

Gate Question

EE:1205 Signals and Systems
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I. QUESTION GATE PH 56

Consider the complex function

$$f(z) = \frac{z^2 \sin z}{(z - \pi)^4}$$

At $z = \pi$, which of the following options is (are) correct?

- (A) The order of the pole is 4
- (B) The order of the pole is 3
- (C) The residue at the pole is $\frac{\pi}{6}$
- (D) The residue at the pole is $\frac{2\pi}{3}$

(GATE PH 2023)

$$\text{Res}(f, \pi) = \frac{1}{2!} \frac{d^2}{dz^2} \left[(z - \pi)^3 \frac{z^2 \sin z}{(z - \pi)^4} \right] \Bigg|_{z=\pi} \quad (6)$$

$$\text{Res}(f, \pi) = \frac{1}{2!} \frac{d^2}{dz^2} \left[\frac{z^2 \sin z}{(z - \pi)} \right] \Bigg|_{z=\pi} \quad (7)$$

$$= \frac{1}{2!} \left(-\frac{z^2 - 2z(z - \pi)}{(z - \pi)^2} \right) \cos(z) + \left(\frac{z^2 - 2}{z - \pi} - \frac{2z^2}{(z - \pi)^3} - \frac{4z}{(z - \pi)^2} \right) \sin(z) \quad (8)$$

Applying L'Hopital rule Since $\sin(\pi) = 0$ and $\cos(\pi) = -1$, this simplifies to:

$$\text{Res}(f, \pi) = \frac{1}{2!} \frac{\pi^2 - 6}{3} = \frac{\pi^2 - 6}{6} \quad (9)$$

II. SOLUTION

TABLE 1
INPUT PARAMETERS

Parameter	Used to denote	Values
m	order of pole at $z = \pi$?
$\text{Res}(f, \pi)$	Residue of pole	?

To calculate m :

$$\lim_{z \rightarrow \pi} (z - \pi)^3 f(z) \quad (1)$$

$$= \lim_{z \rightarrow \pi} (z - \pi)^3 \left(\frac{z^2 \sin z}{(z - \pi)^4} \right) \quad (2)$$

$$= \lim_{z \rightarrow \pi} \left(\frac{z^2 \sin z}{(z - \pi)} \right) \quad (3)$$

$$= \lim_{z \rightarrow \pi} \left(\frac{2z \sin(z) - z^2 \cos(z)}{1} \right) \quad (4)$$

Which is a finite value , so

$m = 3$

$$\text{Res}(f, \pi) = \frac{1}{(m - 1)!} \frac{d^{m-1}}{dz^{m-1}} [(z - \pi)^m f(z)] \Bigg|_{z=\pi} \quad (5)$$