

**MA 322: Scientific Computing  
LAB 7 Report  
Abheek Ghosh - 140123047**

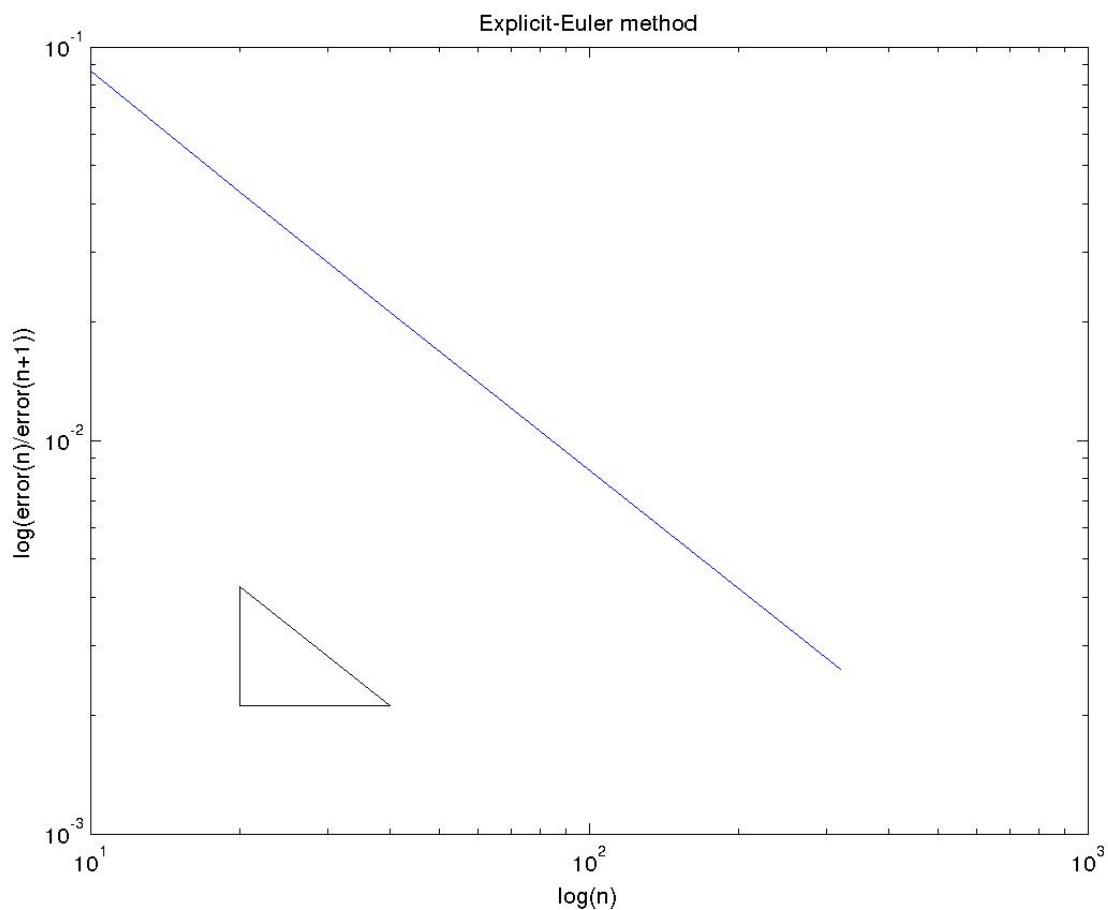
**NOTE: The right angled triangle for comparison is of same length on both perpendicular and base. So, the hypotenuse has a slope of -1. The distortion in graph is due to change in scale of graph. The comparison of slope of the method should be compared with relative to unit slope of the triangle.**

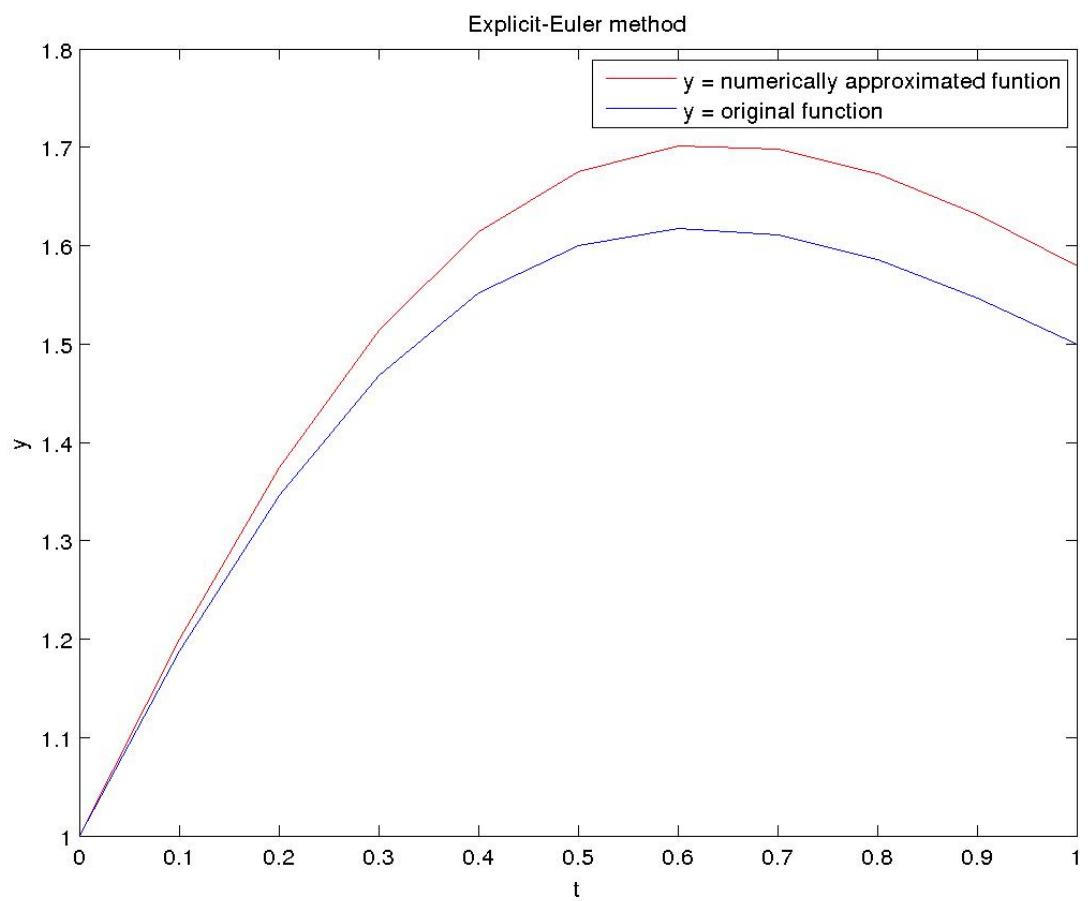
**Question 1**

**(A)**

Explicit-Euler method

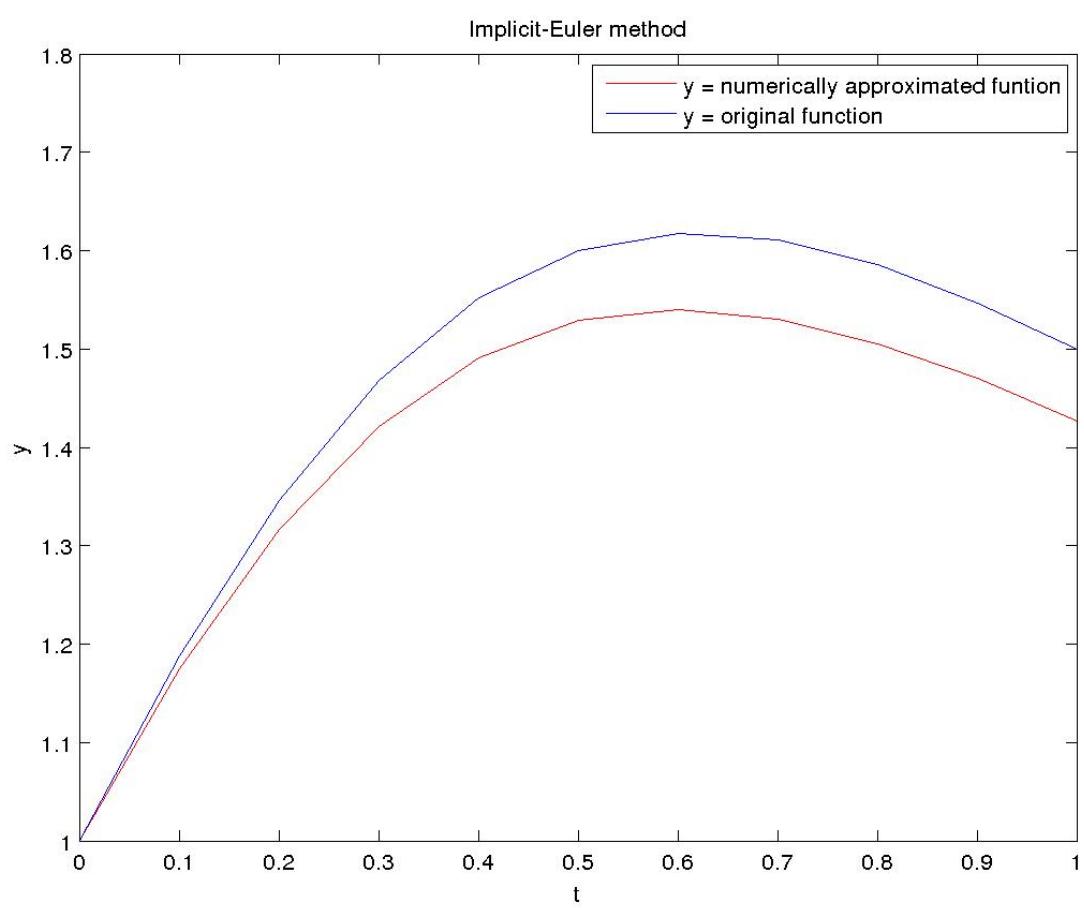
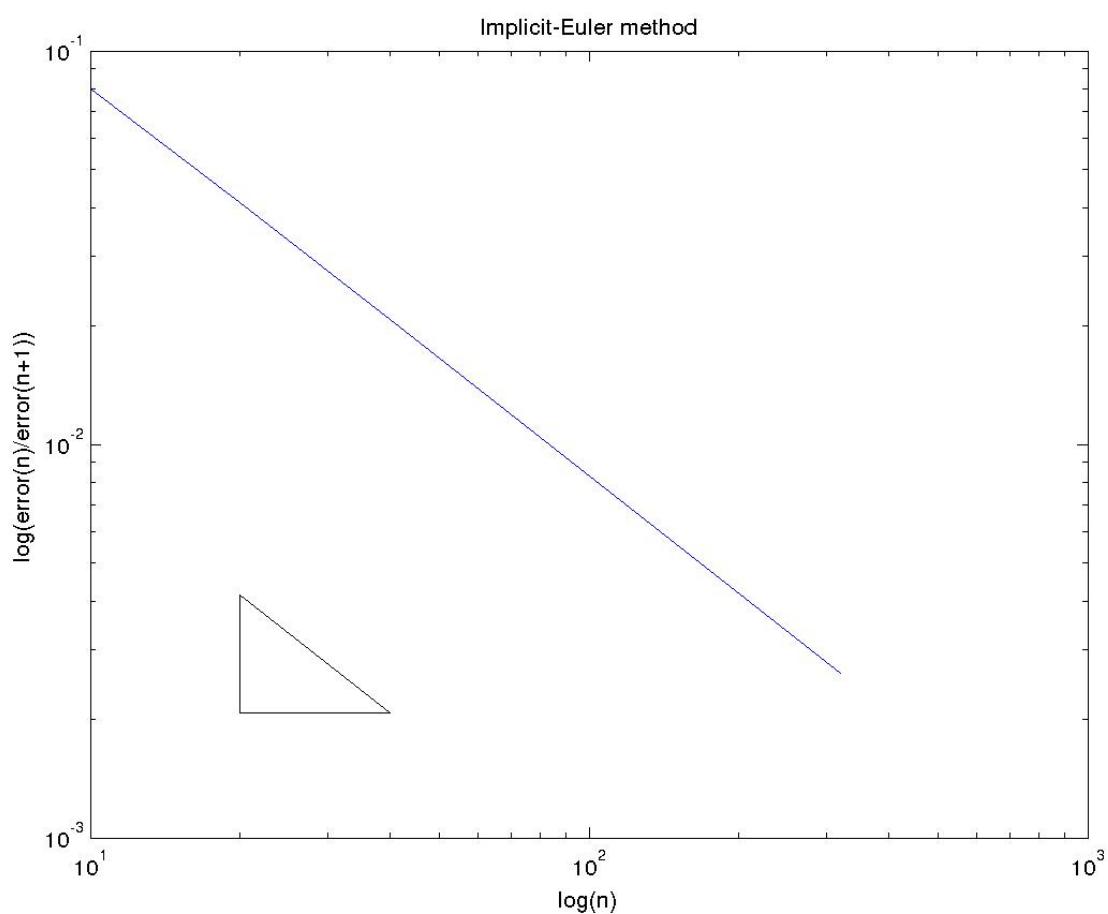
N	Error	Order
10	8.728002e-02	1.026022
20	4.285993e-02	1.015965
40	2.119413e-02	1.007824
80	1.053975e-02	1.003868
160	5.255765e-03	1.001947





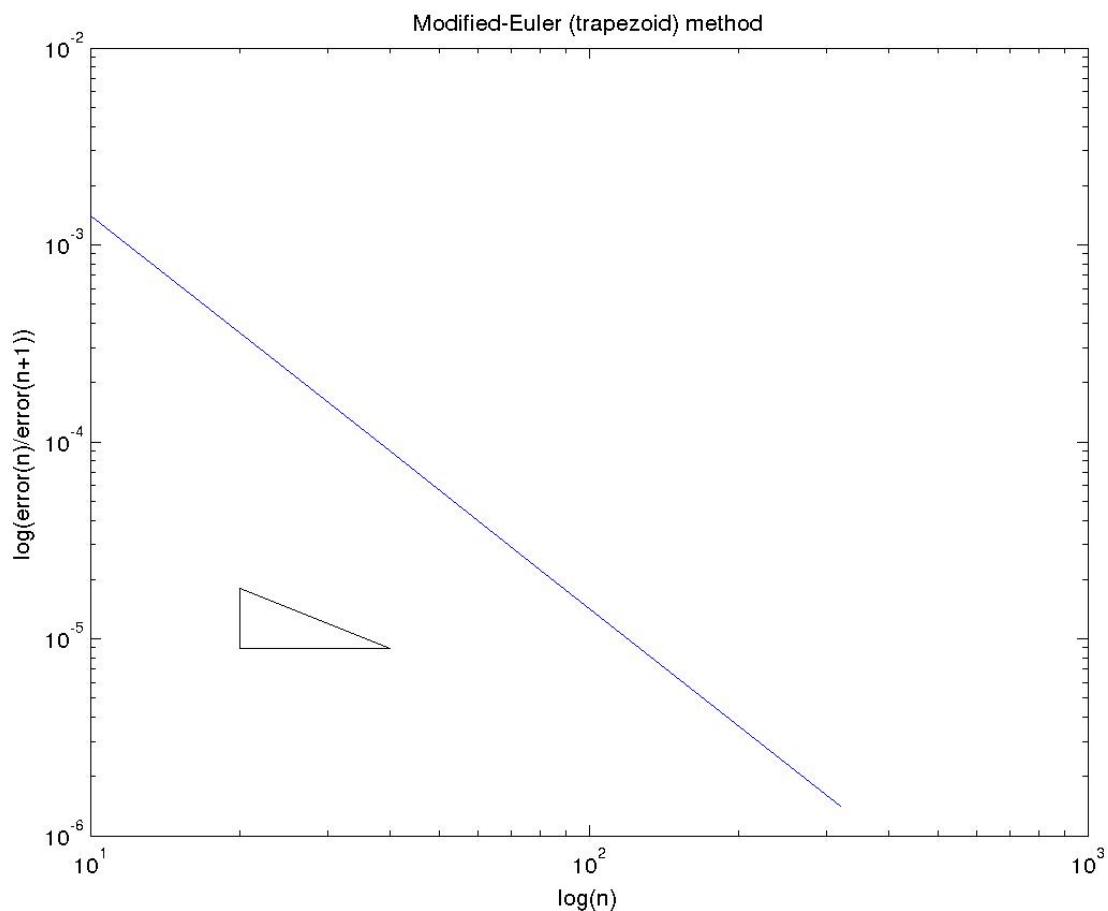
### Implicit-Euler method

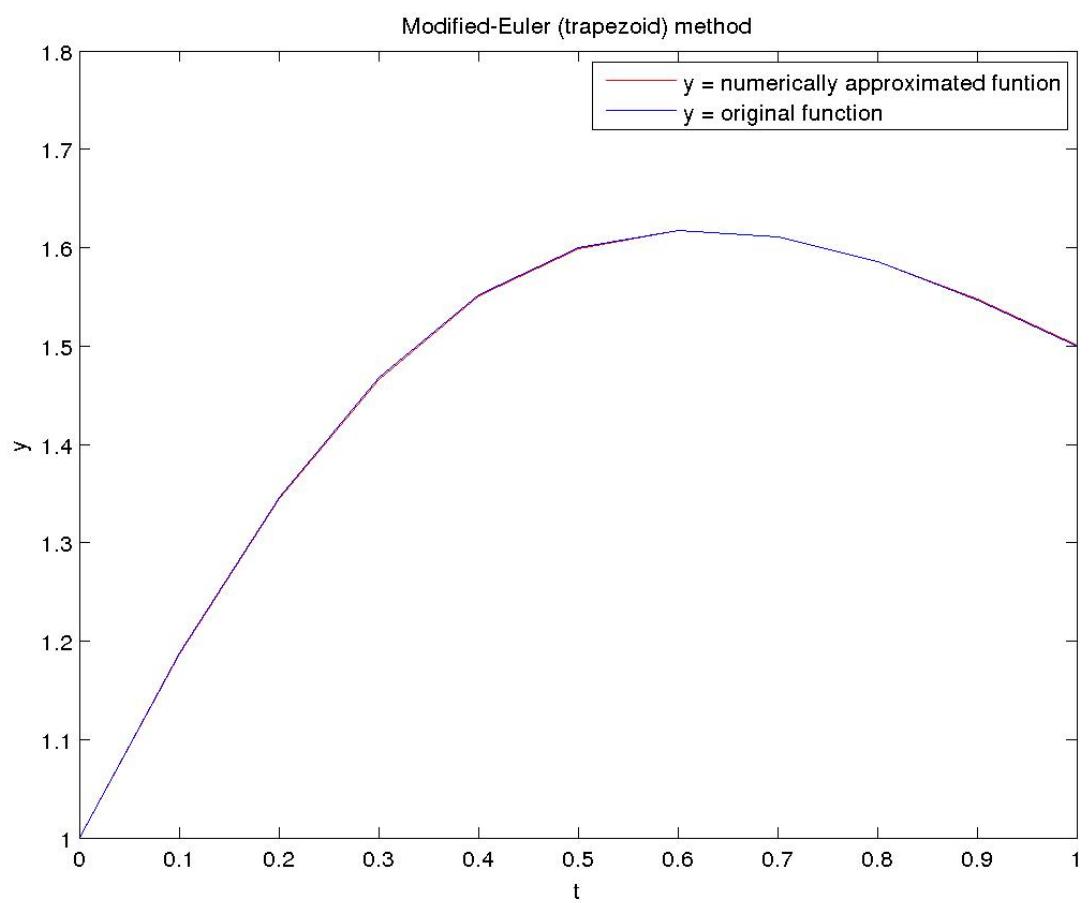
N	Error	Order
10	8.022849e-02	0.967347
20	4.103251e-02	0.984525
40	2.073751e-02	0.991968
80	1.042664e-02	0.996121
160	5.227355e-03	0.998028



### Modified-Euler (trapezoid) method

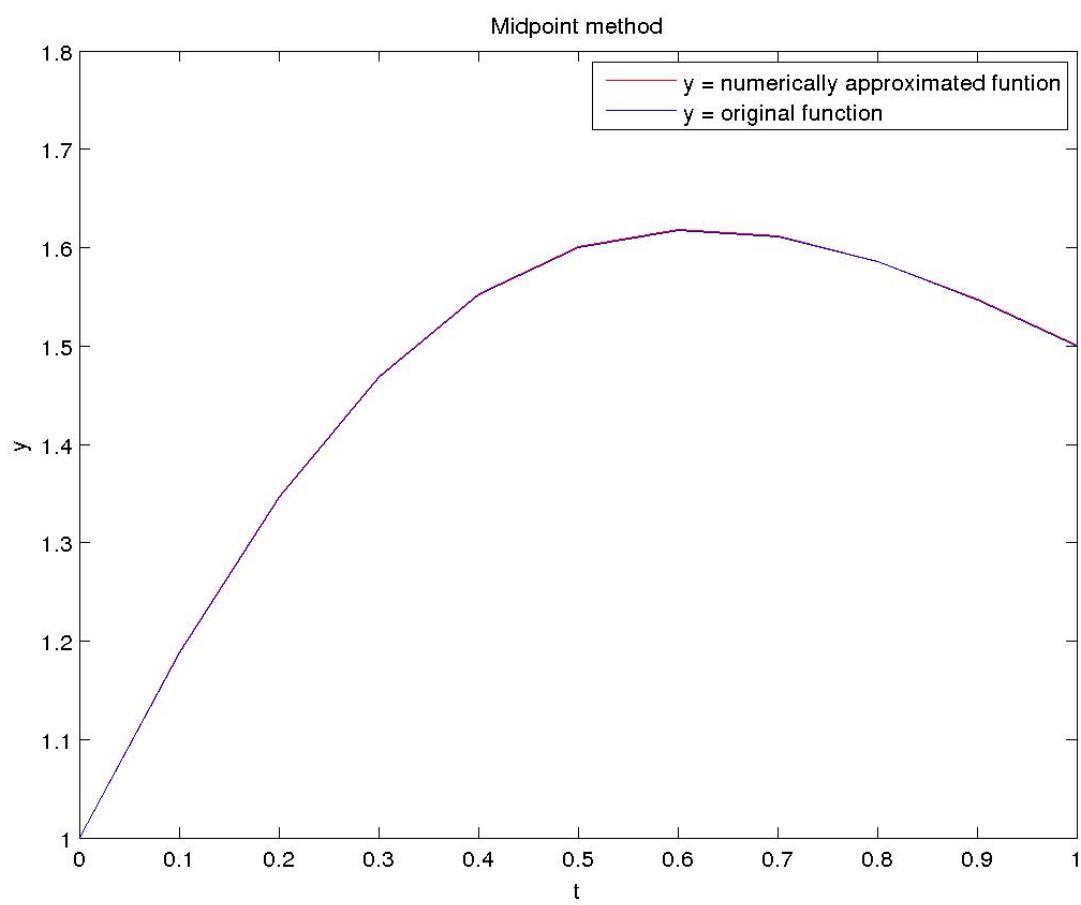
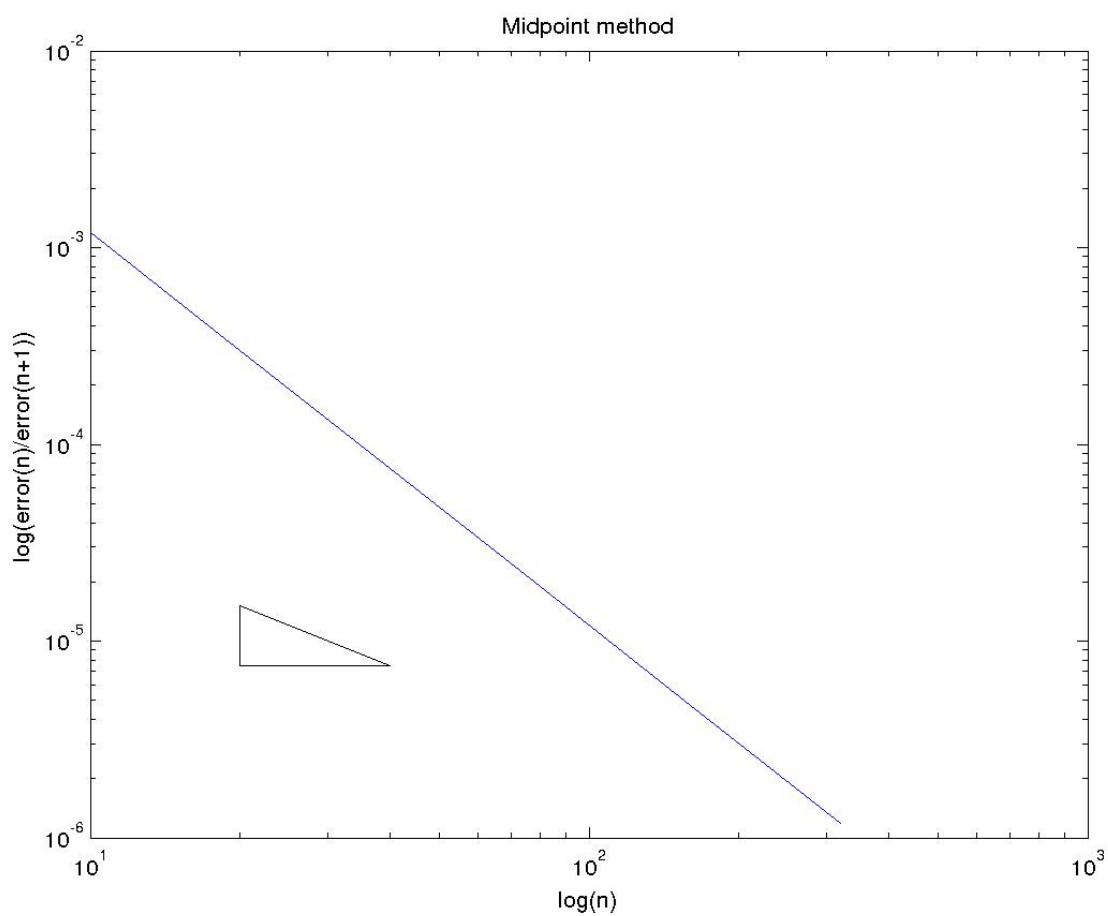
N	Error	Order
10	1.414167e-03	1.980782
20	3.582828e-04	1.998082
40	8.968986e-05	1.997962
80	2.245416e-05	2.000010
160	5.613502e-06	1.999948





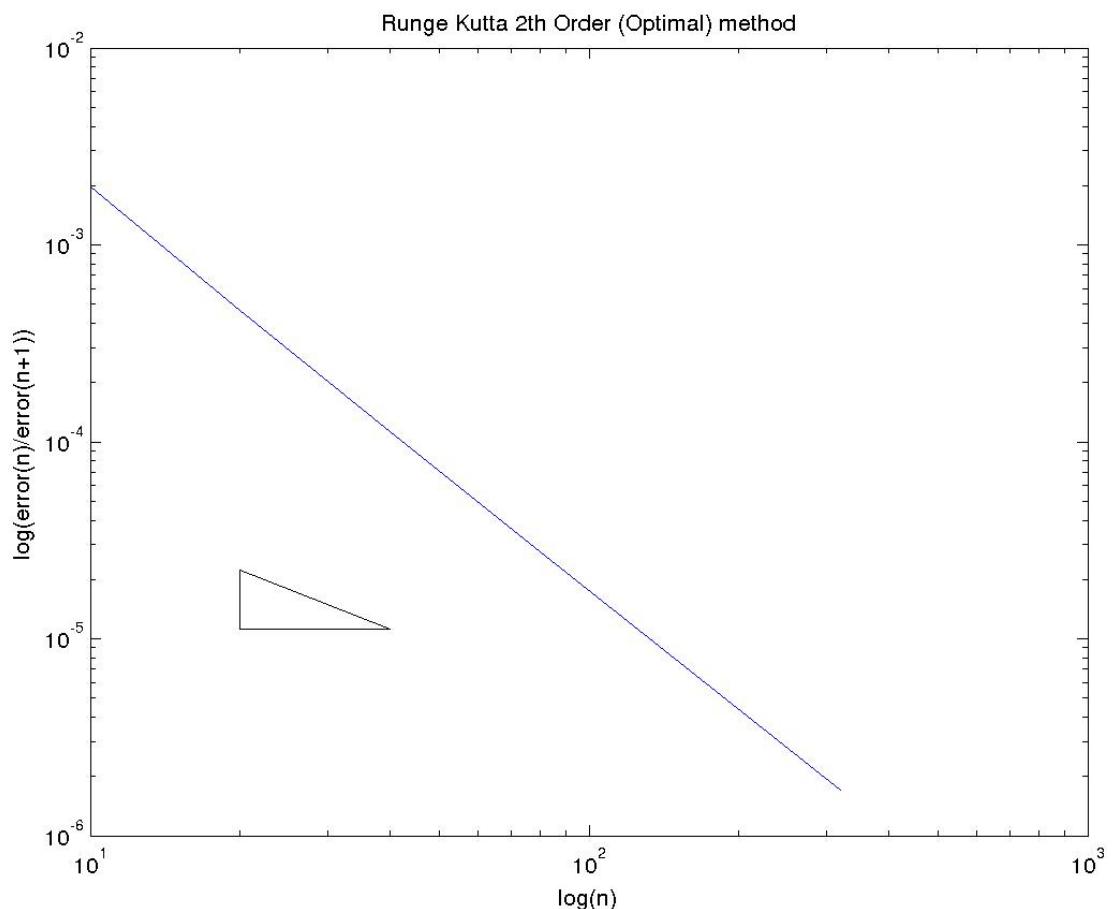
### Midpoint method

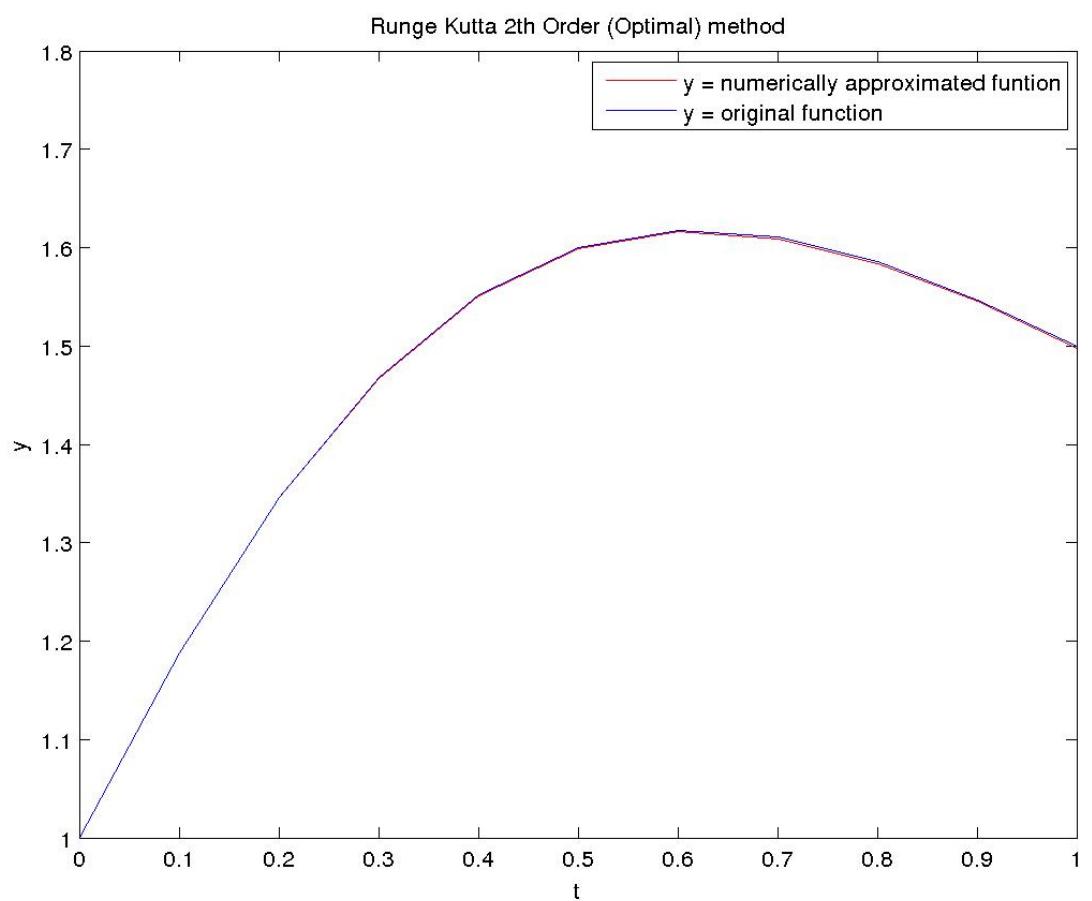
N	Error	Order
10	1.202405e-03	1.995053
20	3.016337e-04	2.000562
40	7.537903e-05	1.999535
80	1.885083e-05	2.000036
160	4.712592e-06	2.000004



