

# Assignment-1

EE:1205 Signals and Systems  
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## I. QUESTION 11.14.8

A spring balance has a scale that reads from 0 to 50 kg. The length of the scale is 20cm. A body is suspended from this balance, when displaced and released, oscillates with a period of 0.6s. What is weight of the body?

## II. SOLUTION

TABLE 0  
INPUT PARAMETERS

Parameter	Value	Description
$M$	50 kg	Mass of block
$l$	0.2 m	Maximum displacement of spring
$T$	0.6 s	Time period of oscillation
$F$	490 N	Force
$k$	2450 N/m	Spring Constant

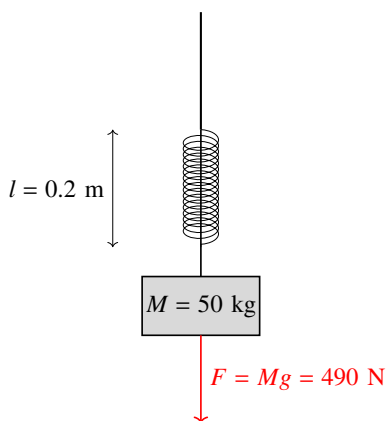


Fig. 0. spring-mass system

$$F = ma = -kx \quad (1)$$

This equation can be rearranged as:

$$ma = -kx \quad (2)$$

$$m \frac{d^2 x}{dt^2} = -kx \quad (3)$$

## III. DERIVATION OF SIMPLE HARMONIC MOTION PERIOD USING LAPLACE TRANSFORM

$$\frac{d^2 x}{dt^2} + \frac{k}{m}x = 0 \quad (4)$$

Take the Laplace transform of both sides:

$$s^2 X(s) - sx(0) - x'(0) + \frac{k}{m}X(s) = 0 \quad (5)$$

Rearrange terms to solve for  $X(s)$ :

$$X(s)(s^2 + \frac{k}{m}) = sx(0) + x'(0) \quad (6)$$

Solve for  $X(s)$ :

$$X(s) = \frac{sx(0) + x'(0)}{s^2 + \frac{k}{m}} \quad (7)$$

Find the roots of the characteristic equation:

$$s^2 + \frac{k}{m} = 0 \quad (8)$$

Let  $\omega^2 = \frac{k}{m}$ , then  $s = \pm j\omega$ .

Express  $\omega$  in terms of  $T$ :

$$T = \frac{2\pi}{\omega} \quad (9)$$

Solve for  $\omega$  in terms of  $T$ :

$$\omega = \frac{2\pi}{T} \quad (10)$$

Substitute  $\omega$  back into the characteristic equation:

$$s = \pm j \frac{2\pi}{T} \quad (11)$$

Now, the Laplace transform solution  $X(s)$  becomes:

Express  $s$  in terms of  $\omega$ :

$$X(s) = \frac{s(x_0) + x'(0)}{(j\omega)^2 + \omega^2} \quad (12)$$

Simplify and take the inverse Laplace transform to obtain the displacement  $x(t)$  in the time domain:

$$x(t) = A \cos(\omega t + \phi) \quad (13)$$

$$A = l = 0.2m \quad (14)$$

$$\phi = 0 \quad (15)$$

$$\omega = \frac{2\pi}{T} = 10.47\text{rad/s} \quad (16)$$

Where  $A$  is the amplitude,  $\omega$  is the angular frequency, and  $\phi$  is the phase angle. The period  $T$  is related to the angular frequency by  $T = \frac{2\pi}{\omega}$ , giving the desired result:

$$T = 2\pi \sqrt{\frac{m}{k}} \quad (17)$$

The weight of the body is defined as:

$$\text{Weight} = mg = 22.36 \times 9.8 = 219.16 \text{ N} \quad (18)$$

Therefore, the weight of the body is approximately 219 N.