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# SIGNAL PROCESSING

## Through GATE

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# Introduction

This book provides solutions to signal processing problems in GATE.



## Chapter 1

# Harmonics

- 1.1 Let  $y(t)=x(4t)$ , where  $x(t)$  is a continuous-time periodic signal of 100s. the fundamental period of  $y(t)$  is (**rounded off to the nearest integer**) (GATE IN 2023)

**Solution:**

- 1.2 In the circuit shown below, it is observed that the amplitude of voltage across the resistor is the same as the amplitude of the source voltage. What is the angular frequency  $\omega_0$  (in rad/s)?



(GATE BM 2023) **Solution:**



- 1.3 For a regular sinusoidal wave propagating in deep water having wave height of 3.5 m and wave period of 9 s, the wave steepness is \_\_\_\_\_ (round off to three decimal places).

Gate 2023 NM 33 **Solution:**



## Chapter 2

# Filters

2.1 For the circuit given below, choose the angular frequency  $\omega_0$  at which voltage across capacitor has maximum amplitude?

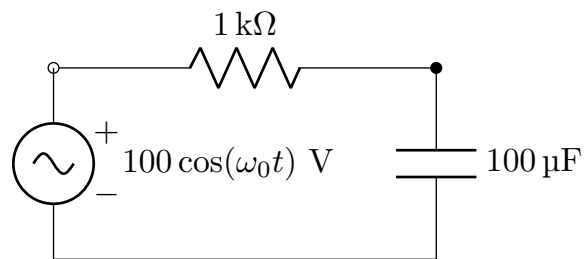


Figure 2.1: circuit

- (A) 1000
- (B) 100
- (C) 1
- (D) 0

(GATE BM 2023 Question 16)

**Solution:**

Parameter	Description	Value
$V_i(j\omega)$	Input voltage	100
$v_c(t)$	Potential difference across Capacitor	?
$V_c(s)$	Potential difference across Capacitor	$V_c(s)$
$H(s)$	Transfer function	$\frac{V_c(s)}{V_i(s)}$
$V_o$	Amplitude of input voltage	100 V
$R$	Resistance in circuit	1 k $\Omega$
$C$	Capacitance in circuit	100 $\mu$ F
$\omega_o$	Angular frequency of input voltage	$\omega_o$

Table 2.1: input values

$$V_c(s) = \frac{V_i(s) \frac{1}{sC}}{R + \frac{1}{sC}} \quad (2.1)$$

$$\Rightarrow H(s) = \frac{1}{1 + sRC} \quad (2.2)$$

$$\Rightarrow H(j\omega) = \frac{1}{1 + j\omega RC} \quad (2.3)$$

$$|H(j\omega)| = \frac{1}{\sqrt{1 + (\omega RC)^2}} \quad (2.4)$$

$$v_c(t) = \frac{100}{\sqrt{1 + (\omega_o RC)^2}} \left( \cos \omega_o t + \arctan \left( \frac{1}{\omega_o RC} \right) \right) \quad (2.5)$$

Maximum amplitude of  $v_c(t)$  occurs at  $\omega_o = 0$

$$\therefore \omega_o = 0 \quad (2.6)$$

$\therefore$  maximum value of  $v_c(t)$  at steady state is 100 Volts.

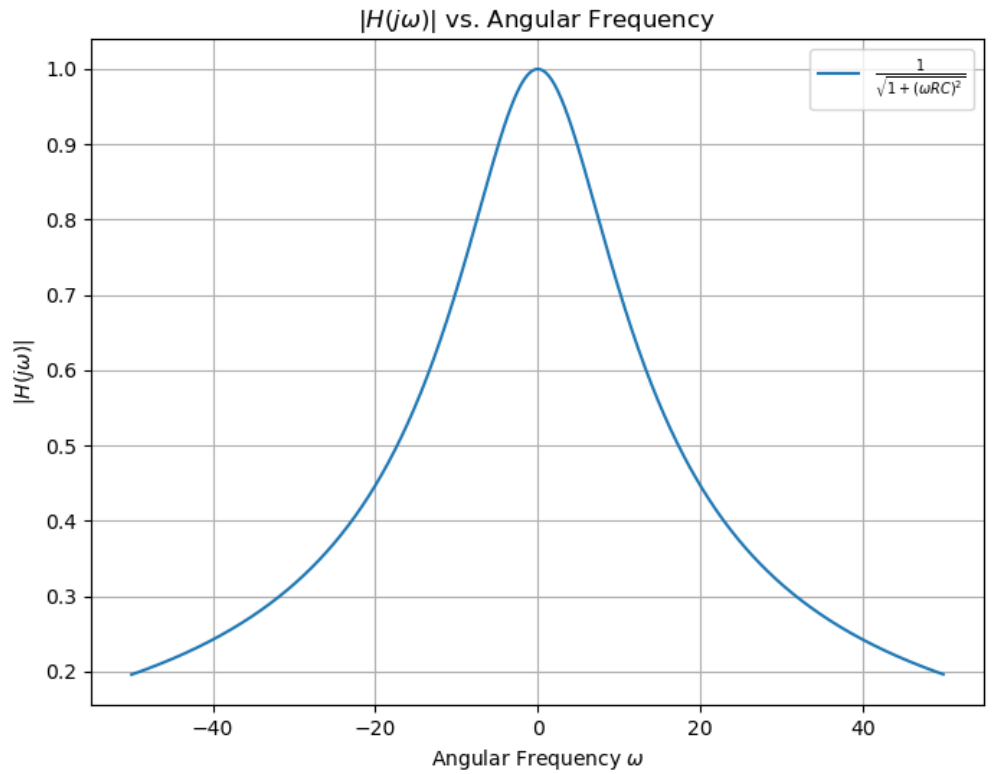


Figure 2.2:  $|H(j\omega)|$

2.2 In the following circuit, the switch S is open for  $t < 0$  and closed for  $t \geq 0$ . What is the steady state voltage (in Volts) across the capacitor when the switch is closed?

(GATE BM 2023 Question 30)

2.3 A finite impulse response (FIR) filter has only two non-zero samples in its impulse response  $h[n]$ , namely  $h[0] = h[1] = 1$ . The Discrete Time Fourier Transform (DTFT) of  $h[n]$  equals  $H(e^{j\omega})$ , as a function of the normalized angular frequency  $\omega$ . For the range  $|\omega| \leq \pi$ ,  $|H(e^{j\omega})|$  is equal to

