Discrete Assignment

EE:1205 Signals and Systems

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I. Question GATE AG 14

$y = e^{mx} + e^{-mx}$ is the solution of which differential equation?

II. Solution

$$y = e^{mx} + e^{-mx} \tag{1}$$

The Laplace transform of $e^{mx(t)}$ is given by:

$$\mathcal{L}\{e^{mx(t)}\} = \frac{1}{s-m} \tag{2}$$

Similarly, the Laplace transform of $e^{-mx(t)}$ is:

$$\mathcal{L}\{e^{-mx(t)}\} = \frac{1}{s+m} \tag{3}$$

Now, applying the linearity property, the Laplace transform of $y(t)$ is the sum of the Laplace transforms of the individual terms:

$$\mathcal{L}\{y(t)\} = \mathcal{L}\{e^{mx(t)}\} + \mathcal{L}\{e^{-mx(t)}\} \tag{4}$$

$$= \frac{1}{s-m} + \frac{1}{s+m} \tag{5}$$

$$= \frac{2s}{s^2 - m^2} \quad -m < Re(s) < m \tag{6}$$

Taking the inverse laplace transform :

$$\frac{d^2y}{dx^2} - m^2 y(t) = \mathcal{L}^{-1}\left(\frac{2s}{s^2 - m^2}\right) \tag{7}$$

To find the inverse Laplace transform of $\frac{2s}{s^2-m^2}$, let's express it in partial fraction form:

$$\frac{2s}{s^2 - m^2} = \frac{A}{s-m} + \frac{B}{s+m} \tag{8}$$

Multiplying through by the common denominator:

$$2s = A(s+m) + B(s-m) \tag{9}$$

Now, solving for A and B:

$$2s = As + Am + Bs - Bm \tag{10}$$

Equating coefficients:

$$A + B = 2 \tag{11}$$

$$A - B = 0 \tag{12}$$

Solving this system of equations gives $A = B = 1$. Now, we can express the original fraction in partial fraction form:

$$\frac{2s}{s^2 - m^2} = \frac{1}{s-m} + \frac{1}{s+m} \tag{13}$$

Now, the inverse Laplace transform of each term:

$$\mathcal{L}^{-1}\left\{\frac{2s}{s^2 - m^2}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s-m}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{s+m}\right\} \tag{14}$$

This yields the following differential equation:

$$\frac{d^2y}{dx^2} - m^2 y(t) = \delta(t-m) + \delta(t+m) \tag{15}$$

where $\delta(t-m)$ and $\delta(t+m)$ are Dirac delta functions.

$$\frac{d^2y}{dx^2} - m^2 y = 0 \tag{16}$$