### Runge Kutta 2th Order (Optimal) method

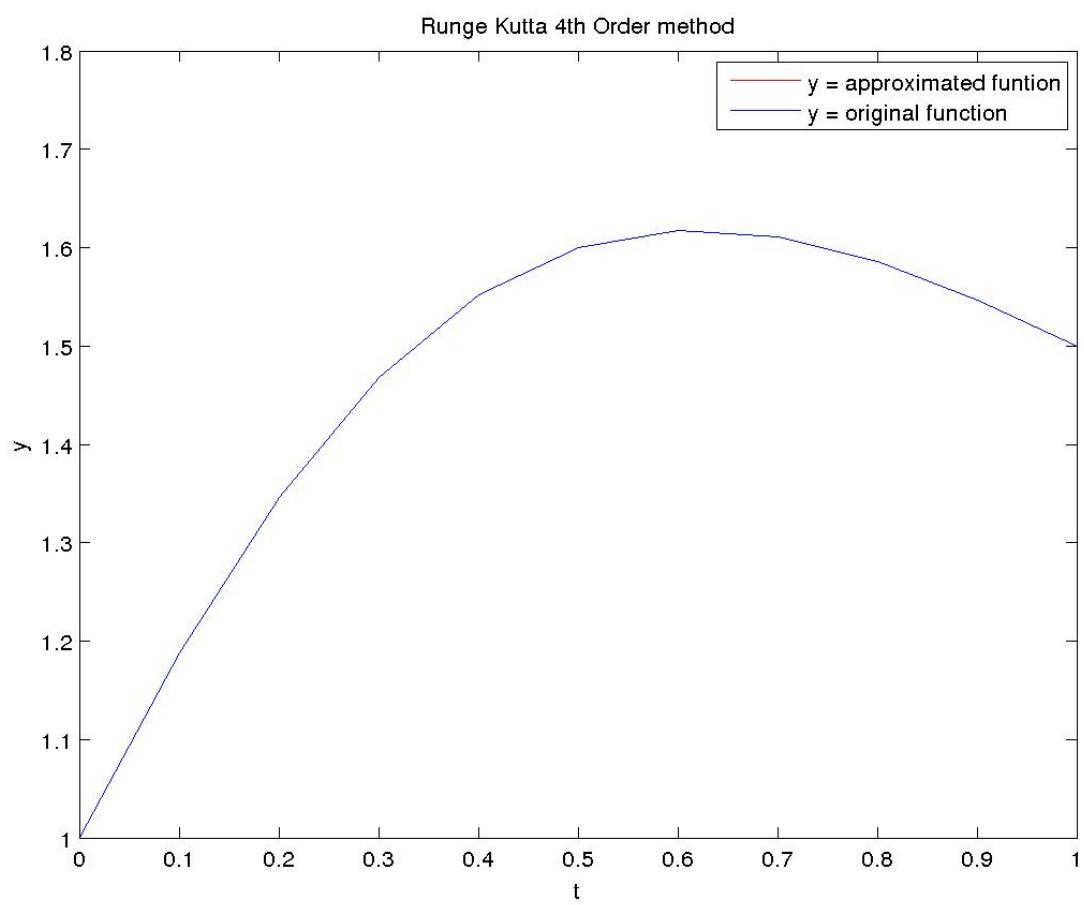
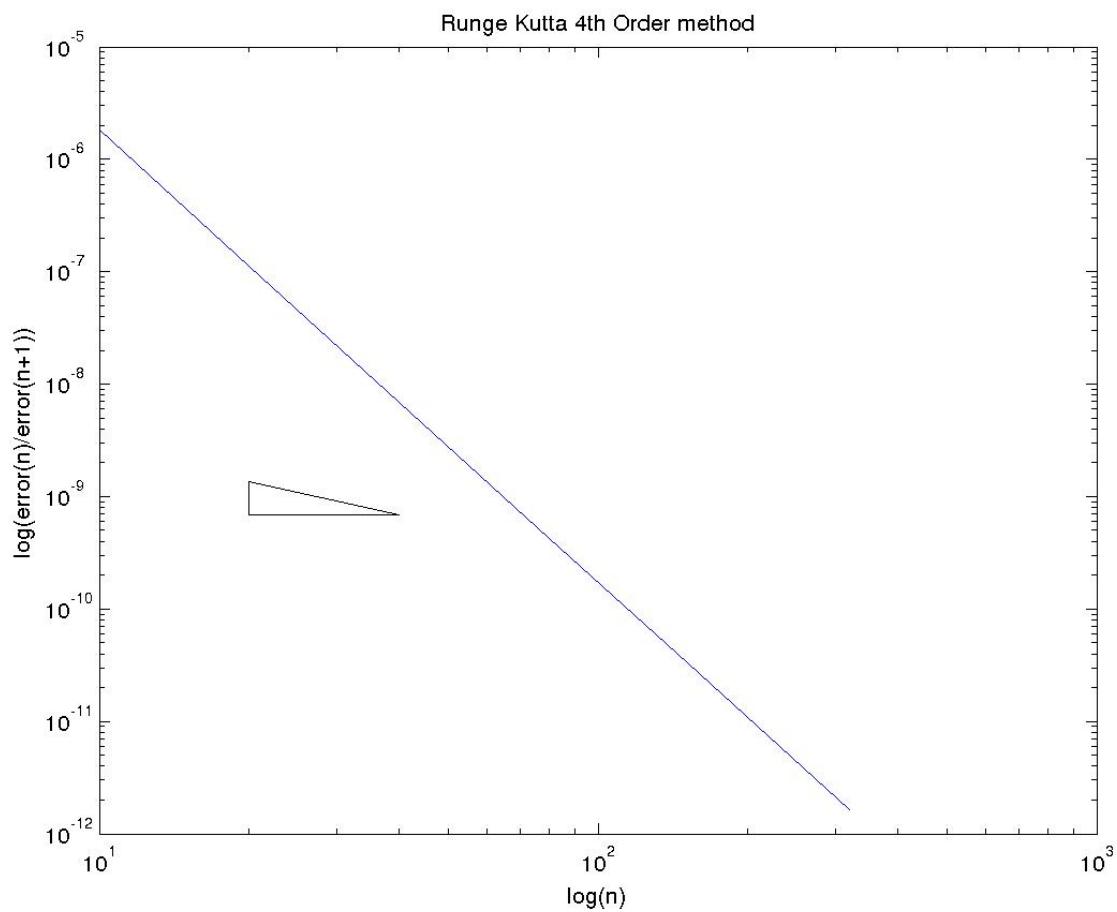
N	Error	Order
10	1.980428e-03	2.092101
20	4.644874e-04	2.049850
40	1.121780e-04	2.025210
80	2.755871e-05	2.012637
160	6.829590e-06	2.006348





### Runge Kutta 4th Order method

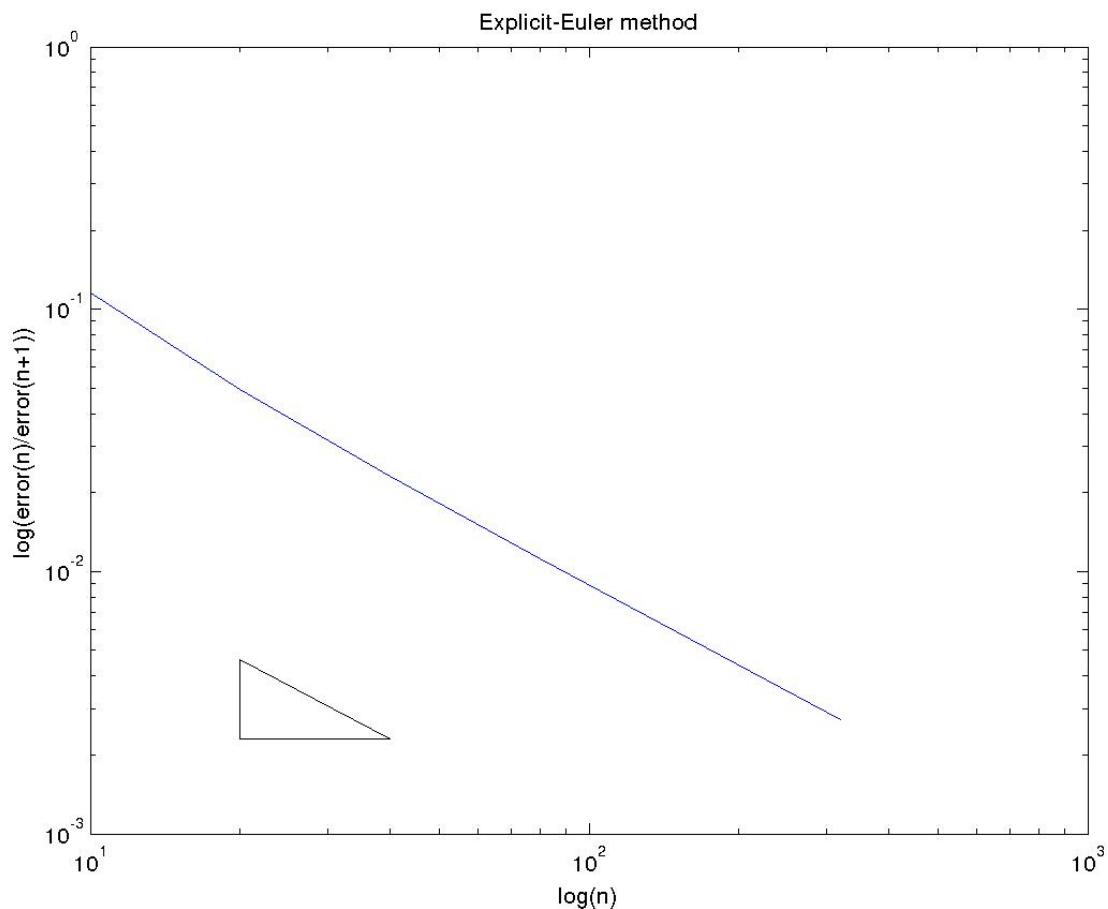
N	Error	Order
10	1.823453e-06	4.036679
20	1.111049e-07	4.018364
40	6.856224e-09	4.008934
80	4.258685e-10	4.004488
160	2.653411e-11	4.002259



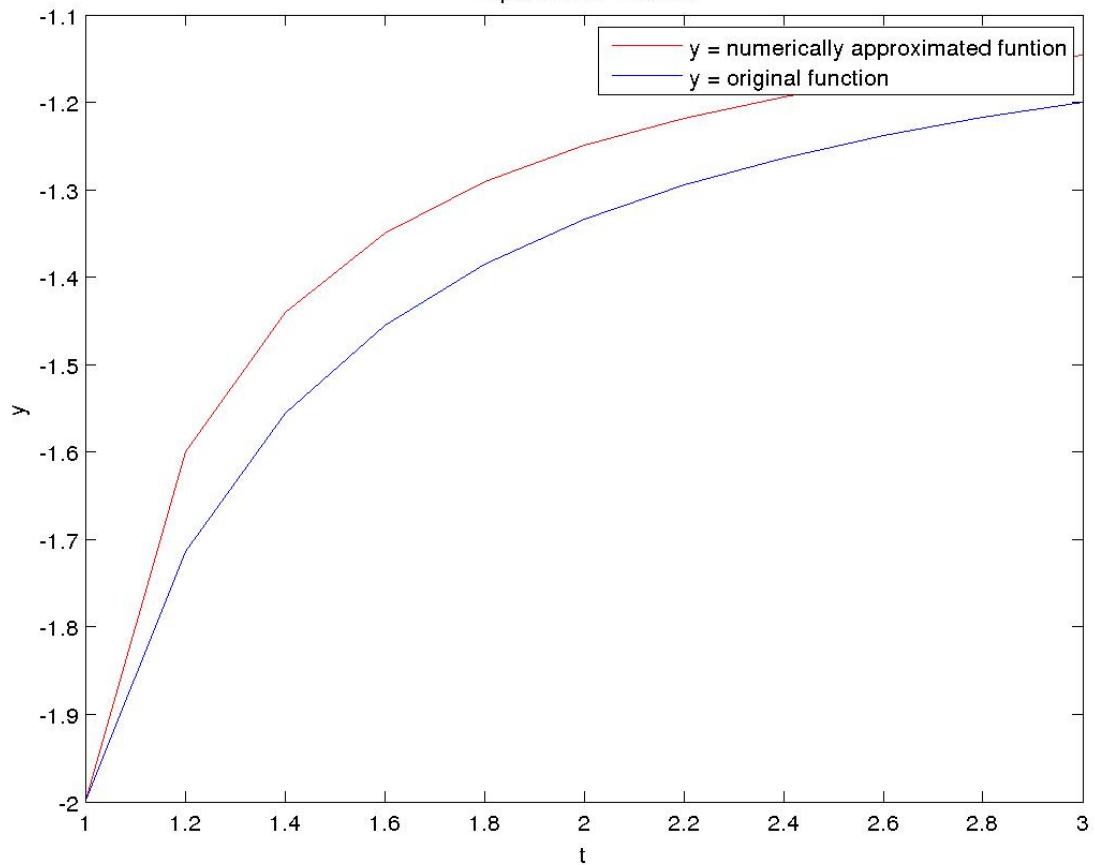
**(B)**

Explicit-Euler method

N	Error	Order
10	1.155556e-01	1.227102
20	4.936241e-02	1.096493
40	2.308443e-02	1.044949
80	1.118814e-02	1.021197
160	5.512480e-03	1.010424

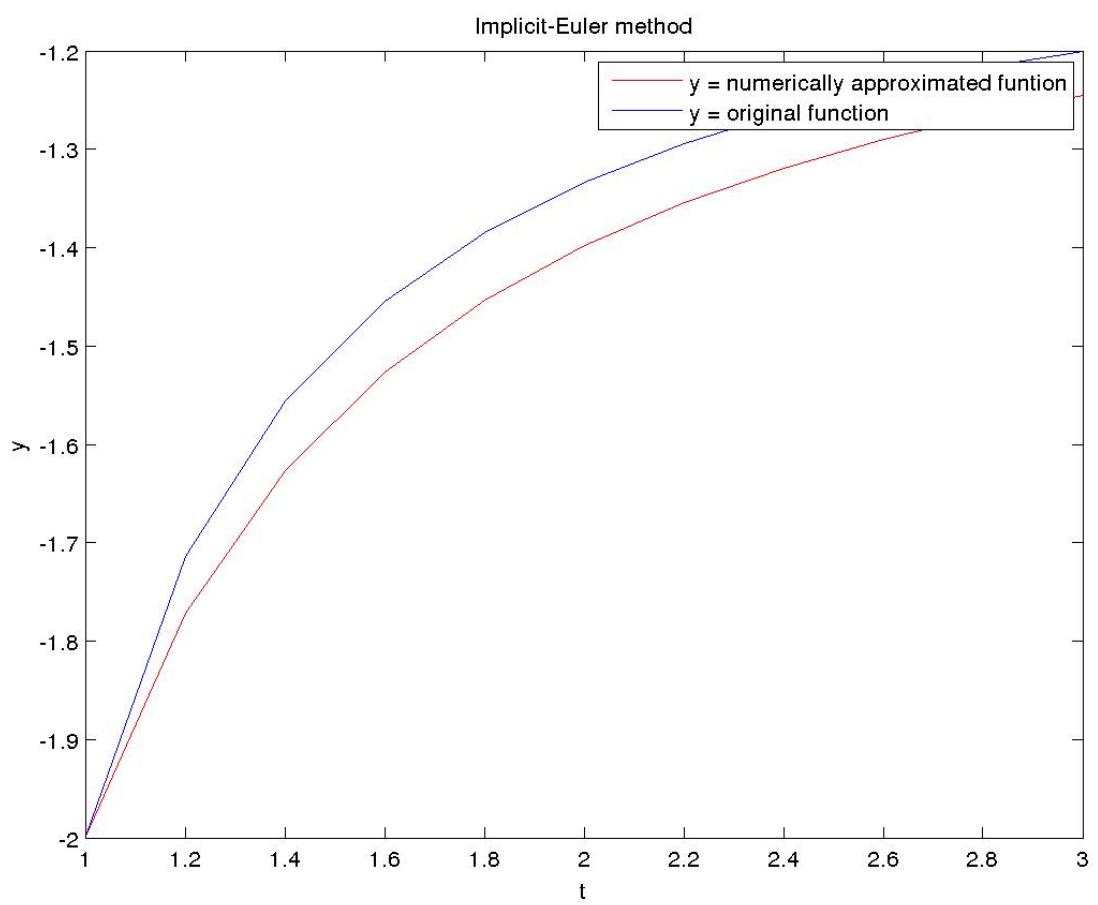
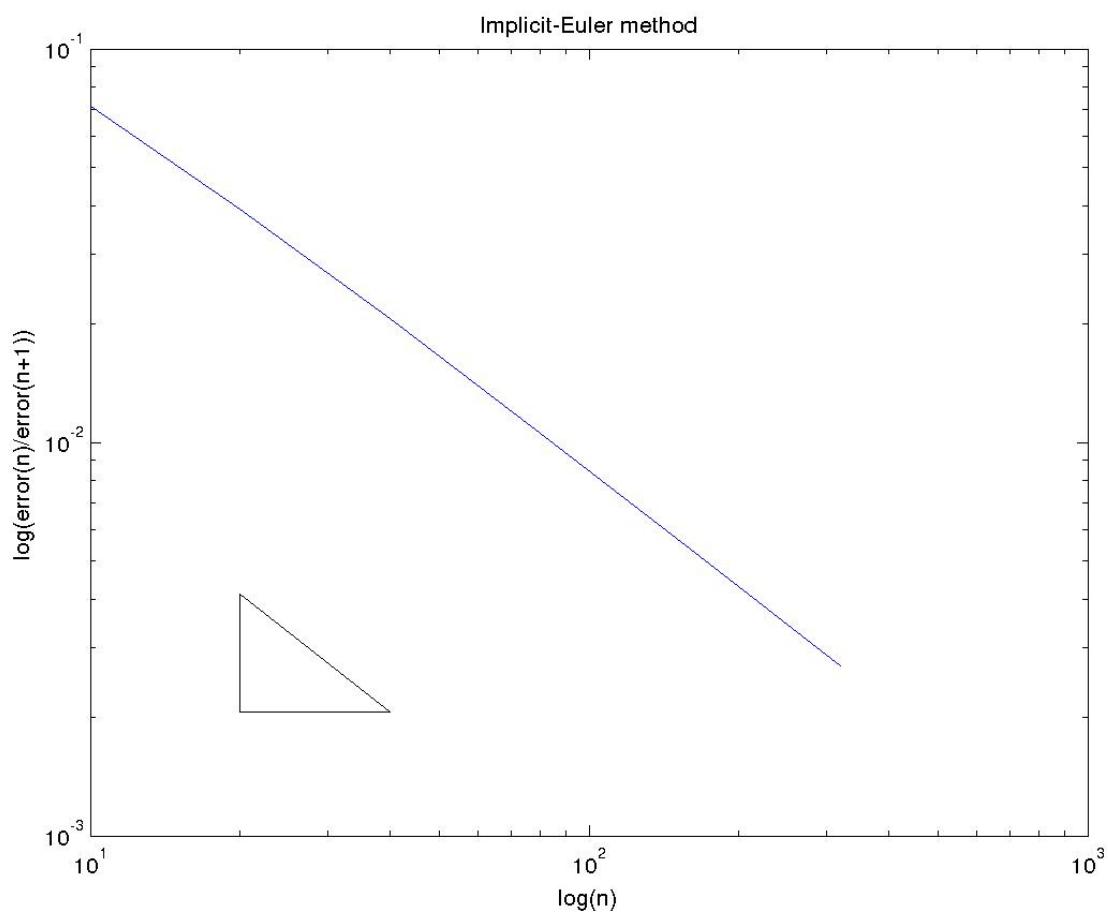


Explicit-Euler method



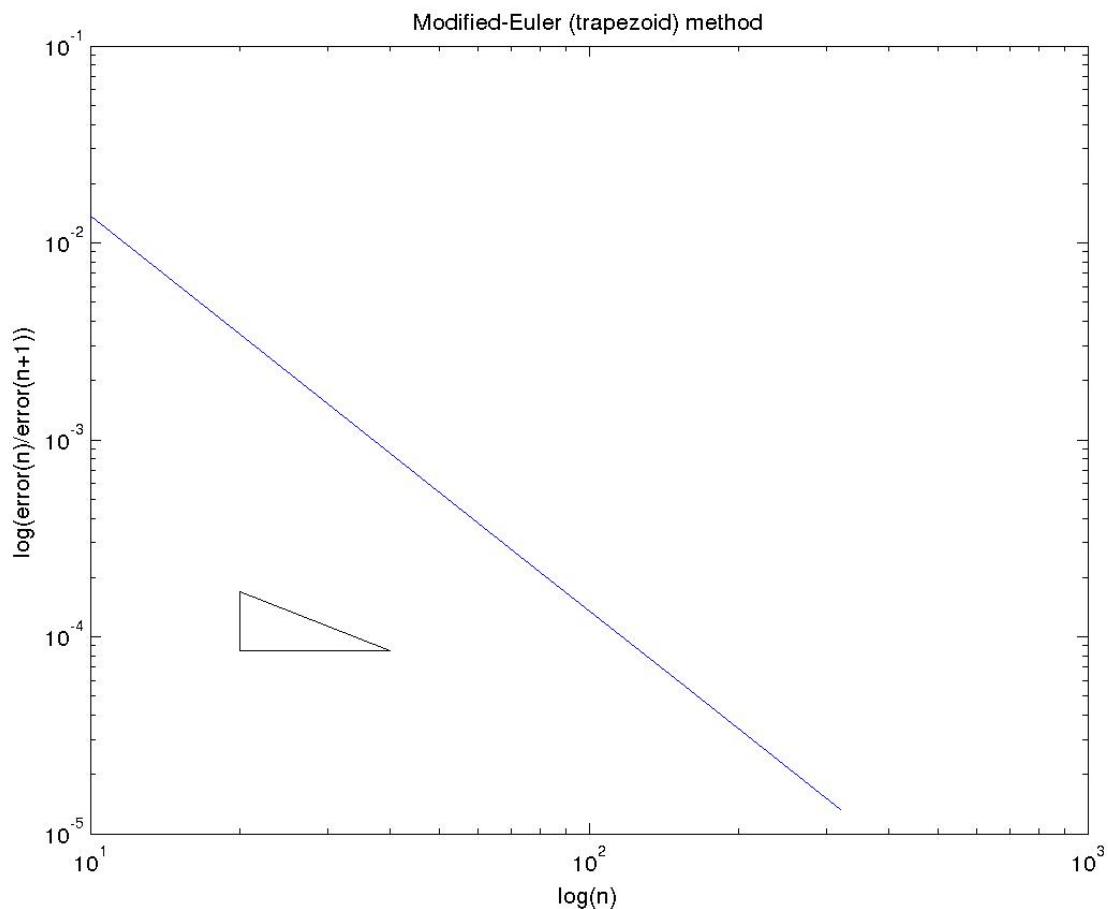
Implicit-Euler method

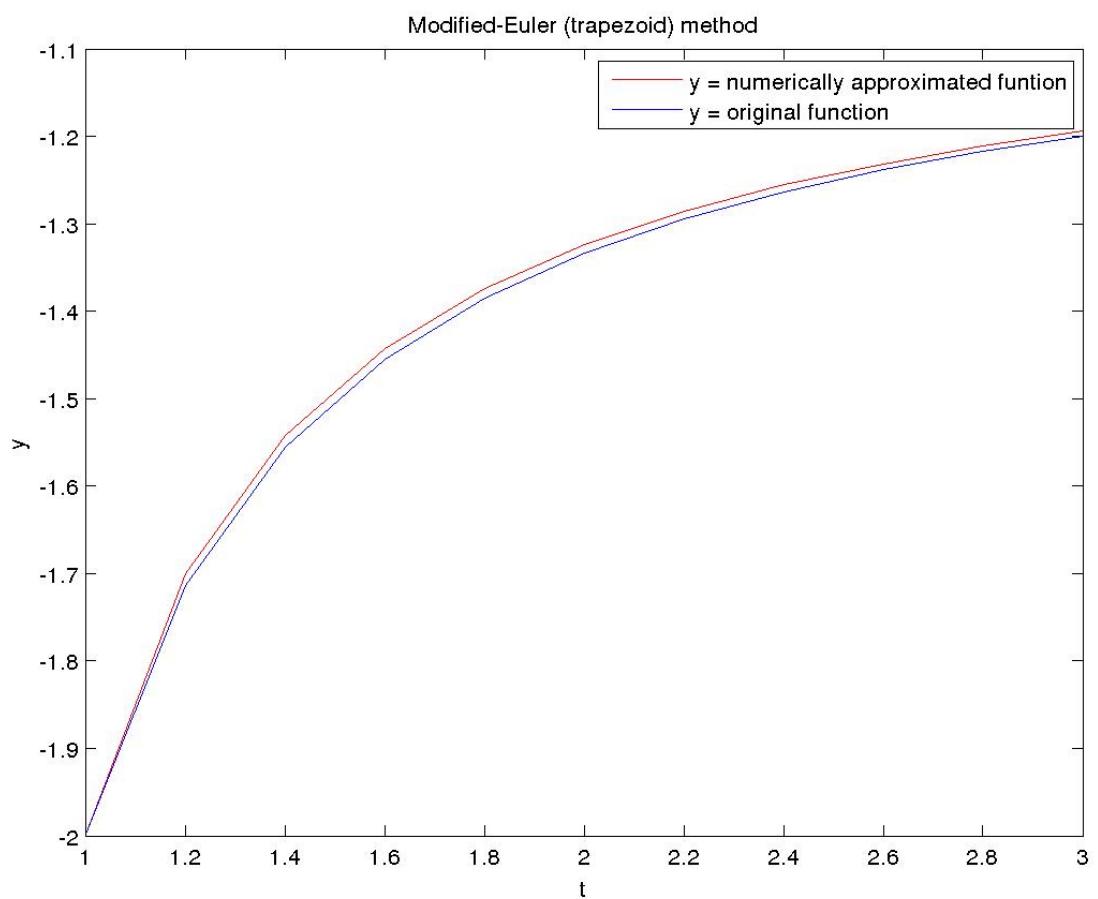
N	Error	Order
10	7.154419e-02	0.867985
20	3.919990e-02	0.928349
40	2.059795e-02	0.962450
80	1.057056e-02	0.980201
160	5.358312e-03	0.989956



### Modified-Euler (trapezoid) method

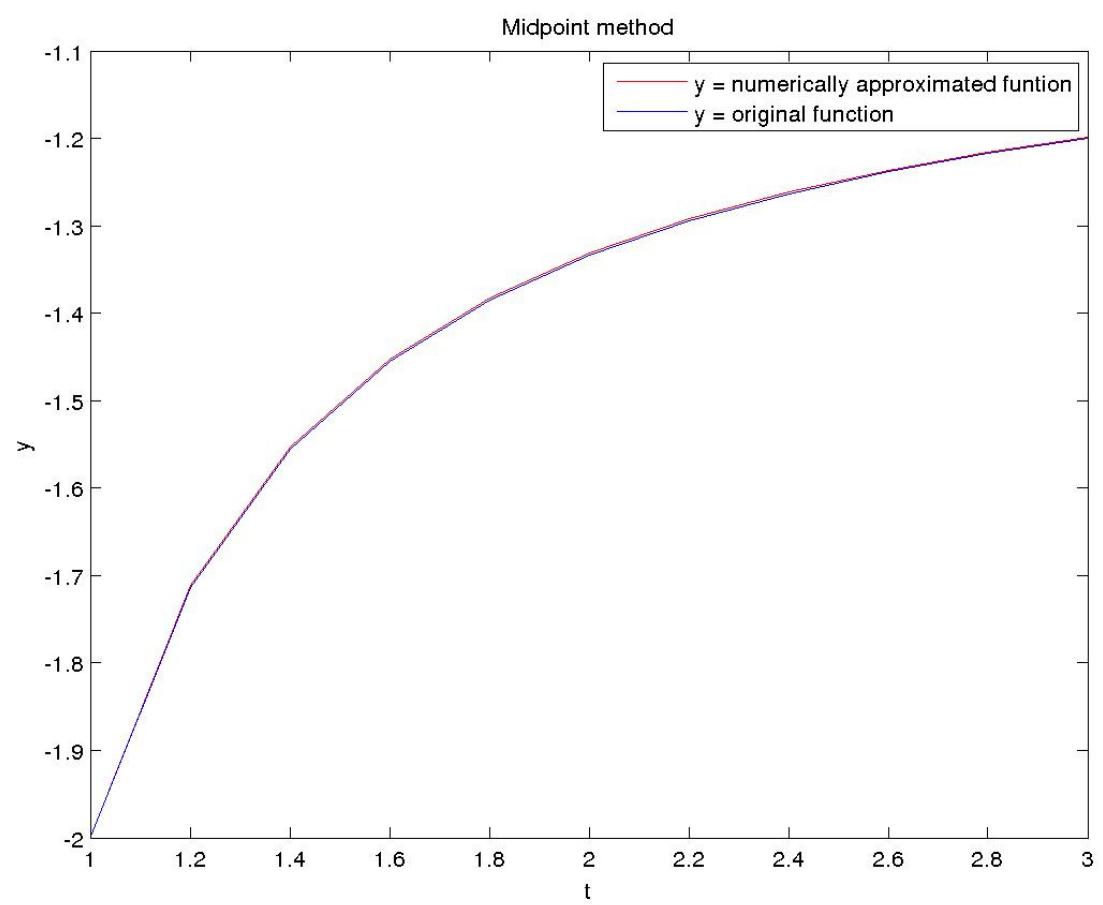
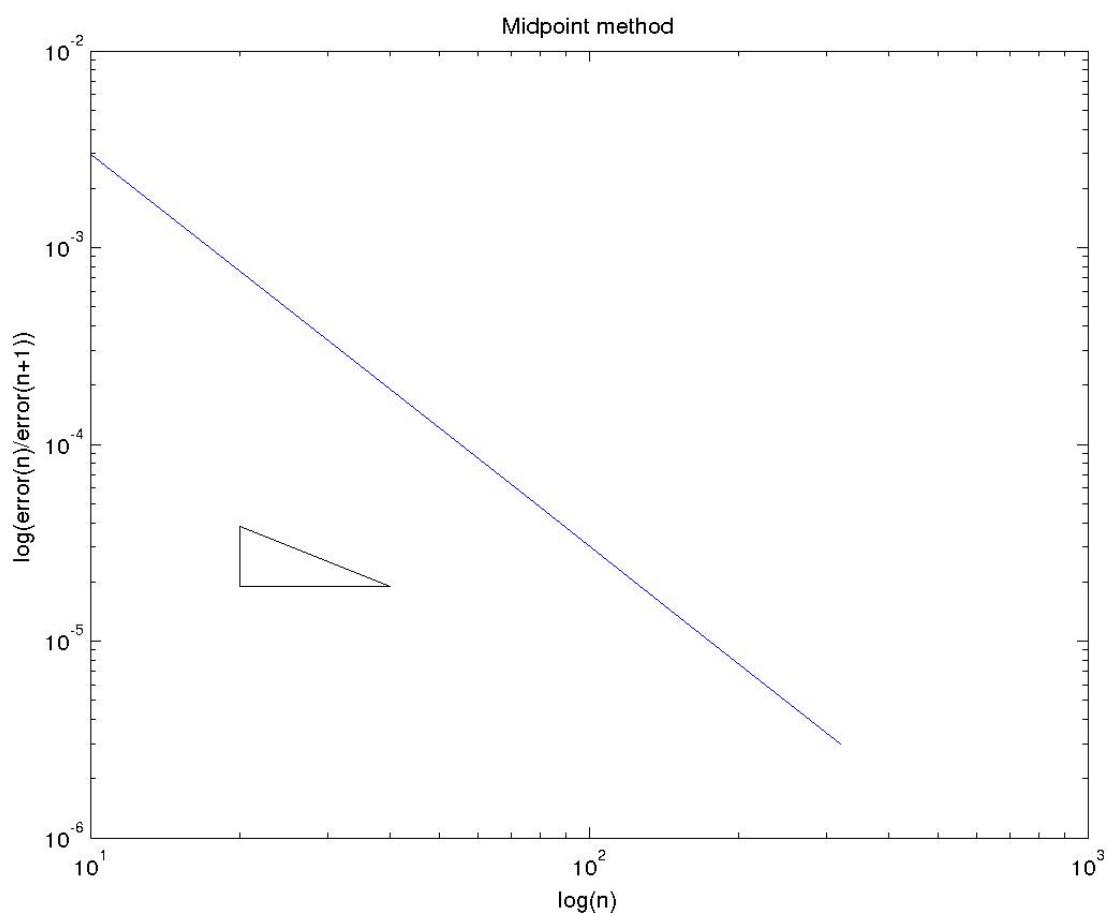
N	Error	Order
10	1.382418e-02	2.008651
20	3.435384e-03	2.011356
40	8.521125e-04	2.002841
80	2.126091e-04	2.000262
160	5.314263e-05	2.000172





