Assignment-2

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A custom header file with some functions used in both question. Code for C++

```
#include <iostream>
#include <iomanip>

using namespace std;

long base_generator(long a, long x, long q, long r, long m) {
long k=x/q;
x = (a * (x - (k * q))) - (k * r);
while(x<0) x+=m;
return x;
}
```

1 Question 1

Code for C++

```
#include<iostream>
#include<iomanip>
#include"lab2.h"

using namespace std;

long generate(long a, long m, long x, long n, int range) {
double u;
unsigned long Density[range];
for(int i=0;i<range;++i) {
Density[i] = 0;
}
cout<<"Initial values, a = "<<a<<", m = "<<m<<", x_0 = "<<x<<", n = "<<n<<"\n";
long q=m/a, r=m%a, k;
for(int j=0;j<n;++j) {
u = (double)x/(double)m;
```

```
x = base_generator(a,x,q,r,m);
Density[(int)(u*range)]++ ;
cout<<"x"<<j<<"= "<<x<<"\tu"<<j<<"= "<<setprecision(10)<<u<<"\n";
}

cout<<"$$a = "<<a<<", m = "<<m<<", x_0 = "<<x<<", n = "<<n<<"$$\n";
cout<<"\n\\begin{center} \\begin{tabular}{||c | c||} \\hline\n";
cout<<"Range & Frequency \\\\ [0.5ex] \\hline \\hline";
u = (long double)1/(long double)range;
for(int i=0;i<range;++i) {
cout<<setprecision(2)<<(u*i)<<"-"<<(u*(i+1))<<" & "<<Density[i]<<"\\\\\n \\hline \n";
}
cout<<"\\end{tabular} \n \\end{center}\n";
return 0;
}

int plotdata(long a, long m, long x, long n) {
double u,v;
long q=m/a, r=m%a, k;
cout<<"Initial value, $x_0$ = "<<x<<"\n";
u = (long double)x/(long double)m;
for(int j=0;j<n;++j) {
x = base_generator(a,x,q,r,m);
v = u;
u = (long double)x/(long double)m;
cout<<"u"<<j<<"= "<<fixed<<setprecision(10)<<v<<"\tu"<<j+1<<"= "<<u<<"\n";
}
cout<<endl;
return 0;
}

int main() {
long x=1,a[] = {16807,40692,40014},m[] = {2147483647,2147483399,2147483563},n=1000;

for(int i=0;i<3;++i) {
for(n=1000;n<=100000; n=n*10)
generate(a[i],m[i],x,n,20);
}

plotdata(a[0],m[0],x,100000);
}
```

From the bar graphs we observe that the numbers generated are very random and distribute uniformly as we increase their count.

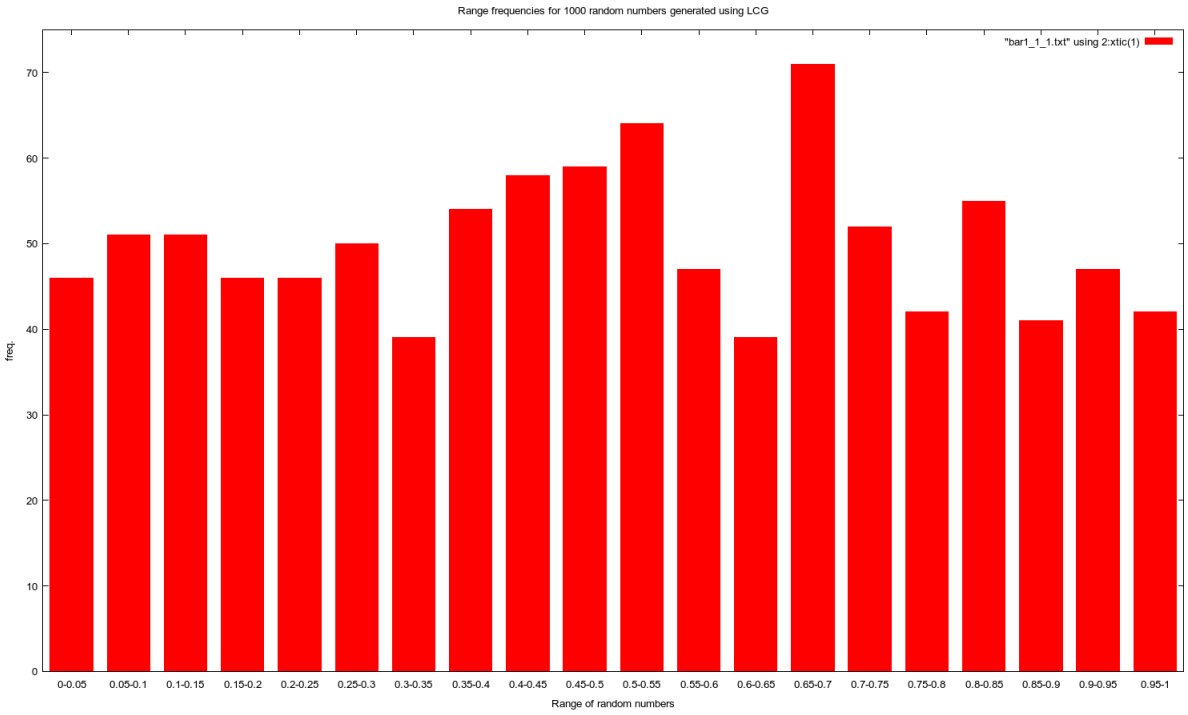
The autocorrelation between $u(i)$ and $u(i+1)$ is very low and they are not (or very less) dependent on each other as we see from the $u(i)$ vs $u(i+1)$ plot

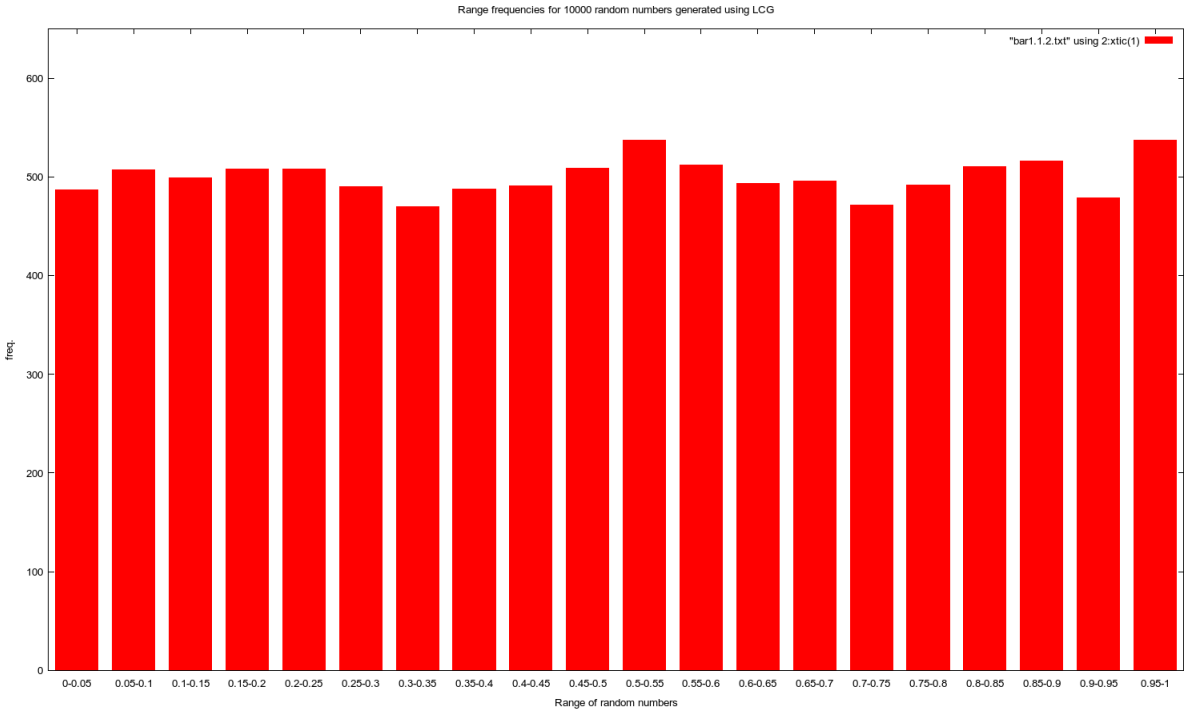
$a = 16807, m = 2147483647, x_0 = 522329230, n = 1000, 10000, 100000$

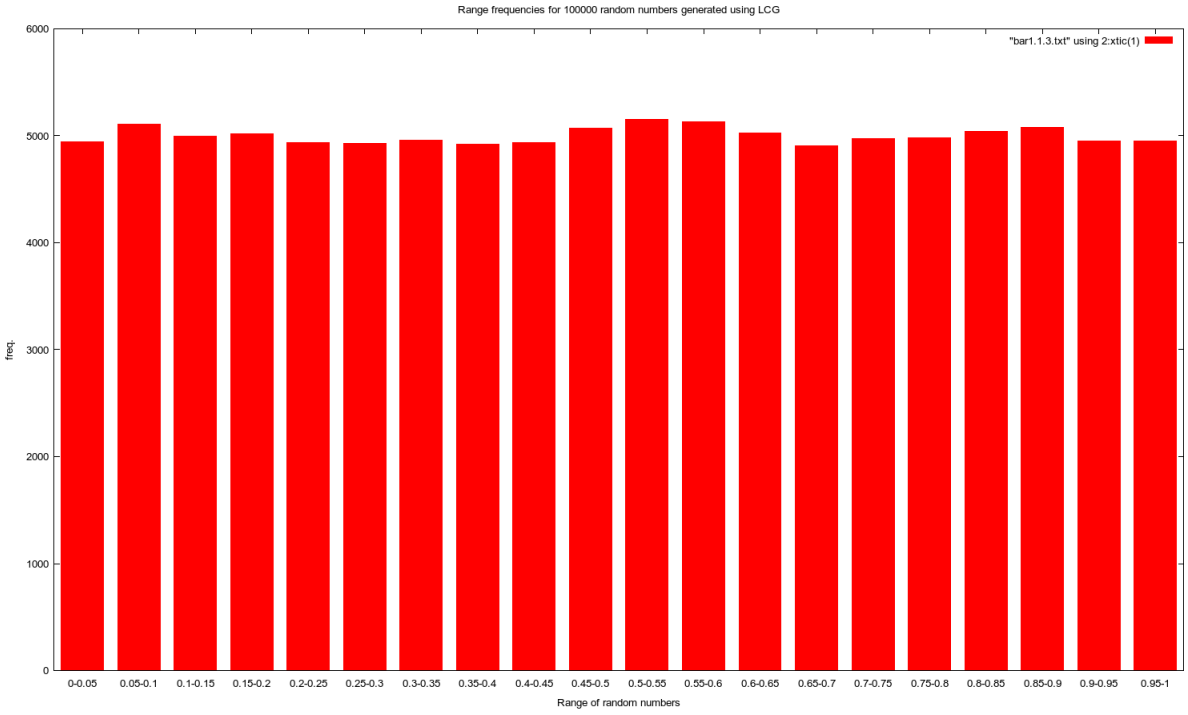
Range	Frequency
0-0.05	46
0.05-0.1	51
0.1-0.15	51
0.15-0.2	46
0.2-0.25	46
0.25-0.3	50
0.3-0.35	39
0.35-0.4	54
0.4-0.45	58
0.45-0.5	59
0.5-0.55	64
0.55-0.6	47
0.6-0.65	39
0.65-0.7	71
0.7-0.75	52
0.75-0.8	42
0.8-0.85	55
0.85-0.9	41
0.9-0.95	47
0.95-1	42

