Q1: To show that if F is Weibull/log normal cumulative distribution function ,then for any λ such that $-1 \leq \lambda \leq 1$,
$F^*(x) = (1+\lambda)F - \lambda F^2$,
F^* is a cumulative distribution function.

Ans:

We know that F is a cumulative distribution function on a probability space (Ω, A, P) if:
$F(x) = P(X^{-1}(-\infty, x])$=$P(\omega : X(\omega) \leq x)$ for $\forall$ x $\in \mathbb{R}$.

Some properties of the cumulative distribution function of a random variable:

1. F is monotonically increasing.
   To show $F(x_1) \leq F(x_2)$ if $x_1 \leq x_2$.
   Since $(-\infty, x_1] \subseteq (-\infty, x_2]$ for $x_1 \leq x_2$, so $X^{-1}(-\infty, x_1] \subseteq X^{-1}(-\infty, x_2]$.
   Hence $F(x_1) = P(X^{-1}(-\infty, x_1]) \leq P(X^{-1}(-\infty, x_2]) = F(x_2)$

2. F is right continuous, that is $lim_{t \to x_{t>x}} F(t) = F(x+)$.
   Let us denote $lim_{t \to x_{t>x}} F(t) = F(x+)$
   Let $A_n = \{\omega : X(\omega) \leq x + \frac{1}{n}\} = X^{-1}(-\infty, x + \frac{1}{n}]$.
   Then $A_1 \supseteq A_2 \supseteq A_3 \supseteq$ ....$A_n \supseteq A_{n+1}$.....
   So A_n is a sequence of contracting (decreasing) events.
   By the continuity theorem
   $lim_{n \to \infty} F(x + \frac{1}{n}) = lim_{n \to \infty} P(A_n) = P(\cap_{n=1}^{\infty} A_n)$
   $= P(\cap_{n=1}^{\infty} X^{-1}(-\infty, x + \frac{1}{n}]) = P(X^{-1}(\cap_{n=1}^{\infty}(-\infty, x + \frac{1}{n}])) = P(X^{-1}(-\infty, x]) = F(x)$
   Since F is a monotonically increasing function on R, hence $F(x+)$ exists $\forall$ x $\in \mathbb{R}$ and is finite (since F is bounded) for every real x,
   so F(x+) $= lim_{n \to \infty} F(x + \frac{1}{n}) = F(x)$

3. $F(-\infty) = lim_{x \to -\infty} F(x) = 0$
   Let $A_n = \{\omega : X(\omega) \leq -n\} = X^{-1}(-\infty, -n]$.
   Then $A_1 \supseteq A_2 \supseteq A_3 \supseteq$ ....$A_n \supseteq A_{n+1}$.....
   is a sequence of contracting (decreasing) events.
   Since F is a bounded, monotonically increasing function $lim_{x \to -\infty} F(x)$ exists and
   $F(-\infty) = lim_{n \to \infty} F(-n) = lim_{n \to \infty} P(A_n) = P(\cap_{n=1}^{\infty} A_n) = P(\cap_{n=1}^{\infty} X^{-1}(-\infty, -n]) = P(X^{-1}(\cap_{n=1}^{\infty}(-\infty, -n])) = P(X^{-1}(\phi)) = 0$

4. $F(\infty) = lim_{x\to\infty} F(x) = 1$
   Let $A_n = \{\omega : X(\omega) \leq n\} \ = \ X^{-1}(-\infty, n]$.
   Then $A_1 \subseteq A_2 \subseteq A_3 \subseteq$ ....$A_n \subseteq A_{n+1}$.....
   is a sequence of increasing events.
   Since F is a bounded, monotonically increasing function $lim_{x\to-\infty} F(x)$ exists and
   $F(\infty) = lim_{n\to\infty} F(n) = lim_{n\to\infty} P(A_n) = P(\cup_{n=1}^{\infty} A_n) = P(\cup_{n=1}^{\infty} X^{-1}(-\infty, n]) = P(X^{-1}(\cup_{n=1}^{\infty}(-\infty, n])) = P(X^{-1}(\mathbb{R})) = 1$

For Weibull Distribution:
$F(x) = 1 - e^{-\alpha x^{\beta}}$ $, x > 0$

$F^*(x) = (1+\lambda)F - \lambda F^2$
$F^*(x) = 1 + (\lambda - 1)e^{-\alpha x^{\beta}} - \lambda e^{-2\alpha x^{\beta}}$

for property 1,
Differentiating $F^*(x)$
$F^{*'}(x) = (1+\lambda)\frac{dF}{dx} - 2\lambda F\frac{dF}{dx}$
$= (1+\lambda - 2\lambda F)\frac{dF}{dx}$
Since, F is CDF
So,$\frac{dF}{dx} > 0$ as F is monotonically increasing.
Since, $0 \leq F \leq 1$
So, $0 \geq -2\lambda F \geq -2\lambda$
So, $1+\lambda \geq (1+\lambda-2\lambda F) \geq 1-\lambda$
As, $\lambda \geq -1$,
So,$F^{*'}(x) > 0$
Hence, it is monotonically increasing function.

for property 2
$F^*(x+) = lim_{n\to\infty} F^*(x + \frac{1}{n}) = F^*(x)$
Now , F is CDF. So , F is right continuous.
So, λF^2 is also right continuous.
$(1+\lambda)F - \lambda F^2$ will also be a right continuous.
[Sum of continuous function is also a continuous function.]
Hence, F^* is right continuous.

for property 3,
$F^*(-\infty) = lim_{x\to-\infty} F^*(x) = 0$
Since, F is a CDF
So, $F(-\infty) = lim_{x\to-\infty} F(x) = 0$
So,$F^2(-\infty) = lim_{x\to-\infty} F^2(x) = 0$
So,$(1+\lambda)F(-\infty) - \lambda F^2(-\infty) = lim_{x\to-\infty}((1+\lambda)F(x) - \lambda F^2(x)) = 0$

So,$F^*(-\infty) = lim_{x\to-\infty}F^*(x) = 0$

for property 4
$F^*(\infty) = lim_{x\to\infty}F^*(x) = 1$
Since, F is a CDF
So, $F(\infty) = lim_{x\to\infty}F(x) = 1$
So,$F^2(\infty) = lim_{x\to\infty}F^2(x) = 1$
So,$(1+\lambda)F(\infty) - \lambda F^2(\infty) = lim_{x\to\infty}((1+\lambda)F(x) - \lambda F^2(x)) = 1+\lambda-\lambda = 1$

So,$F^*(\infty) = lim_{x\to\infty}F^*(x) = 1$

Hence F^* follows all four properties of Cumulative Distribution function.
So, F^* is a Cumulative Distribution Function.
By inverse transform method:
$F^*(x) = (1+\lambda)F - \lambda F^2 = u$ where u ∼ U(0,1)
So, by solvig the quadratic we get two roots:

$$F(x) = \begin{cases} \frac{(1+\lambda)\pm\sqrt{(1+\lambda)^2-4\lambda u}}{2\lambda} & \lambda \neq 0, \lambda \in (-1,1) \\ \\ u & \lambda = 0 \end{cases}$$

Using this we can find out the inverse transformation of $F^*(x)$. For weibull distribution take the root with - sign and for lognormal take the root with + sign as $\forall x \in \mathbb{R}$ we have $F(x) > 0$.

Code for R

```
\#BY WEIBULL DISTRIBUTION

a<-1
b<-2
d<-c(-1,-0.5,0,0.5,1)
x<-1
j<-1

for(i in 1:5)
{
        while(j<=10000)
        {
                u<-runif(1)
                if(d[i]!=0)
                        x[j]=(1/a)*(-1*log(((d[i]-1)+
```

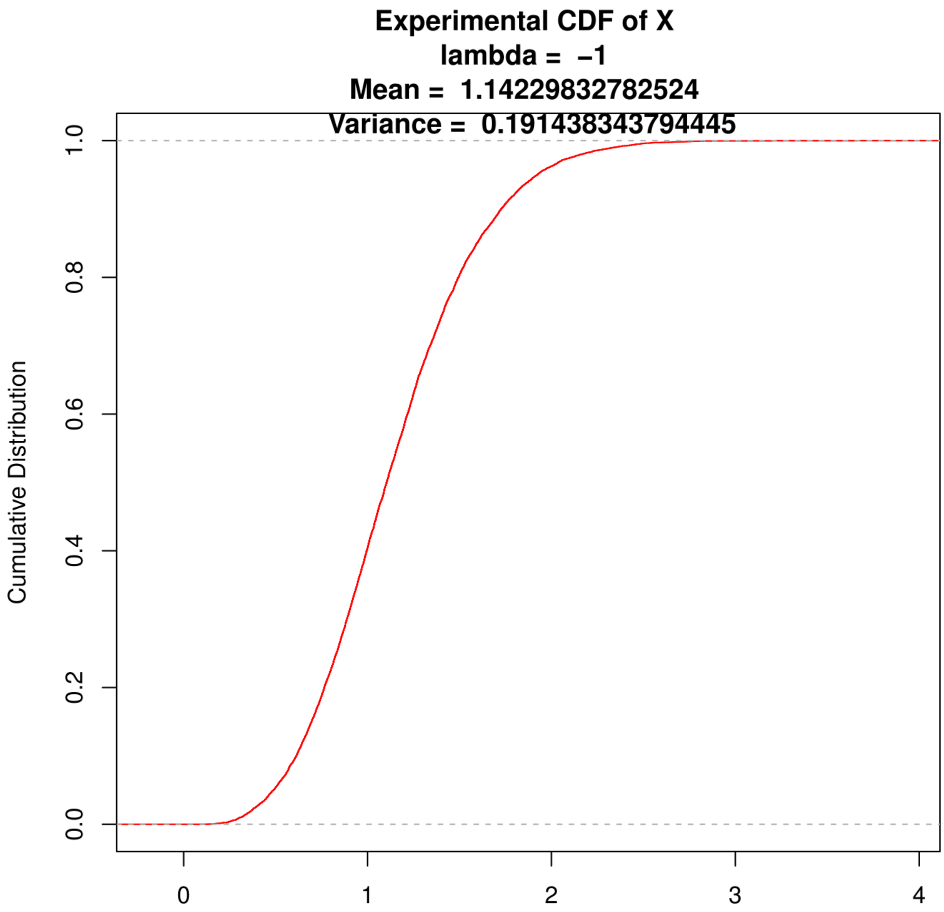
```
                    sqrt((1+d[i])^2 - 4*u*d[i]))/(2*d[i])))^(1/b)
            else
                    x[j]=(1/a)*(-1*log(1-u))^(1/b)
            j=j+1
    }

    j<-1


    cat("\nThe Mean of the Distributon calculated is ",mean(x))
    cat("\nThe Varinace of the Distributon calculated is ",var(x))
    cat("\n")
    h=ecdf(x)
    plot( h,col="red", xlab="", ylab="Cumulative Distribution",
     main=paste("\nExperimental CDF of X \nlambda = ",d[i],
     "\nMean = ",mean(x),"\n Variance = ",var(x)))
    hist(x,probability='TRUE')
    lines(density(x), col='darkorange')
}
```

Here, $\alpha = 1, \beta = 2$

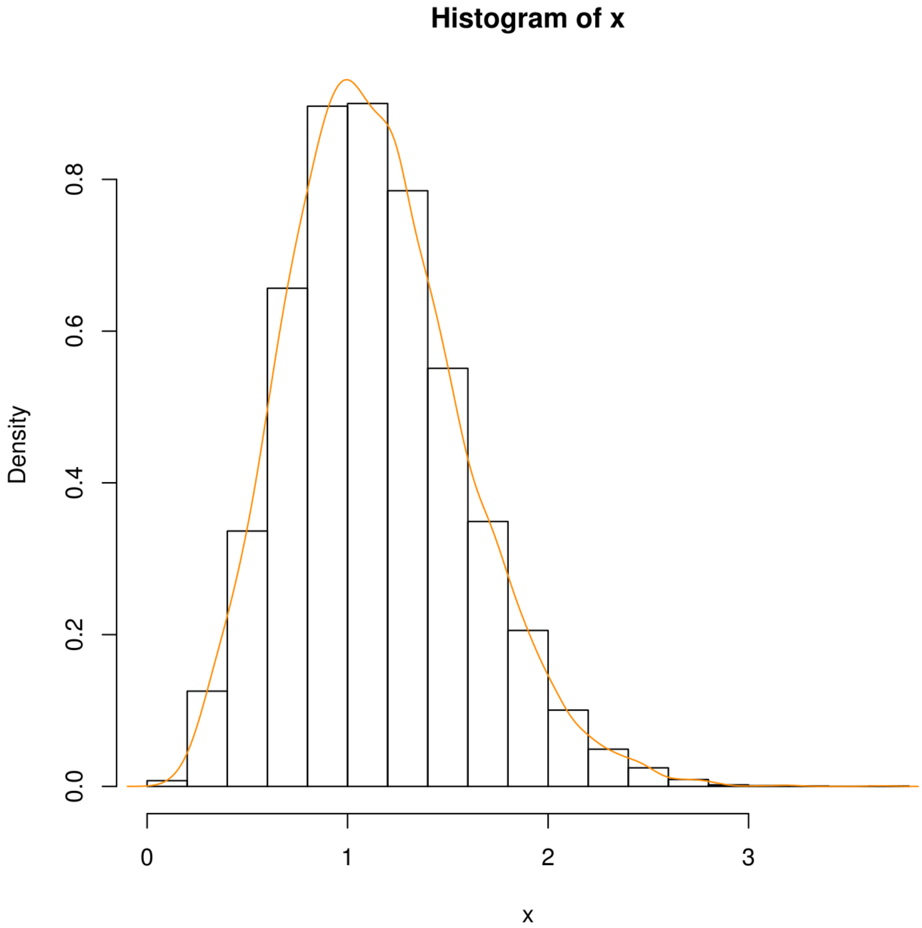
Histograms:

For $\lambda = -1$

CDF of X:

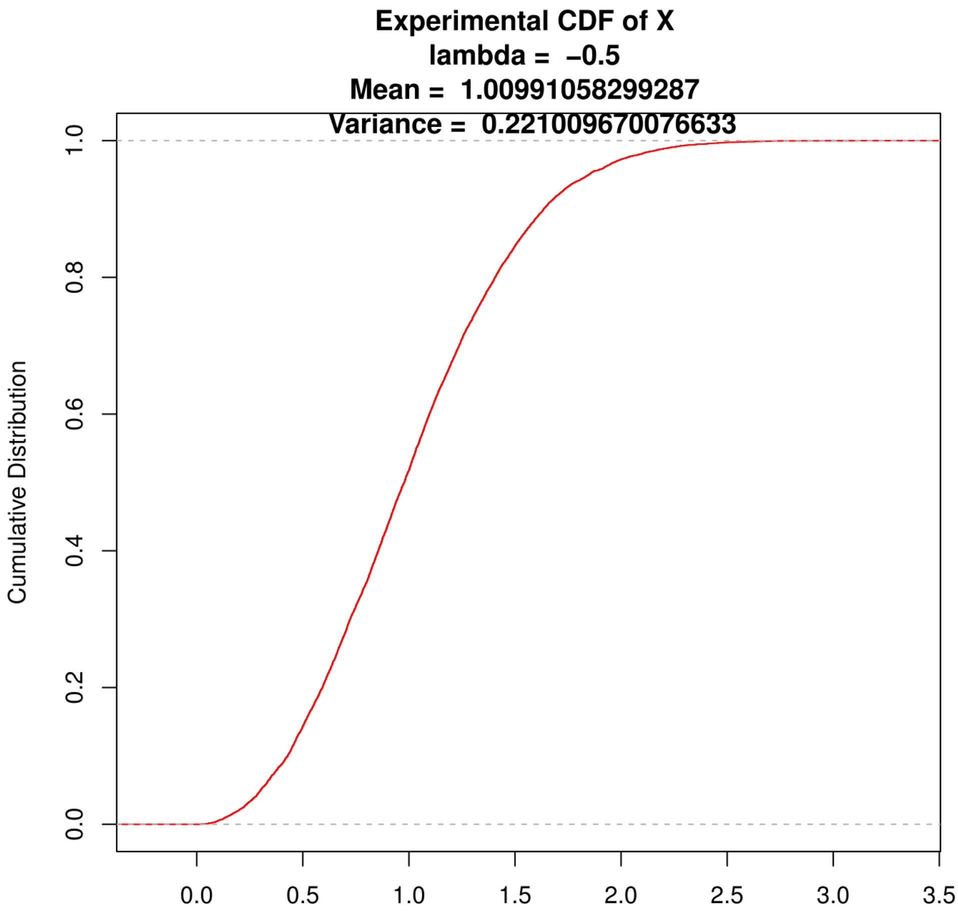

Experimental CDF of X
lambda = −1
Mean = 1.14229832782524
Variance = 0.191438343794445
Cumulative Distribution
0.0
0.2
0.4
0.6
0.8
1.0
0
1
2
3
4

Density of X:

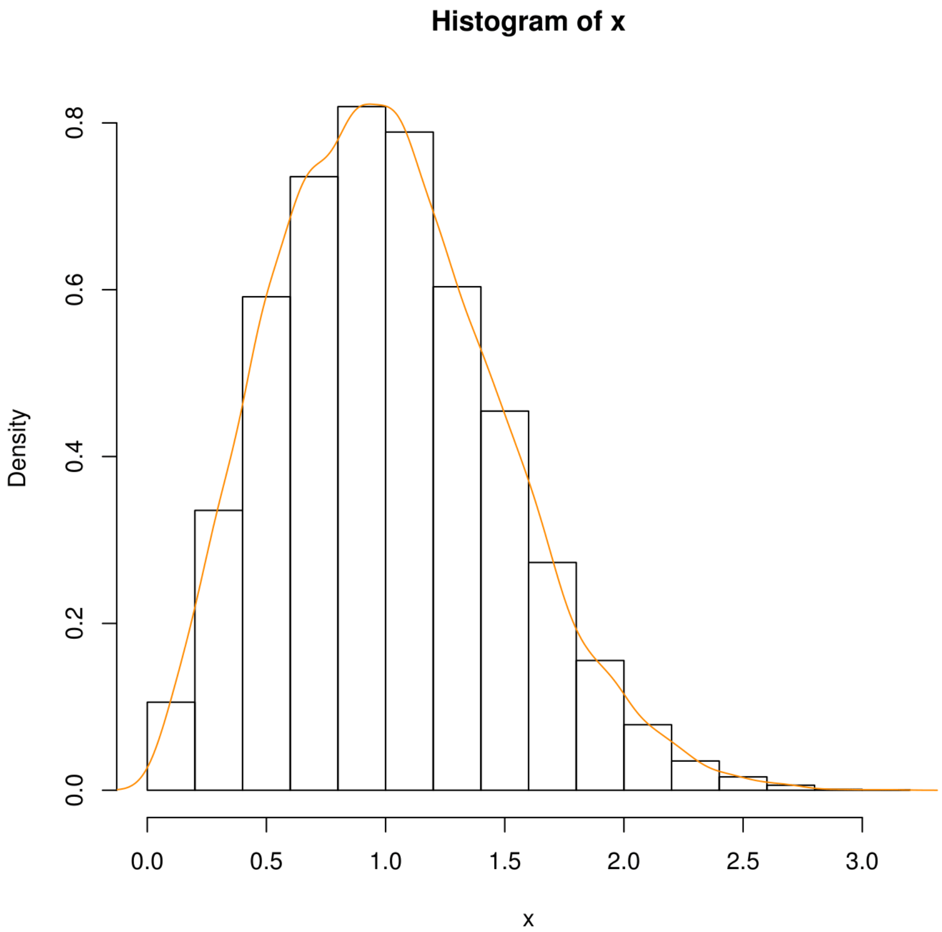


For $\lambda = -0.5$

CDF of X:

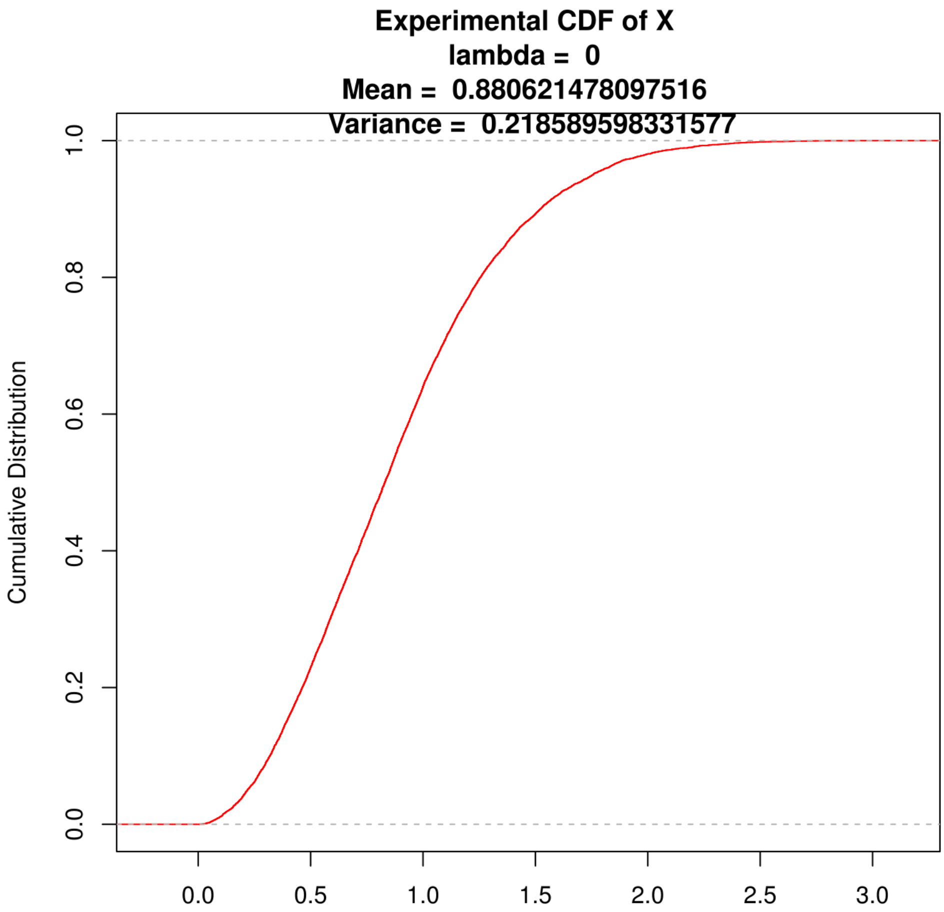

Experimental CDF of X
lambda = −0.5
Mean = 1.00991058299287
Variance = 0.221009670076633
Cumulative Distribution
0.0
0.2
0.4
0.6
0.8
1.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5

Density of X:

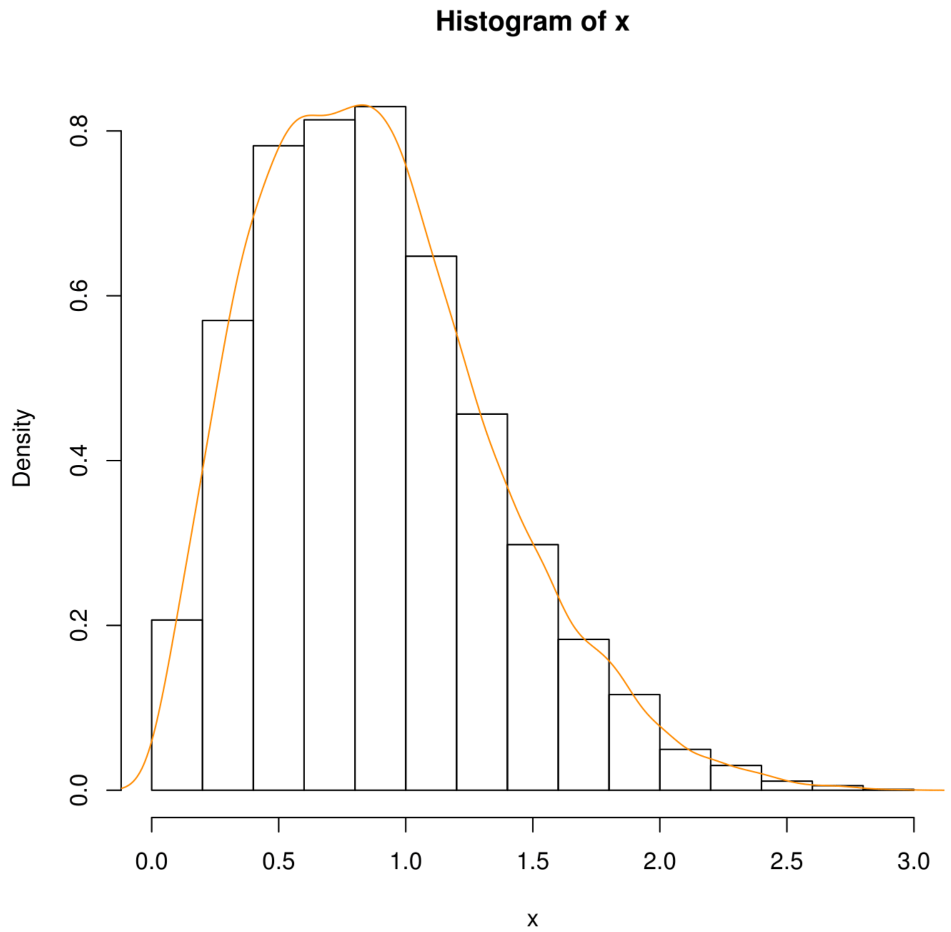


For $\lambda = 0$

CDF of X:

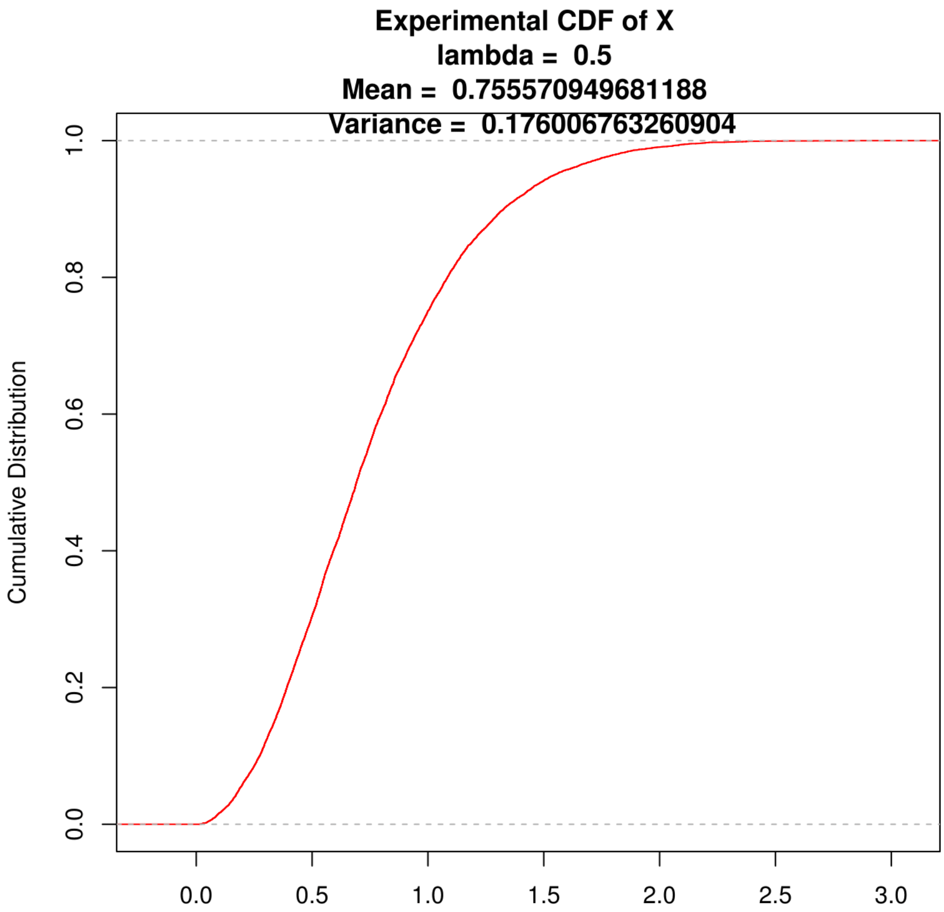

Experimental CDF of X
lambda = 0
Mean = 0.880621478097516
Variance = 0.218589598331577
Cumulative Distribution
0.0
0.2
0.4
0.6
0.8
1.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0

Density of X:

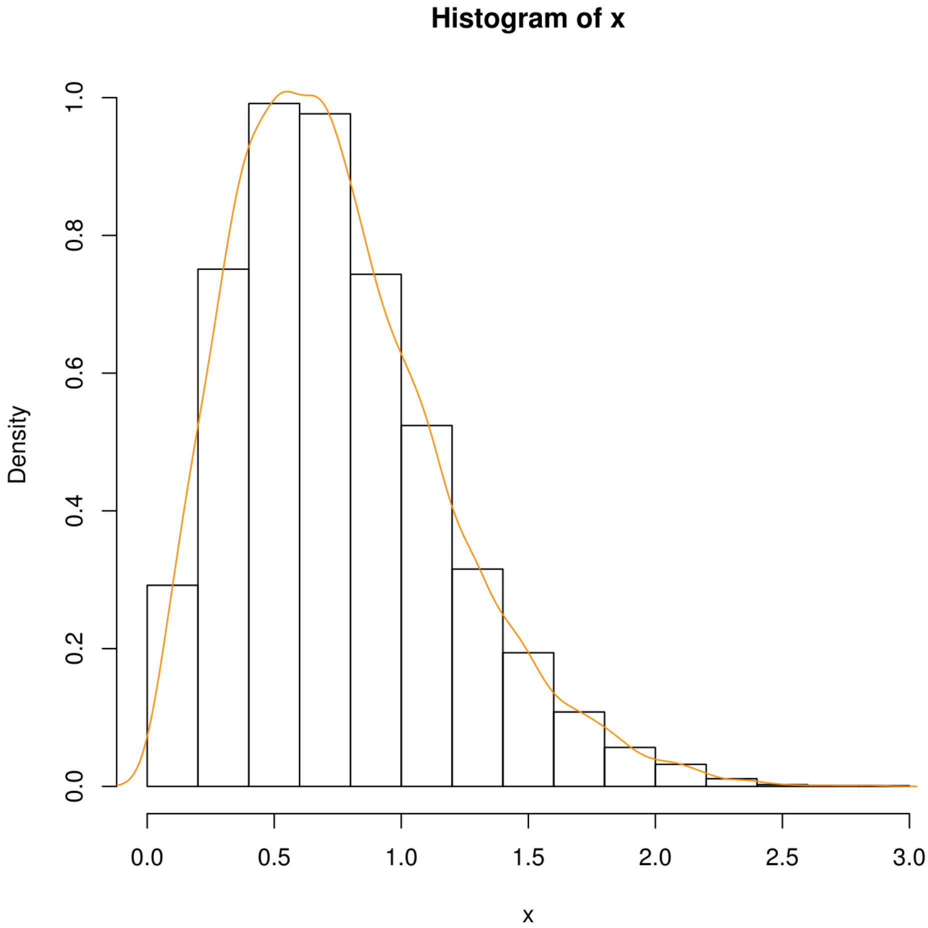


For $\lambda = 0.5$

CDF of X:

