Gate 2022 EC Q50

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Two linear time-invariant systems with transfer functions

$$G_1(s) = \frac{10}{s^2 + s + 1}$$

and

$$G_2(s) = \frac{10}{s^2 + s\sqrt{10} + 10}$$

have unit step responses $y_1(t)$ and $y_2(t)$, respectively. Which of the following statements is/are true?

1) $y_1(t)$ and $y_2(t)$ have the same percentage peak overshoot.
2) $y_1(t)$ and $y_2(t)$ have the same steady state values.
3) $y_1(t)$ and $y_2(t)$ have the same damped frequency of oscillation.
4) $y_1(t)$ and $y_2(t)$ have the same 2% settling time.

Solution: The general second-order transfer function is given by:

Parameter	Description	value
$X_1(s)$	input	$\frac{1}{s}$
$X_2(s)$	input	$\frac{1}{s}$
$G_1(s)$	transfer function	$\frac{10}{s^2+s+1}$
$G_2(s)$	transfer function	$\frac{10}{s^2+s\sqrt{10}+10}$
$y_1(t)$	unit step response	−
$y_2(t)$	unit step response	−
ω_n	natural frequency	−
ζ	damping ratio	−

TABLE 4
Given Parameters

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{1}$$

After comparing the coefficients of $G_1(s)$ and $G_2(s)$, as $\zeta = \frac{1}{2}$ is less than 1, the system is underdamped.

Tranfer function	ω_n	ζ
$G_1(s)$	1	$\frac{1}{2}$
$G_1(s)$	$\sqrt{10}$	$\frac{1}{2}$

TABLE 4
Given Parameters

$$Y(s) = X(s)\,G(s) \tag{2}$$
$$= \frac{1}{s}\left(\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}\right) \tag{3}$$

Applying inverse laplace transform,

$$y(t) = 1 - \frac{e^{-\zeta\omega_n t}}{1-\zeta^2}\sin(\omega_d t + \phi) \tag{4}$$

where ω_d is the damped frequency of oscillation.

$$\omega_d = \omega_n\sqrt{1-\zeta^2} \tag{5}$$

The percentage peak overshoot (PO):

$$PO = \left(\frac{y_{\max} - y_{\text{ss}}}{y_{\text{ss}}}\right) \times 100\% \tag{6}$$

$y_{\max}$ is obtained by differentiating (4) with respect to time and equating it to zero, substituting the value in (4),

$$y_{\max} = 1 + \frac{1}{\sqrt{1-\zeta^2}} \tag{7}$$

y_{ss} is obtained by final value theorem,

$$y_{\text{ss}} = \lim_{s\to 0} sY(s) \tag{8}$$
$$= \lim_{s\to 0} s\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}\frac{1}{s} \tag{9}$$
$$= 1 \tag{10}$$

Substituting the values of $y_{\max}$ and y_{ss} in (6),

$$PO = \frac{1}{\sqrt{1-\zeta^2}} \times 100\% \tag{11}$$

$y_1(t)$ and $y_2(t)$ have same ζ, they have same percentage peak overshoot.So, option (1) is correct.
The steady state value of $y(t)$ is given by final value theorem:

$$y_{1ss} = \lim_{s\to 0} sY_1(s) \tag{12}$$
$$= \lim_{s\to 0} s\frac{10}{s^2 + s + 1}\frac{1}{s} \tag{13}$$
$$= 10 \tag{14}$$
$$y_{2ss} = \lim_{s\to 0} sY_2(s) \tag{15}$$
$$= \lim_{s\to 0} s\frac{10}{s^2 + s\sqrt{10} + 10}\frac{1}{s} \tag{16}$$
$$= 1 \tag{17}$$

as both the unit step responses have different steady state values, option (2) is incorrect.
From (6), as ω_n is different for $y_1(t)$ and $y_2(t)$, they have different damped frequency of oscillation. Hence option (3) is incorrect.
Settling time T_s:

$$T_s = \frac{4}{\zeta\omega_n} \tag{18}$$

As, ω_n is different for $y_1(t)$ and $y_2(t)$, they have different 2% settling time, Hence option (4) is incorrect.
So, only option (1) is correct.