Gate Question

EE:1205 Signals and Systems
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I. Question GATE PH 56

Consider the complex function

$$f(z) = \frac{z^2 \sin z}{(z-\pi)^4}$$

At $z = \pi$, which of the following options is (are) correct?

(A) The order of the pole is 4
(B) The order of the pole is 3
(C) The residue at the pole is $\frac{\pi}{6}$
(D) The residue at the pole is $\frac{2\pi}{3}$

(GATE PH 2023)

II. Solution

TABLE 1
Input Parameters

Parameter	Used to denote	Values
m	order of pole at z = π	?
$Res(f,\pi)$	Residue of pole	?

To calculate m :

$$lim_{z\to\pi}(z-\pi)^3 f(z) \tag{1}$$

$$= \lim_{z\to\pi}(z-\pi)^3 \left(\frac{z^2 \sin z}{(z-\pi)^4}\right) \tag{2}$$

$$= \lim_{z\to\pi}\left(\frac{z^2 \sin z}{(z-\pi)}\right) \tag{3}$$

$$= \lim_{z\to\pi}\left(\frac{2z\sin(z) - z^2\cos(z)}{1}\right) \tag{4}$$

Which is a finite value , so order of pole is 3.

$$\mathrm{Res}(f,\pi) = \frac{1}{(m-1)!}\frac{d^m}{dz^m}\left[(z-\pi)^m f(z)\right]\Big|_{z=\pi} \tag{5}$$

$$\mathrm{Res}(f,\pi) = \frac{1}{2!}\frac{d}{dz}\left[(z-\pi)^3 \frac{z^2\sin z}{(z-\pi)^4}\right]\Big|_{z=\pi} \tag{6}$$

$$\mathrm{Res}(f,\pi) = \frac{1}{2!}\frac{d}{dz}\left[\frac{z^2\sin z}{(z-\pi)}\right]\Big|_{z=\pi} \tag{7}$$

$$\mathrm{Res}(f,\pi) = \frac{1}{2!}\frac{d}{dz}\left[z^2\sin z\right]\Big|_{z=\pi} \tag{8}$$

$$\mathrm{Res}(f,\pi) = \frac{1}{2!}\left(2z\sin z + 2z^2\cos z\right)\Big|_{z=\pi} \tag{9}$$

Now, substitute $z = \pi$:

$$\mathrm{Res}(f,\pi) = \frac{1}{2!}\left(2\pi\sin(\pi) + 2\pi^2\cos(\pi)\right) \tag{10}$$

Since $\sin(\pi) = 0$ and $\cos(\pi) = -1$, this simplifies to:

$$\mathrm{Res}(f,\pi) = \frac{1}{2!}(-2\pi^2) = -\pi^2 \tag{11}$$