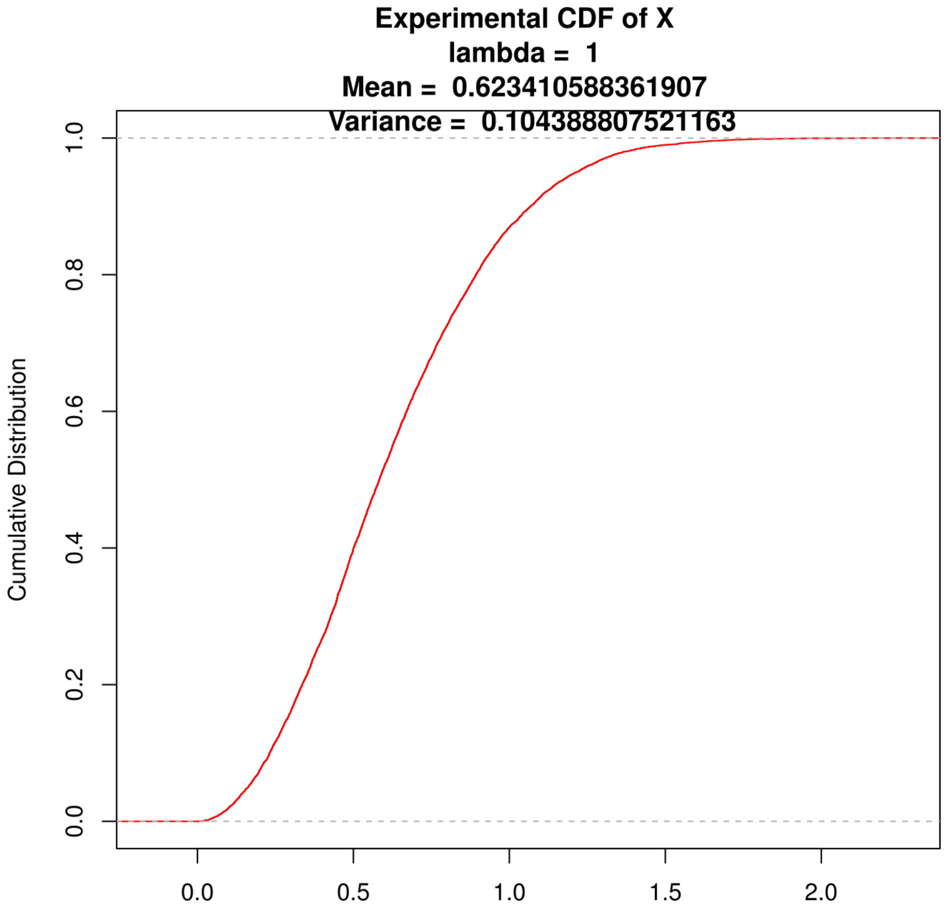

Experimental CDF of X
lambda = 0.5
Mean = 0.755570949681188
Variance = 0.176006763260904
Cumulative Distribution
0.0
0.2
0.4
0.6
0.8
1.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0

Density of X:

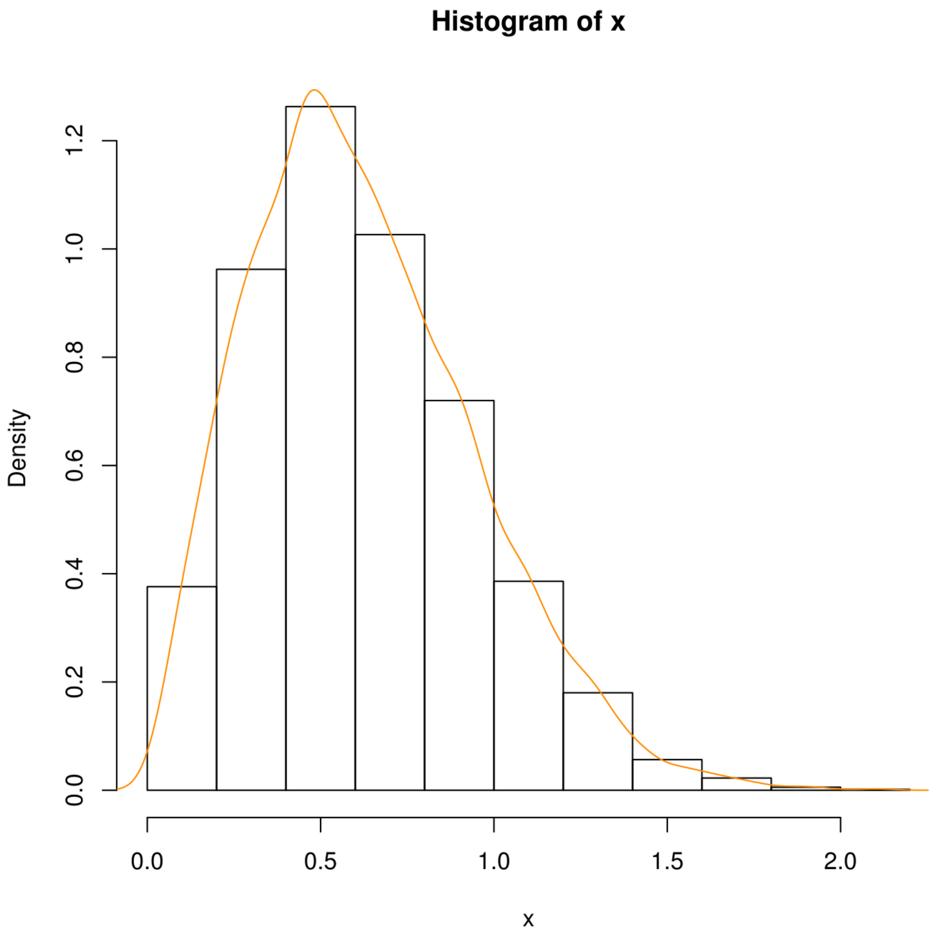


For $\lambda = 1.0$

CDF of X:


Experimental CDF of X
lambda = 1
Mean = 0.623410588361907
Variance = 0.104388807521163
Cumulative Distribution
0.0
0.2
0.4
0.6
0.8
1.0
0.0
0.5
1.0
1.5
2.0

Density of X:


Histogram of x
Density
0.0
0.2
0.4
0.6
0.8
1.0
1.2
0.0
0.5
1.0
1.5
2.0
x

For Lognormal Distribution:

Probability Density function is

$N(lnx;\ \mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(lnx-\mu)^2}{2\sigma^2}}$

Cumulative density function:

$F(x) = \int_0^x lnN(t;\ \mu,\sigma)dt = \frac{1}{2}\text{erfc}[-\frac{lnx-\mu}{\sigma\sqrt{2}}] = \Phi(\frac{lnx-\mu}{\sigma}), x > 0$

erfc=complementary error function

Φ=Cumulative distribution function of Normal Distribution. $F^*(x) = (1+\lambda)F - \lambda F^2$

$F^*(x) = 1 + (\lambda - 1)e^{-\alpha x^\beta} - \lambda e^{-2\alpha x^\beta}$

for property 1,

Differentiating $F^*(x)$

$F^{*'}(x) = (1+\lambda)\frac{dF}{dx} - 2\lambda F\frac{dF}{dx}$

$\quad = (1+\lambda-2\lambda F)\frac{dF}{dx}$

Since, F is CDF

So,$\frac{dF}{dx} > 0$ as F is monotonically increasing.

Since, $0 \leq F \leq 1$

So, $0 \geq -2\lambda F \geq -2\lambda$

So, $1+\lambda \geq (1+\lambda-2\lambda F) \geq 1-\lambda$

As, $\lambda \geq -1$,

So,$F^{*'}(x) > 0$

Hence, it is monotonically increasing function.

for property 2

$F^*(x+) = lim_{n\to\infty}F^*(x+\frac{1}{n}) = F^*(x)$

Now , F is CDF. So , F is right continuous.

So, λF^2 is also right continuous.

$(1+\lambda)F - \lambda F^2$ will also be a right continuous.

[**Sum of continuous function is also a continuous function.**]

Hence, F^* is right continuous.

for property 3,

$F^*(-\infty) = lim_{x\to-\infty}F^*(x) = 0$

Since, F is a CDF

So, $F(-\infty) = lim_{x\to-\infty}F(x) = 0$

So,$F^2(-\infty) = lim_{x\to-\infty}F^2(x) = 0$

So,$(1+\lambda)F(-\infty) - \lambda F^2(-\infty) = lim_{x\to-\infty}((1+\lambda)F(x) - \lambda F^2(x)) = 0$

So,$F^*(-\infty) = lim_{x\to-\infty}F^*(x) = 0$

for property 4

$F^*(\infty) = lim_{x\to\infty}F^*(x) = 1$

Since, F is a CDF

So, $F(\infty) = lim_{x\to\infty} F(x) = 1$
So,$F^2(\infty) = lim_{x\to\infty} F^2(x) = 1$
So,$(1+\lambda)F(\infty) - \lambda F^2(\infty) = lim_{x\to\infty}((1+\lambda)F(x) - \lambda F^2(x)) = 1 + \lambda - \lambda = 1$

So,$F^*(\infty) = lim_{x\to\infty} F^*(x) = 1$

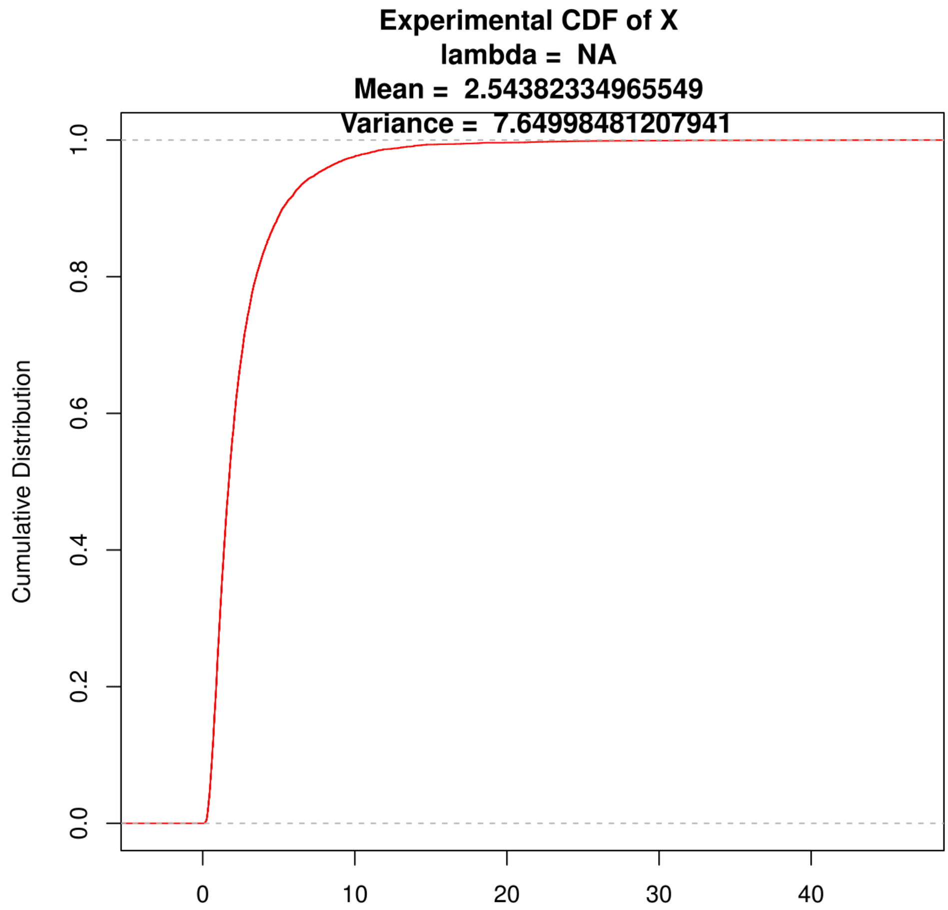
Hence F^* follows all four properties of Cumulative Distribution function.
So, F^* is a Cumulative Distribution Function.
Code for R

```
f<-function(b)
{
        u<-runif(1)
        return (((1+b)-sqrt((1+b)^2 - 4*b*u))/(2*b))
}

d<-c(-1,-0.5,0,0.5,1)
x<-1
j<-1
for(j in 1:5)
{
        for(i in 1:10000)
        {
                if(d[j]!=0)
                        x[i]<-qlnorm(f(d[j]),meanlog=0,sdlog=1,
                lower.tail=TRUE,log.p=FALSE)
                else
                        x[i]<-qlnorm(runif(1),meanlog=0,sdlog=1,
                lower.tail=TRUE,log.p=FALSE)
        }


cat("\nThe Mean of the Distributon calculated is ",mean(x))
cat("\nThe Varinace of the Distributon calculated is ",var(x))
cat("\n")
h=ecdf(x)
plot( h,col="red", xlab="", ylab="Cumulative Distribution",
 main=paste("\nExperimental CDF of X \nlambda = ",d[i],
"\nMean = ",mean(x),"\n Variance = ",var(x)))
hist(x,ylim=c(0,1),probability='TRUE')
lines(density(x), col='darkorange')

}
```

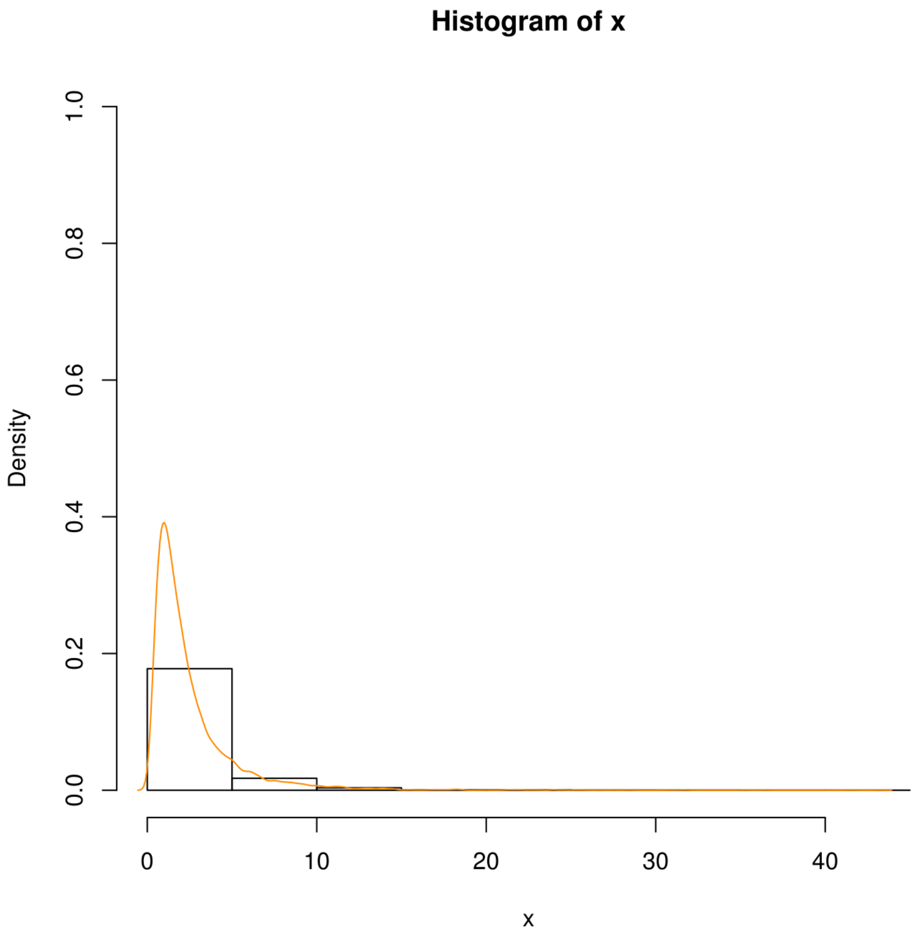
Histograms:

For $\lambda = -1$

CDF of X:

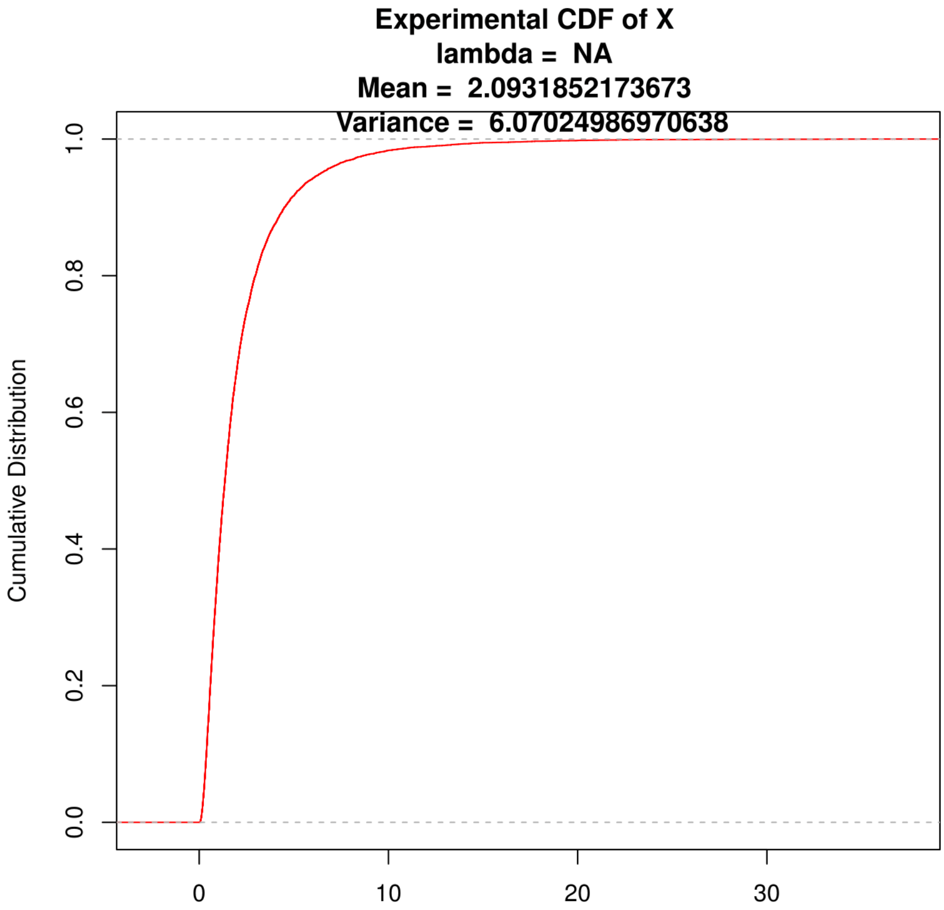


Density of X:



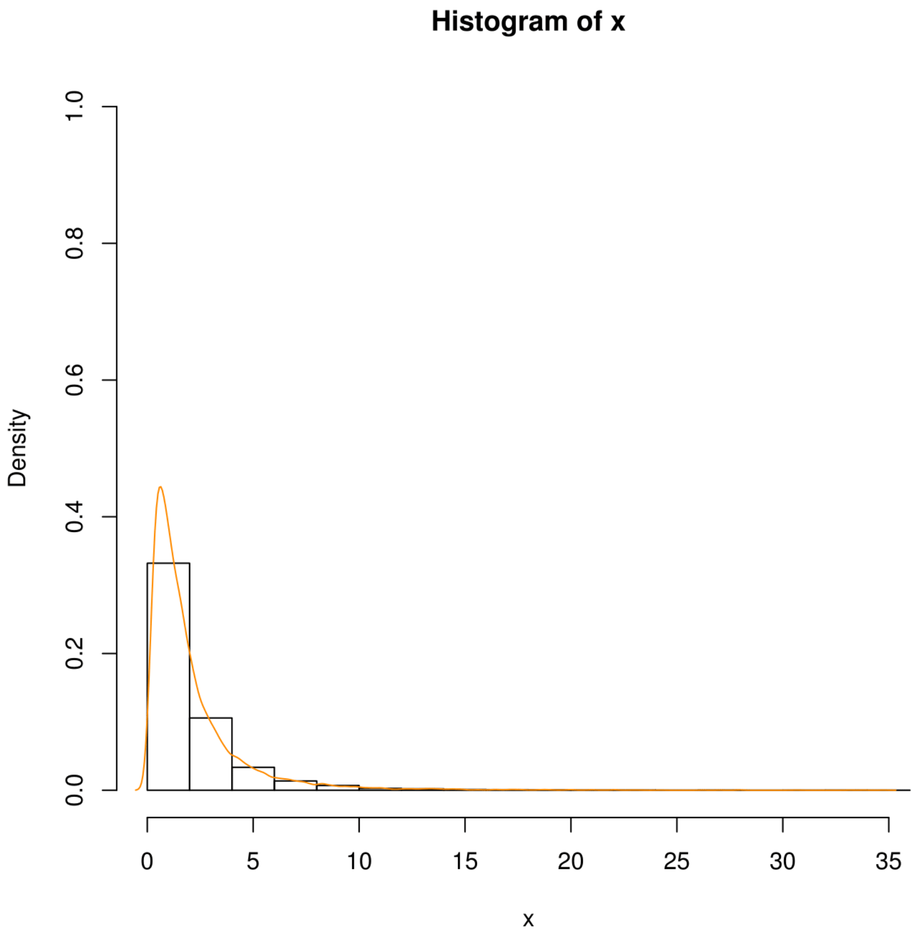
For $\lambda = -0.5$

CDF of X:



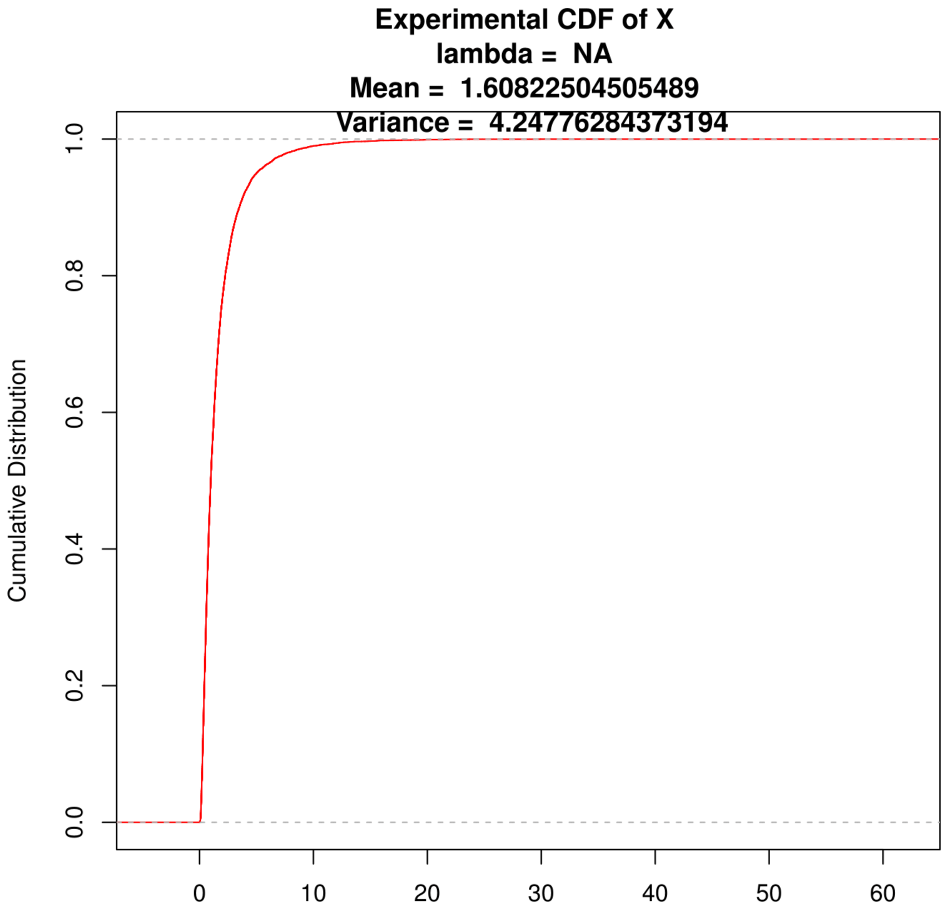
Experimental CDF of X
lambda = NA
Mean = 2.0931852173673
Variance = 6.07024986970638
Cumulative Distribution
0.0
0.2
0.4
0.6
0.8
1.0
0
10
20
30

Density of X:

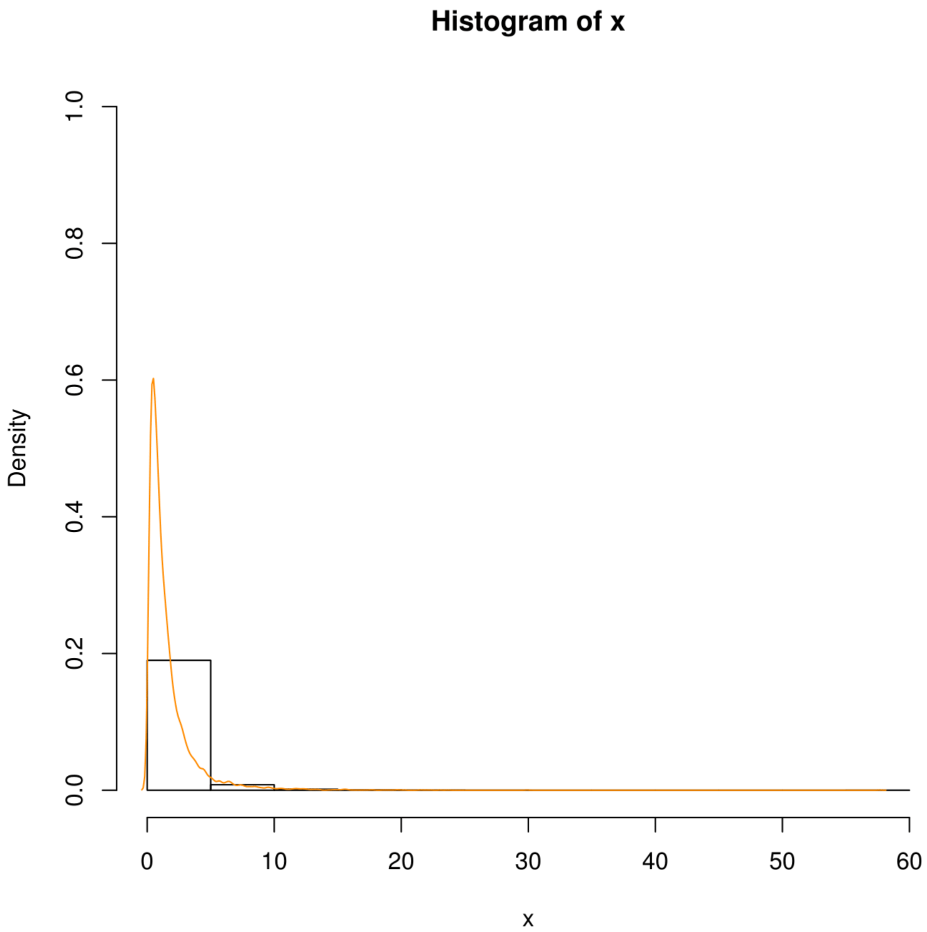


For $\lambda = 0$

CDF of X:

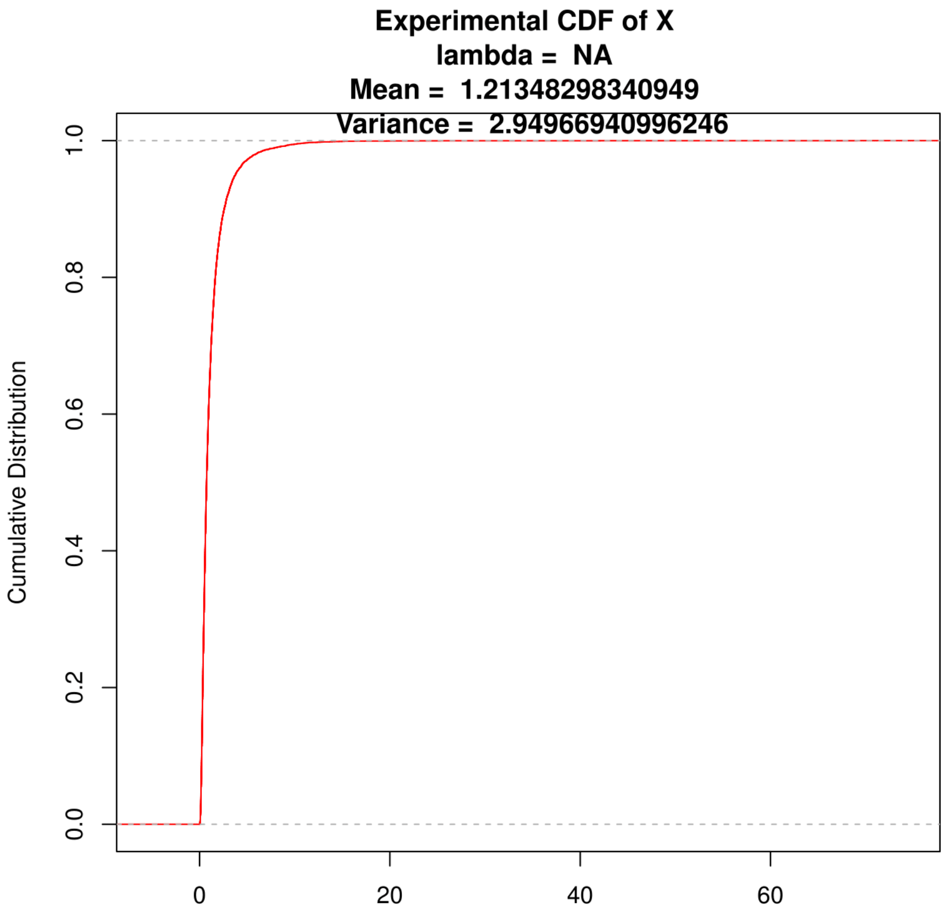

Experimental CDF of X
lambda = NA
Mean = 1.60822504505489
Variance = 4.24776284373194
Cumulative Distribution
0.0
0.2
0.4
0.6
0.8
1.0
0
10
20
30
40
50
60

