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MA 473: Computational Finance Lab – I, January 10, 2018

1. Solve the following parabolic initial-boundary-value problem:
$$\begin{cases} \dfrac{\partial u}{\partial t} - \dfrac{\partial^2 u}{\partial x^2} = \sin(2\pi\, x)\sin(4\pi\, t), \quad (x,t) \in (0,1)\times(0,1), \\ u(0,t) = u(1,t) = 0, \quad t \in (0,1], \\ u(x,0) = 0, \quad x \in (0,1), \end{cases}$$
by two-level finite difference schemes with the spatial step-size $h = 0.025$ and the time steps $k = 0.05$. Calculate the solution for three time levels.

2. Solve the following parabolic PDE by using two-level and three-level finite difference schemes:
$$\begin{cases} \dfrac{\partial u}{\partial t} - \dfrac{\partial^2 u}{\partial x^2} = 0, \quad (x,t) \in (0,1)\times(0,T),\ T > 0 \\ u(0,t) = u(1,t) = 0, \quad t \in (0,T], \\ u(x,0) = f(x), \quad x \in (0,1), \end{cases}$$
for the following values of f:
   (a) $f(x) = \sin(\pi\, x)$
   (b) $f(x) = x\,(1-x)$.

   Here, the values of the step size in the x-direction is $\delta\, x = 0.025$, and in the t-direction is $\delta\, t = 0.0025$. Determine the solution for three time levels.

3. Solve the following parabolic problem:
$$\begin{cases} \dfrac{\partial u}{\partial t} - \dfrac{\partial^2 u}{\partial x^2} = 0, \quad (x,t) \in (0,1)\times(0,t),\ t > 0, \\ u(x,0) = \cos(\pi\, x/2), \quad x \in (0,1), \\ u_x(0,t) = 0, \quad t \geq 0, \\ u(1,t) = 0, \quad t \geq 0, \end{cases}$$
by using the Neumann boundary condition approximation as given below:
$$u_0^{n+1} = u_0^n + 2\frac{k}{h^2}(u_1^n - u_0^n).$$
Use the spatial step-size $h = 0.10$, the time steps $k = 0.005$ and find the solution at $t = 0.1$.

4. Determine the numerical solution of the following one-dimensional parabolic IBVP:
$$\begin{cases} u_t - u_{xx} = 0, \quad (x,t) \in (0,1/2)\times(0,T),\ T > 0 \\ u(0,t) = 0, \quad u_x(1/2,t) = t^2, \quad \forall t \in (0,T] \\ u(x,0) = x(1-x), \quad x \in (0,1/2), \end{cases}$$
by using the Crank-Nicolson scheme with $h = 0.3$ and $k = 0.05$ for three time levels.