Q: The Fourier cosine series of a function is given by: $f(x) = \sum_{n=0}^{\infty} f_n \cos nx$. For $f(x) = \cos^4 x$, the numerical value of $(f_4 + f_5)$ is

Solution:

Parameter	Value	Description
$f(x)$	-	Function
f_{n}	-	Coefficient of $\cos nx$ in Fourier series
C_n	-	Coefficient of $e^{\frac{-jn2\pi t}{T}}$ in Fourier series

TABLE I
INPUT PARAMETERS TABLE

$$C_n = \frac{1}{T}\int_0^T \cos^4(x) e^{\frac{-jn2\pi t}{T}}\, dx \tag{1}$$

$$C_n = \frac{2}{\pi}\int_0^{\frac{\pi}{2}} \cos^4(x)\cos(nx)\, dx + 0 \tag{2}$$

$$C_4 = \frac{2}{\pi}\int_0^{\frac{\pi}{2}} (\cos x)^4 \cos(4x)\, dx \tag{3}$$

$$= \frac{2}{\pi}\left(\frac{3}{8}\int_0^{\frac{\pi}{2}} \cos(4x)\, dx + \frac{1}{2}\int_0^{\frac{\pi}{2}} \cos(2x)\cos(4x)\, dx + \frac{1}{8}\int_0^{\frac{\pi}{2}} \cos(4x)^2\, dx\right) \tag{4}$$

$$= \frac{2}{\pi}\left(\frac{3}{8}\frac{1}{4}\sin(4x)\Big|_0^{\frac{\pi}{2}} + \frac{1}{2}\left[\frac{1}{6}\sin(6x) + \frac{1}{2}\sin(2x)\right]\Bigg|_0^{\frac{\pi}{2}} + \frac{1}{8}\frac{1}{2}\left[x + \frac{1}{8}\sin(8x)\right]\Bigg|_0^{\frac{\pi}{2}}\right) \tag{5}$$

$$= \frac{1}{16} \tag{6}$$

$$C_5 = \frac{2}{\pi}\int_0^{\frac{\pi}{2}} (\cos x)^4 \cos(5x)\, dx \tag{7}$$

$$= \frac{2}{\pi}\left(\frac{3}{8}\int_0^{\frac{\pi}{2}} \cos(5x)\, dx + \frac{1}{2}\int_0^{\frac{\pi}{2}} \cos(2x)\cos(5x)\, dx + \frac{1}{8}\int_0^{\frac{\pi}{2}} \cos(4x)\cos(5x)\, dx\right) \tag{8}$$

$$= \frac{2}{\pi}\left(\frac{3}{8}\frac{1}{5}\sin(5x)\Big|_0^{\frac{\pi}{2}} + \frac{1}{2}\left[\frac{1}{7}\sin(7x) + \frac{1}{3}\sin(3x)\right]\Bigg|_0^{\frac{\pi}{2}} + \frac{1}{2}\left[\frac{1}{9}\sin(9x) + \sin(x)\right]\Bigg|_0^{\frac{\pi}{2}}\right) \tag{9}$$

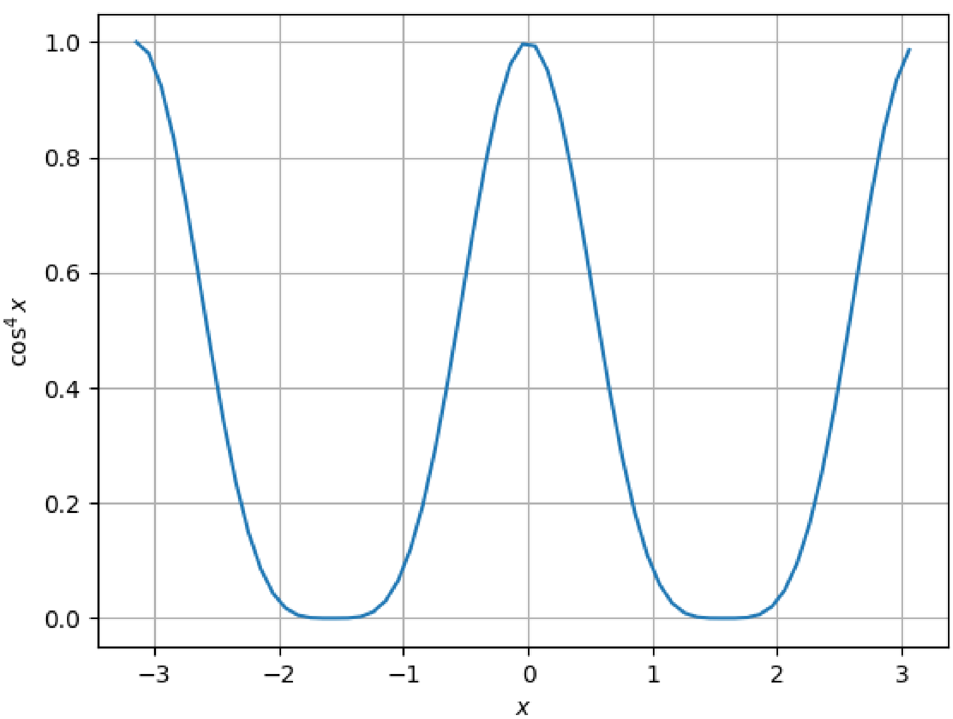
$$= 0 \tag{10}$$

Since the function is even,

$$\cos(\omega t) = \frac{1}{2}(e^{j\omega t} + e^{-j\omega t}) \tag{11}$$

$$f_n = 2C_n \tag{12}$$

$$\therefore f_4 + f_5 = 0.125 \tag{13}$$



1.0
0.8
0.6
0.4
0.2
0.0
$\cos^4 x$
−3
−2
−1
0
1
2
3
x