Density of X:

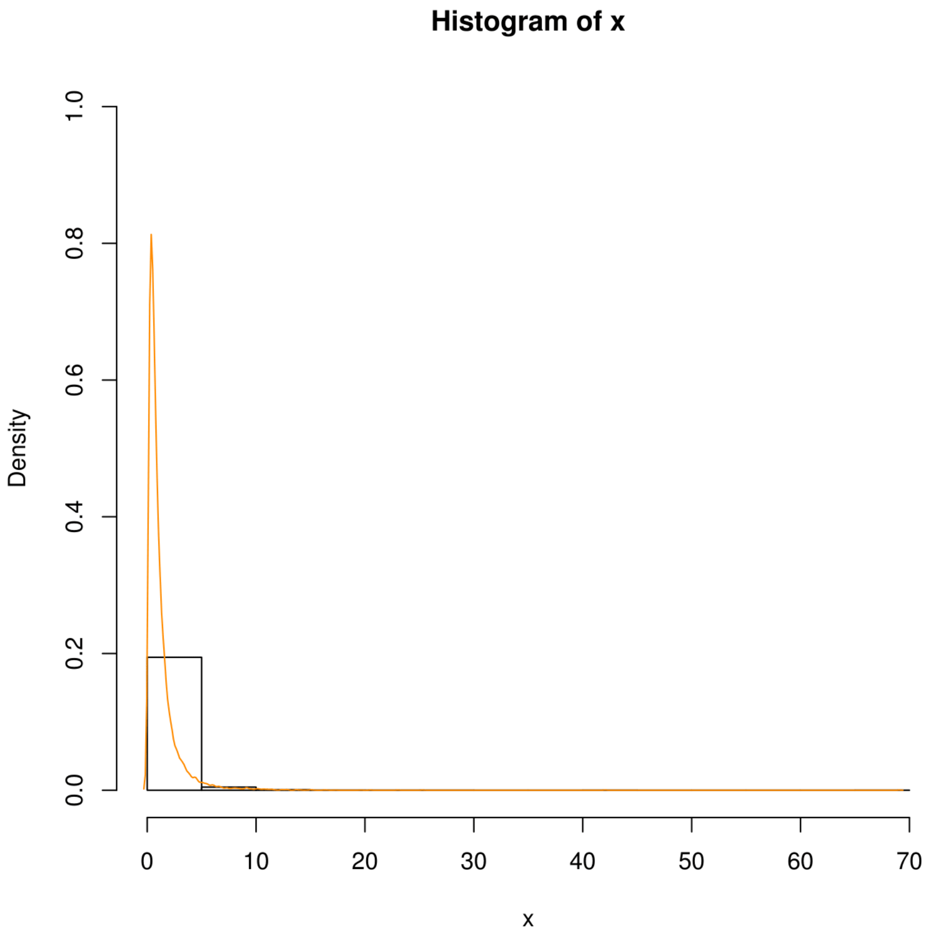


For $\lambda = 0.5$

CDF of X:

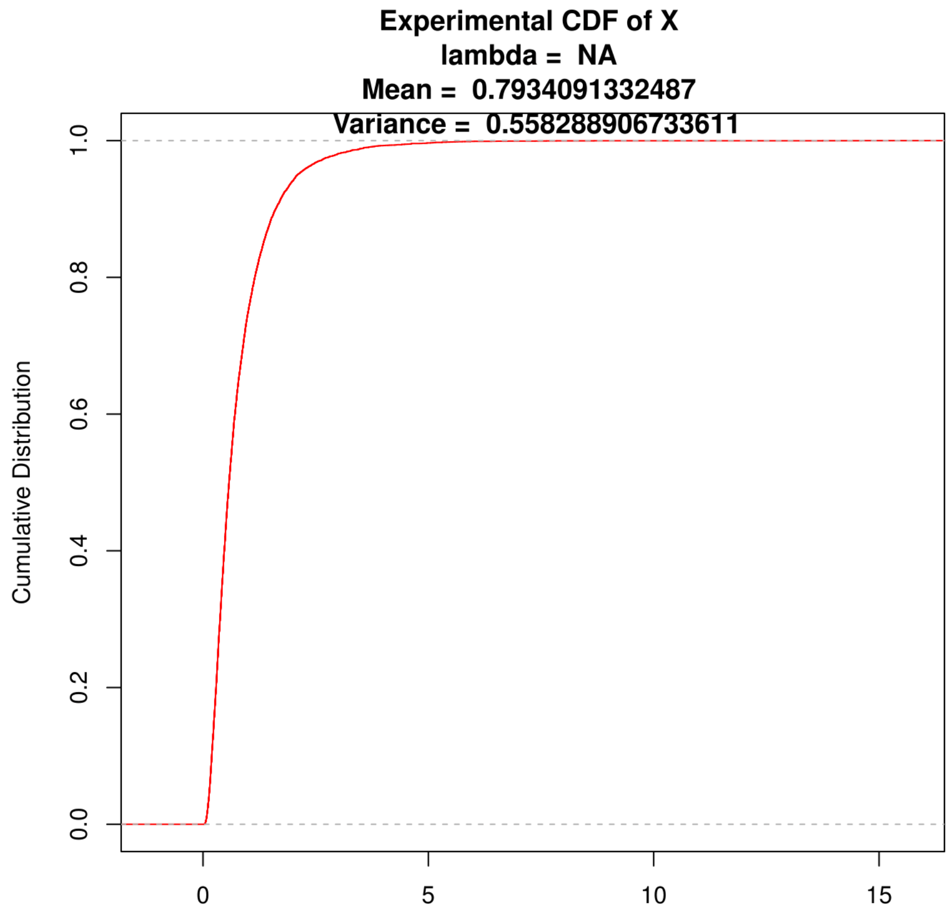

Experimental CDF of X
lambda = NA
Mean = 1.21348298340949
Variance = 2.94966940996246
Cumulative Distribution
0.0
0.2
0.4
0.6
0.8
1.0
0
20
40
60

Density of X:

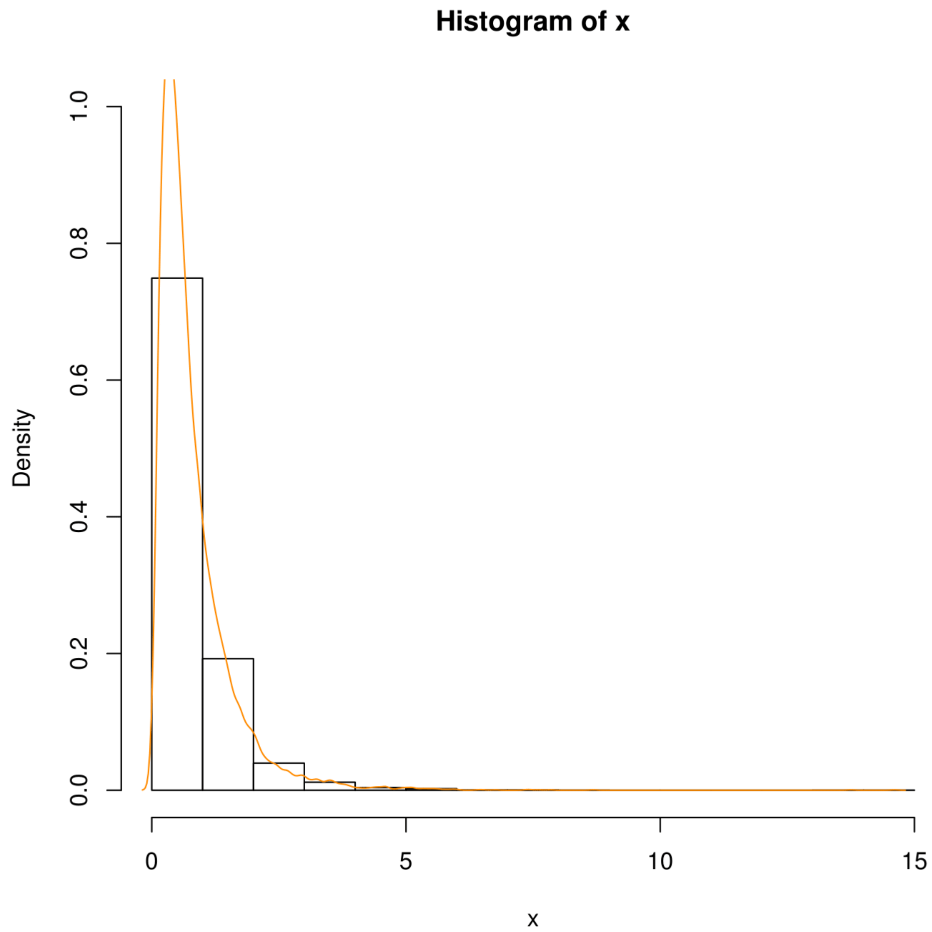


For $\lambda = 1$

CDF of X:


Experimental CDF of X
lambda = NA
Mean = 0.7934091332487
Variance = 0.558288906733611
Cumulative Distribution
0.0
0.2
0.4
0.6
0.8
1.0
0
5
10
15

Density of X:


Histogram of x
Density
x
0.0
0.2
0.4
0.6
0.8
1.0
0
5
10
15

Q2: Use Marshall Olkin Weibull Distribution to generate random numbers having the CDF F^* such that λ satisfies $-1 \leq \lambda \leq 1$,
$F^*(x) = (1+\lambda)F - \lambda F^2$,

Ans:

We have Marshall Olkin Weibull Distribution which is given as follows :

$W_1 = 1 - e^{-\alpha x^{\beta_1}}$
$W_2 = 1 - e^{-\alpha x^{\beta_2}}$
$W_3 = 1 - e^{-\alpha x^{\beta_3}}$
These are 3 webull distributions.
Marshall Olkin Weibull Distribution is bivariate in nature given by two random variables :
$X_1 = min(w_1, w_2)$
$X_2 = min(w_1, w_3)$
(X_1, X_2) follows this distribution.
Now , F is its CDF which depends on $\alpha, \beta_1, \beta_2, \beta_3$.Also , its probability density function depends on these parameters.
So, now $F^*(X) = (1+\lambda)F - \lambda F^2$
$f^*(x)$ is the pdf of the new distribution.
Upon differentiating $F^*(x)$ we get :
$f^*(x) \ = (1+\lambda)f - 2\lambda(Ff + f_x f_y)$
Moreover in Marshall Olkin Weibull Distribution:
$Ff = f_x f_y$
So,$f^*(x) \ = (1+\lambda)f - 2\lambda(2Ff)$
Hence,$f^*(x) \ = (1+\lambda)f - 4\lambda(Ff)$..............**1)**

By acceptance rejection method: $f(x) = \frac{d^2F(x_1,x_2)}{dx_1 dx_2}$
So,$\frac{f^*(x)}{f(x)} \leq c$ where c $\in \mathbb{R}$
Now, from 1)
$\frac{f^*(x)}{f(x)} = \frac{(1+\lambda)f - 4\lambda fF}{f}$.......**2)**

So,maximum value is attained when F=1
Hence,putting F=1 in 2) we get,
$c \geq (1 - 3\lambda)$
So, we get the value of c as atleast 1-3λ.

Code for R

```
library(MASS)
#F* = (1+lamda)F - (lamda)^2

# 1-D Weibull
weibull_pdf_1d <- function(x, alpha, theta)
{
        return (alpha * theta * x^(alpha - 1) * exp(-theta * x^alpha))
}

weibull_se_1d <- function(x, alpha, theta)
{
        return (exp(-theta * x^alpha))
}

weibull_cdf_1d <- function(x, alpha, theta)
{
        return (1 - weibull_se_1d(x, alpha, theta))
}

#Generates n 1-D weibull numbers with given parameters
weibull_generator_1d <- function(alpha, theta, n)
{
        u <- runif(n)
        return (((-1/theta) * log(u))^(1/alpha))
}

# 2-D Marshall-Olkin bivariate Weibull
mobw_pdf_2d <- function(x1, x2, alpha, lamda0, lamda1, lamda2)
{
        if(x1 < x2)
        {
                return (weibull_pdf_1d(x1, alpha, lamda1) *
                 weibull_pdf_1d(x2, alpha, lamda0 + lamda2))
        }
        else if (x1 > x2)
        {
                return (weibull_pdf_1d(x1, alpha, lamda0 + lamda1) *
                 weibull_pdf_1d(x2, alpha, lamda2))
        }
        else
        {
                return ((lamda0/(lamda0 + lamda1 + lamda2)) *
                 weibull_pdf_1d(x1, alpha, lamda0 + lamda1 + lamda2))
```

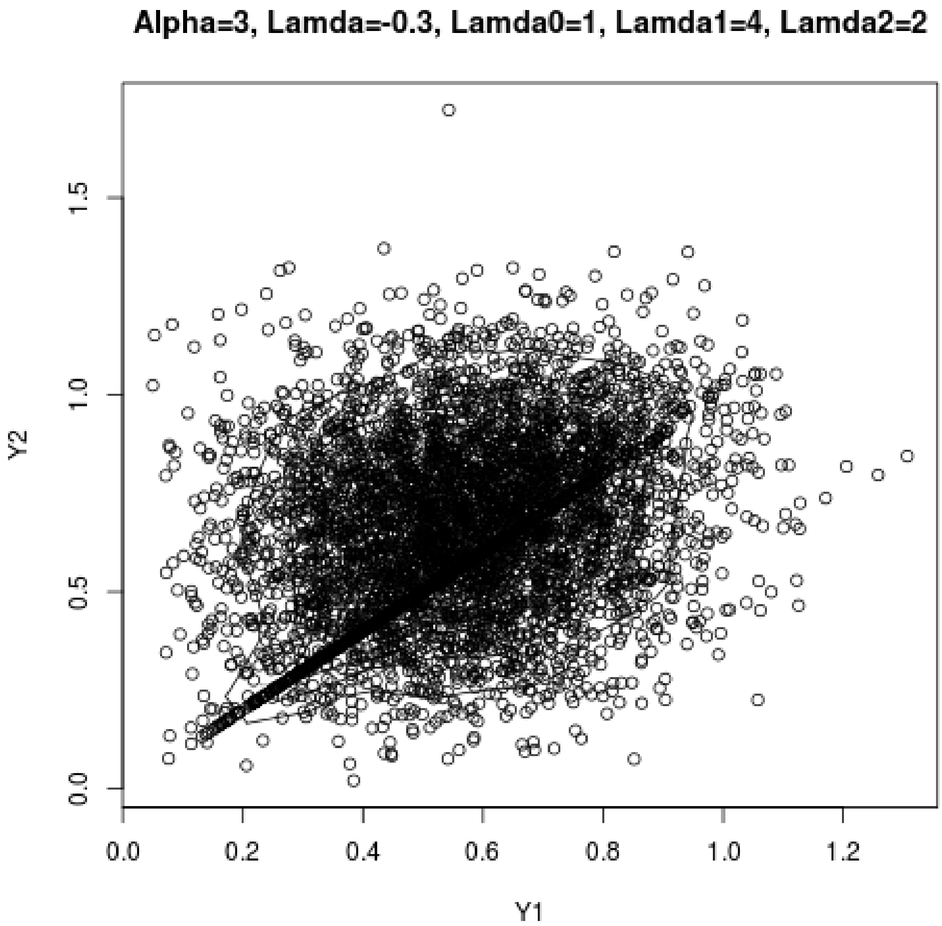
```
        }
}

mobw_se_2d <- function(x1, x2, alpha, lamda0, lamda1, lamda2)
{
        z <- pmax(x1, x2)
        return (weibull_se_1d(x1, alpha, lamda1) *
         weibull_se_1d(x2, alpha, lamda2) * weibull_se_1d(z, alpha, lamda0))
}

mobw_cdf_2d <- function(x1, x2, alpha, lamda0, lamda1, lamda2)
{
        if(x1 < x2)
        {
                return (weibull_cdf_1d(x1, alpha, lamda1) *
                 weibull_cdf_1d(x2, alpha, lamda0 + lamda2))
        }
        else if (x1 > x2)
        {
                return (weibull_cdf_1d(x1, alpha, lamda0 + lamda1) *
                 weibull_cdf_1d(x2, alpha, lamda2))
        }
        else
        {
                return ((lamda0/(lamda0 + lamda1 + lamda2)) *
                 weibull_cdf_1d(x1, alpha, lamda0 + lamda1 + lamda2))
        }
}

mobw_cdf_partial_x1 <- function(x1, x2, alpha, lamda0, lamda1, lamda2)
{
        if(x1 < x2)
        {
                return (weibull_pdf_1d(x1, alpha, lamda1) *
                 weibull_cdf_1d(x2, alpha, lamda0 + lamda2))
        }
        else if (x1 > x2)
        {
                return (weibull_pdf_1d(x1, alpha, lamda0 + lamda1) *
                 weibull_cdf_1d(x2, alpha, lamda2))
        }
        else
        {
```

```
                return ((lamda0/(lamda0 + lamda1 + lamda2)) *
                 weibull_pdf_1d(x1, alpha, lamda0 + lamda1 + lamda2))
        }
}

mobw_cdf_partial_x2 <- function(x1, x2, alpha, lamda0, lamda1, lamda2)
{
        if(x1 < x2)
        {
                return (weibull_cdf_1d(x1, alpha, lamda1) *
                 weibull_pdf_1d(x2, alpha, lamda0 + lamda2))
        }
        else if (x1 > x2)
        {
                return (weibull_cdf_1d(x1, alpha, lamda0 + lamda1) *
                 weibull_pdf_1d(x2, alpha, lamda2))
        }
        else
        {
                return ((lamda0/(lamda0 + lamda1 + lamda2)) *
                 weibull_cdf_1d(x1, alpha, lamda0 + lamda1 + lamda2))
        }
}

# PDF of our distribution function
f <- function(x1, x2, lamda, alpha, lamda0, lamda1, lamda2)
{
        return ( (1 + lamda) *
         mobw_pdf_2d(x1, x2, alpha, lamda0, lamda1, lamda2)
- lamda*2*( (mobw_cdf_partial_x2(x1, x2, alpha, lamda0, lamda1, lamda2)
        * mobw_cdf_partial_x1(x1, x2, alpha, lamda0, lamda1, lamda2))
        + (mobw_cdf_2d(x1, x2, alpha, lamda0, lamda1, lamda2)
        * mobw_pdf_2d(x1, x2, alpha, lamda0, lamda1, lamda2)) ) )
}

#Parameters for bivariate weibull
n <- 10000
alpha <- vector(,2)      #TODO select about 2 values >0
lamda <- vector(,2)      #TODO select some value in 0 to -1
lamda0 <- vector(,2) #TODO select about 2 values >0
lamda1 <- vector(,2)     #TODO select about 2 values >0
lamda2 <- vector(,2)     #TODO select about 2 values >0
alpha[1] <- 3;
```

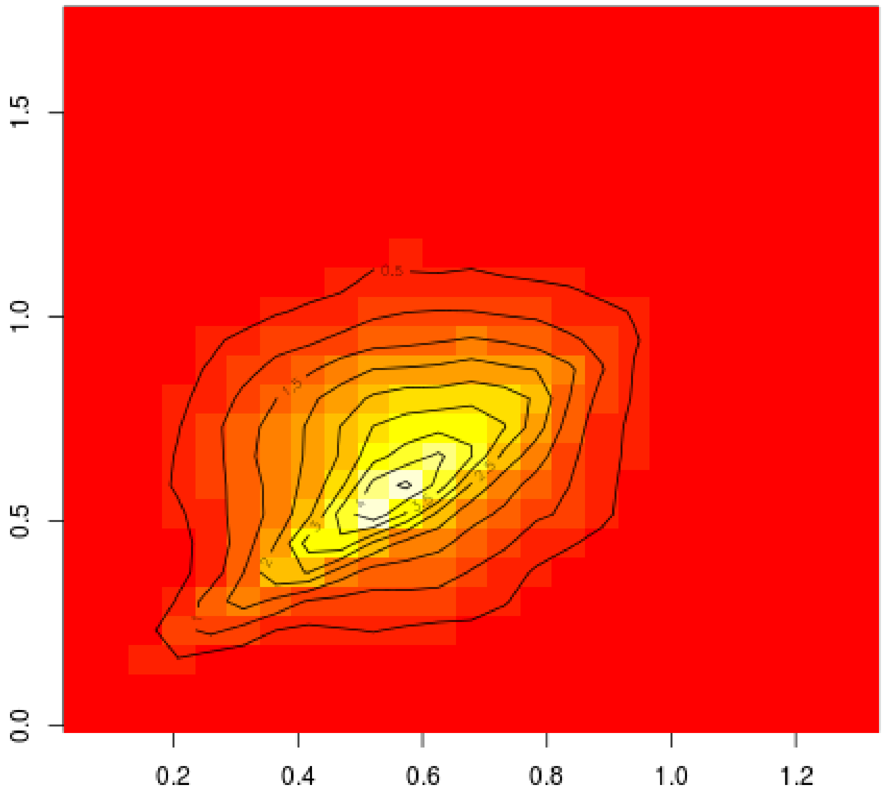
```
alpha[2] <- 7;
lamda[1] <- -0.3;
lamda[2] <- -0.75;
lamda0[1] <- 1;
lamda0[2] <- 5;
lamda1[1] <- 4;
lamda1[2] <- 8;
lamda2[1] <- 2;
lamda2[2] <- 9;
d <-1;

#TODO start loop
for(p in 1:2){
        Alpha <- alpha[p];

for(j in 1:2){
        Lamda <- lamda[j];

for(k in 1:2){
        Lamda0 <- lamda0[k];

for(l in 1:2){
        Lamda1 <- lamda1[l];

for(m in 1:2){
Lamda2 <- lamda2[m];
U0 <- weibull_generator_1d(Alpha, Lamda0, n)
U1 <- weibull_generator_1d(Alpha, Lamda1, n)
U2 <- weibull_generator_1d(Alpha, Lamda2, n)

# Here (X1, X2) follows Marshall-Olkin bivariate Weibull
# distribution with parameters (alpha, lamda0, lamda1, lamda2)
X1 <- pmin(U0, U1)
X2 <- pmin(U0, U2)

# Now using Acceptance-Rejection to find required distribution
# we take the sample distribution MOBW
c <- 1 - 3*Lamda
U <- runif(n)
Y1 <- vector(,0)
Y2 <- vector(,0)

for (i in 1:n) {
```

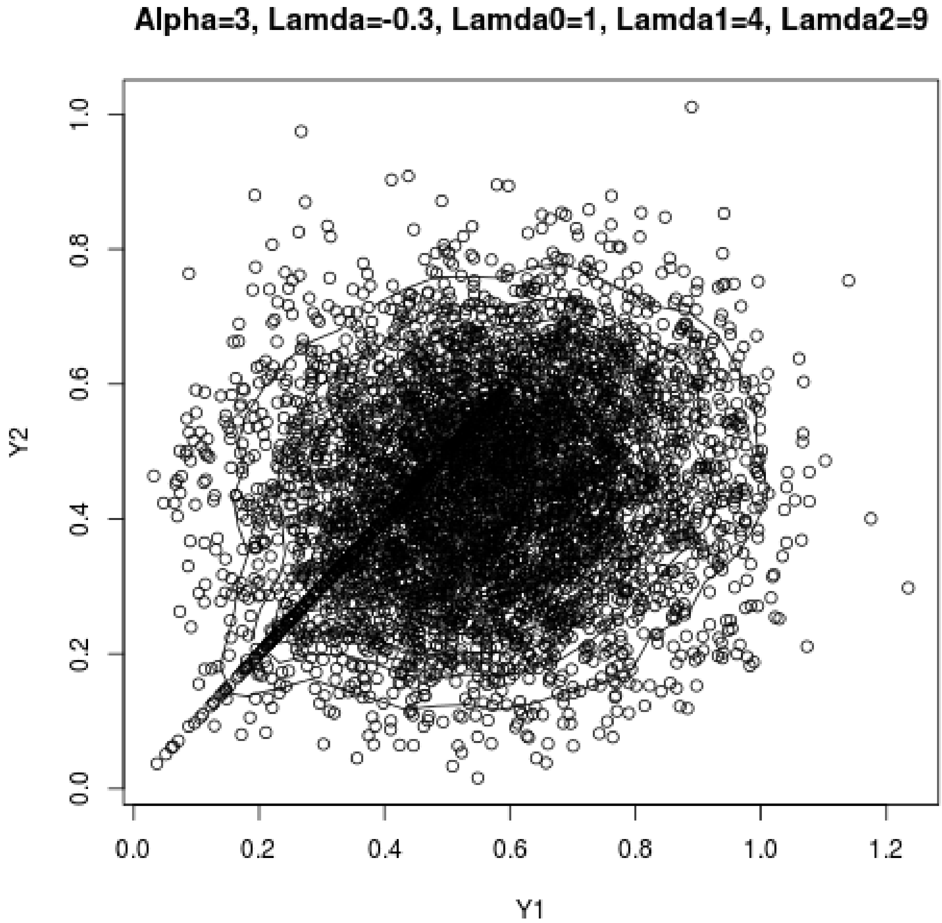
```
        x1 <- X1[i]
        x2 <- X2[i]
        u <- U[i]
        if (f(x1, x2, Lamda, Alpha, Lamda0, Lamda1, Lamda2) > c * u *
         mobw_pdf_2d(x1, x2, Alpha, Lamda0, Lamda1, Lamda2))
        {
                Y1 <- c(Y1, x1)
                Y2 <- c(Y2, x2)
        }
}

#TODO print images
png(paste0("Question2_",toString(d),"a.png"));
plot(Y1,Y2,main=paste0("Alpha=",toString(Alpha), ", Lamda=",
toString(Lamda),", Lamda0=", toString(Lamda0),", Lamda1=",
toString(Lamda1),", Lamda2=", toString(Lamda2) ))
z.kde=kde2d(Y1,Y2)
contour(z.kde, add = TRUE)
dev.off();

png(paste0("Question2_",toString(d),"b.png"));
plot(Y1,Y2,main=paste0("Alpha=",toString(Alpha), ",
 Lamda=", toString(Lamda),", Lamda0=", toString(Lamda0),",
 Lamda1=", toString(Lamda1),", Lamda2=", toString(Lamda2) ))
z.kde=kde2d(Y1,Y2)
contour(z.kde,add=TRUE)
image(z.kde);
contour(z.kde, add = TRUE)
dev.off();

d <- d+1;
#TODO end for loop

}
}
}
}
}

rm(list = ls())
```

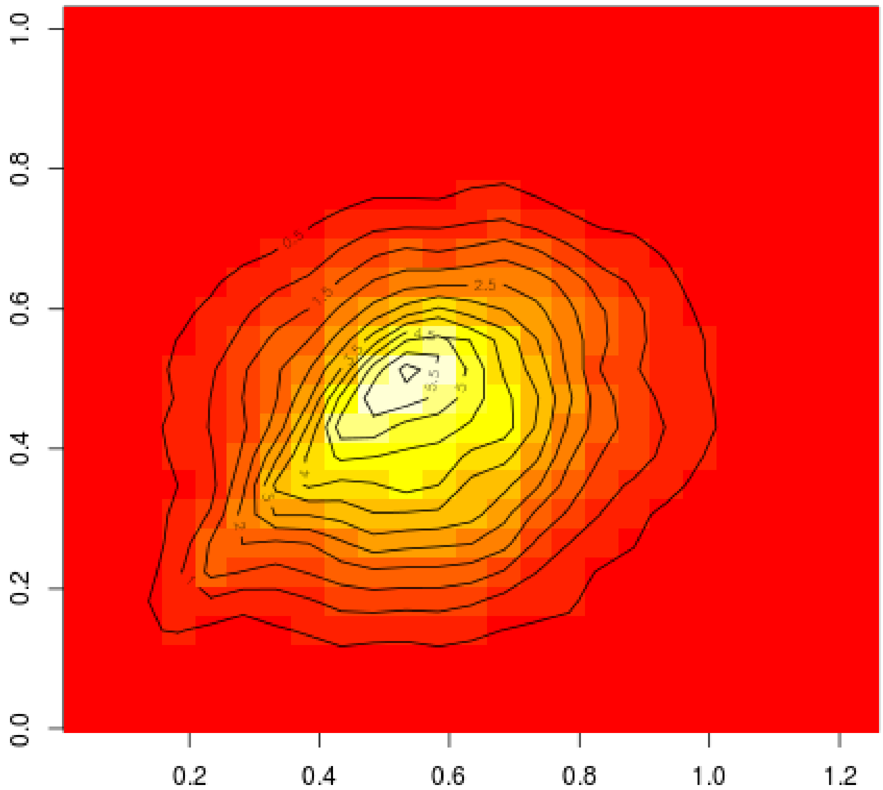
The following are the plots of the distribution of F_*.



