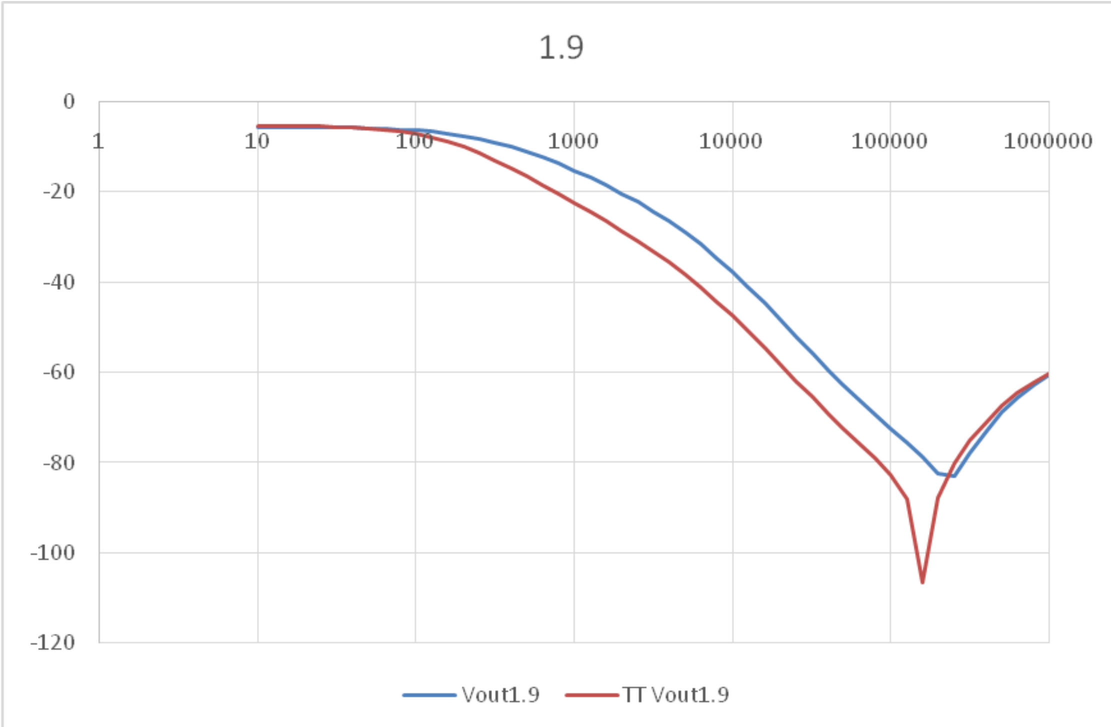

Figure 4.3 Frequnecy vs Voltage Graph for order 1.9

In fig.4.4, we are showing the comparison graph of frequency response between the Modified Tow Thomas Bi-quad Topologies for order 1.1 by blue, 1.5 by orange and 1.9 by grey. Frequency is expressed in log and Voltage is expressed in dB.