Range	Frequency
0-0.05	487
0.05-0.1	507
0.1-0.15	499
0.15-0.2	508
0.2-0.25	508
0.25-0.3	490
0.3-0.35	470
0.35-0.4	488
0.4-0.45	491
0.45-0.5	509
0.5-0.55	537
0.55-0.6	512
0.6-0.65	493
0.65-0.7	496
0.7-0.75	471
0.75-0.8	492
0.8-0.85	510
0.85-0.9	516
0.9-0.95	479
0.95-1	537

Range	Frequency
0-0.05	4940
0.05-0.1	5107
0.1-0.15	4999
0.15-0.2	5017
0.2-0.25	4934
0.25-0.3	4929
0.3-0.35	4959
0.35-0.4	4919
0.4-0.45	4938
0.45-0.5	5074
0.5-0.55	5152
0.55-0.6	5133
0.6-0.65	5024
0.65-0.7	4907
0.7-0.75	4976
0.75-0.8	4979
0.8-0.85	5040
0.85-0.9	5078
0.9-0.95	4948
0.95-1	4947




Range frequencies for 10000 random numbers generated using LCG
"bar1.1.2.txt" using 2:xtic(1)
freq.
0
100
200
300
400
500
600
0-0.05
0.05-0.1
0.1-0.15
0.15-0.2
0.2-0.25
0.25-0.3
0.3-0.35
0.35-0.4
0.4-0.45
0.45-0.5
0.5-0.55
0.55-0.6
0.6-0.65
0.65-0.7
0.7-0.75
0.75-0.8
0.8-0.85
0.85-0.9
0.9-0.95
0.95-1
Range of random numbers

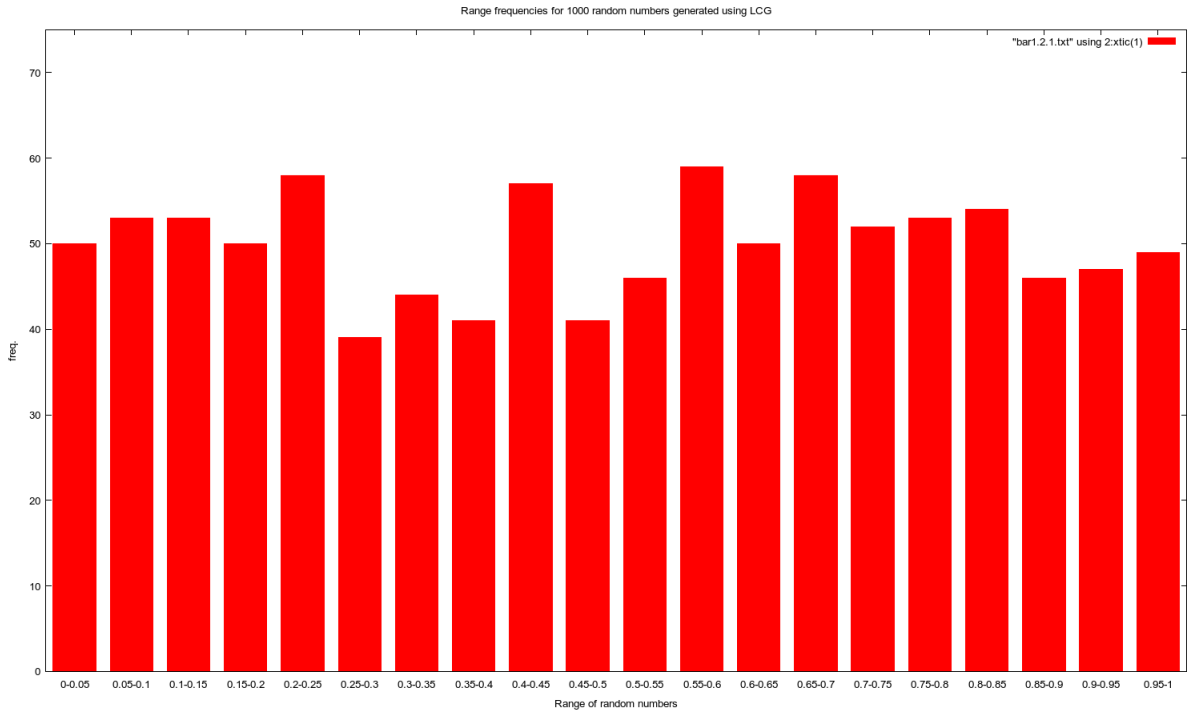

Range frequencies for 100000 random numbers generated using LCG
"bar1.1.3.txt" using 2:xtic(1)
freq.
0
1000
2000
3000
4000
5000
6000
0-0.05
0.05-0.1
0.1-0.15
0.15-0.2
0.2-0.25
0.25-0.3
0.3-0.35
0.35-0.4
0.4-0.45
0.45-0.5
0.5-0.55
0.55-0.6
0.6-0.65
0.65-0.7
0.7-0.75
0.75-0.8
0.8-0.85
0.85-0.9
0.9-0.95
0.95-1
Range of random numbers

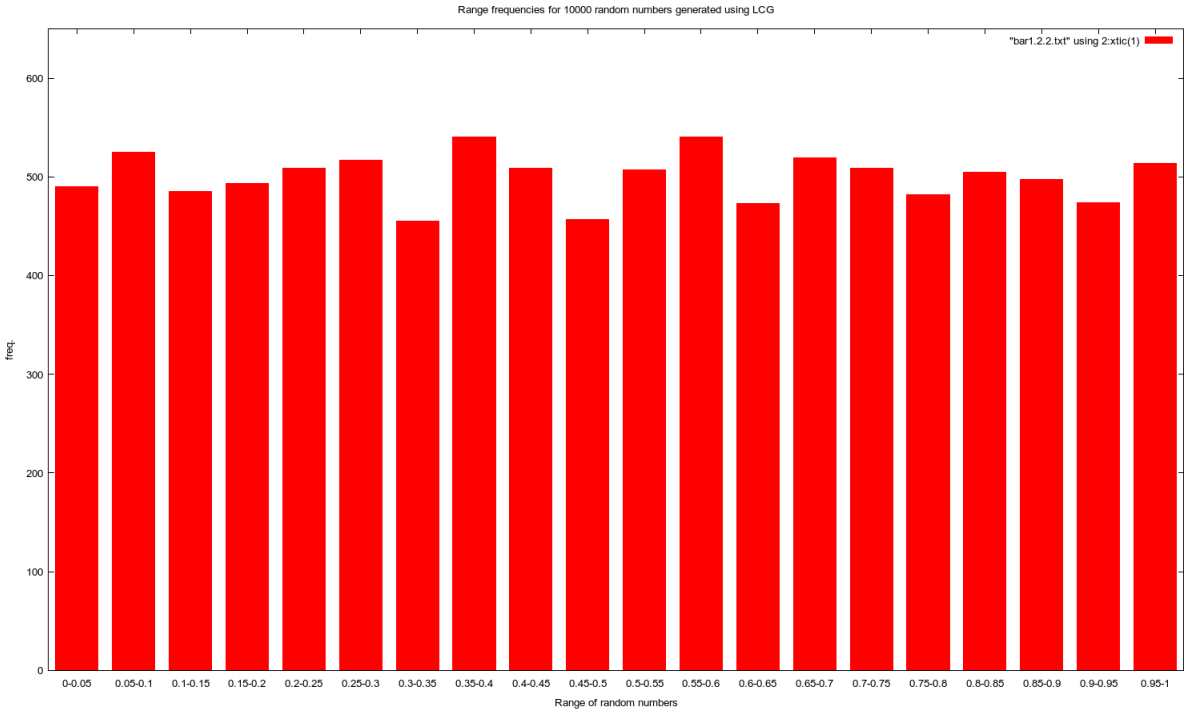
$a = 40692, m = 2147483399, x_0 = 2121278613, n = 1000, 10000, 100000$

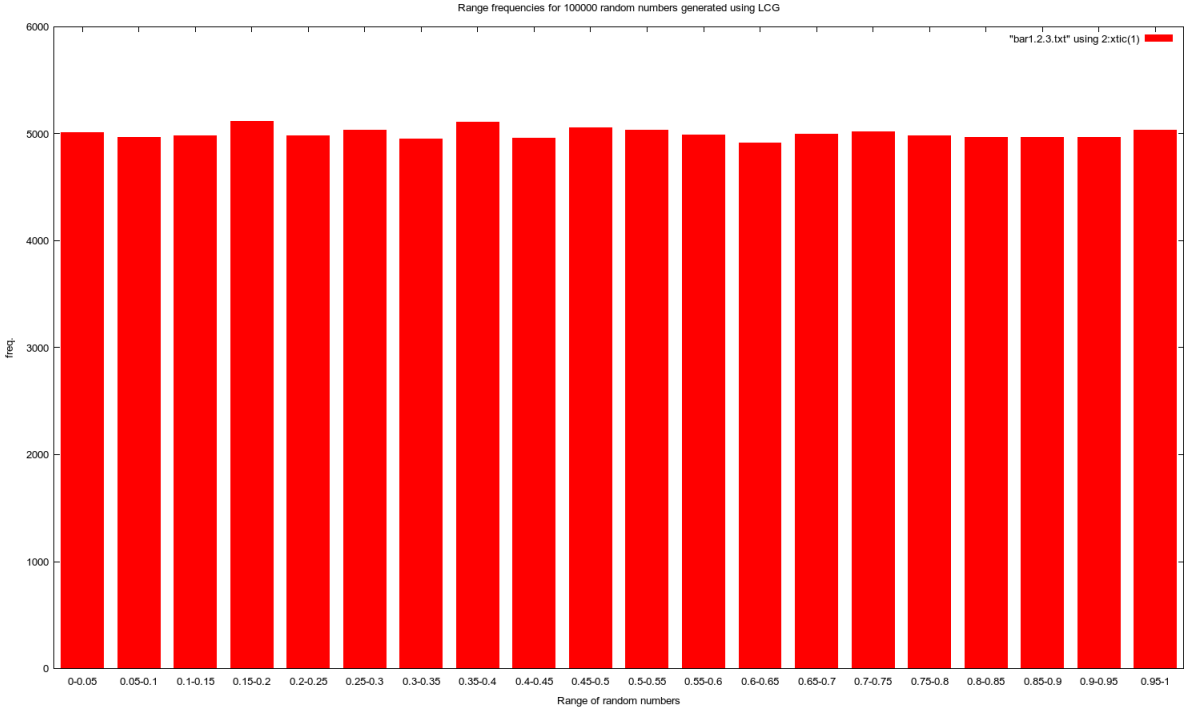
Range	Frequency
0-0.05	50
0.05-0.1	53
0.1-0.15	53
0.15-0.2	50
0.2-0.25	58
0.25-0.3	39
0.3-0.35	44
0.35-0.4	41
0.4-0.45	57
0.45-0.5	41
0.5-0.55	46
0.55-0.6	59
0.6-0.65	50
0.65-0.7	58
0.7-0.75	52
0.75-0.8	53
0.8-0.85	54
0.85-0.9	46
0.9-0.95	47
0.95-1	49

