Lecture - 3

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Outline

Time Schedule

- Thursday (10 - 10:55 a.m.), Friday (11:00 - 11:55 p.m.), Monday (9 - 9:55 a.m.).
- OFFICE HOURS: 3:00pm - 4:00pm. (try to come by appointment)
- LAB HOURS: 3:00pm - 5:00pm. (AL-2)

TEXT BOOKS

The text for this course is

1. "Statistics and Finance : An Introduction" by **David Ruppert**
2. "Statistical Analysis of Financial Data in S-plus" : **René Carmona**

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Statistics

- Turning the data into information
- data-based decision making
- The technology of the "Scientific Method"

Syllabus

This course will cover the following topics:

1. Financial data and fitting different probability distributions.
2. Estimation: Parametric(Maximum likelihood/ EM)/ Non-parameteric (Kernal density/ bootstrap) / bayesian.
3. Basic testing of hypothesis (Critical region/ Confidence interval/ p-value)
4. Different techniques of monte carlo simulation.
5. Financial Time Series Analysis
6. Linear regression models
7. Applied Multivariate Analysis
8. Extreme Value theory
9. Copula based approach

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Package Related Information

We mainly use **"R"** and **"Python"**, but you can use any other packages for example **S-plus** and **Matlab**.
Instructions for **R** will be given.

We come to the details later in lab-classes.

Outline

The grade for this course will be determined based on your performance on

- Assignments (labs) (5%),
- Quiz (10%)
- Midsem exam (30%),
- Project Report, Attendance, Attitude, Class performance etc (labs) (10%),
- Final Examination (45% [10% + 35%]).

Questions ???

Asset Returns

- Let P_t be the price of an asset at time t, and assume no dividend. **One-period simple return:** Gross return $1 + R_t = \frac{P_t}{P_{t-1}}$ or $P_t = P_{t-1}(1 + R_t)$
- **Simple Return:**

$$R_t = \frac{P_t}{P_{t-1}} - 1 = \frac{P_t - P_{t-1}}{P_{t-1}}$$

- **Multiperiod Simple Return:** Gross Return

$$\begin{aligned} 1 + R_t(k) &= \frac{P_t}{P_{t-k}} = \frac{P_t}{P_{t-1}} \times \frac{P_{t-1}}{P_{t-2}} \times \frac{P_{t-2}}{P_{t-3}} \times \cdots \times \frac{P_{t-k+1}}{P_{t-k}} \\ &= (1 + R_t)(1 + R_{t-1}) \cdots (1 + R_{t-k+1}) \end{aligned}$$

The k-th period simple net return is $R_t(k) = \frac{P_t}{P_{t-k}} - 1$.

- **Example:** Suppose the daily closing prices of a stock are

Day	1	2	3	4	5
Price	37.84	38.49	37.12	37.60	36.30

- What is the simple return from day 1 to day 2 ? Ans :

$$R_2 = \frac{38.49 - 37.84}{37.84} = 0.017$$

- What is the simple return from day 1 to day 5 ? Ans :

$$R_5(4) = \frac{36.30 - 37.84}{37.84} = -0.041$$

- Verify that $1 + R_5(4) = (1 + R_2)(1 + R_3) \cdots (1 + R_5)$.

- Annalized (average) return:

$$R_t(k) = [\prod_{j=0}^{k-1}(1 + R_{t-j})]^{\frac{1}{k}} - 1$$

An approximation : Annualized$[R_t(k)] \approx \frac{1}{k}\sum_{j=0}^{k-1} R_{t-j}$

- Continuously compounded (or log) return $r_t = \ln(1 + R_t) = \ln \frac{P_t}{P_{t-1}} = p_t - p_{t-1}$.
- Multiperiod log return:

$$\begin{aligned} r_t &= \log(1 + R_t(k)) \\ &= \log((1 + R_t)(1 + R_{t-1}) \cdots (1 + R_{t-k+1})) \\ &= \log(1 + R_t) + \log(R_{t-1}) + \cdots + \log(1 + R_{t-k+1}) \\ &= r_t + r_{t-1} + \cdots + r_{t-k+1}. \end{aligned}$$

where $r_t = \log(\frac{P_t}{P_{t-1}})$

- Example (continued). Use the previous daily prices.
- What is the log return from day 1 to day 2 ?
  Ans : $r_2 = \ln(38.49) - \ln(37.48) = 0.017.$
- What is the log return from Day 1 to Day 5 ?
  Ans : $r_5(4) = \ln(36.3) - \ln(37.84) = -0.042.$
- It is easy to verify $r_5(4) = r_2 + r_3 + \cdots + r_5$

- N assets

$$R_{p,t} = \sum_{i=1}^{N} w_i R_{it}$$

- **Example :** An investor holds stocks of IBM, Microsoft and Citi-Group. Assume that her capital allocation is 30%, 30% and 40%. The monthly simple returns for the three companies are 1.42, 3.37 and 2.20. What is the mean simple return of her stock portfolio ?
  Answer : $E(R_t) = 0.3 \times 1.42 + 0.3 \times 3.37 + 0.4 \times 2.20 = 2.32$
- Divident payment: $R_t = \frac{P_t + D_t}{P_{t-1}} - 1$, $r_t = \ln(D_t + P_t) - \ln(P_{t-1})$.
- Excess Return: (adjusting for risk) $Z_t = R_t - R_{0t}$, $z_t = r_t - r_{0t}$, where r_{0t} denotes the log-return of a reference asset (e.g. risk-free interest rate).

Relationship

$$r_t = \ln(1 + R_t), \quad R_t = e^{r_t} - 1.$$

If returns are in **percentage**, then

$$r_t = 100 \times \ln(1 + \frac{R_t}{100}), \quad R_t = [exp(r_t/100) - 1] \times 100$$

Temporal aggregation of the returns produces

$$1 + R_t(k) = (1 + R_t)(1 + R_{t-1}) \cdots (1 + R_{t-k+1}),$$
$$r_t(k) = r_t + r_{t-1} + \cdots + r_{t-k+1}.$$

These two relations are important in practice, e.g. obtain annual returns from monthly returns.

Example :

If monthly log-returns of an asset are 4.46%, -7.34% and 10.77%, then what is the corresponding quarterly return ?
Answer : $4.46 - 7.34 + 10.77 = 7.89\%$.

If the monthly simple returns of an asset are 4.46%, -7.34% and 10.77%, then what is the corresponding quarterly simple return ?
Ans : $(1 + 0.0446)(1 - 0.0734)(1 + 0.1077) - 1 = 1.0721 - 1 = 0.0721 = 7.21\%$