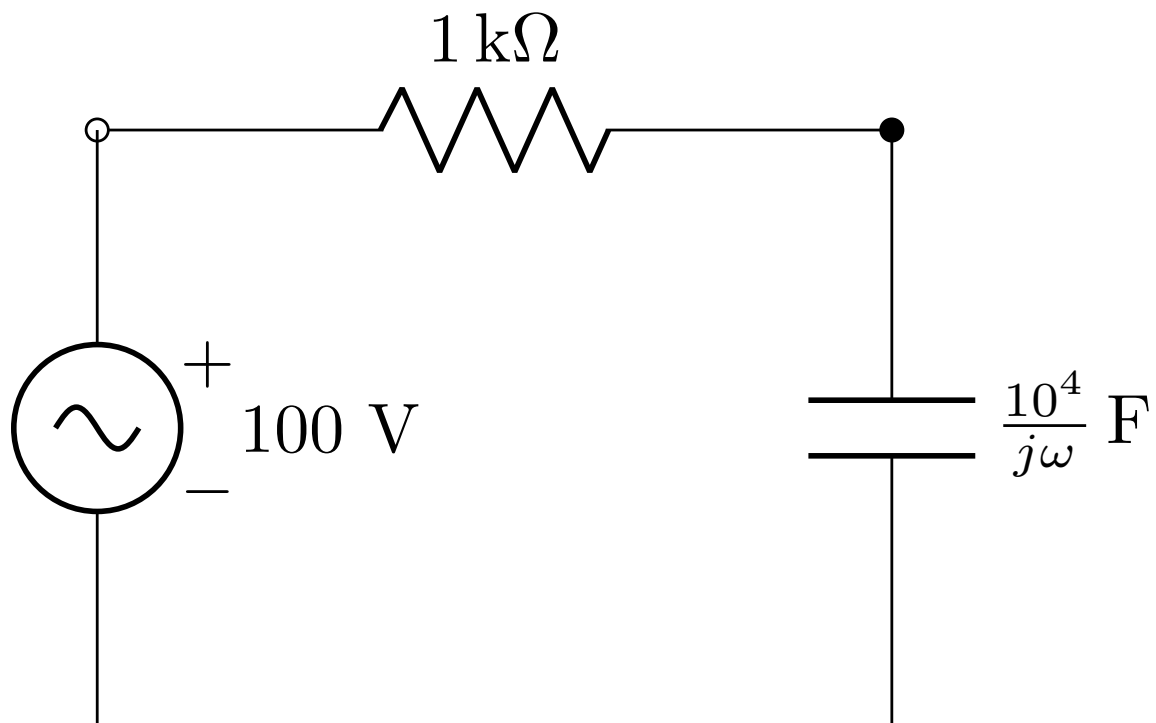


Figure 2.3: circuit in  $\omega$ -domain

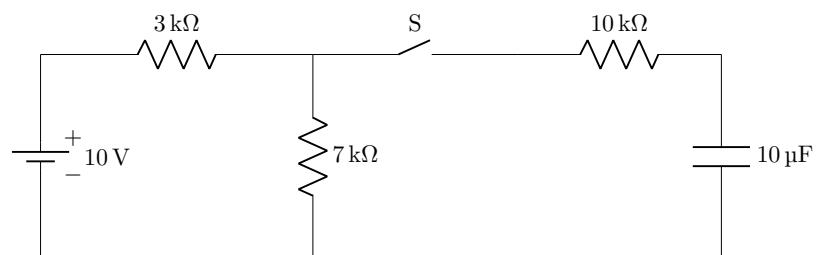


Figure 2.4: circuit

(A)  $2|\cos(\omega)|$

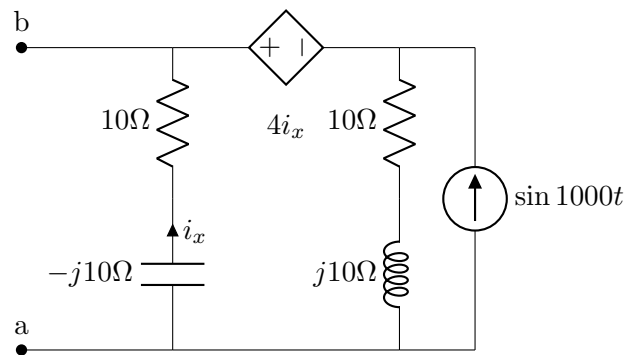
(B)  $2|\sin(\omega)|$

(C)  $2\left|\cos\left(\frac{\omega}{2}\right)\right|$

(D)  $2\left|\sin\left(\frac{\omega}{2}\right)\right|$

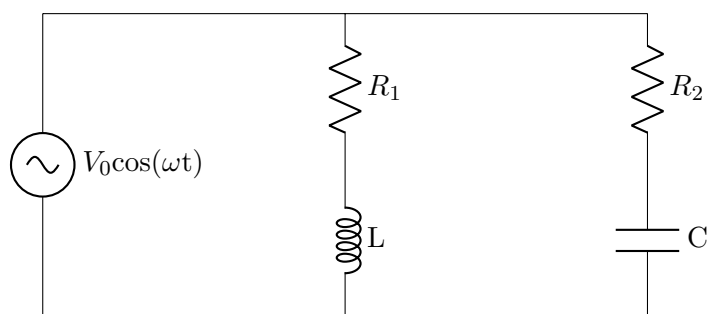
(GATE BM 2023 Question 17)

2.4 For the circuit shown, if  $i = \sin 1000t$ , the instantaneous value of the Thevenin's voltage (in volts) across the terminals a and b at time  $t=5\text{ms}$  is



(GATE EE 2023 Question 51)

- 2.5 In the circuit shown,  $\omega = 100\pi \text{ rad/s}$ ,  $R_1 = R_2 = 2.2\Omega$  and  $L = 7\text{mH}$ . the capacitance  $C$  for which  $Y_{in}$  is purely real is mF



(GATE IN 2023 Q46)

**Solution:**



2.6 An input voltage in the form of a square wave of frequency  $1\text{ kHz}$  is given to a circuit, which results in the output shown schematically below. Which one of the following options is the CORRECT representation of the circuit? (GATE PH 2023 Q37)

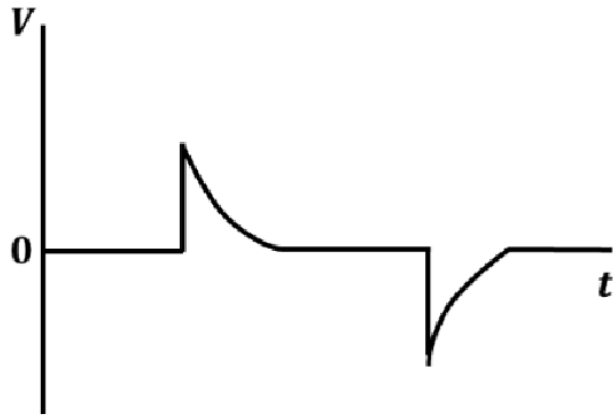
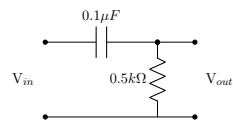
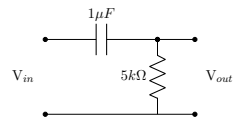


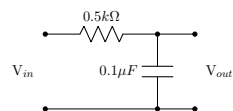
Figure 2.5:



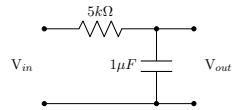
(a)



(b)



(c)



(d)

(GATE 2023 PH 37) **Solution:**

- 2.7 In the circuit shown below, switch S was closed for long time. If the switch is opened at  $t = 0$ , the maximum magnitude of the voltage  $V_R$ , in volts is (rounded off to the nearest integer) (GATE 2023 EC 35)

**Solution:**

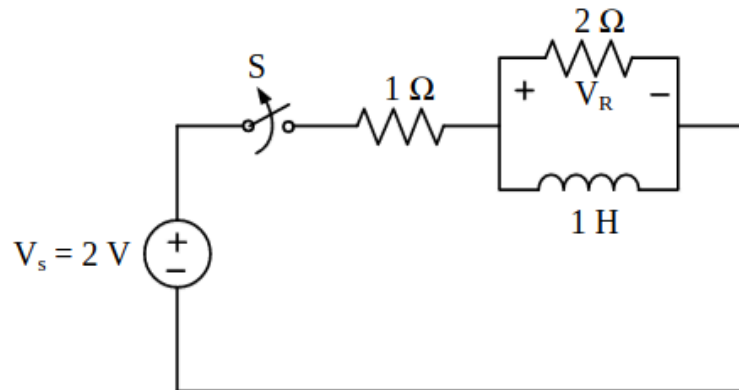


Figure 2.6:

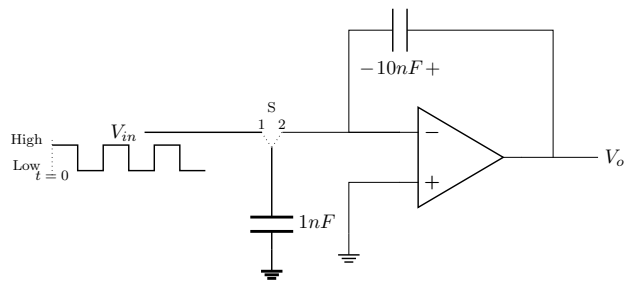
2.8 A signal  $x(t) = 2 \cos(180\pi t) \cos(60\pi t)$  is sampled at 200 Hz and then passed through an ideal low pass filter having cut-off frequency of 100 Hz.

The maximum Frequency present in the filtered signal in Hz is \_\_\_\_\_ (Round off to the nearest integer.) (GATE 2023 EE) **Solution:**

2.9 In the circuit shown, the input voltage  $V_{in} = 100mV$ . The switch and the opamp are ideal. At time  $t = 0$ , the initial charge stored in the  $10nF$  capacitor is  $1nC$ , with the polarity as indicated in the figure. The switch  $S$  is controlled using a  $1KHz$  square-wave voltage signal  $V_s$  as shown. Whenever  $V_s$  is 'High',  $S$  is in position '1' and when  $V_s$  is 'Low',  $S$  is in position '2'.

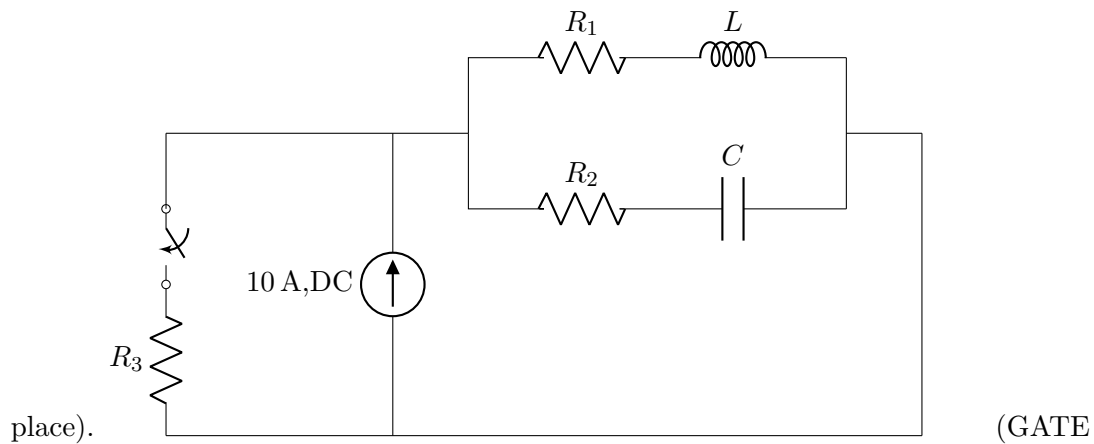
At  $t = 20ms$ , the magnitude of the voltage  $V_o$  will be

(GATE IN 2023)



**Solution:**

2.10 The value of parameters of the circuit shown in the figure are  $R_1 = 2\Omega, R_2 = 2\Omega, R_3 = 3\Omega, L = 10\text{mH}, C = 100\mu\text{F}$ . For time  $t < 0$ , the circuit is at steady state with the switch ' $K$ ' in closed condition. If the switch is opened at  $t = 0$ , the value of the voltage across the inductor ( $V_L$ ) at  $t = 0^+$  in Volts is \_\_\_\_\_ (Round off to 1 decimal