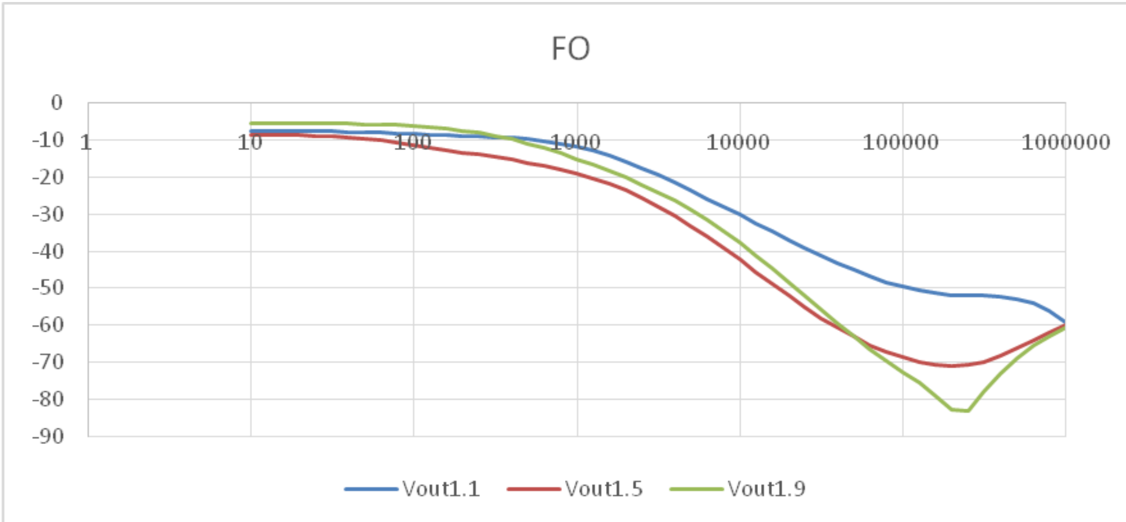

Figure 4.4 Modified Tow Thomas graph for fractional order 1.1,1.5,1.9

In fig.4.5, we are showing the comparison graph of frequency response between the Modified Tow Thomas Bi-quad Topologies for order 1.1 by blue, 1.5 by orange and 1.9 by grey. Frequency is expressed in log and Voltage is expressed in dB.

