Gate 2021 Assignment

EE:1205 Signals and Systems
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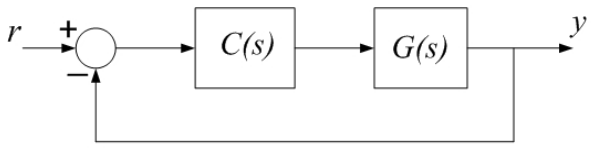
I. Question IN 02

Consider a unity feedback configuration with a plant and a PID controller as shown in the figure. $G(s) = \frac{1}{(s+1)(s+3)}$ and $C(s) = \frac{K(s+3+j)(s+3-j)}{s}$ with K being scalar . The closed loop is :



A only stable for $K < 0$
B stable for all value of K
C only stable for $K > 0$
D only stable for K between –1 and +1

II. Solution

$$[X(s) - Y(s)]G(s)C(s) = Y(s) \tag{1}$$

$$Y(s) = \frac{X(s)G(s)C(s)}{1 + G(s)C(s)} \tag{2}$$

Characteristic equation is:

$$1 + G(s)C(s) = 0 \tag{3}$$

$$s(s+1)(s+3) + K((s+3)^2 - j^2) = 0 \tag{4}$$

$$s^3 + s^2(K+4) + s(3+6K) + 10 = 0 \tag{5}$$

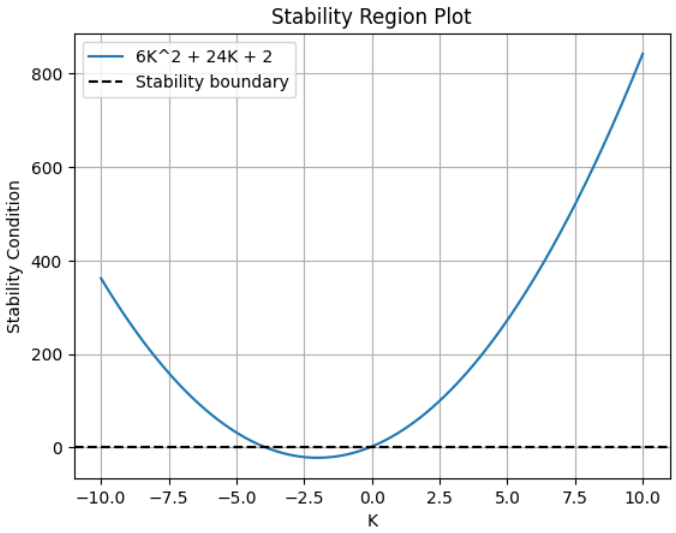
For stability we need :

$$(K+4)(3+6K) > 10 \tag{6}$$

$$[\because as^3 + bs^2 + cs + d = 0 \text{ for stability } bc > ad] \tag{7}$$

$$3K + 6K^2 + 12 + 24K > 10 \tag{8}$$

$$6K^2 + 24K + 2 > 0 \tag{9}$$



If k =1 then above equation is valid, hence option A is wrong.
If k = −1 then above equation is invalid, hence option B is wrong.
If k = 2 then also above equation is valid, hence option D is wrong.
If $k > 0$ then always above equation is valid, hence option C is correct.