2023 EE 29Q) **Solution:**

## Chapter 3

# Z-transform





## Chapter 4

# Systems

- 4.1 Consider a unity-gain negative feedback system consisting of the plant  $G(s)$  and a proportional-integral controller. Let the proportional gain and integral gain be 3 and 1, respectively. For a unit step reference input, the final values of the controller output and the plant output, respectively, are

$$G(s) = \frac{1}{(s-1)}$$

(GATE EE 2023)

**Solution:**

Parameter	Description	Value
$K_p$	Proportional Gain	3
$K_i$	Integral Gain	1
$r(t)$	Reference Input	$u(t)$
$w(t)$	Controller Output	?
$y(t)$	Plant Output	?
$e(t)$	Error Input	$r(t) - y(t)$

Table 1: Parameter Table

From the Fig. 4.1:

$$E(s) = U(s) - Y(s) \quad (4.1)$$

$$W(s) = 3E(s) + \frac{1}{s}E(s) \quad (4.2)$$

$$Y(s) = G(s)W(s) \quad (4.3)$$

Some results:

$$tx(t) \xleftrightarrow{\mathcal{L}} -\frac{dX(s)}{ds} \quad (4.4)$$

$$e^{-at}x(t) \xleftrightarrow{\mathcal{L}} X(s+a) \quad (4.5)$$

By using (4.4) and (4.5):

$$e^{-t}u(t) \xleftrightarrow{\mathcal{L}} \frac{1}{s+1}, \text{Re}(s) > -1 \quad (4.6)$$

$$te^{-t}u(t) \xleftrightarrow{\mathcal{L}} \frac{1}{(s+1)^2}, \text{Re}(s) > -1 \quad (4.7)$$

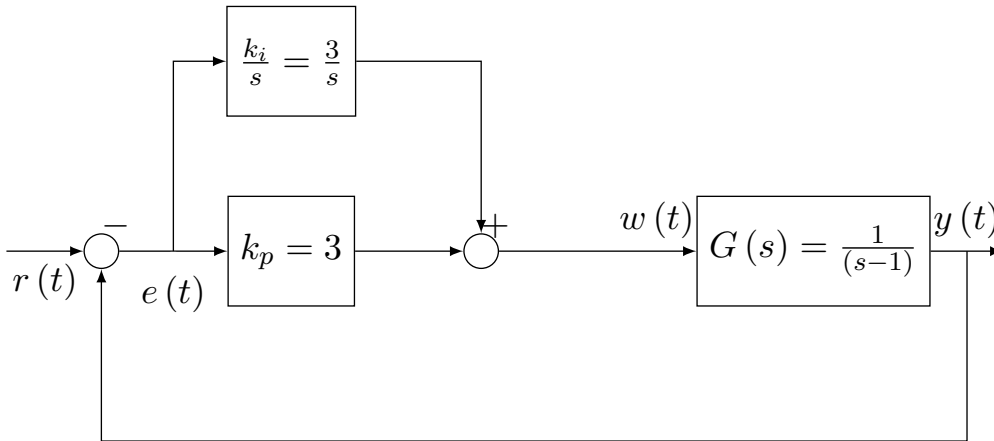


Figure 4.1: Block Diagram of System

(a) **Plant Output:**

From (4.1) , (4.2) and (4.3):

$$Y(s) = \frac{3s+1}{s(s+1)^2}, \text{Re}(s) > -1 \quad (4.8)$$

Final Value Theorem:

$$\lim_{t \rightarrow \infty} x(t) = \lim_{s \rightarrow 0} sX(s) \quad (4.9)$$

Using (4.9) on Y(s):

$$\lim_{t \rightarrow \infty} y(t) = \lim_{s \rightarrow 0} sY(s) \quad (4.10)$$

$$= 1 \quad (4.11)$$

Taking partial fraction of (4.8) :

$$Y(s) = \frac{1}{s} + \frac{2}{(s+1)^2} - \frac{1}{s+1} \quad (4.12)$$

Using (4.6) and (4.7):

$$\therefore y(t) = u(t) + 2te^{-t}u(t) - e^{-t}u(t) \quad (4.13)$$

(b) **Controller Output:**

From (4.2)

$$W(s) = \frac{3}{s} + \frac{1}{s^2} - Y(s) \left( 3 + \frac{1}{s} \right) \quad (4.14)$$

Substituting (4.8)

$$W(s) = \frac{(s-1)(3s+1)}{s(s+1)^2}, \operatorname{Re}(s) > -1 \quad (4.15)$$

Using (4.9) on  $W(s)$

$$\lim_{t \rightarrow \infty} w(t) = \lim_{s \rightarrow 0} sW(s) \quad (4.16)$$

$$= -1 \quad (4.17)$$

Taking partial fraction of equation(4.15) :

$$W(s) = -\frac{1}{s} - \frac{4}{(s+1)^2} + \frac{4}{s+1} \quad (4.18)$$

Using equations (4.6) and (4.7) and taking inverse lapalace transform:

$$w(t) = -u(t) - 4te^{-t}u(t) + 4e^{-t}u(t) \quad (4.19)$$

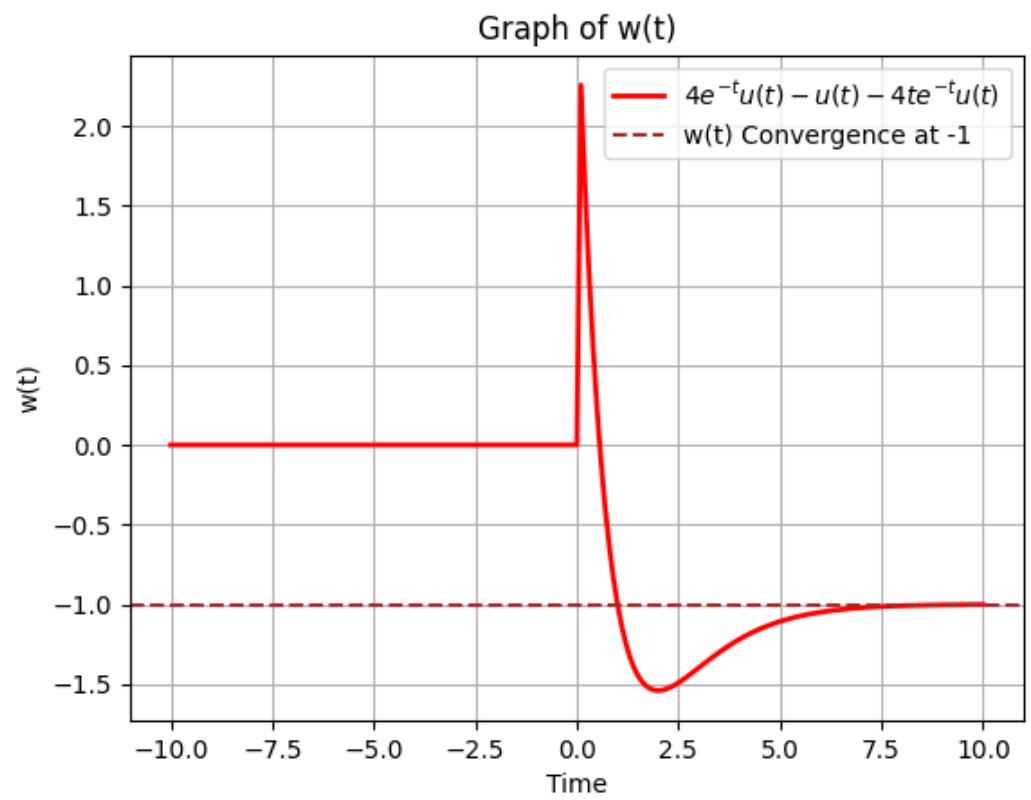


Figure 4.2:  $w(t)$  converges at -1.

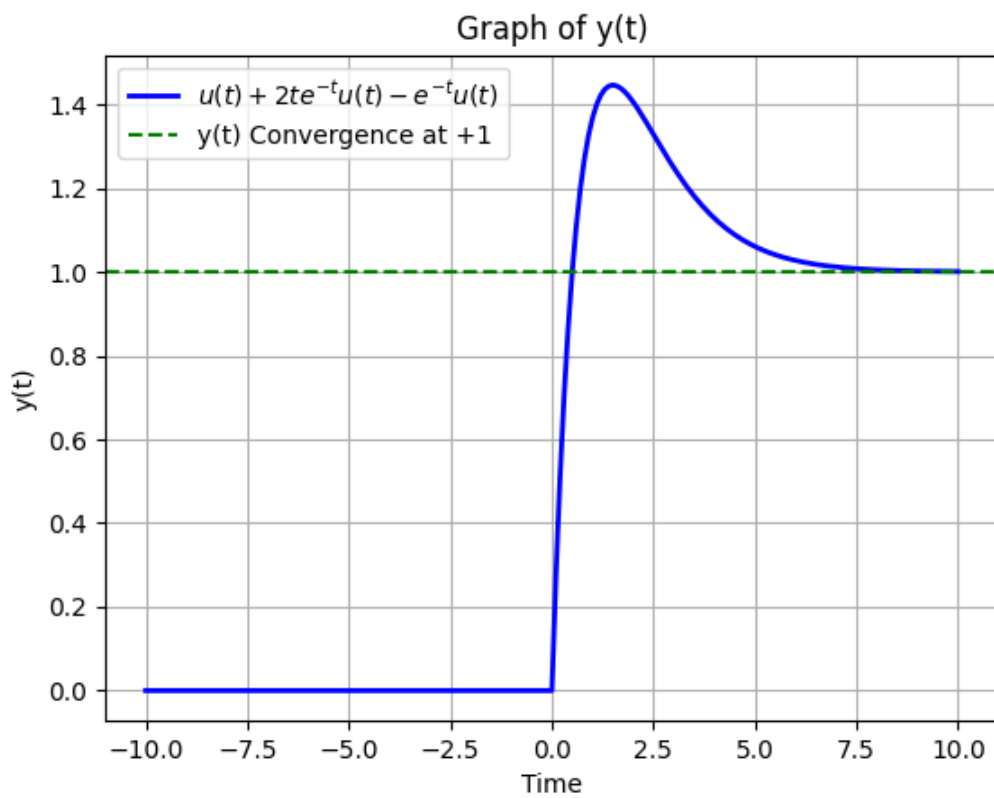


Figure 4.3:  $y(t)$  converges at +1

4.2 Level ( $h$ ) in a steam boiler is controlled by manipulating the flow rate ( $F$ ) of the break-up(fresh) water using a proportional ( $P$ ) controller. The transfer function between the output and the manipulated input is

$$\frac{h(s)}{F(s)} = \frac{0.25(1-s)}{s(2s+1)}$$

The measurement and the valve transfer functions are both equal to 1. A process engineer wants to tune the controller so that the closed loop response gives the decaying oscillations under the servo mode. Which one of the following is the CORRECT value of the controller gain to be used by the engineer?

