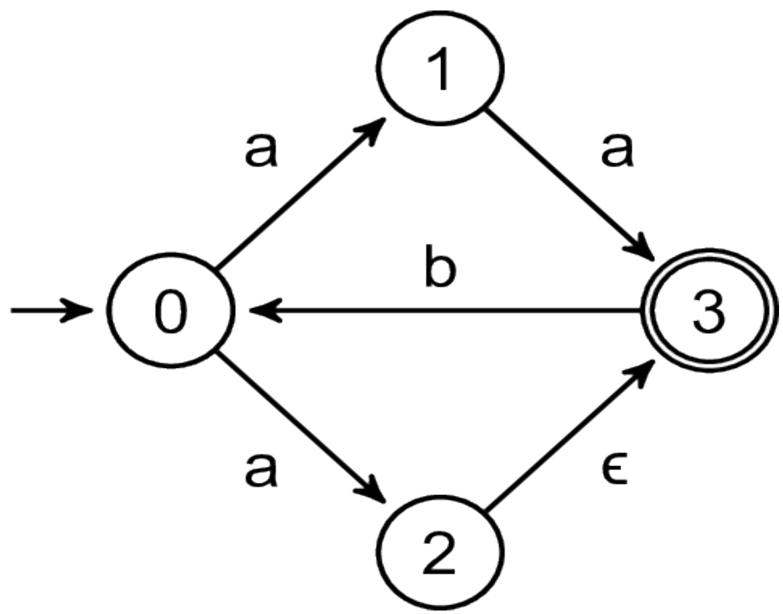
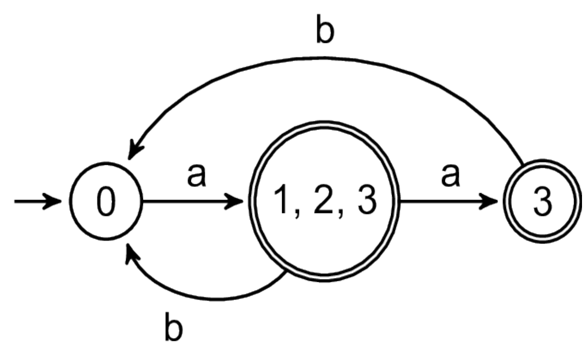
NFA → DFA Practice





Analyzing the reduction (cont'd)

- Can reduce any NFA to a DFA using subset alg.
- How many states in the DFA?
  - Each DFA state is a subset of the set of NFA states
  - Given NFA with n states, DFA may have 2^n states
    - Since a set with n items may have 2^n subsets
  - Corollary
    - Reducing a NFA with n states may be $O(2^n)$

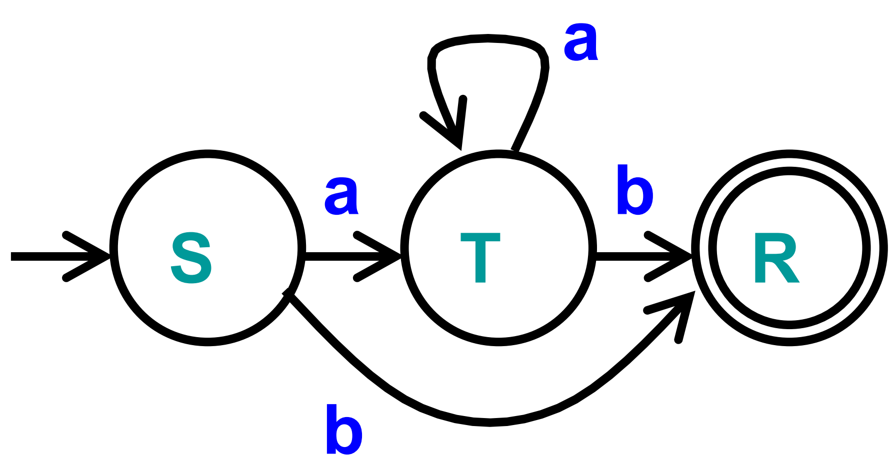
J. Hopcroft, “An n log n algorithm for minimizing states in a finite automaton,” 1971

Minimizing DFA: Hopcroft Reduction

- Intuition
  - Look to distinguish states from each other
    - End up in different accept / non-accept state with identical input
- Algorithm
  - Construct initial partition
    - Accepting & non-accepting states
  - Iteratively refine partitions (until partitions remain fixed)
    - Split a partition if members in partition have transitions to different partitions for same input
      - Two states x, y belong in same partition if and only if for all symbols in Σ they transition to the same partition
  - Update transitions & remove dead states