Range	Frequency
0-0.05	490
0.05-0.1	525
0.1-0.15	485
0.15-0.2	493
0.2-0.25	509
0.25-0.3	517
0.3-0.35	455
0.35-0.4	540
0.4-0.45	509
0.45-0.5	457
0.5-0.55	507
0.55-0.6	540
0.6-0.65	473
0.65-0.7	519
0.7-0.75	509
0.75-0.8	482
0.8-0.85	505
0.85-0.9	497
0.9-0.95	474
0.95-1	514

Range	Frequency
0-0.05	5009
0.05-0.1	4964
0.1-0.15	4984
0.15-0.2	5114
0.2-0.25	4977
0.25-0.3	5031
0.3-0.35	4948
0.35-0.4	5109
0.4-0.45	4958
0.45-0.5	5054
0.5-0.55	5031
0.55-0.6	4987
0.6-0.65	4912
0.65-0.7	4995
0.7-0.75	5015
0.75-0.8	4981
0.8-0.85	4965
0.85-0.9	4963
0.9-0.95	4968
0.95-1	5035




Range frequencies for 10000 random numbers generated using LCG
"bar1.2.2.txt" using 2:xtic(1)
freq.
0
100
200
300
400
500
600
0-0.05
0.05-0.1
0.1-0.15
0.15-0.2
0.2-0.25
0.25-0.3
0.3-0.35
0.35-0.4
0.4-0.45
0.45-0.5
0.5-0.55
0.55-0.6
0.6-0.65
0.65-0.7
0.7-0.75
0.75-0.8
0.8-0.85
0.85-0.9
0.9-0.95
0.95-1
Range of random numbers

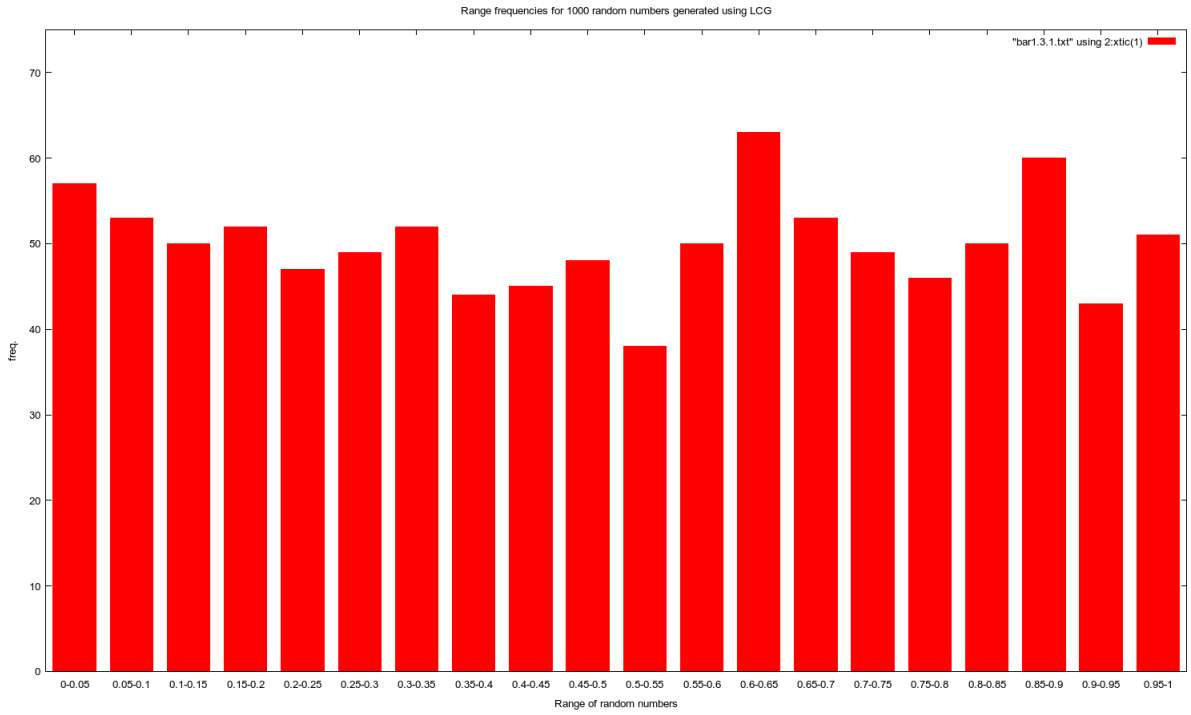

Range frequencies for 100000 random numbers generated using LCG
"bar1.2.3.txt" using 2:xtic(1)
freq.
0
1000
2000
3000
4000
5000
6000
0-0.05
0.05-0.1
0.1-0.15
0.15-0.2
0.2-0.25
0.25-0.3
0.3-0.35
0.35-0.4
0.4-0.45
0.45-0.5
0.5-0.55
0.55-0.6
0.6-0.65
0.65-0.7
0.7-0.75
0.75-0.8
0.8-0.85
0.85-0.9
0.9-0.95
0.95-1
Range of random numbers

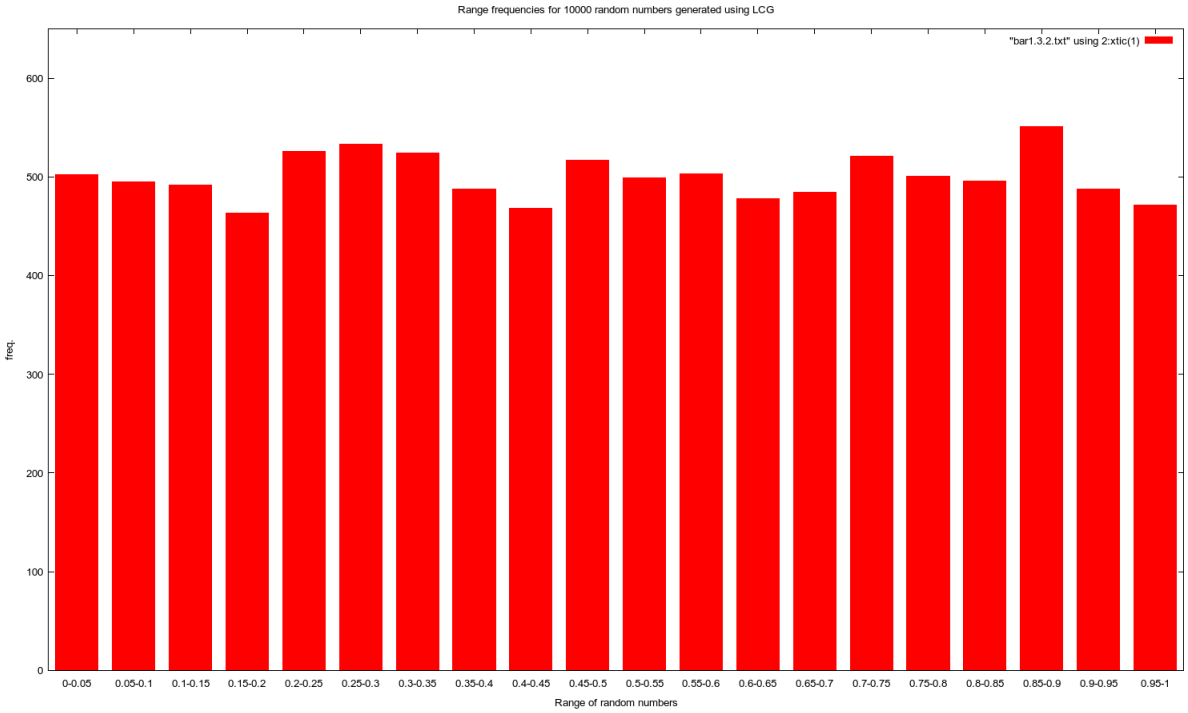
$a = 40014, m = 2147483563, x_0 = 1487191379, n = 1000, 10000, 100000$

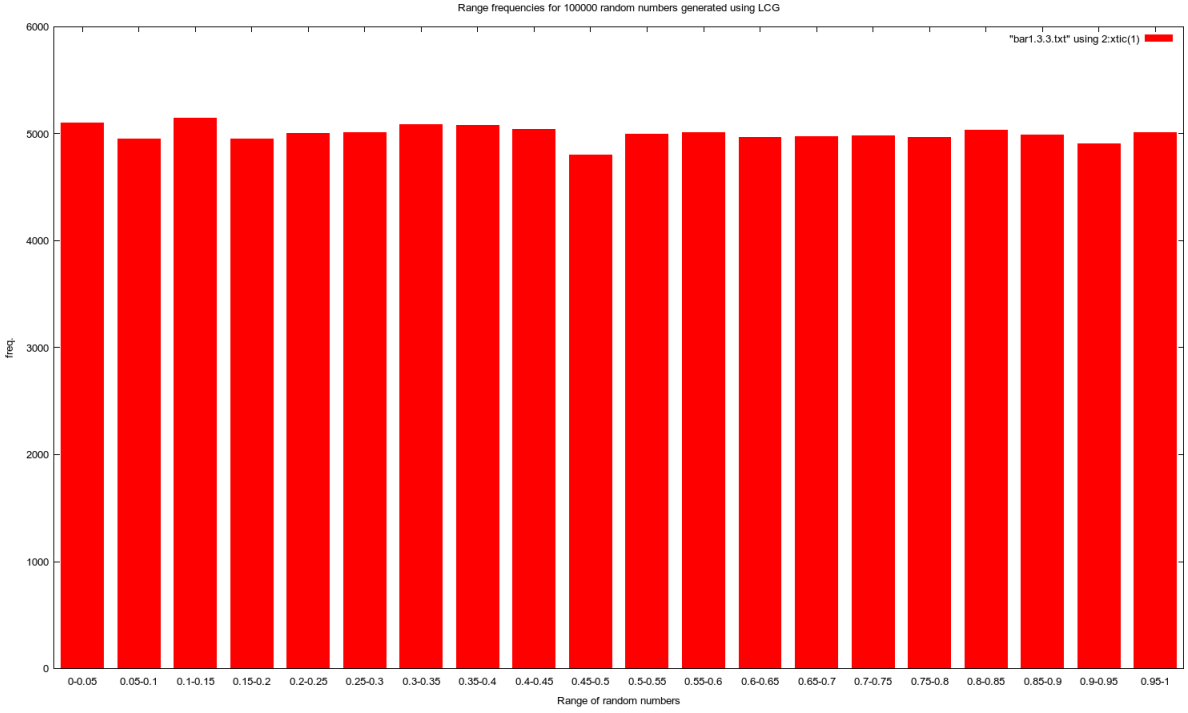
Range	Frequency
0-0.05	57
0.05-0.1	53
0.1-0.15	50
0.15-0.2	52
0.2-0.25	47
0.25-0.3	49
0.3-0.35	52
0.35-0.4	44
0.4-0.45	45
0.45-0.5	48
0.5-0.55	38
0.55-0.6	50
0.6-0.65	63
0.65-0.7	53
0.7-0.75	49
0.75-0.8	46
0.8-0.85	50
0.85-0.9	60
0.9-0.95	43
0.95-1	51

