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# SIGNAL PROCESSING

## Through GATE

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# Contents

|                              |           |
|------------------------------|-----------|
| Introduction                 | iii       |
| <b>1 Harmonics</b>           | <b>1</b>  |
| <b>2 Filters</b>             | <b>3</b>  |
| <b>3 Z-transform</b>         | <b>5</b>  |
| <b>4 Sequences</b>           | <b>7</b>  |
| <b>5 Contour Integration</b> | <b>9</b>  |
| <b>6 Laplace Transform</b>   | <b>11</b> |



# Introduction

This book provides solutions to signal processing problems in GATE.



## Chapter 1

# Harmonics





## Chapter 2

# Filters



## Chapter 3

# Z-transform



## Chapter 4

# Sequences

4.1 Consider the discrete time signal  $x[n] = u[-n + 5] - u[n + 3]$ , where

$$u[n] = \begin{cases} 1; n \geq 0 \\ 0; n < 0 \end{cases}$$

The smallest  $n$  for which  $x[n] = 0$  is? **Solution:** From Fig. 1, the minimum value of  $n$  is given as

$$n = -3 \tag{4.1}$$



Figure 1: Plot of function  $x(n)$  taken from python3

## Chapter 5

# Contour Integration





## Chapter 6

# Laplace Transform

- 6.1 The number of zeroes of the polynomial  $P(s) = s^3 + 2s^2 + 5s + 80$  in the right side of the plane? (GATE IN 2023)

**Solution:** The table below shows the Routh array of the  $n^{th}$ - order characteristic polynomial :

$$a_0s^n + a_1s^{n-1} + \dots + a_{n-1}s^1 + a_ns^0 \quad (6.1)$$

|           |                                     |                                     |       |     |
|-----------|-------------------------------------|-------------------------------------|-------|-----|
| $s^n$     | $a_0$                               | $a_2$                               | $a_4$ | ... |
| $s^{n-1}$ | $a_1$                               | $a_3$                               | $a_5$ | ... |
| $s^{n-2}$ | $b_1 = \frac{a_1a_2 - a_3a_0}{a_1}$ | $b_2 = \frac{a_1a_4 - a_5a_0}{a_1}$ | ...   | ..  |
| $s^{n-3}$ | $c_1 = \frac{b_1a_3 - b_2a_1}{b_1}$ | $\vdots$                            |       |     |
| $\vdots$  | $\vdots$                            | $\vdots$                            |       |     |
| $s^1$     | $\vdots$                            | $\vdots$                            |       |     |
| $s^0$     | $a_n$                               |                                     |       |     |

Table 6.1: Routh Array

Characteristic Equation:

$$s^3 + 2s^2 + 5s + 80 = 0 \quad (6.2)$$

From Table 6.1:

|       |  |    |
|-------|--|----|
| $s^3$ | 1  | 5  |
| $s^2$ | 2  | 80 |
| $s^1$ | $\frac{2 \times 5 - 80 \times 1}{2} = -35$ |    |
| $s^0$ | $\frac{-35 \times 80}{-35} = 80$           |    |

Table 6.2:

From Table 6.2:

Since there are 2 sign changes in the first column of the Routh tabulation. So, the number of zeros in the right half of the s-plane will be 2.

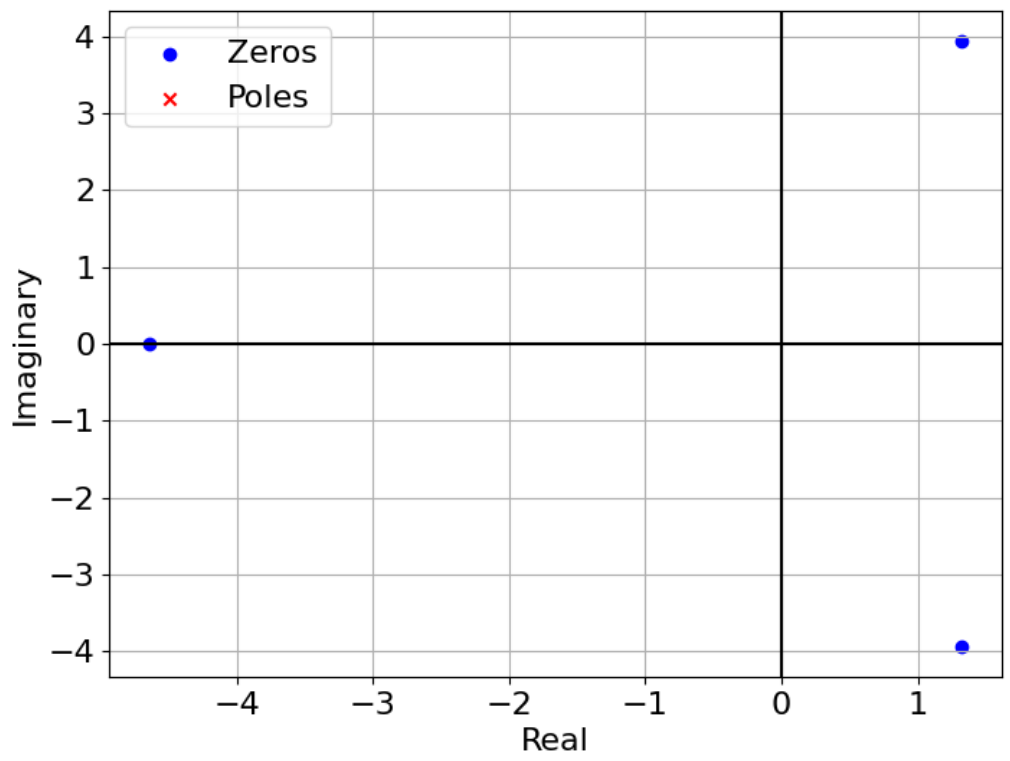


Figure 6.1:

