Gate 2023 EC 58

HIBA MUHAMMED
EE23BTECH11026

Problem Statement

Let $x_1(t) = u(t+1.5) - u(t-1.5)$ and $x_2(t)$ is shown in the figure below. For $y(t) = x_1(t) * x_2(t)$, the $\int_{-\infty}^{\infty} y(t)\,dt$ is __________.

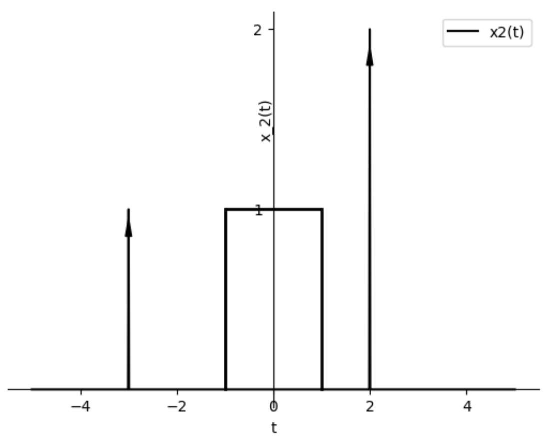

Fig. 1. Figure

Solution

Input Parameters		
Function	**Expression**	**Description**
$x_1(t)$	$u(t+1.5) - u(t-1.5)$	Step function with delay and width parameters.
$X_1(f)$		Fourier Transform of $x_1(t)$.
$x_2(t)$	$\delta(t+3) + \text{rect}\left(\frac{t}{2}\right) + 2\delta(t-2)$	Impulse function followed by a rectangle and two impulses.
$X_2(f)$		Fourier Transform of $x_2(t)$.

TABLE I
Input Parameters

$$x_1(t) = u(t+1.5) - u(t-1.5) \tag{1}$$

$$x_1(t) = \text{rect}\left(\frac{t}{3}\right) \tag{2}$$

$$\tag{3}$$

The Fourier Transform of $rect\left(\frac{t}{a}\right)$:

$$\text{rect}\left(\frac{t}{a}\right) \overset{\mathcal{F}}{\leftrightarrow} a \times \sin(2\pi f \frac{a}{2}) \tag{4}$$

$$\text{where rect}\left(\frac{t}{a}\right) = \begin{cases} 1 & \text{if } |t| < \frac{a}{2} \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

$$X_1(f) = 3\text{sinc}(1.5 \cdot 2\pi f) \tag{6}$$

$$x_2(t) = \delta(t+3) + \text{rect}\left(\frac{t}{2}\right) + 2\delta(t-2) \tag{7}$$

$$X_2(f) = e^{3j \cdot 2\pi f} + 2\text{sinc}(2\pi f) + 2e^{-2j \cdot 2\pi f} \tag{8}$$

$$Y(f) = X_1(f) \cdot X_2(f) \tag{9}$$

$$Y(f) = 3\text{sinc}(1.5 \cdot 2\pi f) \cdot (e^{3j \cdot 2\pi f} + 2\text{sinc}(2\pi f) + 2e^{-2j \cdot 2\pi f}) \tag{10}$$

$$y(t) = x_1(t) * x_2(t) \tag{11}$$

$$Y(f) \overset{\mathcal{F}}{\leftrightarrow} y(t) \tag{12}$$

$$y(t) = \text{rect}\left(\frac{t+3}{3}\right) + 2\text{rect}\left(\frac{t-2}{3}\right) + (t+2.5)u(t+2.5) + (t-2.5)u(t-2.5) \tag{13}$$

