Gate 2021 Assignment

EE:1205 Signals and Systems

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I. Question EC 13

Two continuous random variables X and Y are related as $Y = 2X + 3$. Let σ_x^2 and σ_y^2 denote the variances of X and Y , respectively. The variances are related as :
(A) $\sigma_y^2 = 2\sigma_x^2$
(B) $\sigma_y^2 = 4\sigma_x^2$
(C) $\sigma_y^2 = 5\sigma_x^2$
(D) $\sigma_y^2 = 25\sigma_x^2$

II. Solution

$$Y = 2X + 3 \tag{1}$$

Take the laplace transform:

$$\mathcal{L}\{Y\} = \mathcal{L}\{2X + 3\} = 2\mathcal{L}\{X\} + 3\mathcal{L}\{1\} \tag{2}$$

$$\mathcal{L}\{Y\} = 2\mathcal{L}\{X\} + 3\frac{1}{s} \tag{3}$$

Now, the variance of a random variable A is given by:

$$\text{Var}(A) = \mathcal{L}\{E[A^2]\} - [\mathcal{L}\{E[A]\}]^2 \tag{4}$$

Let $A = X$ and $B = Y$. We want to find $\text{Var}(Y)$.

$$\text{Var}(Y) = \mathcal{L}\{E[Y^2]\} - [\mathcal{L}\{E[Y]\}]^2 \tag{5}$$

Since $Y = 2X + 3$, we can express $E[Y]$ and $E[Y^2]$ in terms of X.

$$E[Y] = 2E[X] + 3 \tag{6}$$

$$E[Y^2] = 4E[X^2] + 12E[X] + 9 \tag{7}$$

Substitute these into the variance formula:

$$\text{Var}(Y) = \mathcal{L}\{4E[X^2] + 12E[X] + 9\} - [\mathcal{L}\{2E[X] + 3\}]^2 \tag{8}$$

Let $\mathcal{L}\{E[X]\} = \mu_x$ (the mean of X) and $\mathcal{L}\{E[X^2]\} = \mu_{x^2}$ (the mean of X^2).

$$\text{Var}(Y) = 4\mu_{x^2} + 12\mu_x + 9 - [2\mu_x + 3]^2 \tag{9}$$

$$\text{Var}(Y) = 4\mu_{x^2} - 4\mu_x^2 \tag{10}$$

Now, remember that $\text{Var}(X) = \mu_{x^2} - \mu_x^2$, so we can substitute this in:

$$\text{Var}(Y) = 4(\text{Var}(X)) \tag{11}$$

Finally, substitute back :

$$\sigma_y^2 = 4\sigma_x^2 \tag{12}$$