Range	Frequency
0-0.05	502
0.05-0.1	495
0.1-0.15	492
0.15-0.2	463
0.2-0.25	526
0.25-0.3	533
0.3-0.35	524
0.35-0.4	488
0.4-0.45	468
0.45-0.5	517
0.5-0.55	499
0.55-0.6	503
0.6-0.65	478
0.65-0.7	484
0.7-0.75	521
0.75-0.8	501
0.8-0.85	496
0.85-0.9	551
0.9-0.95	488
0.95-1	471

Range	Frequency
0-0.05	5103
0.05-0.1	4947
0.1-0.15	5142
0.15-0.2	4948
0.2-0.25	5006
0.25-0.3	5014
0.3-0.35	5088
0.35-0.4	5081
0.4-0.45	5044
0.45-0.5	4798
0.5-0.55	4994
0.55-0.6	5007
0.6-0.65	4968
0.65-0.7	4971
0.7-0.75	4983
0.75-0.8	4967
0.8-0.85	5036
0.85-0.9	4990
0.9-0.95	4905
0.95-1	5008



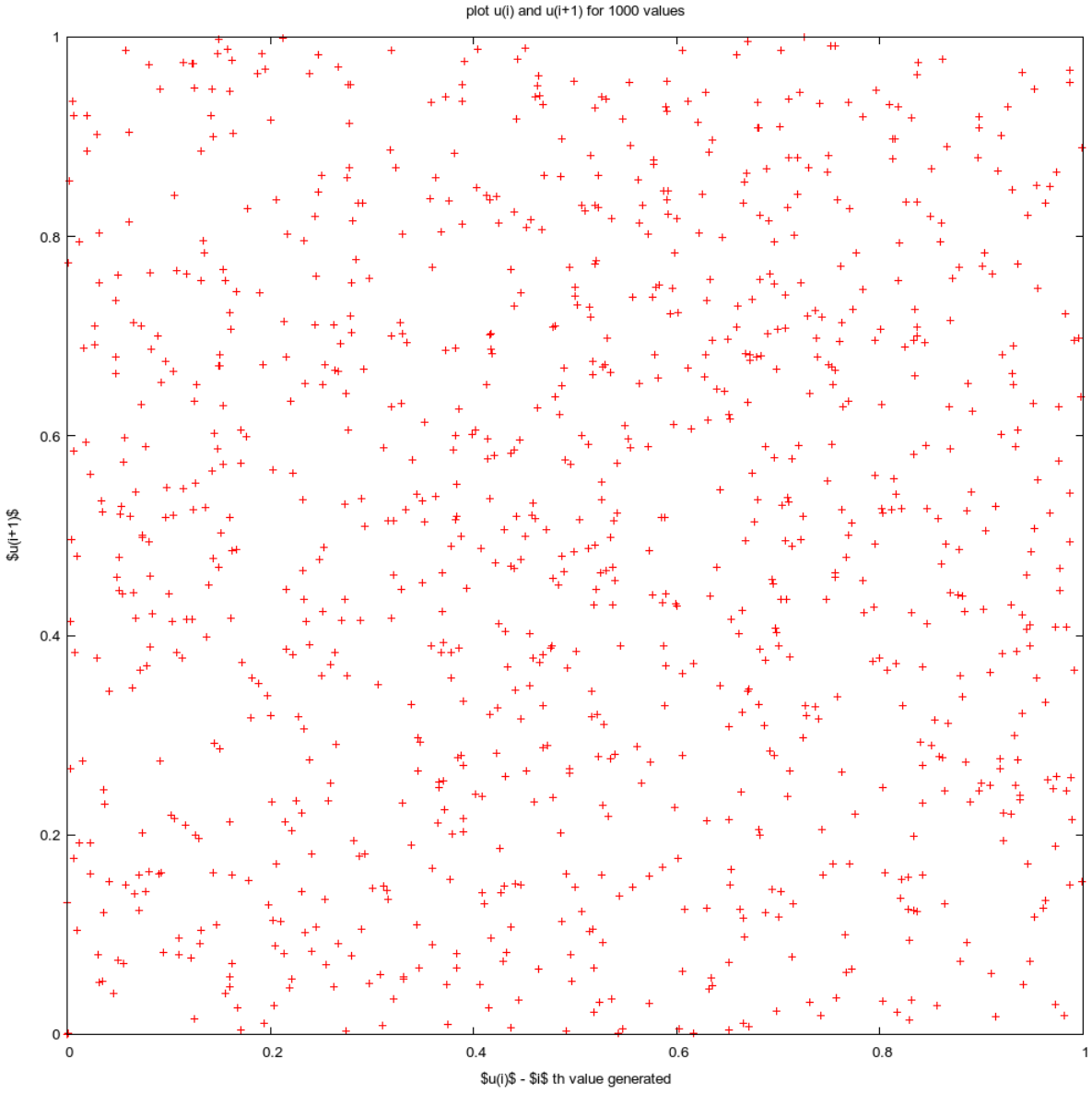

Range frequencies for 10000 random numbers generated using LCG
"bar1.3.2.txt" using 2:xtic(1)
freq.
0
100
200
300
400
500
600
0-0.05
0.05-0.1
0.1-0.15
0.15-0.2
0.2-0.25
0.25-0.3
0.3-0.35
0.35-0.4
0.4-0.45
0.45-0.5
0.5-0.55
0.55-0.6
0.6-0.65
0.65-0.7
0.7-0.75
0.75-0.8
0.8-0.85
0.85-0.9
0.9-0.95
0.95-1
Range of random numbers

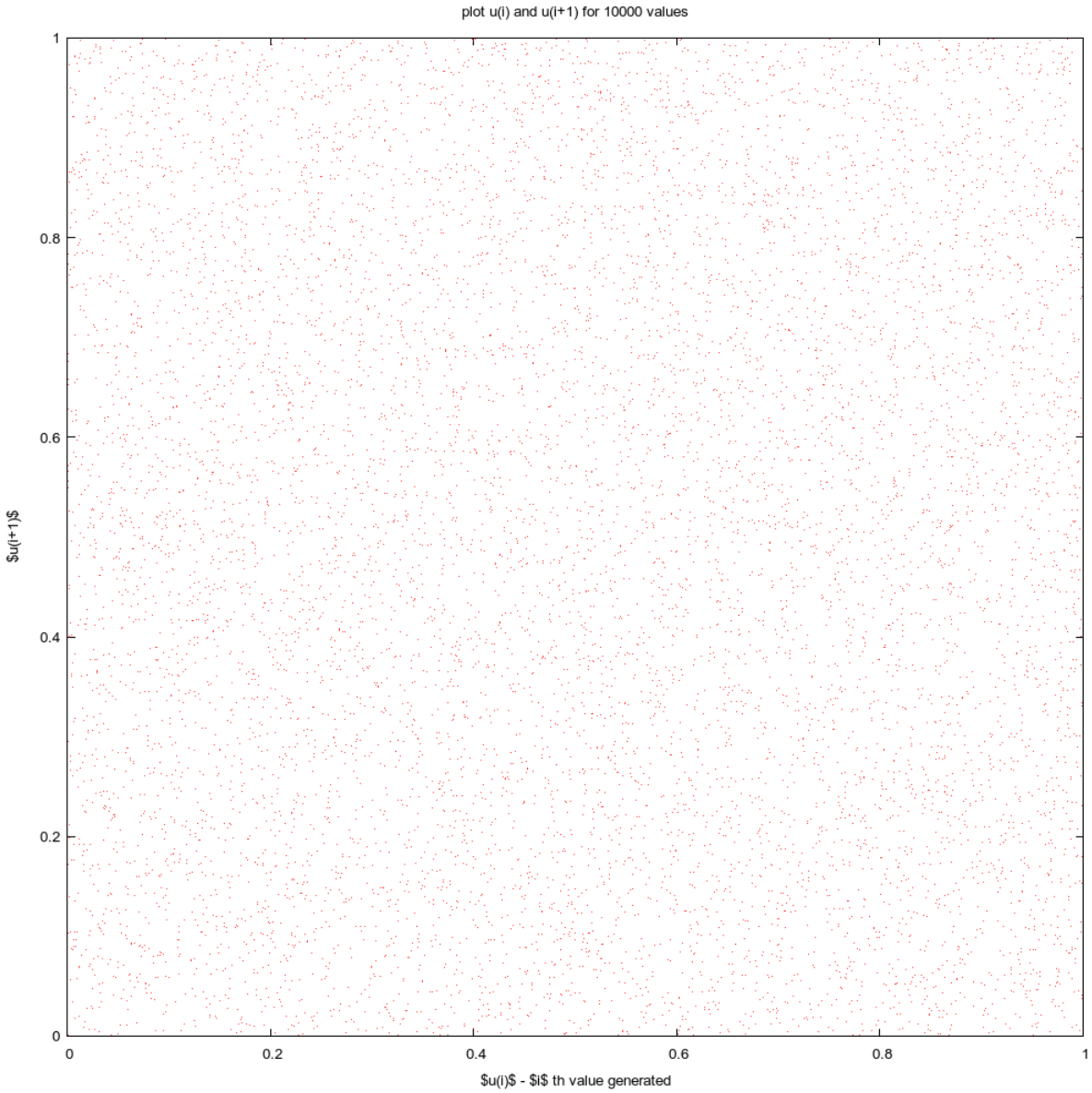

Range frequencies for 100000 random numbers generated using LCG
"bar1.3.3.txt" using 2:xtic(1)
freq.
0
1000
2000
3000
4000
5000
6000
0-0.05
0.05-0.1
0.1-0.15
0.15-0.2
0.2-0.25
0.25-0.3
0.3-0.35
0.35-0.4
0.4-0.45
0.45-0.5
0.5-0.55
0.55-0.6
0.6-0.65
0.65-0.7
0.7-0.75
0.75-0.8
0.8-0.85
0.85-0.9
0.9-0.95
0.95-1
Range of random numbers

