Assignment-9

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1 Question 1

Code for R

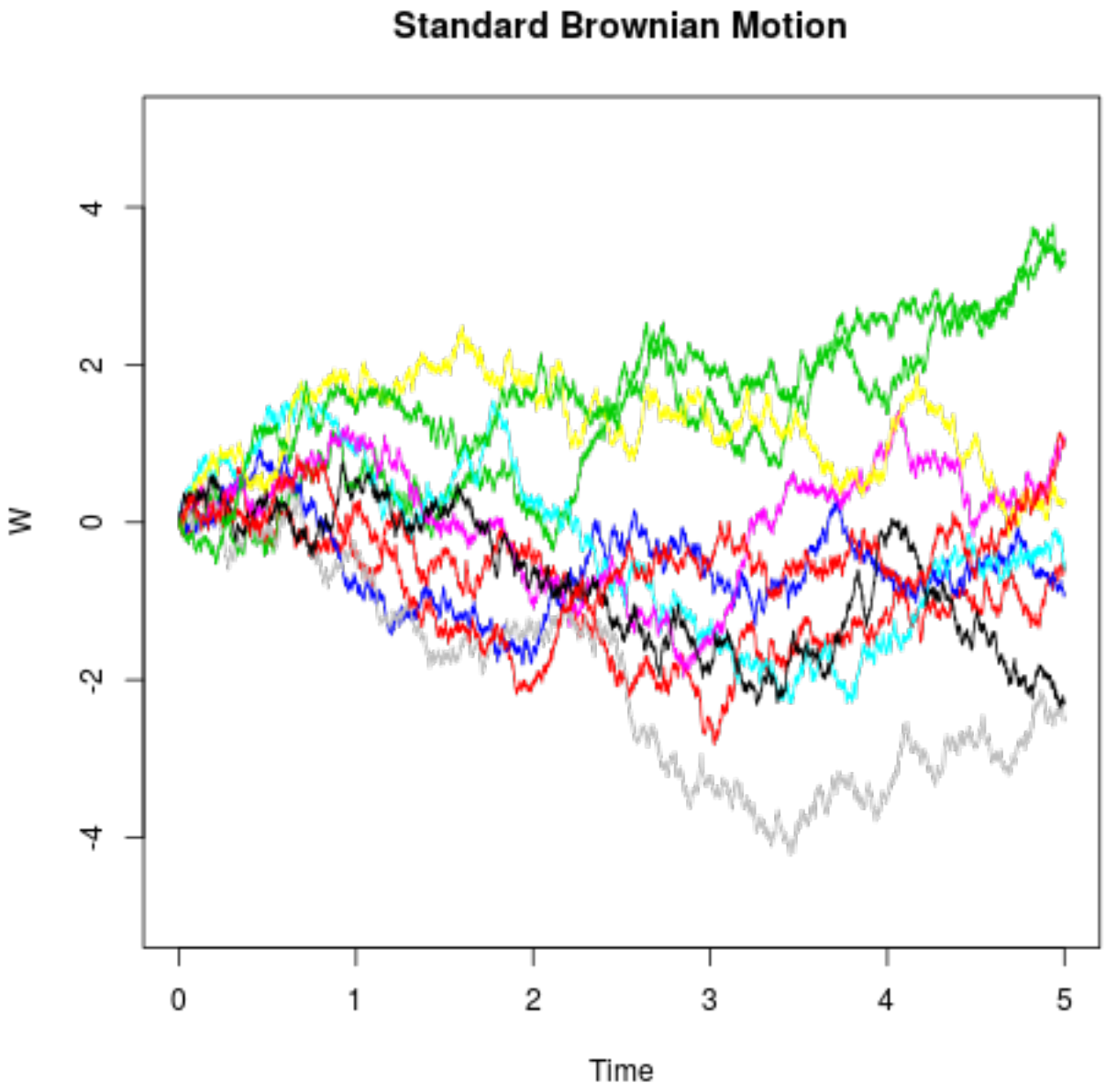
```
m <- 10  # No. of paths
n <- 5000   # No of time points
t <- 5
dt <- t/n

w2 <- vector(,m)
w5 <- vector(,m)
T <- seq(0, t, dt)
pal <- palette()

for (i in 1:m) {
   Z <- rnorm(n)
   W <- cumsum(c(0, dt^(1/2)*Z))
   w2[i] <- W[n*(2/t) + 1]
   w5[i] <- W[n + 1]
   if(i == 1) {
      plot(T, W, ylim=c(-5, 5), col=pal[i %% 8 + 1], cex=0.0001, main="Standard Brownian
          Motion", xlab="Time", ylab="W", type="l")
   } else {
      lines(T, W, col=pal[i %% 8 + 1], cex=0.0001)
   }
}

dev.copy(png,"plot1.png");
dev.off ();

cat(" E[W(2)] = ",mean(w2),"\n")
cat(" E[W(5)] = ",mean(w5),"\n")

rm(list = ls())
```

