Gate Question
EE:1205 Signals and Systems
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I. QUESTION GATE PH 56

Consider the complex function

$$f(z) = \frac{z^2 \sin z}{(z - \pi)^4}$$

At $z = \pi$, which of the following options is (are) correct?
- **(A)** The order of the pole is 4
- **(B)** The order of the pole is 3
- **(C)** The residue at the pole is $\frac{\pi}{6}$
- **(D)** The residue at the pole is $\frac{2\pi}{3}$

(GATE PH 2023)

II. SOLUTION

TABLE 1
INPUT PARAMETERS

Parameter	Used to denote	Values
m	order of pole at $z = \pi$	?
$Res(f,\pi)$	Residue of pole	?

(a) As the power of $(z - \pi)$ in denominator is 4 , so the order of the pole is 4.

(b)

$$\text{Res}(f, \pi) = \frac{1}{(m - 1)!} \frac{d^{m-1}}{dz^{m-1}} \left[(z - \pi)^m f(z) \right] \Bigg|_{z=\pi} \tag{1}$$

$$\text{Res}(f, \pi) = \frac{1}{3!} \frac{d^3}{dz^3} \left[(z - \pi)^4 \frac{z^2 \sin z}{(z - \pi)^4} \right] \Bigg|_{z=\pi} \tag{2}$$

$$\text{Res}(f, \pi) = \frac{1}{3!} \frac{d^3}{dz^3} z^2 \sin z \Bigg|_{z=\pi} \tag{3}$$

$$= \frac{1}{3!} (6 \cos z - 6z \sin z - z^2 \cos z) \Bigg|_{z=\pi} \tag{4}$$

$$\tag{5}$$

Since $\sin(\pi) = 0$ and $\cos(\pi) = -1$, this simplifies to:

$$\text{Res}(f, \pi) = \frac{\pi^2 - 6}{3!} = \frac{\pi^2 - 6}{6} \tag{6}$$