Assignment-1

EE:1205 Signals and Systems
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I. Question 11.14.8

A spring balance has a scale that that reads fro 0 to 50 kg. The length of the scale is 20cm. A body is suspended from this balance , when displaced and released, oscillates with a period of 0.6s. What is weight of the body?

II. Solution

TABLE 0
Input Parameters

Parameter	Value	Description
M	50 kg	Mass of block
l	0.2 m	Maximum displacement of spring
T	0.6 s	Time period of oscillation
F	490 N	Force

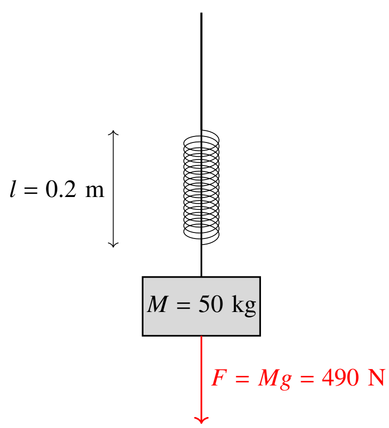

Fig. 0. spring-mass system

The spring constant, k, is calculated as:

$$k = \frac{F}{l} = \frac{490}{0.2} = 2450\,\text{N/m}^{-1} \tag{1}$$

Given the equation of motion for a mass-spring system with mass m and spring constant k is given by:

$$F = ma = -kx \tag{2}$$

This equation can be rearranged as:

$$ma = -kx \tag{3}$$

$$m\frac{d^2x}{dt^2} = -kx \tag{4}$$

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0 \tag{5}$$

$$50\frac{d^2x}{dt^2} + 2450x = 0 \tag{6}$$

III. Derivation of Simple Harmonic Motion Period using Laplace Transform

$$m\frac{d^2x}{dt^2} + kx = 0 \tag{7}$$

Rewrite the equation as:

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0 \tag{8}$$

Take the Laplace transform of both sides:

$$s^2X(s) - sx(0) - x'(0) + \frac{k}{m}X(s) = 0 \tag{9}$$

Rearrange terms to solve for $X(s)$:

$$X(s)(s^2 + \frac{k}{m}) = sx(0) + x'(0) \tag{10}$$

Solve for $X(s)$:

$$X(s) = \frac{sx(0) + x'(0)}{s^2 + \frac{k}{m}} \tag{11}$$

Find the roots of the characteristic equation:

$$s^2 + \frac{k}{m} = 0 \tag{12}$$

Let $\omega^2 = \frac{k}{m}$, then $s = \pm j\omega$.
Express ω in terms of T:

$$T = \frac{2\pi}{\omega} \tag{13}$$

Solve for ω in terms of T:

$$\omega = \frac{2\pi}{T} \tag{14}$$

Substitute ω back into the characteristic equation:

$$s = \pm j\frac{2\pi}{T} \tag{15}$$

Now, the Laplace transform solution $X(s)$ becomes:

$$X(s) = \frac{s(x_0) + x'(0)}{s^2 + \frac{k}{m}} \tag{16}$$

Express s in terms of ω:

$$X(s) = \frac{s(x_0) + x'(0)}{(j\omega)^2 + \omega^2} \tag{17}$$

Simplify and take the inverse Laplace transform to obtain the displacement $x(t)$ in the time domain:

$$x(t) = A\cos(\omega t + \phi) \tag{18}$$

Where A is the amplitude, ω is the angular frequency, and ϕ is the phase angle. The period T is related to the angular frequency by $T = \frac{2\pi}{\omega}$, giving the desired result:

$$T = 2\pi\sqrt{\frac{m}{k}} \tag{19}$$

The weight of th body is defined as:

$$\text{Weight} = mg = 22.36 \times 9.8 = 219.16\,\text{N} \tag{20}$$

Therefore, the weight of the body is approximately 219 N.