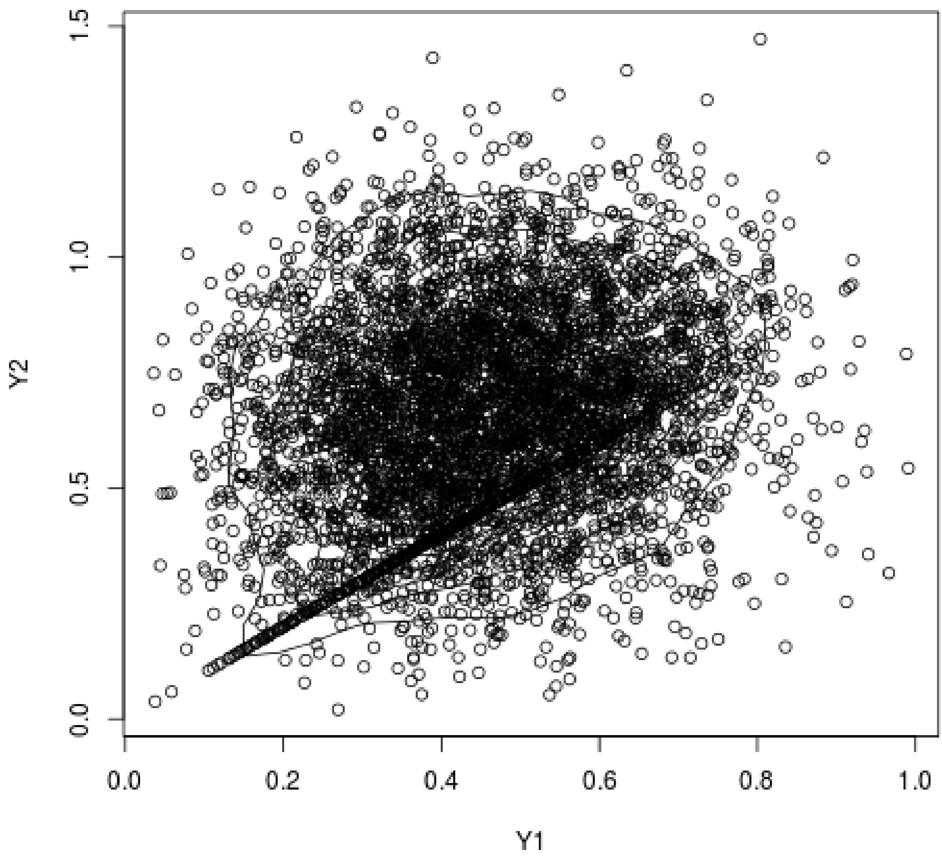
Alpha=3, Lamda=-0.3, Lamda0=1, Lamda1=4, Lamda2=2
Y2
0.0
0.5
1.0
1.5
Y1
0.0
0.2
0.4
0.6
0.8
1.0
1.2

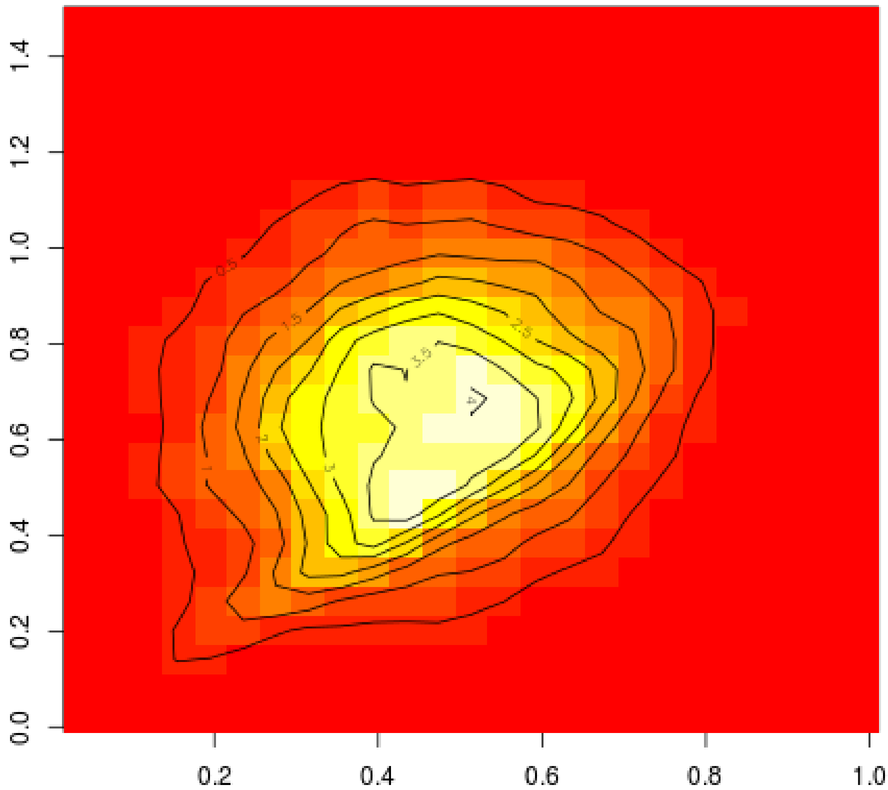

0.0
0.5
1.0
1.5
0.2
0.4
0.6
0.8
1.0
1.2
0.5
1.5

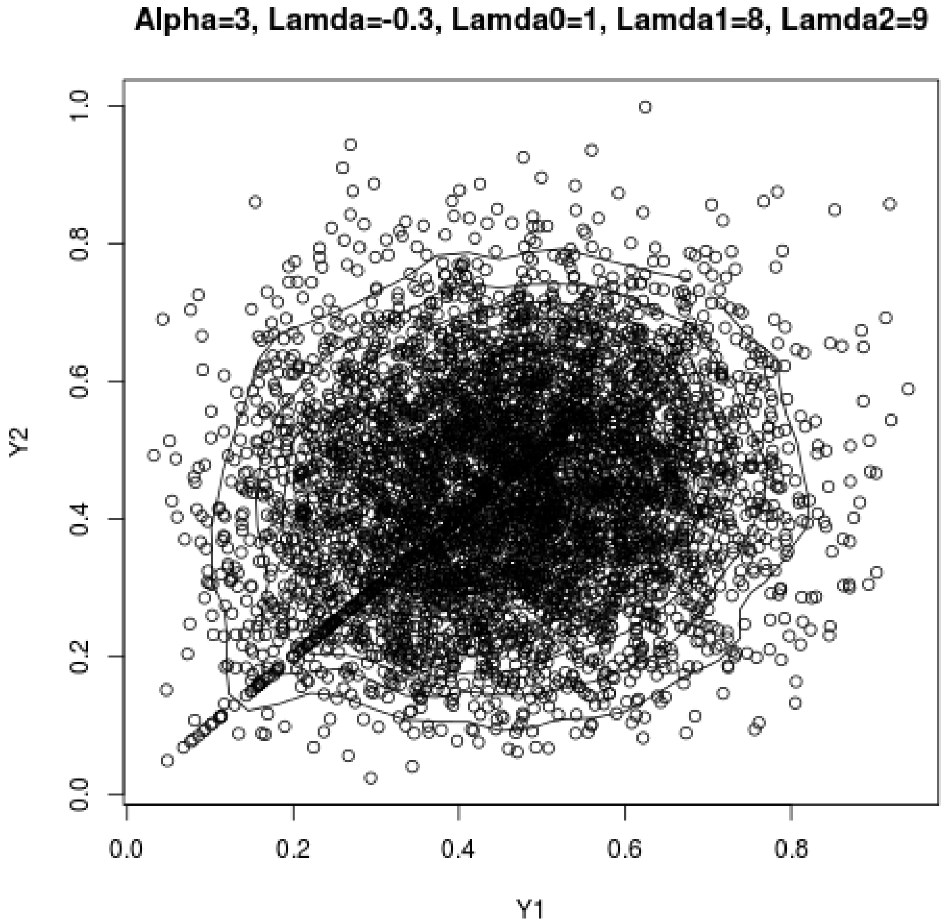

Alpha=3, Lamda=-0.3, Lamda0=1, Lamda1=4, Lamda2=9
Y2
Y1
0.0
0.2
0.4
0.6
0.8
1.0
1.2

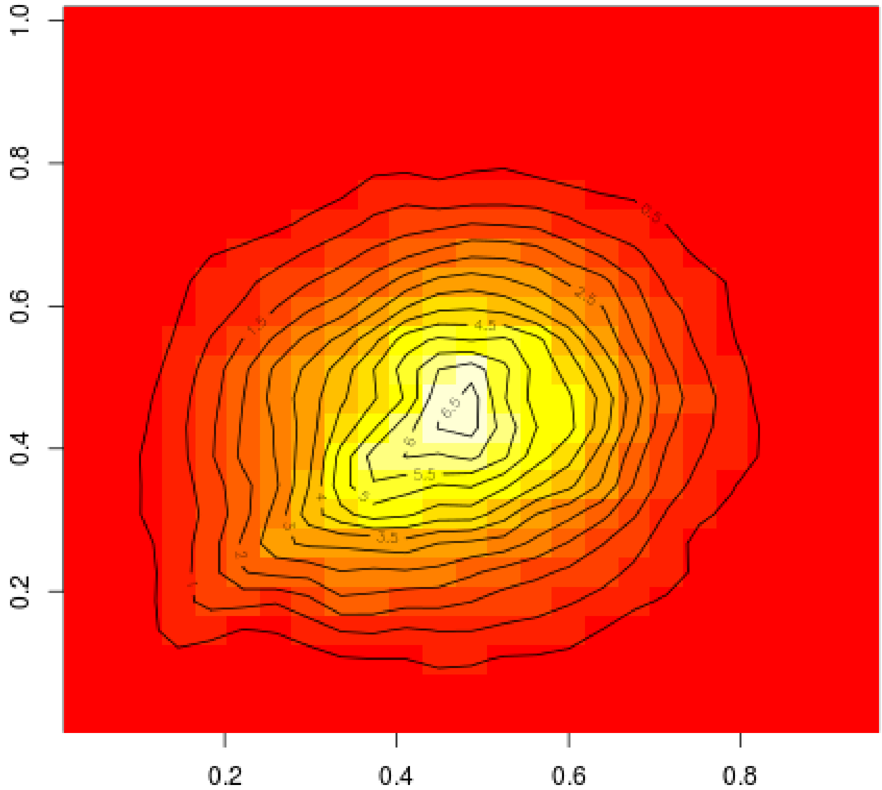

0.0
0.2
0.4
0.6
0.8
1.0
0.2
0.4
0.6
0.8
1.0
1.2

Alpha=3, Lamda=-0.3, Lamda0=1, Lamda1=8, Lamda2=2

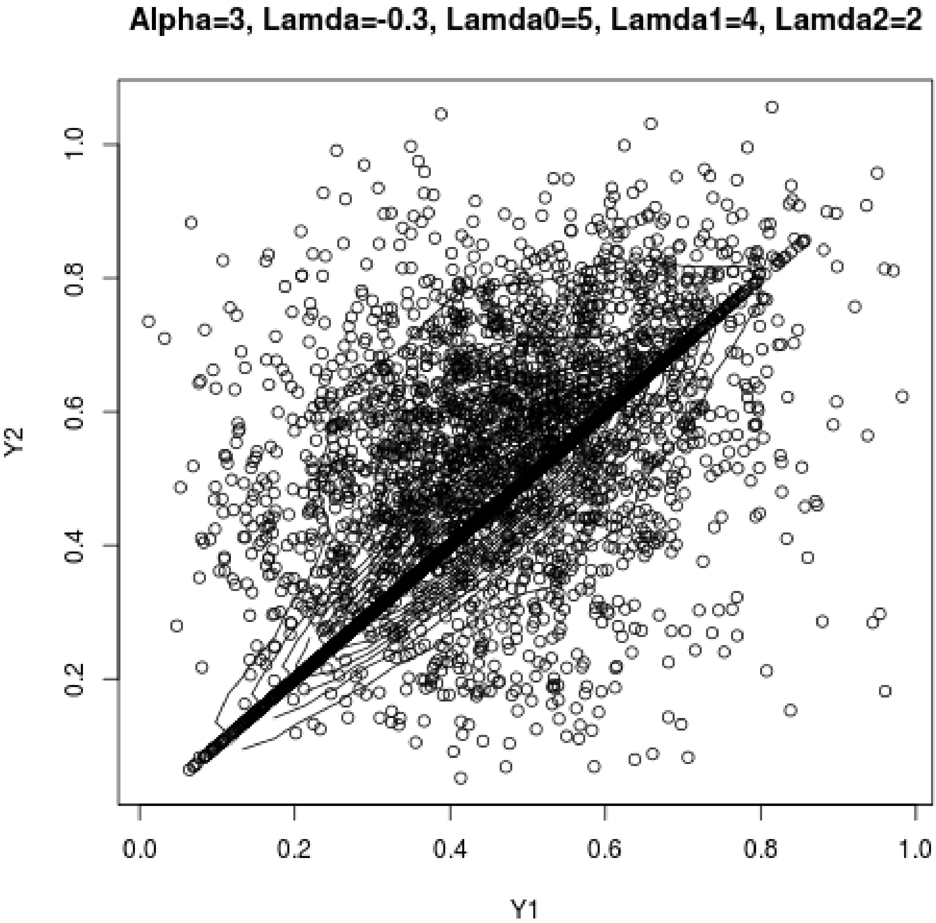


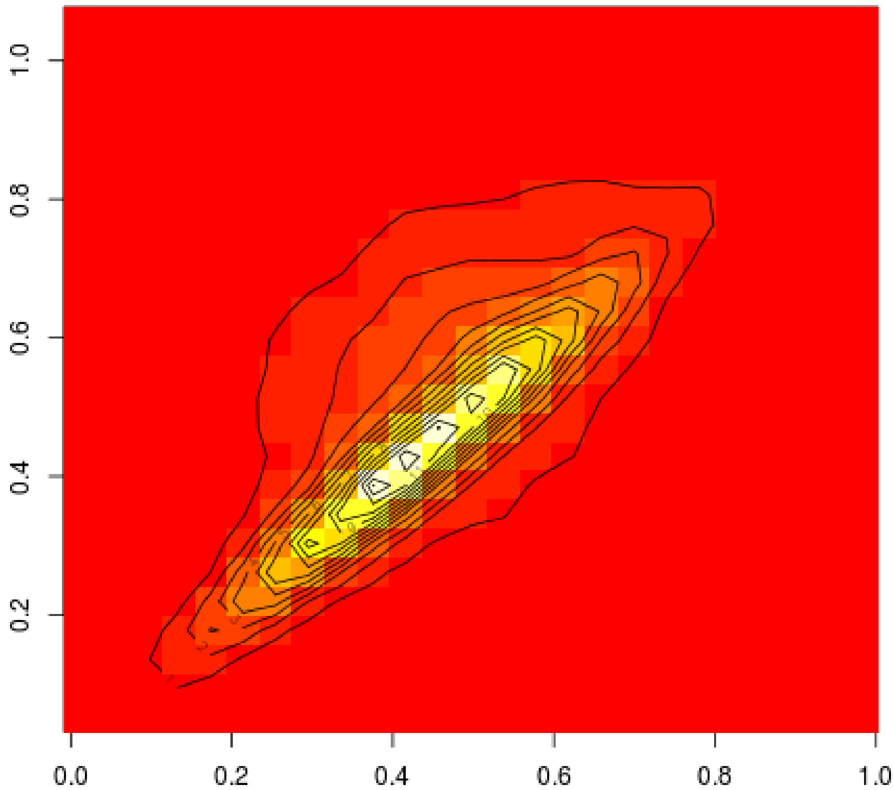

0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
0.2
0.4
0.6
0.8
1.0

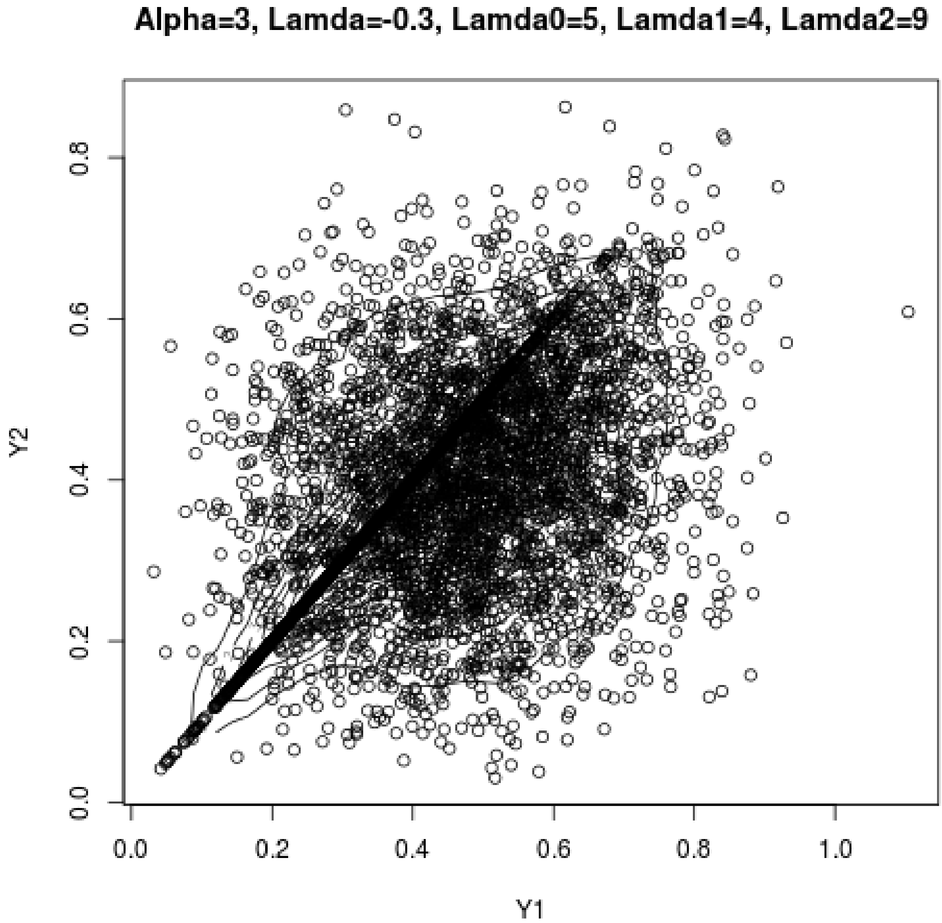

Alpha=3, Lamda=-0.3, Lamda0=1, Lamda1=8, Lamda2=9
Y2
0.0
0.2
0.4
0.6
0.8
1.0
Y1

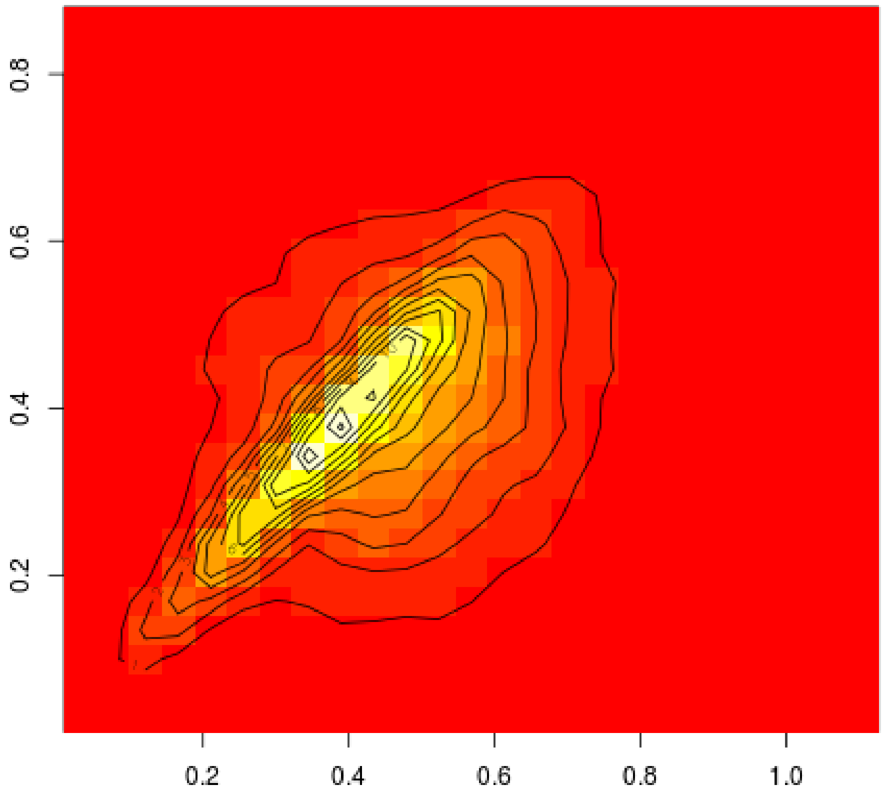


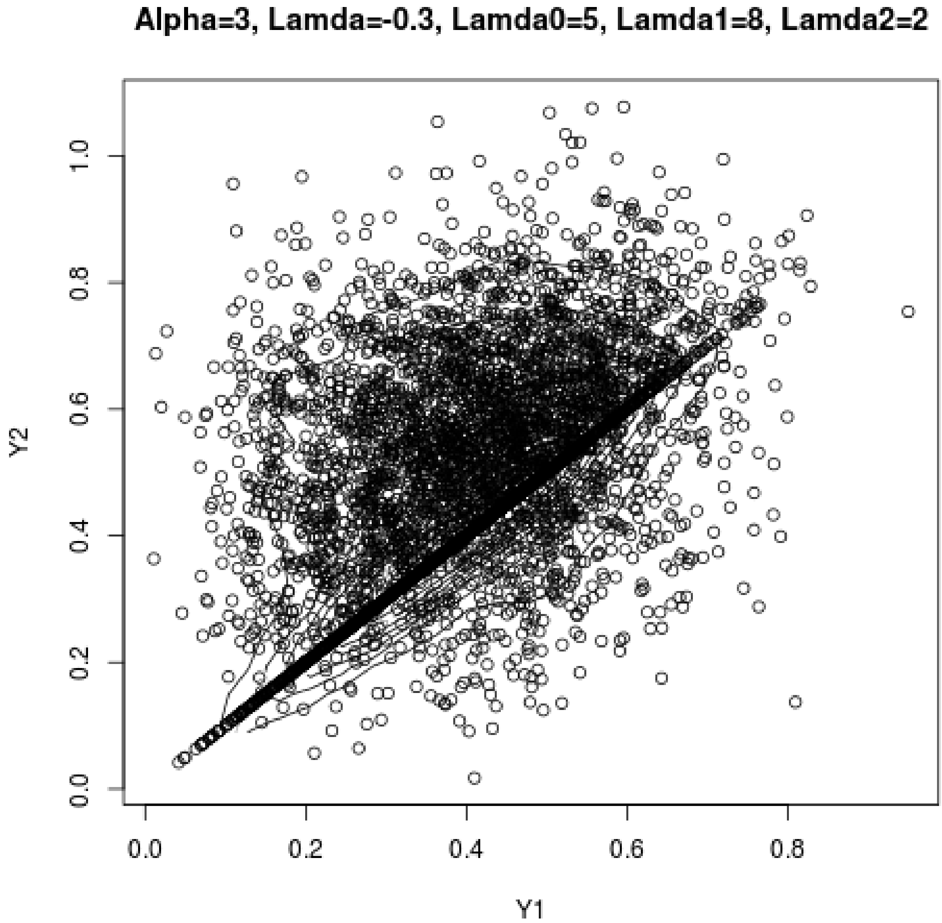
1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8

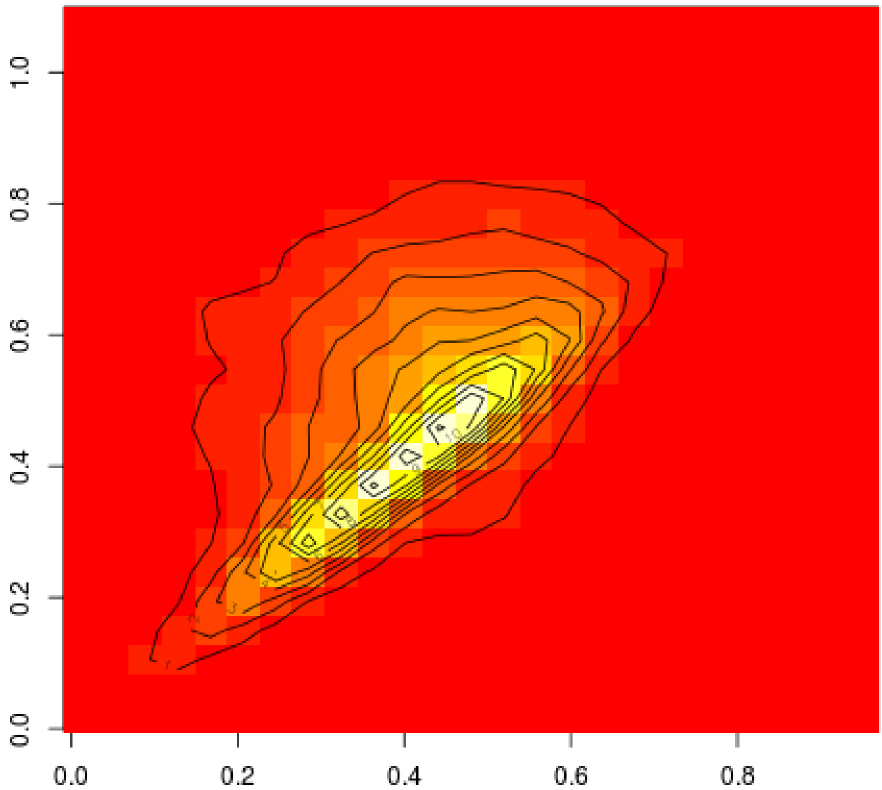

Alpha=3, Lamda=-0.3, Lamda0=5, Lamda1=4, Lamda2=2
Y2
1.0
0.8
0.6
0.4
0.2
0.0
0.2
0.4
0.6
0.8
1.0
Y1

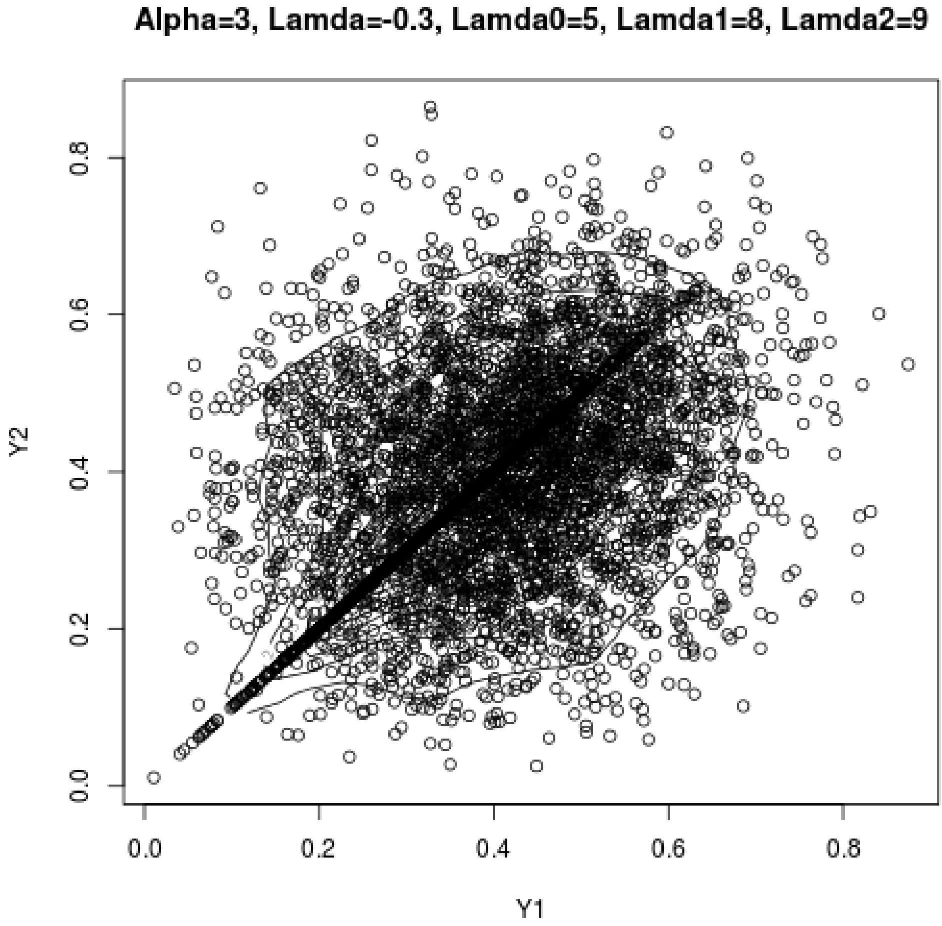

0.0
0.2
0.4
0.6
0.8
1.0
0.2
0.4
0.6
0.8
1.0


Alpha=3, Lamda=-0.3, Lamda0=5, Lamda1=4, Lamda2=9
Y2
0.0
0.2
0.4
0.6
0.8
Y1
0.0
0.2
0.4
0.6
0.8
1.0