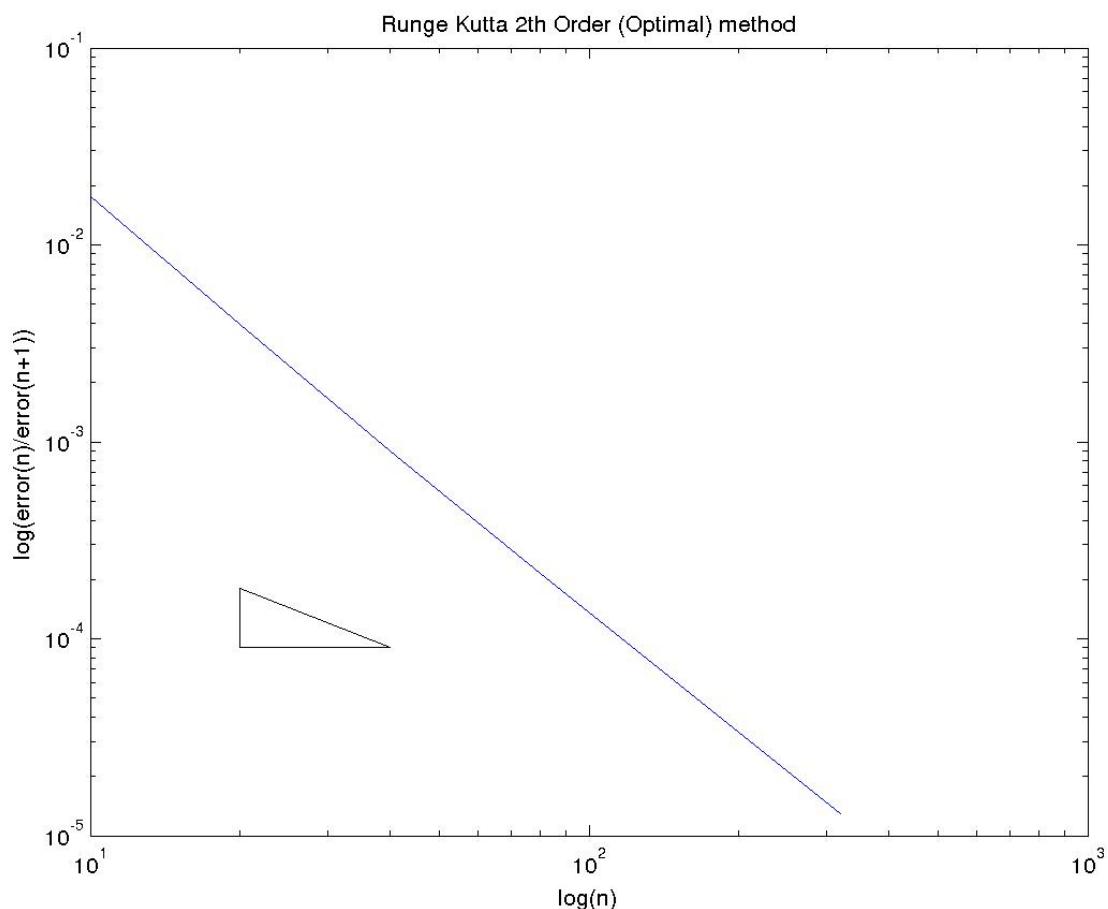
Midpoint method

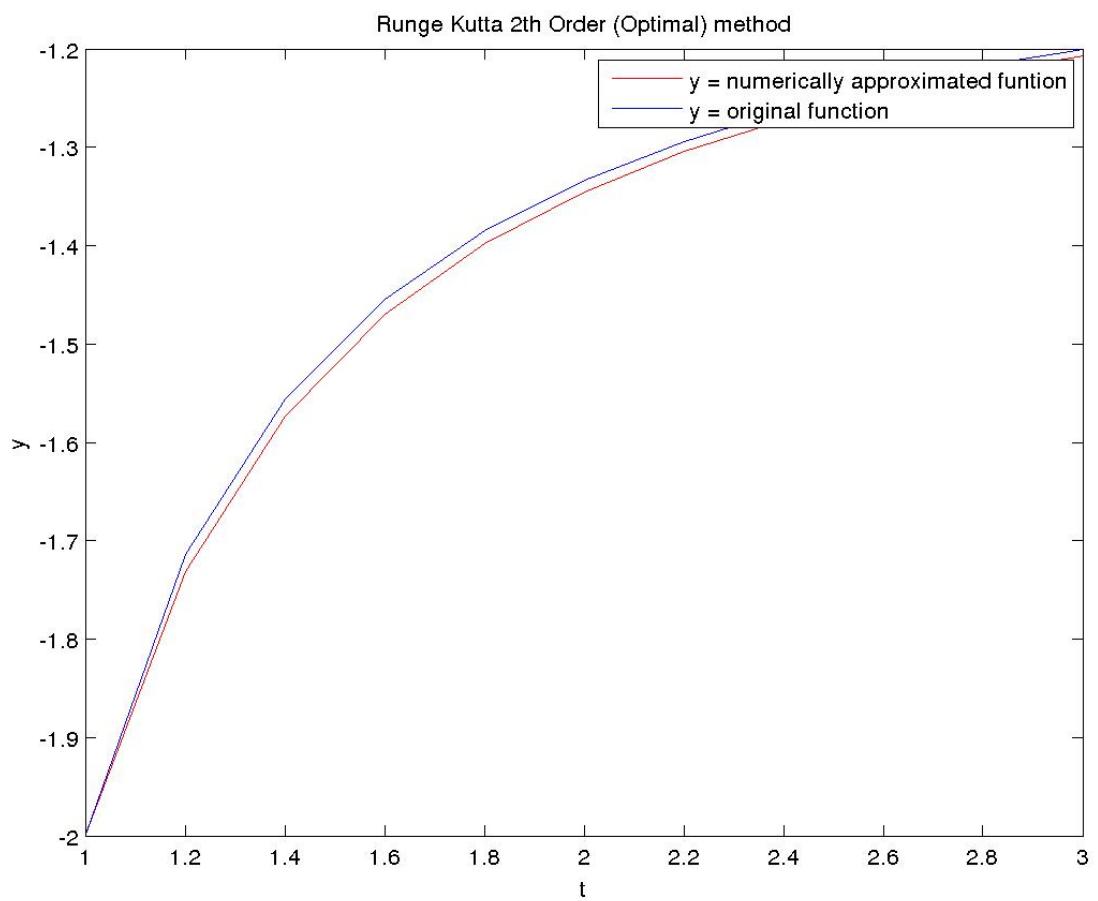
N	Error	Order
10	2.976513e-03	1.964880
20	7.624653e-04	1.998627
40	1.907979e-04	1.996826
80	4.780453e-05	1.999980
160	1.195130e-05	1.999991



### Runge Kutta 2th Order (Optimal) method

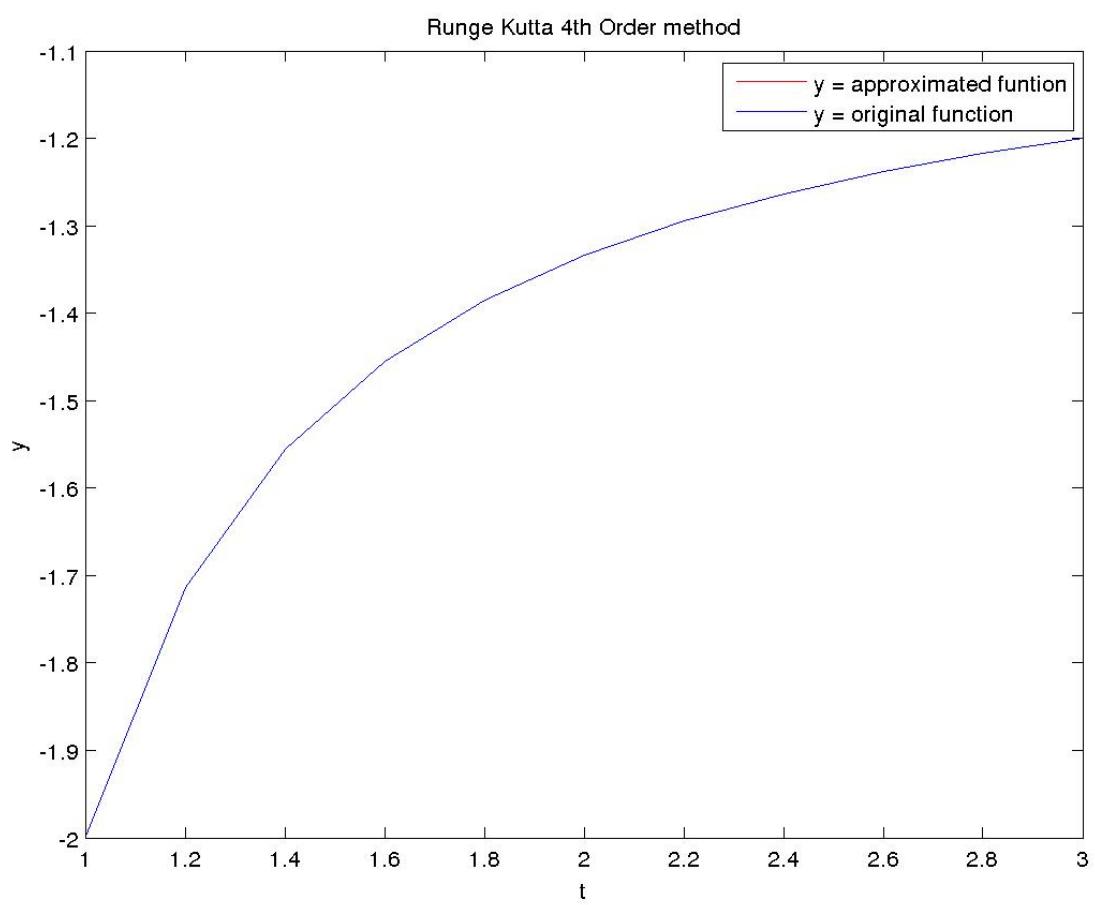
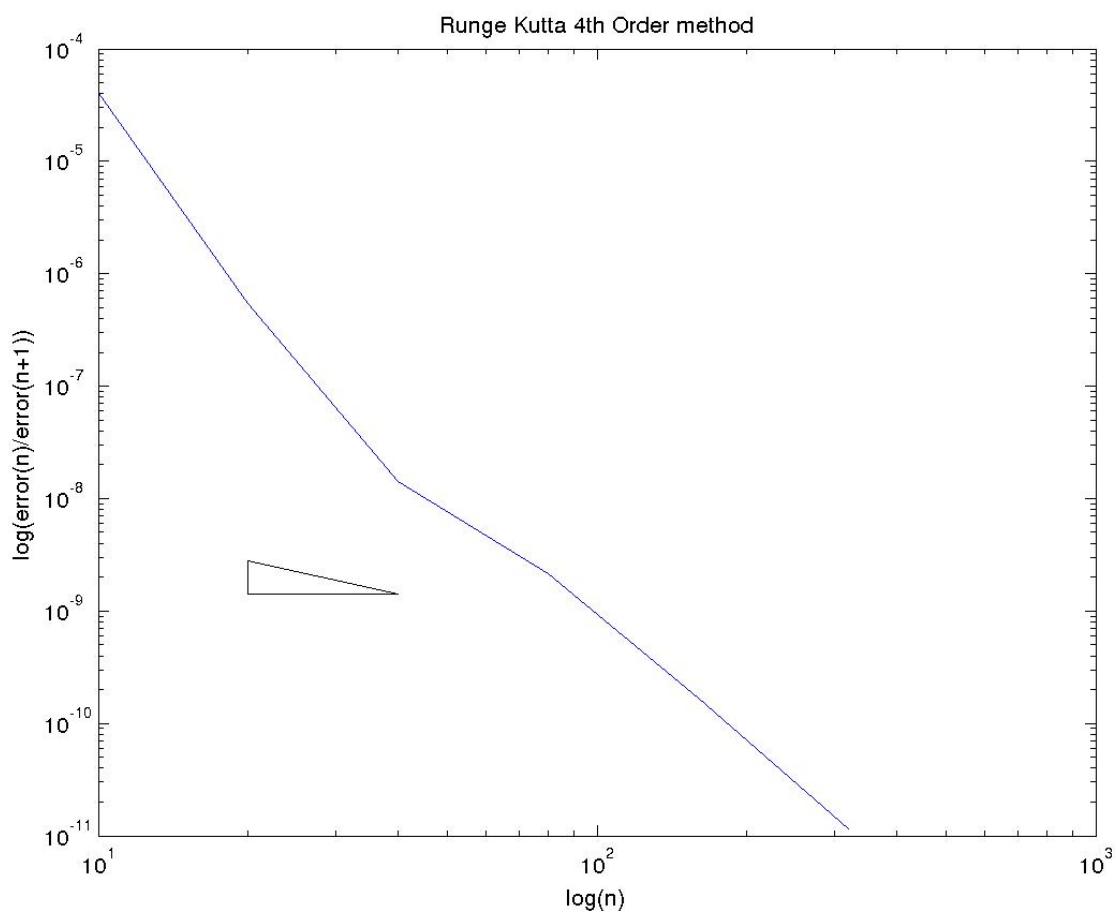
N	Error	Order
10	1.763556e-02	2.167027
20	3.926893e-03	2.125475
40	8.999482e-04	2.065738
80	2.149653e-04	2.033370
160	5.251254e-05	2.016781





### Runge Kutta 4th Order method

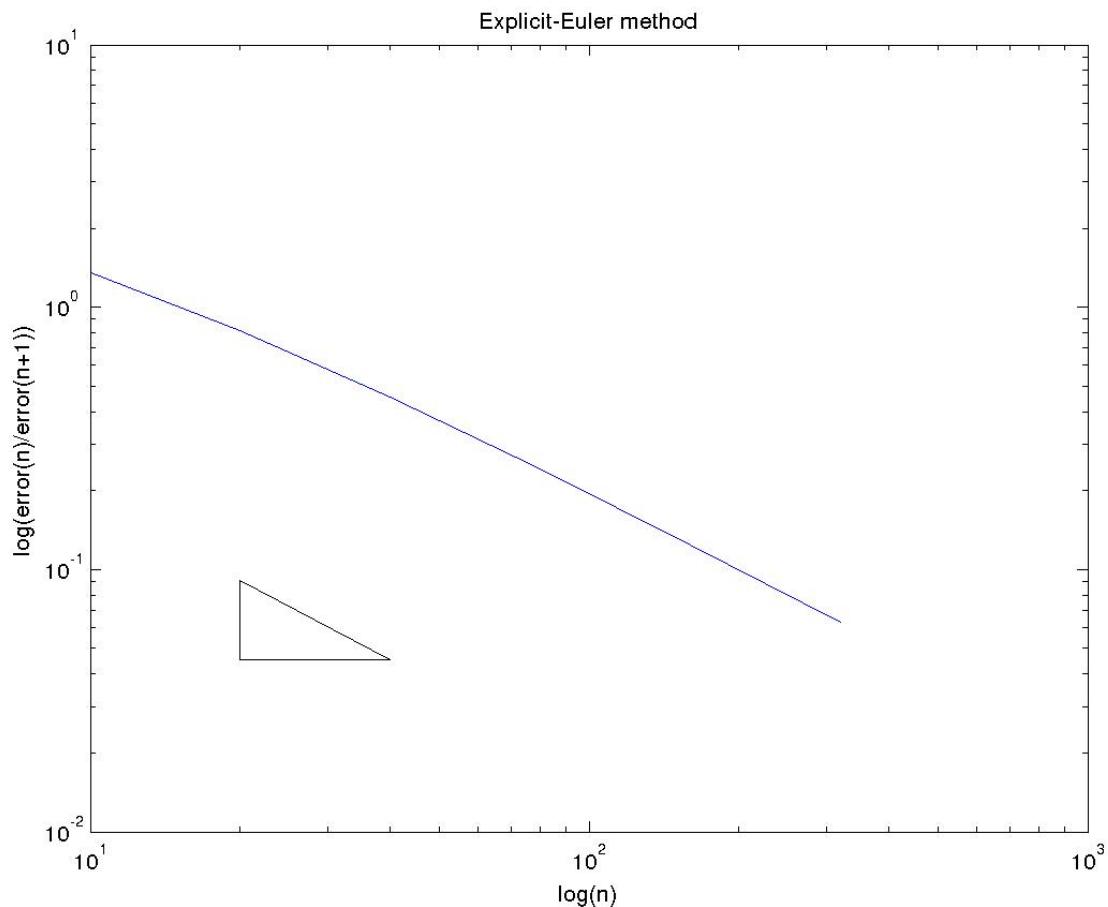
N	Error	Order
10	4.053383e-05	6.218198
20	5.444444e-07	5.271688
40	1.409345e-08	2.718562
80	2.141158e-09	3.661933
160	1.691598e-10	3.864334

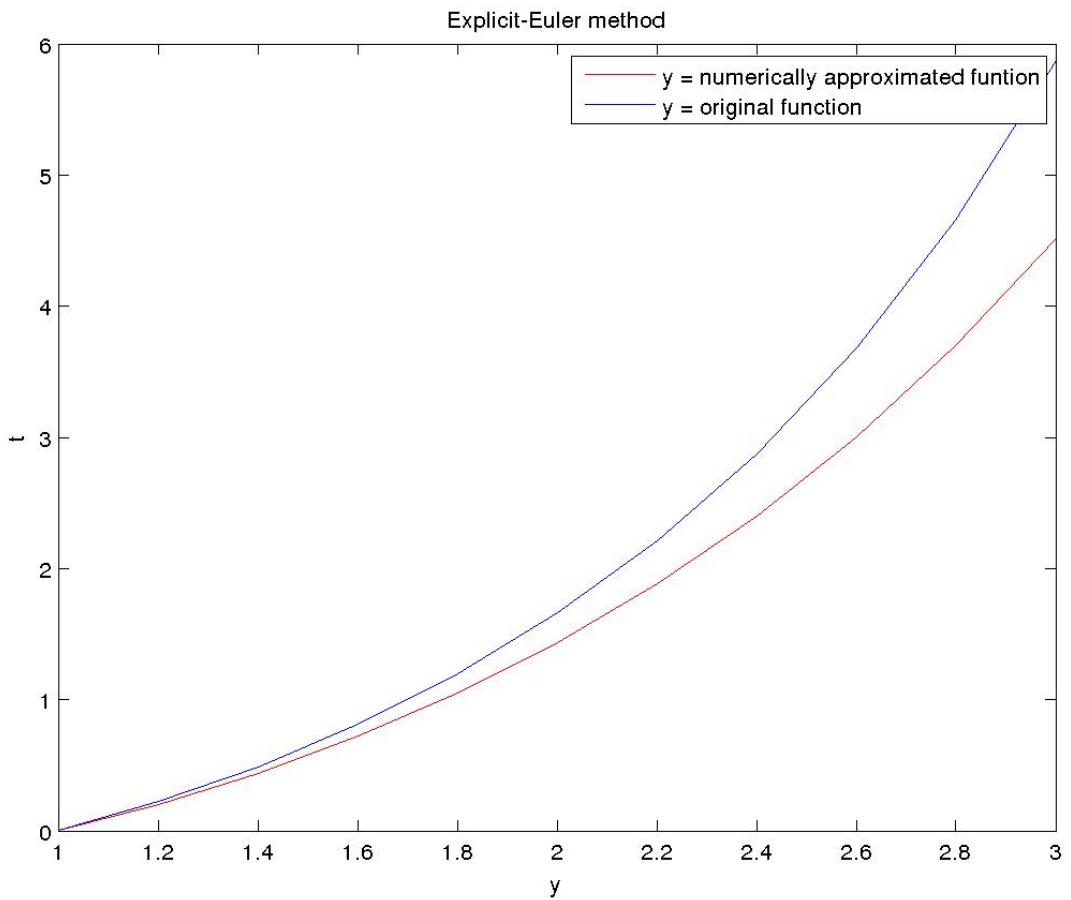


**(C)**

Explicit-Euler method

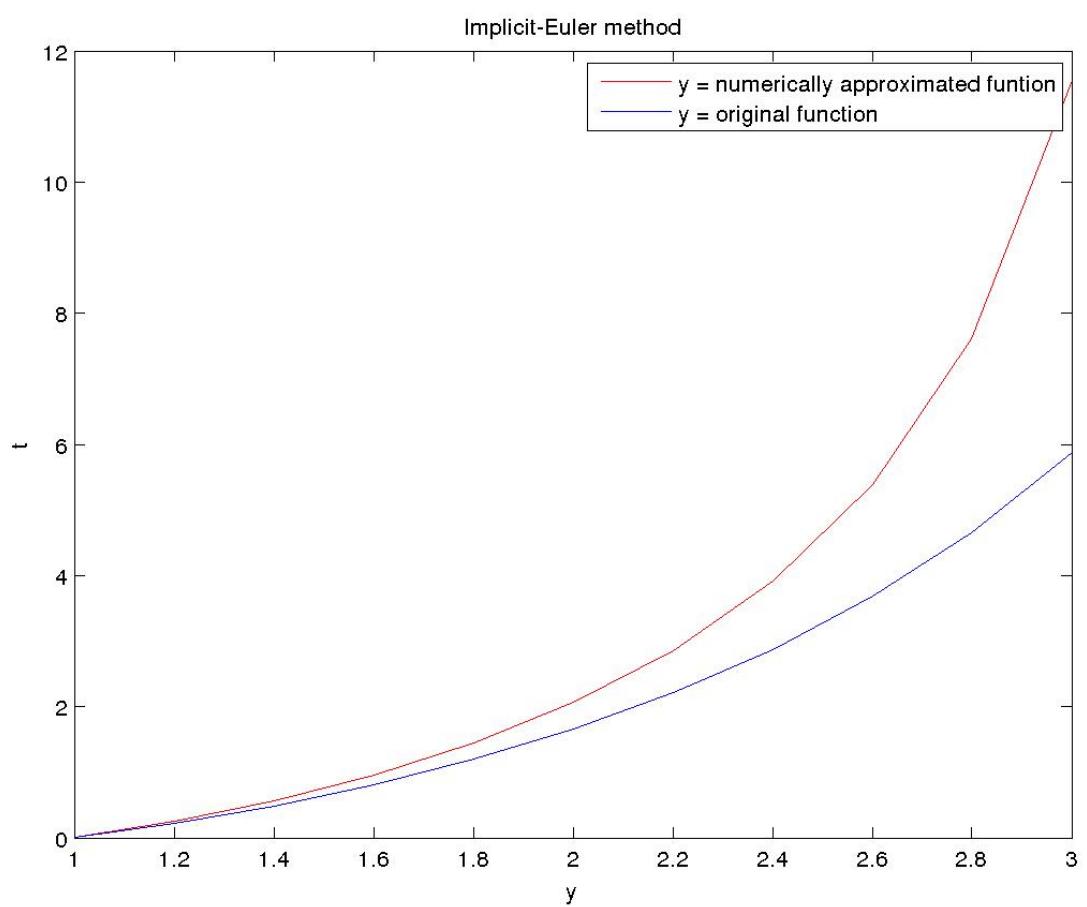
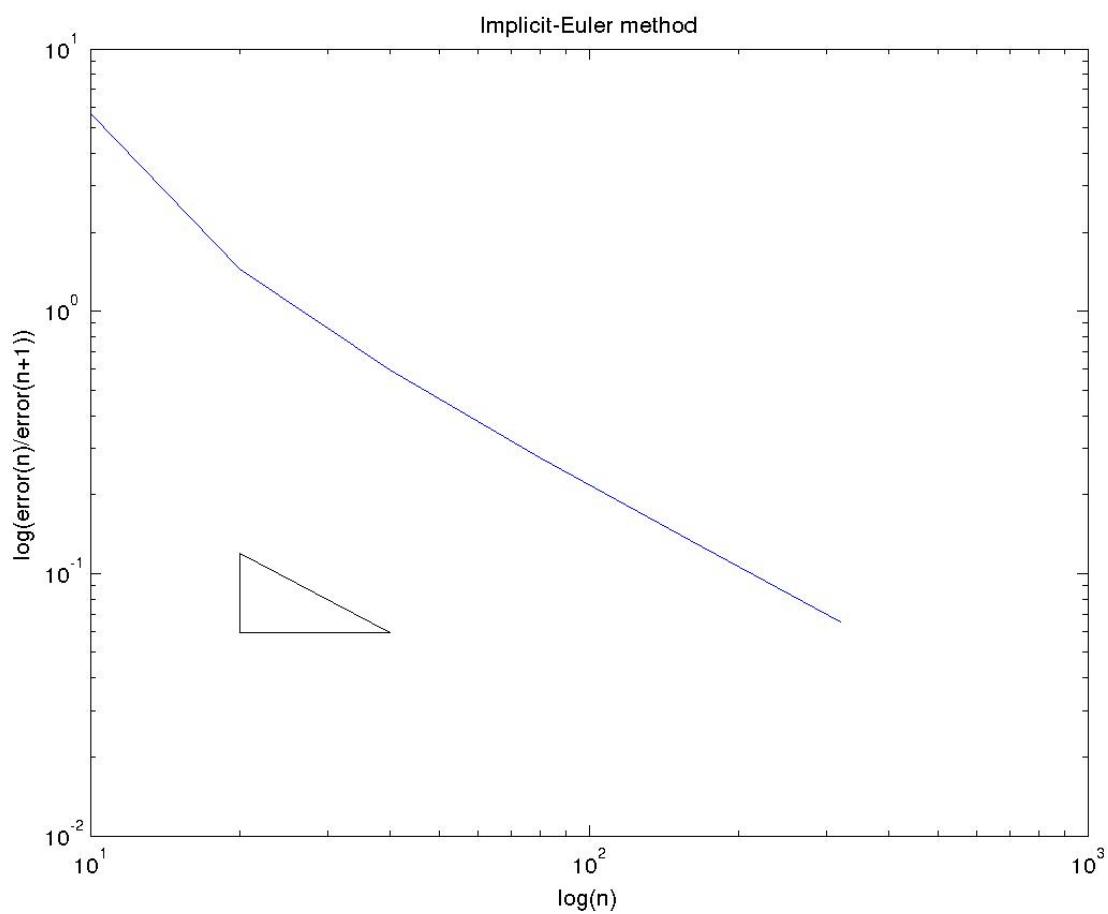
N	Error	Order
10	1.359823e+00	0.739928
20	8.142197e-01	0.843863
40	4.536422e-01	0.913033
80	2.409145e-01	0.953853
160	1.243726e-01	0.976192





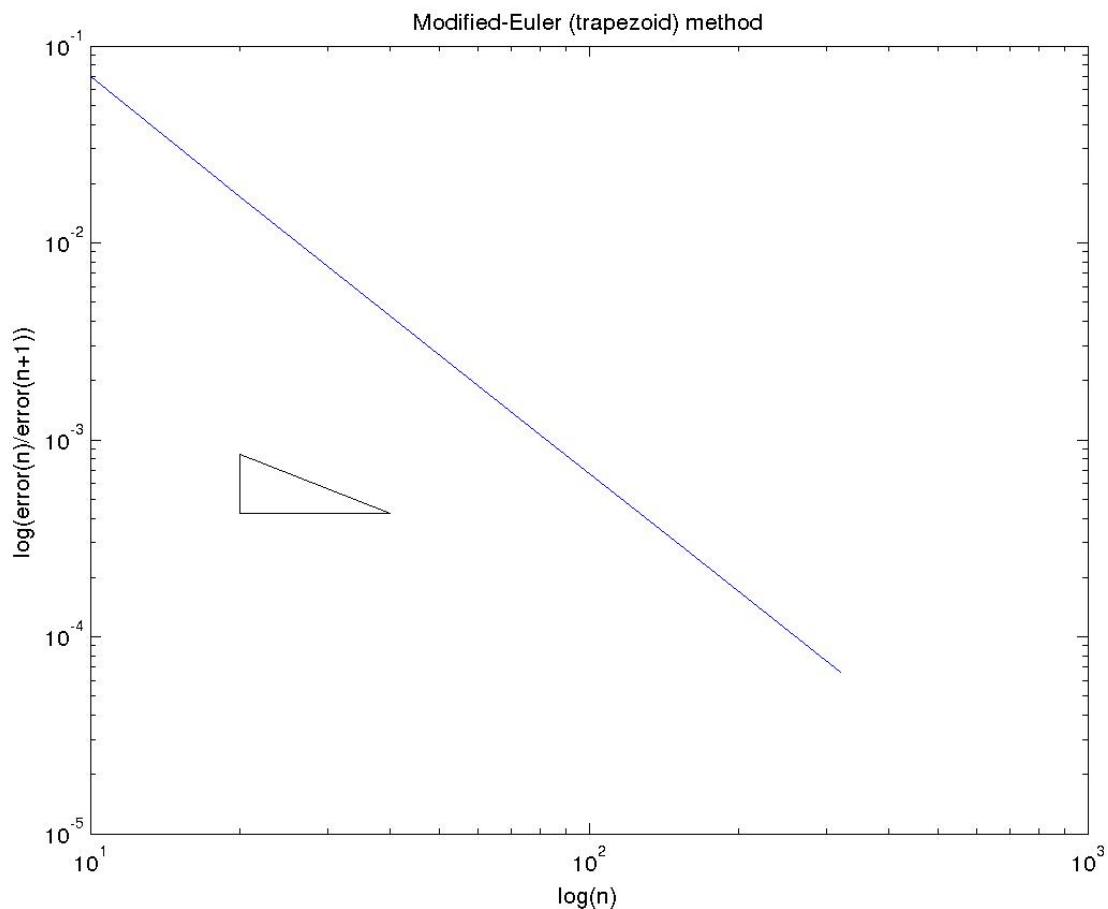
### Implicit-Euler method

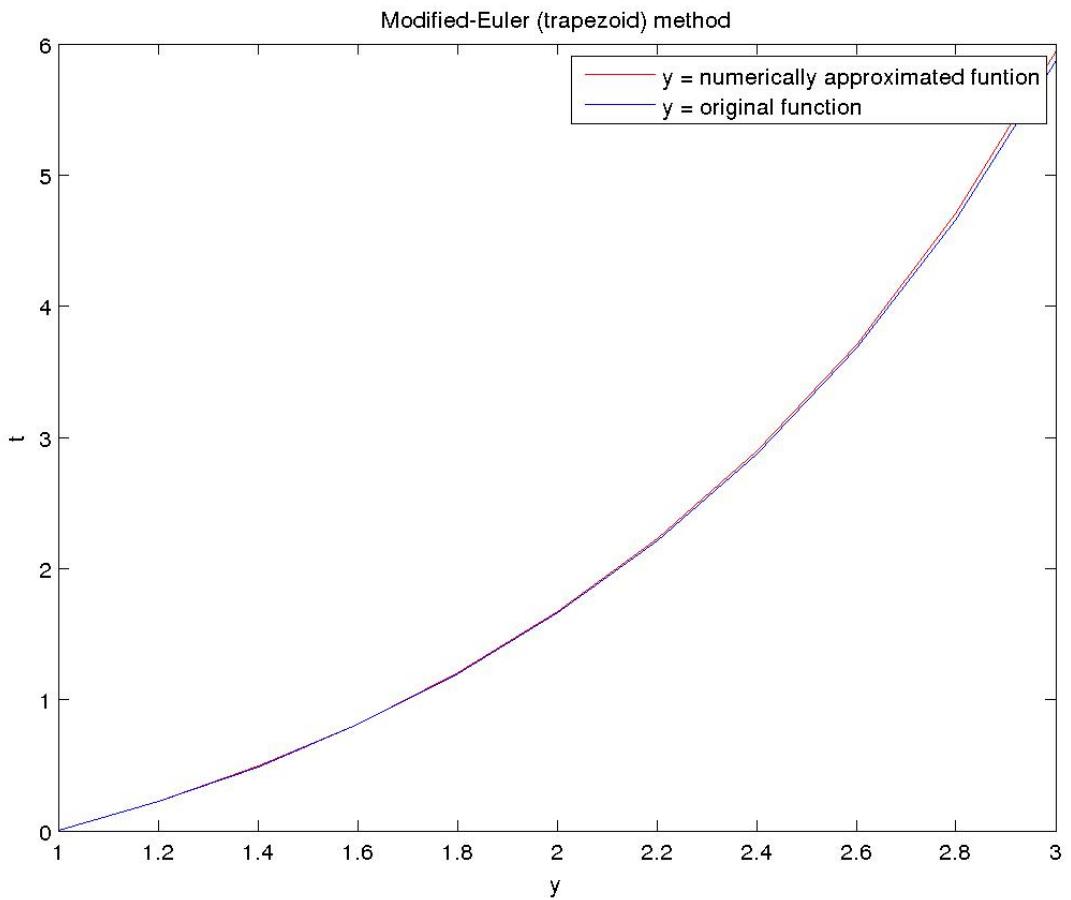
N	Error	Order
10	5.674011e+00	1.975610
20	1.442687e+00	1.271163
40	5.977421e-01	1.113688
80	2.762233e-01	1.052714
160	1.331563e-01	1.025443