Minimizing DFA: Example 1

- DFA

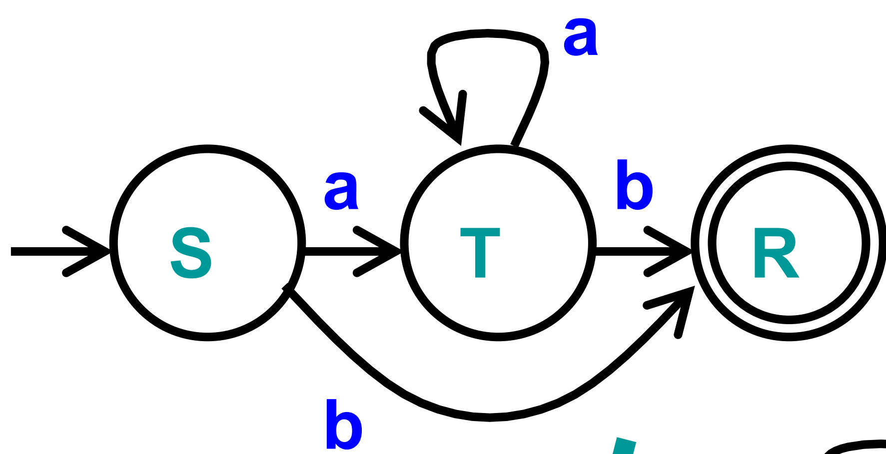


- Initial partitions

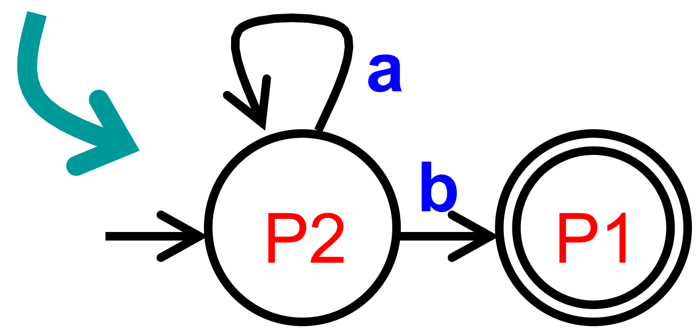
- Split partition

Minimizing DFA: Example 1

- DFA

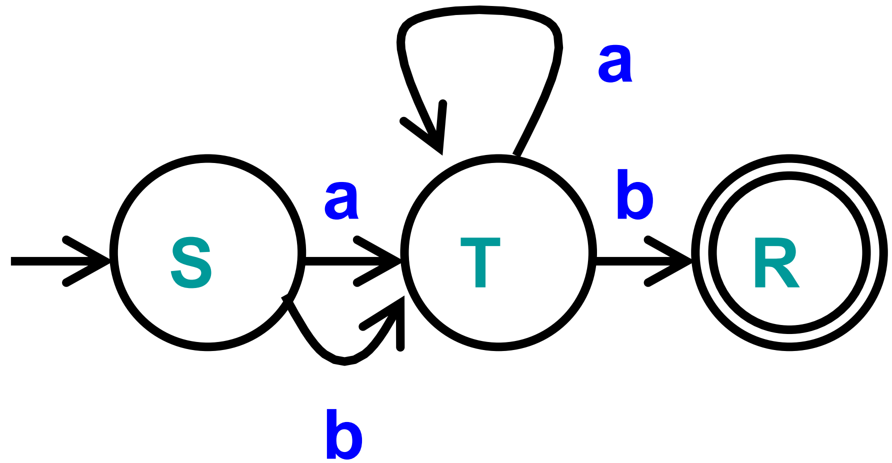


- Initial partitions
  - Accept { R } = P1
  - Reject { S, T } = P2



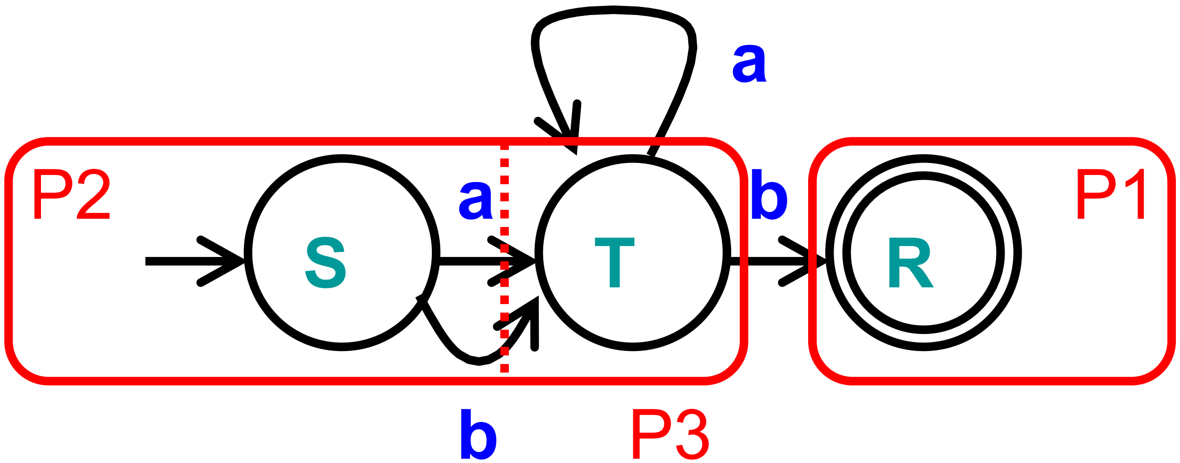
- Split partition? → Not required, minimization done
  - move(S,a) = T ∈ P2
    - move(S,b) = R ∈ P1
  - move(T,a) = T ∈ P2
    - move (T,b) = R ∈ P1