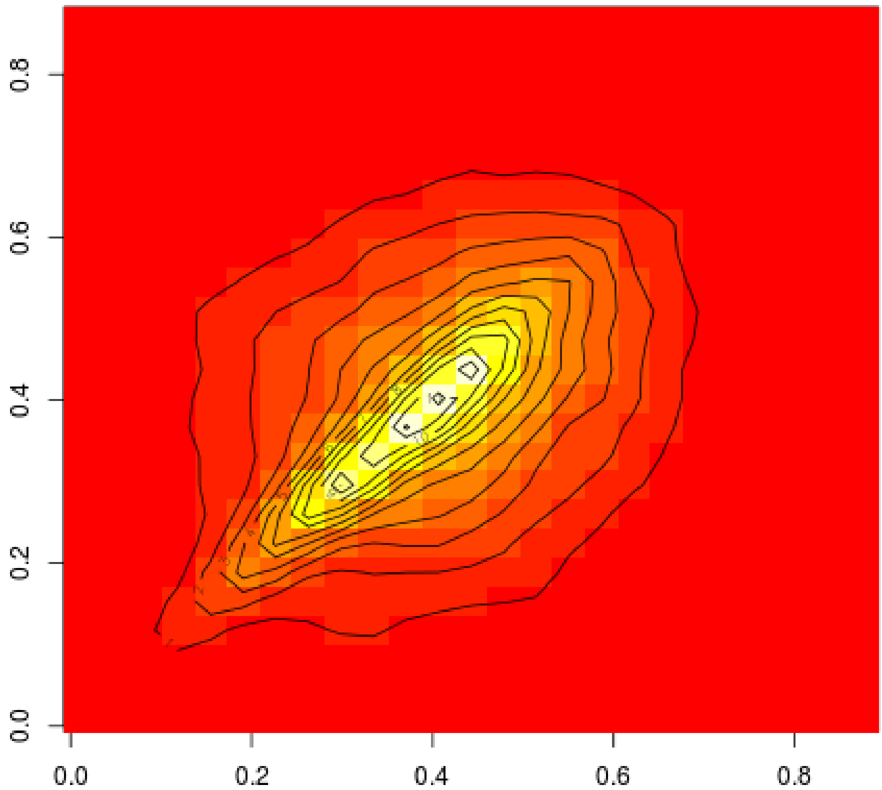

0.2
0.4
0.6
0.8
1.0
0.2
0.4
0.6
0.8

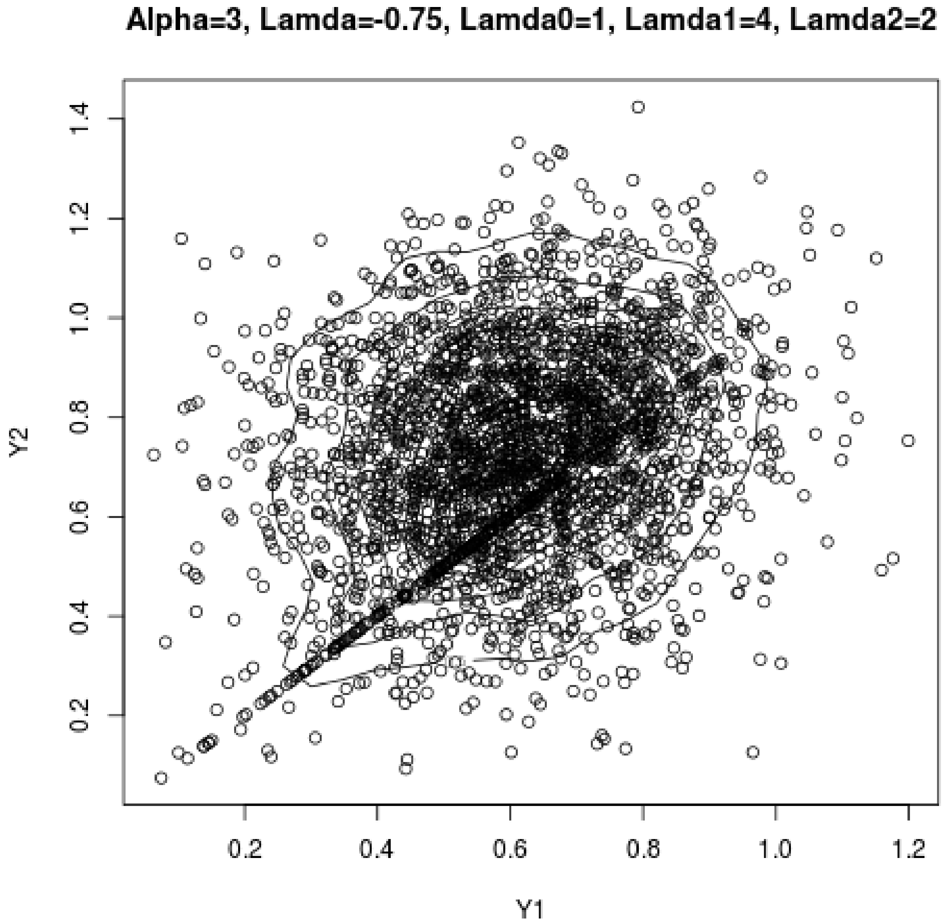

Alpha=3, Lamda=-0.3, Lamda0=5, Lamda1=8, Lamda2=2
Y2
Y1
0.0
0.2
0.4
0.6
0.8
1.0

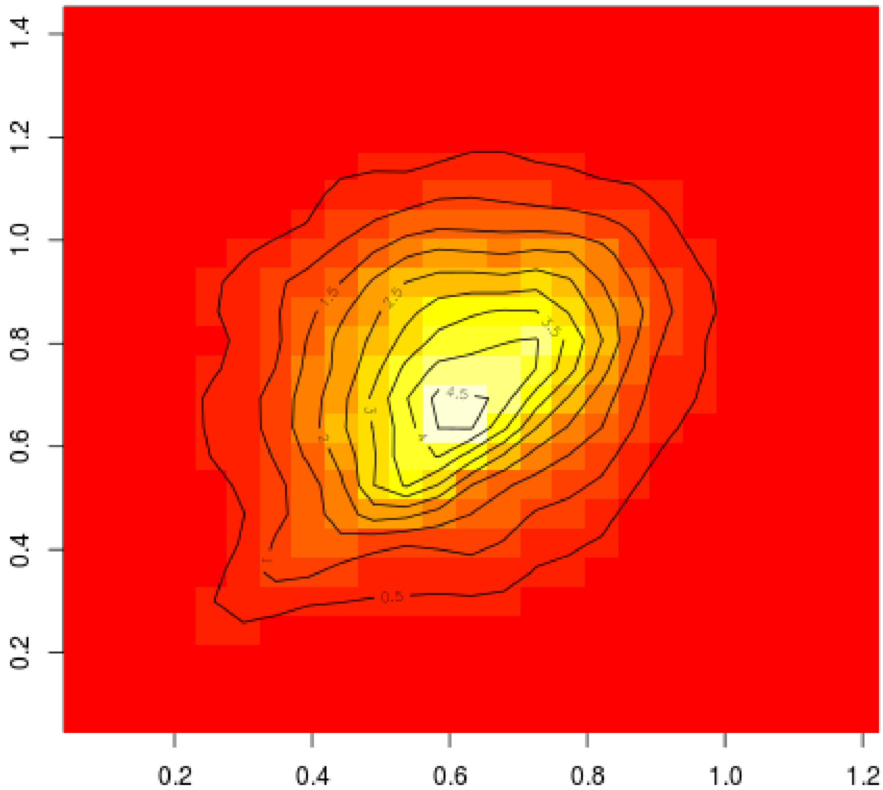

1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8



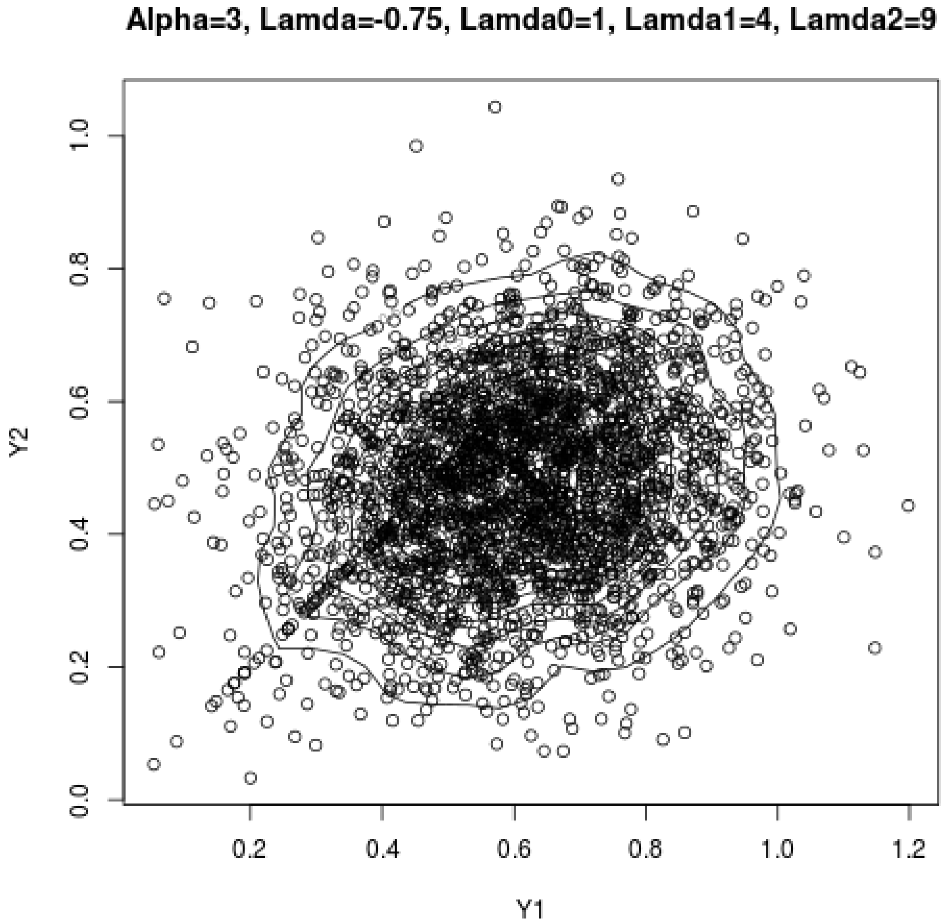
Alpha=3, Lamda=-0.3, Lamda0=5, Lamda1=8, Lamda2=9
Y2
0.0
0.2
0.4
0.6
0.8
Y1
0.0
0.2
0.4
0.6
0.8


0.0
0.2
0.4
0.6
0.8
0.0
0.2
0.4
0.6
0.8

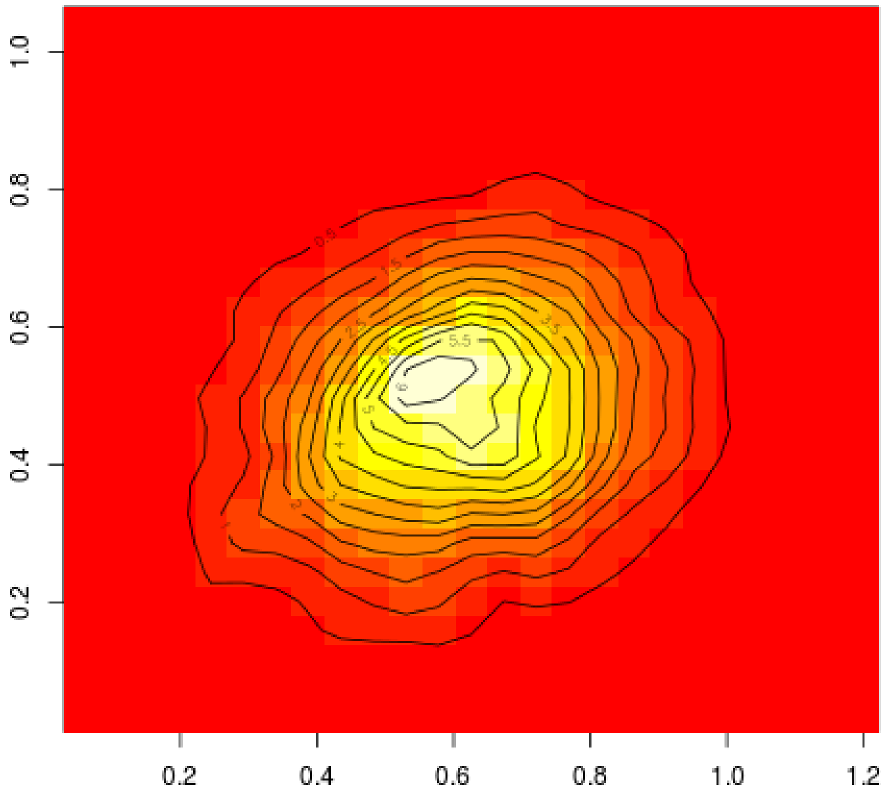

Alpha=3, Lamda=-0.75, Lamda0=1, Lamda1=4, Lamda2=2
Y2
Y1
0.2
0.4
0.6
0.8
1.0
1.2
1.4

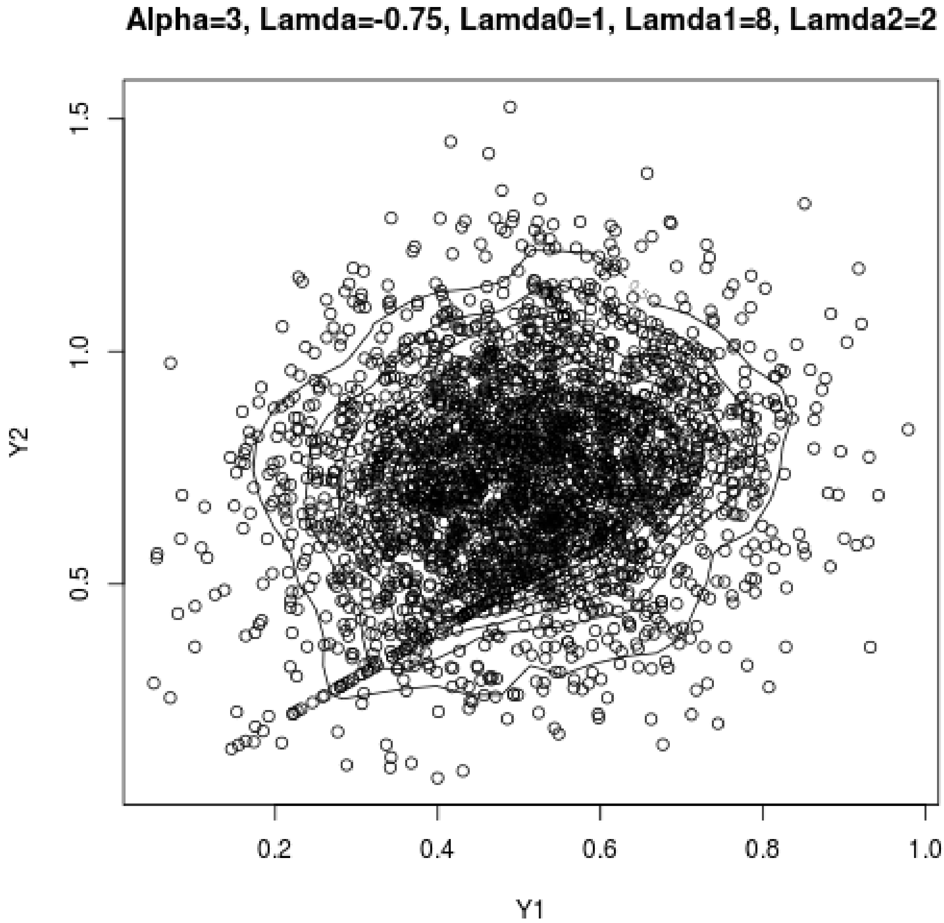


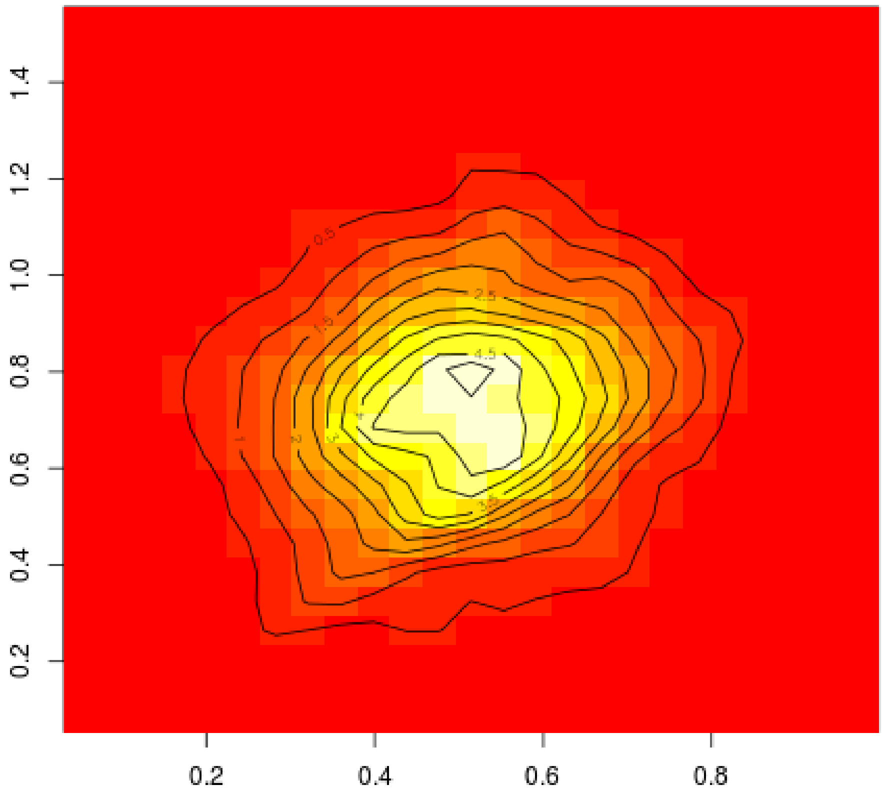
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0
1.2

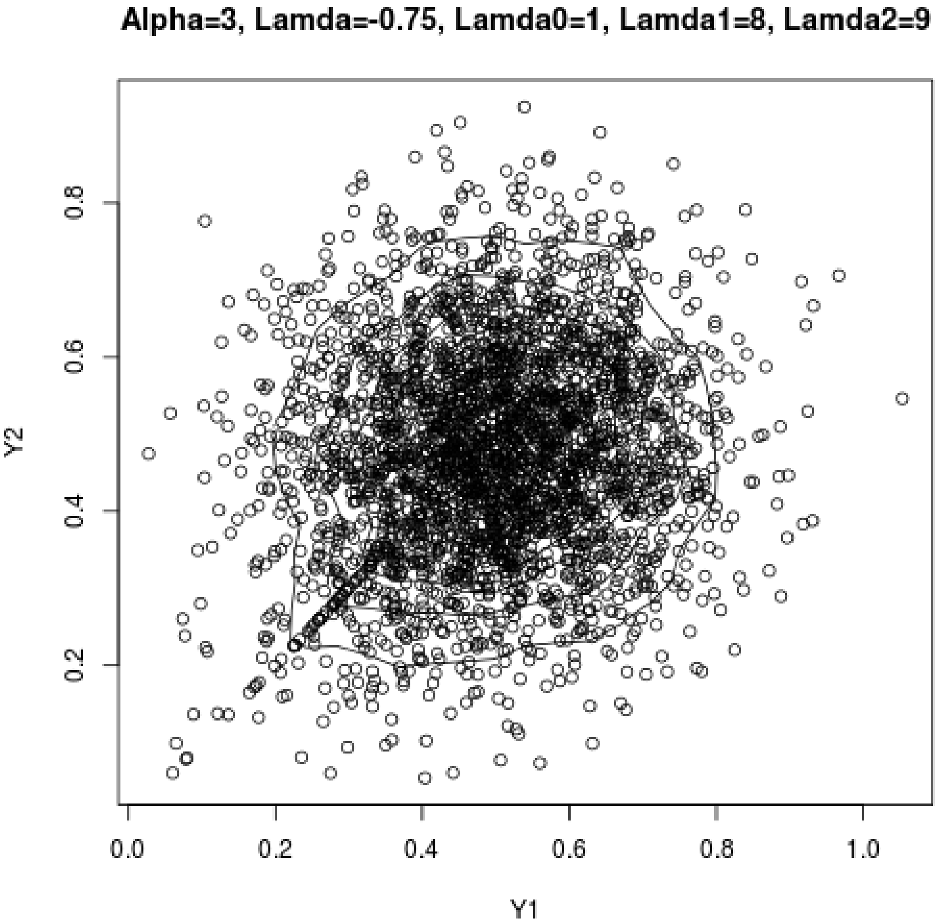


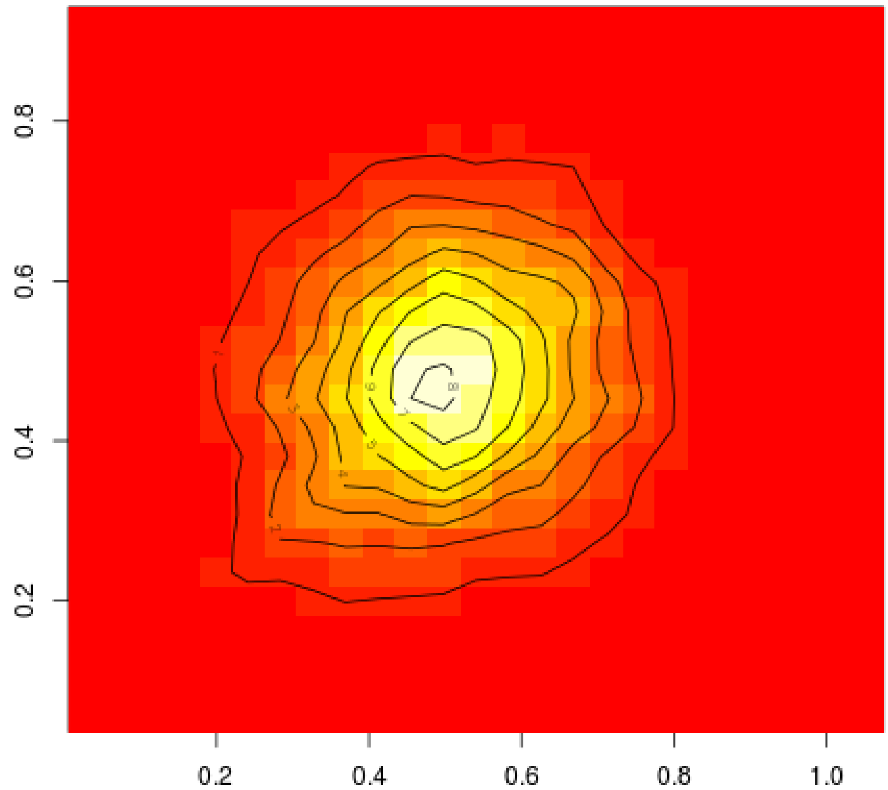
Alpha=3, Lamda=-0.75, Lamda0=1, Lamda1=4, Lamda2=9
Y2
Y1
0.0
0.2
0.4
0.6
0.8
1.0
0.2
0.4
0.6
0.8
1.0
1.2

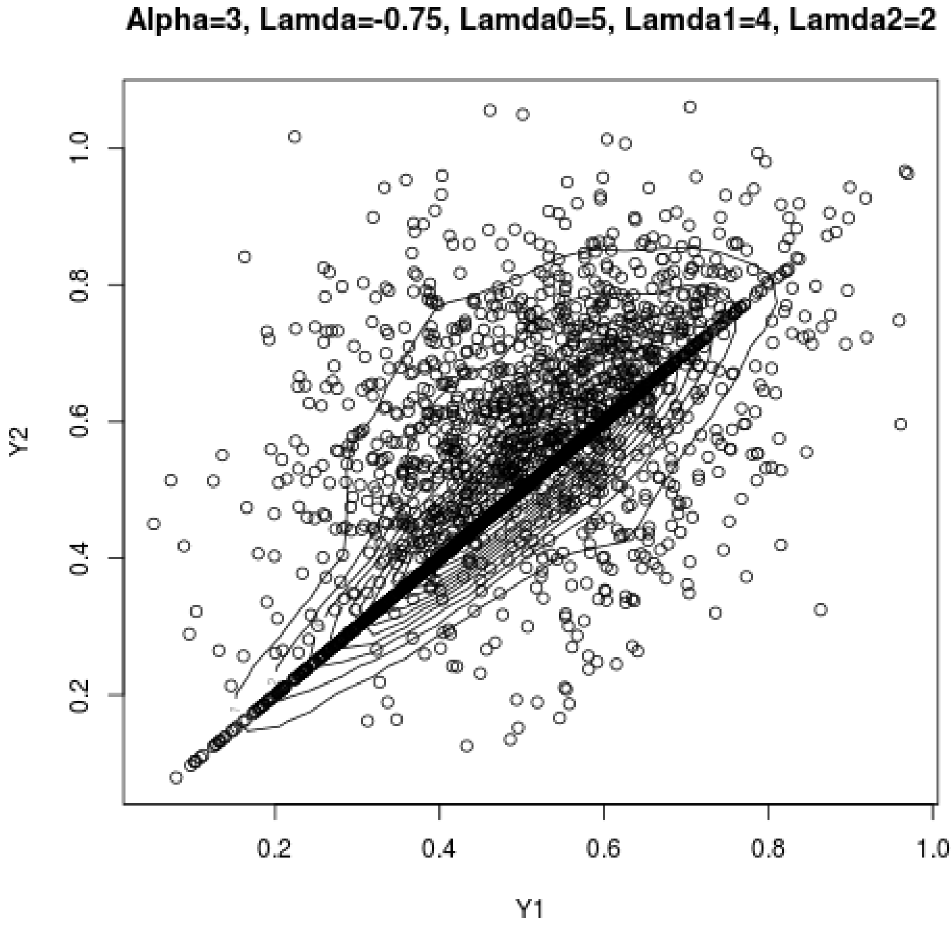

1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0
1.2


Alpha=3, Lamda=-0.75, Lamda0=1, Lamda1=8, Lamda2=2
Y2
1.5
1.0
0.5
0.2
0.4
0.6
0.8
1.0
Y1

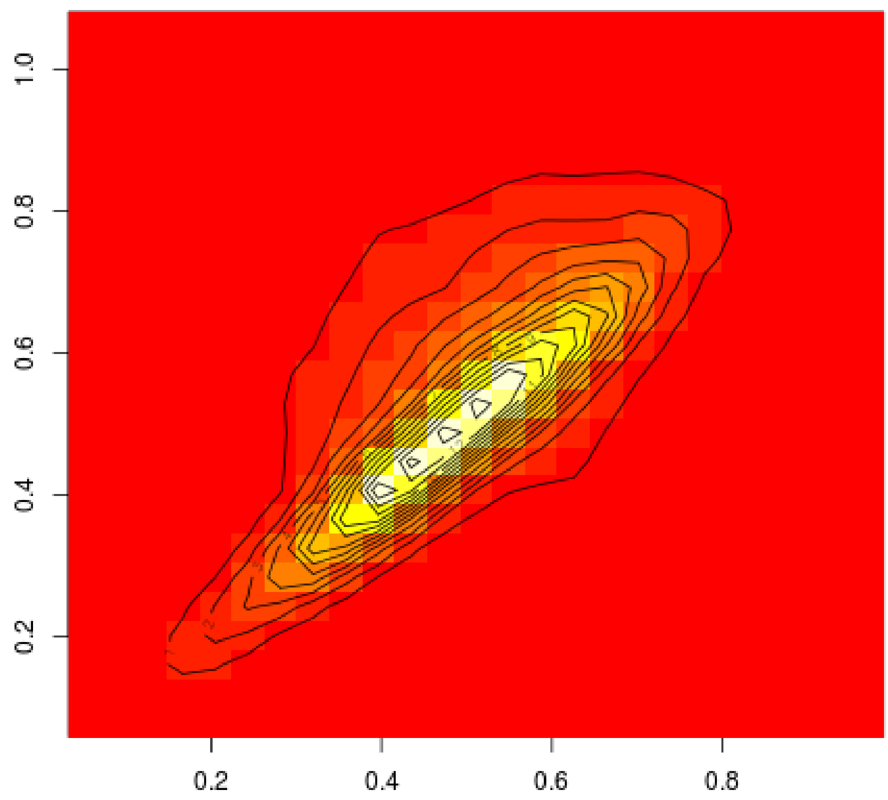

0.2
0.4
0.6
0.8
1.0
1.2
1.4
0.2
0.4
0.6
0.8

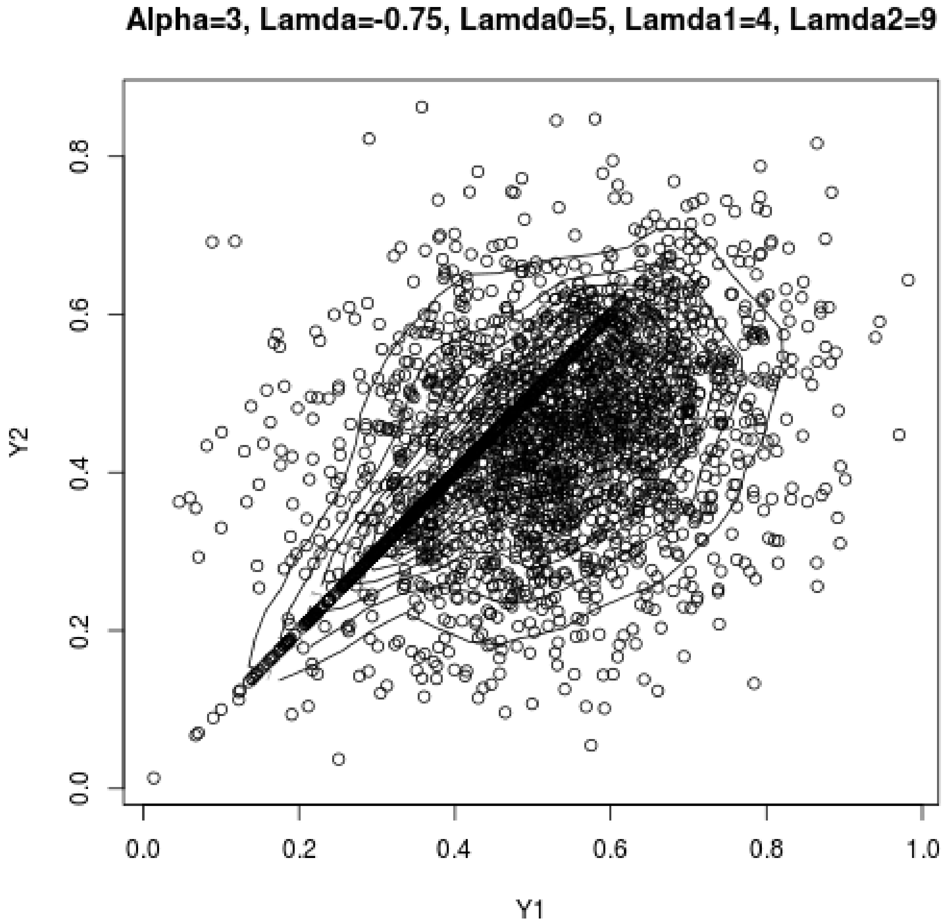

Alpha=3, Lamda=-0.75, Lamda0=1, Lamda1=8, Lamda2=9
Y2
0.2
0.4
0.6
0.8
Y1
0.0
0.2
0.4
0.6
0.8
1.0

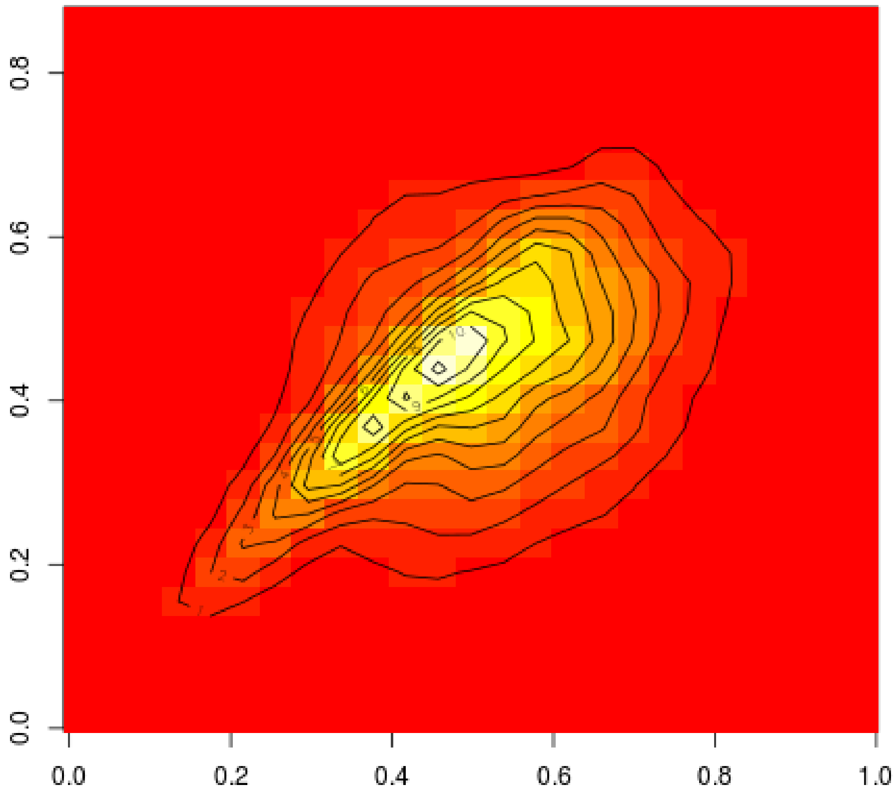

0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0



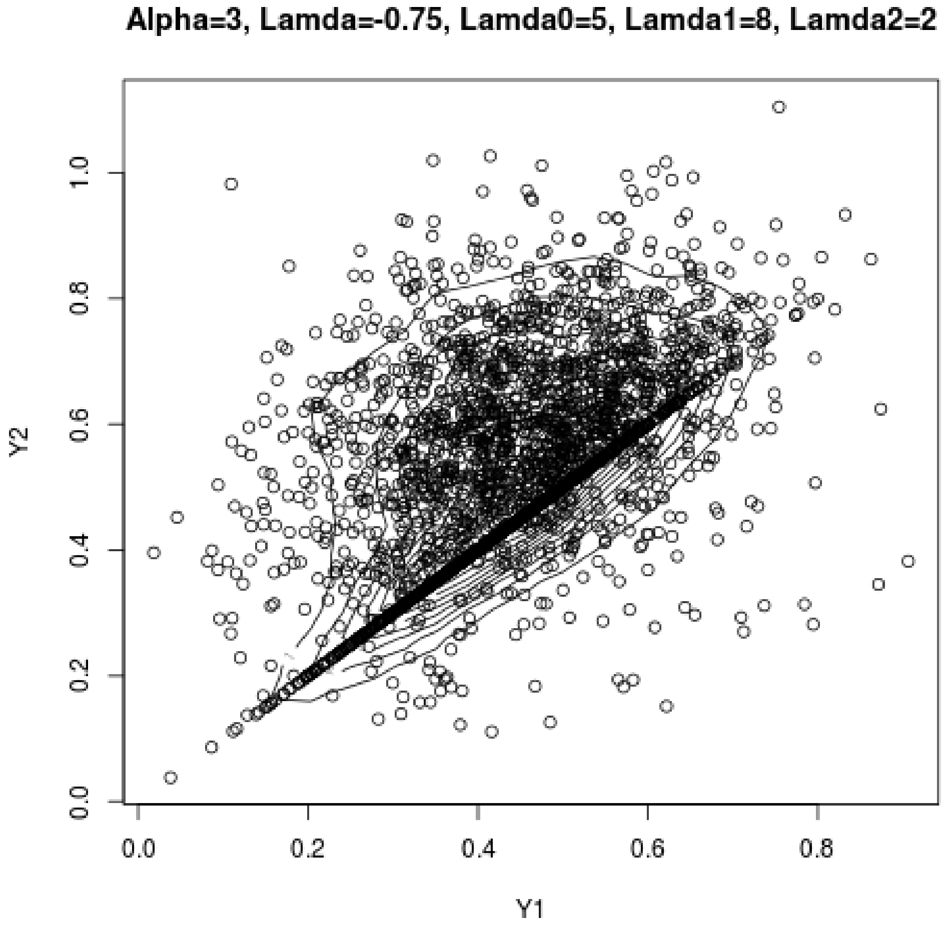
Alpha=3, Lamda=-0.75, Lamda0=5, Lamda1=4, Lamda2=2
Y2
1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0
Y1

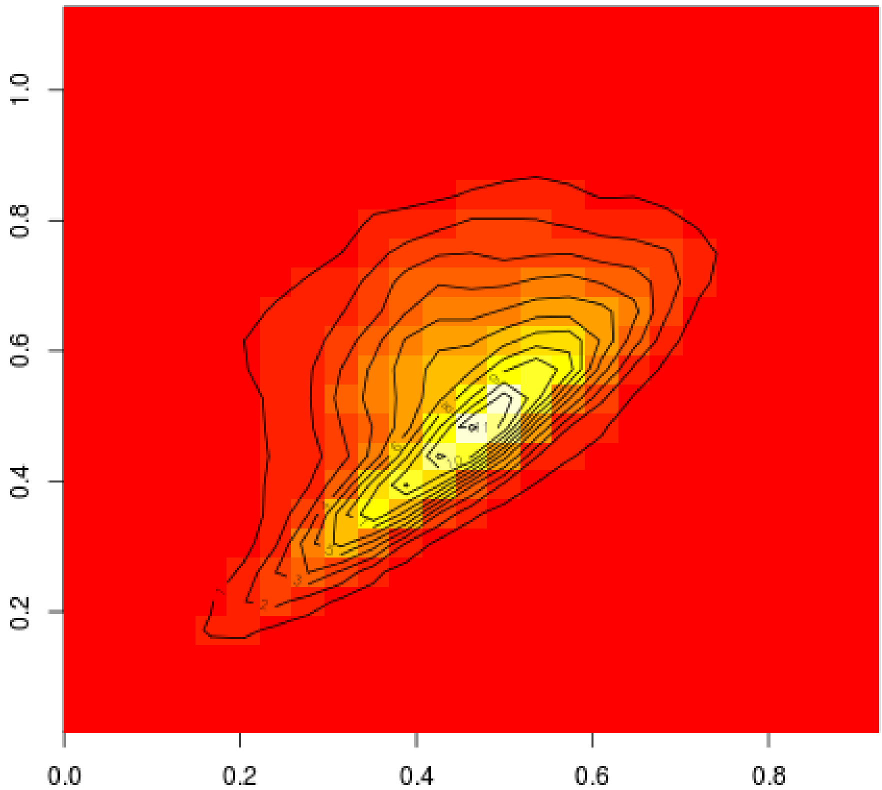

1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8

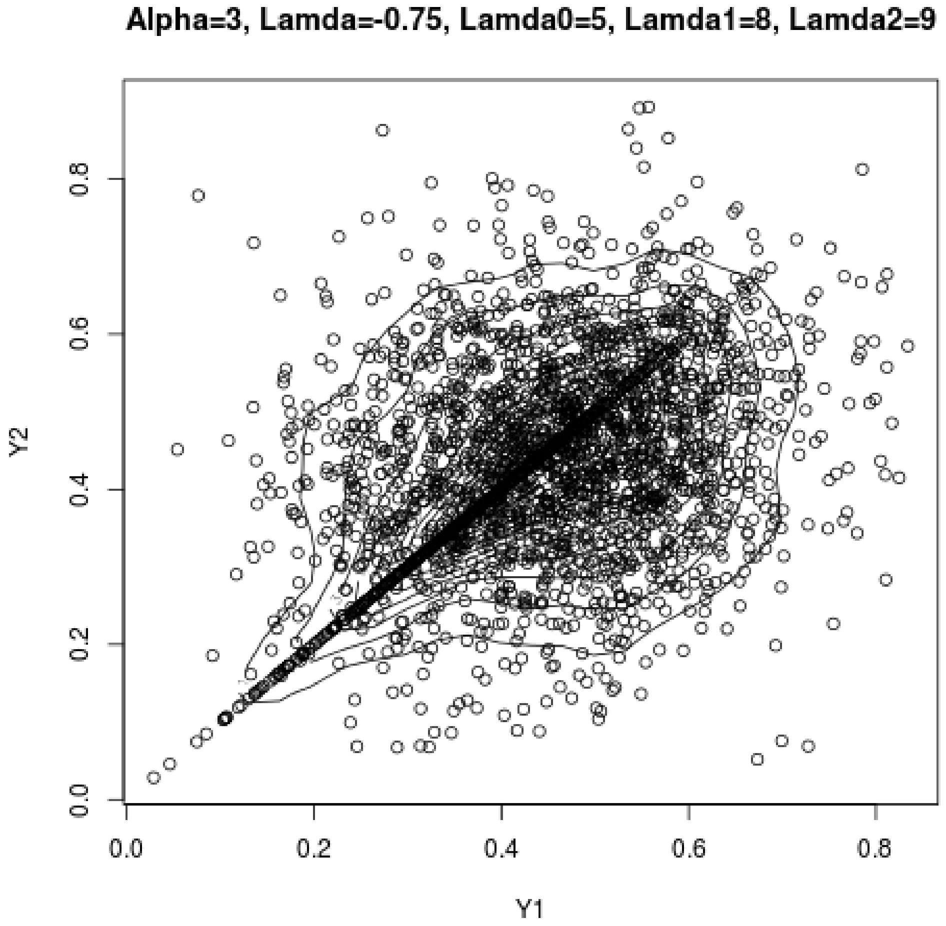

Alpha=3, Lamda=-0.75, Lamda0=5, Lamda1=4, Lamda2=9
Y2
0.0
0.2
0.4
0.6
0.8
0.0
0.2
0.4
0.6
0.8
1.0
Y1

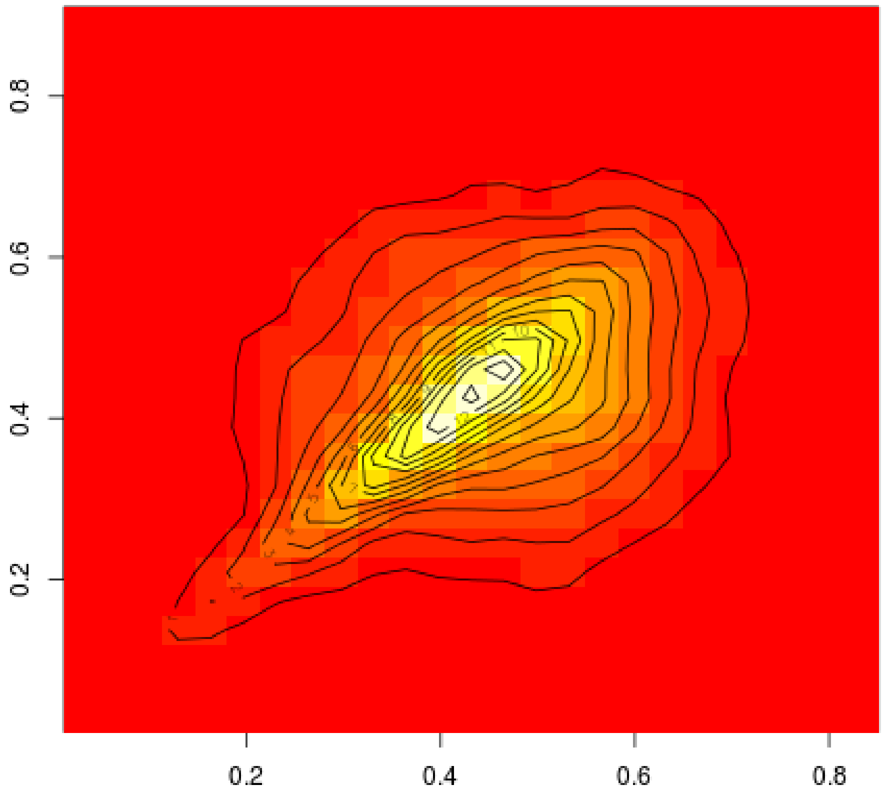


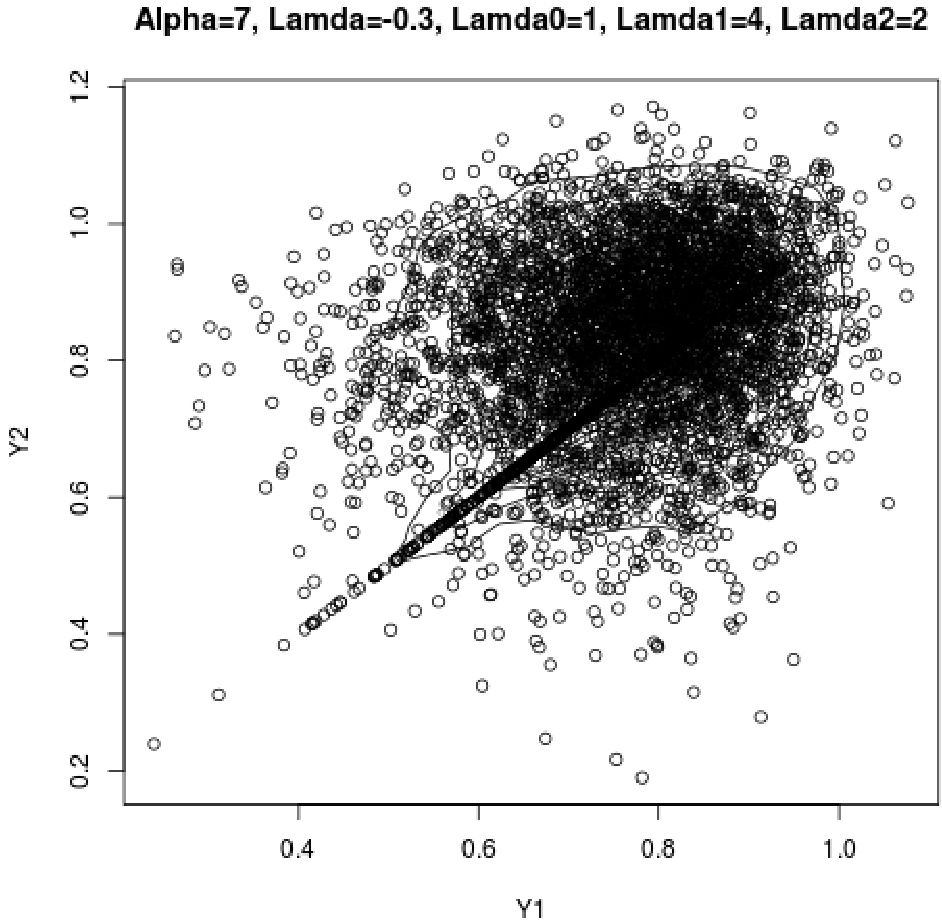
0.0
0.2
0.4
0.6
0.8
0.0
0.2
0.4
0.6
0.8
1.0


Alpha=3, Lamda=-0.75, Lamda0=5, Lamda1=8, Lamda2=2
Y2
Y1
0.0
0.2
0.4
0.6
0.8
1.0

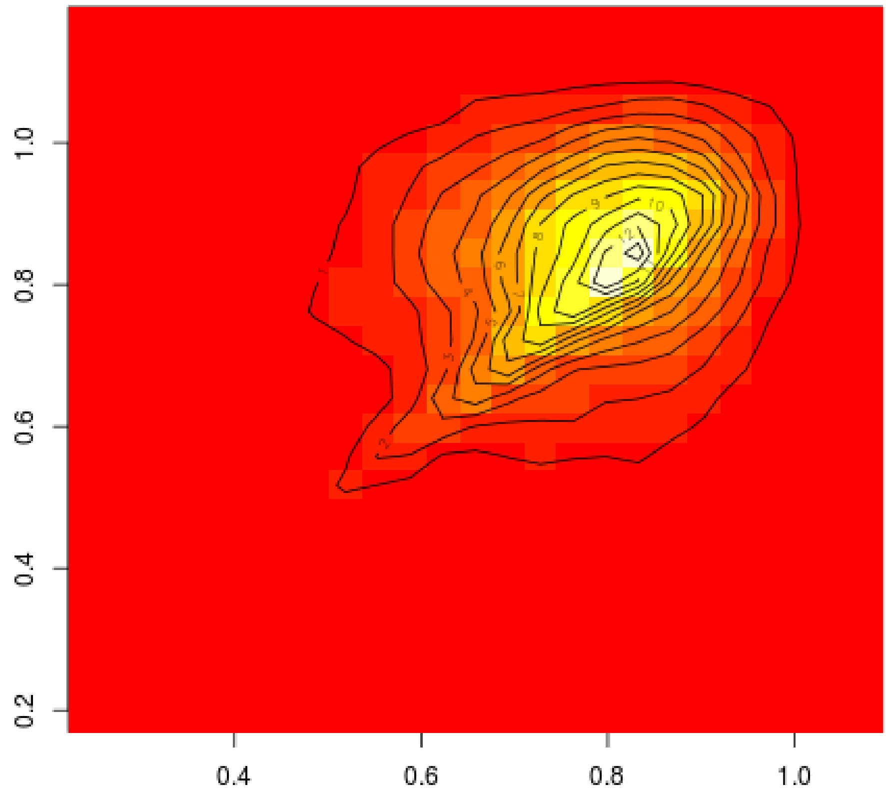

1.0
0.8
0.6
0.4
0.2
0.0
0.2
0.4
0.6
0.8


Alpha=3, Lamda=-0.75, Lamda0=5, Lamda1=8, Lamda2=9
Y2
Y1
0.0
0.2
0.4
0.6
0.8
0.0
0.2
0.4
0.6
0.8

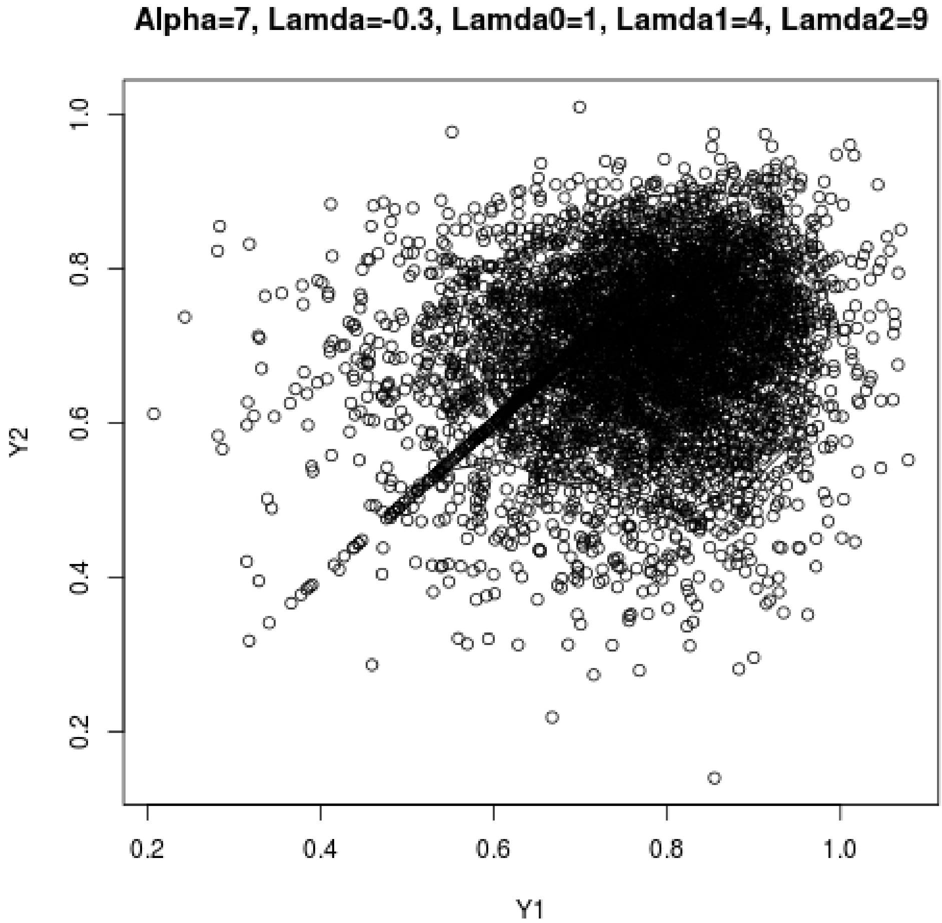

0.2
0.4
0.6
0.8
0.2
0.4
0.6
0.8



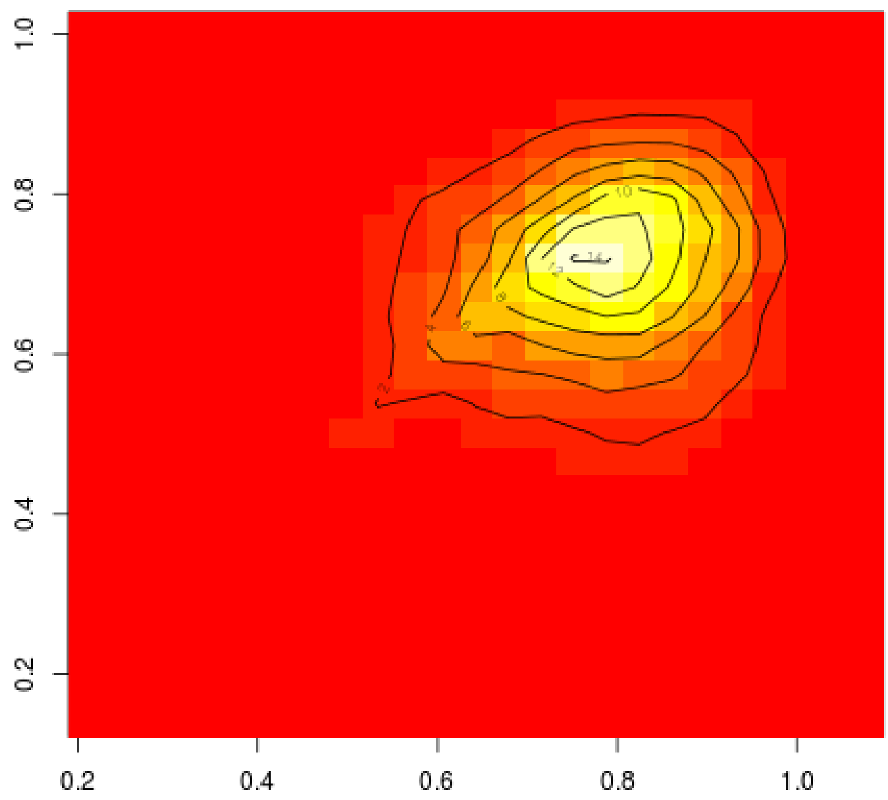
Alpha=7, Lamda=-0.3, Lamda0=1, Lamda1=4, Lamda2=2
Y2
Y1
0.2
0.4
0.6
0.8
1.0
1.2
0.4
0.6
0.8
1.0



1.0
0.8
0.6
0.4
0.2
0.4
0.6
0.8
1.0



Alpha=7, Lamda=-0.3, Lamda0=1, Lamda1=4, Lamda2=9
Y2
1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0
Y1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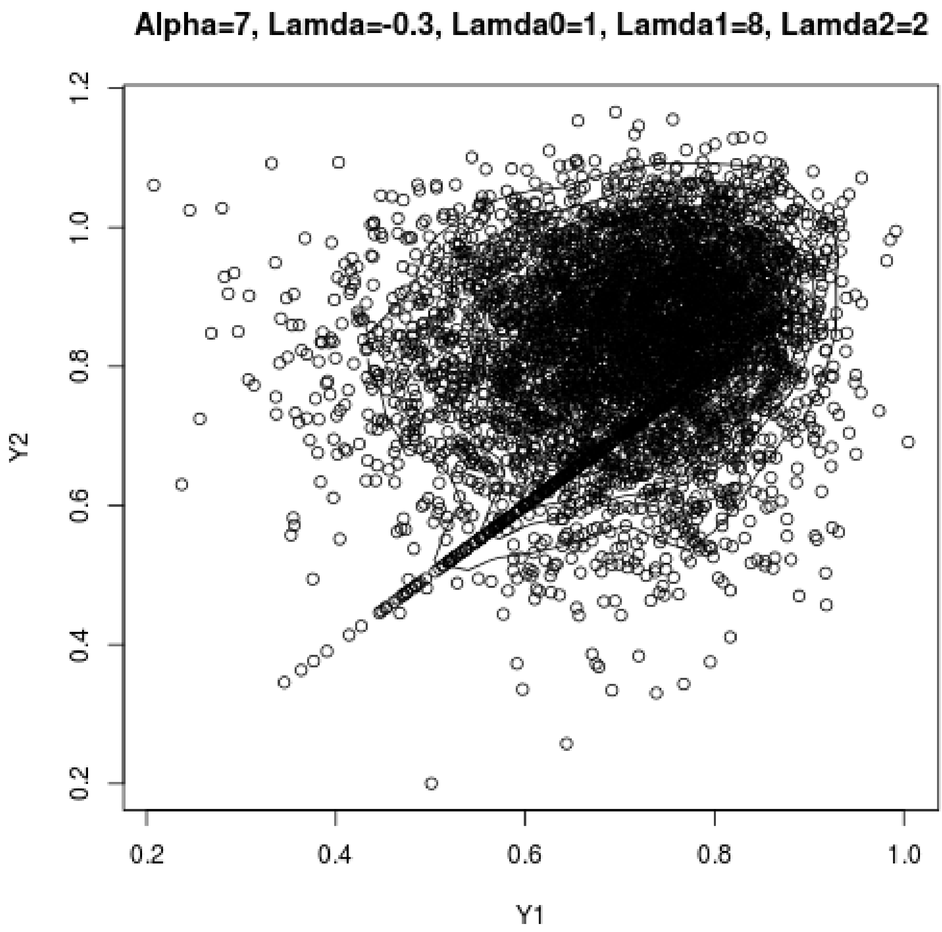

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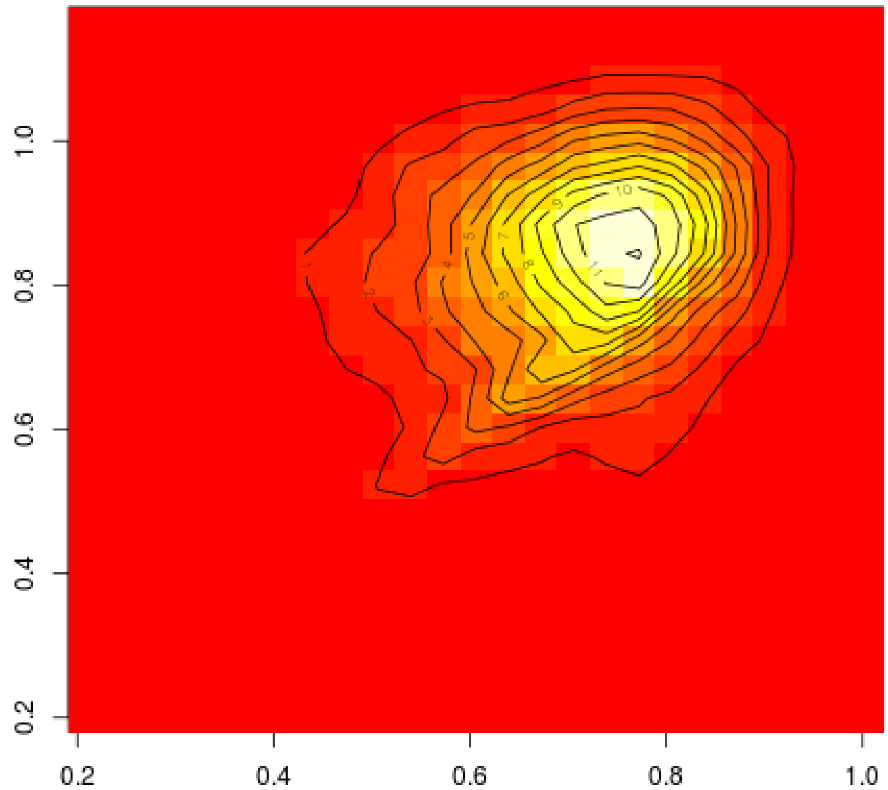

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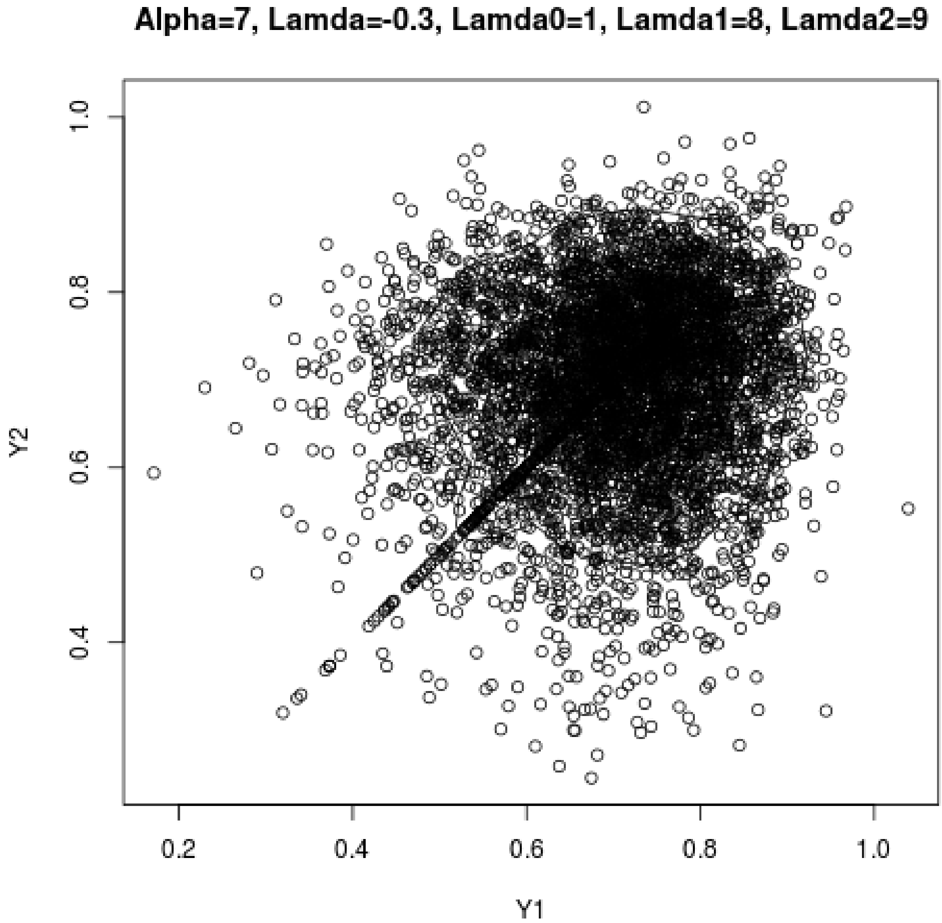
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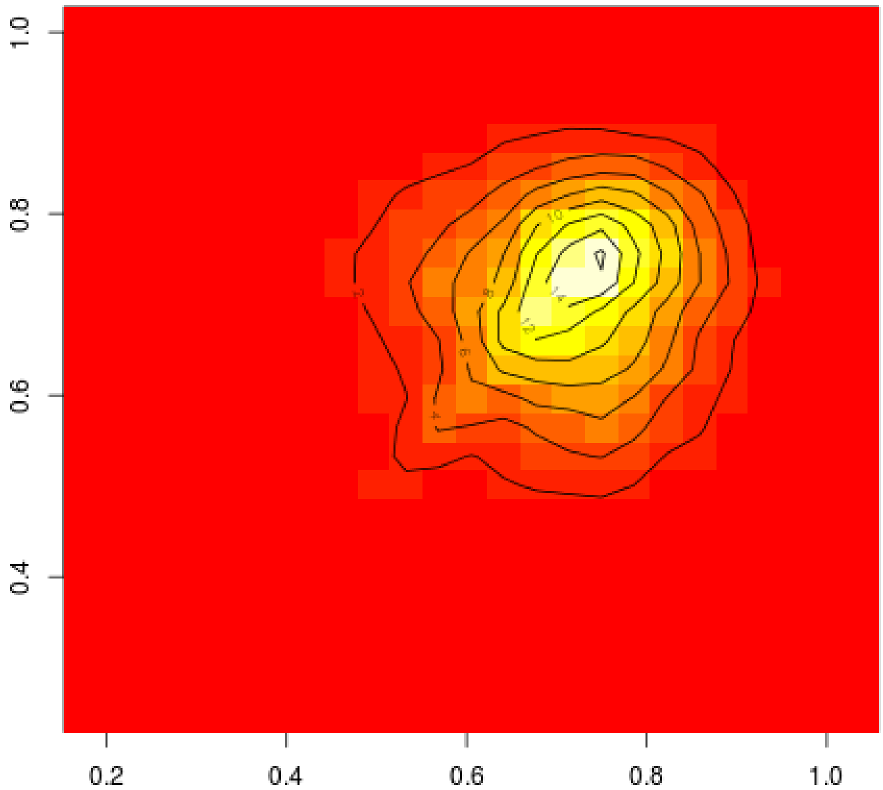
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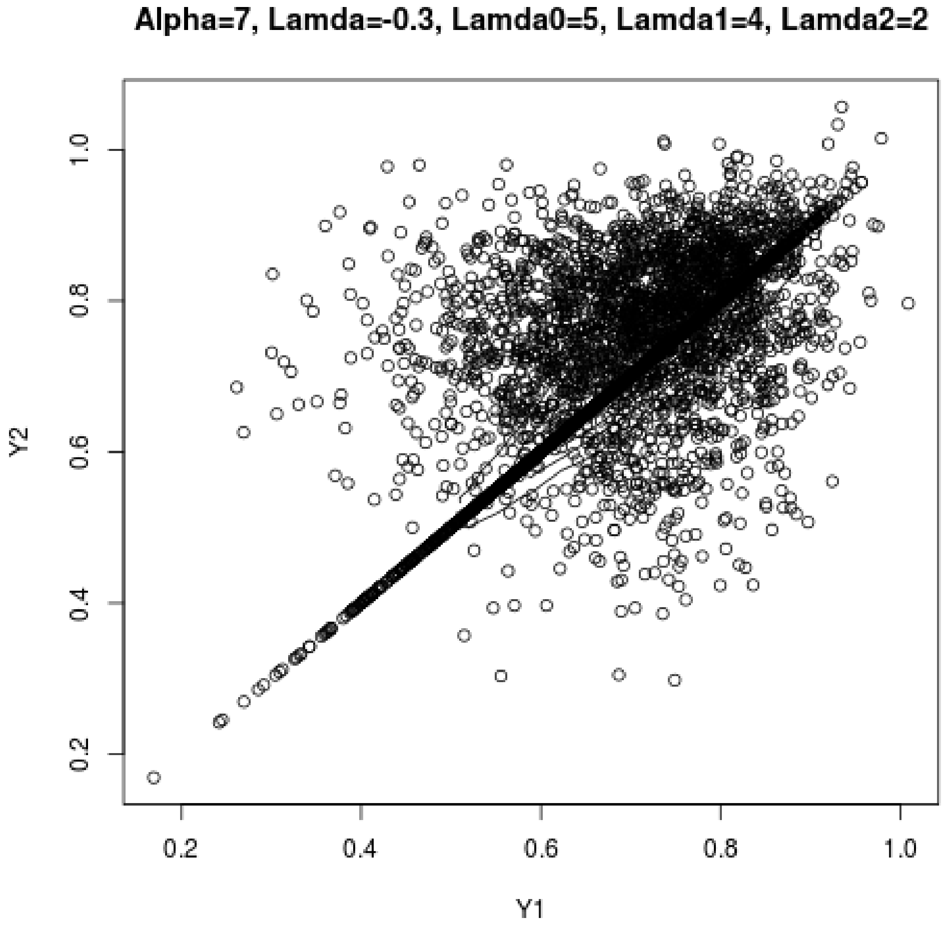