### Modified-Euler (trapezoid) method

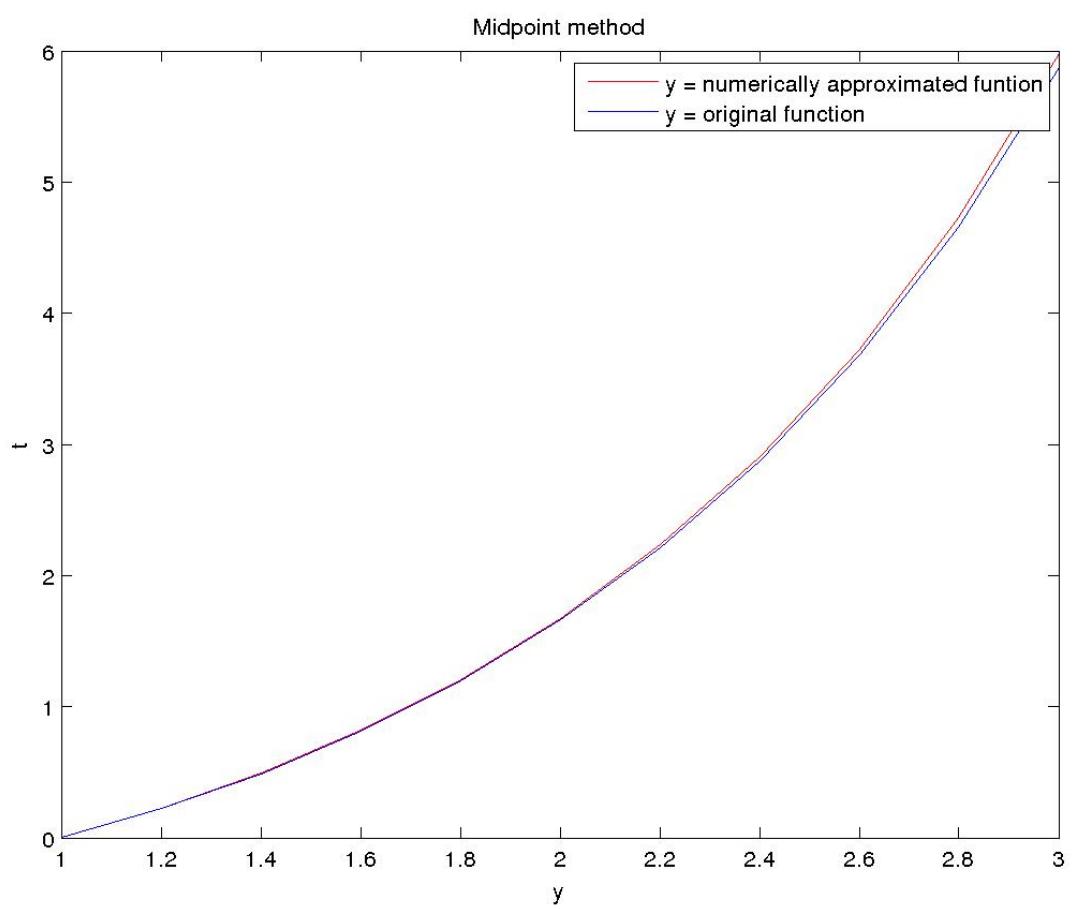
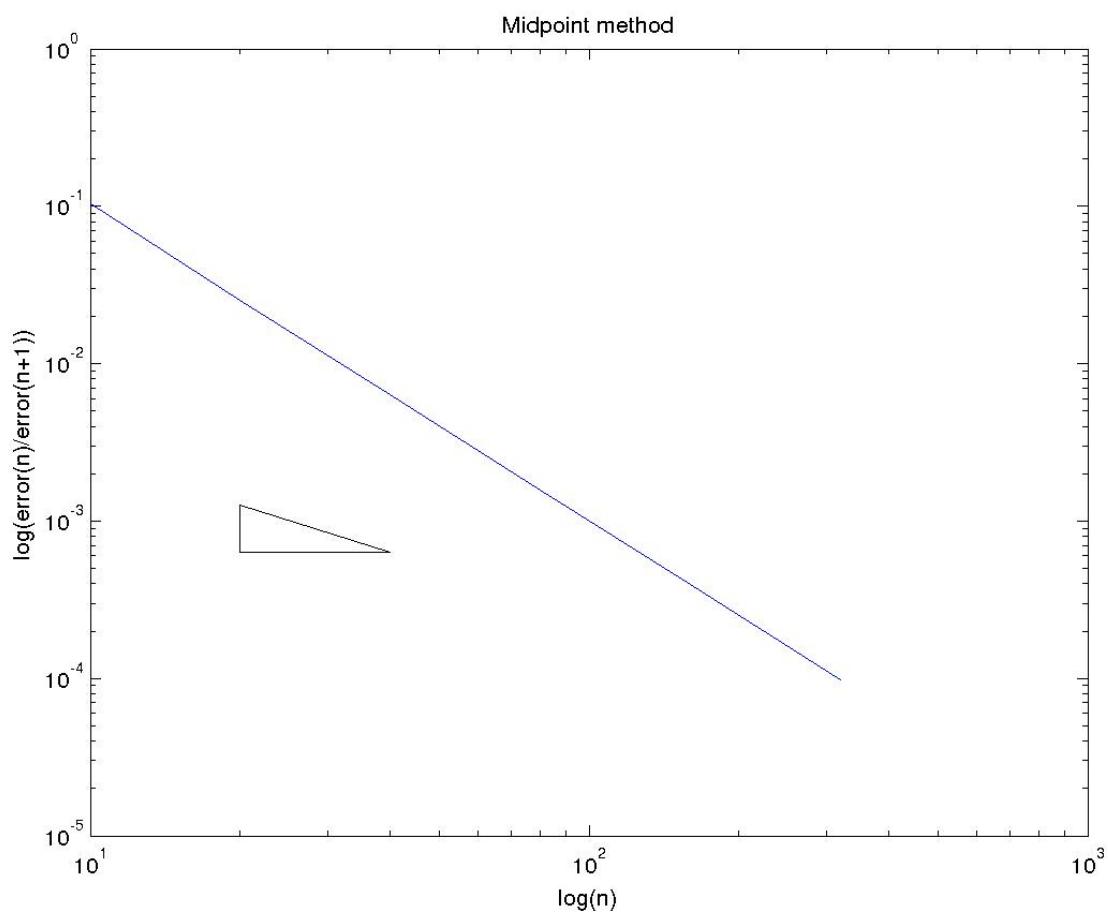
N	Error	Order
10	6.985429e-02	2.032640
20	1.707290e-02	2.007953
40	4.244762e-03	2.001976
80	1.059738e-03	2.000494
160	2.648439e-04	2.000123





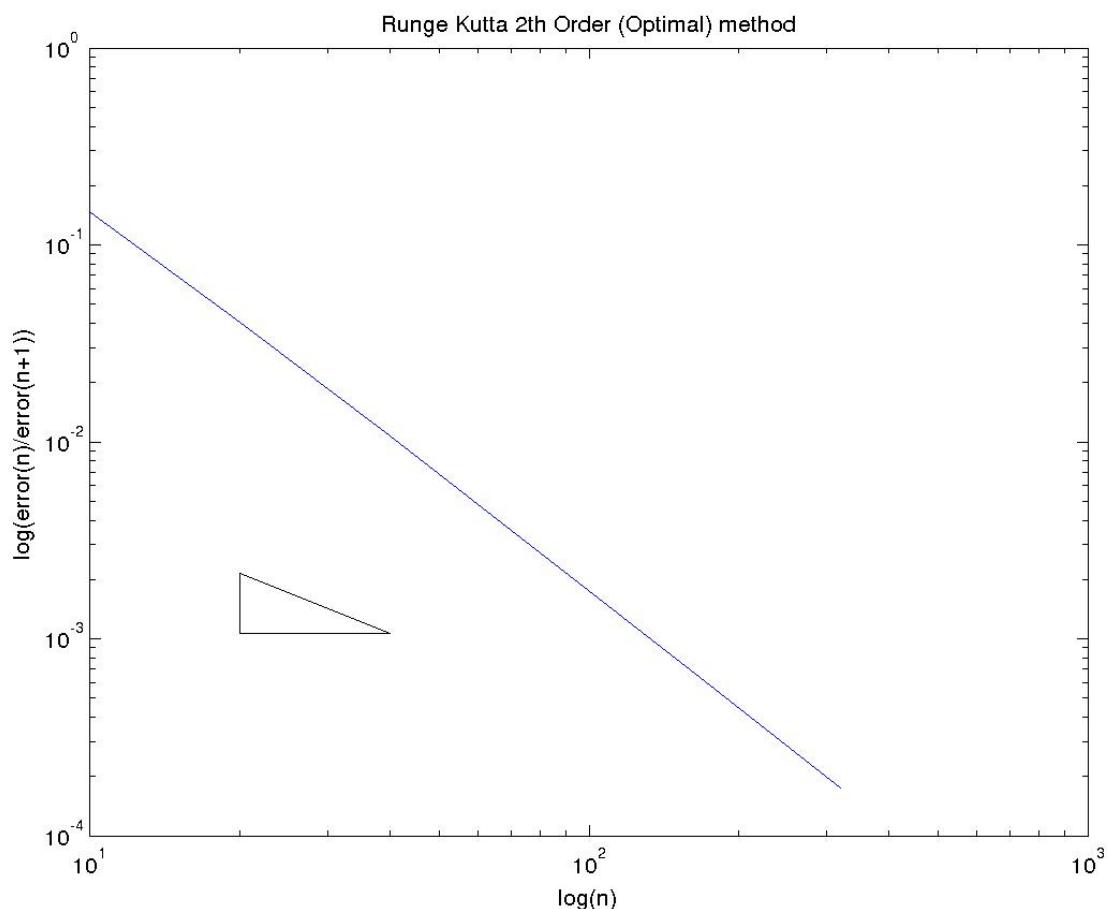
### Midpoint method

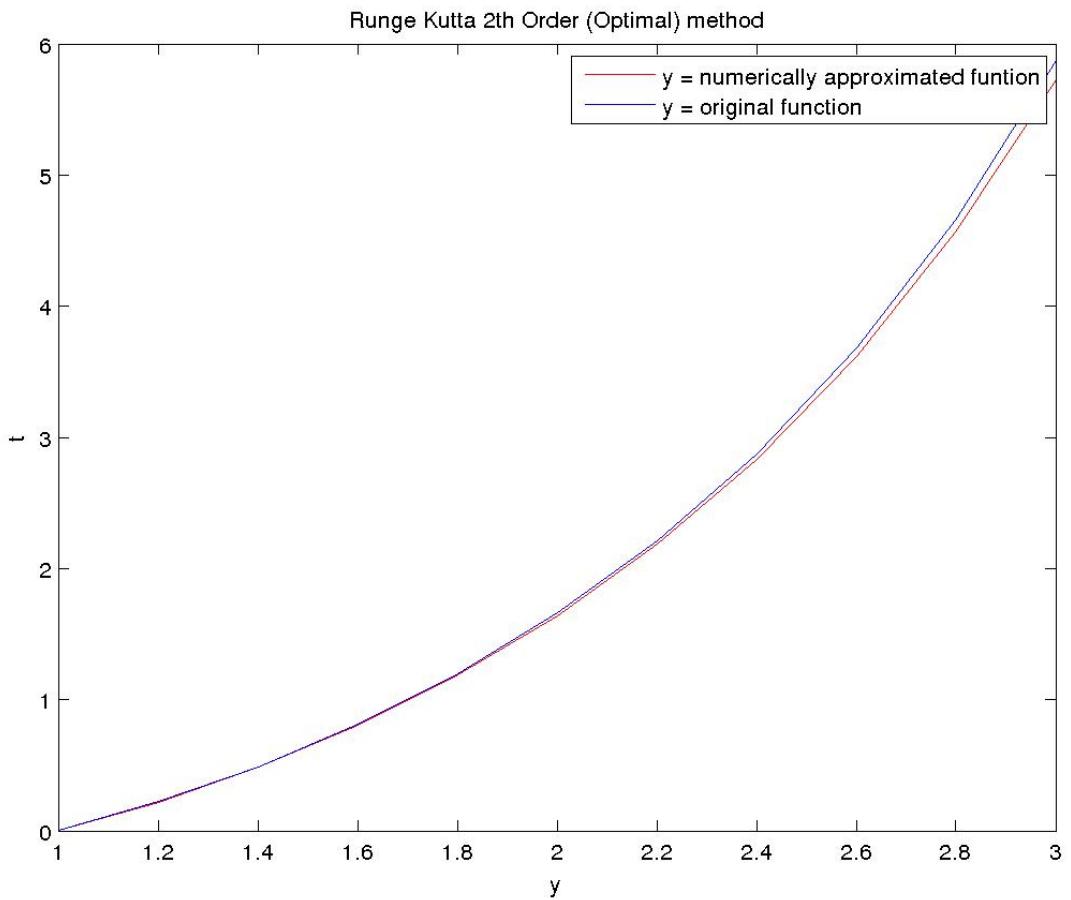
N	Error	Order
10	1.045453e-01	2.040836
20	2.540690e-02	2.009994
40	6.307877e-03	2.002486
80	1.574255e-03	2.000621
160	3.933942e-04	2.000155



### Runge Kutta 2th Order (Optimal) method

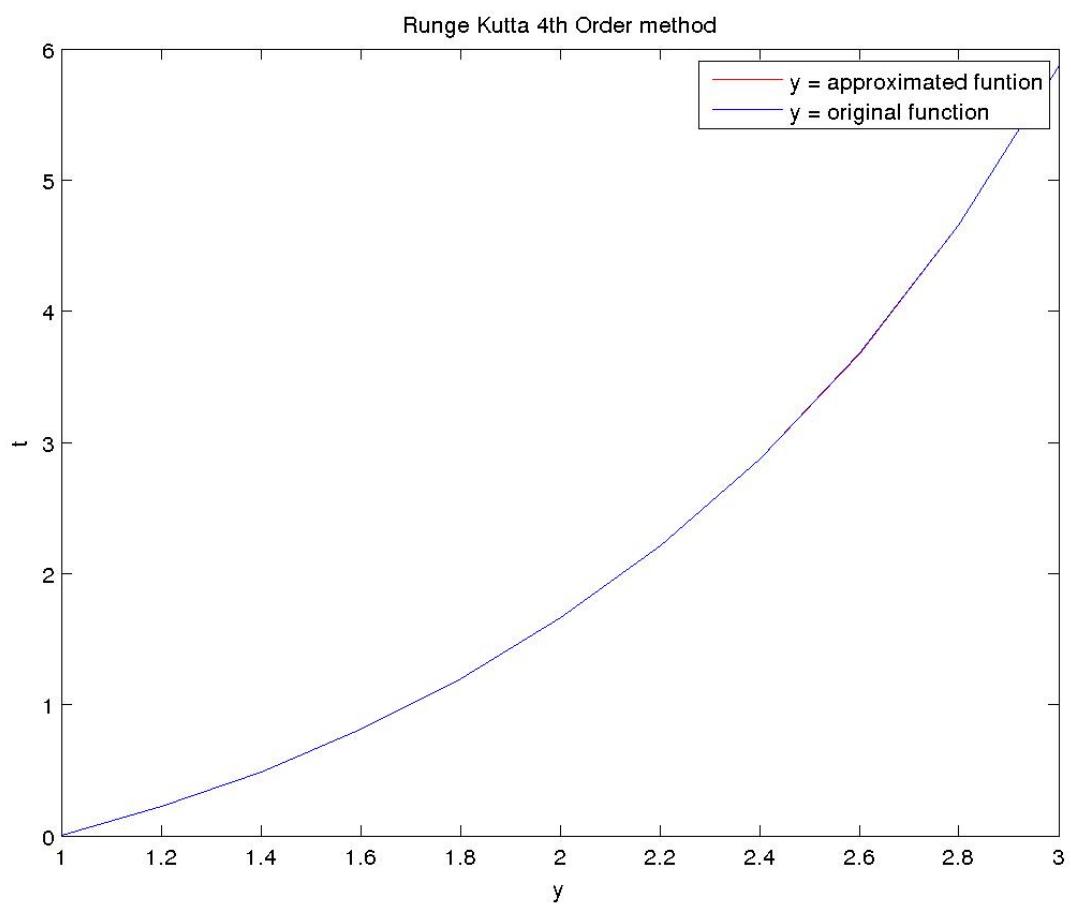
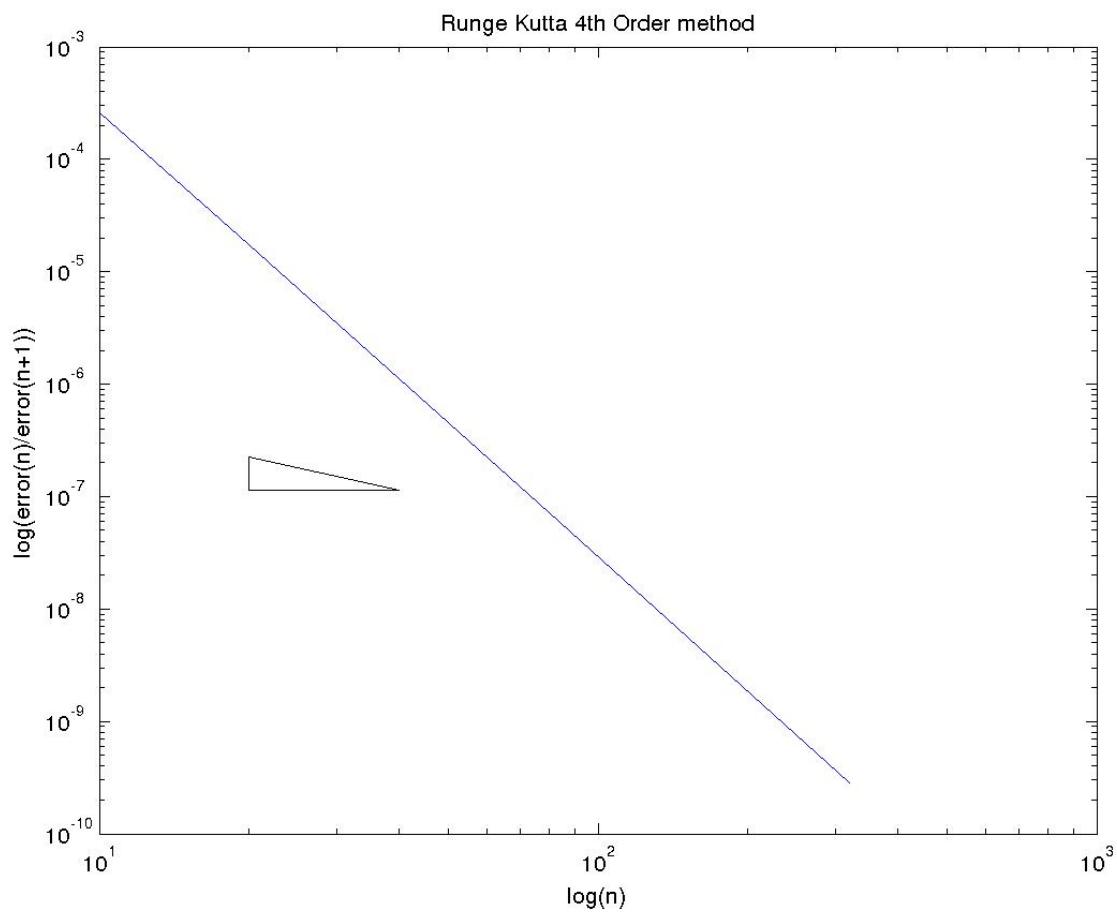
N	Error	Order
10	1.454753e-01	1.838099
20	4.068799e-02	1.926147
40	1.070627e-02	1.965506
80	2.741333e-03	1.983452
160	6.932395e-04	1.991912





### Runge Kutta 4th Order method

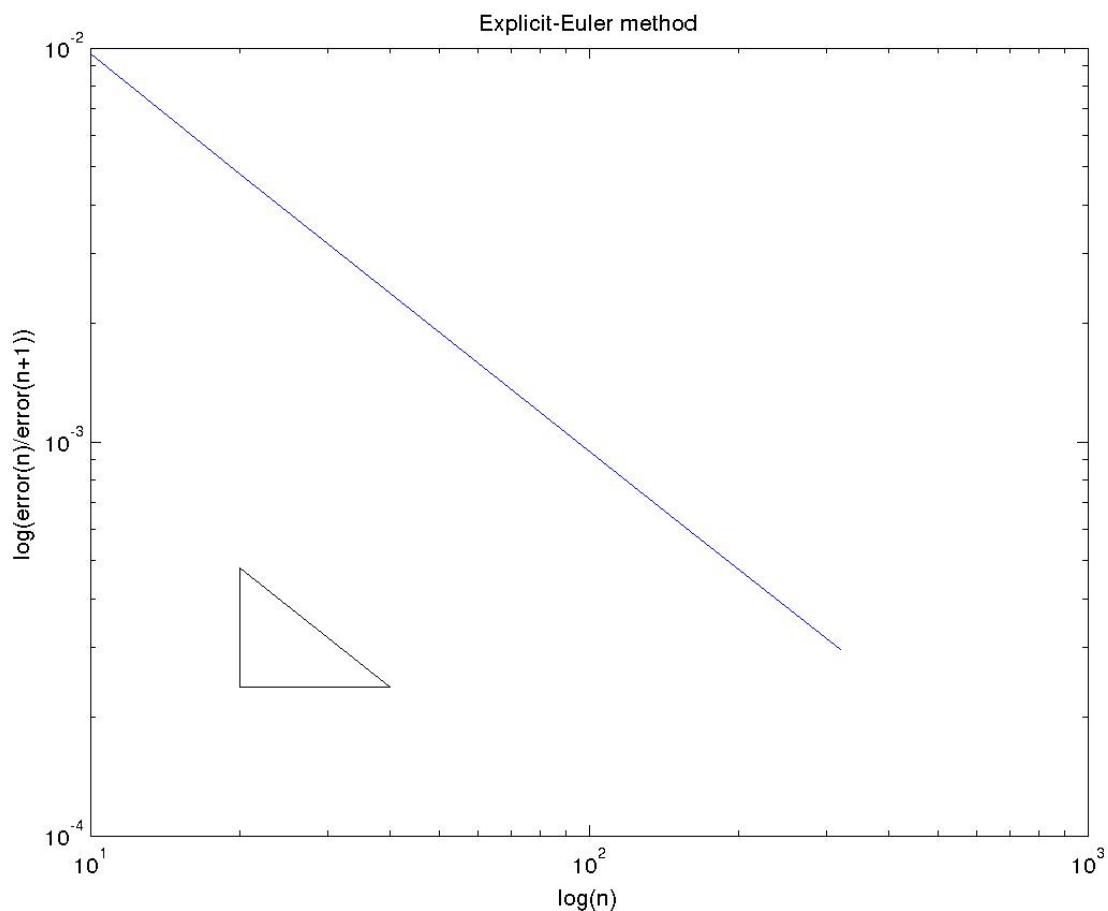
N	Error	Order
10	2.614084e-04	3.904348
20	1.745797e-05	3.951604
40	1.128346e-06	3.975832
80	7.171296e-08	3.987953
160	4.519643e-09	3.993964

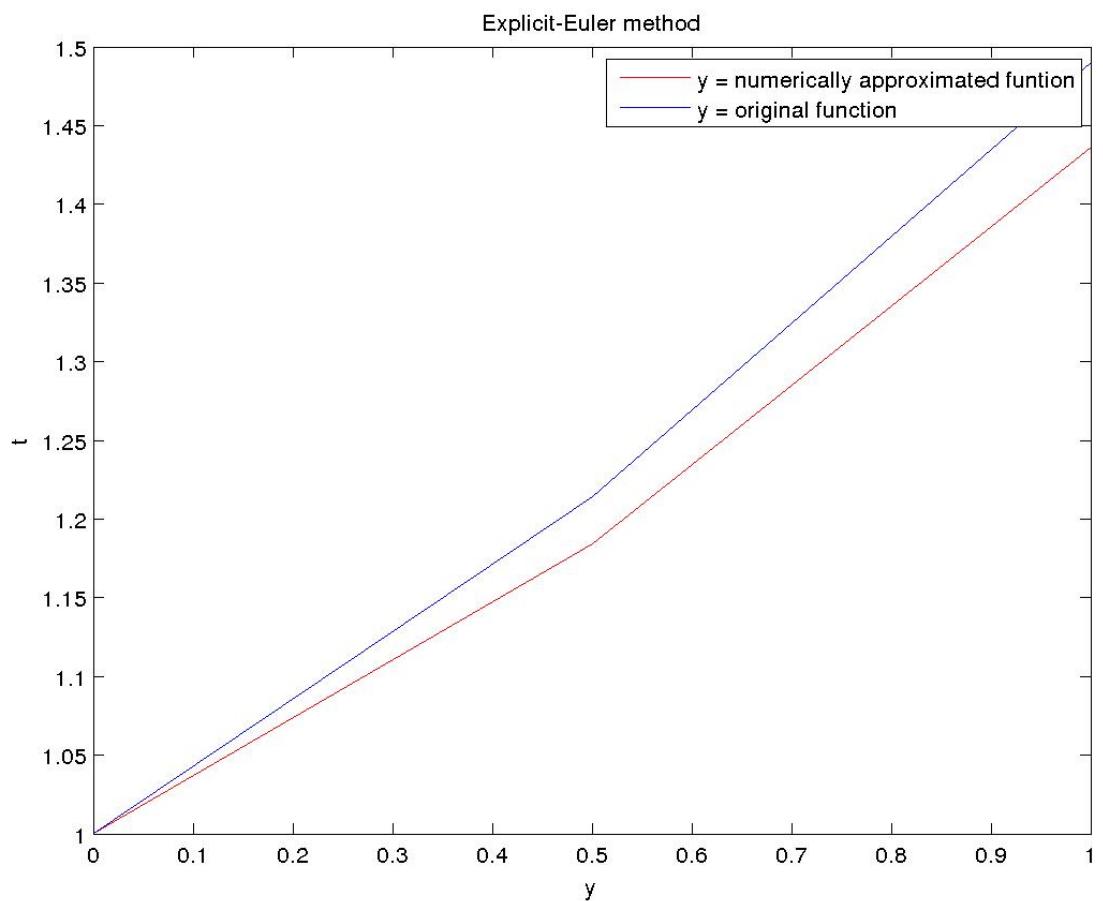


**(D)**

Explicit-Euler method

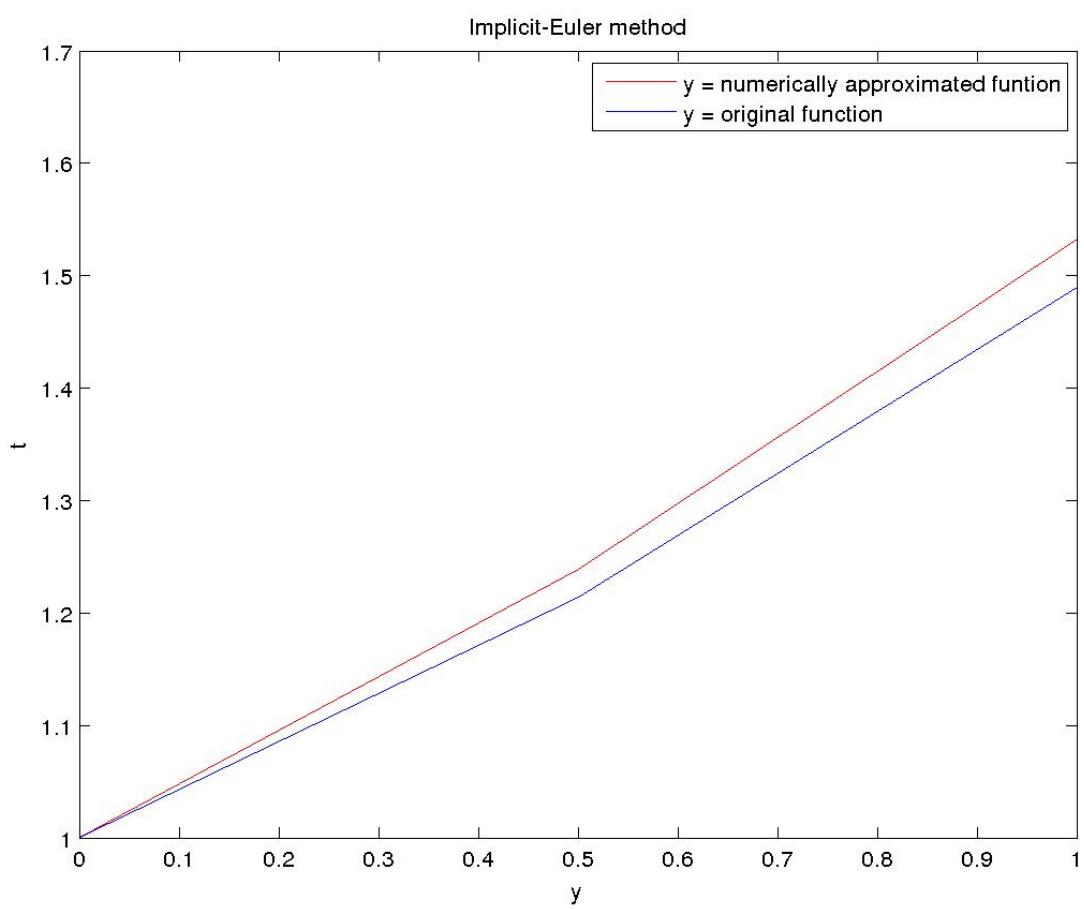
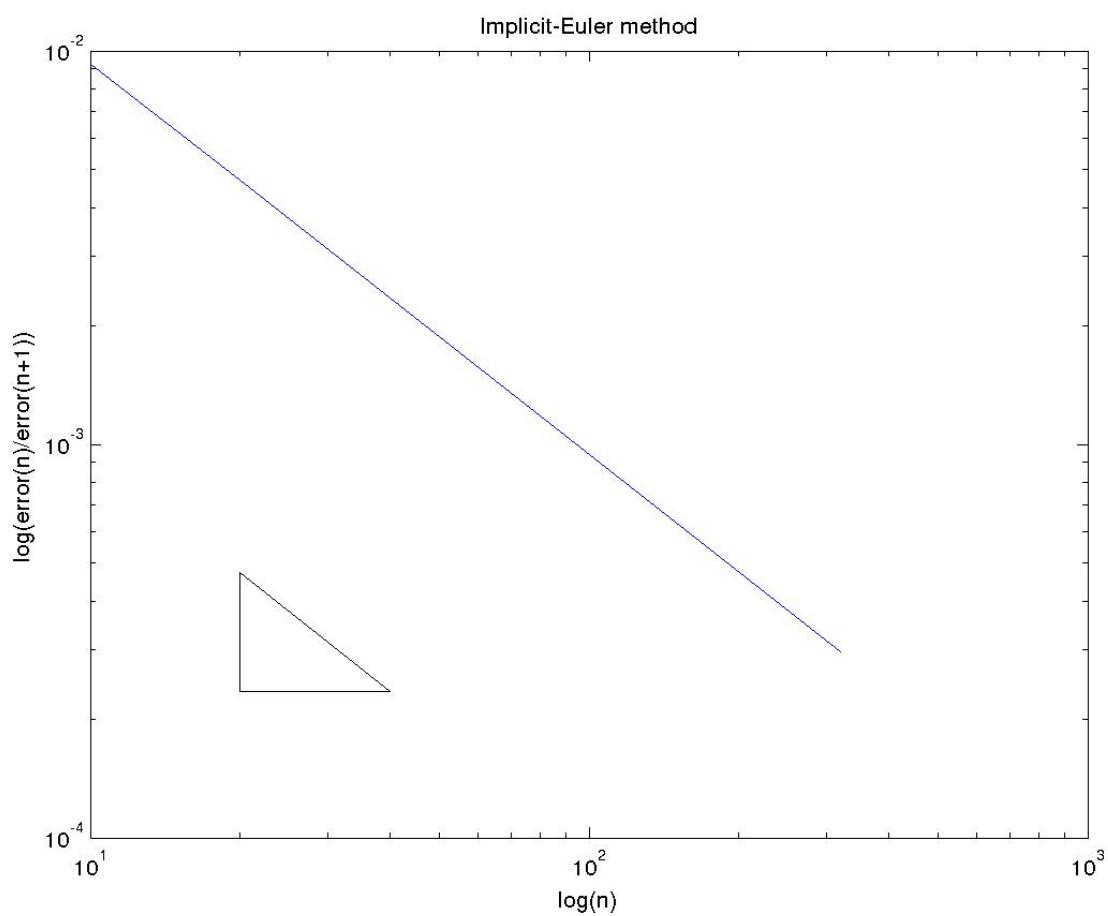
N	Error	Order
10	9.711427e-03	1.017006
20	4.798811e-03	1.008424
40	2.385436e-03	1.004193
80	1.189257e-03	1.002091
160	5.937671e-04	1.001044