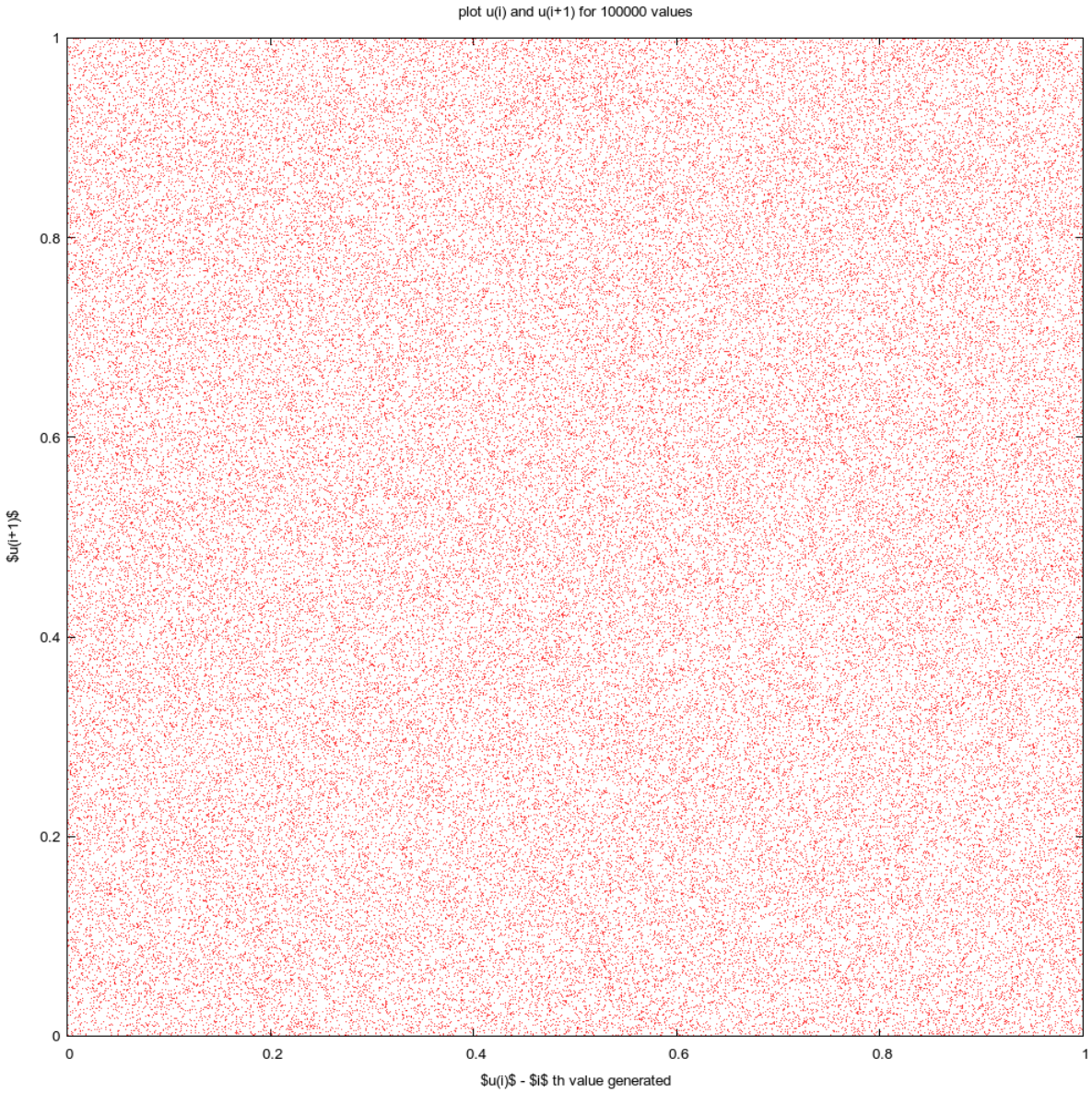
The autocorrelation between $u(i)$ and $u(i+1)$ is very low and they are not(very less) dependent on each other as we see from the $u(i)$ vs $u(i+1)$ plots.

$$a = 16807, m = 2147483647, x_0 = 522329230, n = 1000, 10000, 100000$$

$$x = [0:1],\ y = [0:1]$$



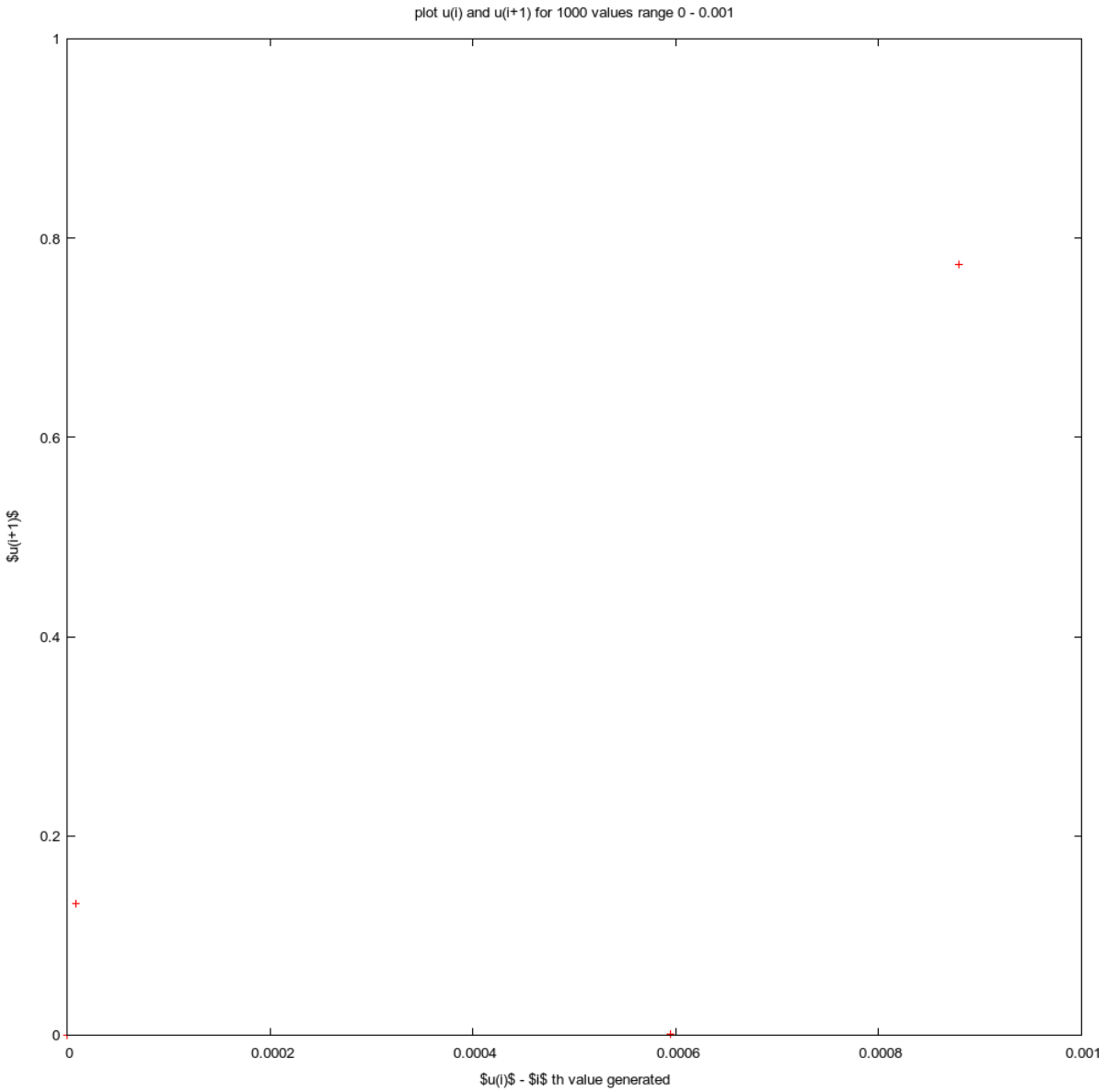

plot u(i) and u(i+1) for 10000 values
$u(i+1)$
$u(i)$ - i th value generated
0
0.2
0.4
0.6
0.8
1