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Y1
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0.8
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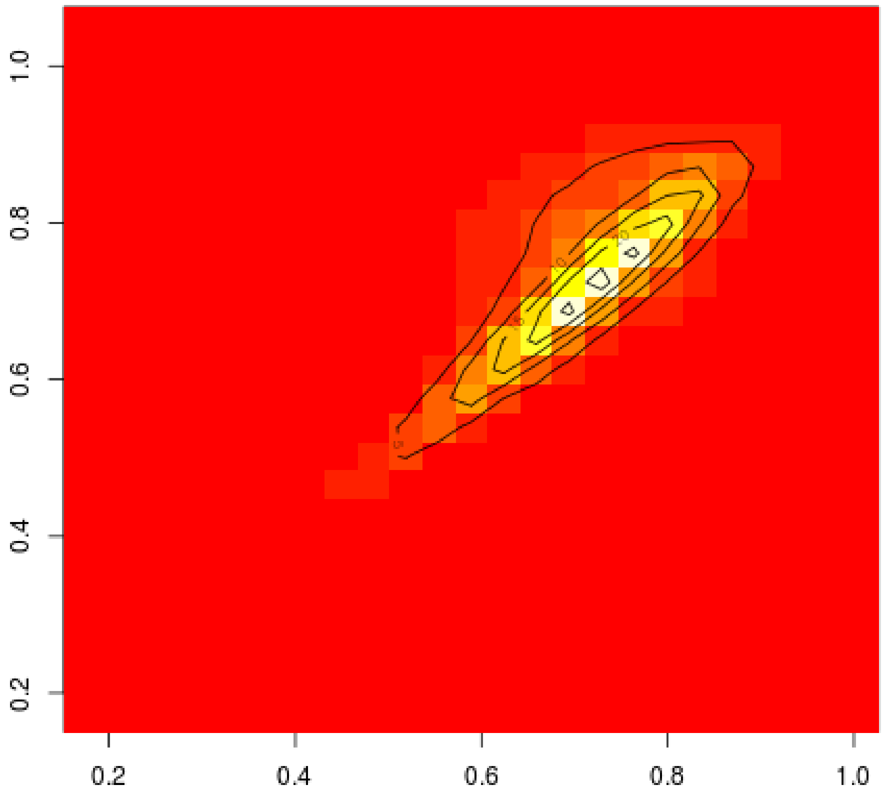
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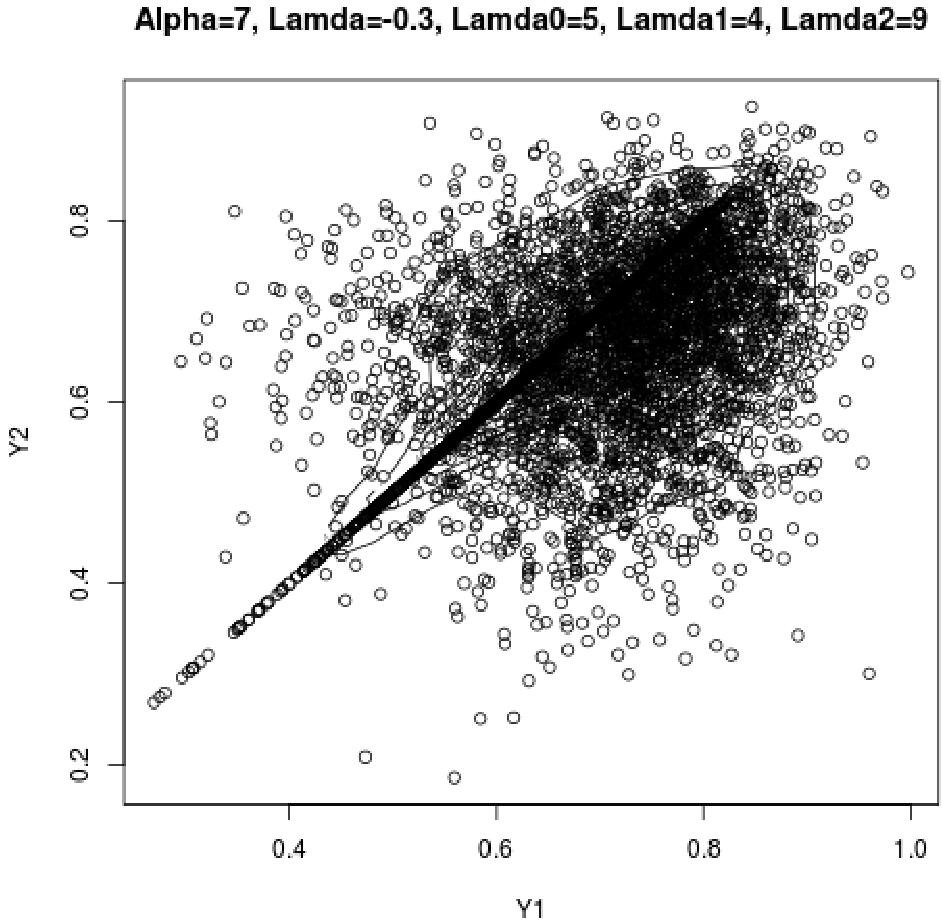
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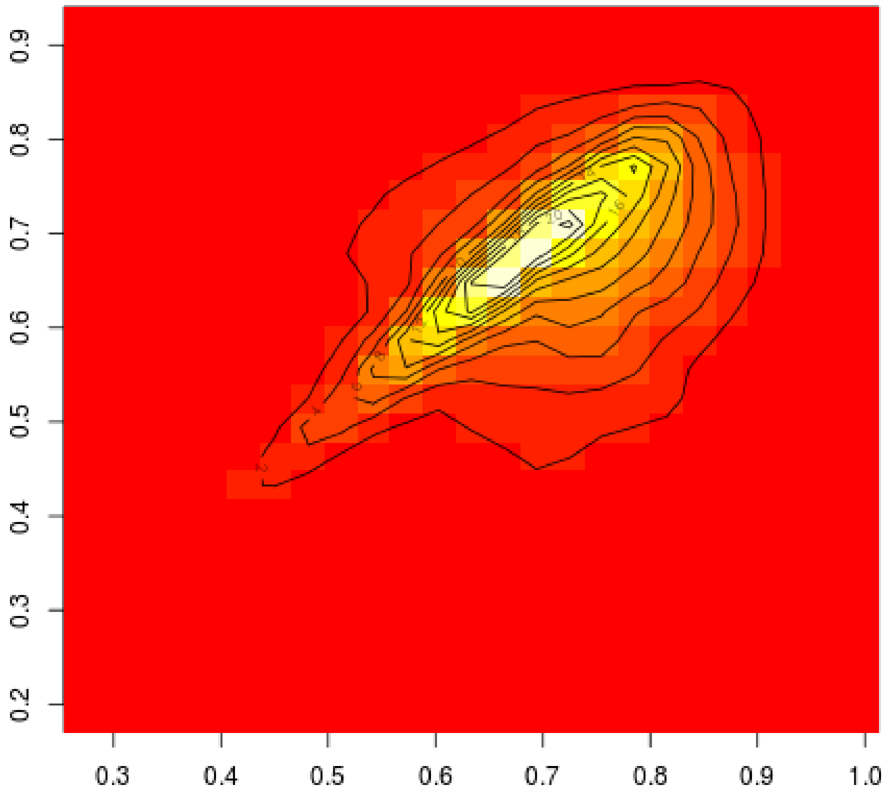
Y1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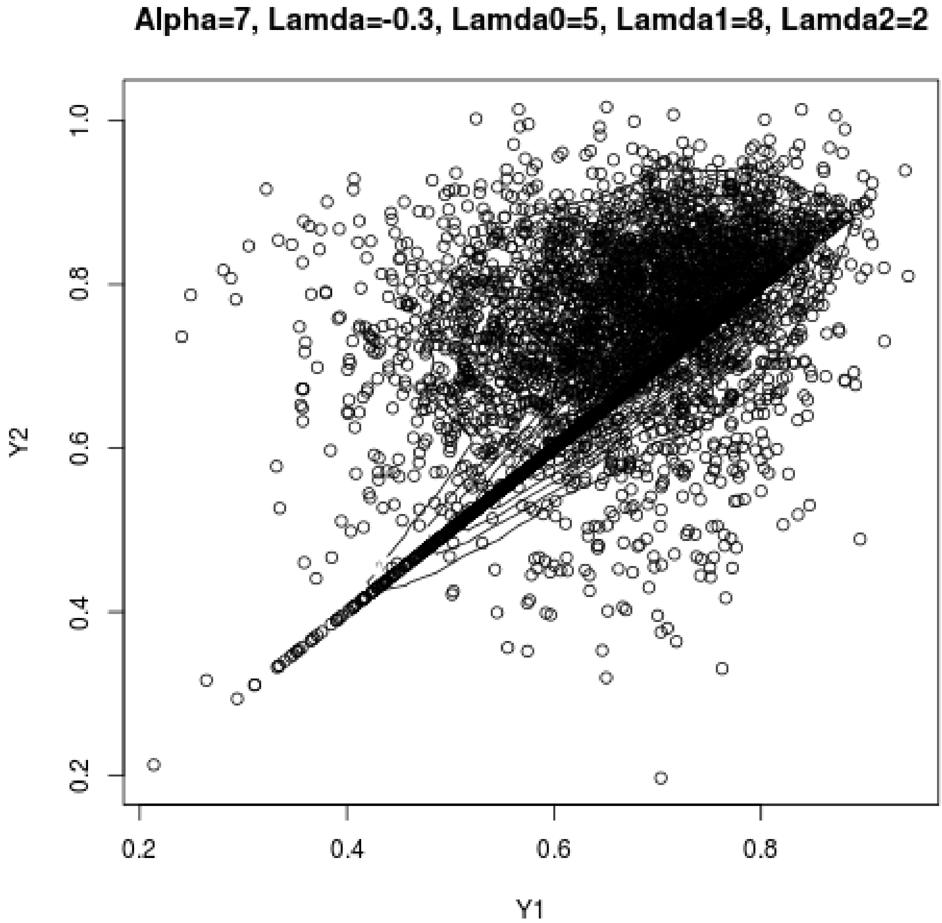
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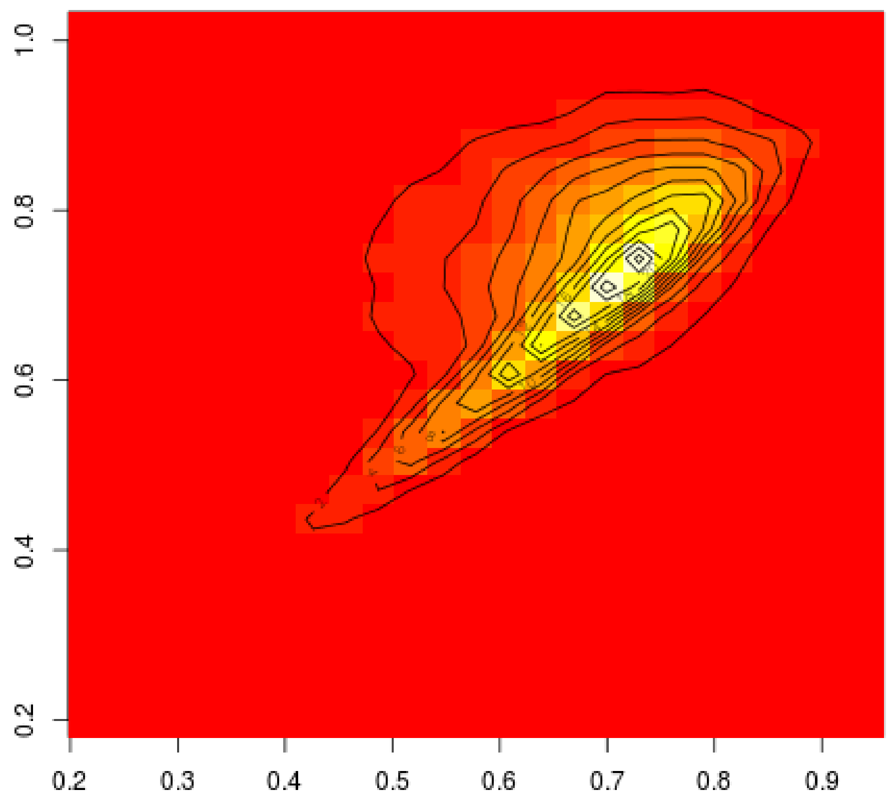

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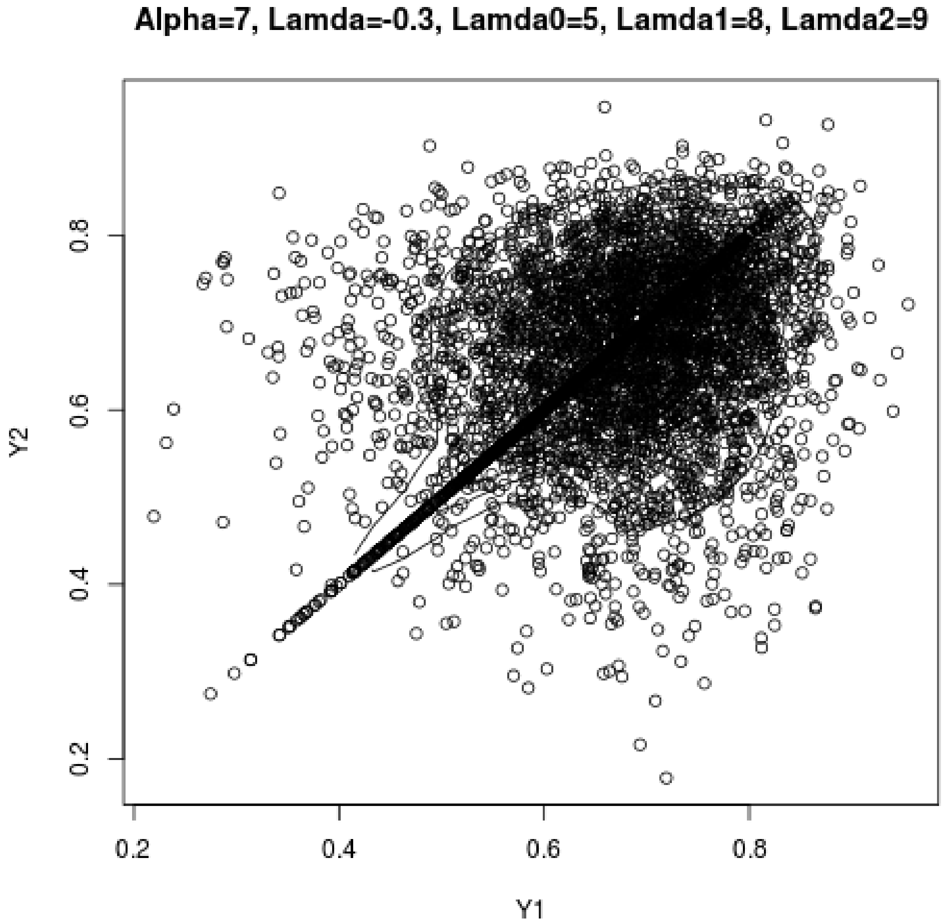
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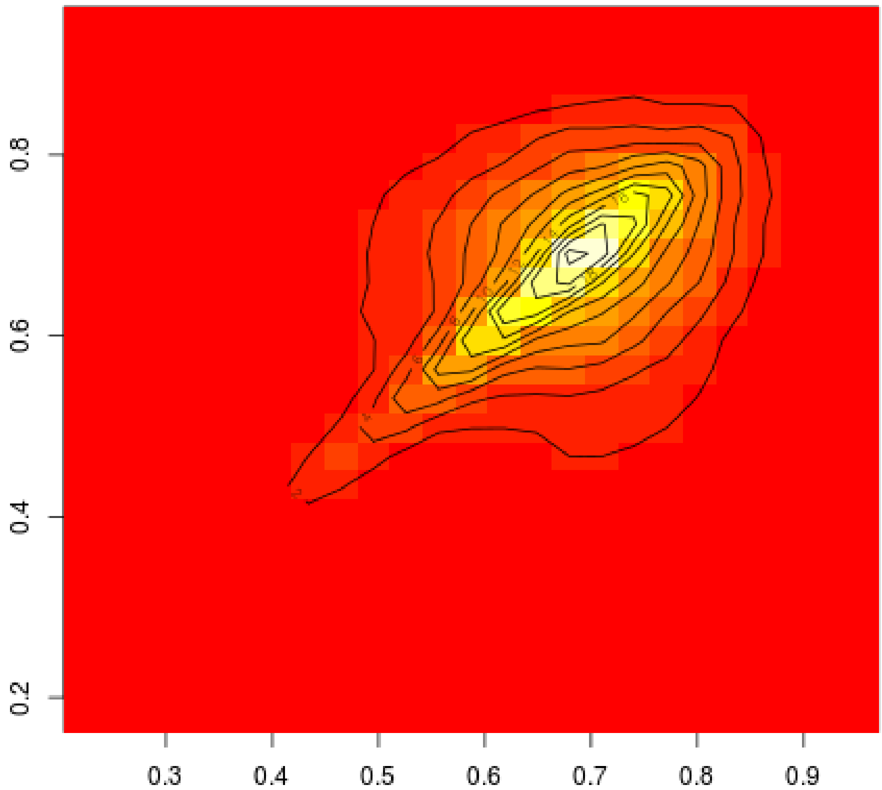
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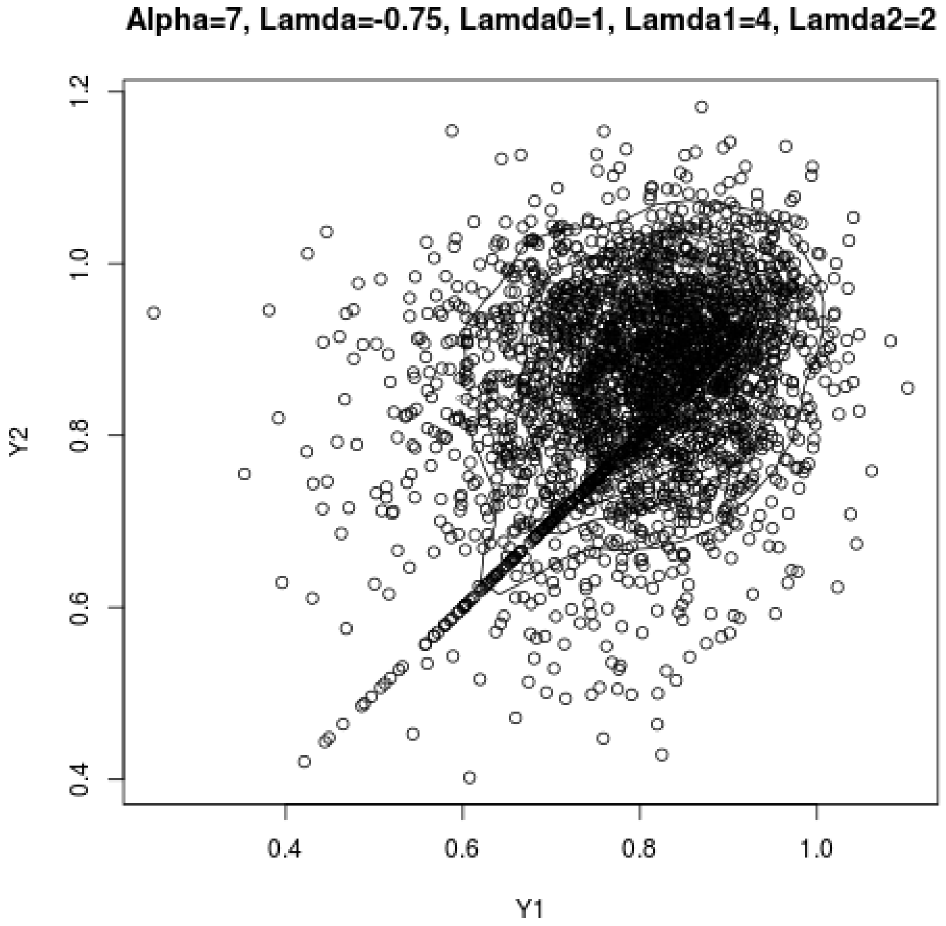
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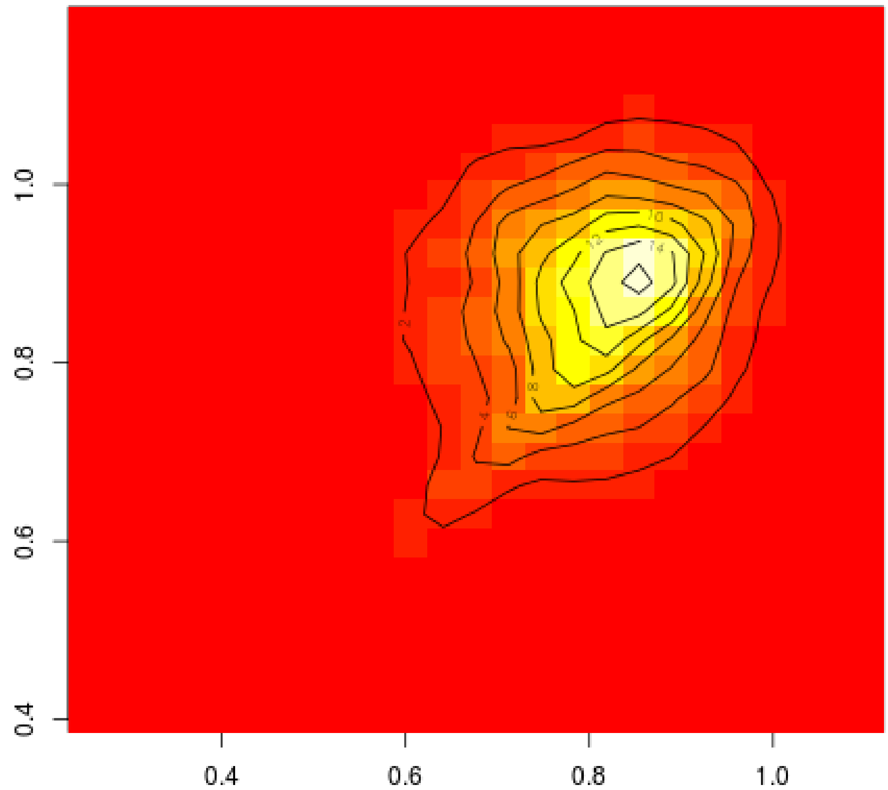
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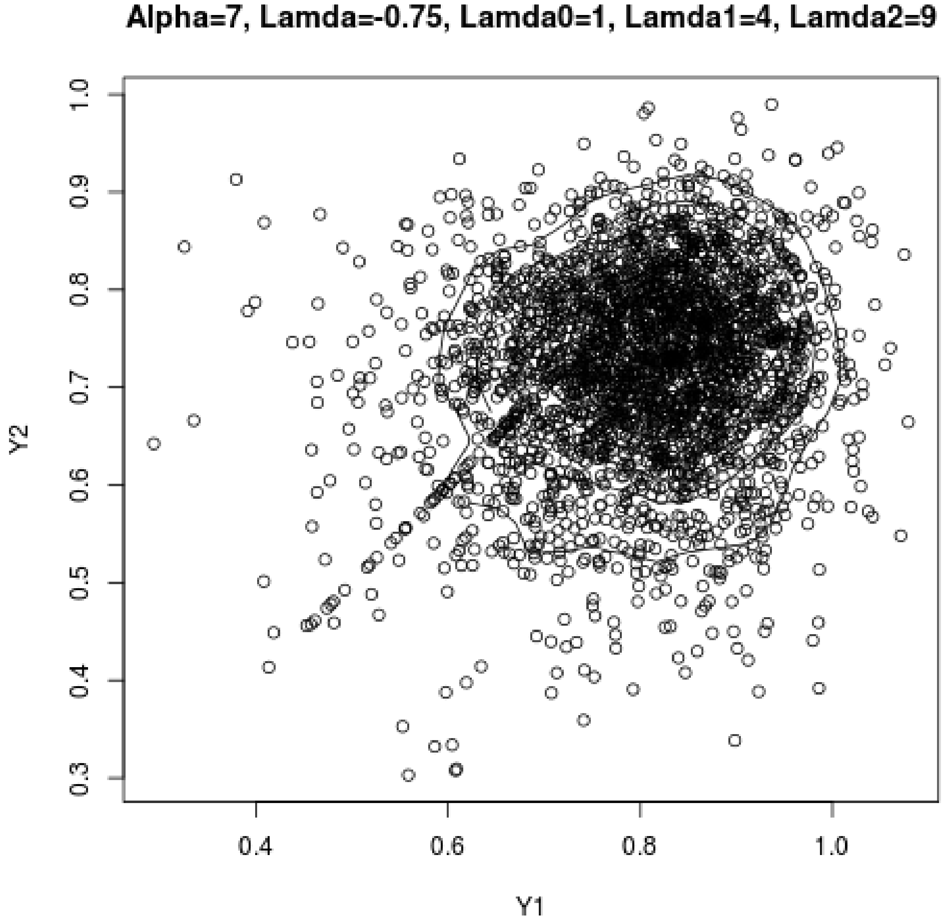

