

Gate Question

EE:1205 Signals and Systems
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I. QUESTION GATE PH 56

Consider the complex function

$$f(z) = \frac{z^2 \sin z}{(z - \pi)^4}$$

At $z = \pi$, which of the following options is (are) correct?

- (A) The order of the pole is 4
- (B) The order of the pole is 3
- (C) The residue at the pole is $\frac{\pi}{6}$
- (D) The residue at the pole is $\frac{2\pi}{3}$

(GATE PH 2023)

II. SOLUTION

TABLE 1
INPUT PARAMETERS

Parameter	Used to denote	Values
m	order of pole at $z = \pi$?
$Res(f, \pi)$	Residue of pole	?

(a) $m =$ minimum k such that $\lim_{z \rightarrow \pi} (z - \pi)^k f(z)$ is finite

$$= \lim_{z \rightarrow \pi} (z - \pi)^3 \left(\frac{z^2 \sin z}{(z - \pi)^4} \right) \quad (1)$$

$$= \lim_{z \rightarrow \pi} \left(\frac{z^2 \sin z}{(z - \pi)} \right) \quad (2)$$

$$= \lim_{z \rightarrow \pi} \left(\frac{2z \sin(z) + z^2 \cos(z)}{1} \right) \quad (3)$$

Which is a finite value, so $m = 3$

(b) Residue of function at $z = \pi$ is the coefficient of $\frac{1}{z - \pi}$

$$\text{let } z - \pi = t \quad (4)$$

$$z = t + \pi \quad (5)$$

$$f(t + \pi) = \frac{(t + \pi)^2 \sin(t + \pi)}{t^4} \quad (6)$$

Residue is the coefficient of $\frac{1}{t}$

$$= \frac{(t^2 + 2t\pi + \pi^2)(-1)(t - \frac{t^3}{6} + \dots)}{t^4} \quad (7)$$

$$= \frac{-t^3 + \frac{\pi^2 t^3}{6} + \text{terms with coefficient } t^n \text{ where } n \neq 3}{t^4} \quad (8)$$

so coefficient of $1/t$ is $\frac{\pi^2}{6} - 1$