Assignment-8

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1 Question 1

Code for R

```
m <- 100
z <- 1.96    #for 95 \% confidence interval

while (m<=100000) {
   U <- runif(m)
   Y <- exp(U^(1/2))
   I <- mean(Y)
   SD <- sd(Y)
   cat("m = ", m,"\n")
   cat("Expected value, I = ", I, "\n")
   cat("Variance = ", SD^2, "\n")
   cat("95 \% confidence interval = (", I - z*SD/(m^(1/2)), ", ",I + z*SD/(m^(1/2)), ")\n\n")
   m <- m*10
}

rm(list = ls())
```

Using standard method

m = 100

Expected value, I = 2.046535

Variance = 0.182022

95 % confidence interval = (1.962914 , 2.130157)

m = 1000

Expected value, I = 1.999513

Variance = 0.2015493

95 % confidence interval = (1.971687 , 2.027339)

m = 10000

Expected value, I = 2.006749

Variance = 0.1921842

95 % confidence interval = (1.998157 , 2.015341)

m = 1e+05

Expected value, I = 2.00195

Variance = 0.1951535

95 % confidence interval = (1.999211 , 2.004688)

2 Question 2

Code for R

```
m <- 100
z <- 1.96   #for 95 \% confidence interval

while (m<=100000) {
   U <- runif(m)
   Y1 <- exp(U^(1/2))
   Y2 <- exp((1-U)^(1/2))
   Y <- (Y1 + Y2)/2
   I <- mean(Y)
   std_dev <- sd(Y)
   cat("m = ", m,"\n")
   cat("Expected value, I = ", I, "\n")
   cat("Variance = ", std_dev^2, "\n")
   cat("95 \% confidence interval = (", I - z*std_dev/(m^(1/2)), ", ",I + z*std_dev/(m^(1/2)),
       ")\n")
   cat("Variance reduction = ", 100*(1 - var(Y)/var(Y1)) ,"%\n\n")
   m <- m*10
}

rm(list = ls())
```

Using antithetic variate

m = 100

Expected value, I = 2.004638

Variance = 0.0008439335

95 % confidence interval = (1.998944 , 2.010332)

Variance reduction = 99.50857 %

m = 1000

Expected value, I = 2.001343

Variance = 0.0009813989

95 % confidence interval = (1.999401 , 2.003284)

Variance reduction = 99.47258 %

m = 10000

Expected value, I = 1.999538

Variance = 0.001085473

95 % confidence interval = (1.998892 , 2.000184)

Variance reduction = 99.44871 %

m = 1e+05

Expected value, I = 1.999922

Variance = 0.001080687

95 % confidence interval = (1.999718 , 2.000126)

Variance reduction = 99.44492 %

3 Question 3

Code for R

```
m <- 100
z <- 1.96   #for 95 \% confidence interval

while (m<=100000) {
   U <- runif(m)
   Y1 <- exp(U^(1/2))

   X <- U
   X <- X^(1/2)
   c <- -cov(X,Y1)/var(X)
   mean_x <- mean(X)
   Y <- Y1 + c*(X - mean_x)
   I <- mean(Y)
   std_dev <- sd(Y)
   cat("m = ", m,"\n")
   cat("Expected value, I = ", I, "\n")
   cat("Variance = ", std_dev^2, "\n")
   cat("95 \% confidence interval = (", I - z*std_dev/(m^(1/2)), ", ",I + z*std_dev/(m^(1/2)),
       ")\n")
   cat("Variance reduction = ", 100*(1 - var(Y)/var(Y1)) ,"%\n\n")
   m <- m*10
}

rm(list = ls())
```

m = 100

Expected value, I = 1.941239

Variance = 0.002987869

95 % confidence interval = (1.930525 , 1.951952)

Variance reduction = 98.49517 %

m = 1000

Expected value, I = 1.992915

Variance = 0.00266213

95 % confidence interval = (1.989718 , 1.996113)

Variance reduction = 98.58274 %

m = 10000

Expected value, I = 2.011091

Variance = 0.002700158

95 % confidence interval = (2.010072 , 2.012109)

Variance reduction = 98.61463 %

m = 1e+05

Expected value, I = 2.000766

Variance = 0.002692681

95 % confidence interval = (2.000444 , 2.001088)

Variance reduction = 98.61088 %