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GATE NM-50 2022

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Q: Let y(x) be the solution of the differential equation

$$y'' - 4y' - 12y = 3e^{5x}$$
 (1)

satisfying $y(0) = \frac{18}{7}$ and $y'(0) = \frac{-1}{7}$.

Then y(1) is _____ (rounded off to nearest integer). GATE NM 2022

Solution:

Parameter	Description	Value
$y'' - 4y' - 12y = 3e^{5x}$	Differential equation	none
y(x)	Solution of differential equation	$y(0) = \frac{18}{7}$
y'(x)	First order derivative of solution of differential equation	$y'(0) = \frac{-1}{7}$

IABLE 0
INPUT PARAMETERS

$$\frac{1}{s-a} \stackrel{\mathcal{L}^{-1}}{\longleftrightarrow} e^{at} \tag{11}$$

Now finding Inverse Laplace Transform on both sides of (10),

From (11)

$$\implies y(t) = \left(e^{6t} - \frac{3}{7}e^{5t} + 2e^{-2t}\right)u(t)$$
 (12)

$$\implies y(1) = e^6 - \frac{3}{7}e^5 + 2e^{-2} \tag{13}$$

$$y(1) = 340$$
 (14)

$$y''(t) \stackrel{\mathcal{L}}{\longleftrightarrow} s^{2}Y(s) - sy(0) - y'(0) \tag{2}$$

$$y'(t) \stackrel{\mathcal{L}}{\longleftrightarrow} sY(s) - y(0)$$
 (3)

$$y(t) \stackrel{\mathcal{L}}{\longleftrightarrow} Y(s)$$
 (4)

$$e^{at} \stackrel{\mathcal{L}}{\longleftrightarrow} \frac{1}{s-a} \tag{5}$$

Applying Laplace transform on both sides of (1),

$$\mathcal{L}\left(y''(t) - 4y'(t) - 12y(t)\right) = \mathcal{L}\left(3e^{5x}\right) \tag{6}$$

From (2), (3), (4), (5)

$$Y(s)(s^2-4s-12)-y(0)(s-4)$$

$$-y'(0) = \frac{3}{s - 5} \tag{7}$$

$$Y(s)\left(s^2 - 4s - 12\right) - \frac{(18s - 73)}{7} = \frac{3}{(s - 5)} \tag{8}$$

$$Y(s) = \frac{3}{(s-5)(s^2-4s-12)} + \frac{(18s-73)}{7(s^2-4s-12)}$$
(9)

$$\implies Y(s) = \frac{1}{(s-6)} - \frac{3}{7(s-5)} + \frac{1}{(s+2)}$$
 (10)

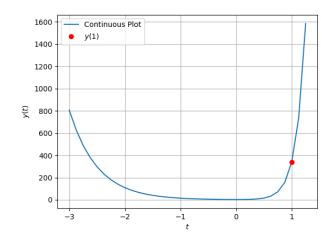


Fig. 0.