

Discrete Assignment

EE:1205 Signals and Systems
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I. QUESTION 11.9.3.24

Show that the ratio of the sum of the first n terms of a geometric progression (G.P.) to the sum of terms from $(n+1)$ th to $(2n)$ th term is $\frac{1}{r^n}$.

II. SOLUTION

TABLE 0
INPUT PARAMETERS

Variable	Description	Value
$x(0)$	First term of G.P	
r	Common ratio of G.P	
$x(n)$	General Term	$x(0) \cdot r^n \cdot u(n)$

$$x(n) = x(0)r^n u(n) \quad (1)$$

where

$$u(n) = \begin{cases} 0 & \text{for } n < 0 \\ 1 & \text{for } n \geq 0 \end{cases}$$

$$y(n) = \sum_{k=-\infty}^n x(k) \quad (2)$$

$$y(n) = x(n) * u(n) \quad (3)$$

Taking z transform

$$Y(z) = X(z)U(z) \quad (4)$$

$$= \left(\frac{x(0)}{1 - rz^{-1}} \right) \left(\frac{1}{1 - z^{-1}} \right) \quad |z| > |r| \cap |z| > 1 \quad (5)$$

$$= \frac{x(0)}{(1 - rz^{-1})(1 - z^{-1})} \quad |z| > |r| \quad (6)$$

which can be expressed as

$$Y(z) = \frac{x(0)}{r-1} \left(\frac{r}{1 - rz^{-1}} - \frac{1}{1 - z^{-1}} \right) \quad (7)$$

Using partial fractions, again the inverse of the above can be expressed as

$$y(n) = x(0) \left(\frac{r^n - 1}{r - 1} \right) u(n) \quad (8)$$

$$y(2n) = x(0) \left(\frac{r^{2n-1} - 1}{r - 1} \right) u(2n) \quad (9)$$

Now we have to find $\frac{y(n)}{y(2n)-y(n)}$

$$\frac{y(n)}{y(2n) - y(n)} = \frac{x(0) \left(\frac{r^n - 1}{r - 1} \right) u(n)}{x(0) \left(\frac{r^{2n-1} - 1}{r - 1} \right) u(2n) - x(0) \left(\frac{r^n - 1}{r - 1} \right) u(n)} \quad (10)$$

$$= \frac{\left(\frac{r^n - 1}{r - 1} \right)}{\left(\frac{r^{2n-1} - 1}{r - 1} \right) - \left(\frac{r^n - 1}{r - 1} \right)} \quad (11)$$

$$= \frac{r^n - 1}{(r^{2n} - 1) - (r^n - 1)} \quad (12)$$

$$= \frac{r^n}{(r^{2n}) - (r^n)} \quad (13)$$

$$= \frac{1}{r^n} \quad (14)$$

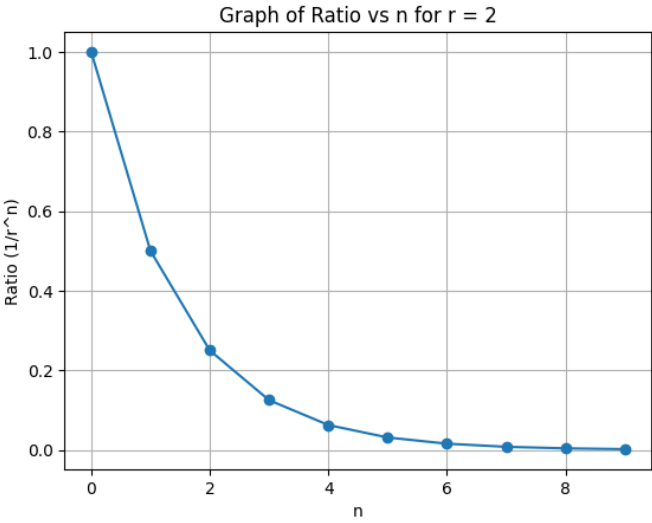


Fig. 0. Plot of ratio vs $1/r^n$ for $r = 2$