- (a) 0.25
- (b) 2
- (c) 4
- (d) 6

GATE CH 2023

**Solution:**

Closed loop signal transfer function of the above block diagram can be given by,

$$T(s) = \frac{G(s)}{1 + G(s)H(s)} \quad (4.20)$$

From Fig. 4.4 and Table 4.2 for a unit impulse,  $X(s) = 1$

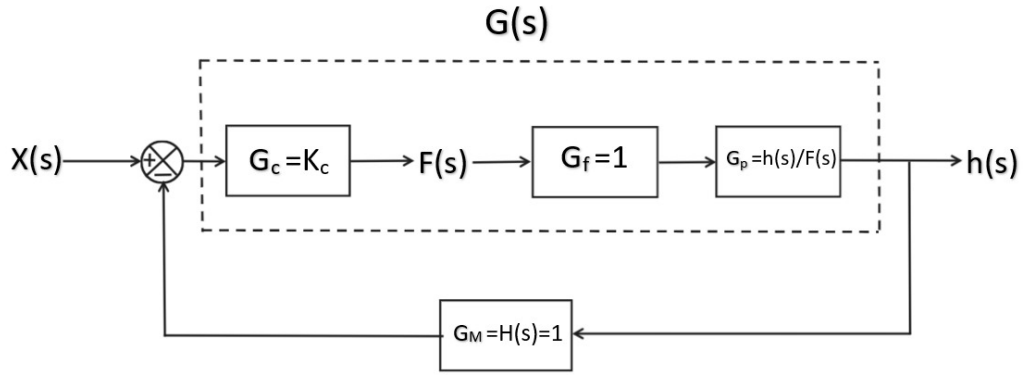


Figure 4.4: Closed loop Block diagram

$$h(s) = T(s) \times X(s) \quad (4.21)$$

$$h(s) = \frac{(1-s)K_c}{8s^2 + (4-K_c)s + K_c} \quad (4.22)$$

$$\Rightarrow h(s) = \frac{(1-s)K_c}{8(s-s_1)(s-s_2)} \quad (4.23)$$

Where,

$$s_1 = \frac{(K_c - 4)}{16} + \sqrt{\left(\frac{K_c - 4}{16}\right)^2 - \frac{K_c}{8}} \quad (4.24)$$

$$s_2 = \frac{(K_c - 4)}{16} - \sqrt{\left(\frac{K_c - 4}{16}\right)^2 - \frac{K_c}{8}} \quad (4.25)$$



From (4.23) we get,

$$h(s) = \frac{K_c}{8(s_1 - s_2)} \left( \frac{1 - s_1}{s - s_1} - \frac{1 - s_2}{s - s_2} \right) \quad (4.26)$$

Now taking the inverse laplace transform we have,

$$h(t) = \frac{K_c}{8(s_1 - s_2)} [(1 - s_1)e^{s_1 t} - (1 - s_2)e^{s_2 t}] u(t) \quad (4.27)$$

$$\Rightarrow h(t) = e^{\frac{K_c - 4}{16}} \left( A_1 e^{\sqrt{\left(\frac{K_c - 4}{16}\right)^2 - \frac{K_c}{8}}} - A_2 e^{-\sqrt{\left(\frac{K_c - 4}{16}\right)^2 - \frac{K_c}{8}}} \right) u(t) \quad (4.28)$$

Where,

$$A_1 = \frac{K_c}{8} \left( \frac{1 - s_1}{s_1 - s_2} \right) \quad (4.29)$$

$$A_2 = \frac{K_c}{8} \left( \frac{1 - s_2}{s_1 - s_2} \right) \quad (4.30)$$

Now applying the condition for underdamped oscillations,

$$\left( \frac{K_c - 4}{16} \right)^2 - \frac{K_c}{8} < 0 \quad (4.31)$$

$$\Rightarrow K_c \in (20 - \sqrt{384}, 20 + \sqrt{384}) \quad (4.32)$$

For the system to be stable,

$$\frac{K_c - 4}{8} < 0 \quad (4.33)$$

$$\Rightarrow K_c < 4 \quad (4.34)$$

From (4.32) and (4.34)

$$K_c \in (0.4, 4) \quad (4.35)$$

(4.35) represents the  $ROC, (Re\{s\} < 0)$

$$\implies K_c = 2 \quad (4.36)$$

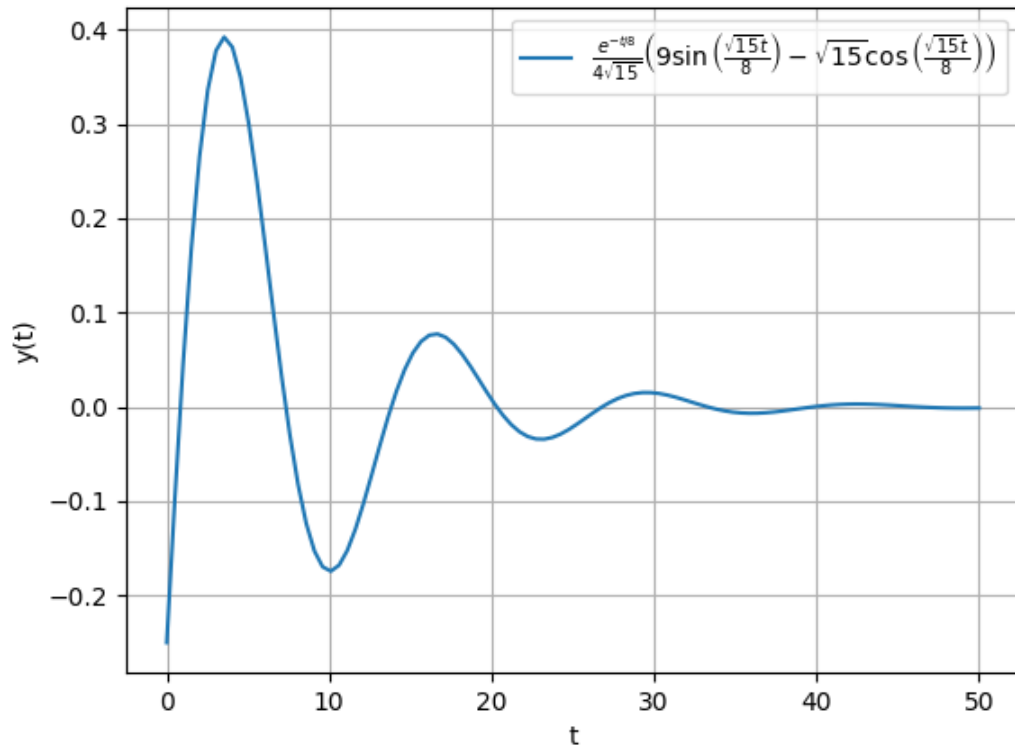
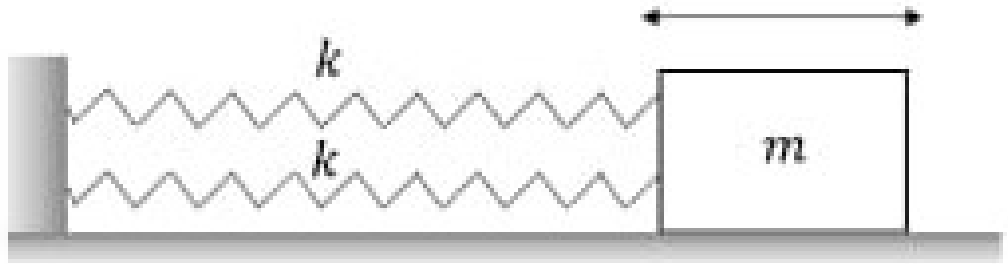


Figure 4.5:  $y(t)$  vs  $t$  graph

- 4.3 The figure shows a block of mass  $m = 20$  kg attached to a pair of identical linear springs, each having a spring constant  $k = 1000$  N/m. The block oscillates on a frictionless horizontal surface. Assuming free vibrations, the time taken by the block to complete ten oscillations is \_\_\_\_\_ seconds . (Rounded off to two decimal places)  
Take  $\pi = 3.14$ . (GATE ME 2023)



- 4.4 A system has transfer function

$$\frac{Y(s)}{X(s)} = \frac{s - \pi}{s + \pi}$$

let  $u(t)$  be the unit step function. The input  $x(t)$  that results in a steady-state output  $y(t) = \sin(\pi t)$  is \_\_\_\_\_. (GATE IN 2023)

**Solution:**

4.5 The state equation of a second order system is

$$\dot{\mathbf{x}}(t) = A\mathbf{x}(t), \quad \mathbf{x}(0) \text{ is the initial condition.}$$

Suppose  $\lambda_1$  and  $\lambda_2$  are two distinct eigenvalues of  $A$ , and  $\nu_1$  and  $\nu_2$  are the corresponding eigenvectors. For constants  $\alpha_1$  and  $\alpha_2$ , the solution,  $\mathbf{x}(t)$ , of the state equation is

(A)  $\sum_{i=1}^2 \alpha_i e^{\lambda_i t} \nu_i$

(B)  $\sum_{i=1}^2 \alpha_i e^{2\lambda_i t} \nu_i$

(C)  $\sum_{i=1}^2 \alpha_i e^{3\lambda_i t} \nu_i$

(D)  $\sum_{i=1}^2 \alpha_i e^{4\lambda_i t} \nu_i$

GATE 2023 EC Question 43

4.6 Consider the complex function

$$f(z) = \frac{z^2 \sin z}{(z - \pi)^4}$$

At  $z = \pi$ , which of the following options is (are) correct?