Minimizing DFA: Example 3



Minimizing DFA: Example 3

- DFA

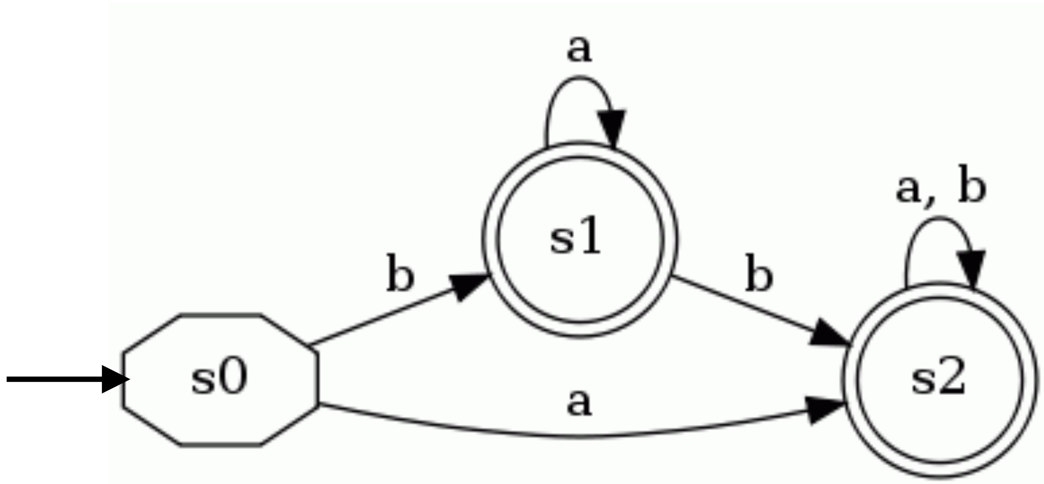


- Initial partitions
  - Accept { R } = P1
  - Reject { S, T } = P2

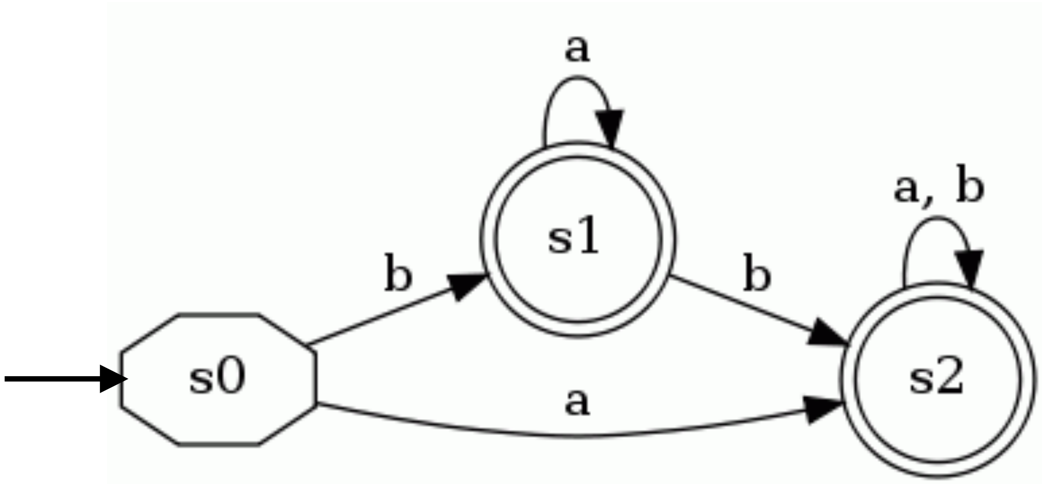
DFA already minimal

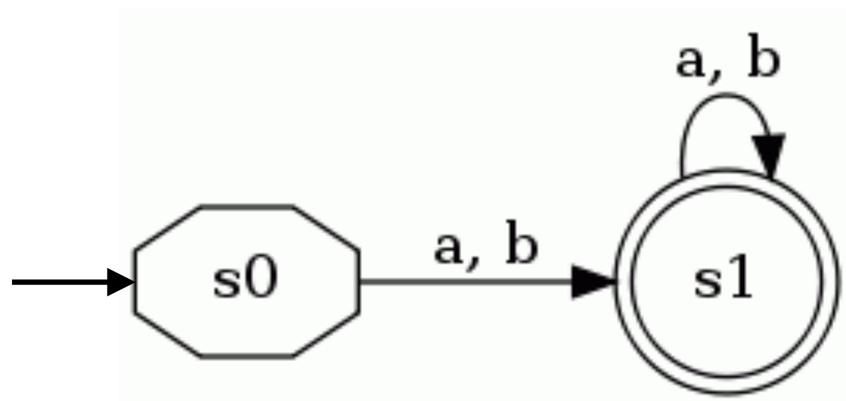
- Split partition? → Yes, different partitions for B
  - move(S,a) = T ∈ P2
