EC-2022

EE23BTECH11210-Dhyana Teja Machineni*

QUESTION:
Consider the signals $x(n)=2^{n-1}u(-n+2)$ and $y(n)=2^{-n+2}u(n+1)$, where $u(n)$ is the unit step sequence. Let $X\left(e^{j\omega}\right)$ and $Y\left(e^{j\omega}\right)$ be the discrete-time Fourier of $x(n)$ and $y(n)$,respectively. The value of the integral $\frac{1}{2\pi}\int_0^{2\pi} X\left(e^{j\omega}\right)Y\left(e^{-j\omega}\right)d\omega$ (rounded off to one decimal place) is ________________
(GATE EC 2022)

Solution:

$$V = \frac{1}{2\pi}\int_0^{2\pi} X\left(e^{j\omega}\right)Y\left(e^{-j\omega}\right)d\omega \tag{1}$$

$$Z\left(e^{j\omega}\right) = X\left(e^{j\omega}\right)Y\left(e^{-j\omega}\right) \tag{2}$$

$$z(n) \xrightarrow{\mathcal{F}} Z(e^{j\omega}) \tag{3}$$

$$z(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} Z\left(e^{j\omega}\right)e^{j\omega n}d\omega \tag{4}$$

$$z(0) = \frac{1}{2\pi}\int_0^{2\pi} Z\left(e^{j\omega}\right)d\omega \tag{5}$$

$$z(n) = x(n) * y(-n) \tag{6}$$

$$= \sum_{k=-\infty}^{\infty} 2^{k-1}u(-k+2)\,2^{n-k+2}u(-n+k+1) \tag{7}$$

$$= \sum_{k=-\infty}^{2} 2^{n+1}u(k-n+1) \tag{8}$$

$$z(0) = \sum_{k=-\infty}^{2} 2u(k+1) \tag{9}$$

$$= 2\sum_{k=-1}^{2} u(k+1) \tag{10}$$

$$\therefore z(0) = 8 \tag{11}$$

$\therefore \frac{1}{2\pi}\int_0^{2\pi} X\left(e^{j\omega}\right)Y\left(e^{-j\omega}\right)d\omega = 8$

Parameter	Description
$u(n)$	unit step function
$z(n)$	$x(n) * y(-n)$
$Z\left(e^{j\omega}\right)$	$X\left(e^{j\omega}\right)Y\left(e^{-j\omega}\right)$

TABLE I
VARIABLES AND THEIR DESCRIPTIONS

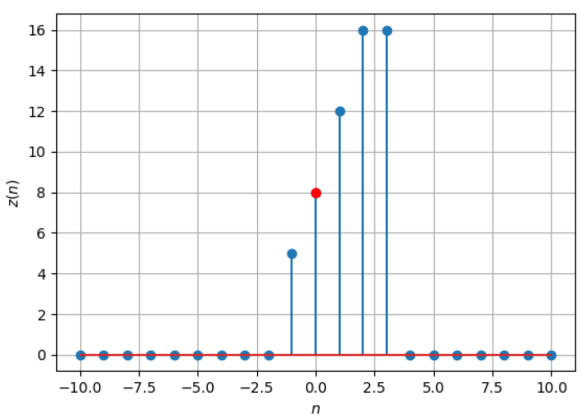

Fig. 0. Stem Plot of $z(n)$