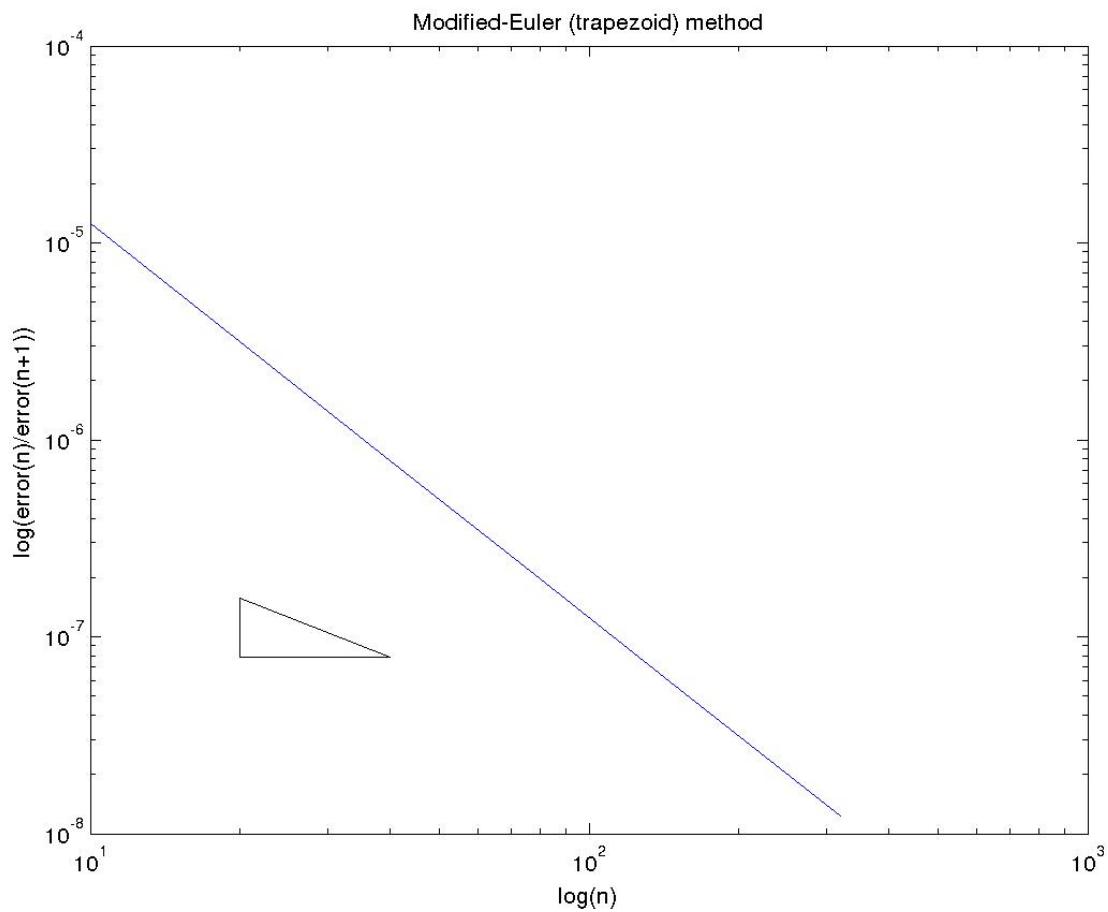
### Implicit-Euler method

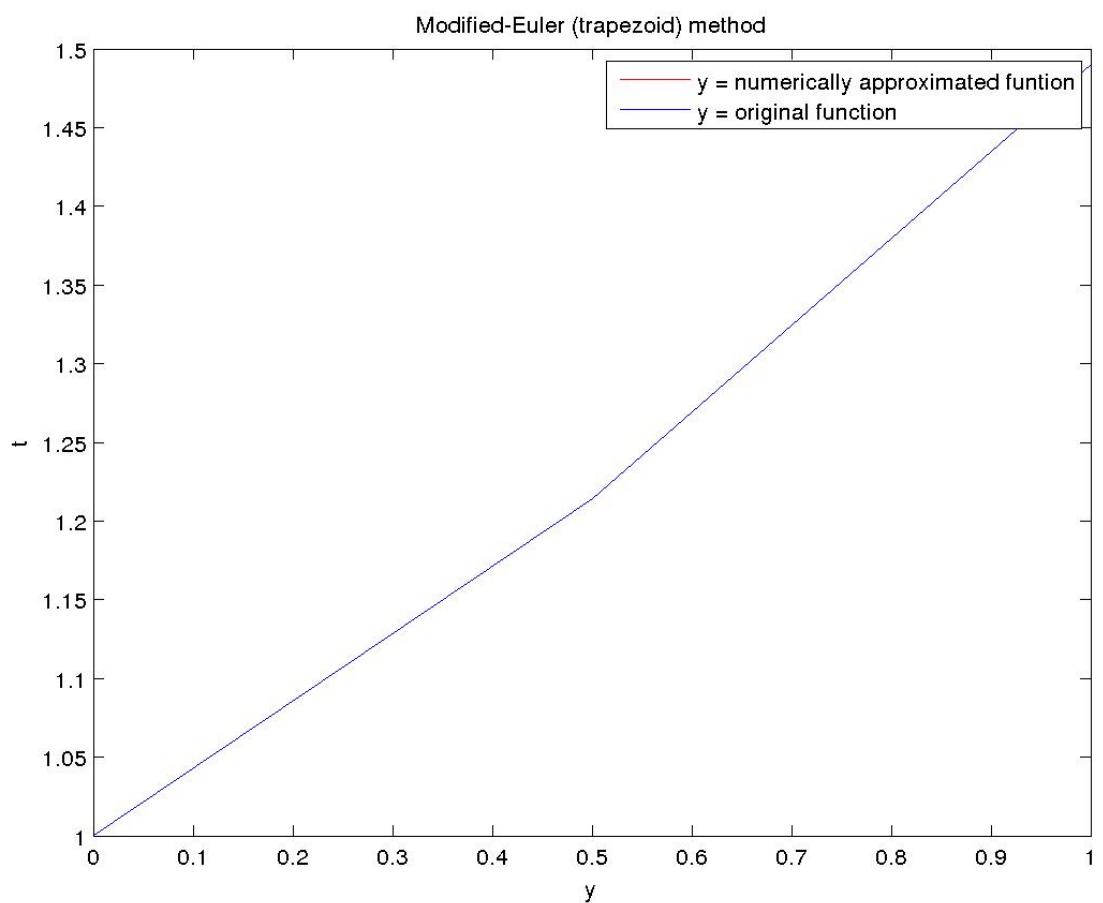
N	Error	Order
10	9.272210e-03	0.983620
20	4.689043e-03	0.991732
40	2.357996e-03	0.995847
80	1.182397e-03	0.997918
160	5.920521e-04	0.998958



### Modified-Euler (trapezoid) method

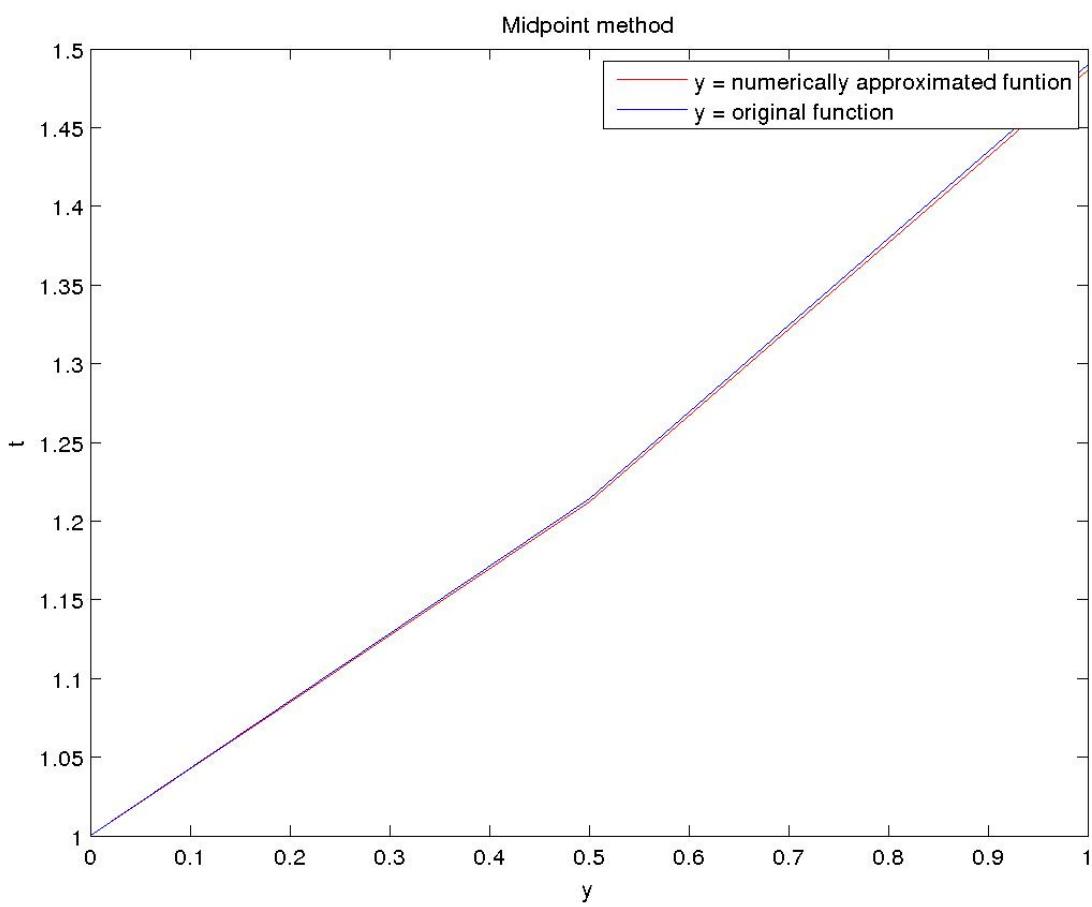
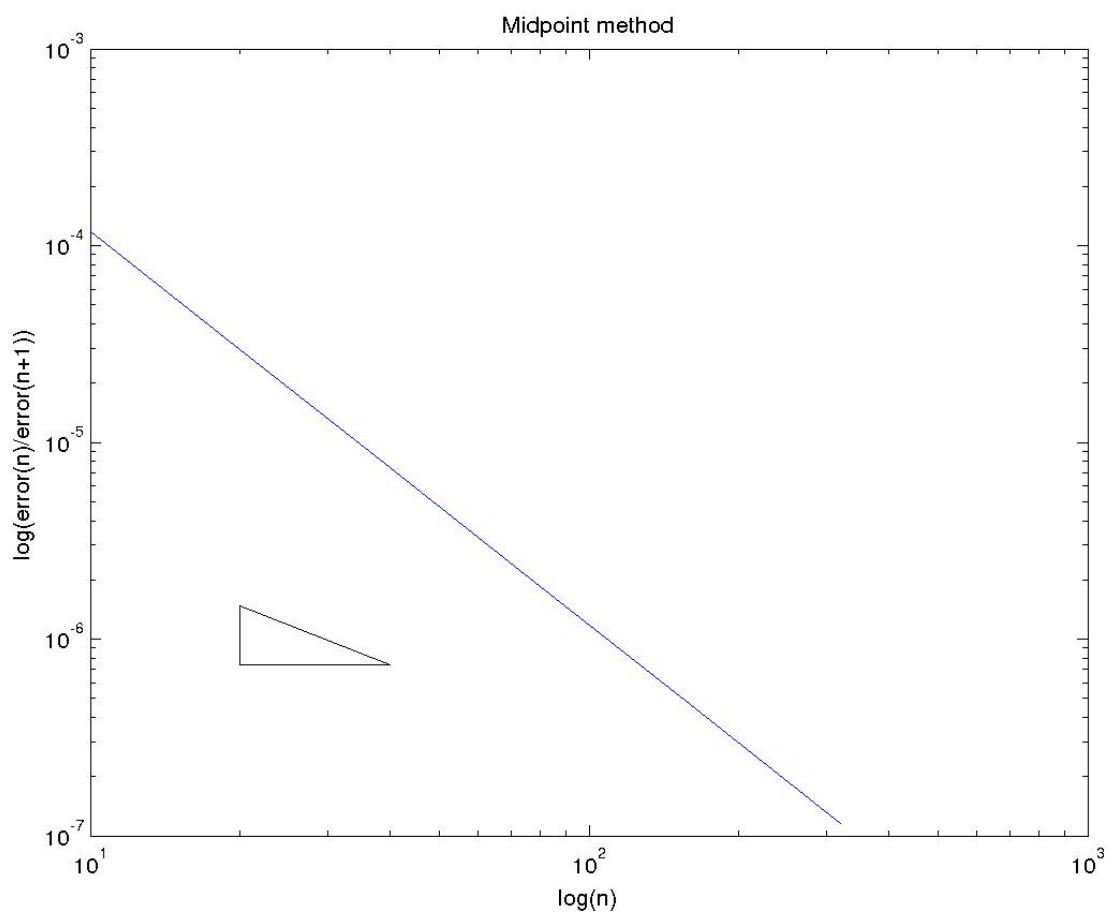
N	Error	Order
10	1.258717e-05	2.001433
20	3.143670e-06	1.998869
40	7.865335e-07	1.999899
80	1.966472e-07	1.999883
160	4.916579e-08	1.999991





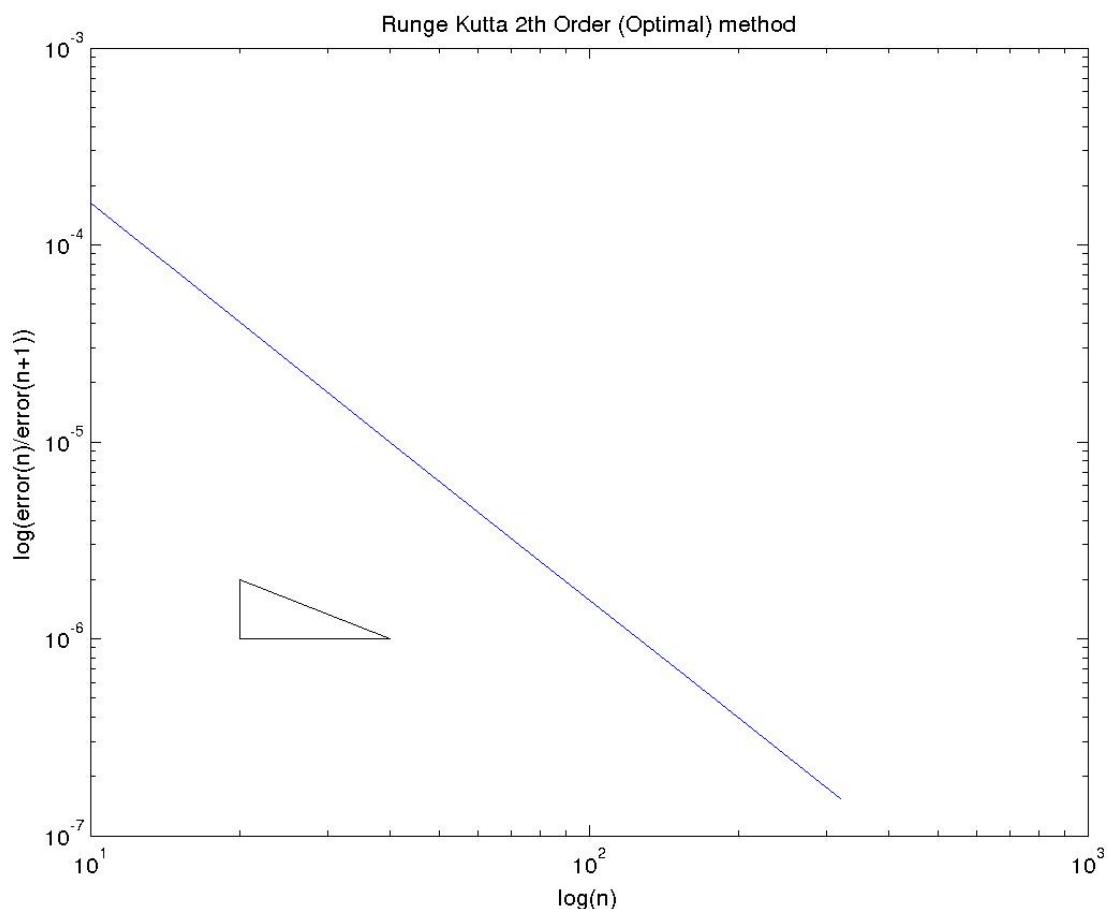
### Midpoint method

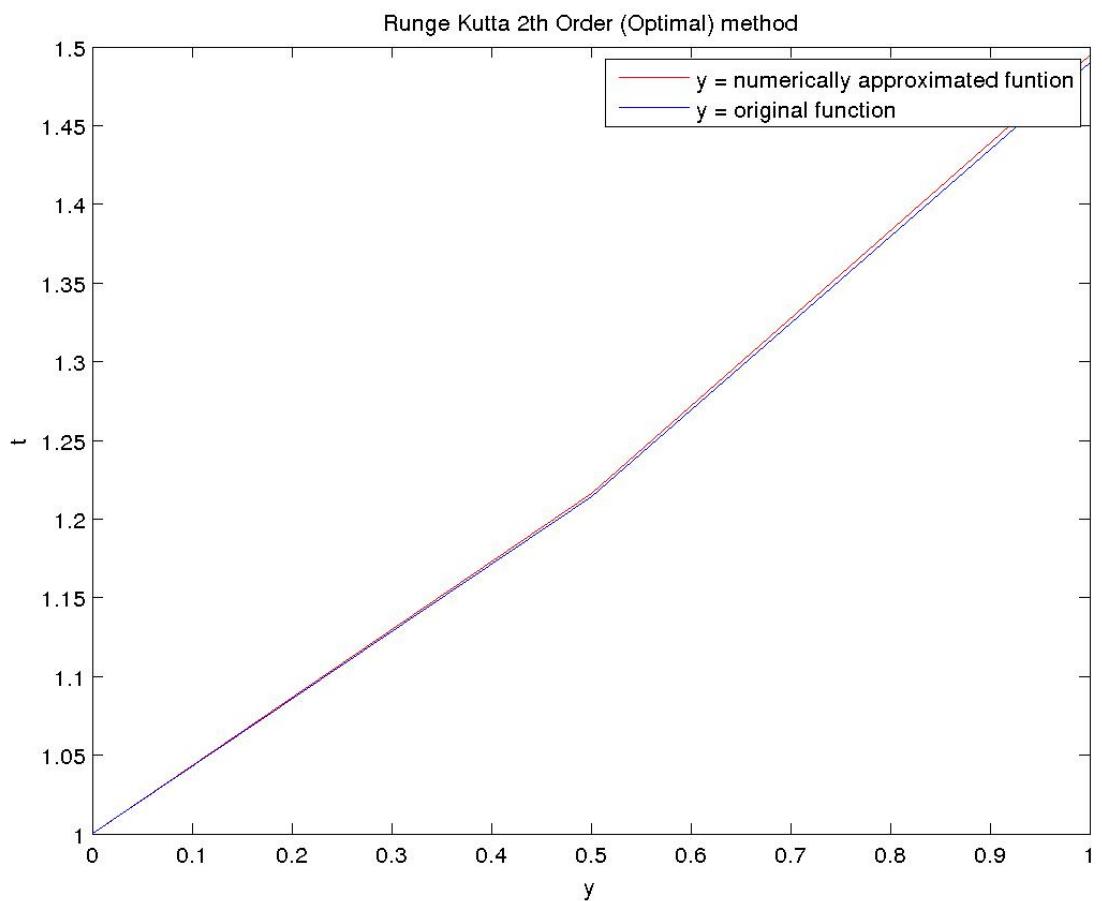
N	Error	Order
10	1.186375e-04	2.000333
20	2.965253e-05	2.000084
40	7.412698e-06	2.000018
80	1.853151e-06	1.999998
160	4.632884e-07	2.000008



### Runge Kutta 2th Order (Optimal) method

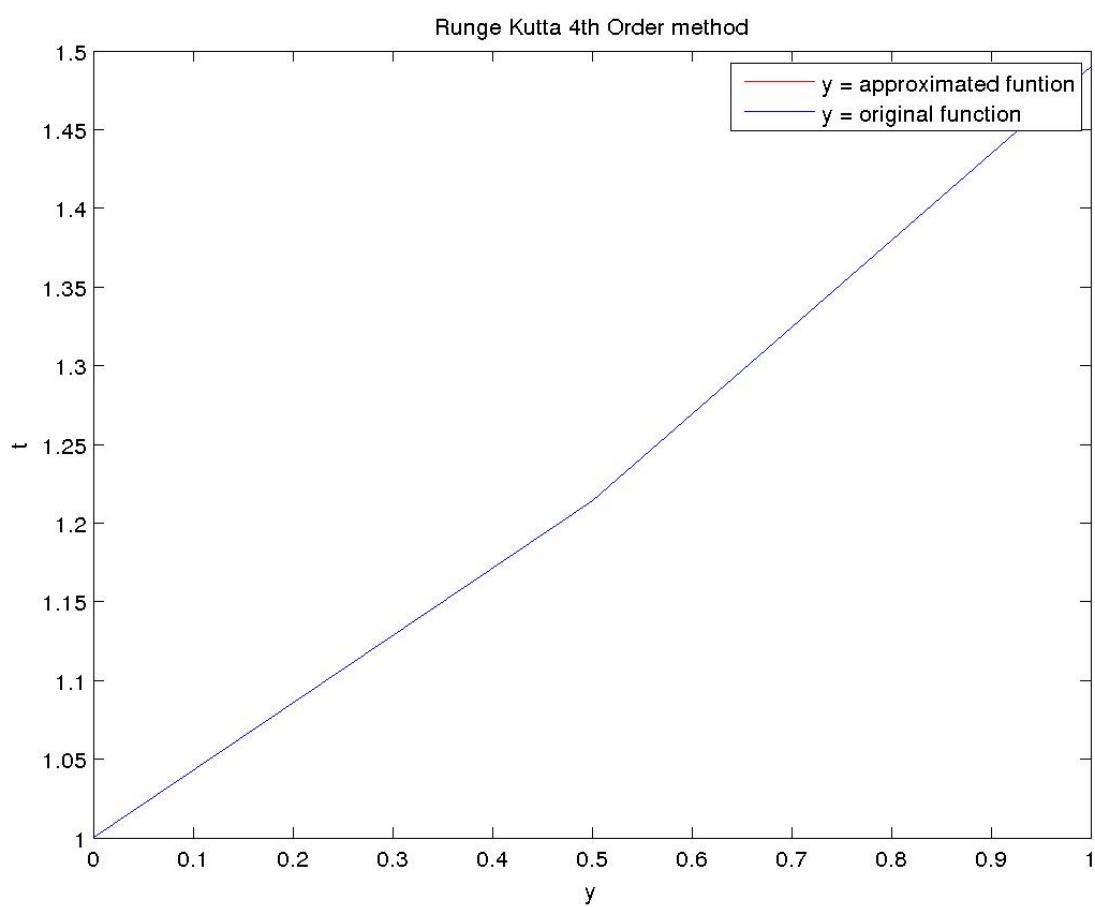
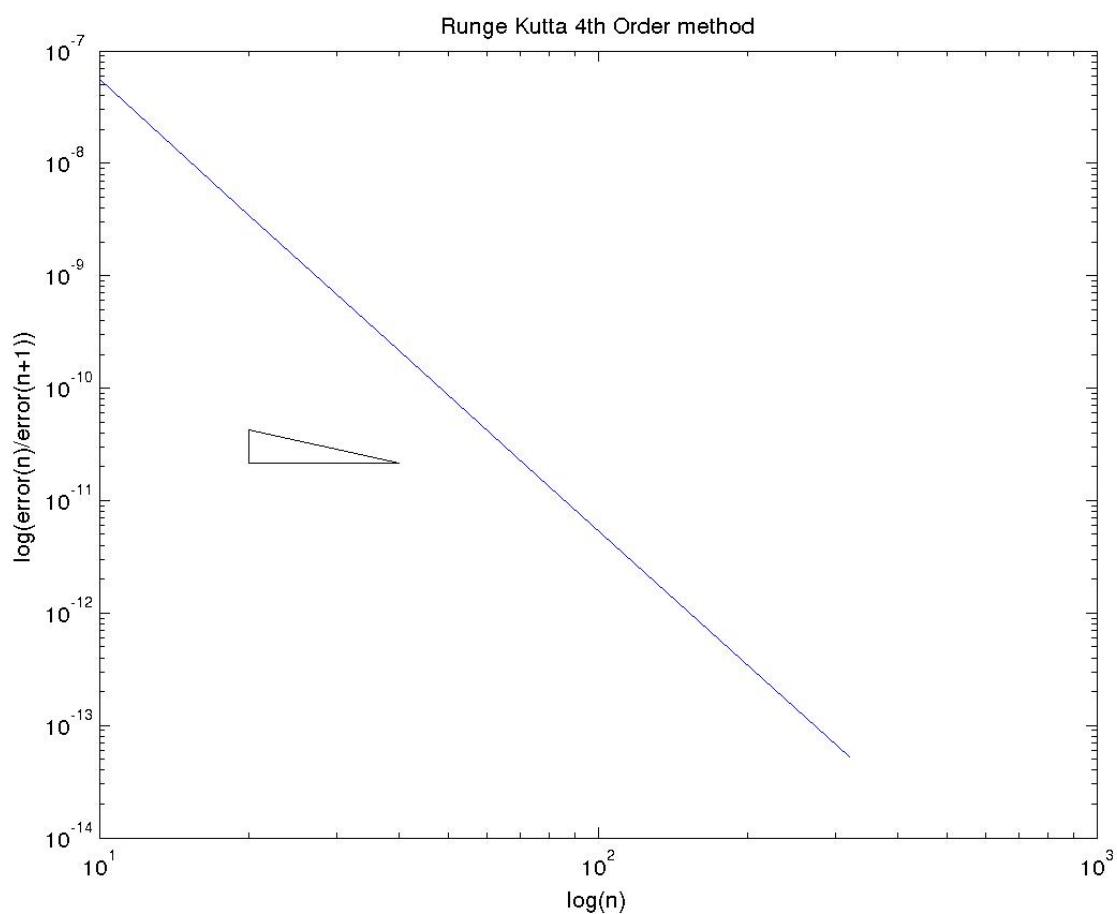
N	Error	Order
10	1.641263e-04	2.026594
20	4.028213e-05	2.013296
40	9.978148e-06	2.006647
80	2.483070e-06	2.003323
160	6.193394e-07	2.001661





### Runge Kutta 4th Order method

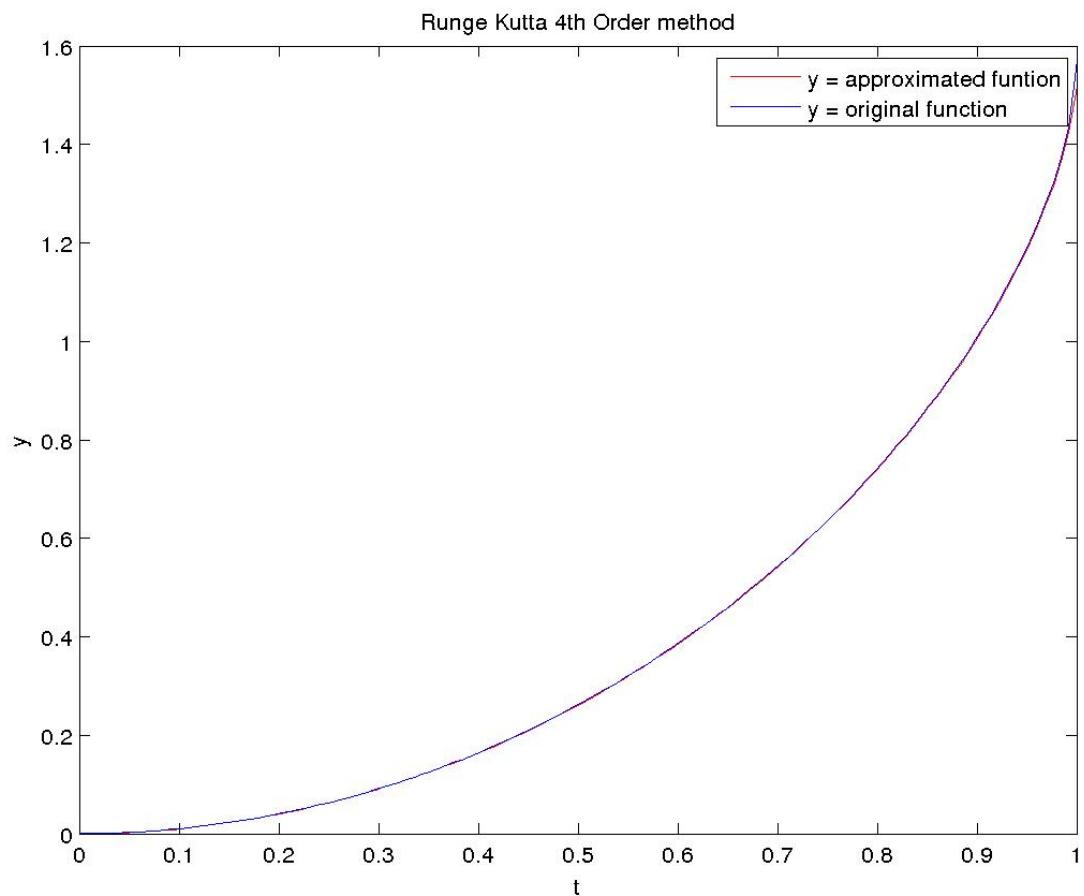
N	Error	Order
10	5.643600e-08	4.029736
20	3.455292e-09	4.014998
40	2.137224e-10	4.007514
80	1.328826e-11	4.005918
160	8.271162e-13	3.986508



