Assignment-1

EE:1205 Signals and Systems

Indian Institute of Technology, Hyderabad

Abhey Garg
EE23BTECH11202

I. Question 11.14.8

A spring balance has a scale that that reads fro 0 to 50 kg. The length of the scale is 20cm. A body is suspended from this balance , when displaced and released, oscillates with a period of 0.6s. What is weight of the body?

II. Solution

TABLE 0
Input Parameters

Parameter	Value	Description
Mass (M)	50 kg	Mass of block
Maximum displacement of the spring (l)	0.2 m	Maximum elongation of spring
Time period (T)	0.6 s	Period of one oscillation
Spring Constant (k)	2450 N/m	Ratio of force applied and elongation

The maximum force exerted on the spring is calculated using:

$$F = Mg \tag{1}$$

where g is the acceleration due to gravity ($9.8\,\mathrm{m/s^2}$):

$$F = 50 \times 9.8 = 490\,\mathrm{N} \tag{2}$$

The spring constant, k, is calculated as:

$$k = \frac{F}{l} = \frac{490}{0.2} = 2450\,\mathrm{N/m^{-1}} \tag{3}$$

Given the equation of motion for a mass-spring system with mass m and spring constant k is given by:

$$F = ma = -k\pi x \tag{4}$$

This equation can be rearranged as:

$$ma = -kx \tag{5}$$

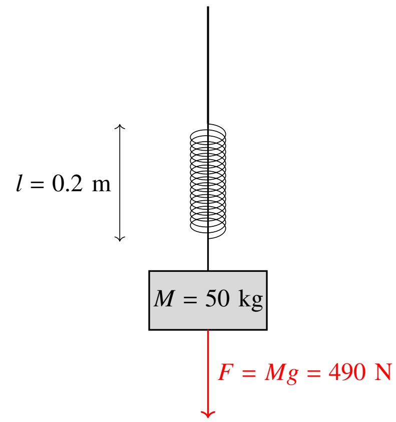

Fig. 0. Illustration of another spring-mass system

$$m\frac{d^2x}{dt^2} = -kx \tag{6}$$

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0 \tag{7}$$

The solution to this second-order differential equation is of the form:

$$x(t) = A\cos(\omega t) + B\sin(\omega t) \tag{8}$$

Where: - A and B are constants determined by initial conditions. - ω represents the angular frequency.

Substituting this solution into the equation, we get:

$$\frac{d^2x}{dt^2} + \omega^2 x = 0 \tag{9}$$

Comparing this with the differential equation for the mass-spring system, we get:

$$\omega^2 = \frac{k}{m} \tag{10}$$

$$\omega = \sqrt{\frac{k}{m}} \tag{11}$$

The natural frequency f_0 of the system is related to the angular frequency as:

$$f_0 = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \tag{12}$$

Therefore, $f_0 = 2\pi\sqrt{\frac{m}{k}}$, which gives the frequency of free oscillations (natural frequency) of the mass-spring system based on its mass m and spring constant k.

The time period, t, for mass m suspended from the balance is given by:

$$t = 2\pi\sqrt{\frac{m}{k}} \tag{13}$$

Hence, solving for m:

$$m = \left(\frac{T}{2\pi}\right)^2 \times k = \left(\frac{0.6}{2 \times 3.14}\right)^2 \times 2450 = 22.36\,\text{kg} \tag{14}$$

The weight of th body is defined as:

$$\text{Weight} = mg = 22.36 \times 9.8 = 219.16\,\text{N} \tag{15}$$

Therefore, the weight of the body is approximately 219 N.

Starting with Newton's second law of motion ($F = ma$) and Hooke's law ($F = -kx$) for the mass attached to a spring:

Newton's second law: F = ma and Hooke's law: F = -kx

Given values:

$$m = 50\,\text{kg}$$
$$k = 2450\,\text{N/m}$$

Equating these forces, where m is the mass, a is the acceleration, k is the spring constant, and x is the displacement from the equilibrium position:

$$ma = -kx \tag{16}$$

Here, a represents the second derivative of displacement with respect to time, denoted as $\frac{d^2x}{dt^2}$.

Therefore, substituting $a = \frac{d^2x}{dt^2}$ into the equation $ma = -kx$, we get the differential equation governing the motion of the mass on the spring:

$$50\frac{d^2x}{dt^2} + 2450x = 0 \tag{17}$$

This equation describes the simple harmonic motion of the mass attached to the spring with the given mass and spring constant values.