GATE-EC-Q46

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Question: The outputs of four systems (S_1, S_2, S_3, S_4) corresponding to the input signal $\sin(t)$, for all time t , are shown in the figure. Based on the given information, which of the four systems is/are definately NOT LTI(linear and time-invariant)?

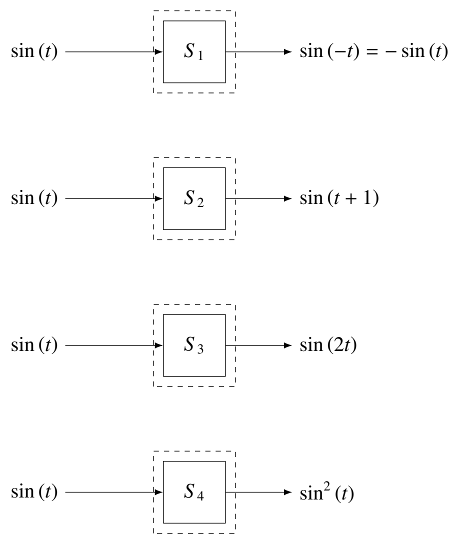

Fig. 1. Block Diagram of Systems

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Solution:

Parameter	Description
(S_1, S_2, S_3, S_4)	Systems Given
$\sin(t)$	Input
$H(\omega)$	Transfer Function
$X(\omega)$	Fourier-Transform of input
$Y(\omega)$	Fourier-Transform of output
$\Phi(\omega)$	Phase of Transfer Function

TABLE I
PARAMETER TABLE

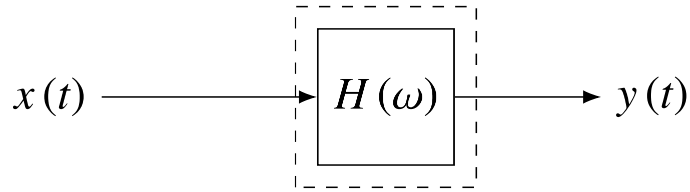

Fig. 2. Block Diagram of LTI System

For an LTI system :

$$y(t) = h(t) * x(t) \tag{1}$$

$$Y(\omega) = H(\omega) X(\omega) \tag{2}$$

$H(\omega)$ is a complex exponential :

$$H(j\omega) = |H(j\omega)| e^{j\Phi(\omega)} \tag{3}$$

$x(t) = \sin(t)$, and $w_o = 1 rad/sec$

$$X(\omega) = j\pi\left(\delta(\omega+\omega_0) - \delta(\omega-\omega_0)\right) \tag{4}$$

Now,

$$Y(\omega) = \left(\delta(\omega+\omega_0) - \delta(\omega-\omega_0)\right)\pi\, |H(\omega)|\, e^{j\Phi(\omega)} \tag{5}$$

$$x(t)\,\delta(t-t_o) = x(t_0)\,\delta(t-t_o) \tag{6}$$

Using property (6) in (5) :

$$\begin{aligned} Y(\omega) = & j\pi\, |H(-\omega_0)|\, e^{j\Phi(-\omega_0)}\delta(\omega+\omega_0) \\ & - j\pi\, |H(\omega_0)|\, e^{j\Phi(j\omega_0)}\delta(\omega-\omega_0) \end{aligned} \tag{7}$$

By definition of the Fourier transform,

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt \tag{8}$$

$$X^*(\omega) = \int_{-\infty}^{\infty} x^*(t) e^{j\omega t}\, dt \tag{9}$$

$$X^*(-\omega) = \int_{-\infty}^{\infty} x^*(t) e^{-j\omega t}\, dt \tag{10}$$

For real-time domain signal :

$$x(t) = x^*(t) \tag{11}$$

Therefore , from (10):

$$X(\omega) = X^{*}(-\omega) \tag{12}$$

By (12) , Given $h(t)$ a real-time domain signal, $H(\omega)$ is conjugate symmetric.

$$|H(\omega)| = |H(-\omega)| \tag{13}$$

$$\Phi(-\omega) = -\Phi(\omega) \tag{14}$$

Therefore using (13) and (14) in (7),

$$Y(\omega) = j\pi |H(\omega_0)| \left(e^{-j\Phi(\omega_0)}\delta(\omega + \omega_0) - e^{j\Phi(\omega_0)}\delta(\omega - \omega_0)\right) \tag{15}$$

Taking Inverse Fourier Transform,

$$\delta(\omega - \omega_0) \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{2\pi} e^{j\omega_0 t} \tag{16}$$

$$\delta(\omega + \omega_0) \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{2\pi} e^{-j\omega_0 t} \tag{17}$$

$$\implies y(t) = j\pi |H(\omega_0)| \frac{1}{2} \left(e^{-j(\omega_0 t + \Phi(\omega_0))} - e^{j(\omega_0 t + \Phi(\omega_0))}\right) \tag{18}$$

$$\implies y(t) = |H(\omega_0)| \sin(\omega_0 t + \Phi(\omega_0)) \tag{19}$$

$w_0 = 1$ rad/sec :

$$y(t) = |H(1)| \sin(t + \Phi(1)) \tag{20}$$

From (20) we can see output cant have scaled frequency nor a squared output. But can have a shifted output or amplitude-scaled output.

So, S_3 and S_4 cannot be LTI system.