Standard Brownian Motion

t = 5

E[W(2)] = -0.3271348

E[W(5)] = 0.2187029



2 Question 2

Code for R

```
m <- 10  # No. of paths
n <- 5000   # No of time points
t <- 5
dt <- t/n
mu <- 0.06
sigma <- 0.3
w0 <- 5

w2 <- vector(,m)
w5 <- vector(,m)
T <- seq(0, t, dt)
pal <- palette()

for (i in 1:m) {
   Z <- rnorm(n)
   X <- cumsum(c(w0, mu*dt + sigma*dt^(1/2)*Z))
   w2[i] <- X[n*(2/t) + 1]
   w5[i] <- X[n + 1]
   if(i == 1) {
      plot(T, X, ylim=c(3.5, 7), col=pal[i %% 8 + 1], cex=0.0001, main="General Brownian
          Motion, mu = 0.06, sigma = 0.3", xlab="Time", ylab="X", type="l")
   } else {
      lines(T, X, col=pal[i %% 8 + 1], cex=0.0001)
   }
}

dev.copy(png,"plot2.png");
dev.off ();

cat(" E[X(2)] = ",mean(w2),"\n")
cat(" E[X(5)] = ",mean(w5),"\n")

rm(list = ls())
```

General Brownian Motion

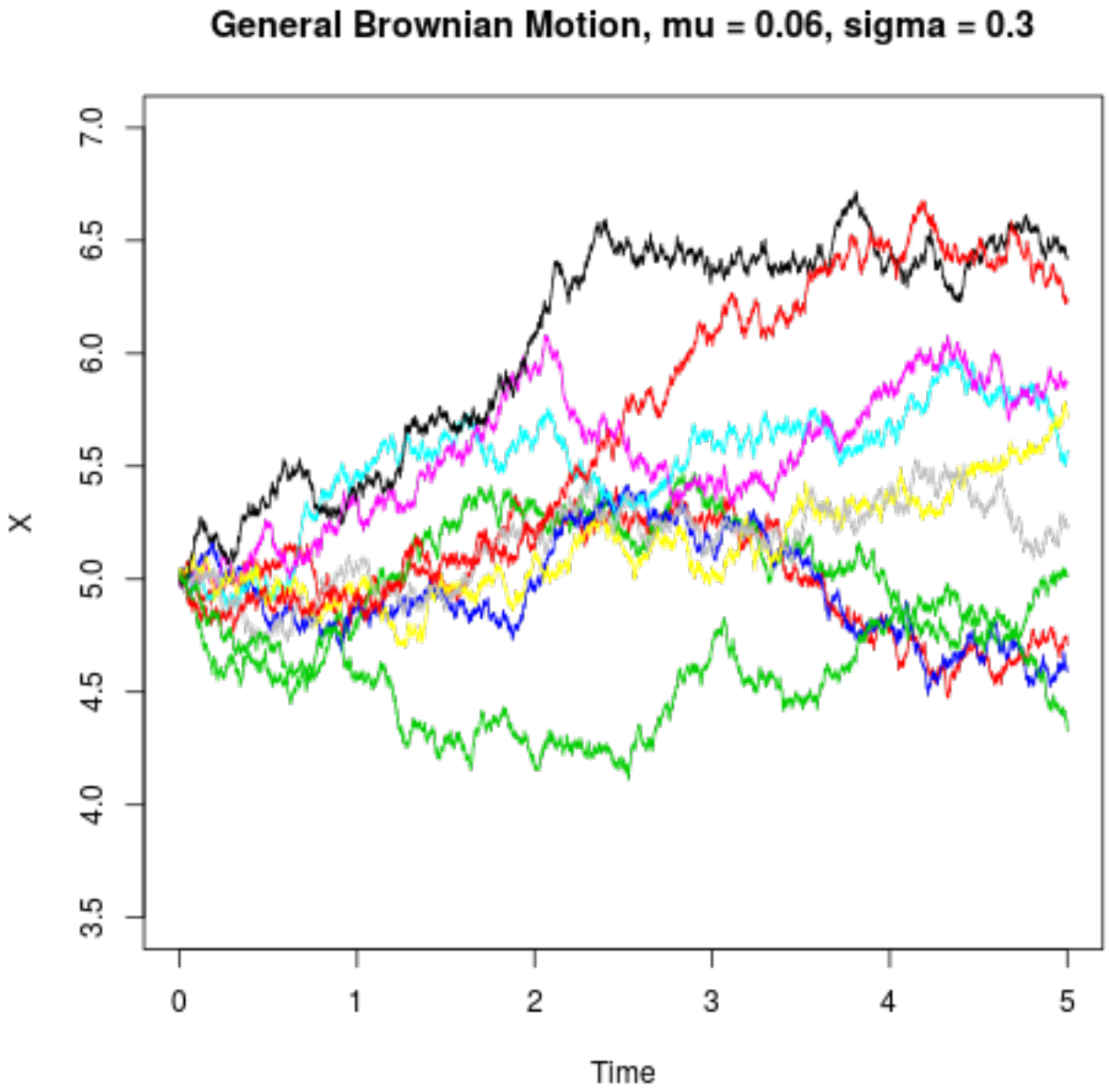
mu = 0.06

sigma = 0.3

t = 5

E[X(2)] = 5.279188

E[X(5)] = 5.369947


General Brownian Motion, mu = 0.06, sigma = 0.3