- (A) The order of the pole is 4
- (B) The order of the pole is 3
- (C) The residue at the pole is  $\frac{\pi}{6}$
- (D) The residue at the pole is  $\frac{2\pi}{3}$

(GATE PH 2023)

4.7 A buoy of virtual mass 30 kg oscillates in a fluid medium as a single degree of freedom system. If the total damping in the system is set as 188.5 N-s/m, such that the oscillation just ceases to occur, then the natural period of the system is \_\_\_\_\_ s (round off to one decimal place) (GATE MN 2023 question 63)

4.8 Which of the following statement(s) is/are true?

- (a) If an LTI system is causal, it is stable.
- (b) A discrete time LTI system is causal if and only if its response to a step input  $u[n]$  is 0 for  $n < 0$ .
- (c) If a discrete time LTI system has an impulse response  $h[n]$  of finite duration the system is stable.
- (d) If the impulse response  $0 < |h[n]| < 1$  for all  $n$ , then the LTI system is stable.

(GATE EE 2023 question 27)

**Solution:**

4.9 The outlet concentration  $C_A$  of a plug flow reactor (PFR) is controlled by manipulating the inlet concentration  $C_{A0}$ . The following transfer function describes the dynamics of this PFR.

$$\frac{C_A(s)}{C_{A0}(s)} = e^{-(\frac{V}{F})(k+s)}$$

In the above question,  $V=1m^3$ ,  $F=0.1m^3min^{-1}$  and  $k=0.5min^{-1}$ . The measurement and valve transfer functions are both equal to 1. The ultimate gain, defined as the proportional controller gain that produces sustained oscillations, for this system is (GATE 2023 CH 61)

**Solution:**

4.10 For the block diagram shown in the figure, the transfer function  $\frac{Y(s)}{R(s)}$  is

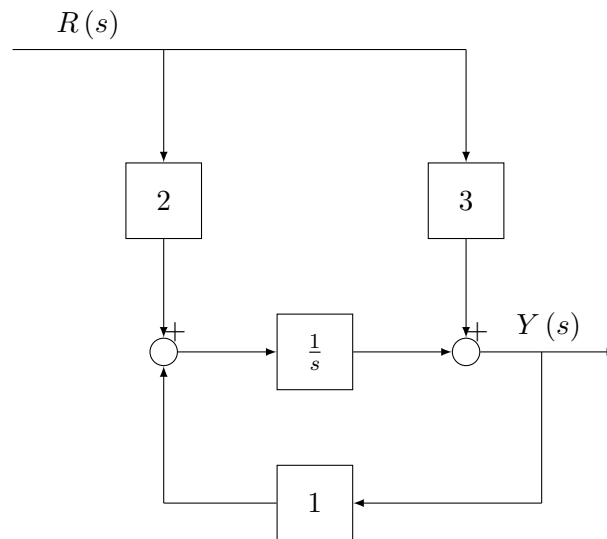


Figure 4.6: Block diagram

(GATE EE 2023)

**Solution:**

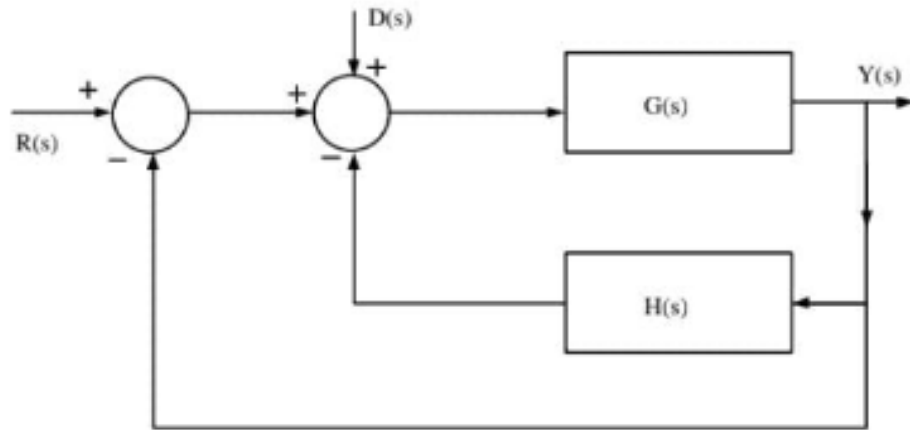


4.11 In the following block diagram,  $R(s)$  and  $D(s)$  are two inputs. The output  $Y(s)$  is expressed as  $Y(s) = G_1(s)R(s) + G_2(s)D(s)$ .

$G_1(s)$  and  $G_2(s)$  are given by

GATE 2023 EC Question 42

**Solution:**



4.12 In the table shown below, match the signal type with its spectral characteristics.(gate EC 29)

**Solution:**

4.13 The impulse response of an LTI system is  $h(t) = \delta(t) + 0.5\delta(t - 4)$ , where  $\delta(t)$  is continuous-time unit impulse signal. If the input signal  $x(t) = \cos\left(\frac{7\pi t}{4}\right)$ , the output is (GATE IN 2023)

**Solution:**

4.14 Consider the state-space description of an LTI system with matrices

$$A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 3 & -2 \end{bmatrix}, \quad D = 1$$

For the input,  $\sin(\omega t)$ ,  $\omega > 0$ , the value of  $\omega$  for which the steady-state output of the system will be zero, is \_\_\_\_\_ (Round off to the nearest integer). (GATE 2023 EE Q46)

**Solution:**

PARAMETER	DESCRIPTION
$G_c$	Proportional controller's transfer function
$G_f$	Valve transfer function
$G_p$	Process transfer function
$G_M$	Measurement transfer function
$G(s)$	Open loop transfer function
$T(s)$	Transfer function of system

Table 4.2: PARAMETER TABLE 1

Signal Type	Spectral Characteristics
(i) Continuous, aperiodic	(a) Continuous, aperiodic
(ii) Continuous, periodic	(b) Continuous, periodic
(iii) Discrete, aperiodic	(c) Discrete, aperiodic
(iii) Discrete, periodic	(c) Discrete, periodic

## Chapter 5

# Sequences

5.1 Consider the discrete time signal  $x[n] = u[-n + 5] - u[n + 3]$ , where

$$u[n] = \begin{cases} 1; n \geq 0 \\ 0; n < 0 \end{cases}$$

The smallest  $n$  for which  $x[n] = 0$  is?

**Solution:** From Fig. 1, the minimum value of  $n$  is given as

$$n = -3 \tag{5.1}$$



Figure 1: Plot of function  $x(n)$  taken from python3

5.2 Two sequences  $x_1[n]$  and  $x_2[n]$  are described as follows:

$$x_1[0] = x_2[0] = 1 \quad (5.2)$$

$$x_1[1] = x_2[2] = 2 \quad (5.3)$$

$$x_1[2] = x_2[1] = 1 \quad (5.4)$$

$$x_1[n] = x_2[n] = 0 \text{ for all } n < 0 \text{ and } n > 2$$

If  $x[n]$  is obtained by convoluting  $x_1[n]$  with  $x_2[n]$ , which of the following equa-



tions is/are TRUE?

(A)  $x[2] = x[3]$

(B)  $x[1] = 2$

(C)  $x[4] = 3$

(D)  $x[2] = 5$

**Solution:**

5.3 A series ( $S$ ) is given as  $S=1+3+5+7+9+\dots$ . The sum of the first 50 terms of  $S$  is

\_\_\_\_\_

(GATE 2023 BT 32) **Solution:**

5.4 For the signals  $x(t)$  and  $y(t)$  shown in the figure,  $z(t) = x(t) * y(t)$  is maximum at  $t = T_1$ . Then  $T_1$  in seconds is ..... (Round off to the nearest integer) **Solution:**

5.5 A series of natural numbers  $F_1, F_2, F_3, F_4, F_5, F_6, F_7, \dots$  obeys  $F_{n+1} = F_n + F_{n-1}$  for all integers  $n \geq 2$ . If  $F_6 = 37$ , and  $F_7 = 60$ , then what is  $F_1$ ?  
[GATE CS 2023]

**Solution:**

## Chapter 6

# Sampling

6.1 An 8 bit ADC converts analog voltage in the range of 0 to  $+5\text{ V}$  to the corresponding digital code as per the conversion characteristics shown in figure. For  $V_{in} = 1.9922\text{ V}$ , which of the following digital output, given in hex, is true?