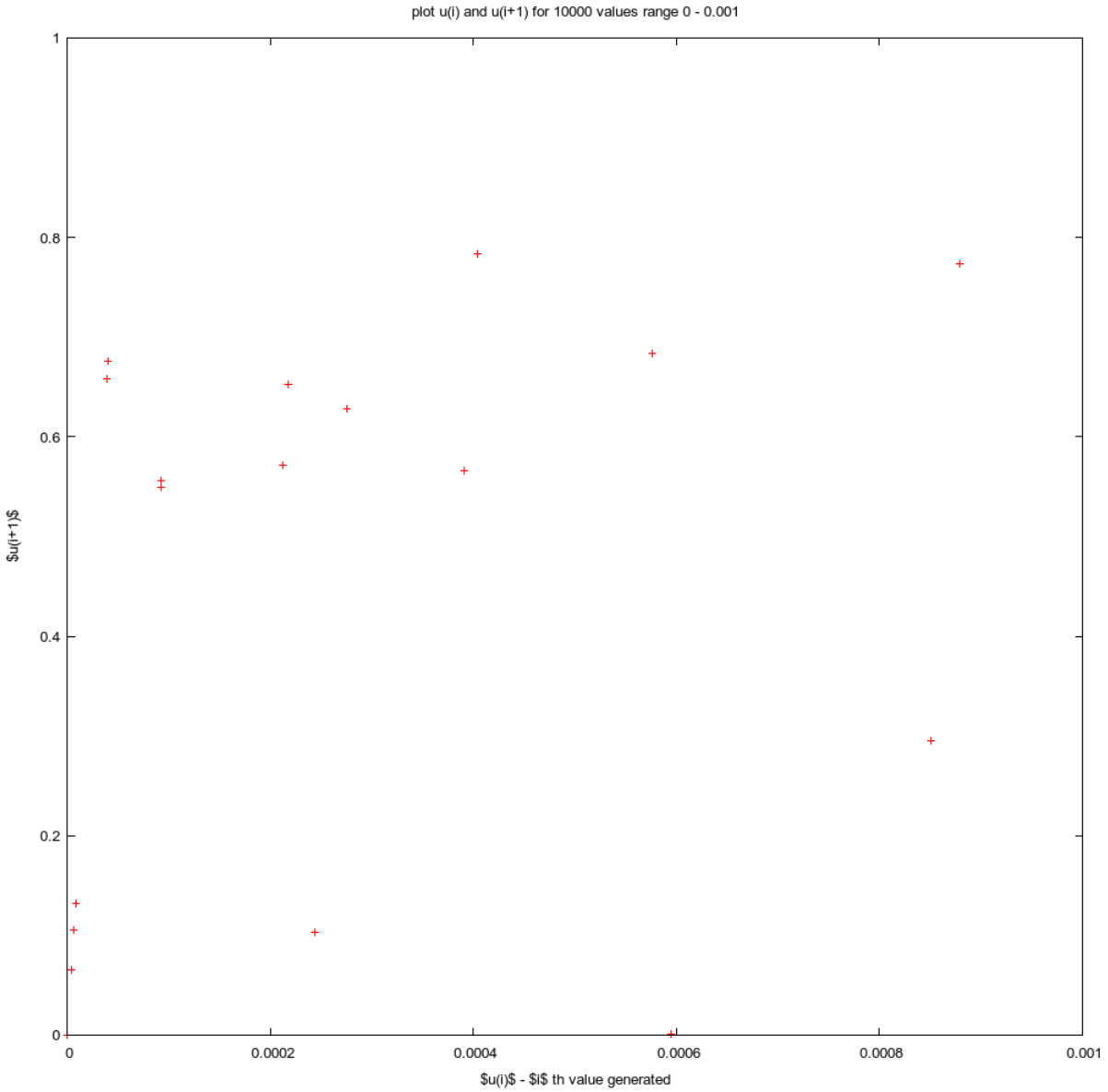

plot u(i) and u(i+1) for 100000 values
$u(i+1)$
$u(i)$ - i th value generated
0
0.2
0.4
0.6
0.8
1

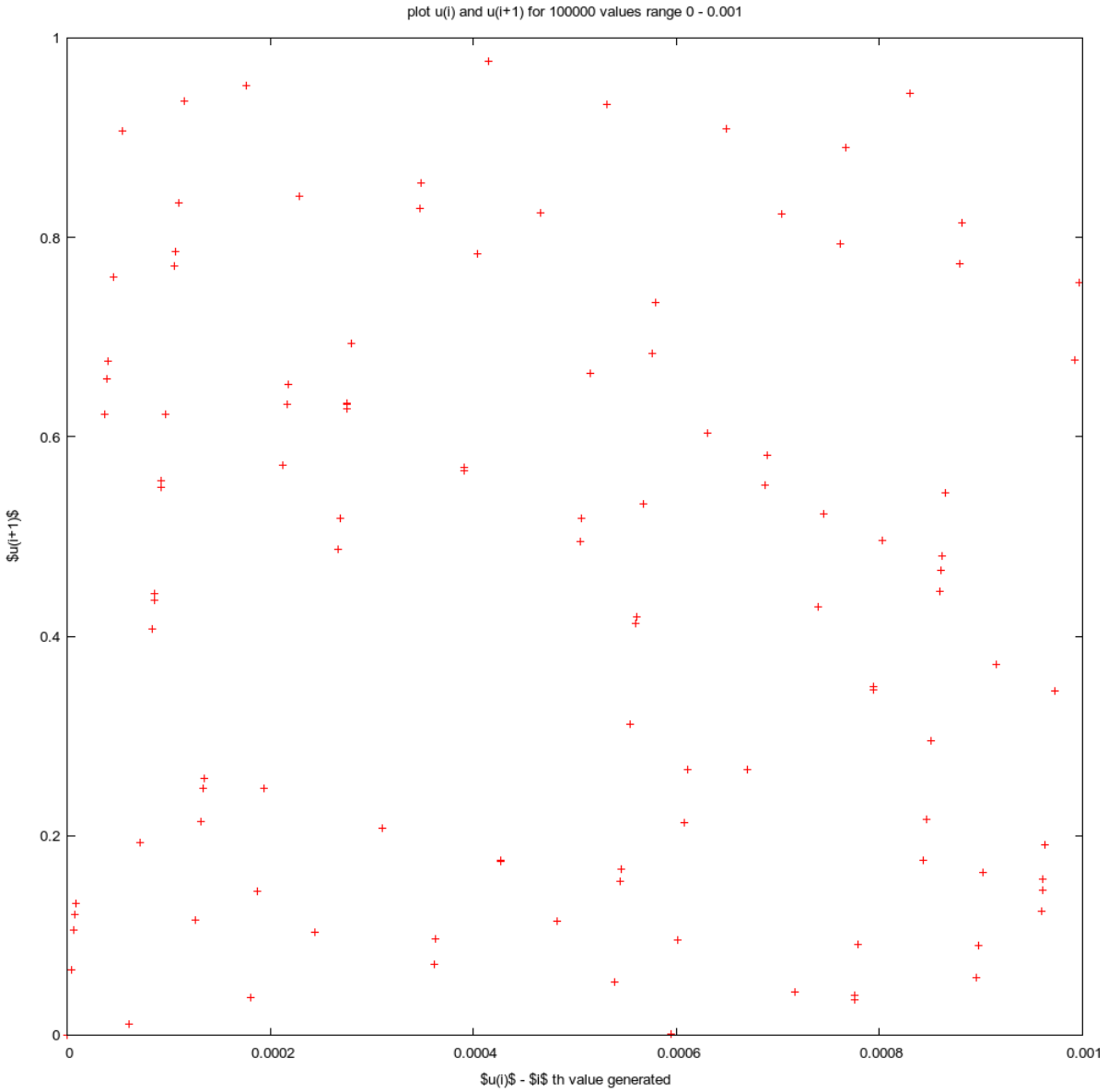
Now zooming into [0:0.001] for $u(i)$. In the zoomed version also they are not related and uniformly distributed. Low autocorrelation. As we have divided the area by 1000 times. we see only 1 point for n=1000 and around 10 for n=10000 and 100 for n=100000, which is as expected (for random numbers).

$$a = 16807, m = 2147483647, x_0 = 522329230, n = 1000, 10000, 100000$$

$$x = [0 : 0.001],\ y = [0 : 1]$$




plot u(i) and u(i+1) for 10000 values range 0 - 0.001
$u(i+1)$
$u(i)$ - i th value generated
0
0.2
0.4
0.6
0.8
1
0
0.0002
0.0004
0.0006
0.0008
0.001


plot u(i) and u(i+1) for 100000 values range 0 - 0.001
$u(i+1)$
$u(i)$ - i th value generated
0
0.2
0.4
0.6
0.8
1
0
0.0002
0.0004
0.0006
0.0008
0.001

2 Question 2

Code for C++

```
#include <iostream>
#include <iomanip>
#include "lab2.h"

using namespace std;

void generator(long A[], long m, int range, int n) {
long *Density, x;
long double u,v;
long double auto_cor[5]= {0,0,0,0,0}, mean = 0, variance = 0, absolute_var = 0;
Density = new long[range];

for(int i=0; i<range; ++i) {
Density[i] = 0;
}

x = A[16];

for(int i=0;i<n;++i) {
for(int j=0;j<16;++j) {
A[j] = A[j+1];
}
A[16] = x;
u = (long double)x/(long double)m;
x = (A[0] + A[12]) % m;
Density[(int)(u*range)]++ ;
v = u;
u = (long double)x/(long double)m;
cout<<fixed<<setprecision(10)<<v<<" "<<u<<"\n";
mean = ((mean * i) + u)/(i+1);
variance = ((variance * i) + ((u - mean) * (u - mean)))/(i+1);
absolute_var = ((absolute_var*i) + ((u - 0.5) * (u - 0.5))) / (i+1);
for(int k=0;k<5;++k) {
auto_cor[k] = auto_cor[k] + (long double)(((long double)x/(long double)m) - mean)* (((long
    double)A[16-k]/(long double)m) - mean);
}
}

cout<<"\n\nProbability in each interval\n";

u = (long double)1/(long double)range;
for(int i=0;i<range;++i) {
cout<<setprecision(2)<<(u*i)<<"-"<<(u*(i+1))<<" "<<setprecision(5)<<(long double)Density[i]/n
    <<"\n";
```

```
43 }
44
45 cout<<"\n\n";
46 cout<<"Mean = "<<setprecision(15)<<mean<<"\n";
47 cout<<"Variance (with changing mean) = "<<variance<<"\n";
48 cout<<"Variance (with mean = 0.5) = "<<absolute_var<<"\n";
49 for(int i = 0; i <5; ++i) {
50 cout<<"Autocorrelation (with lag = "<<i+1<<") = "<<auto_cor[i]/(variance * n)<<"\n";
51 }
52 cout<<"\n\n\n";
53
54 }
55
56
57 int main() {
58 long A[17], m = 2147483648, n = 1000;
59
60 //We can take 1000 and 10000 values from the 100000 generated ones.
61 for(int j=0; j<3; ++j, n*=10) {
62 A[0] = 522329230;
63 for(int i=1; i<17; ++i) {
64 A[i] = base_generator(16807, A[i-1], 2147483647/16807, 2147483647%16807, 2147483647);
65 }
66 generator(A, m, 20 ,n);
67 }
68 }
```

The theoretical value for Uniform Distribution [0:1]

Mean = 0.500

Variance = 0.0833

Autocorrelation (with lag = 1) = 0.000

Autocorrelation (with lag = 2) = 0.000

Autocorrelation (with lag = 3) = 0.000

Autocorrelation (with lag = 4) = 0.000

Autocorrelation (with lag = 5) = 0.000

The values we get are:

$n = 1000$

Mean = 0.503160163763445

Variance (with changing mean) = 0.085404971138431

Variance (with mean = 0.5) = 0.085949722276135

Autocorrelation (with lag = 1) = -0.095406440772602

Autocorrelation (with lag = 2) = -0.018487854412370

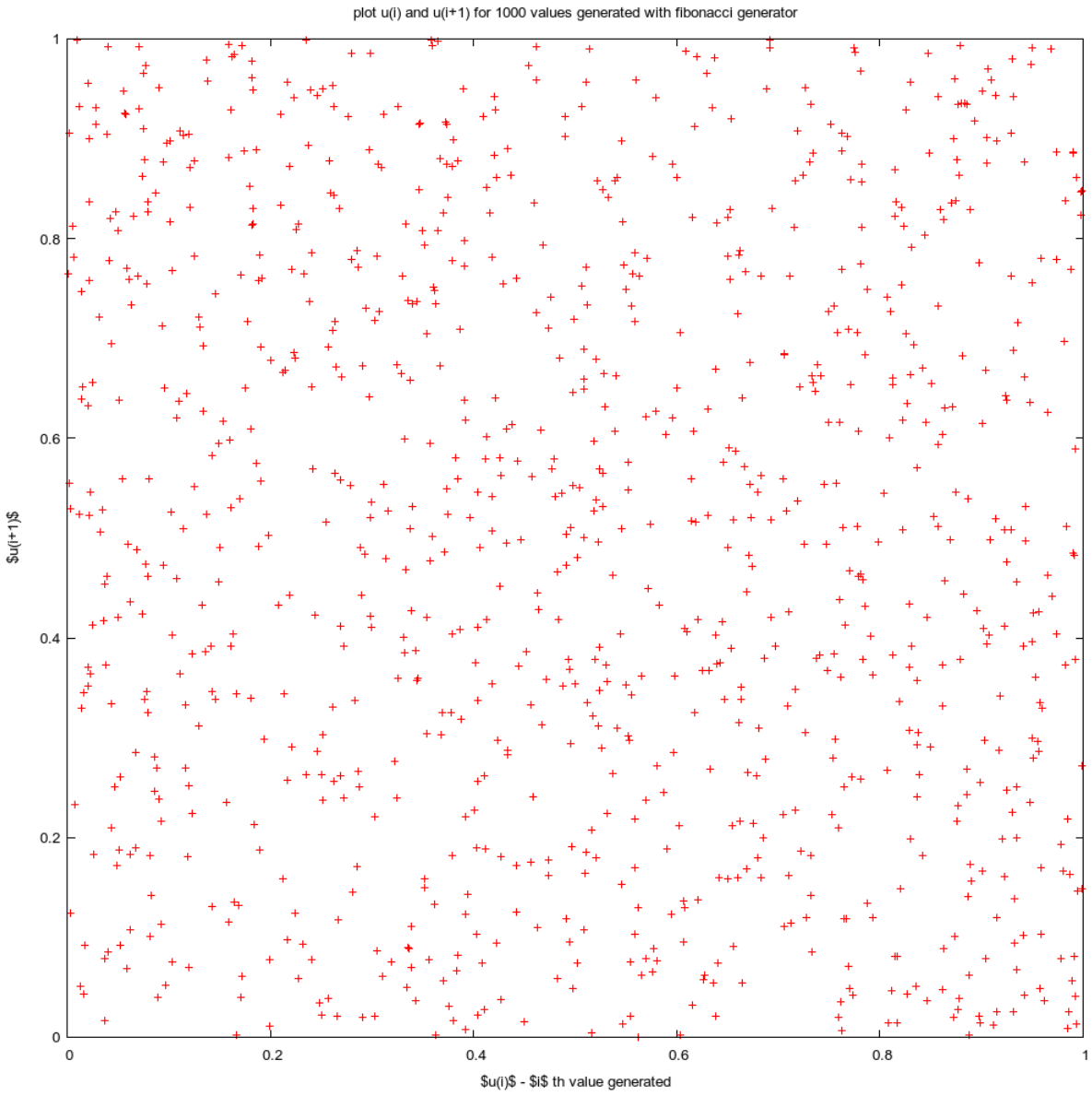
Autocorrelation (with lag = 3) = -0.019750444395744
Autocorrelation (with lag = 4) = -0.014113850867633
Autocorrelation (with lag = 5) = 0.020604150580792

n = 10000
Mean = 0.501647835427616
Variance (with changing mean) = 0.083823841706965
Variance (with mean = 0.5) = 0.083899149061973
Autocorrelation (with lag = 1) = -0.012280387122665
Autocorrelation (with lag = 2) = -0.005113407582166
Autocorrelation (with lag = 3) = 0.012370106172613
Autocorrelation (with lag = 4) = 0.003169611338368
Autocorrelation (with lag = 5) = 0.002564686227511

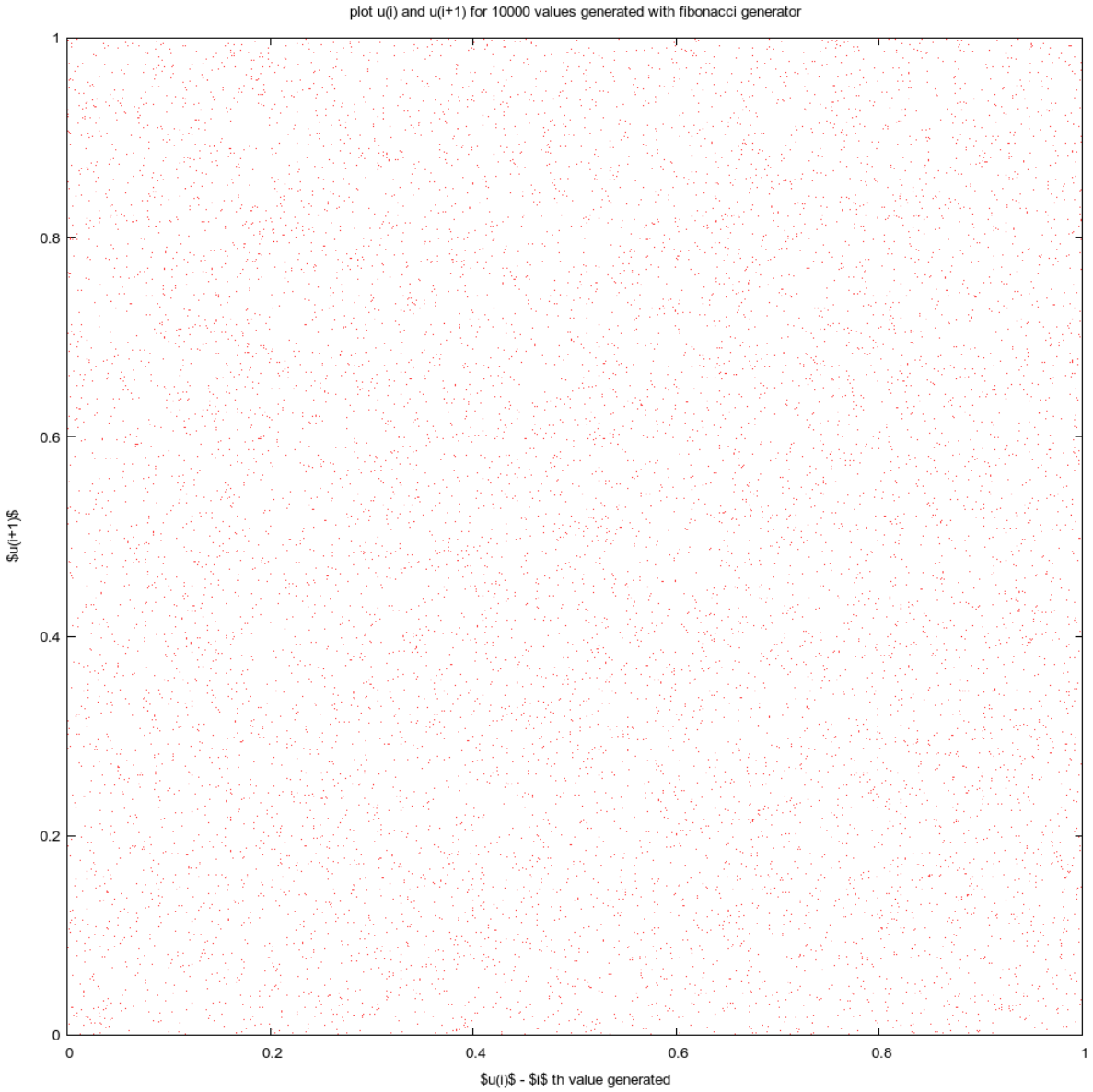
n = 100000
Mean = 0.502687283740165
Variance (with changing mean) = 0.083380568166621
Variance (with mean = 0.5) = 0.083396966929464
Autocorrelation (with lag = 1) = -0.002872745450105
Autocorrelation (with lag = 2) = 0.000516204804362
Autocorrelation (with lag = 3) = 0.003310354488332
Autocorrelation (with lag = 4) = 0.000596430719589
Autocorrelation (with lag = 5) = 0.006116675018030

Plots between $u(i)$ and $u(i+1)$.