    - move(S,b) = T ∈ P2
  - move(T,a) = T ∈ P2
    - move (T,b) = R ∈ P1

Minimizing DFA: Example 3



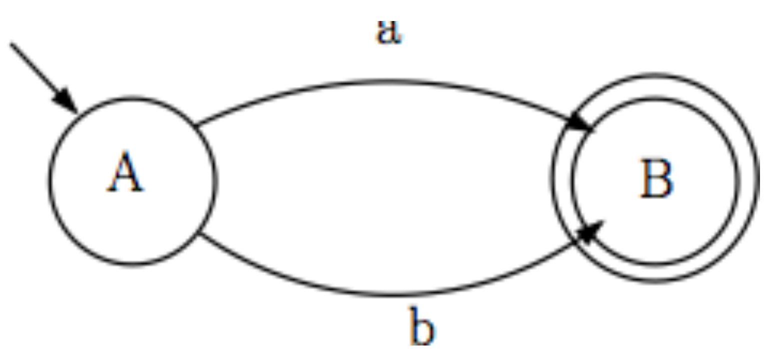
Minimizing DFA: Example 3





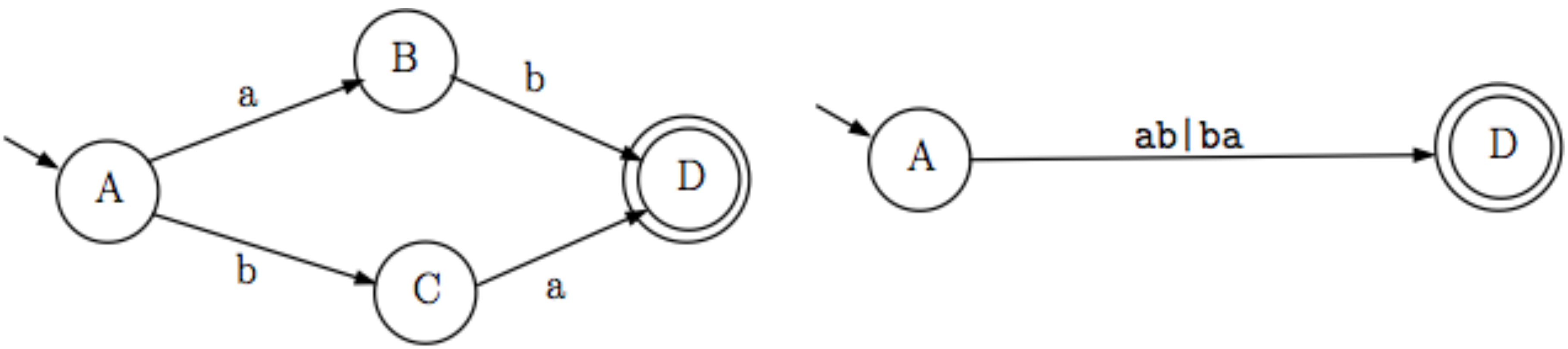
Complement of DFA

- Given a DFA accepting language L
  - How can we create a DFA accepting its complement?
  - Example DFA
    - $\Sigma = \{a,b\}$



