SEQUENCE AND SERIES

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A series (S) is given as S=1+3+5+7+9+..... The sum of the first 50 terms of S is ____________

Solution:

Variable	Description	Value
$x(0)$	First term of AP	1
$x(1)$	Second term of AP	3
d	Common difference of AP $(x(2)-x(1))$	2
$x(n)$	n^{th} term of sequence	$(2n+1)u(n)$

TABLE 0

INPUT PARAMETERS

For an AP,

$$X(z) = \frac{x(0)}{1-z^{-1}} + \frac{dz^{-1}}{(1-z^{-1})^2} \tag{1}$$

$$\implies X(z) = \frac{1}{(1-z^{-1})} + \frac{2z^{-1}}{(1-z^{-1})^2}, |z| > 1 \tag{2}$$

$$y(n) = x(n) * u(n) \tag{3}$$

$$Y(z) = X(z)\,U(z) \tag{4}$$

$$Y(z) = \frac{1}{(1-z^{-1})^2} + \frac{2z^{-1}}{(1-z^{-1})^3} \tag{5}$$

$$\implies Y(z) = \frac{(z^{-1}+1)}{(1-z^{-1})^3}, |z| > 1 \tag{6}$$

Using Contour Integration to find the inverse Z-transform,

$$y(49) = \frac{1}{2\pi j}\oint_C Y(z)\, z^{48}\, dz \tag{7}$$

$$= \frac{1}{2\pi j}\oint_C \frac{(z^{-1}+1)z^{48}}{(1-z^{-1})^3}\, dz \tag{8}$$

We can observe that the pole is repeated 3 times and thus $m = 3$,

$$R = \frac{1}{(m-1)!}\lim_{z\to a}\frac{d^{m-1}}{dz^{m-1}}\left((z-a)^m f(z)\right) \tag{9}$$

$$\implies R = \frac{1}{(2)!}\lim_{z\to 1}\frac{d^2}{dz^2}\left((z-1)^3\frac{(z^{-1}+1)z^{51}}{(z-1)^3}\right) \tag{10}$$

$$\implies R = \frac{1}{2}\lim_{z\to 1}\frac{d^2}{dz^2}(z^{50}+z^{51}) \tag{11}$$

$$\implies R = 2500 \tag{12}$$

$$\therefore y(50) = 2500 \tag{13}$$

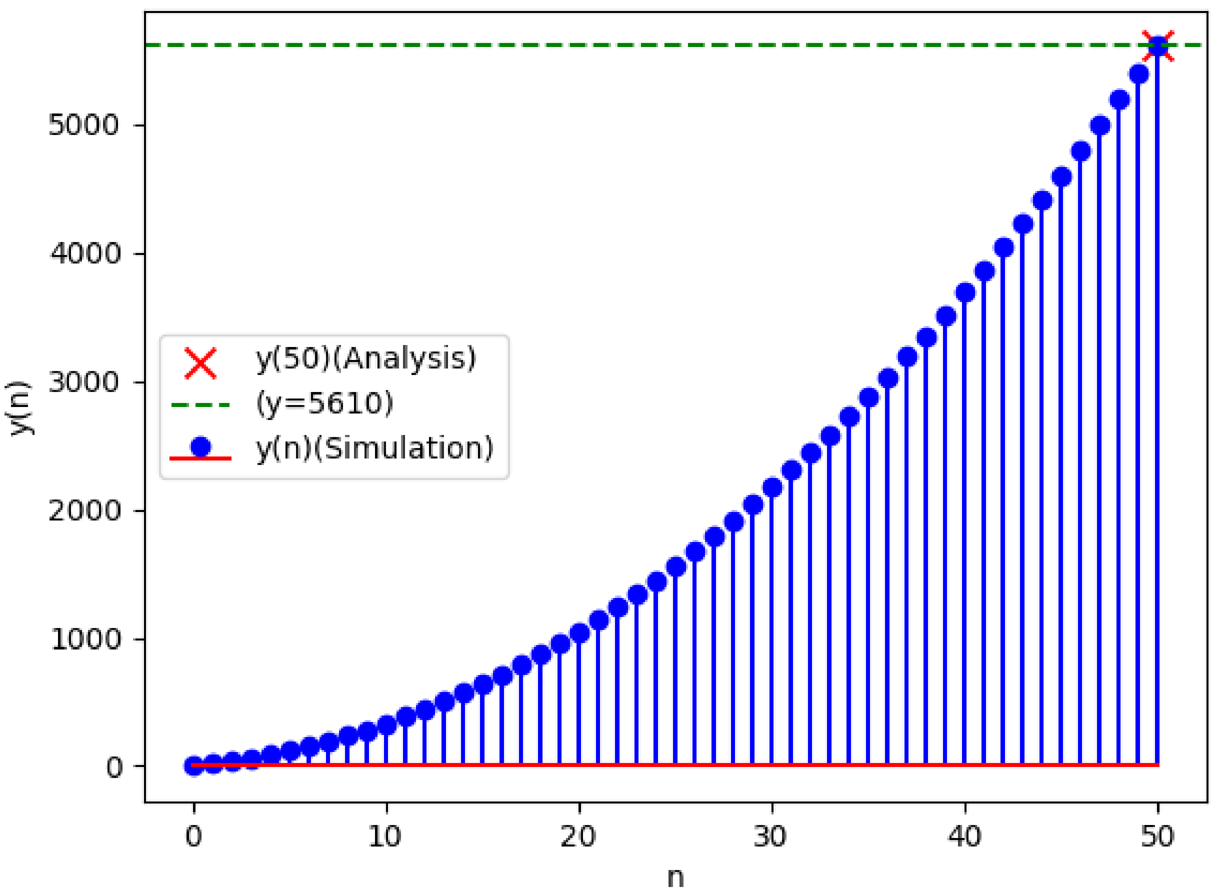

Fig. 0. Analysis vs Simulation