(a)  $64H$

(b)  $65H$

(c)  $66H$

(d)  $67H$

(GATE EE 40)

**Solution:**

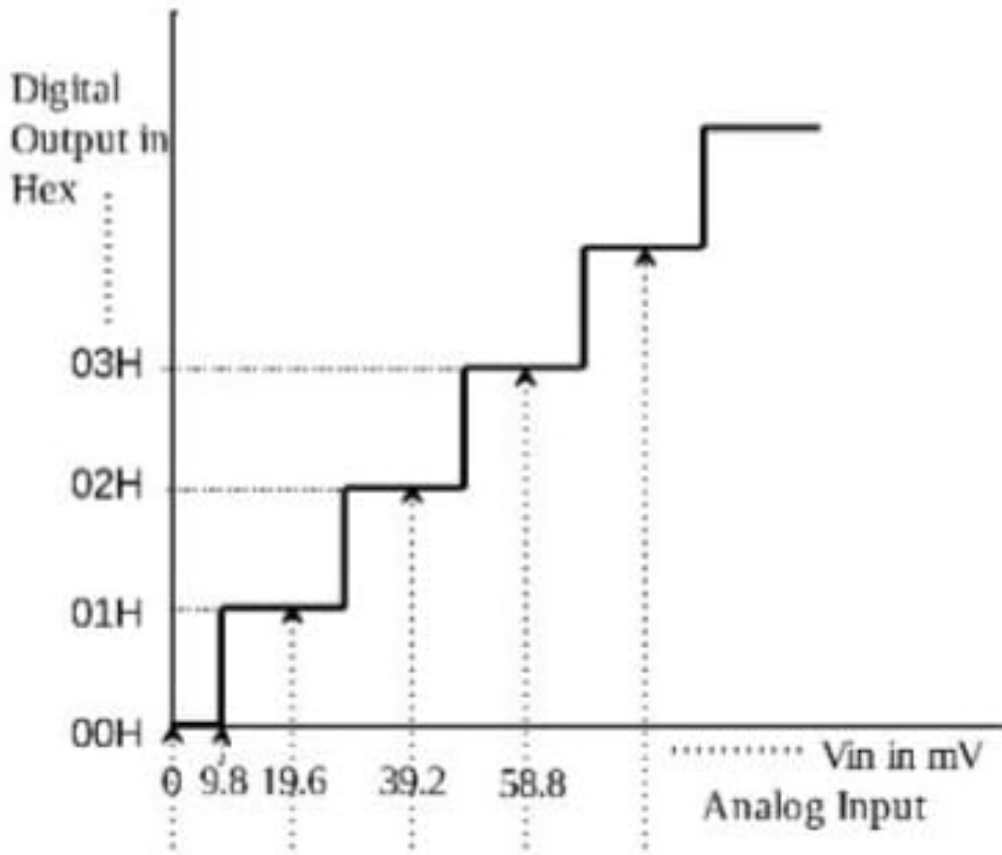


Figure 6.1:

Calculating the step-size:

$$\Delta V_{in} = \frac{V_{max} - V_{min}}{2^n - 1} \quad (6.1)$$

$$= \frac{5 - 0}{2^8 - 1} \quad (6.2)$$

$$= \frac{5}{255} \quad (6.3)$$

$$\Rightarrow V_{out} = \frac{V_{in}}{\Delta V_{in}} \quad (6.4)$$

$$= \frac{1.9922 \times 255}{5} \quad (6.5)$$

$$= 101.59 \quad (6.6)$$

$$\approx 102_{10} \quad (6.7)$$

Symbol	Value	Description
$n$	8	Number of bits of ADC
$V_{min}$	0V	Minimum Analog Voltage
$V_{max}$	5V	Maximum Analog Voltage
$V_{in}$	1.9922V	Input Voltage
$V_{out}$		Output Voltage

Table 6.1: Given Parameters

$\therefore$  correct answer is option (c).

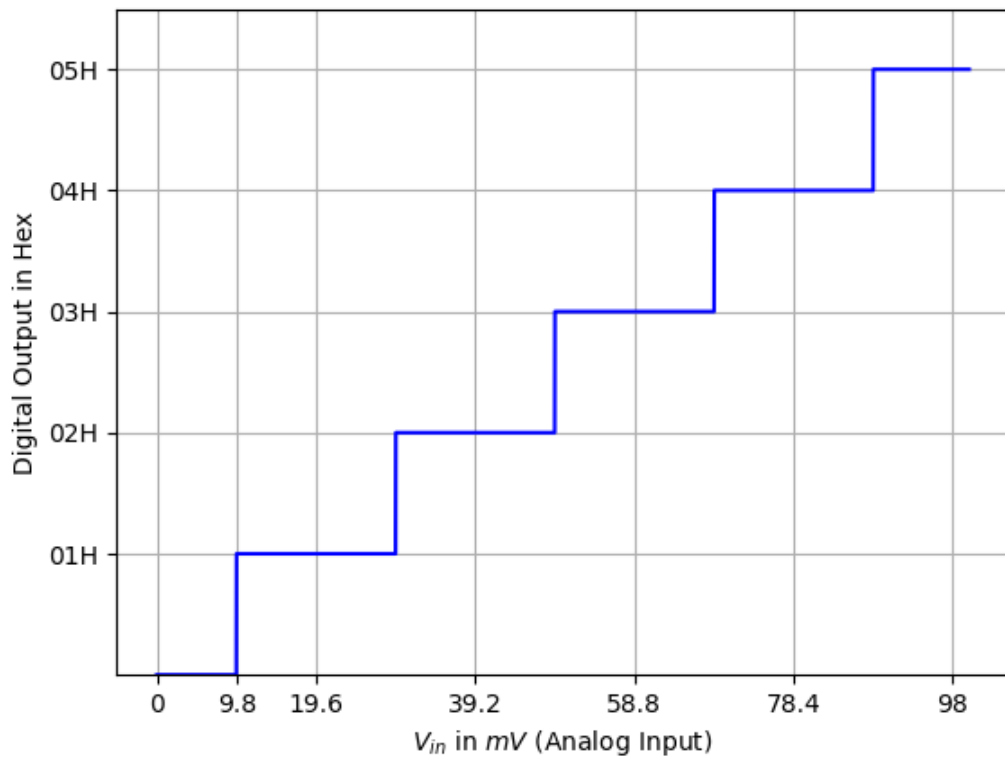
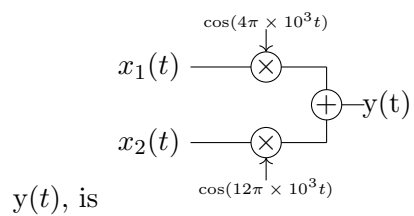


Figure 6.2:

6.2 Let  $x_1(t)$  and  $x_2(t)$  be two band-limited signals having bandwidth  $B = 4\pi \times 10^3$  rad/s each. In the figure below, the Nyquist sampling frequency, in rad/s, required to sample



(a)  $20\pi \times 10^3$



(b)  $40\pi \times 10^3$

(c)  $8\pi \times 10^3$

(d)  $32\pi \times 10^3$

(GATE EC 50)

**Solution:**

6.3 An 8 bit successive approximation Analog to Digital Converter (ADC) has a clock frequency of 1 MHz. Assume that the start conversion and end conversion signals occupy one clock cycle each. Among the following options, what is the maximum frequency that this ADC can sample without aliasing?

- a) 0.9 kHz
- b) 9.9 kHz
- c) 49.9 kHz
- d) 99.9 kHz

(GATE BM 2023)

**Solution:**

6.4 The period of the discrete-time signal  $x[n]$  described by the equation below is  $N =$   
(Round off to the nearest integer).

$$x[n] = 1 + 3 \sin \left( \frac{15\pi}{8}n + \frac{3\pi}{4} \right) - 5 \sin \left( \frac{\pi}{3}n - \frac{\pi}{4} \right)$$

(GATE 2023 EE) **Solution:**



## Chapter 7

# Contour Integration

7.1 The value of the contour integral,  $\oint_C \frac{z+2}{z^2+2z+2} dz$ , where the contour  $C$  is  $\{z : |z+1-\frac{3}{2}i| = 1\}$ , taken in the counter clockwise direction, is

(A)  $-\pi(1+j)$

(B)  $\pi(1+j)$

(C)  $\pi(1-j)$

(D)  $-\pi(1-j)$

(GATE EC 2023)

**Solution:**

7.2 The function  $f(Z) = \frac{1}{Z-1}$  of a complex variable  $Z$  is integrated on a closed contour in an anti-clockwise direction. For which of the following contours, does this integral have a non-zero value?