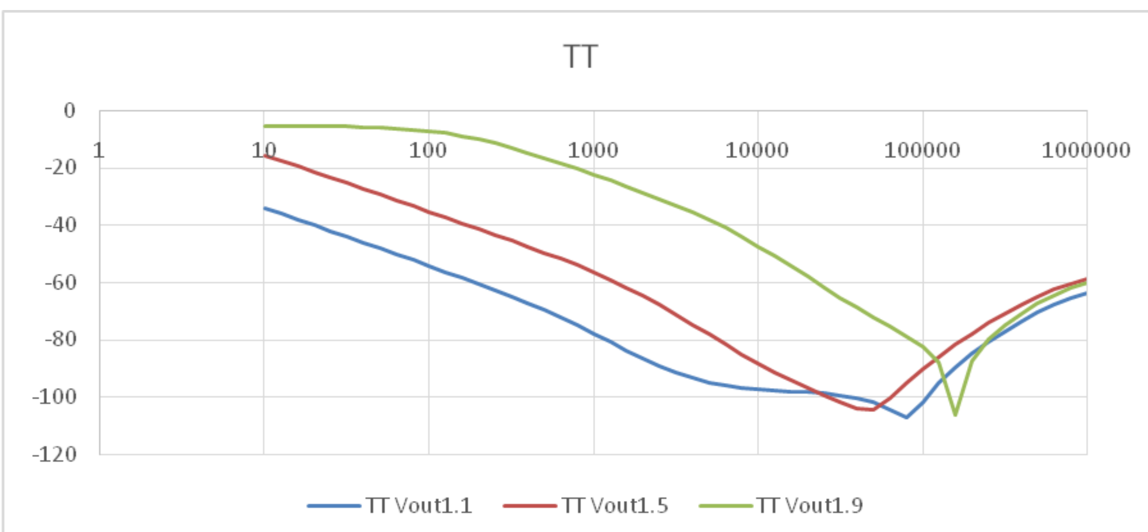

Figure 4.5 Tow Thomas graph for fractional order 1.1,1.5,1.9

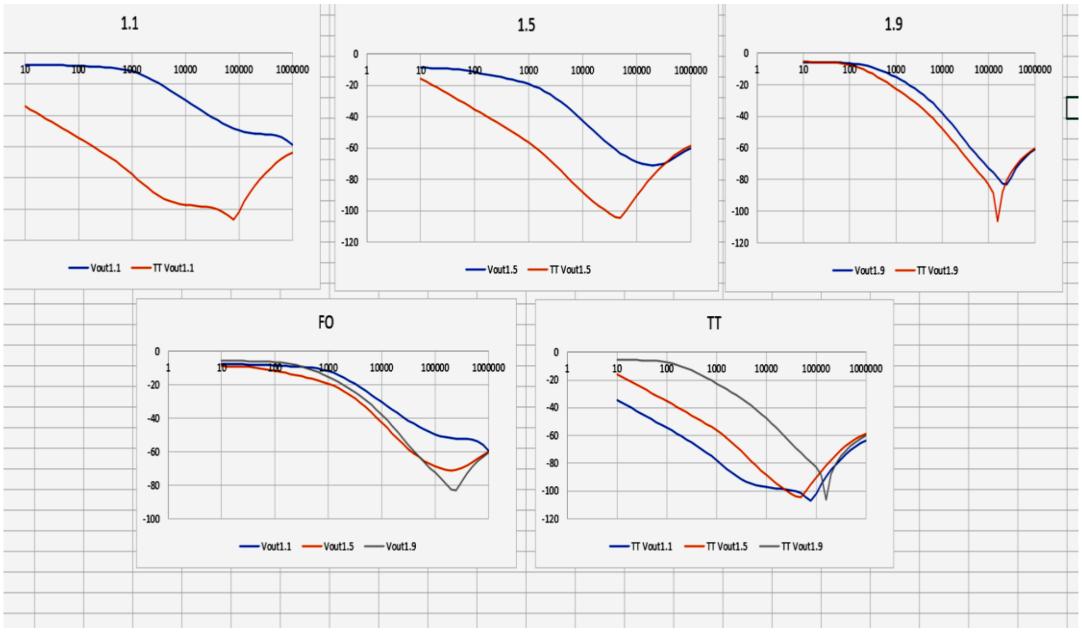

Figure 4.6 Tow Thomas and fractional order graph for fractional order 1.1,1.5,1.9 combined