$$-(t+0.5)u(t+0.5) - (t-0.5)u(t-0.5) \tag{14}$$

$$Y(f) = \int_{-\infty}^{\infty} y(t) e^{-j2\pi f t}\, dt \tag{15}$$

$$\int_{-\infty}^{\infty} y(t)\, dt = Y(0) \tag{16}$$

$$Y(0) = 3\text{sinc}(0) \cdot (e^0 + 2\text{sinc}(0) + 2e^0) \tag{17}$$

$$= 3 \cdot (1 + 2 + 2) \tag{18}$$

$$= 15 \tag{19}$$

Therefore, the value of $\int_{-\infty}^{\infty} y(t)\, dt$ is 15

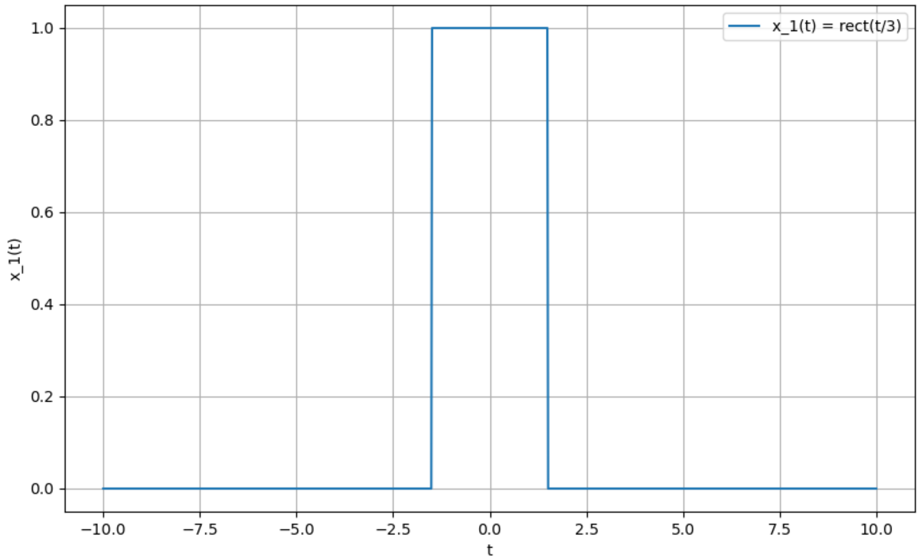

Fig. 2. Graph of $x_1(t) = \text{rect}\left(\frac{t}{3}\right)$

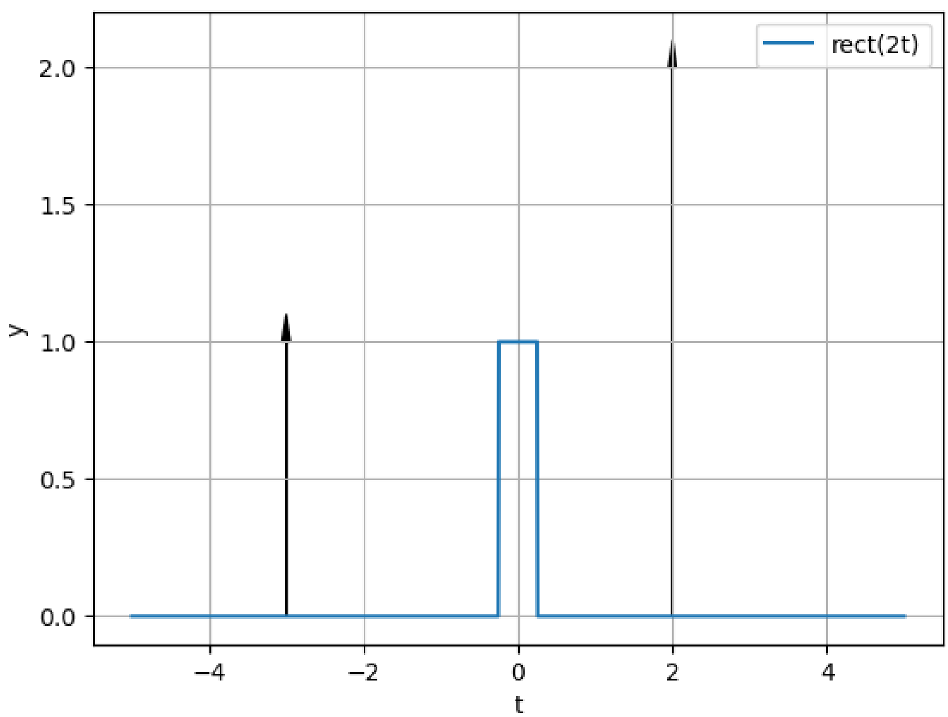

Fig. 3. Graph of $x_2(t) = \delta(t+3) + \text{rect}(2t) + 2\delta(t-2)$

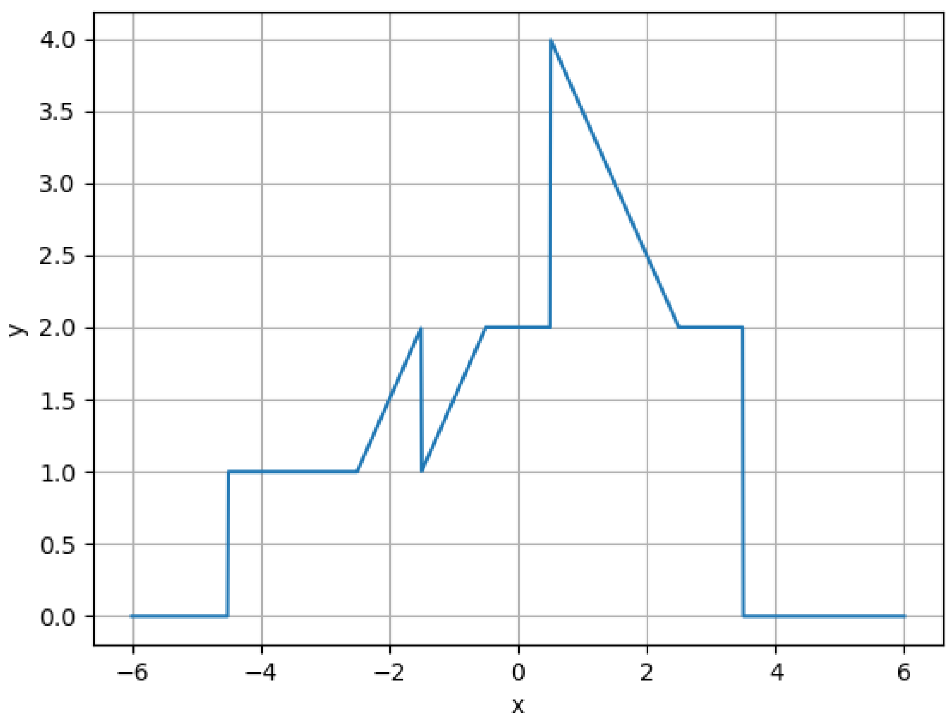

Fig. 4. Graph of $y(t)$