## Question 2

Runge Kutta 4th Order method

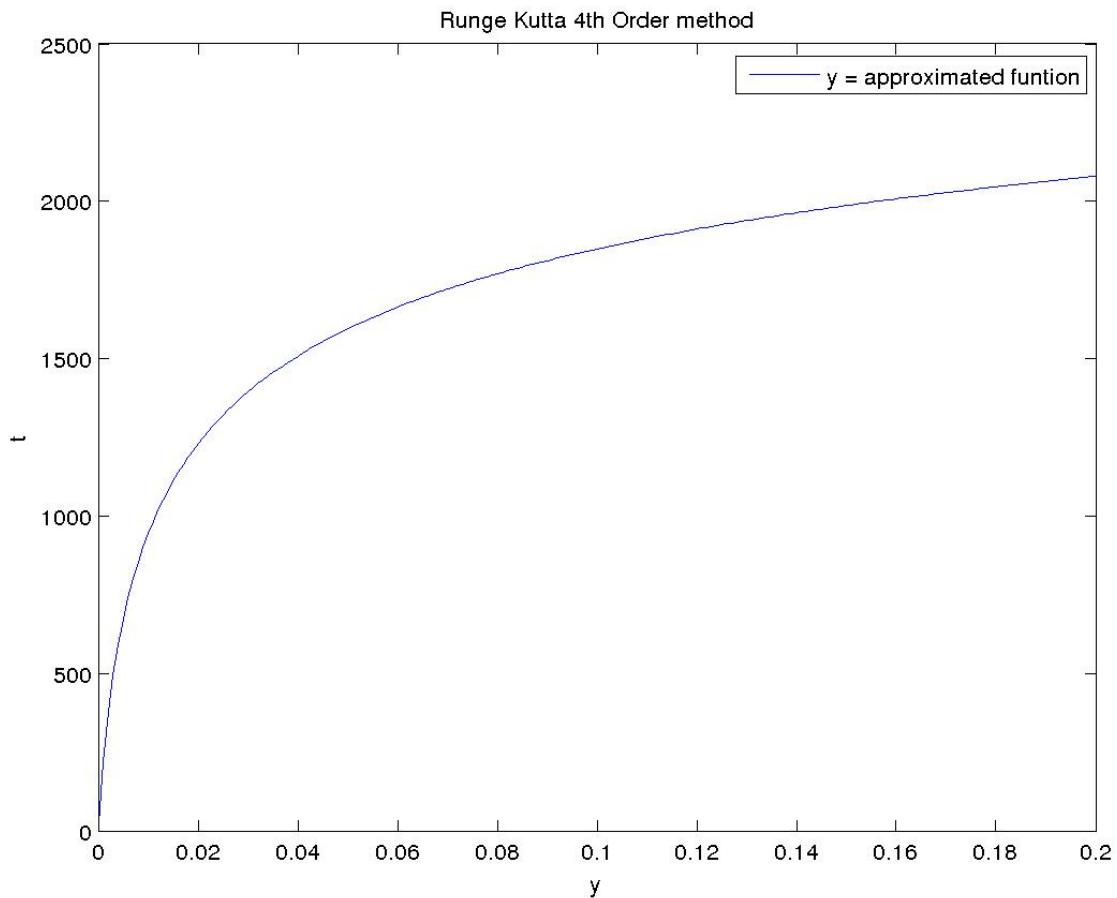
Max-error = 5.167121e-02



### Question 3

Runge Kutta 4th Order method

After 0.200000 seconds amount of KOH formed = 2.079409e+03



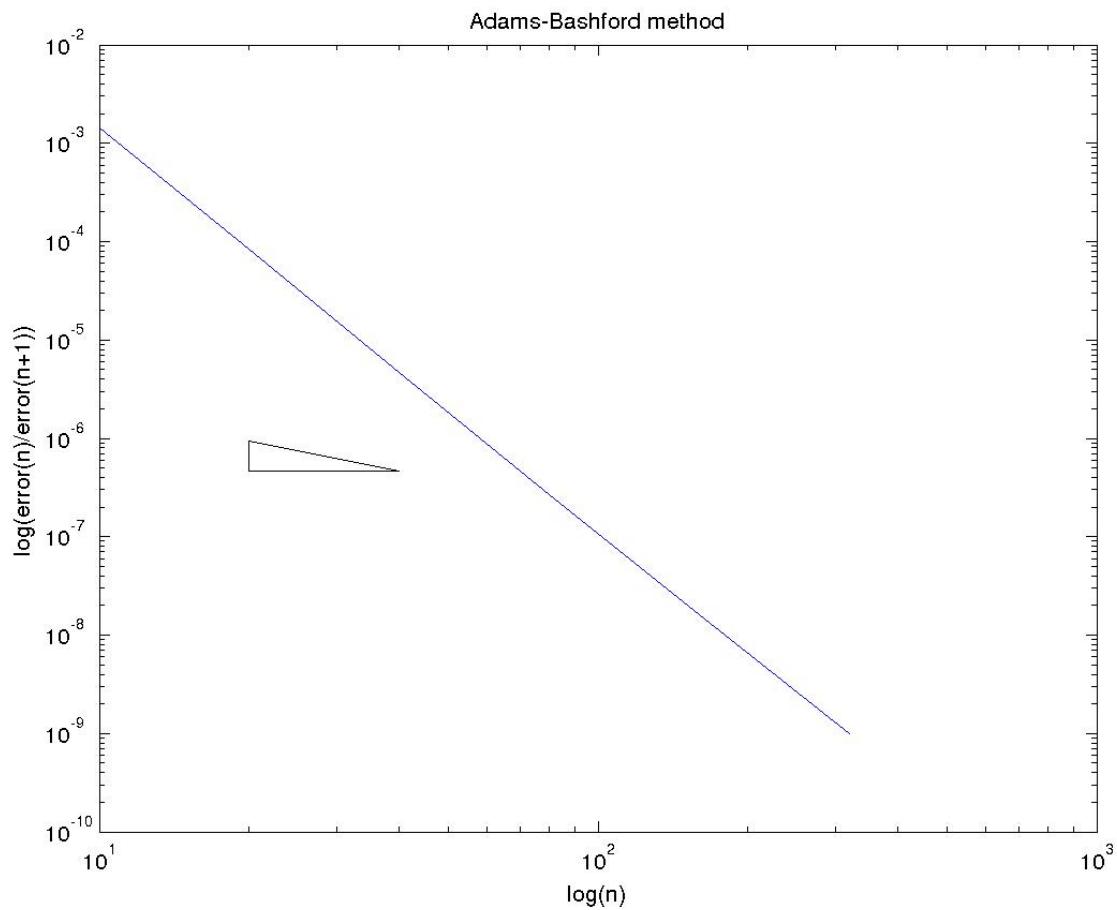
## Question 4

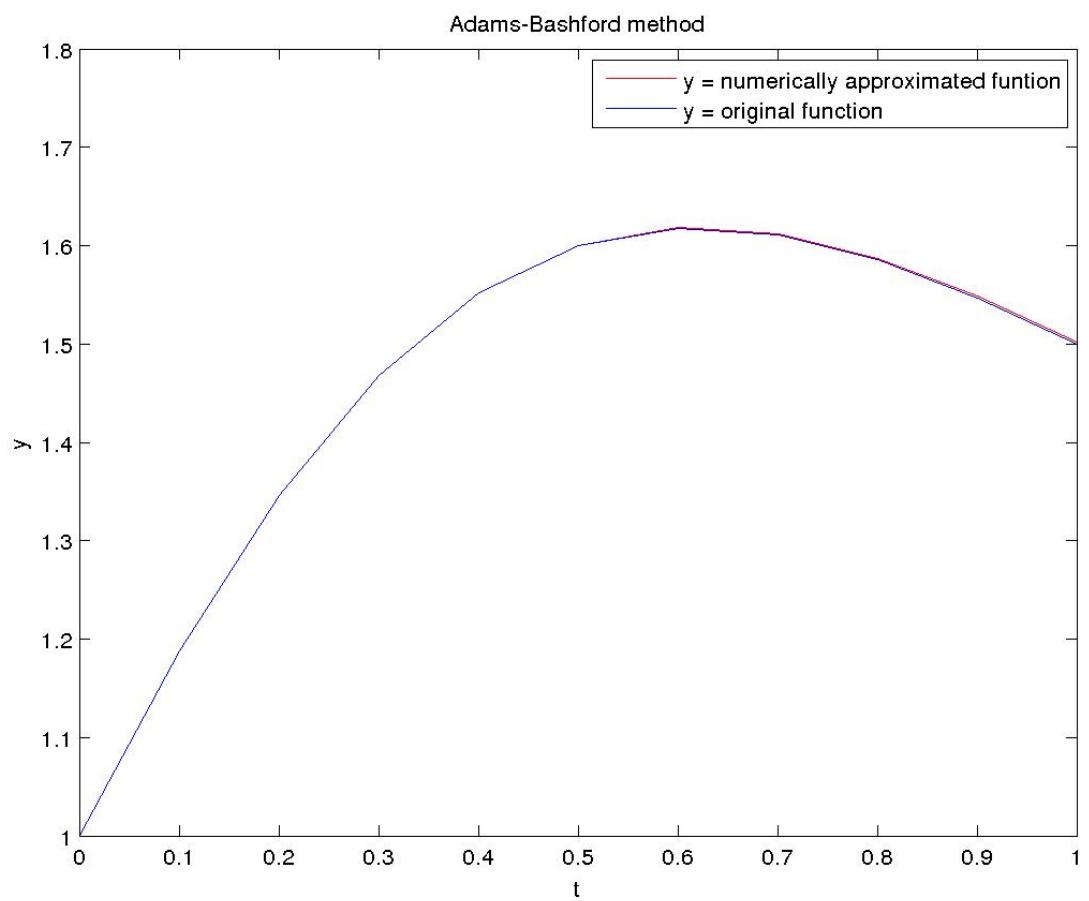
(A)

Using exact starting values

Adams-Bashford method

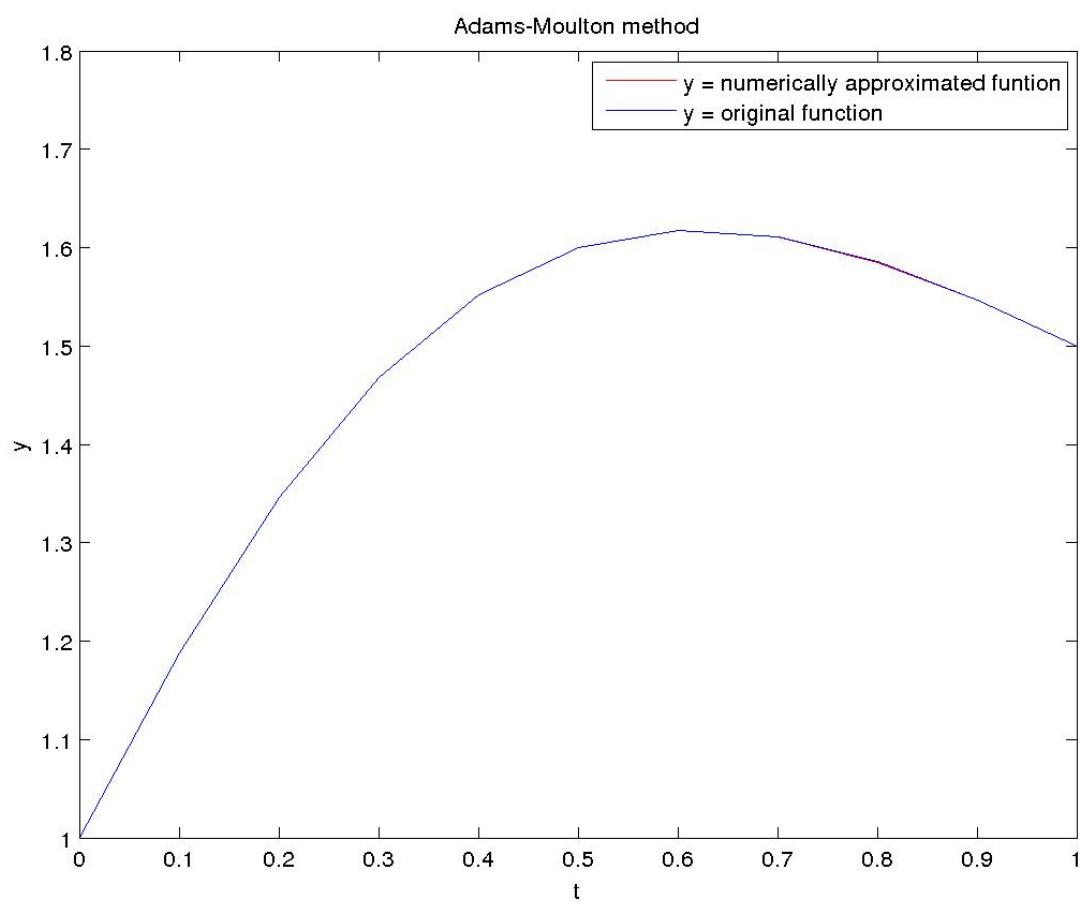
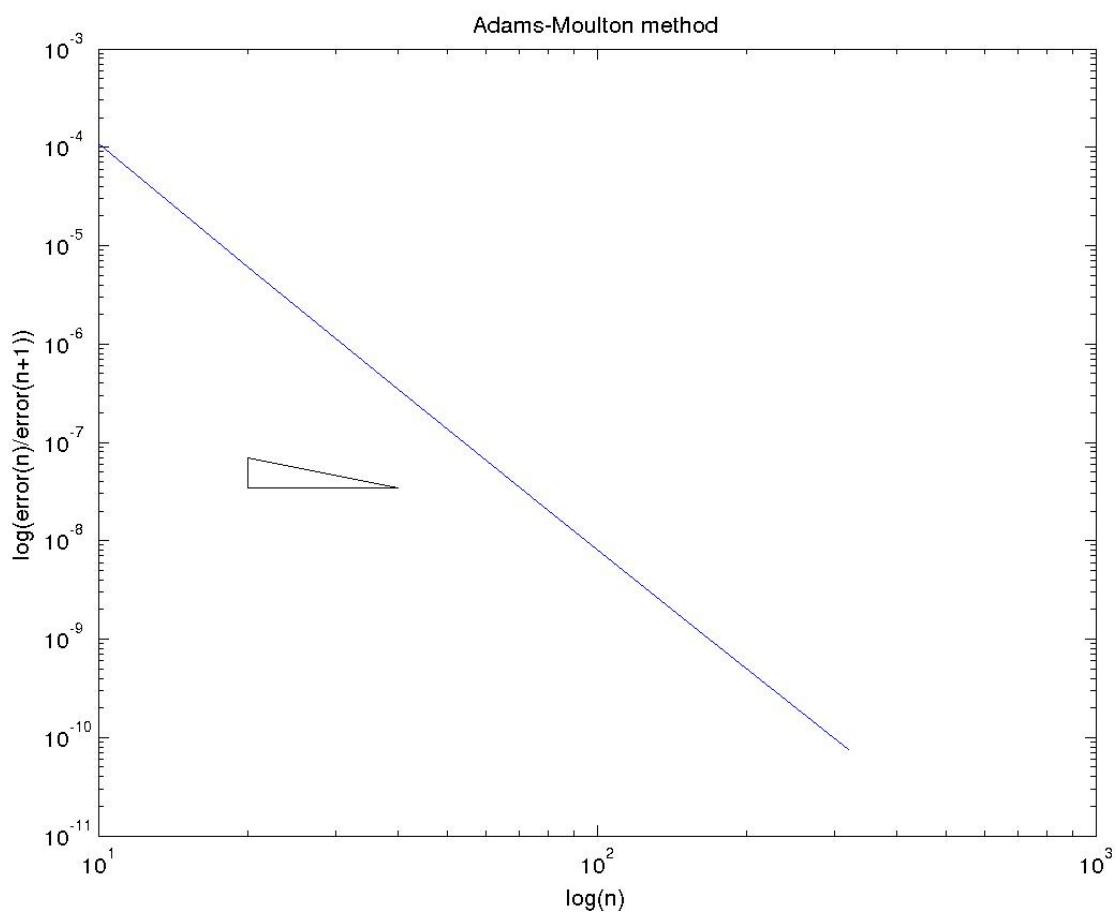
N	Error	Order
10	1.450997e-03	4.127114
20	8.303887e-05	4.155690
40	4.659016e-06	4.106124
80	2.705377e-07	4.061157
160	1.620681e-08	4.032912





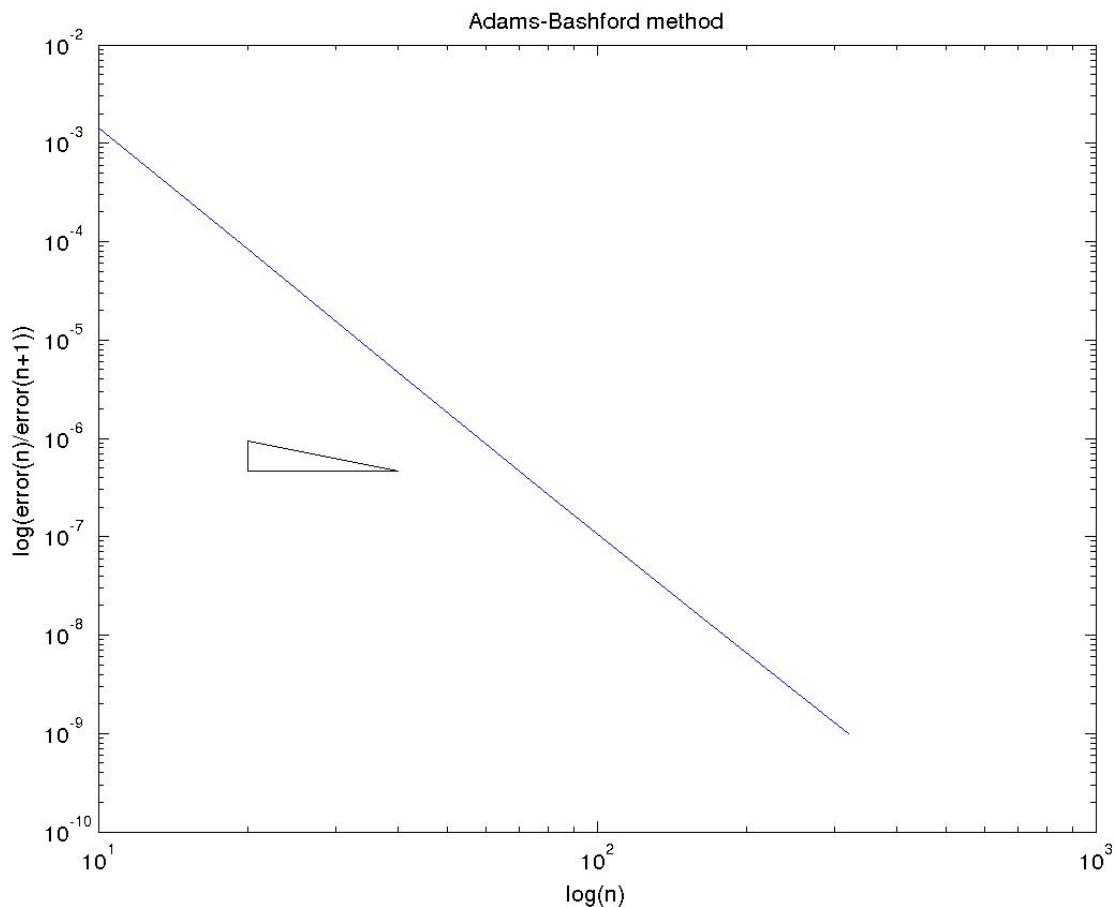
### Adams-Moulton method

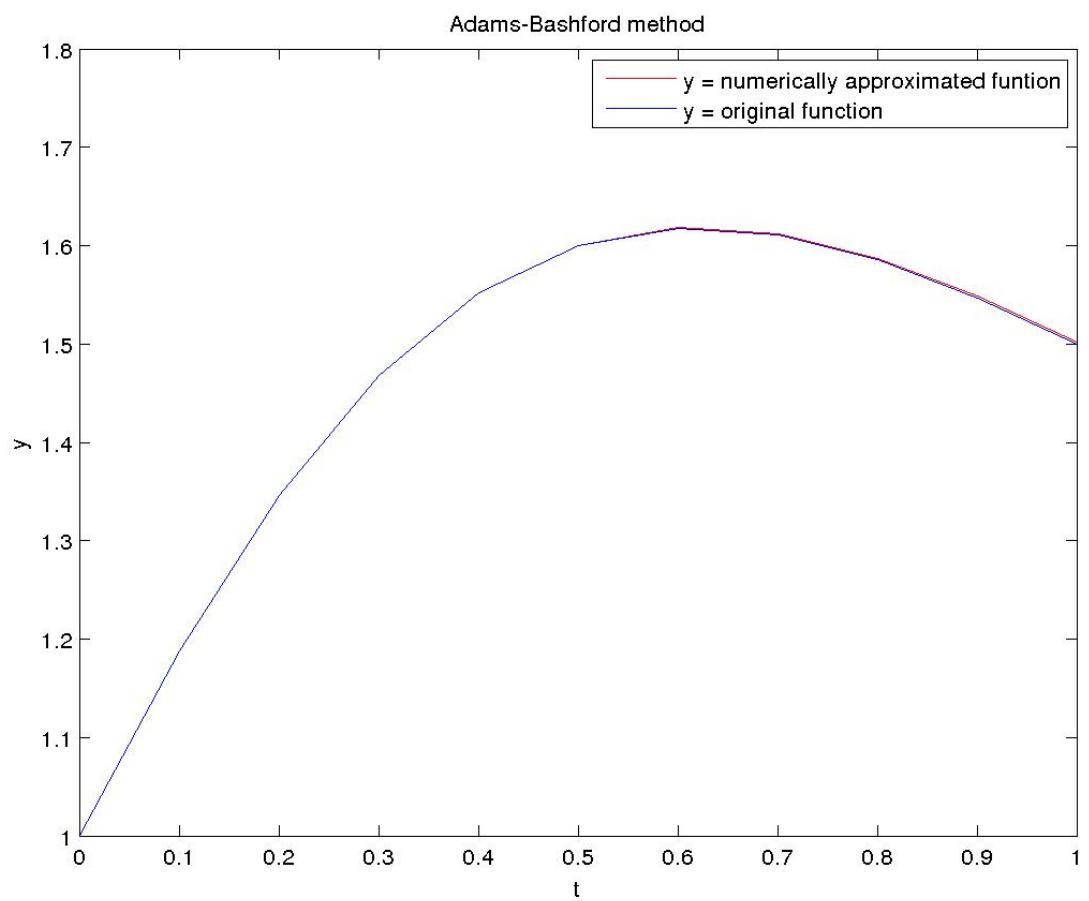
N	Error	Order
10	1.099951e-04	4.168575
20	6.116562e-06	4.147725
40	3.450783e-07	4.094071
80	2.020596e-08	4.051972
160	1.218188e-09	4.027468



Using starting values from Runge-Kutta 4th order  
Adams-Bashford method

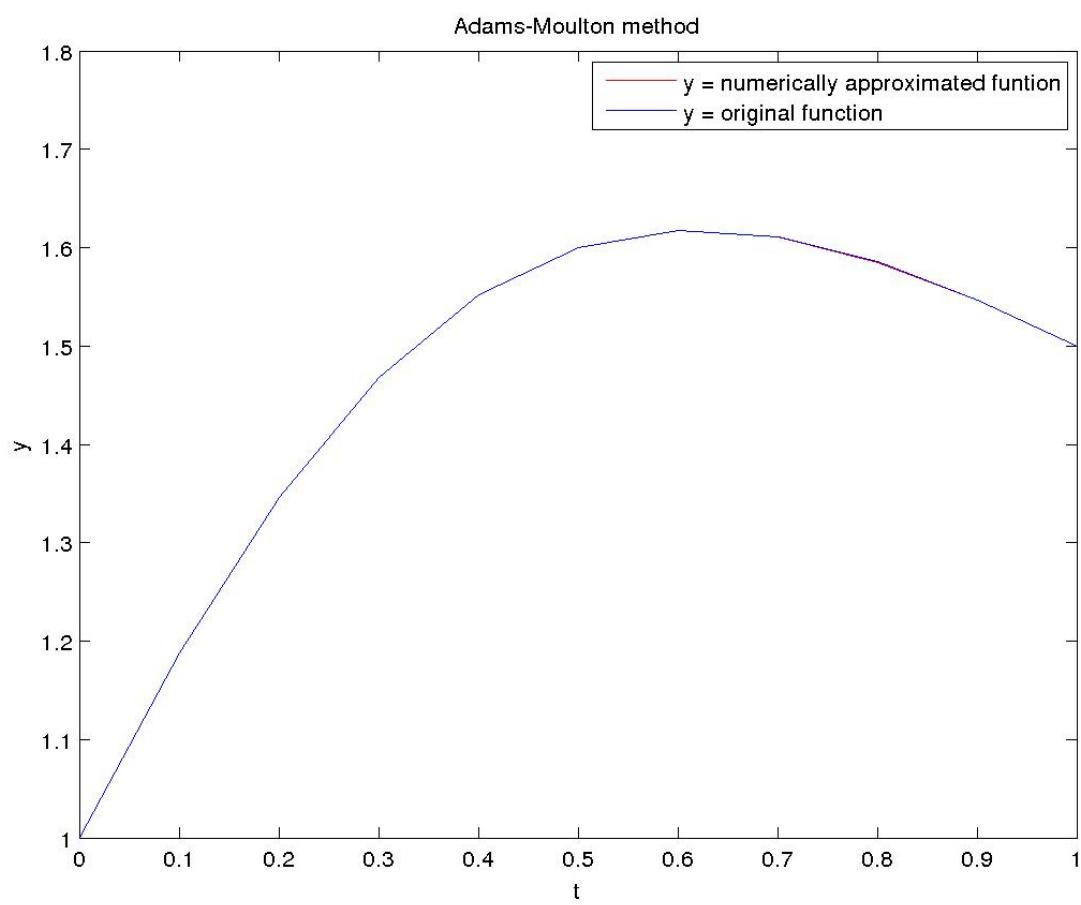
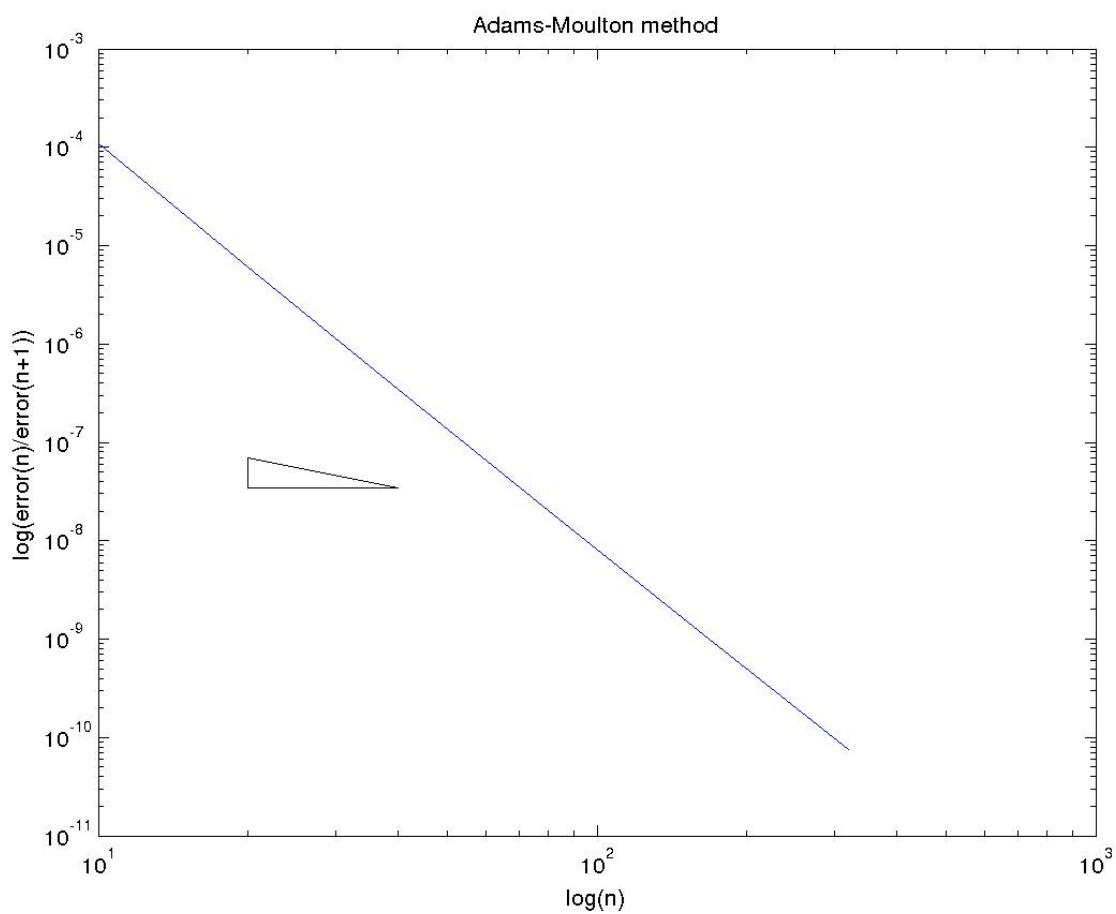
N	Error	Order
10	1.450657e-03	4.126865
20	8.303378e-05	4.155623
40	4.658946e-06	4.106107
80	2.705367e-07	4.061153
160	1.620680e-08	4.032911





### Adams-Moulton method

N	Error	Order
10	1.101454e-04	4.170044
20	6.118685e-06	4.148101
40	3.451081e-07	4.094164
80	2.020640e-08	4.051996
160	1.218195e-09	4.027475