Frequency response

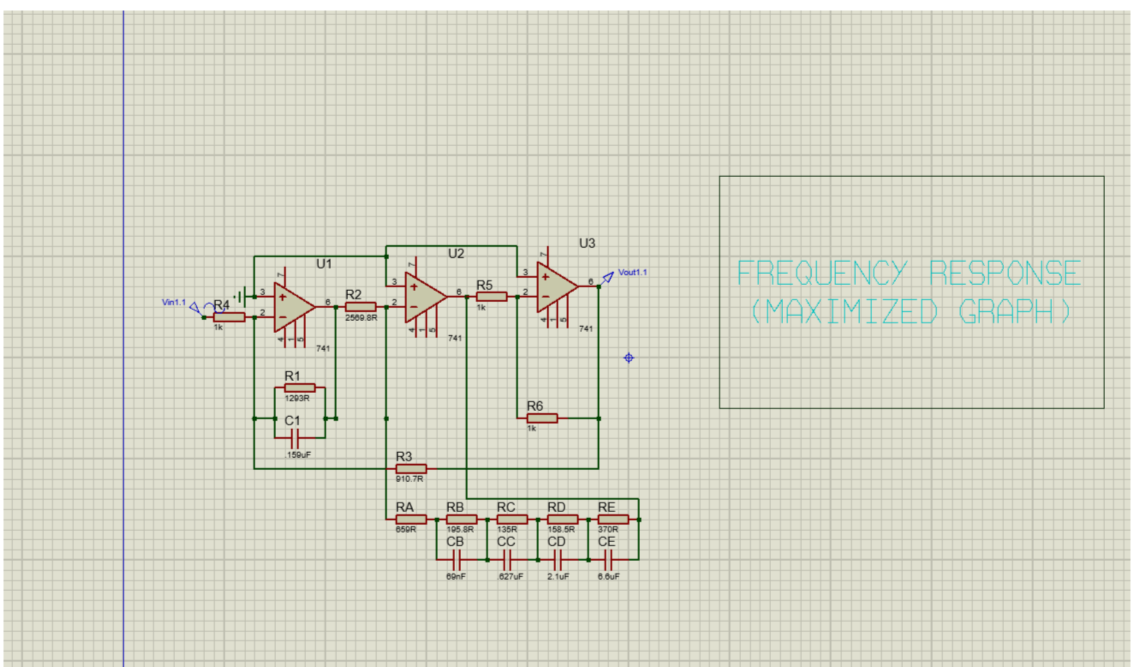

Figure 4.7 Frequency response

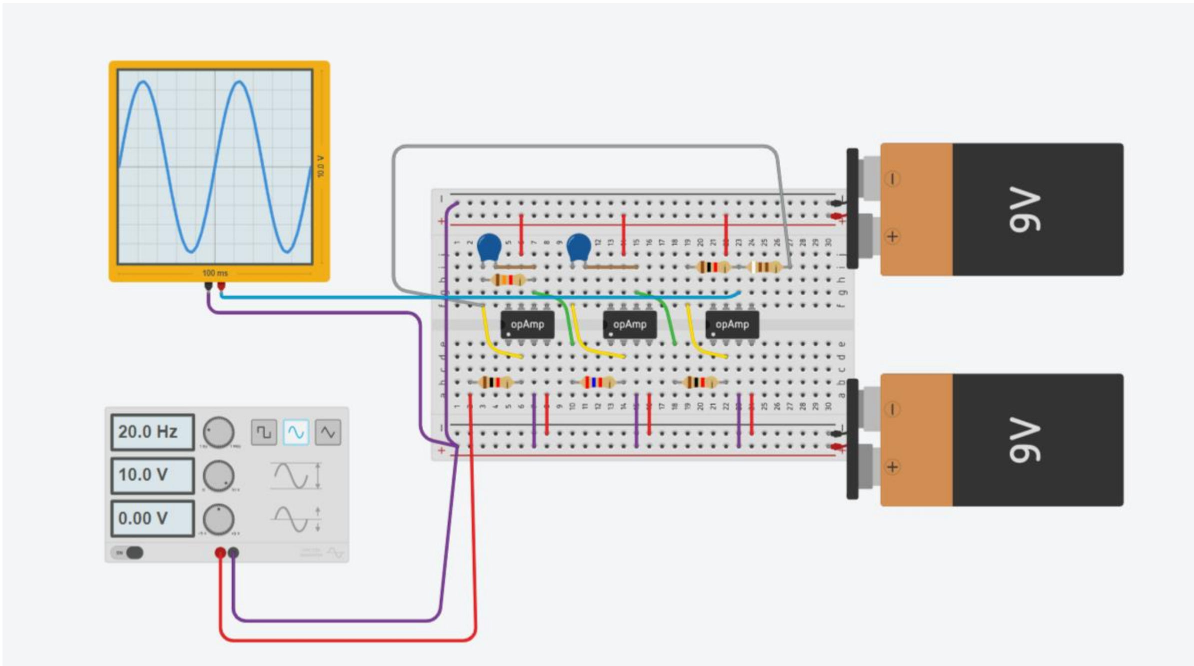

Figure 4.8 Tow Thomas Biquad topology

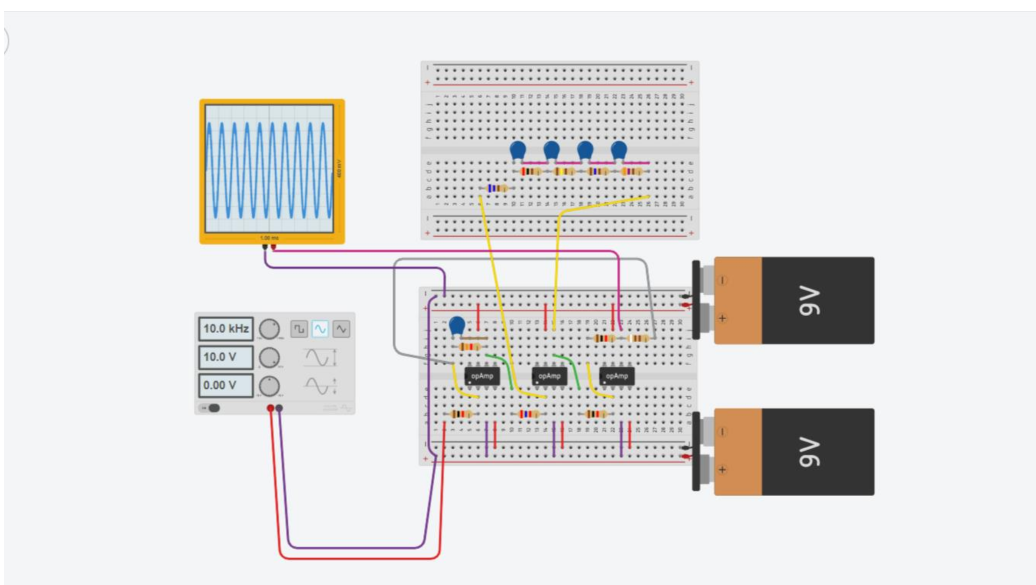

Figure 4.9 Fractinal Order Filter

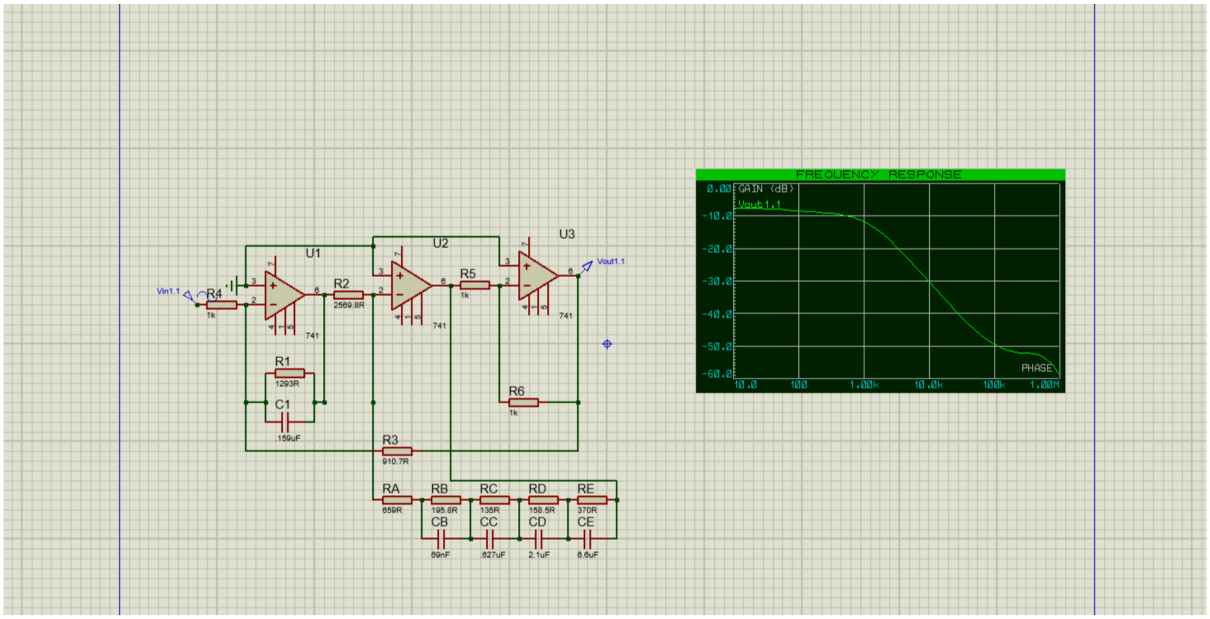

Figure 4.10 1.1 Order Tow Thomas biquad

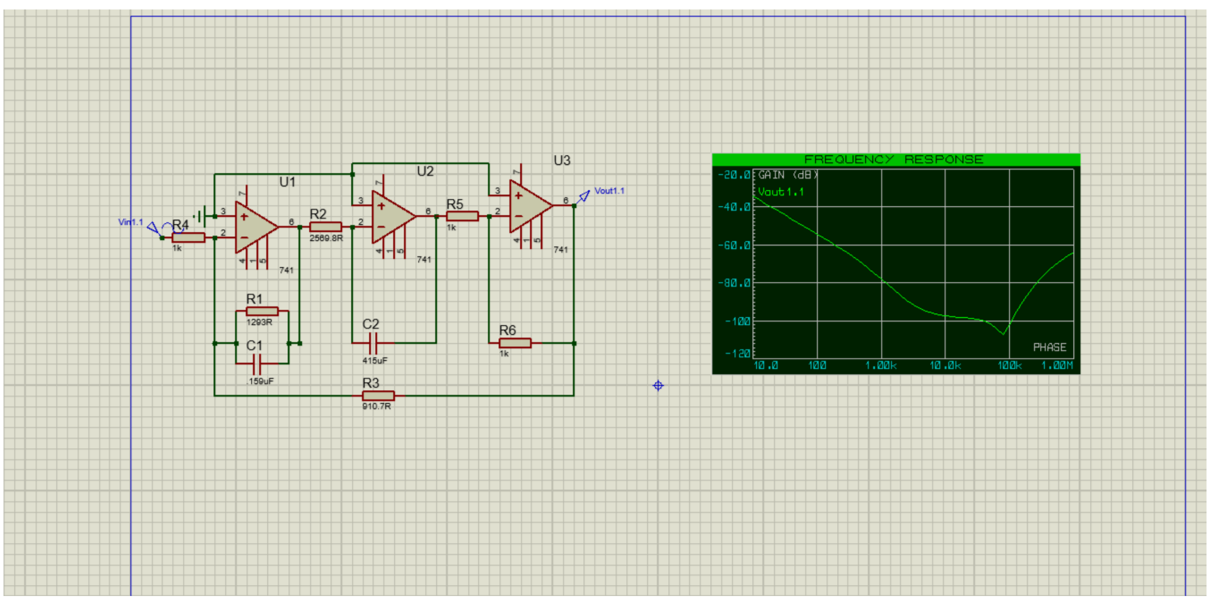

Figure 4.11 1.1 order Modified Fractional Order Fiter

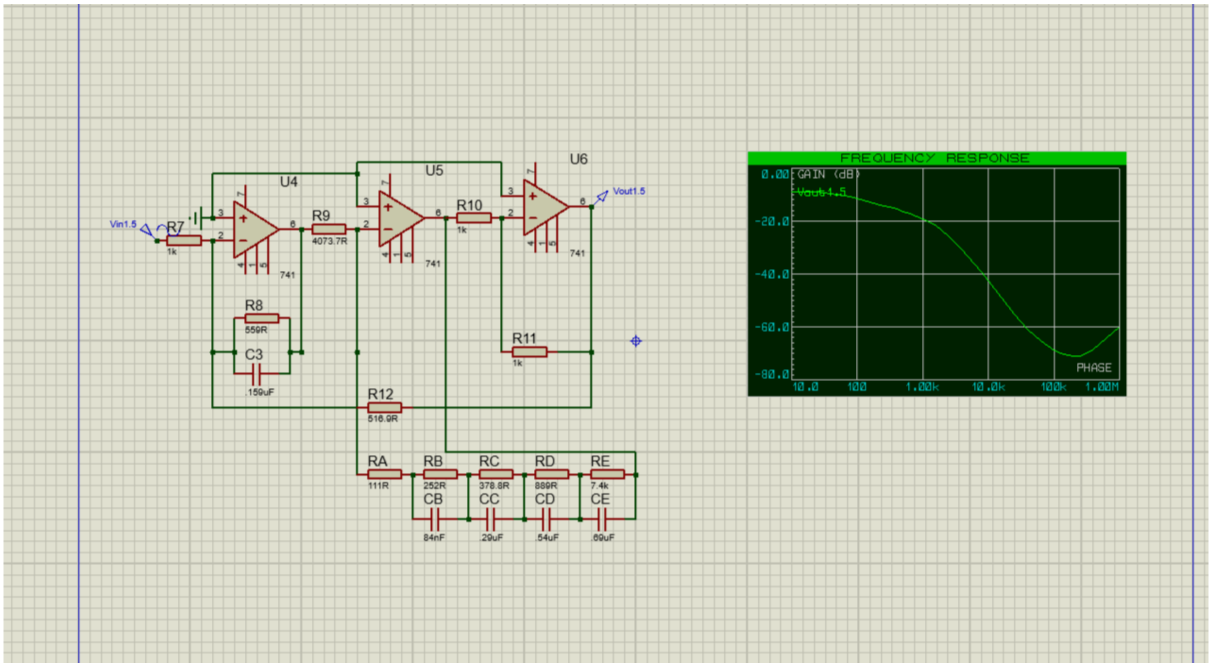