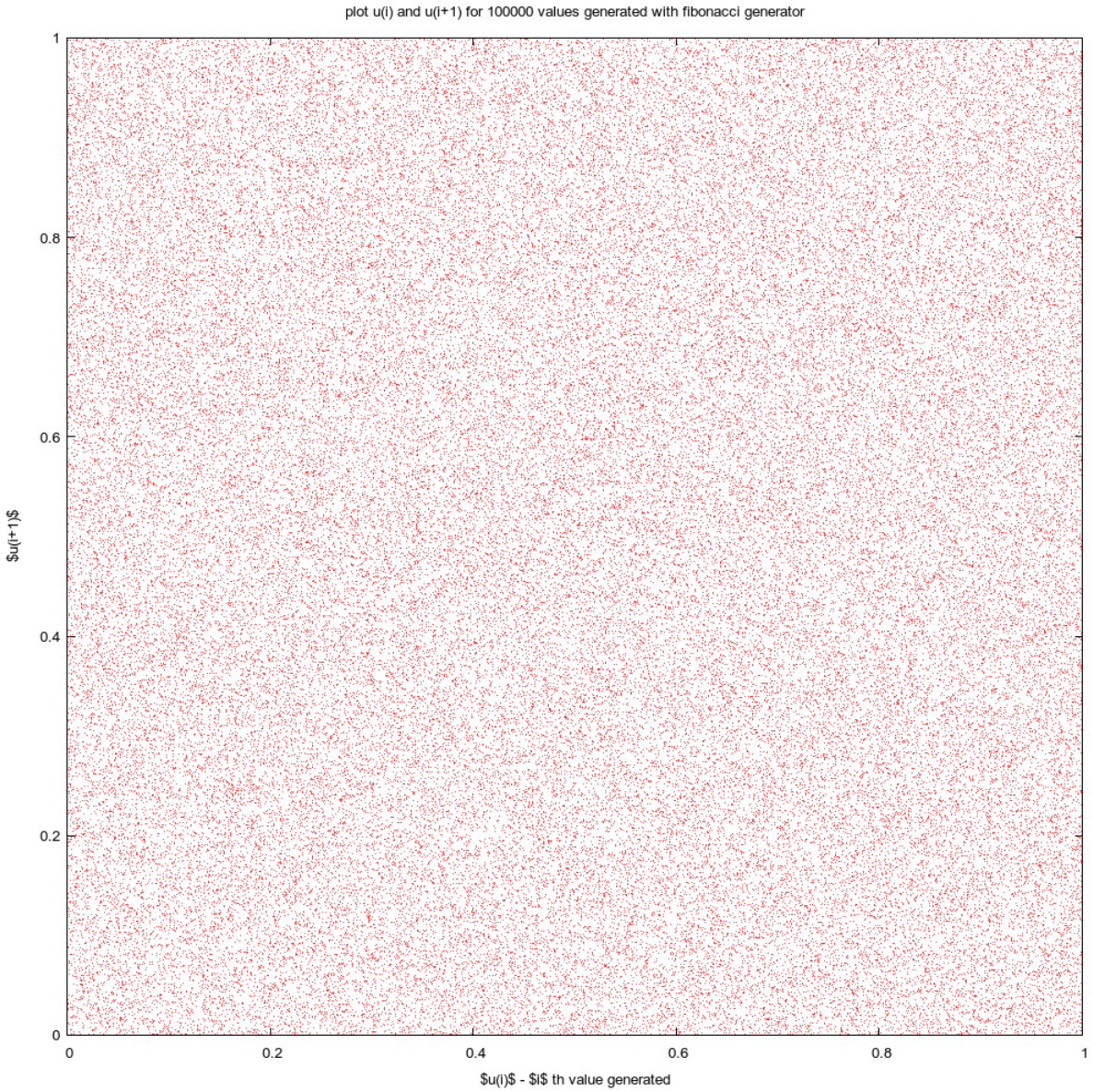
$$n = 1000$$



$n = 10000$

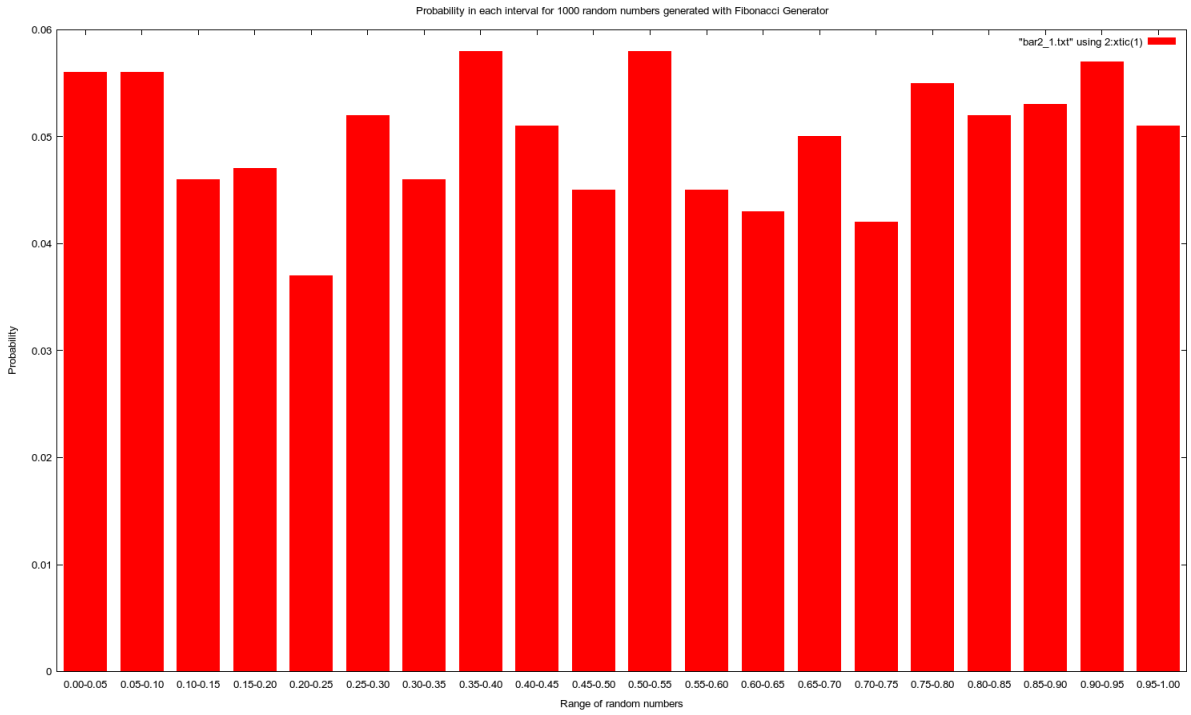

plot u(i) and u(i+1) for 10000 values generated with fibonacci generator
$u(i+1)$
$u(i)$ - i th value generated
0
0.2
0.4
0.6
0.8
1

$n = 100000$

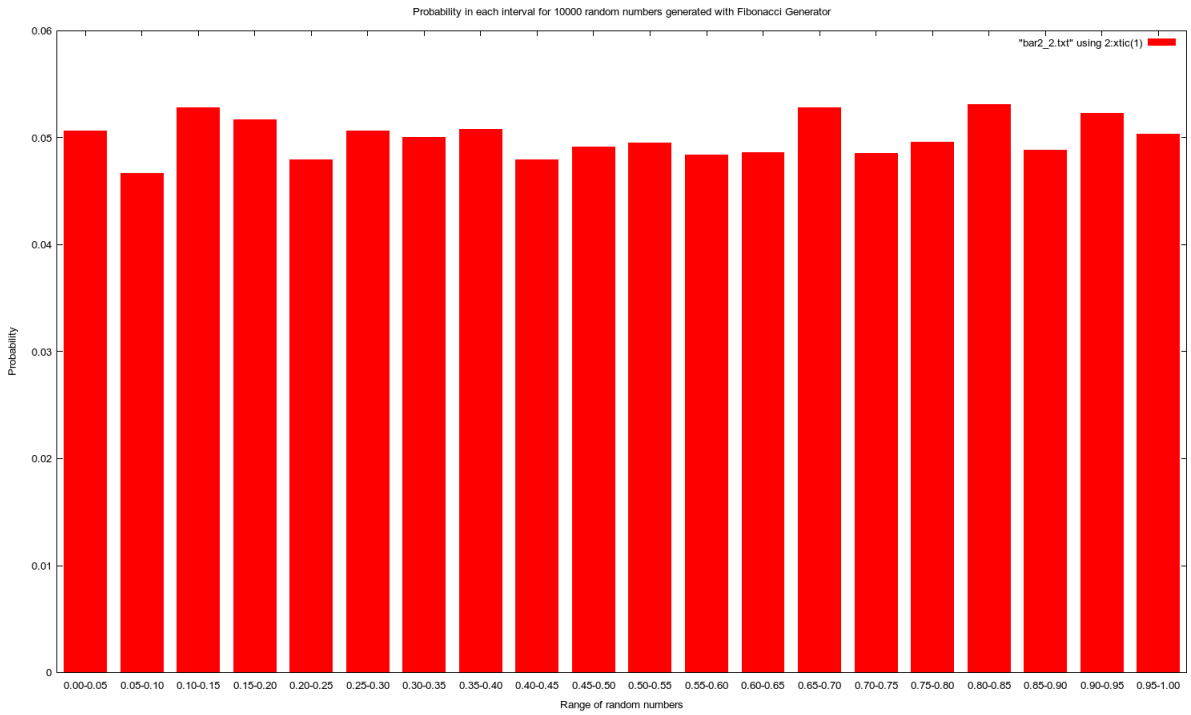

plot u(i) and u(i+1) for 100000 values generated with fibonacci generator
$u(i+1)$
$u(i)$ - i th value generated
0
0.2
0.4
0.6
0.8
1

Probability distribution.

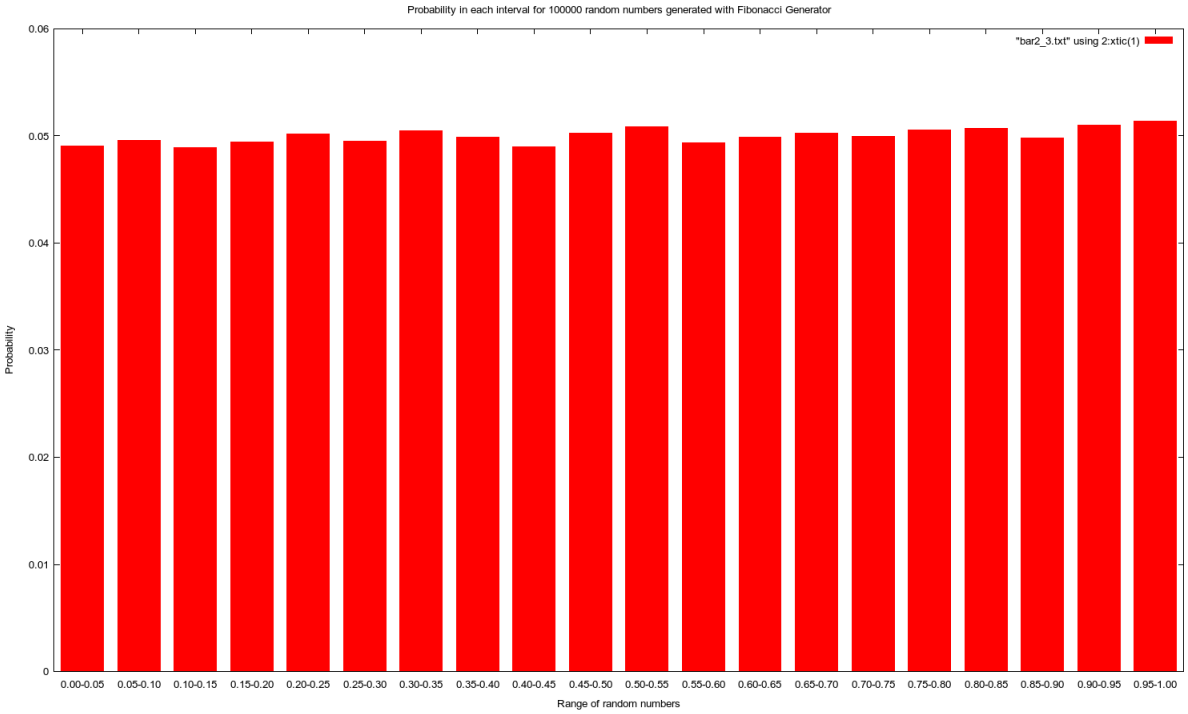
$$n = 1000$$



$$n = 10000$$



$$n = 100000$$


Probability in each interval for 100000 random numbers generated with Fibonacci Generator
"bar2_3.txt" using 2:xtic(1)
Probability
0.06
0.05
0.04
0.03
0.02
0.01
0
0.00-0.05
0.05-0.10
0.10-0.15
0.15-0.20
0.20-0.25
0.25-0.30
0.30-0.35
0.35-0.40
0.40-0.45
0.45-0.50
0.50-0.55
0.55-0.60
0.60-0.65
0.65-0.70
0.70-0.75
0.75-0.80
0.80-0.85
0.85-0.90
0.90-0.95
0.95-1.00
Range of random numbers