

Assignment-1

EE:1205 Signals and Systems
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I. QUESTION 11.14.8

A spring balance has a scale that reads from 0 to 50 kg. The length of the scale is 20cm. A body is suspended from this balance, when displaced and released, oscillates with a period of 0.6s. What is weight of the body?

II. SOLUTION

TABLE 0
INPUT PARAMETERS

Parameter	Value	Description
Mass (M)	50 kg	Mass of block
Maximum displacement of the spring (l)	0.2 m	Maximum elongation of spring
Time period (T)	0.6 s	Period of one oscillation
Force (F)	490 N	Product of mass and acceleration

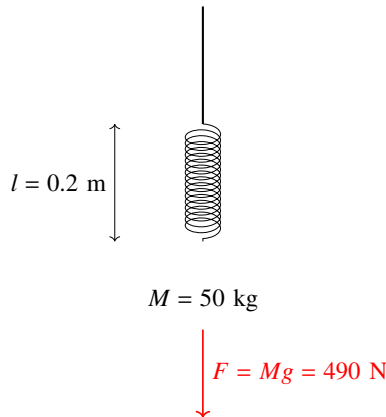


Fig. 0. spring-mass system

The spring constant, k , is calculated as:

$$k = \frac{F}{l} = \frac{490}{0.2} = 2450 \text{ N/m}^{-1} \quad (1)$$

Given the equation of motion for a mass-spring system with mass m and spring constant k is given by:

$$F = ma = -kx \quad (2)$$

This equation can be rearranged as:

$$ma = -kx \quad (3)$$

$$m \frac{d^2x}{dt^2} = -kx \quad (4)$$

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0 \quad (5)$$

$$50 \frac{d^2x}{dt^2} + 2450x = 0 \quad (6)$$

III. DERIVATION OF SIMPLE HARMONIC MOTION PERIOD USING LAPLACE TRANSFORM

$$m \frac{d^2x}{dt^2} + kx = 0 \quad (7)$$

Rewrite the equation as:

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0 \quad (8)$$

Take the Laplace transform of both sides:

$$s^2X(s) - sx(0) - x'(0) + \frac{k}{m}X(s) = 0 \quad (9)$$

Rearrange terms to solve for $X(s)$:

$$X(s)(s^2 + \frac{k}{m}) = sx(0) + x'(0) \quad (10)$$

Solve for $X(s)$:

$$X(s) = \frac{sx(0) + x'(0)}{s^2 + \frac{k}{m}} \quad (11)$$

Find the roots of the characteristic equation:

$$s^2 + \frac{k}{m} = 0 \quad (12)$$

Let $\omega^2 = \frac{k}{m}$, then $s = \pm j\omega$.

Express ω in terms of T :

$$T = \frac{2\pi}{\omega} \quad (13)$$

Solve for ω in terms of T :

$$\omega = \frac{2\pi}{T} \quad (14)$$

Substitute ω back into the characteristic equation:

$$s = \pm j \frac{2\pi}{T} \quad (15)$$

Now, the Laplace transform solution $X(s)$ becomes:

$$X(s) = \frac{s(x_0) + x'(0)}{s^2 + \frac{k}{m}} \quad (16)$$

Express s in terms of ω :

$$X(s) = \frac{s(x_0) + x'(0)}{(j\omega)^2 + \omega^2} \quad (17)$$

Simplify and take the inverse Laplace transform to obtain the displacement $x(t)$ in the time domain:

$$x(t) = A \cos(\omega t + \phi) \quad (18)$$

Where A is the amplitude, ω is the angular frequency, and ϕ is the phase angle. The period T is related to the angular frequency by $T = \frac{2\pi}{\omega}$, giving the desired result:

$$T = 2\pi \sqrt{\frac{m}{k}} \quad (19)$$

The weight of the body is defined as:

$$\text{Weight} = mg = 22.36 \times 9.8 = 219.16 \text{ N} \quad (20)$$

Therefore, the weight of the body is approximately 219 N.