Figure 4.12 1.5 Order Tow Thomas biquad

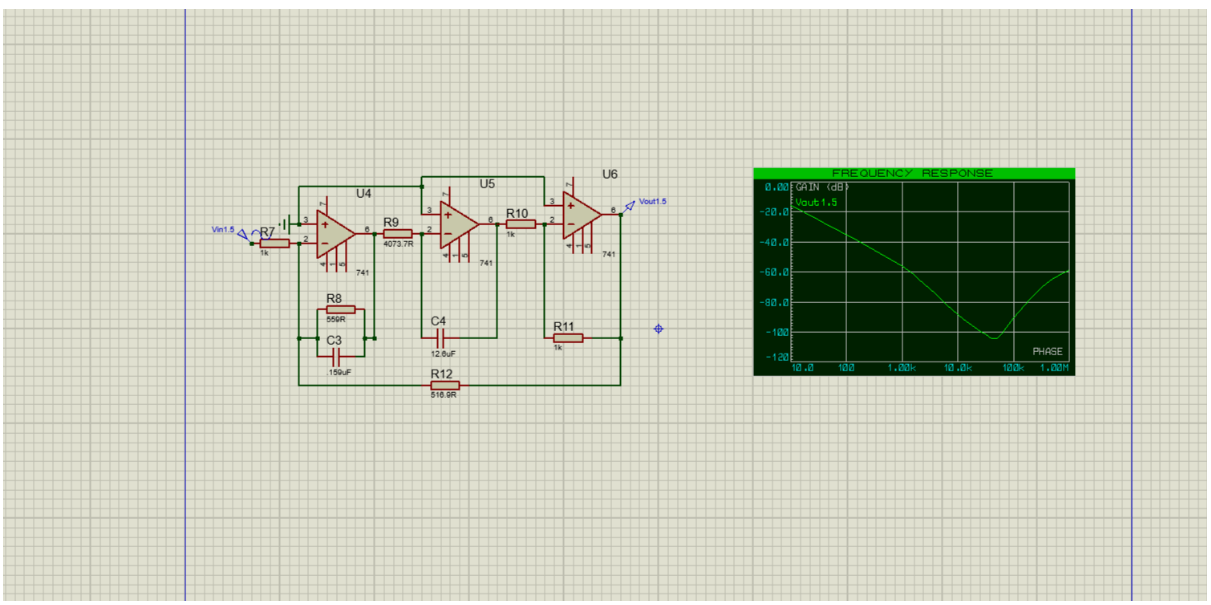

Figure 4.13 1.5 Order Modified Fractional Order Filter

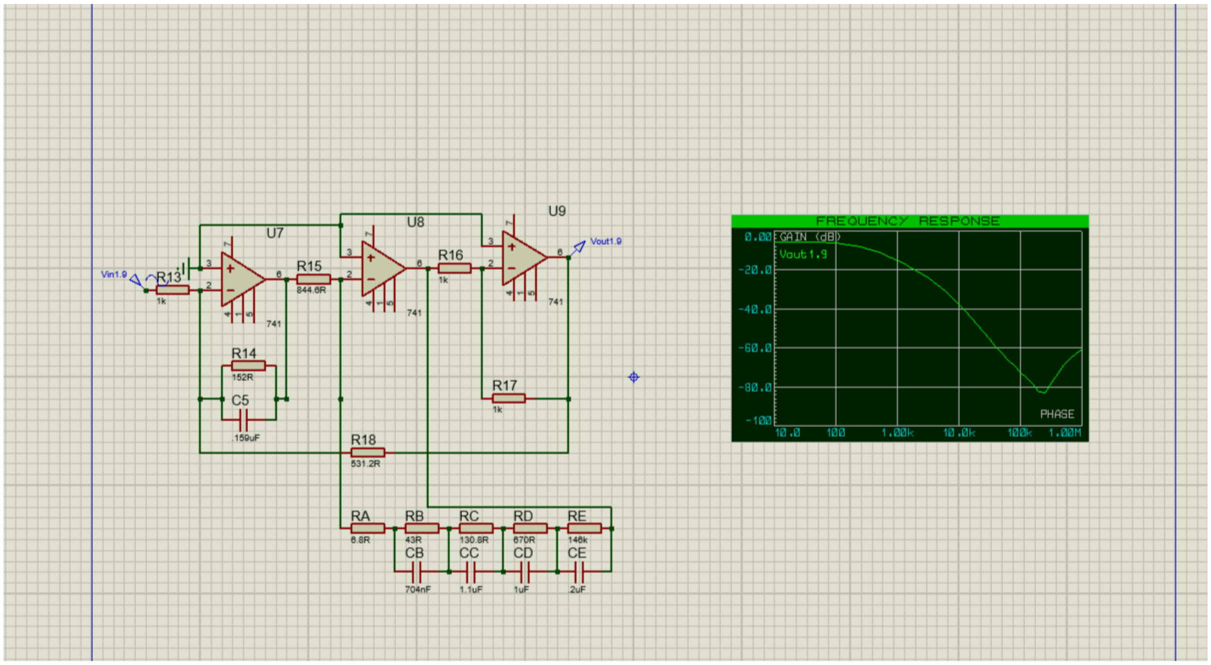

Figure 4.14 1.9 Order Tow Thomas biquad

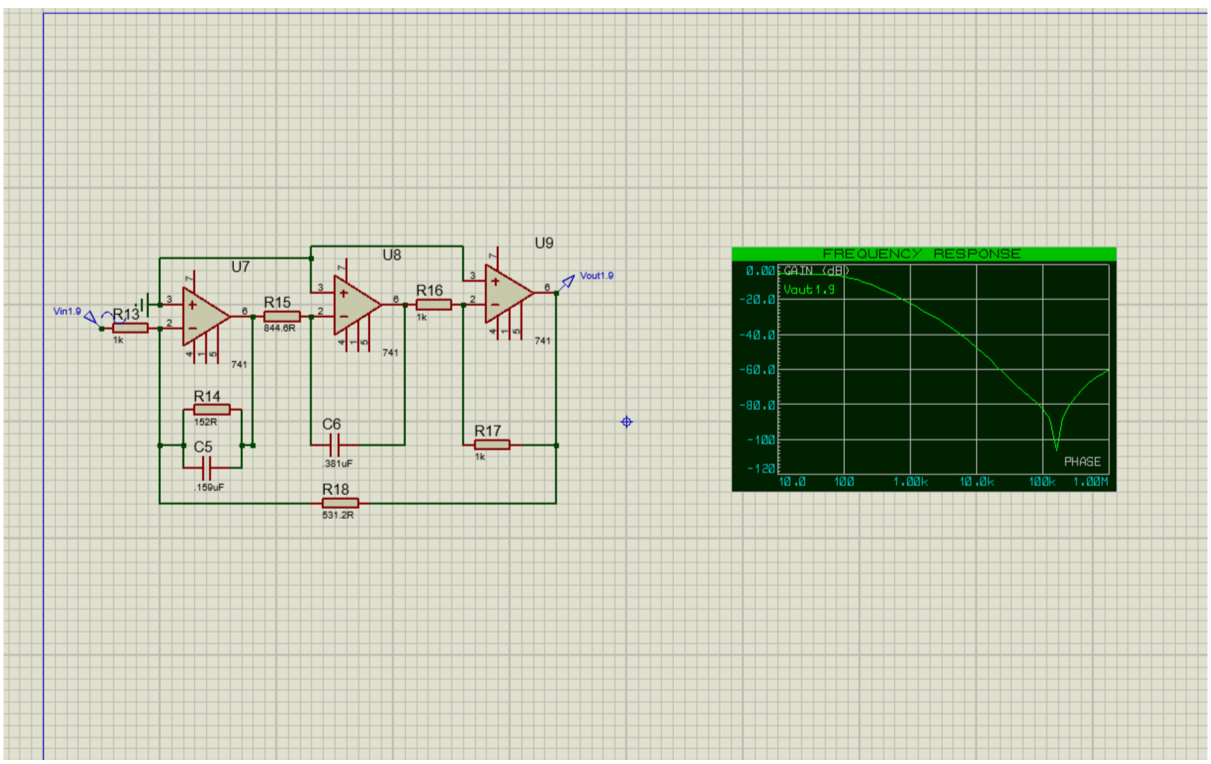

Figure 4.15 1.9 Order Modified Fractional Order Fiter

5. CONCLUSION

Improved designs of fractional order low pass filters of order 1.1, 1.5 and 1.9 are proposed by using simulated annealing, and suitable scaling optimization techniques.

. The simulated magnitude response obtained using proteus is analyzed and verified at circuit level. The optimized results for the order 1.1, 1.5 and 1.9 are designed and verified

Results show significant improvement near the cut off frequency in the passband and slight improvement in the stopband region of designed filters as compared to the existing ones. The proposed designs are realized using the Tow-Thomas bi-quad topology and continued fraction expansion.

Finally, the Proteus simulated results are also shown to show the effectiveness of the proposed fractional order low pass filter with Butterworth approximation.

Therefore, these maximally flat low pass filter circuits may replace the existing filter circuits for applications in signal processing.

REFERENCES

[1] T. J. Freeborn, "Comparison of (1+α) fractional-order transfer functions to approximate low pass Butterworth magnitude responses," Circuits Syst. Signal Process, vol. 35, pp. 1983-2002, 2016.

[2] A. G. Radwan and M. E. Fouda, "Optimization of fractional-order RLC filters," Circuits Syst Signal Process, vol. 32, pp. 2097-2118, 2013.

[3] L. A. Said, S. M. Ismail, A. G. Radwan, et al., "On the optimization of fractional order low pass filters," Circuits Syst. Signal Process, vol. 35, pp. 2017-2039, 2016.