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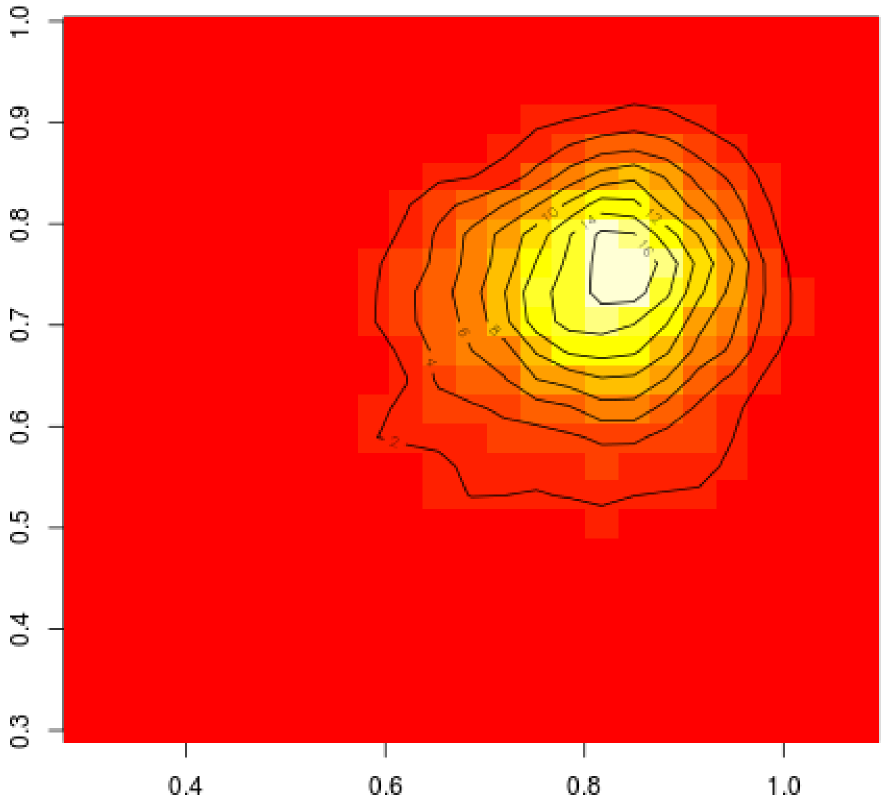
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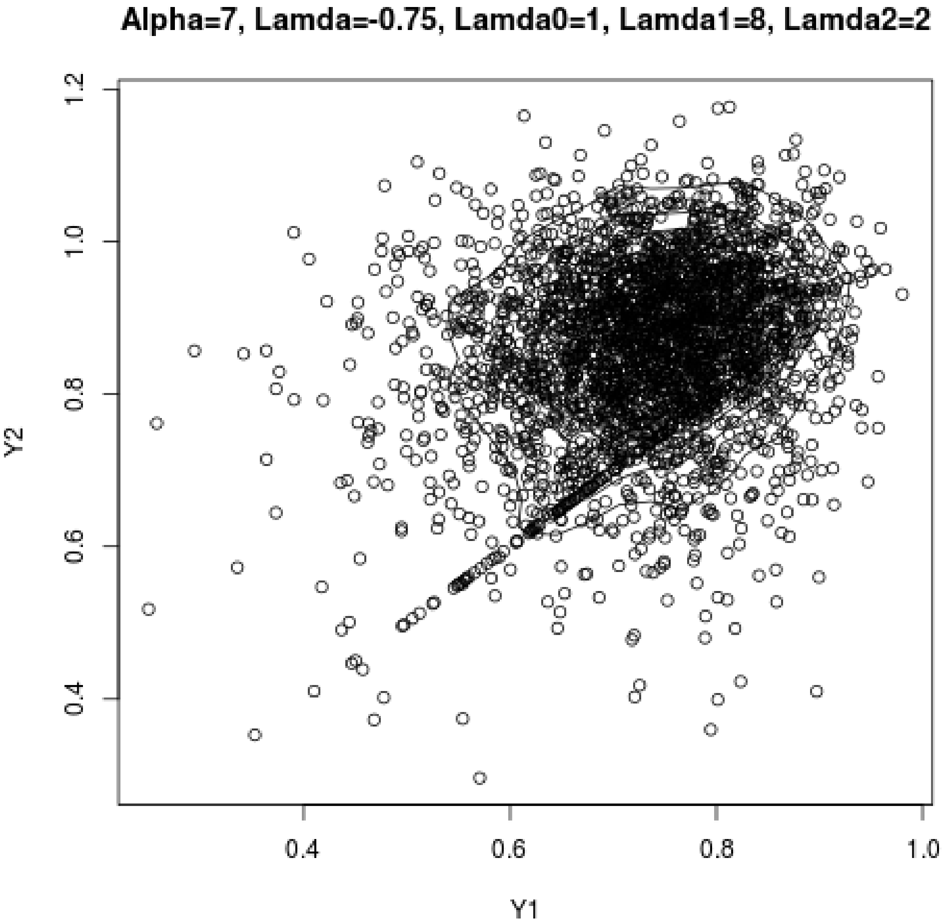
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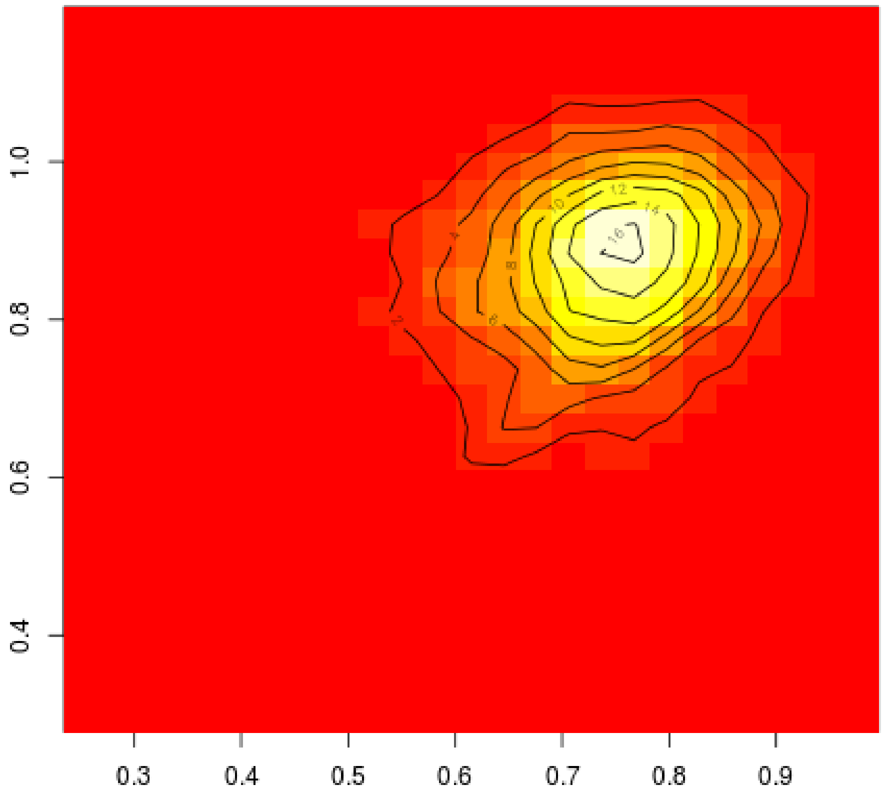

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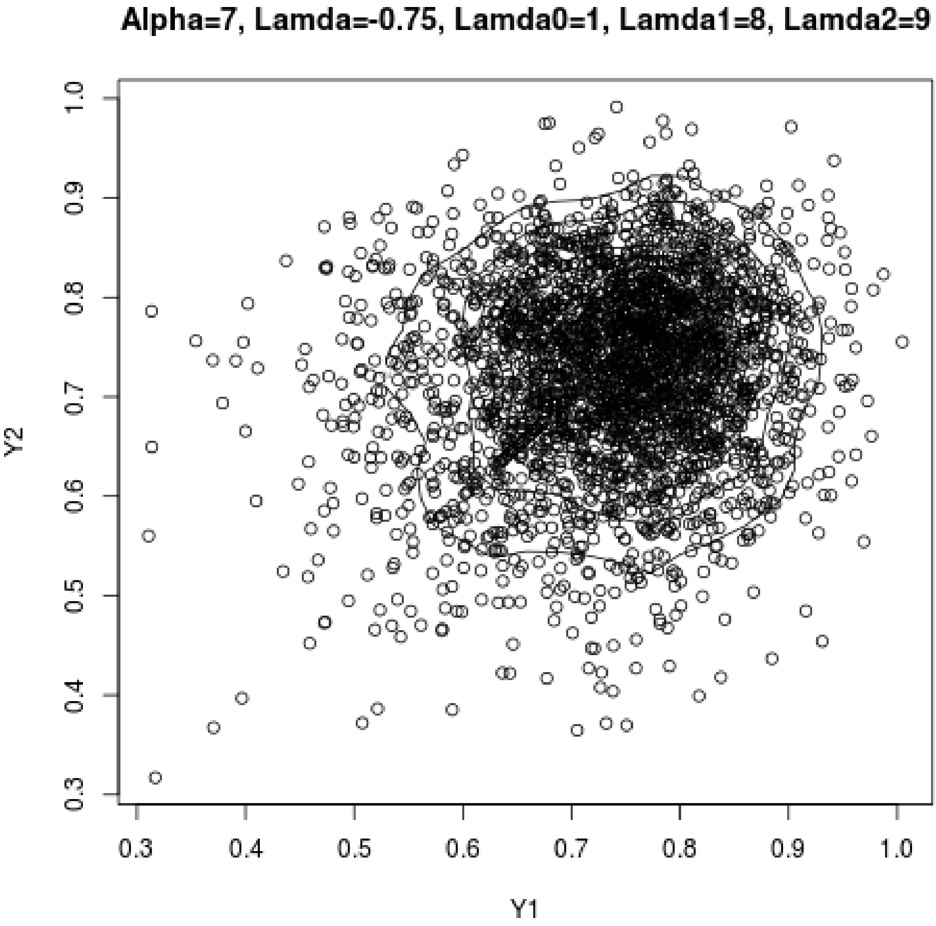
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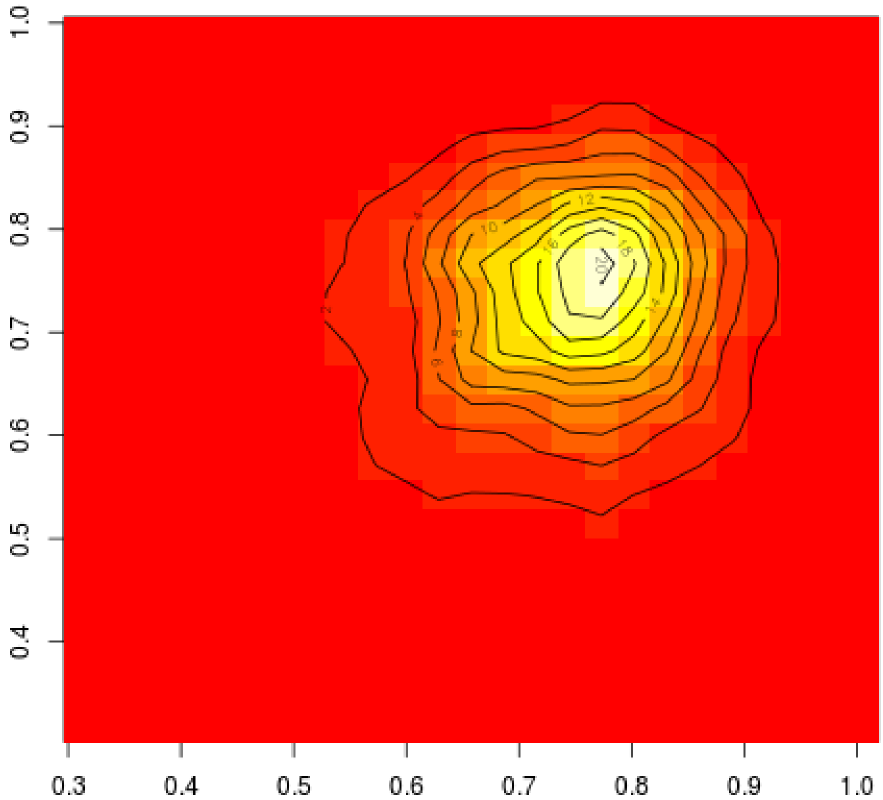
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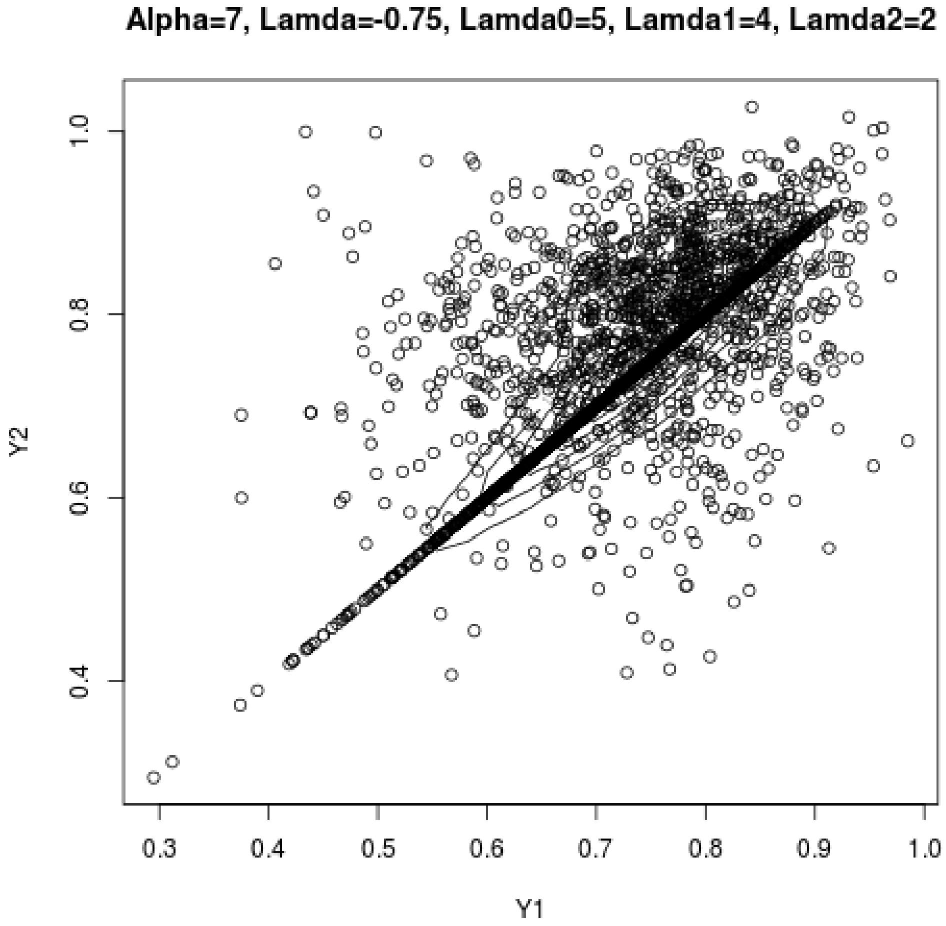
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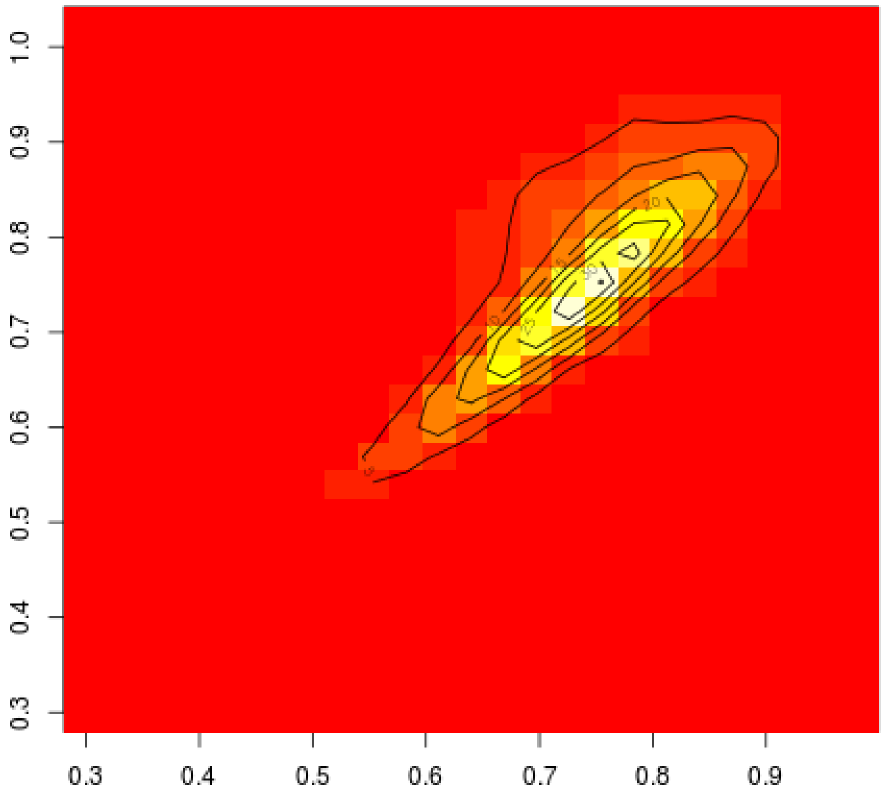
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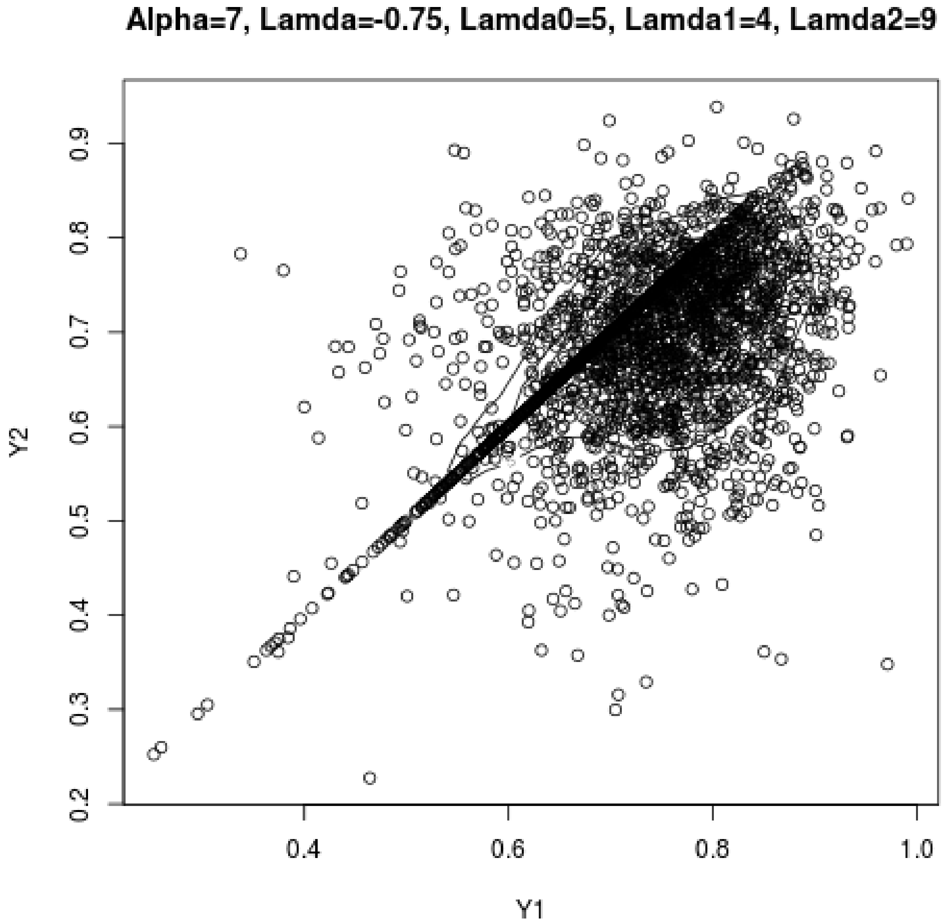
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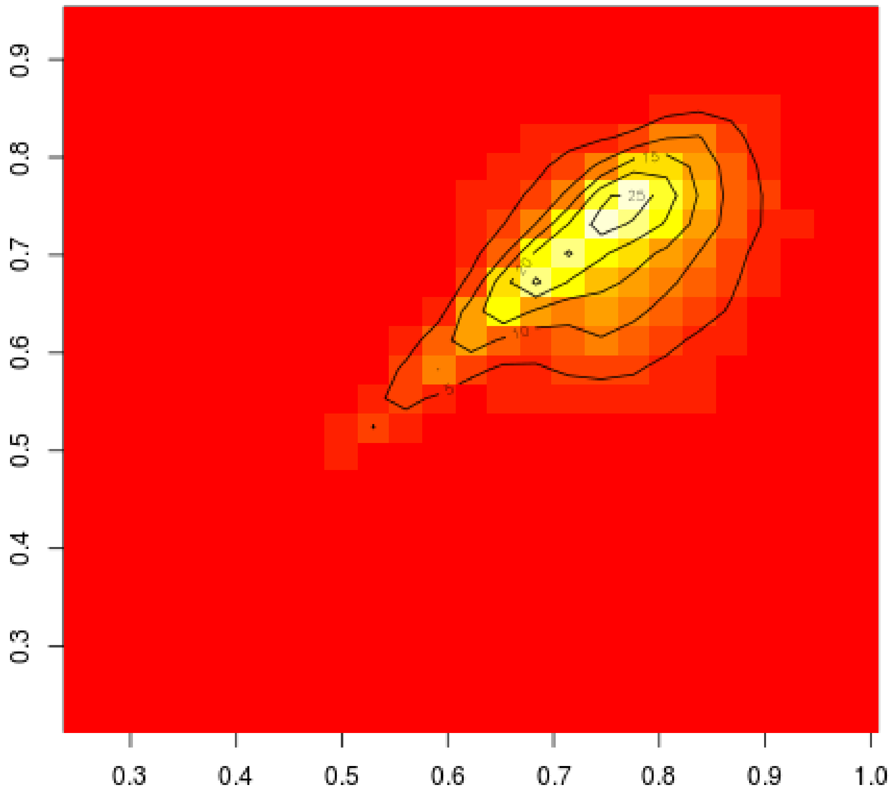
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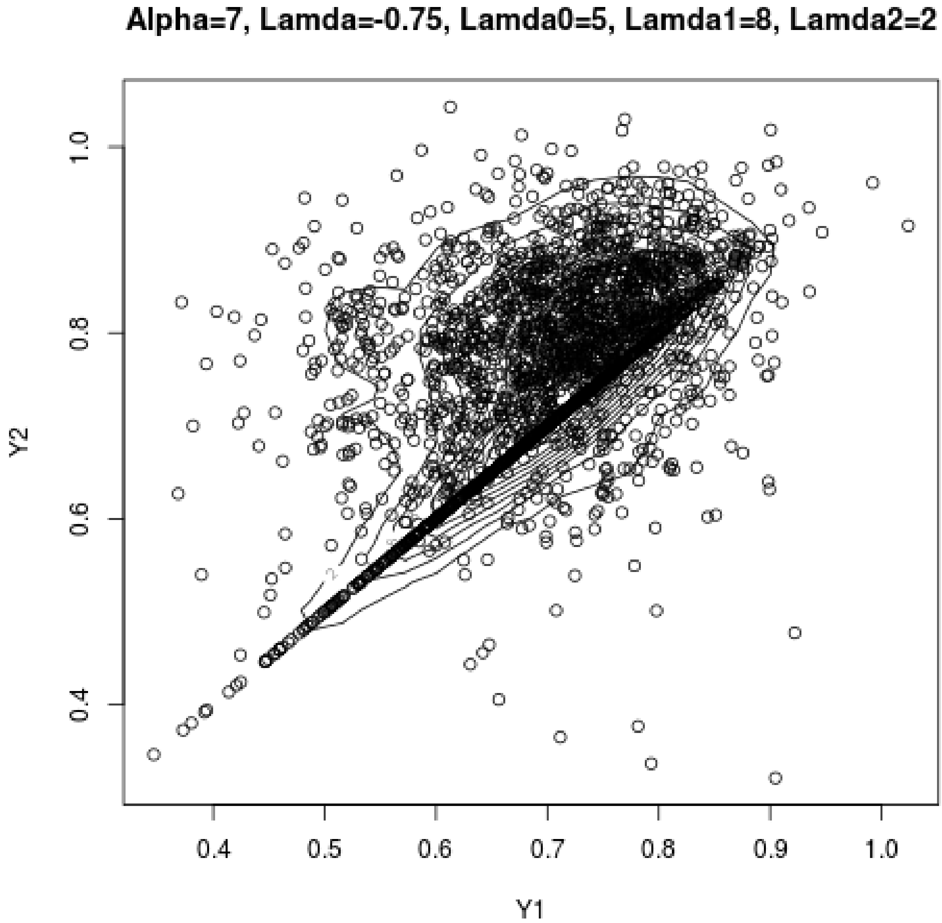

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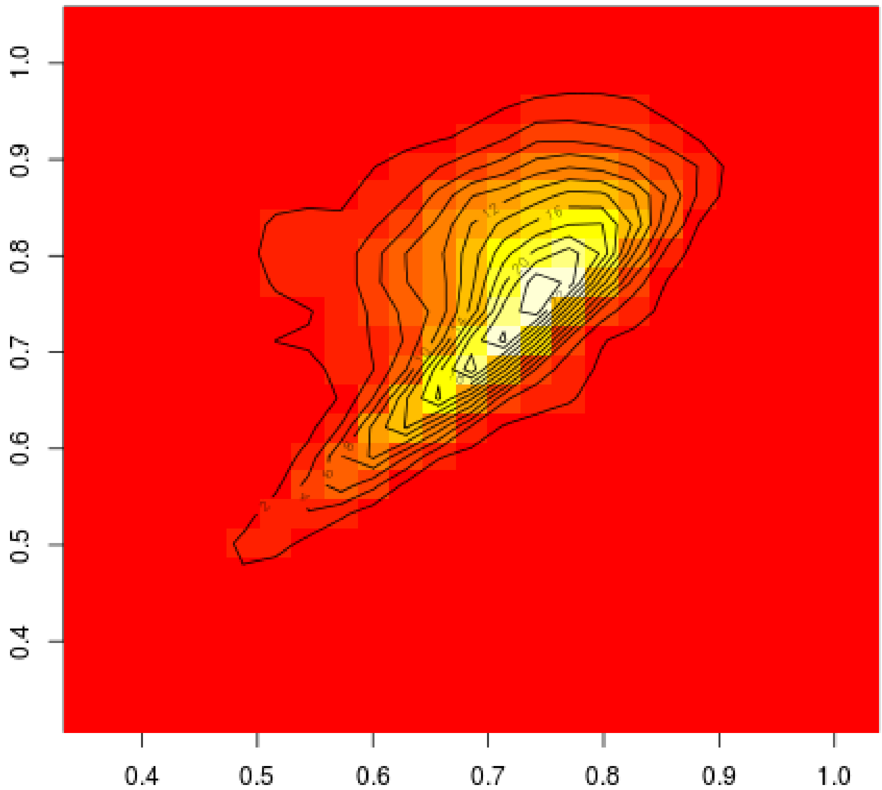

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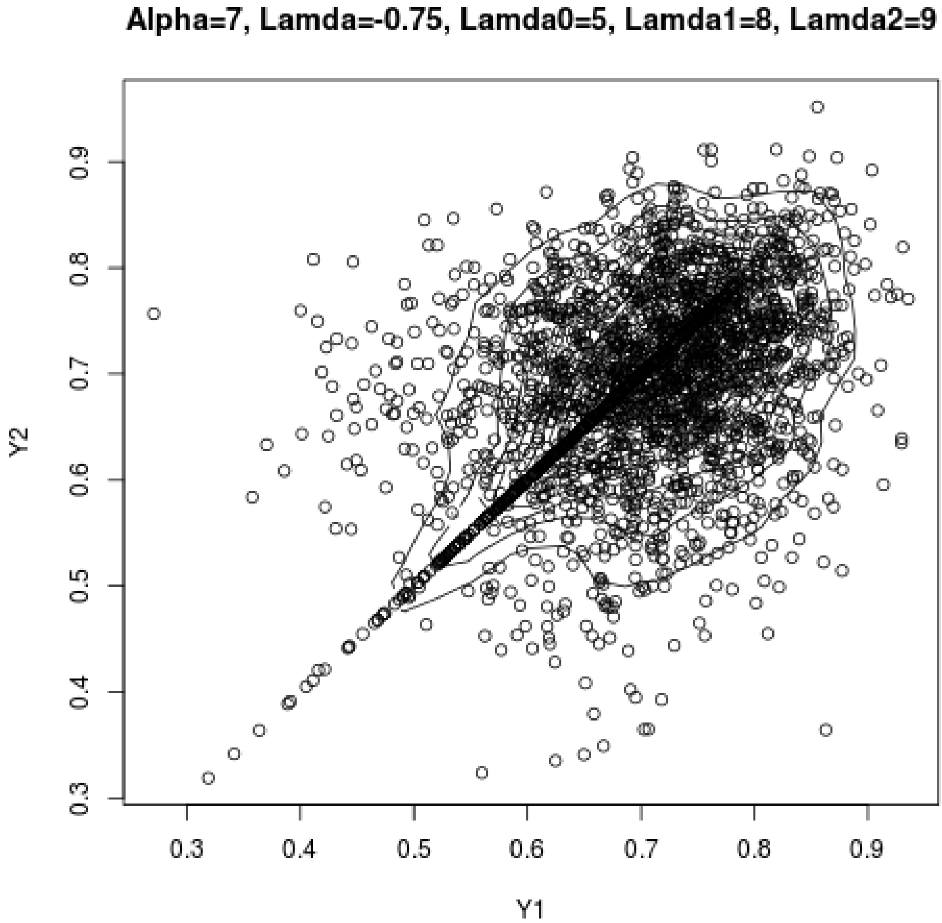
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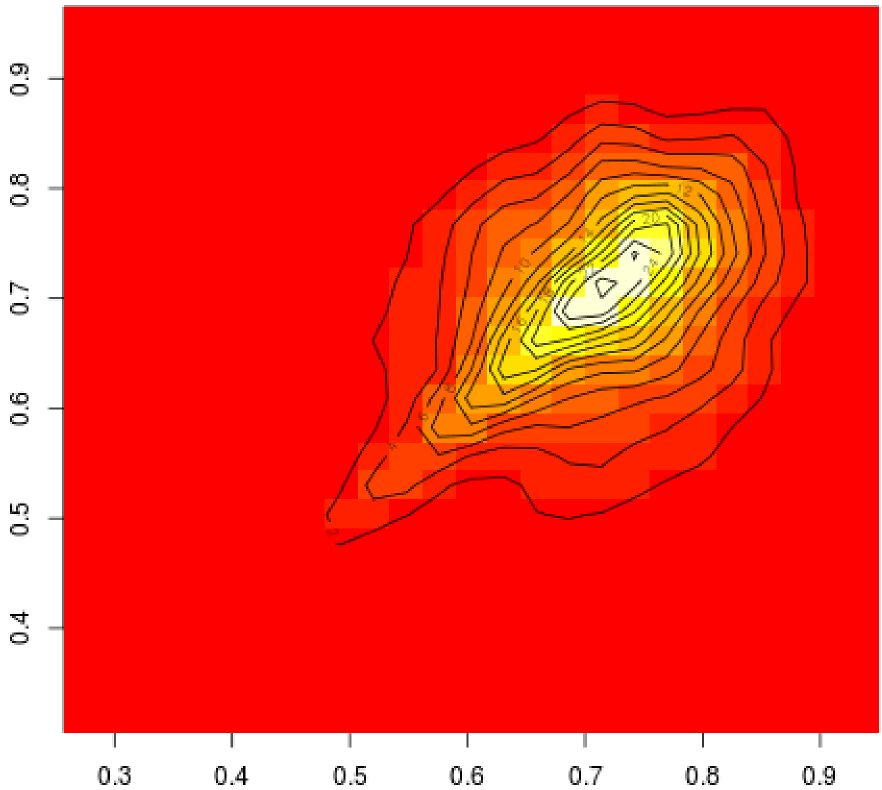
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MADE BY:
Abheek Ghosh (140123047)
Pawan Vaishnav Madandas (140123048)
Divyae Vats (140123049)
Chandan Kumar (140123050)
Prakhaar Bhargava (140123051)