X
7.0
6.5
6.0
5.5
5.0
4.5
4.0
3.5
0
1
2
3
4
5
Time

3 Question 3

Code for R

```
m <- 10  # No. of paths
n <- 5000   # No of time points
t <- 5
dt <- t/n

w2 <- vector(,m)
w5 <- vector(,m)
T <- seq(0, t, dt)
pal <- palette()
y0 <- 5

Mu <- 0.0325 - 0.05*T[2:(n+1)]
Sigma <- 0.012 + 0.0138*T[2:(n+1)] + 0.00125*(T[2:(n+1)]^2)

for (i in 1:m) {
   Z <- rnorm(n)
   Y <- cumsum(c(y0, Mu*dt + Sigma*dt^(1/2)*Z))
   w2[i] <- Y[n*(2/t) + 1]
   w5[i] <- Y[n + 1]
   if(i == 1) {
      plot(T, Y, ylim=c(4.3, 5.1), col=pal[i %% 8 + 1], cex=0.0001, main="Euler Approximation"
          , xlab="Time", ylab="Y", type="l")
   } else {
      lines(T, Y, col=pal[i %% 8 + 1], cex=0.0001)
   }
}

dev.copy(png,"plot3.png");
dev.off ();

cat(" E[Y(2)] = ",mean(w2),"\n")
cat(" E[Y(5)] = ",mean(w5),"\n")

rm(list = ls())
```

Euler approximation

t = 5

Mu = 0.0325 - 0.05*t

Sigma = 0.012 + 0.0138*t + 0.00125*t

E[Y(2)] = 4.980067

E[Y(5)] = 4.541004