GATE 2022 EC

EE:1205 Signals and System
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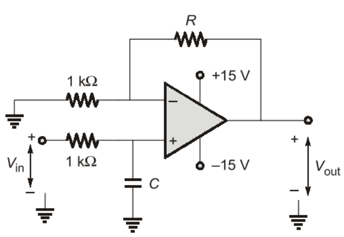
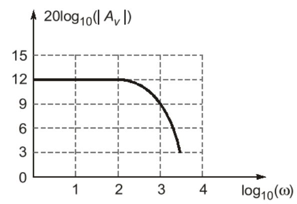
Question 42: A circuit with an ideal OPAMP is shown. The Bode plot for the magnitude (in dB) of the gain transfer function $(A(j\omega)) = \frac{V_{out}(j\omega)}{V_{in}(j\omega)}$ of the circuit is also provided (here, ω is the angular frequency in rad/s). The values of R and C are





(A) $R = 3k\Omega$, $C = 1\mu F$

(B) $R = 1k\Omega$, $C = 3\mu F$

(C) $R = 4k\Omega$, $C = 1\mu F$

(D) $R = 3k\Omega$, $C = 2\mu F$

Solution

Parameter	Description	Value
R	Resistance	?
C	Capacitance	?
R_1	Resistance	1000
ω_{dB}	Cut-off frequency	1000
A_V	Gain Transfer	$\frac{V_{out}}{V_{in}}$

TABLE 1: Given Parameters

On applying KVL,

$$sR_1 i_1(s) + \frac{i_1(s)}{C} = V_{IN}(s)\, s \tag{1}$$

$$\frac{i_1(s)}{C} - sRi_2(s) = sV_0(s) \tag{2}$$

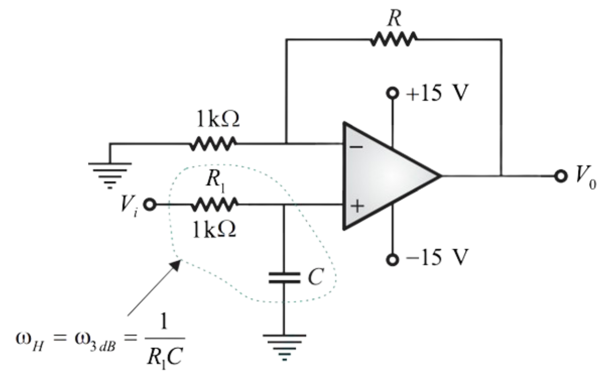


Fig. 1: Active Low Pass Filter

From (1) and (2),

$$\frac{sV_{IN}(s)}{sR_1C + 1} - sRi_2(s) = sV_0(s) \tag{3}$$

$$-\frac{i_1(s)}{C} = sR_1 i_2(s) \tag{4}$$

From (1), (2) and (4) ,

$$V_{OUT}(s) = \frac{1 + 10^{-3}R}{1 + sC10^3} V_{IN}(s) \tag{5}$$

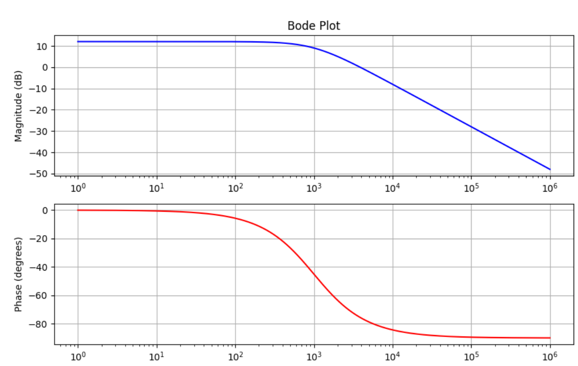


Fig. 2: bode plot

The 3-dB frequency from bode magnitude plot,

$$\implies \omega_{3dB} = 1000 \; rad/sec \tag{6}$$

$$\omega_{3dB} = \frac{1}{R_1 C} \tag{7}$$

$$\implies C = 1\mu F \tag{8}$$

$$\implies A(s) = \frac{V_{OUT}(s)}{V_{IN}(s)} \tag{9}$$

$$= \frac{1 + 10^{-3}R}{1 + sC10^3} \tag{10}$$

$$|A(s)| = \frac{1 + 10^{-3}R}{\sqrt{1 + \omega^2 10^{-6}}} \tag{11}$$

$$\tag{12}$$

A_V at low frequency,

$$|A_V| = 1 + 10^{-3}R \tag{13}$$

$$1 + 10^{-3}R = 10^{\frac{3}{5}} \tag{14}$$

$$R = 3k\Omega \tag{15}$$

Hence, The correct option is (A).