

Gate 2022- Instrumentation Engineering

EE23BTECH11058 - Sindam Ananya*

Question 11: The input $x(t)$ to a system is related to its output $y(t)$ as

$$\frac{dy(t)}{dt} + y(t) = 3x(t-3)u(t-3)$$

Here $u(t)$ represents a unit-step function. The transfer function of this system is

- (A) $\frac{e^{-3s}}{s+3}$
 (B) $\frac{3e^{-3s}}{s+1}$
 (C) $\frac{3e^{-(s/3)}}{s+1}$
 (D) $\frac{e^{-(s/3)}}{s+3}$

(GATE IN 2022)

Solution:

$$\frac{dy(t)}{dt} + y(t) = 3x(t-3)u(t-3) \quad (1)$$

By applying Laplace Transform on both sides

$$x(t) \xleftrightarrow{\mathcal{L}} X(s) \quad (2)$$

$$x(t-t_0) \xleftrightarrow{\mathcal{L}} X(s)e^{-st_0} \quad (3)$$

$$sY(s) + Y(s) = 3X(s)e^{-3s} \quad (4)$$

$$Y(s)(s+1) = 3X(s)e^{-3s} \quad (5)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{3e^{-3s}}{s+1} \quad (Re(s) > 0) \quad (6)$$

$$H(j\omega) = \frac{3e^{-3j\omega}}{1+j\omega} \quad (7)$$

$$= \frac{3(\cos 3\omega - j\sin 3\omega)}{1+j\omega} \quad (8)$$

$$|H(j\omega)| = \frac{3}{\sqrt{1+\omega^2}} \quad (9)$$

$$phase = \tan^{-1} \left(\frac{\omega \cos(3\omega) + \sin(3\omega)}{\omega \sin(3\omega) - \cos(3\omega)} \right) \quad (10)$$

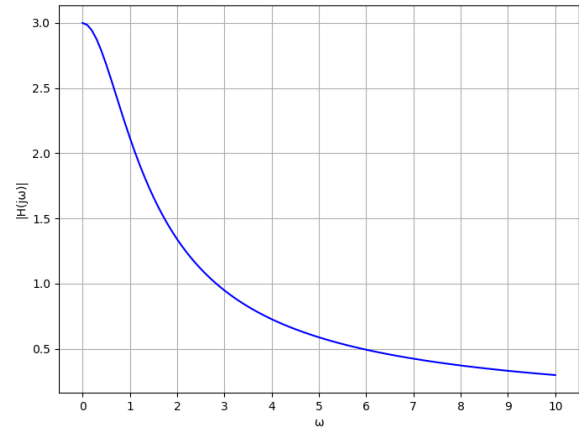


Fig. 0. Plot for magnitude of transfer function

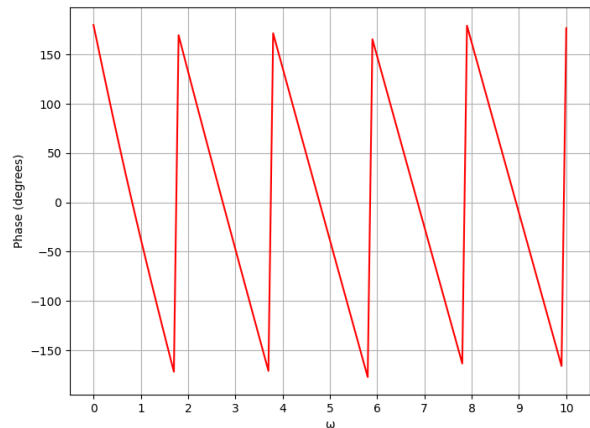


Fig. 0. Plot for phase of transfer function