

1. Consider the following European call option problem:

$$\begin{cases} \frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2(S)S^2\frac{\partial^2 V}{\partial S^2} + (r - \delta)S\frac{\partial V}{\partial S} - rV = 0, & 0 \leq S < \infty, t \leq T, \\ V(S, t) = V_T(S), & 0 \leq S < \infty. \end{cases}$$

With the following transformation

$$\begin{cases} \xi = \frac{S}{S + P_m}, \\ \tau = T - t, \\ V(S, T) = (S + P_m)\bar{V}(\xi, \tau). \end{cases}$$

the above European call option problem becomes the following parabolic PDE:

$$\begin{cases} \frac{\partial \bar{V}}{\partial \tau} = \frac{1}{2}\sigma^2(\xi)\xi^2(1 - \xi^2)\frac{\partial^2 \bar{V}}{\partial \xi^2} + (r - \delta)\xi(1 - \xi)\frac{\partial \bar{V}}{\partial \xi} - [r(1 - \xi) + \delta\xi]\bar{V}, & 0 \leq \xi \leq 1, 0 \leq \tau \leq T, \\ \bar{V}(\xi, 0) = \max(2\xi - 1, 0), & 0 \leq \xi \leq 1, \\ \bar{V}(0, \tau) = \bar{V}(0, 0)e^{-r\tau}, & \bar{V}(1, \tau) = \bar{V}(1, 0)e^{-\delta\tau}, & 0 \leq \tau \leq T. \end{cases}$$

Solve the transformed PDE by the following schemes:

- (i) Forward-Euler for time & central difference for space (FTCS) scheme.
- (ii) Backward-Euler for time & central difference for space (BTCS) scheme.
- (iii) Crank-Nicolson finite difference scheme

The values of the parameters are $T = 1$, $r = 0.04$, $\sigma = 0.25$, $\delta = 0.1$.

To solve the system of linear algebraic equations arising from the implicit schemes use the iterative methods (Jacobi method, Gauß-Seidel method and SOR method).

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2. Consider the following European put option problem:

$$\begin{cases} \frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2(S)S^2\frac{\partial^2 V}{\partial S^2} + (r - \delta)S\frac{\partial V}{\partial S} - rV = 0, & 0 \leq S < \infty, \quad t \leq T, \\ V(S, t) = V_T(S), & 0 \leq S < \infty. \end{cases}$$

With the following transformation

$$\begin{cases} \xi = \frac{S}{S + P_m}, \\ \tau = T - t, \\ V(S, T) = (S + P_m)\bar{V}(\xi, \tau). \end{cases}$$

the above European put option problem becomes the following parabolic PDE:

$$\begin{cases} \frac{\partial \bar{V}}{\partial \tau} = \frac{1}{2}\bar{\sigma}^2(\xi)\xi^2(1 - \xi^2)\frac{\partial^2 \bar{V}}{\partial \xi^2} + (r - \delta)\xi(1 - \xi)\frac{\partial \bar{V}}{\partial \xi} - [r(1 - \xi) + \delta\xi]\bar{V}, & 0 \leq \xi \leq 1, \quad 0 \leq \tau \leq T, \\ \bar{V}(\xi, 0) = \max(1 - 2\xi, 0), & 0 \leq \xi \leq 1, \\ \bar{V}(0, \tau) = \bar{V}(0, 0)e^{-r\tau}, \quad \bar{V}(1, \tau) = \bar{V}(1, 0)e^{-\delta\tau}, & 0 \leq \tau \leq T. \end{cases}$$

Solve the transformed PDE by the following schemes:

- (i) Forward-Euler for time & central difference for space (FTCS) scheme.
- (ii) Backward-Euler for time & central difference for space (BTCS) scheme.
- (iii) Crank-Nicolson finite difference scheme

The values of the parameters are $T = 1$, $r = 0.04$, $\sigma = 0.25$, $\delta = 0.1$.

To solve the system of linear algebraic equations arising from the implicit schemes use the iterative methods (Jacobi method, Gauß-Seidel method and SOR method).
