

Gate Question

EE:1205 Signals and Systems
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I. QUESTION GATE PH 56

Consider the complex function

$$f(z) = \frac{z^2 \sin z}{(z - \pi)^4}$$

At $z = \pi$, which of the following options is (are) correct?

- (A) The order of the pole is 4
- (B) The order of the pole is 3
- (C) The residue at the pole is $\frac{\pi}{6}$
- (D) The residue at the pole is $\frac{2\pi}{3}$

(GATE PH 2023)

Which is a finite value, so order of pole is 3.

$$\text{Res}(f, \pi) = \frac{1}{(m-1)!} \frac{d^m}{dz^m} [(z - \pi)^m f(z)] \Big|_{z=\pi} \quad (5)$$

$$\text{Res}(f, \pi) = \frac{1}{2!} \frac{d}{dz} \left[(z - \pi)^3 \frac{z^2 \sin z}{(z - \pi)^4} \right] \Big|_{z=\pi} \quad (6)$$

Simplify the expression:

$$\text{Res}(f, \pi) = \frac{1}{2!} \frac{d}{dz} \left[\frac{z^2 \sin z}{(z - \pi)} \right] \Big|_{z=\pi} \quad (7)$$

Compute the derivative:

$$\text{Res}(f, \pi) = \frac{1}{2!} \frac{d}{dz} [z^2 \sin z] \Big|_{z=\pi} \quad (8)$$

$$\text{Res}(f, \pi) = \frac{1}{2!} (2z \sin z + 2z^2 \cos z) \Big|_{z=\pi} \quad (9)$$

Now, substitute $z = \pi$:

$$\text{Res}(f, \pi) = \frac{1}{2!} (2\pi \sin(\pi) + 2\pi^2 \cos(\pi)) \quad (10)$$

Since $\sin(\pi) = 0$ and $\cos(\pi) = -1$, this simplifies to:

$$\text{Res}(f, \pi) = \frac{1}{2!} (-2\pi^2) = -\pi^2 \quad (11)$$

II. SOLUTION

TABLE 1
INPUT PARAMETERS

Parameter	Used to denote	Values
m	order of pole at $z = \pi$?
$\text{Res}(f, \pi)$	Residue of pole	?

$$\lim_{z \rightarrow \pi} (z - \pi)^3 f(z) \quad (1)$$

$$= \lim_{z \rightarrow \pi} (z - \pi)^3 \left(\frac{z^2 \sin z}{(z - \pi)^4} \right) \quad (2)$$

$$= \lim_{z \rightarrow \pi} \left(\frac{z^2 \sin z}{(z - \pi)} \right) \quad (3)$$

$$= \lim_{z \rightarrow \pi} \left(\frac{2z \sin z - z^2 \cos z}{1} \right) \quad (4)$$