

1. Consider the following Black-Scholes diffusion equation:

$$\begin{cases} dX(t) = \mu X dt + \sigma X dW(t) \\ X(0) = X_0. \end{cases}$$

- (a) Obtain the exact solution of the above SDE.
- (b) The values of the parameters are $\mu = 0.75$, $\sigma = 0.30$ and $X_0 = 307$, and $t \in (0, 1)$.
- (c) Solve the above SDE by the following methods:
 - i. Euler-Maruyama method and Milstein method.
 - ii. Runge-Kutta method of weak-order two and Strong-order one.
 - iii. Taylor method of weak-order two and strong-order 1.5.
- (d) Plot the order of convergence in a loglog plot (Δt vs. the mean error).

2. Consider the following Langevin SDE:

$$\begin{cases} dX(t) = -\mu X(t)dt + \sigma dW(t) \\ X(0) = X_0. \end{cases}$$

- (a) The values of the parameters are $\mu = 10$, $\sigma = 1$ and $X_0 = 0$, and $t \in (0, 4)$.
 - (b) Solve the above SDE by the following methods:
 - i. Euler-Maruyama method and Milstein method.
 - ii. Runge-Kutta method of weak-order two and Strong-order one.
 - iii. Taylor method of weak-order two and strong-order 1.5.
 - (c) Plot the order of convergence in a loglog plot (Δt vs. the mean error).
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