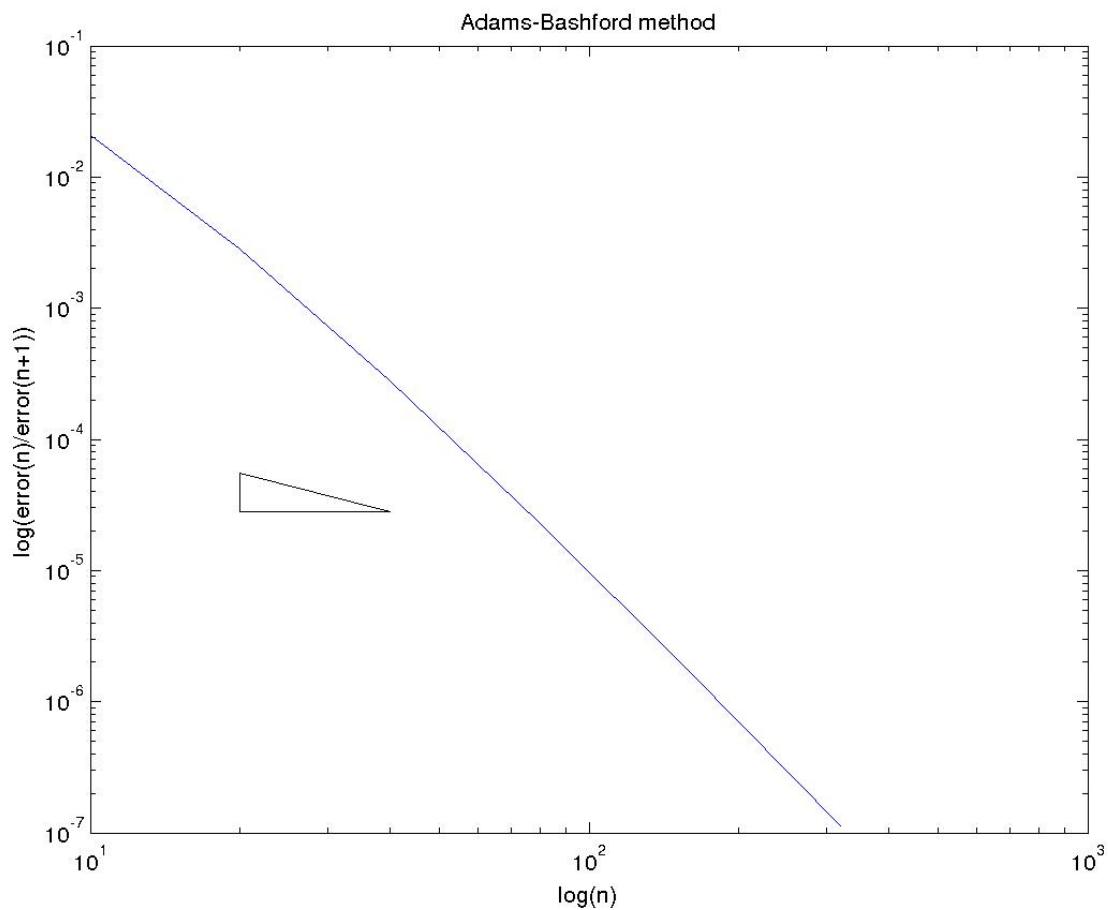


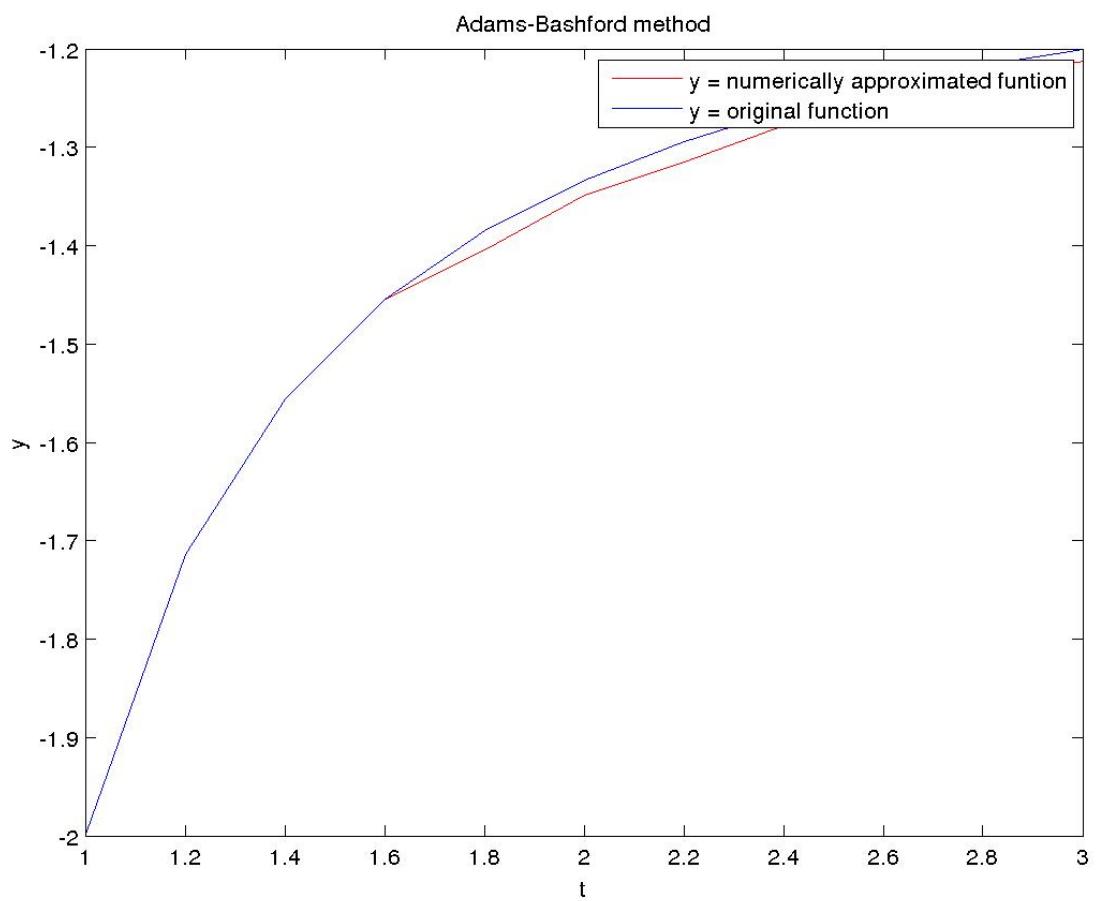
**(B)**

Using exact starting values

Adams-Bashford method

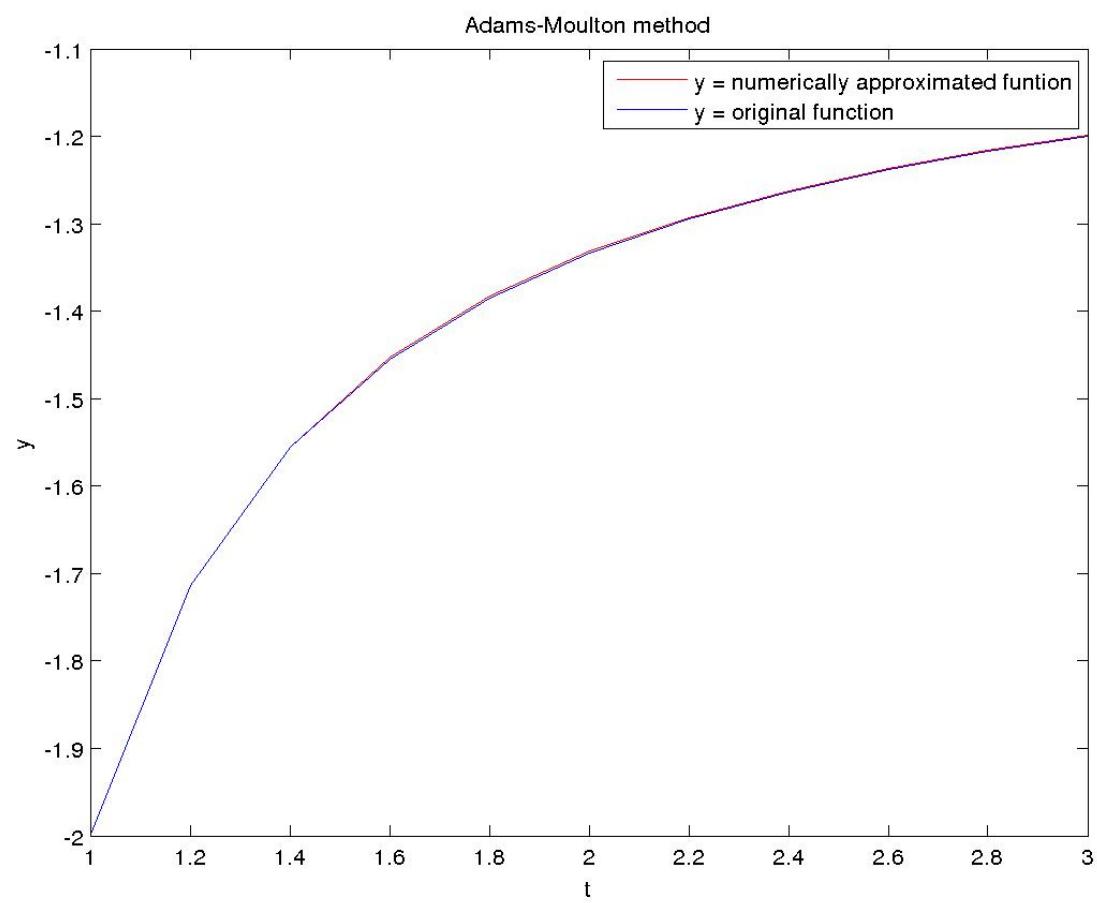
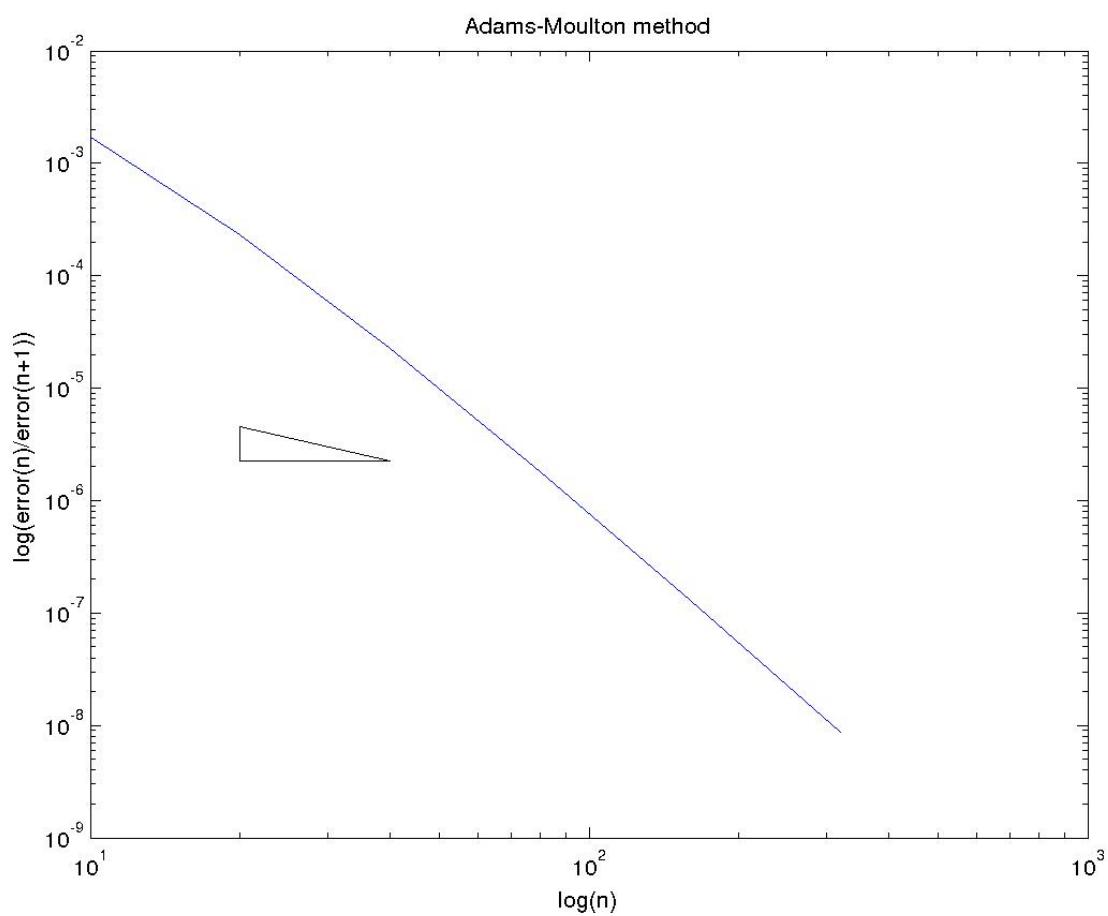
N	Error	Order
10	2.079273e-02	2.868599
20	2.846933e-03	3.355458
40	2.781530e-04	3.596418
80	2.299608e-05	3.777475
160	1.676952e-06	3.884273





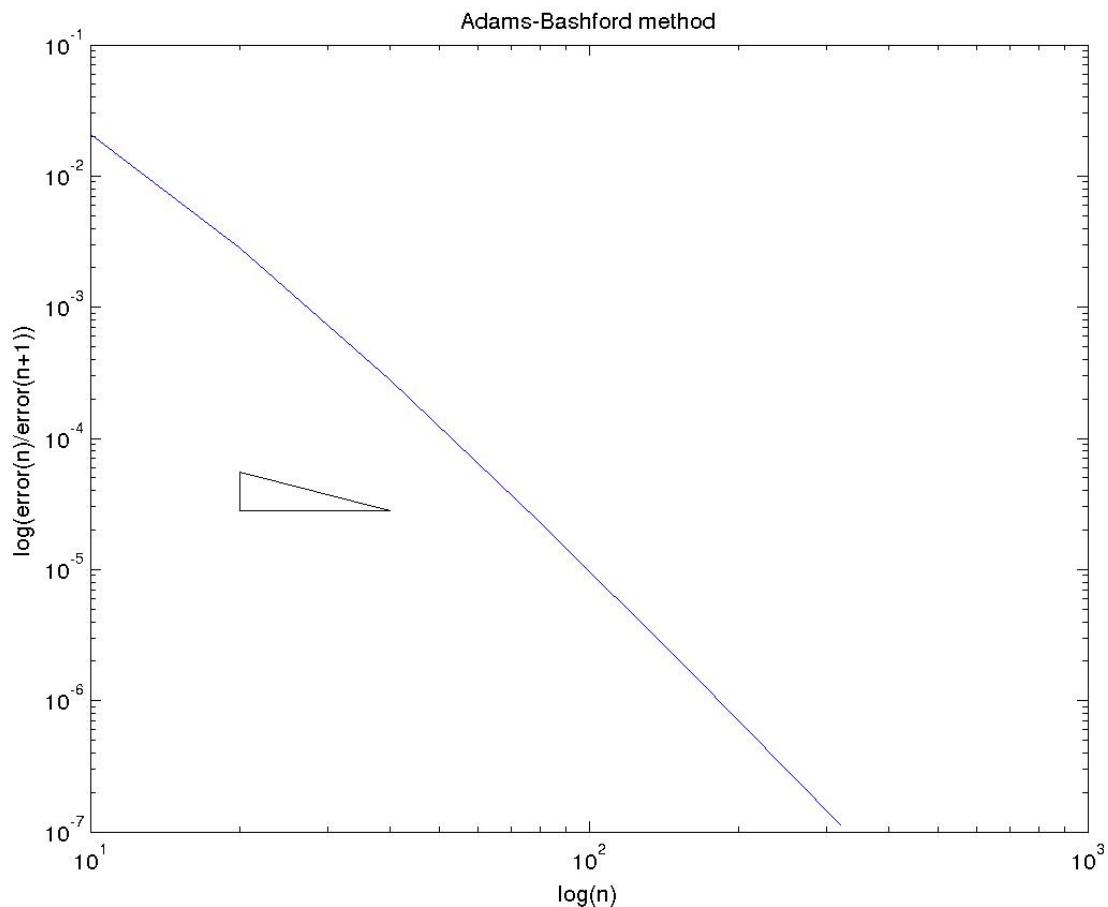
### Adams-Moulton method

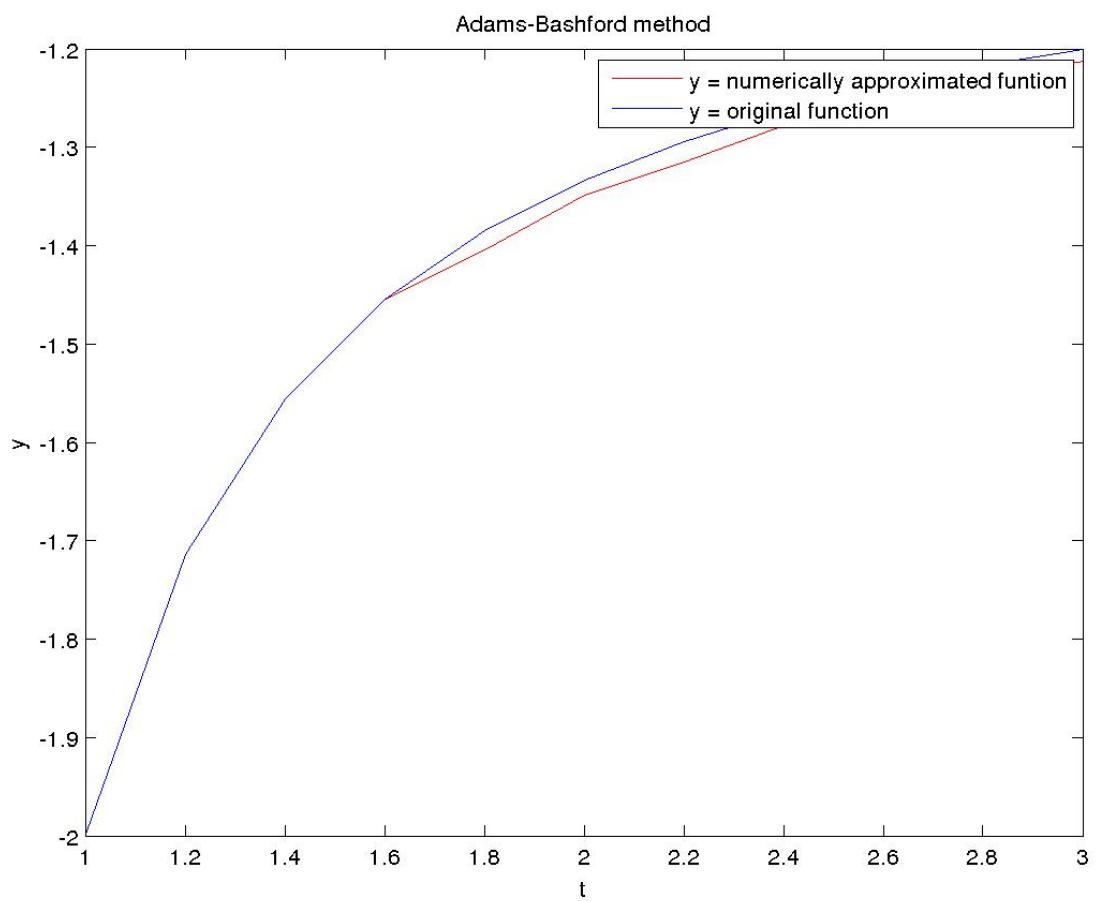
N	Error	Order
10	1.732877e-03	2.910375
20	2.304930e-04	3.350827
40	2.259219e-05	3.637163
80	1.815779e-06	3.806457
160	1.297793e-07	3.899485



Using starting values from Runge-Kutta 4th order  
Adams-Bashford method

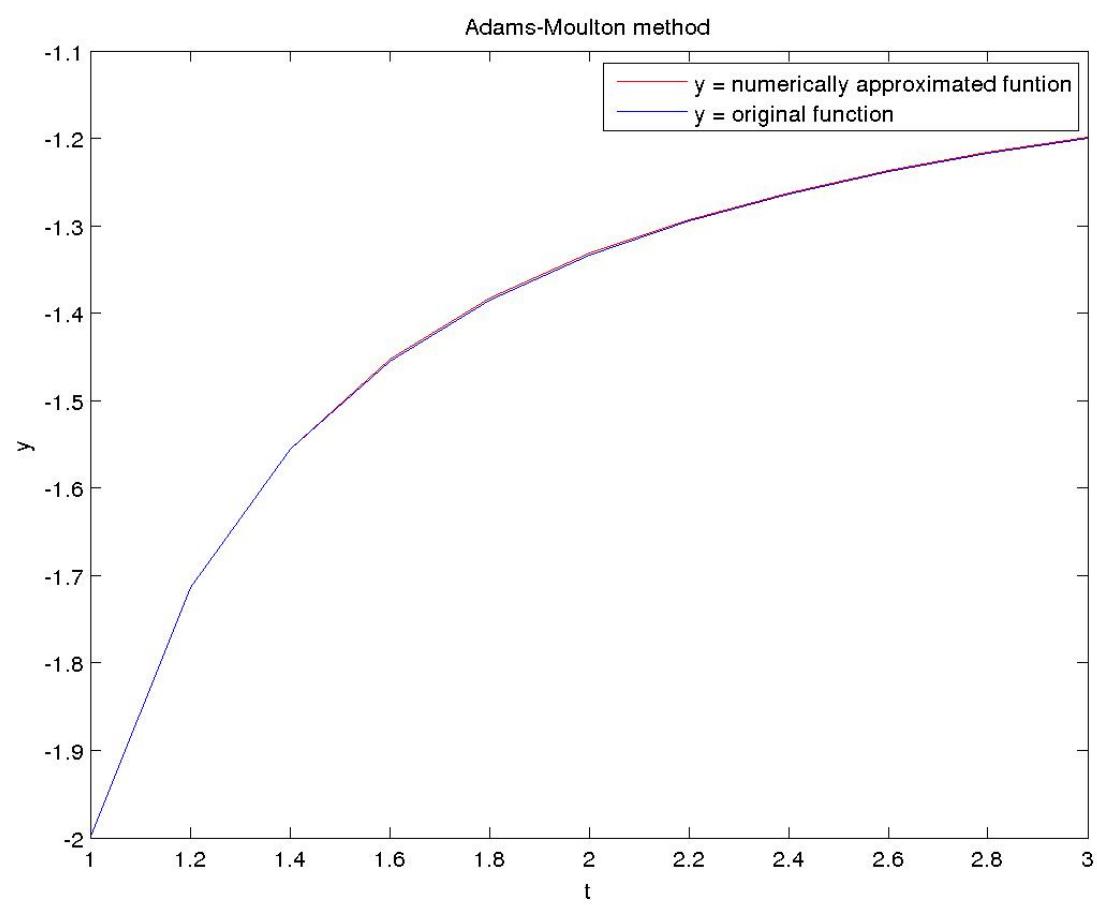
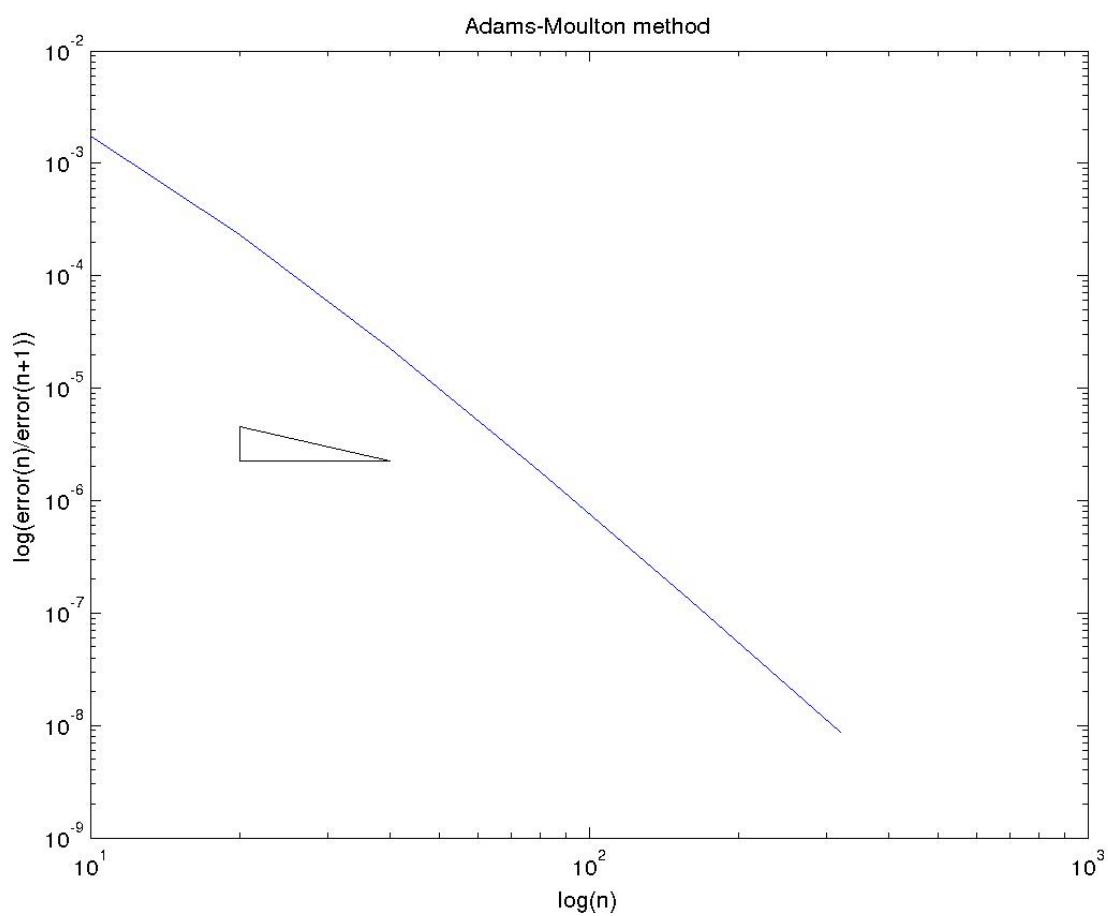
N	Error	Order
10	2.078609e-02	2.868283
20	2.846647e-03	3.355268
40	2.781617e-04	3.596395
80	2.299718e-05	3.777495
160	1.677009e-06	3.884293





### Adams-Moulton method

N	Error	Order
10	1.753780e-03	2.925572
20	2.308289e-04	3.353440
40	2.258418e-05	3.637345
80	1.814907e-06	3.806221
160	1.297381e-07	3.899287

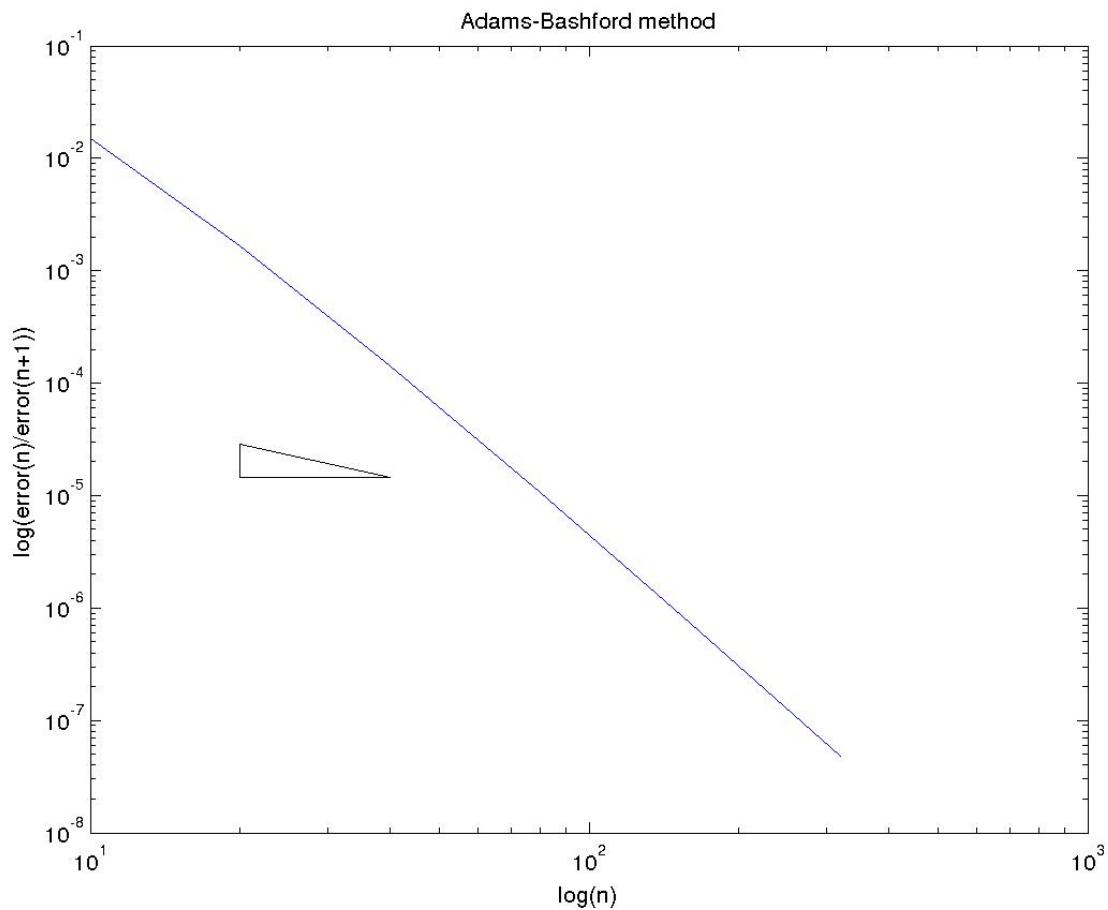


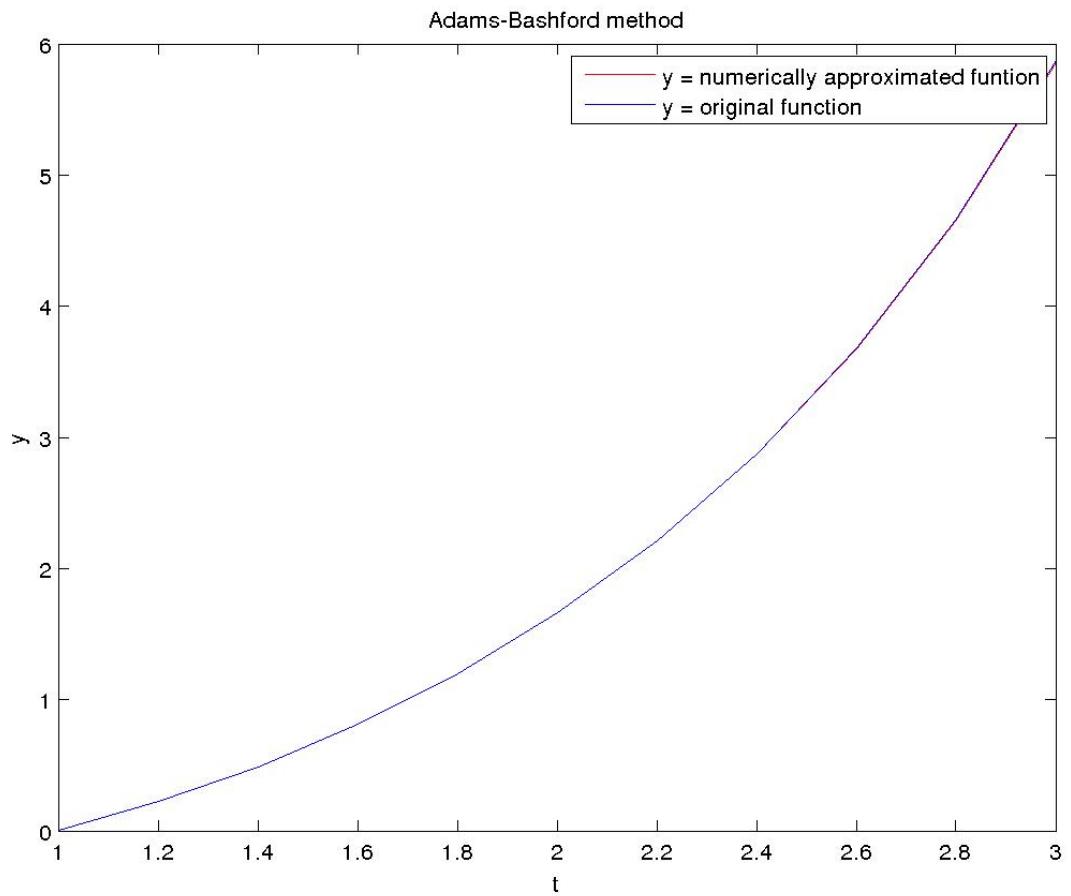
**(C)**

Using exact starting values

Adams-Bashford method

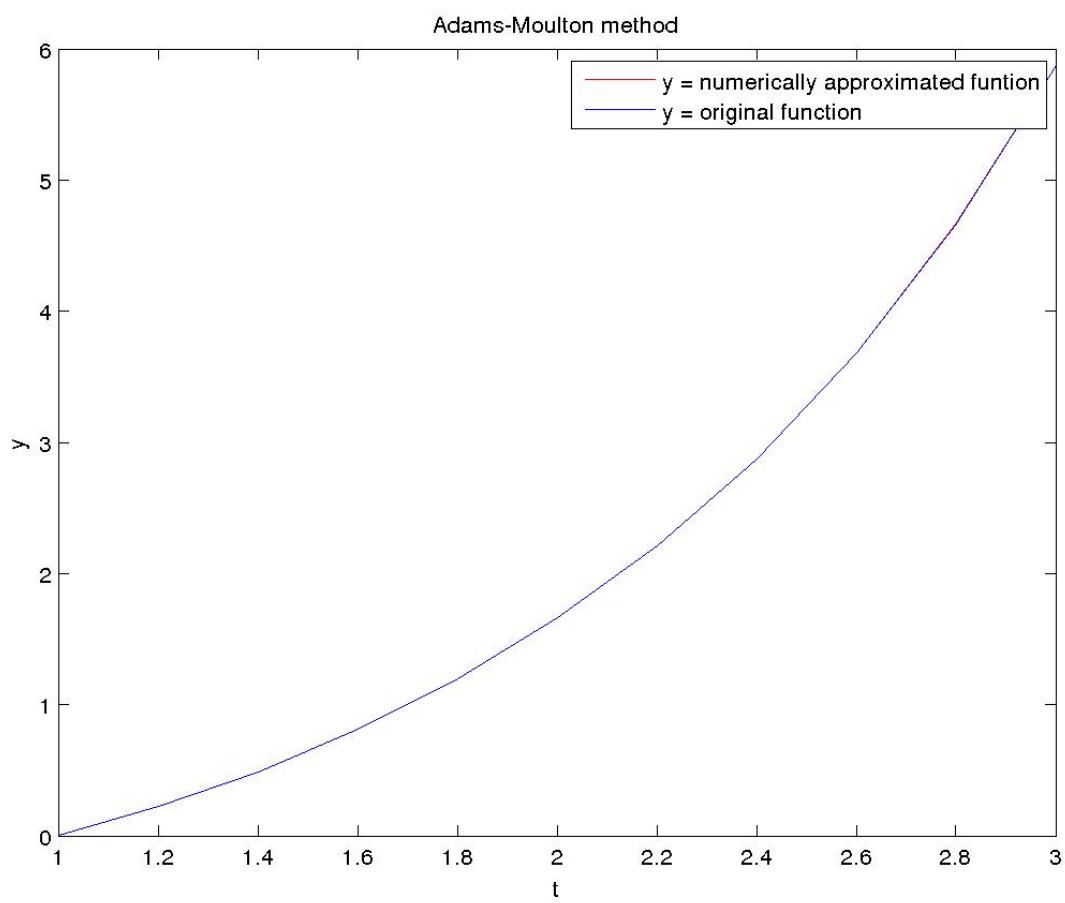
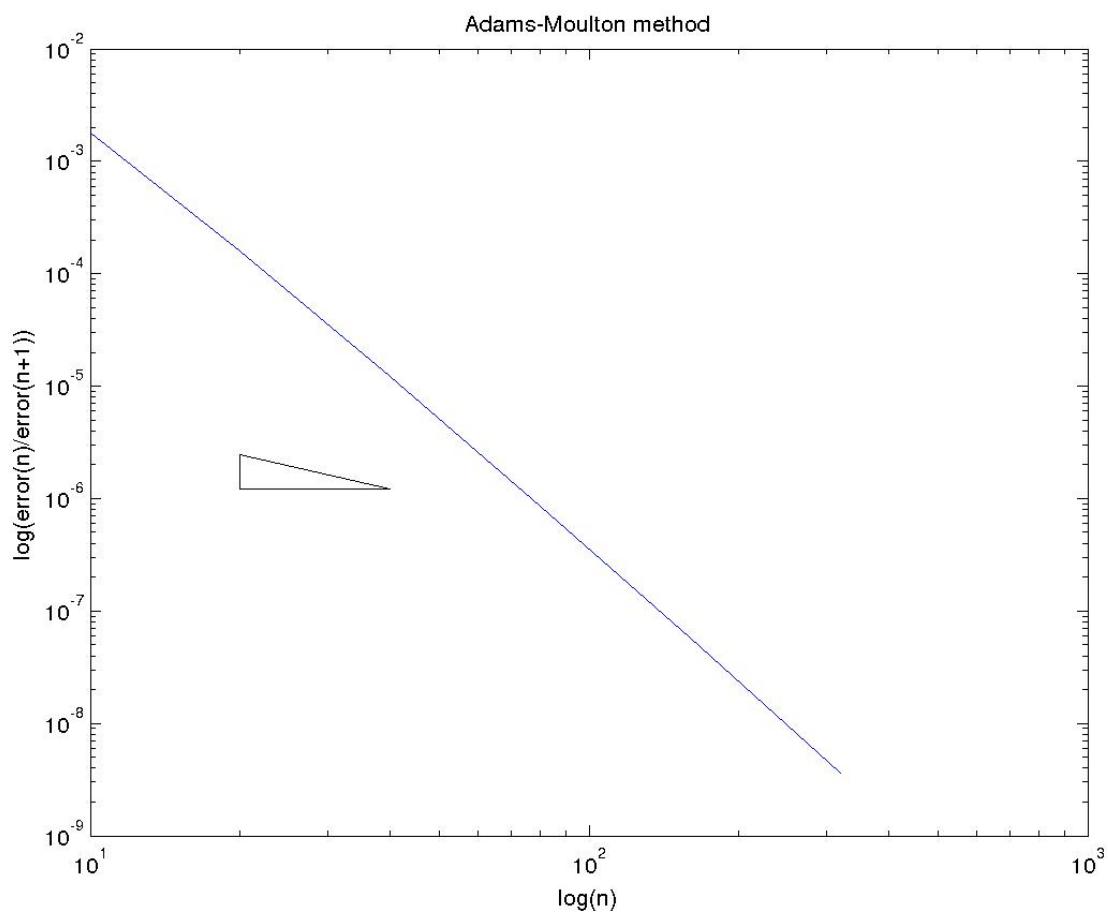
N	Error	Order
10	1.510053e-02	3.169750
20	1.678040e-03	3.543827
40	1.438812e-04	3.750966
80	1.068688e-05	3.868520
160	7.316616e-07	3.932281