(A)  $|z-2| = 0.01$

(B)  $|z-1| = 0.1$

(C)  $|z-3| = 5$

(D)  $|z| = 2$

(GATE 2023 BM)

**Solution:**

## Chapter 8

# Laplace Transform

8.1 The number of zeroes of the polynomial  $P(s) = s^3 + 2s^2 + 5s + 80$  in the right side of the plane? (GATE IN 2023)

**Solution:** The table below shows the Routh array of the  $n^{th}$ - order characteristic polynomial :

$$a_0s^n + a_1s^{n-1} + \dots + a_{n-1}s^1 + a_ns^0 \quad (8.1)$$

$s^n$	$a_0$	$a_2$	$a_4$	...
$s^{n-1}$	$a_1$	$a_3$	$a_5$	...
$s^{n-2}$	$b_1 = \frac{a_1a_2 - a_3a_0}{a_1}$	$b_2 = \frac{a_1a_4 - a_5a_0}{a_1}$	...	..
$s^{n-3}$	$c_1 = \frac{b_1a_3 - b_2a_1}{b_1}$	$\vdots$		
$\vdots$	$\vdots$	$\vdots$		
$s^1$	$\vdots$	$\vdots$		
$s^0$	$a_n$			

Table 8.1: Routh Array

Characteristic Equation:

$$s^3 + 2s^2 + 5s + 80 = 0 \quad (8.2)$$

From Table 8.1:

$s^3$	1	5
$s^2$	2	80
$s^1$	$\frac{2 \times 5 - 80 \times 1}{2} = -35$	
$s^0$	$\frac{-35 \times 80}{-35} = 80$	

Table 8.2:

From Table 8.2:

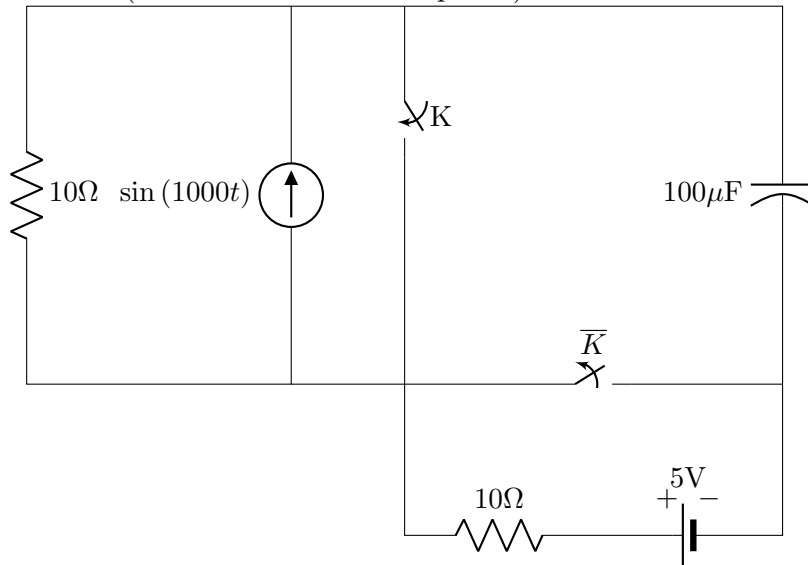
Since there are 2 sign changes in the first column of the Routh tabulation. So, the number of zeros in the right half of the s-plane will be 2.





Figure 8.1:

8.2 The circuit shown in the figure is initially in the steady state with the switch K in open condition and  $\overline{K}$  in closed condition. The switch K is closed and  $\overline{K}$  is opened simultaneously at the instant  $t = t_1$ , where  $t_1 > 0$ . The minimum value of  $t_1$  in milliseconds such that there is no transient in the voltage across the  $100\ \mu F$  capacitor, is \_\_\_\_ (Round off to 2 decimal places) (GATE EE 2023)



**Solution:**

(a) Switch K is open and  $\overline{K}$  is closed.

Using Current divider rule,

$$I_1(j\omega) = \frac{10}{10 + \frac{1}{j\omega C}} \quad (8.3)$$

$$V_1(j\omega) = \frac{10}{1 + 10j\omega C} \quad (8.4)$$

$$|V_1(j\omega)| = 5\sqrt{2} \quad (8.5)$$

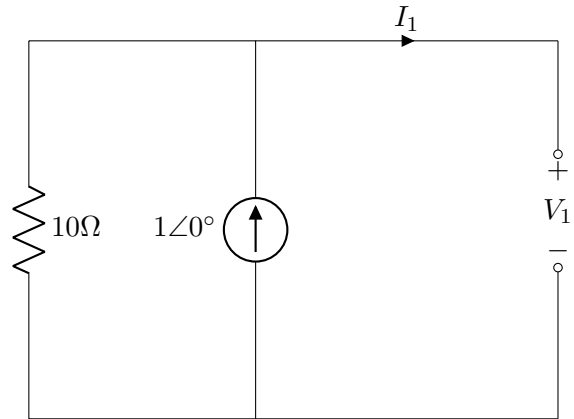


Figure 8.2: K is open and  $\overline{K}$  is closed

From Table 8.3

$$V_1(t) = 5\sqrt{2} \sin\left(\omega t - \frac{\pi}{4}\right) \quad (8.6)$$

(b) Switch K is closed and  $\overline{K}$  is open.

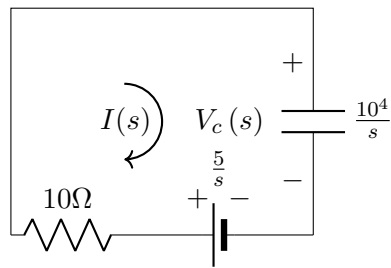


Figure 8.3: K is closed and  $\overline{K}$  is open

The capacitor is charged. Thus, acts as a voltage source.

From eq(8.6) and Table 8.4

$$V_1(s) = \frac{5000 - 5s}{s^2 + 10^6} \quad (8.7)$$

$$I(s) = \frac{\frac{5}{s} - V_1(s)}{10 + \frac{10^4}{s}} \quad (8.8)$$

$$V_c(s) = \frac{5}{s} - 10 \left( \frac{5 - V_1(s)}{1 + 10^{-3}s} \right) \quad (8.9)$$

For transient analysis,

$$\frac{5 - V_1(s)}{1 + 10^{-3}s} = 0 \quad (8.10)$$

$$\implies V_1(s) = 5 \quad (8.11)$$

$$\frac{10^7}{(s^2 + 10^6)(s + 10^3)} = \frac{5}{s} \quad (8.12)$$

$$\frac{5}{s + 10^3} + \frac{10^3 - s}{s^2 + 10^6} = \frac{5}{s} \quad (8.13)$$

$$\frac{-s}{s^2 + 10^6} + \frac{10^3}{s^2 + 10^6} + \frac{1}{s + 10^3} = \frac{1}{s} \quad (8.14)$$

