# Gate 2021 Assignment

EE:1205 Signals and Systems
Indian Institute of Technology, Hyderabad

Abhey Garg
EE23BTECH11202

## I. Question IN 02

Consider the sequence $x_n = 0.5x_{n-1} + 1, n = 1, 2, ...$ with $x_0 = 0$ Then $\lim_{n\to\infty} x_n$ is :

A 0
B 1
C 2
D $\infty$

## II. Solution

$$x_n = 0.5x_{n-1} + 1 \tag{1}$$

Taking Z-transform:

$$X(z) = 0.5z^{-1}X(z) + \frac{1}{1 - z^{-1}} \tag{2}$$

$$X(z)[1 - 0.5z^{-1}] = \frac{1}{1 - z^{-1}} \tag{3}$$

$$X(z) = \frac{1}{(1 - z^{-1})(1 - 0.5z^{-1})} \tag{4}$$

$$X(z) = \frac{A}{1 - z^{-1}} + \frac{B}{1 - 0.5z^{-1}} \tag{5}$$

$$A + B = 1 \tag{6}$$

$$A = -2B \tag{7}$$

$$A = 2 \tag{8}$$

$$B = -1 \tag{9}$$

$$X(z) = \frac{2}{1 - z^{-1}} + \frac{-1}{1 - 0.5z^{-1}} \tag{10}$$

$$\tag{11}$$

Taking inverse Z-transform:

$$x(n) = 2u(n) + (-1)(0.5^n)u(n) \tag{12}$$

$$\lim_{n\to\infty} x(n) = 2 \tag{13}$$