Discrete Assignment

EE:1205 Signals and Systems

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I. Question 11.9.3.24

Show that the ratio of the sum of the first n terms of a geometric progression (G.P.) to the sum of terms from $(n+1)$th to $(2n)$th term is $\frac{1}{r^n}$.

II. Solution

TABLE 0
Input Parameters

Variable	Description	Value
$x(0)$	First term of G.P	
r	Common ratio of G.P	
$x(n)$	General Term	$x(0) \cdot r^n \cdot u(n)$

$$x(n) = x(0)r^n u(n) \tag{1}$$

where

$$u(n) = \begin{cases} 0 & \text{for } n < 0 \\ 1 & \text{for } n \geq 0 \end{cases}$$

$$s(n) = \sum_{k=-\infty}^{n} x(k) \tag{2}$$

$$s(n) = x(n) * u(n) \tag{3}$$

Taking z transform

$$S(z) = X(z)U(z) \tag{4}$$

$$= \left(\frac{x(0)}{1 - rz^{-1}}\right)\left(\frac{1}{1 - z^{-1}}\right) \quad |z| > |r| \cap |z| > 1 \tag{5}$$

$$= \frac{x(0)}{(1 - rz^{-1})(1 - z^{-1})} \quad |z| > |r| \tag{6}$$

which can be expressed as

$$S(z) = \frac{x(0)}{r-1}\left(\frac{r}{1 - rz^{-1}} - \frac{1}{1 - z^{-1}}\right) \tag{7}$$

Using partial fractions, again the inverse of the above can be expressed as

$$s(n) = x(0)\left(\frac{r^n - 1}{r - 1}\right)u(n) \tag{8}$$

$$S(z) = \frac{x(0)}{r-1}\left(\frac{r}{1 - rz^{-1}} - \frac{1}{1 - z^{-1}}\right) \tag{9}$$

$$S(z) = \frac{r-1}{r}x(0)\left(1 - \frac{rz^{-1}}{r} - \frac{1}{1 - z^{-1}}\right) \tag{10}$$

To find S(2z) using the time-shifting property, set k=1 in

$$X_k(z) = z^{-k}X(z) \tag{11}$$

$$S(2z) = z^{-1}S(z) \tag{12}$$

Now, substitute z with 2z in the expression for S(z):

$$S(2z) = \frac{r-1}{r}x(0)\left(1 - \frac{2z}{r} - \frac{1}{1 - 2z^{-1}}\right) \tag{13}$$

$$S(2z) = \frac{r-1}{r}x(0)\left(1 - \frac{2z}{r} - \frac{1}{2z^{-1} - 1}\right) \tag{14}$$

$$S(2z) = \frac{r-1}{r}x(0)\left(2z - r - \frac{2z - 1}{2z - r}\right) \tag{15}$$

Now, to find S(2n) , take the inverse z transform. The expression is:

$$s(2n) = x(0)\left(\frac{r^{2n-1} - 1}{r - 1}\right)u(2n) \tag{16}$$

Now we have to find $\frac{s(n)}{s(2n) - s(n)}$

$$\frac{s(n)}{s(2n) - s(n)} = \frac{x(0)\left(\frac{r^n - 1}{r-1}\right)u(n)}{x(0)\left(\frac{r^{2n}-1}{r-1}\right)u(2n) - x(0)\left(\frac{r^n - 1}{r-1}\right)u(n)} \tag{17}$$

$$= \frac{\left(\frac{r^n-1}{r-1}\right)}{\left(\frac{r^{2n}-1}{r-1}\right) - \left(\frac{r^n-1}{r-1}\right)} \tag{18}$$

$$= \frac{r^n - 1}{(r^{2n} - 1) - (r^n - 1)} \tag{19}$$

$$= \frac{r^n}{(r^{2n}) - (r^n)} \tag{20}$$

$$= \frac{1}{r^n} \tag{21}$$

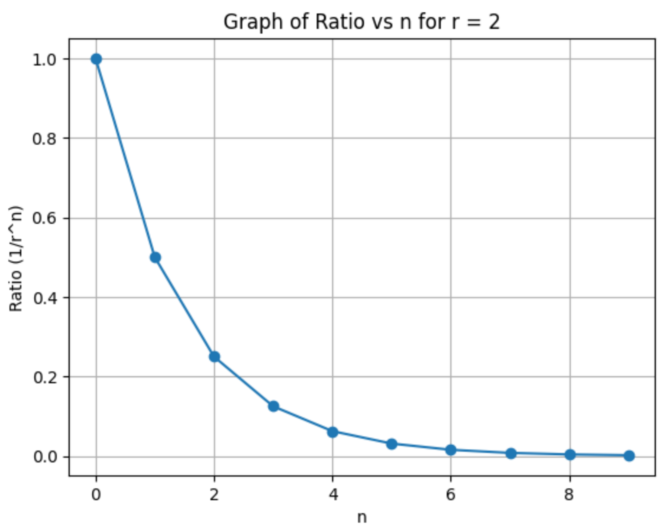

Fig. 0. Plot of ratio vs $1/r^n$ for r = 2