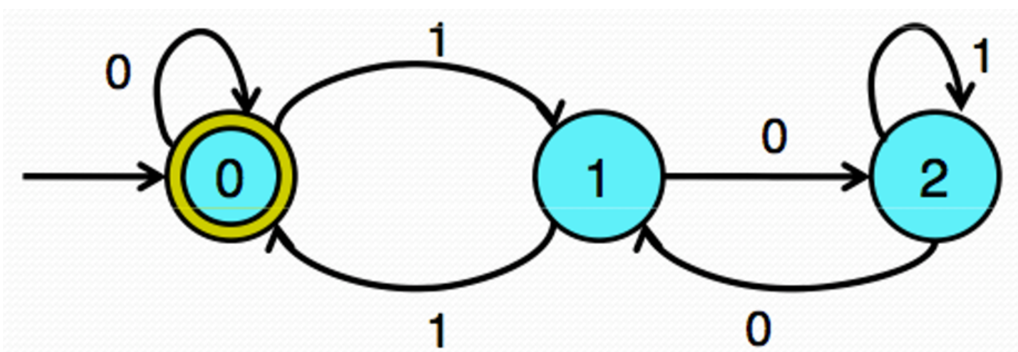
Reducing DFAs to REs

- General idea
  - Remove states one by one, labeling transitions with regular expressions
  - When two states are left (start and final), the transition label is the regular expression for the DFA



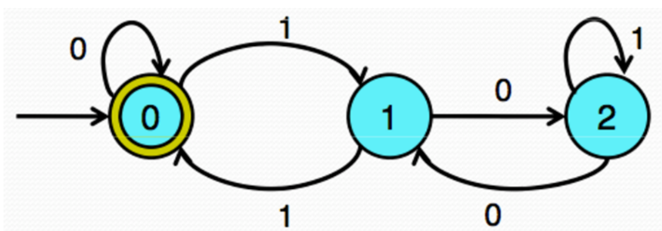
DFA to RE example

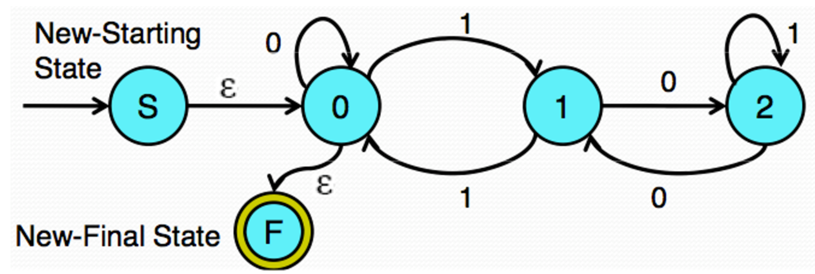
Language over $\Sigma = \{0,1\}$ such that every string is a multiple of 3 in binary

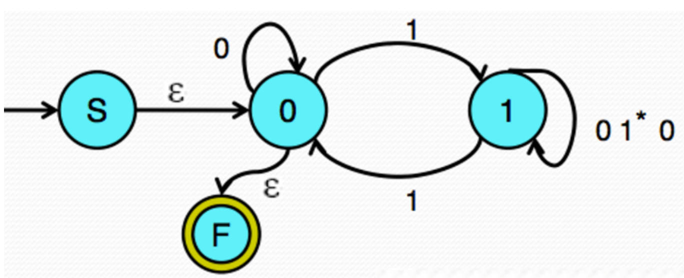


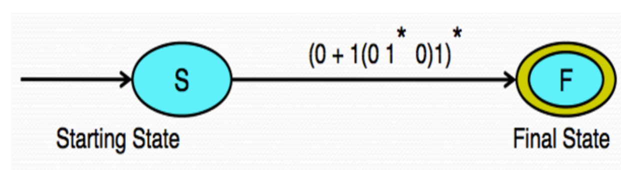
DFA to RE example

Language over $\Sigma = \{0,1\}$ such that every string is a multiple of 3 in binary









$$(0 + 1(0\,1^{*}\,0)1)^{*}$$

Run Time of DFA

- How long for DFA to decide to accept/reject string s?
  - Assume we can compute $\delta(q, c)$ in constant time
  - Then time to process s is $O(|s|)$
    - Can't get much faster!
- Constructing DFA for RE A may take $O(2^{|A|})$ time
  - But usually not the case in practice
- So there's the initial overhead
  - But then processing strings is fast

Summary of Regular Expression Theory

- Finite automata
  - DFA, NFA
- Equivalence of RE, NFA, DFA
  - RE $\rightarrow$ NFA
    - Concatenation, union, closure
  - NFA $\rightarrow$ DFA
    - ε-closure & subset algorithm
- DFA
  - Minimization, complement
  - Implementation