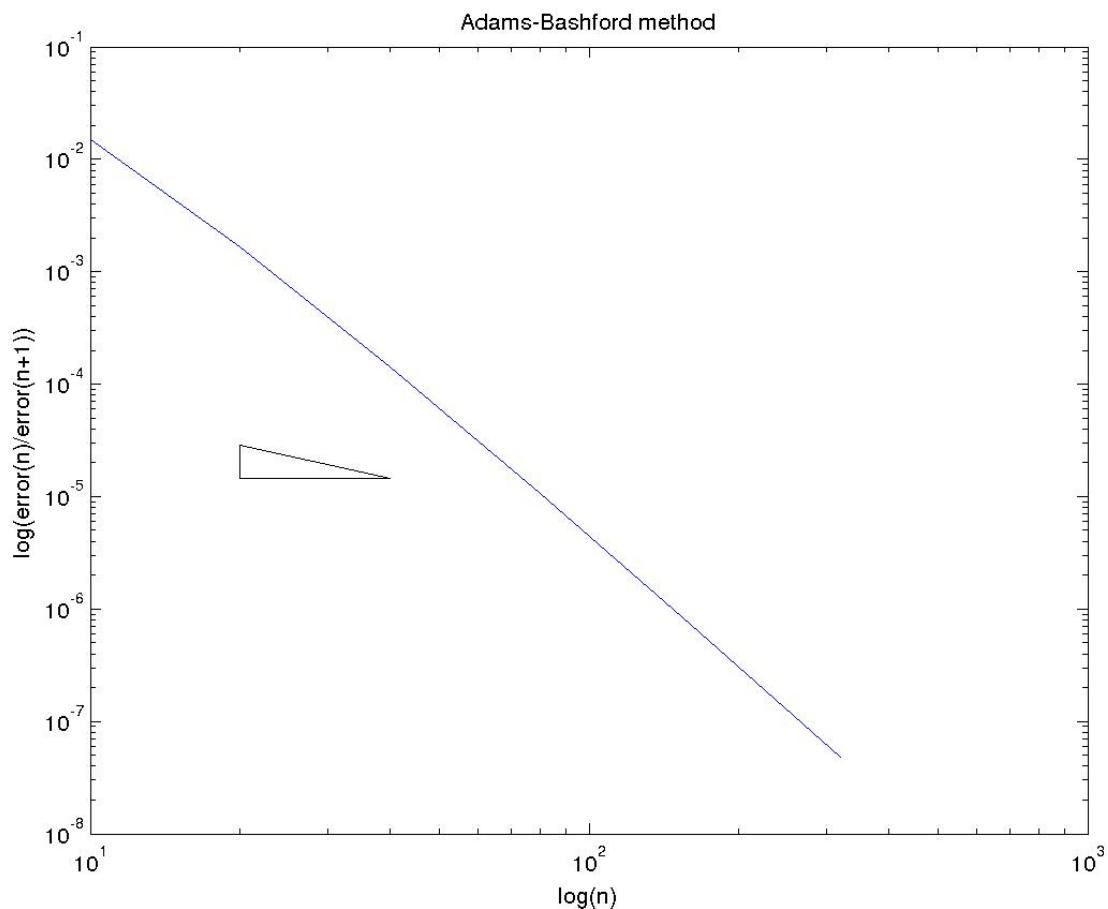
### Adams-Moulton method

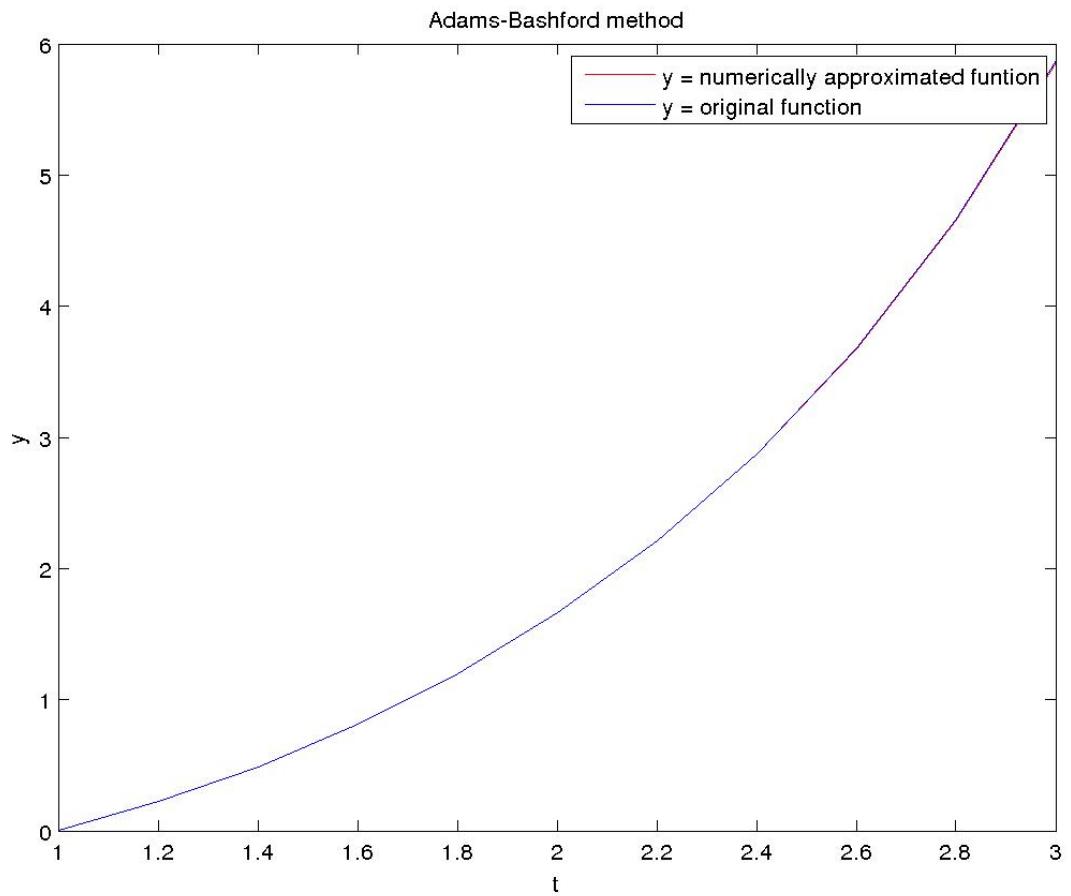
N	Error	Order
10	1.792123e-03	3.501697
20	1.582165e-04	3.698841
40	1.218399e-05	3.830313
80	8.565454e-07	3.912484
160	5.688205e-08	3.962129



Using starting values from Runge-Kutta 4th order  
Adams-Bashford method

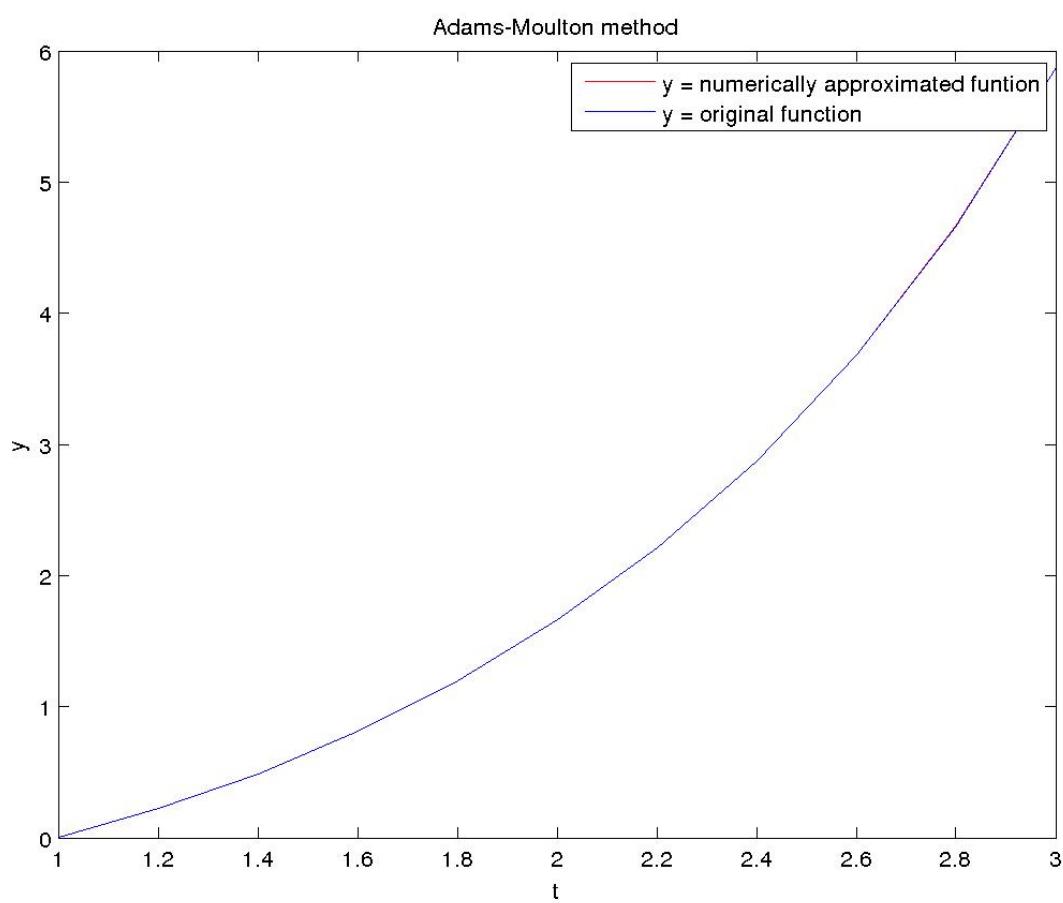
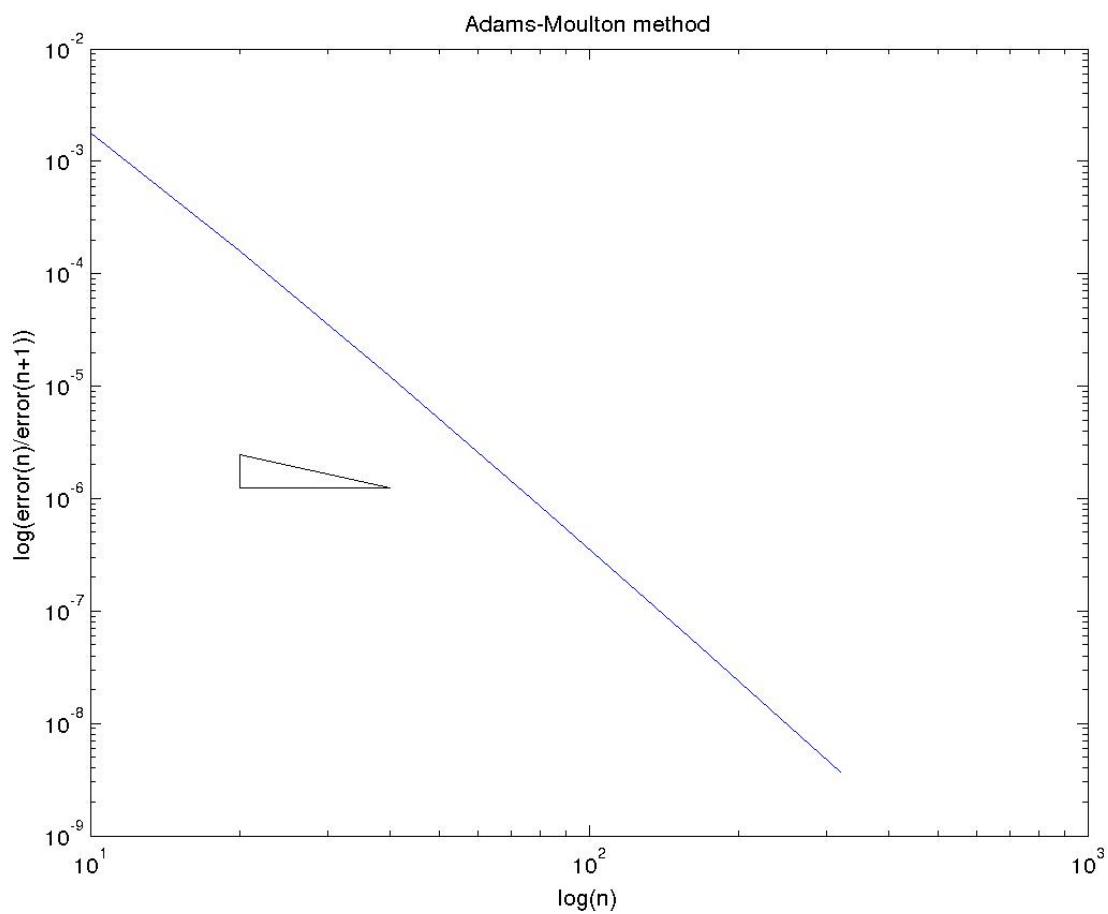
N	Error	Order
10	1.510559e-02	3.172317
20	1.675619e-03	3.543296
40	1.437265e-04	3.750304
80	1.068028e-05	3.868101
160	7.314223e-07	3.932052





### Adams-Moulton method

N	Error	Order
10	1.814826e-03	3.497032
20	1.607398e-04	3.706966
40	1.230879e-05	3.836923
80	8.613634e-07	3.916353
160	5.704882e-08	3.964189

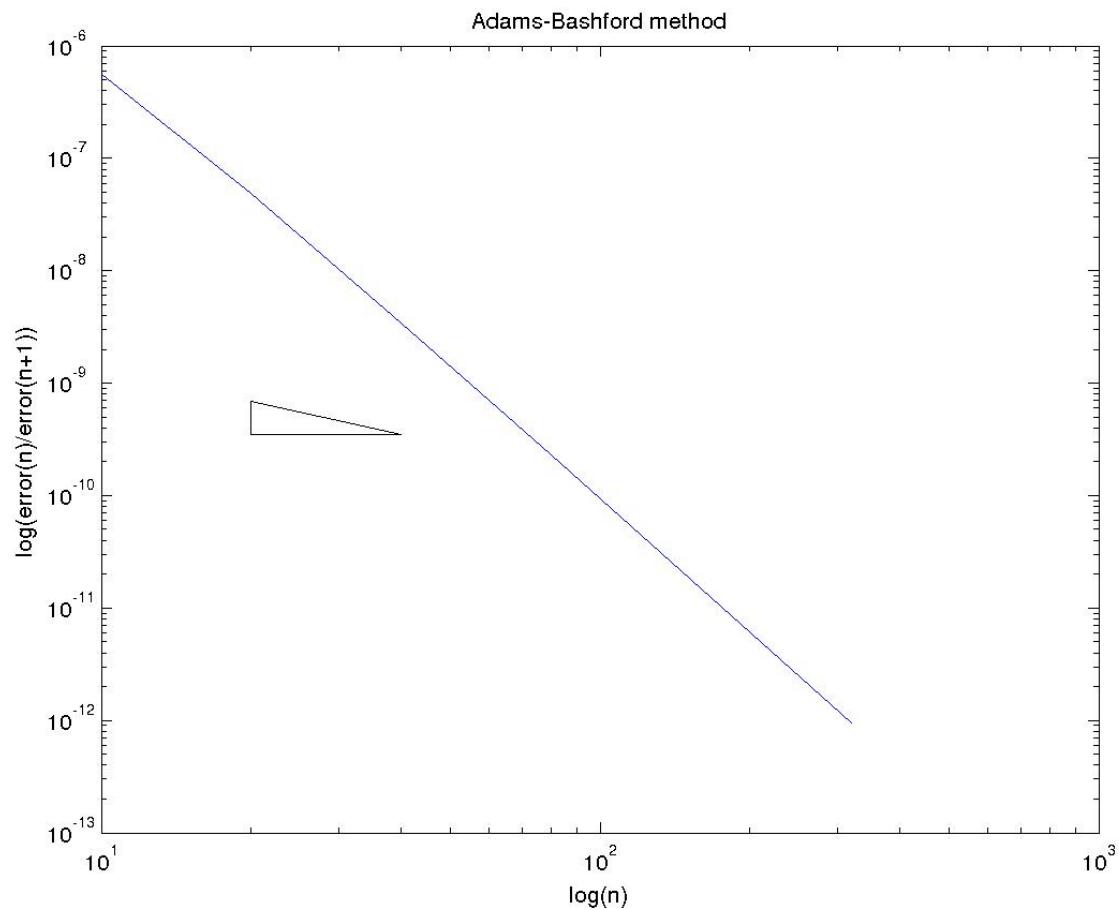


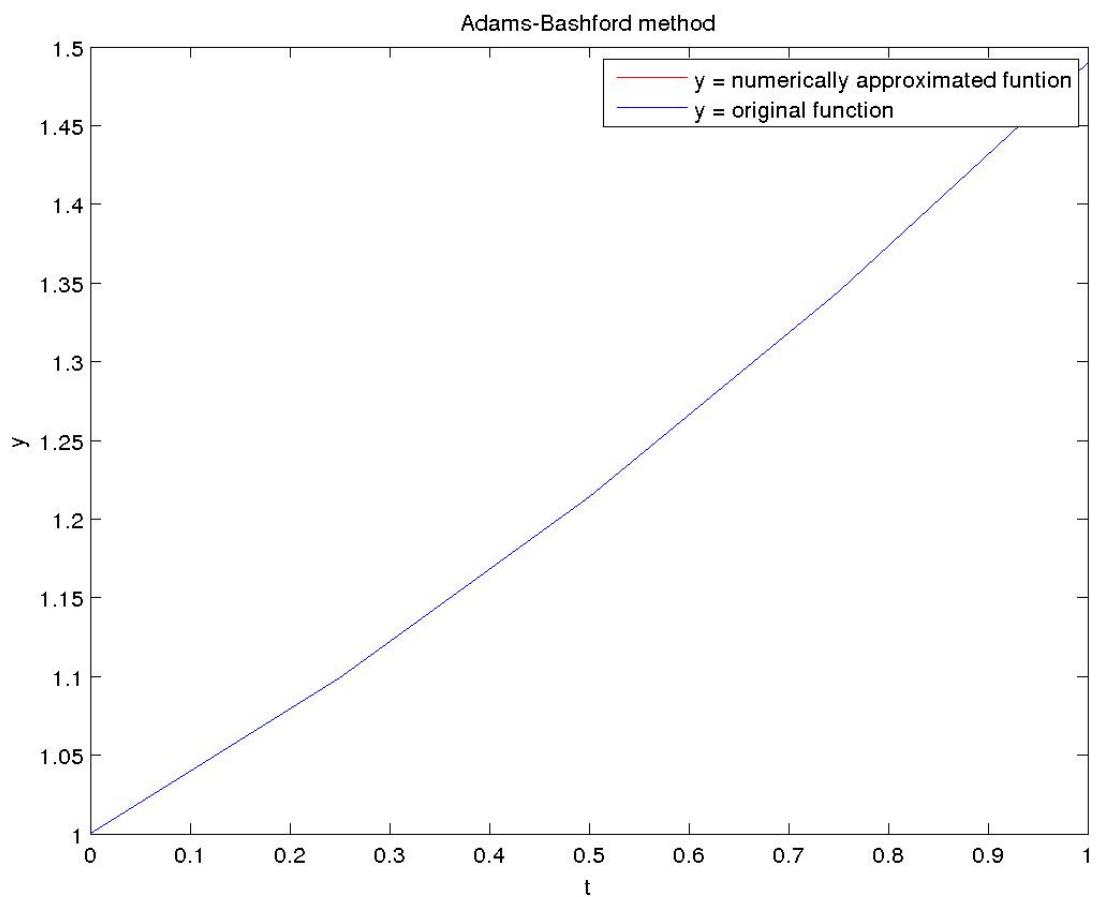
**(D)**

Using exact starting values

Adams-Bashford method

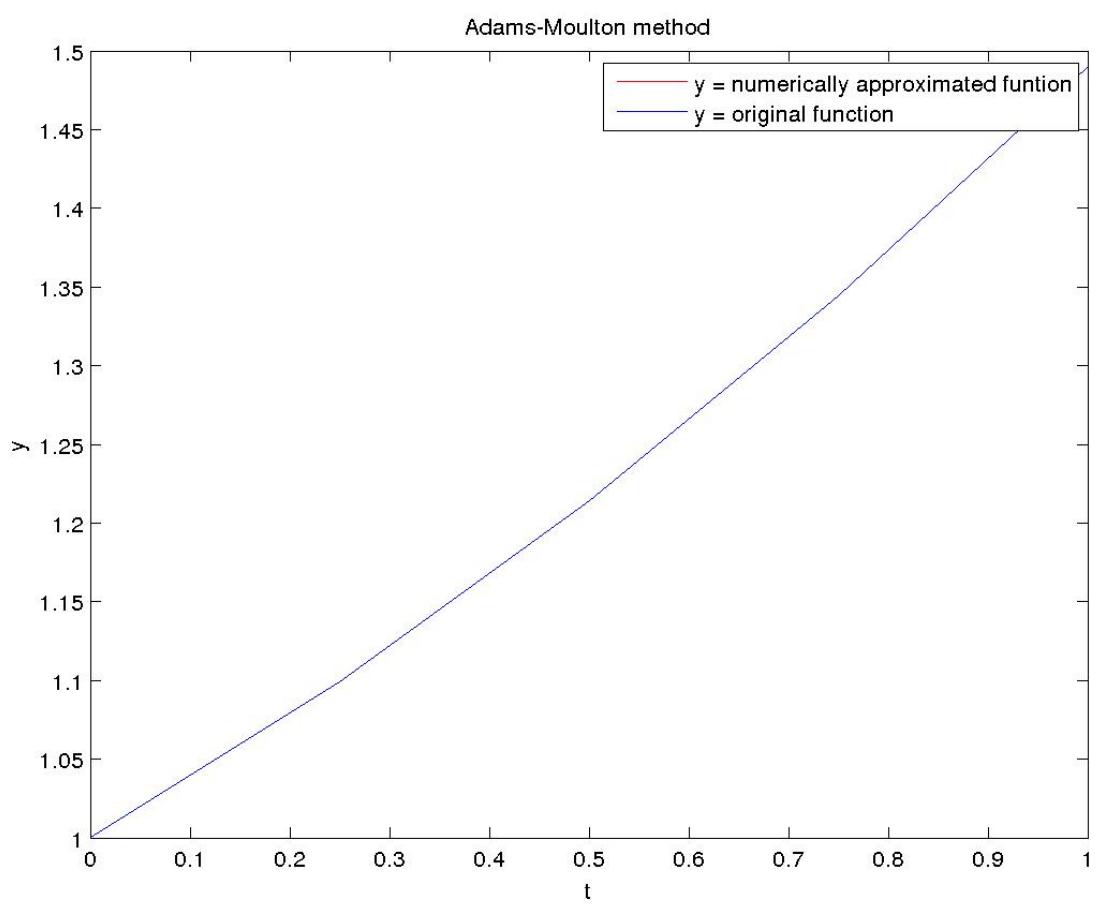
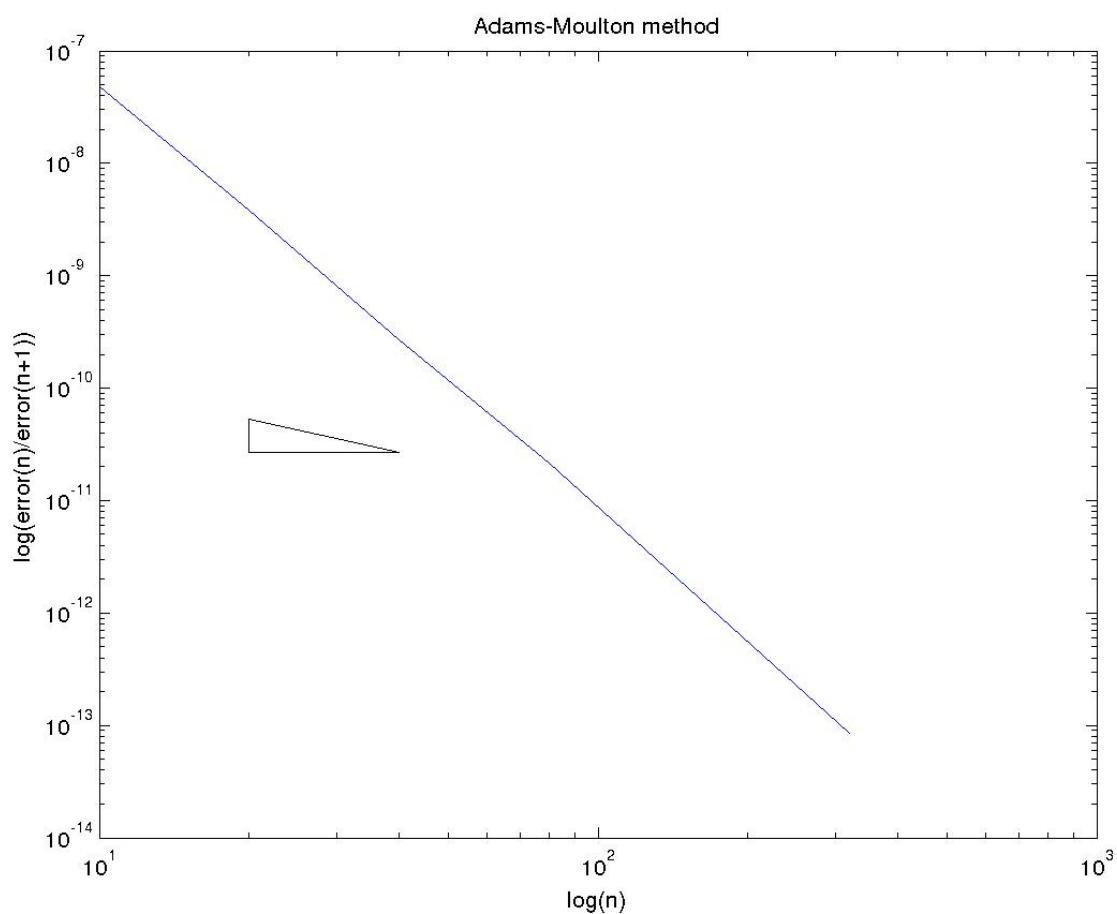
N	Error	Order
10	5.663556e-07	3.550004
20	4.835391e-08	3.805645
40	3.457948e-09	3.911109
80	2.298568e-10	3.956812
160	1.480260e-11	3.977859





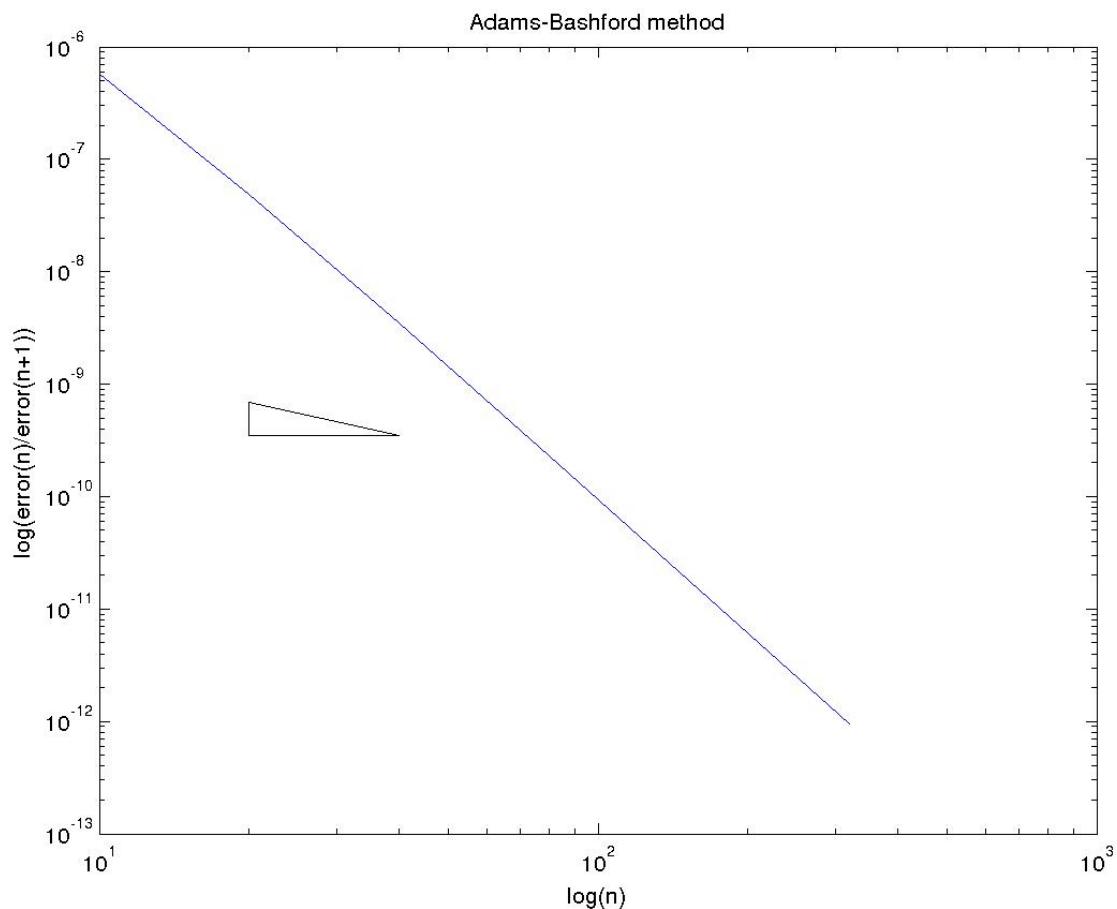
### Adams-Moulton method

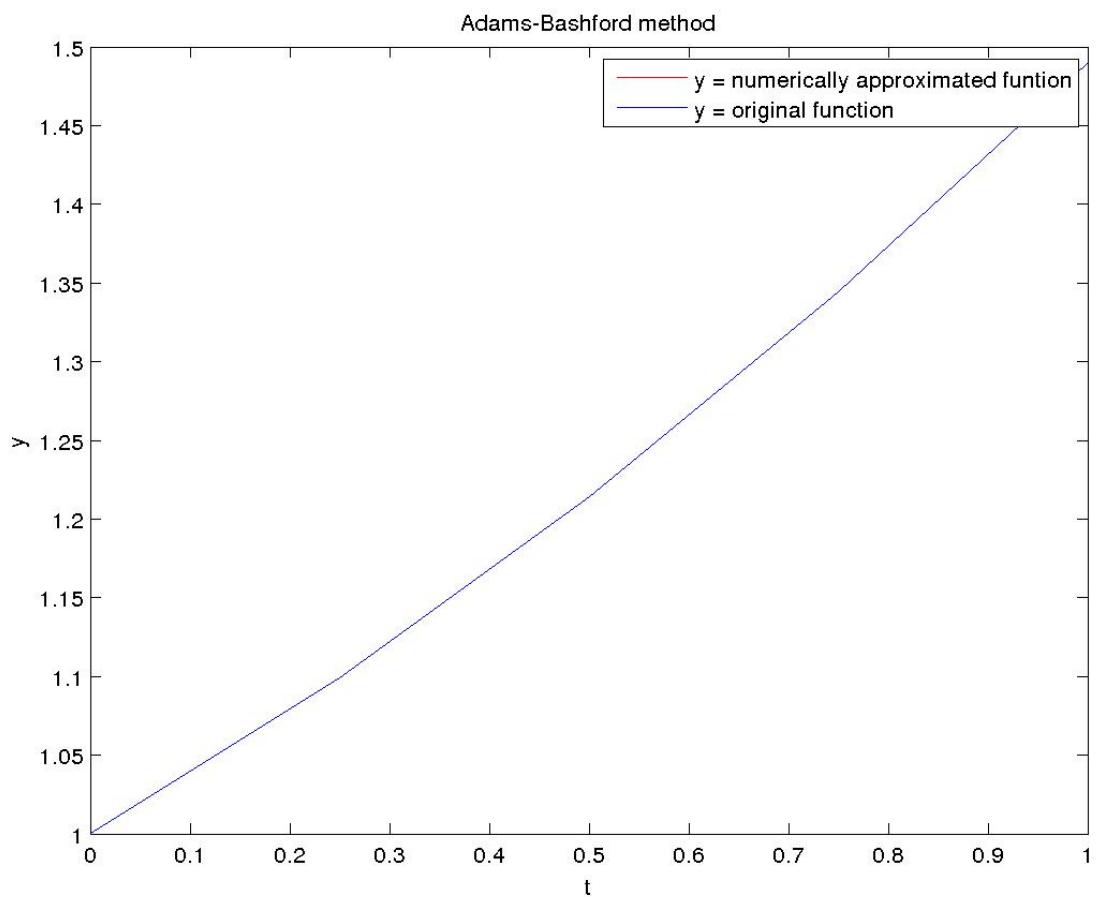
N	Error	Order
10	4.832447e-08	3.654039
20	3.838766e-09	3.845347
40	2.670710e-10	3.634761
80	2.150080e-11	3.976811
160	1.365574e-12	4.008942



Using starting values from Runge-Kutta 4th order  
Adams-Bashford method

N	Error	Order
10	5.781074e-07	3.569301
20	4.870145e-08	3.811686
40	3.468246e-09	3.913438
80	2.301694e-10	3.957886
160	1.481171e-11	3.978405





### Adams-Moulton method

N	Error	Order
10	4.070670e-08	3.492774
20	3.616065e-09	3.796419
40	2.602556e-10	3.610879
80	2.130185e-11	3.969982
160	1.359357e-12	4.002359

