Assignment-1

EE:1205 Signals and Systems

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EE23BTECH11202

I. Question 11.14.8

A spring balance has a scale that that reads fro 0 to 50 kg. The length of the scale is 20cm. A body is suspended from this balance , when displaced and released, oscillates with a period of 0.6s. What is weight of the body?

II. Solution

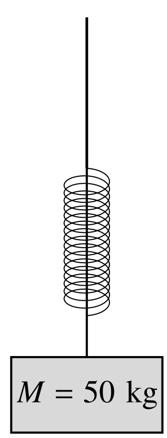
TABLE 0

Input Parameters

Parameter	Value
Mass (M)	50 kg
Maximum displacement of the spring (l)	0.2 m
Time period (T)	0.6 s

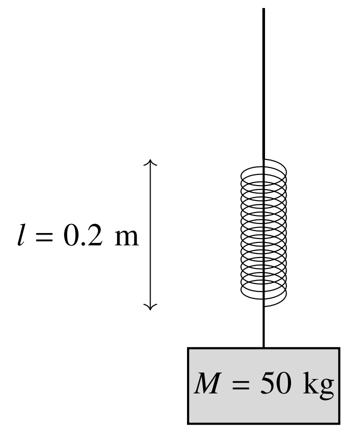
The maximum mass that the scale can read is given by:

$$M = 50\,\text{kg} \tag{1}$$



The maximum displacement of the spring is equal to the length of the scale, given by:

$$l = 20\,\text{cm} = 0.2\,\text{m} \tag{2}$$



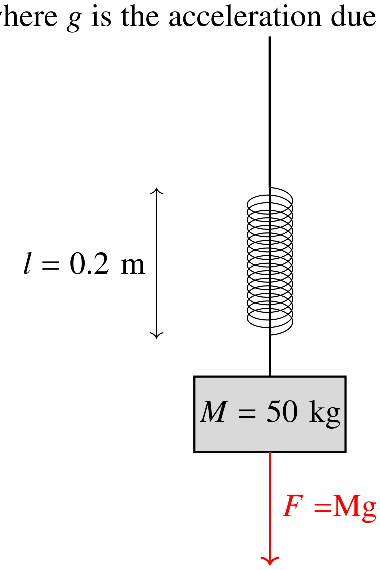
The time period is given as:

$$T = 0.6\,\text{s} \tag{3}$$

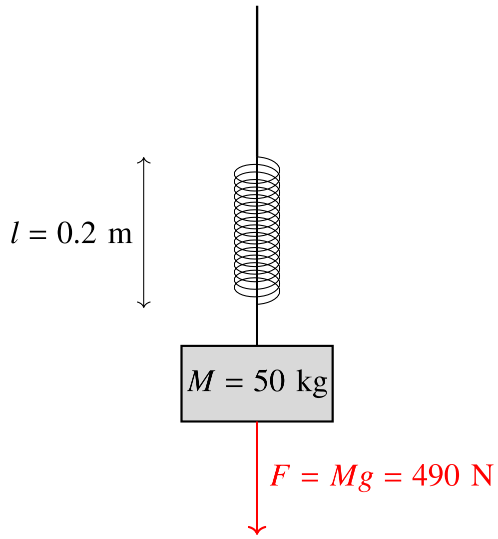
The maximum force exerted on the spring is calculated using:

$$F = Mg \tag{4}$$

where g is the acceleration due to gravity ($9.8\,\text{m/s}^2$):



$$F = 50 \times 9.8 = 490\,\text{N} \tag{5}$$



The spring constant, k, is calculated as:

$$k = \frac{F}{l} = \frac{490}{0.2} = 2450\,\mathrm{N/m^{-1}} \tag{6}$$

The time period, t, for mass m suspended from the balance is given by:

$$t = 2\pi\sqrt{\frac{m}{k}} \tag{7}$$

Hence, solving for m:

$$m = \left(\frac{T}{2\pi}\right)^2 \times k = \left(\frac{0.6}{2 \times 3.14}\right)^2 \times 2450 = 22.36\,\mathrm{kg} \tag{8}$$

The weight of th body is defined as:

$$\text{Weight} = mg = 22.36 \times 9.8 = 219.16\,\mathrm{N} \tag{9}$$

Therefore, the weight of the body is approximately 219 N.

Starting with Newton's second law of motion ($F = ma$) and Hooke's law ($F = -kx$) for the mass attached to a spring:

$$\text{Newton's second law: } F = ma \quad \text{and} \quad \text{Hooke's law: } F = -kx \tag{10}$$

Given values:

$$m = 50\,\mathrm{kg}$$
$$k = 2450\,\mathrm{N/m}$$

Equating these forces, where m is the mass, a is the acceleration, k is the spring constant, and x is the displacement from the equilibrium position:

$$ma = -kx \tag{11}$$

Here, a represents the second derivative of displacement with respect to time, denoted as $\frac{d^2x}{dt^2}$.

Therefore, substituting $a = \frac{d^2x}{dt^2}$ into the equation $ma = -kx$, we get the differential equation governing the motion of the mass on the spring:

$$50\frac{d^2x}{dt^2} + 2450x = 0 \tag{12}$$

This equation describes the simple harmonic motion of the mass attached to the spring with the given mass and spring constant values.