From Table 8.4

$$-\cos(1000t_1) + \sin(1000t_1) + e^{-10^3 t_1} = 1 \quad (8.15)$$

$$\implies t_1 \approx 1.57 \text{msec} \quad (8.16)$$

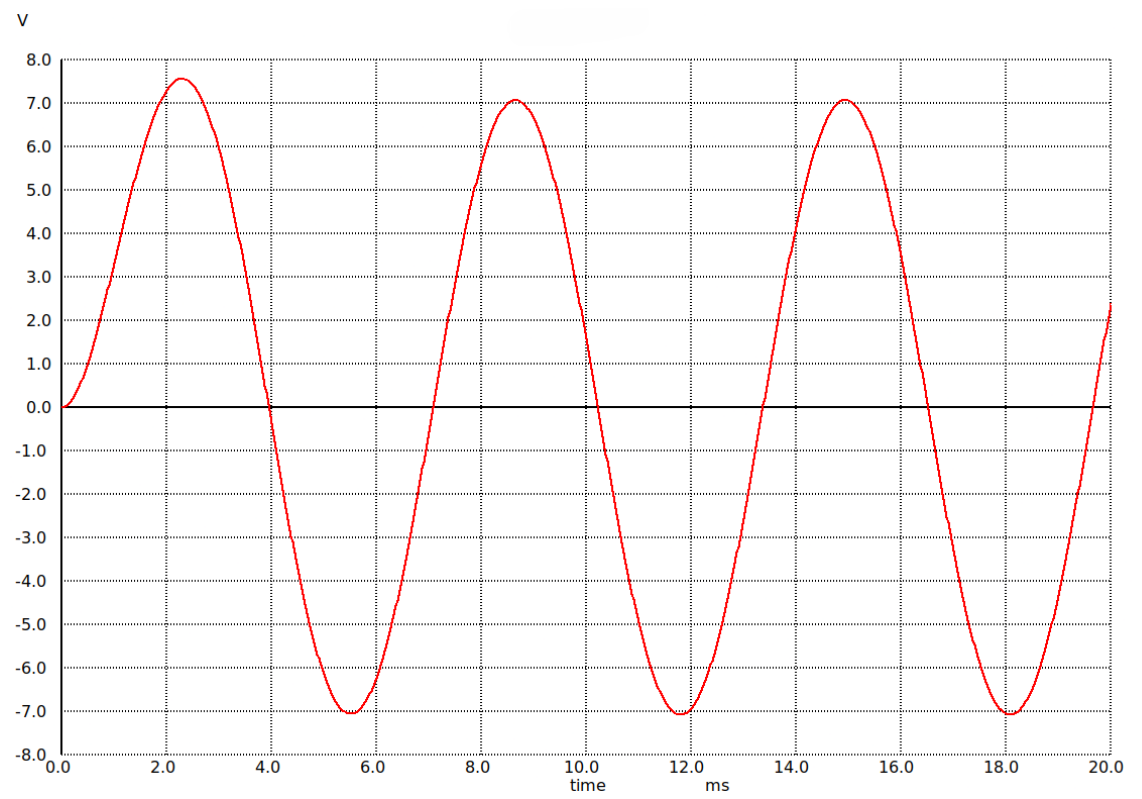


Figure 8.4: plot of  $V_1$  vs time

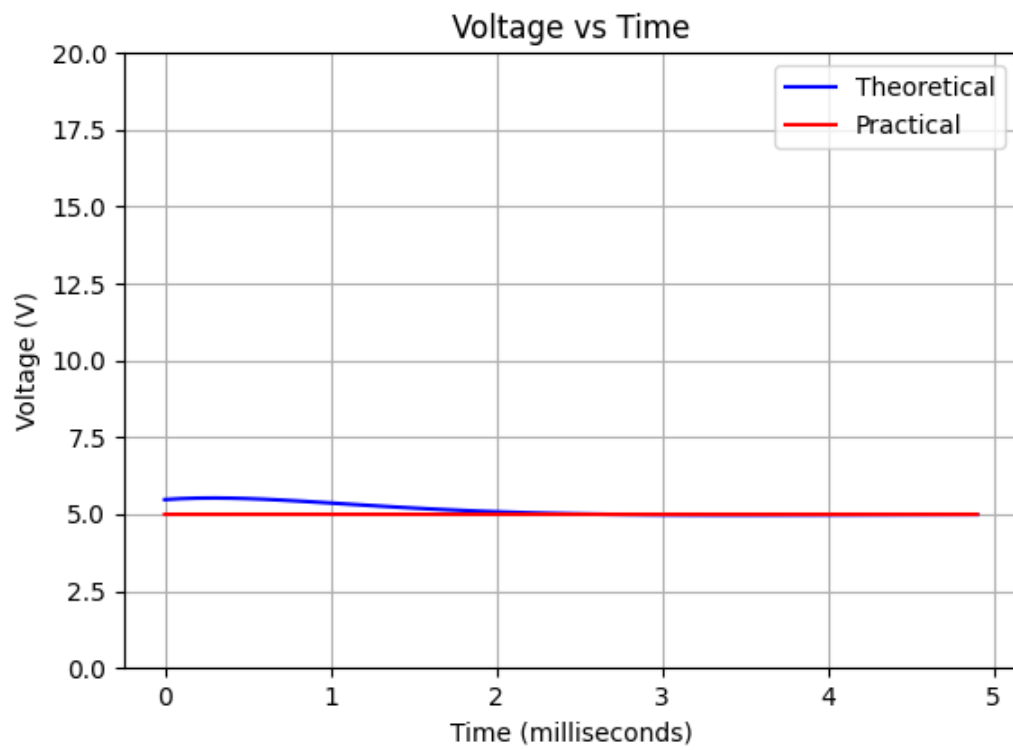


Figure 8.5: plot of  $V_c$  vs time

Parameter	Description	Remarks
$\omega$	frequency of sine-wave	1000 rad s <sup>-1</sup>
$V_1(t)$	Voltage across capacitor	$ V_1(j\omega)  \sin(\omega t - \angle V_1(j\omega))$
$\angle V_1(j\omega)$	phase of $V_1(j\omega)$	$\frac{-\pi}{4}$
$C$	Capacitance	100 $\mu$ F

Table 8.3: Parameters

S Domain	Time Domain
$\frac{1}{s}$	$u(t)$
$\frac{-s}{a^2+s^2}$	$-\cos(at)$
$\frac{a}{a^2+s^2}$	$\sin(at)$
$\frac{1}{s+a}$	$e^{-at}$

Table 8.4: Laplace transforms

8.3  $y = e^{mx} + e^{-mx}$  is the solution of which differential equation?

1.  $\frac{dy}{dx} - my = 0$
2.  $\frac{dy}{dx} + my = 0$
3.  $\frac{d^2y}{dx^2} + m^2y = 0$
4.  $\frac{d^2y}{dx^2} - m^2y = 0$

(GATE AG 2023) **Solution:**

8.4 A cascade control strategy is shown in the figure below. The transfer function between the output ( $y$ ) and the secondary disturbance ( $d_2$ ) is defined as

$$G_{d2}(s) = \frac{y(s)}{d_2(s)}$$

. Which one of the following is the CORRECT expression for the transfer function  $G_{d2}(s)$ ?

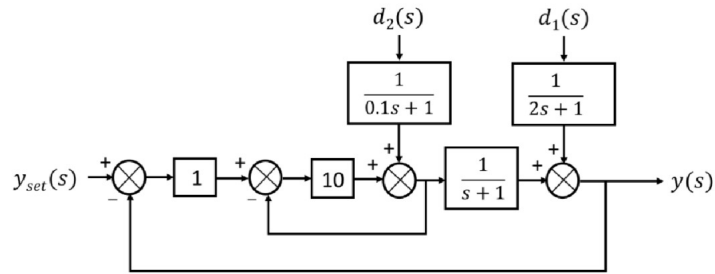


Figure 8.6:

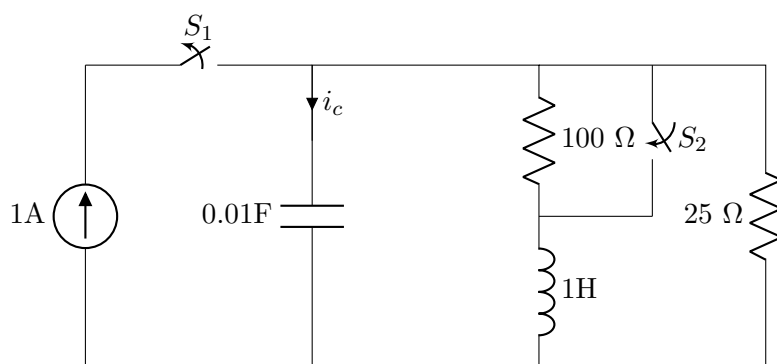
- A.  $\frac{1}{(11s+21)(0.1s+1)}$
- B.  $\frac{1}{(s+1)(0.1s+1)}$
- C.  $\frac{(s+1)}{(s+2)(0.1s+1)}$
- D.  $\frac{(s+1)}{(s+1)(0.1s+1)}$

(GATE CH 2023) **Solution:**



8.5 In the differential equation  $\frac{dy}{dx} + \alpha xy = 0$ ,  $\alpha$  is a positive constant. If  $y = 1.0$  at  $x = 0.0$ , and  $y = 0.8$  at  $x = 1.0$ , the value of  $\alpha$  is (rounded off to three decimal places). (GATE CE 2023) **Solution:**

8.6 The switch  $S_1$  was closed and  $S_2$  was open for a long time. At  $t=0$ , switch  $S_1$  is opened and  $S_2$  is closed, simultaneously. The value of  $i_c(0^+)$ , in amperes, is . (GATE EC 2023)



8.7 The continuous time signal  $x(t)$  is described by:

$$x(t) = \begin{cases} 1, & \text{if } 0 \leq t \leq 1 \\ 0, & \text{elsewhere} \end{cases} \quad (8.17)$$

If  $y(t)$  represents  $x(t)$  convolved with itself, which of the following options is/are TRUE?

A  $y(t) = 0$  for all  $t < 0$

B  $y(t) = 0$  for all  $t > 1$

C  $y(t) = 0$  for all  $t > 3$

D  $\int_{0.1}^{0.75} \frac{dy(t)}{dt} dt \neq 0$

**Solution:**

8.8 The Z-transform of a discrete signal  $x(n)$  is

$$X(z) = \frac{4z}{\left(z - \frac{1}{5}\right)\left(z - \frac{2}{3}\right)(z - 3)} \text{ with ROC} = R \quad (8.18)$$

Which one of the following statements is TRUE?

- (a) Discrete time Fourier transform of  $x[n]$  converges if  $R$  is  $|z| > 3$
- (b) Discrete time Fourier transform of  $x[n]$  converges if  $R$  is  $\frac{2}{3} < |z| < 3$
- (c) Discrete time Fourier transform of  $x[n]$  converges if  $R$  is such that  $x[n]$  is a left-sided sequence.
- (d) Discrete time Fourier transform of  $x[n]$  converges if  $R$  is such that  $x[n]$  is a right-sided sequence.

**GATE EE 2023 Solution:**

8.9 The phase margin of the transfer function  $G(s) = \frac{2(1-s)}{(1+s)^2}$  is \_\_\_\_\_ degrees. (rounded off to the nearest integer). (GATE IN 2023)

**Solution:**

8.10 Consider the second-order linear differential equation

$$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} - y = 0, \quad x \geq 1$$

with the initial conditions

$$y(x=1) = 6, \quad \left. \frac{dy}{dx} \right|_{x=1} = 2.$$

Then the value of  $y$  at  $x = 2$  is \_\_\_\_\_.

GATE ME 2023

**Solution:**

8.11 The transfer function of a measuring instrument is

$$G_m(s) = \frac{1.05}{2s + 1} \exp(-s)$$

At time  $t = 0$ , a step change of +1 unit is introduced in the input of this instrument. The time taken by the instrument to show an increase of 1 unit in its output is (rounded off to two decimal places).

(GATE CH 2023) **Solution:**

8.12 The laplace transform of  $x_1(t) = e^{-t}u(t)$  is  $X_1(s)$ , where  $u(t)$  is the unit step function. The laplace transform of  $x_2(t) = e^t u(-t)$  is  $X_2(s)$ . Which one of the following statements is TRUE?

- (a) The region of convergence of  $X_1(s)$  is  $\text{Re}(s) \geq 0$
- (b) The region of convergence of  $X_2(s)$  is confined to the left half-plane of  $s$ .
- (c) The region of convergence of  $X_1(s)$  is confined to the right half-plane of  $s$ .
- (d) the imaginary axis in the  $s$ -plane is included in both the region of convergence of  $X_1(s)$  and the region of convergence of  $X_2(s)$ .

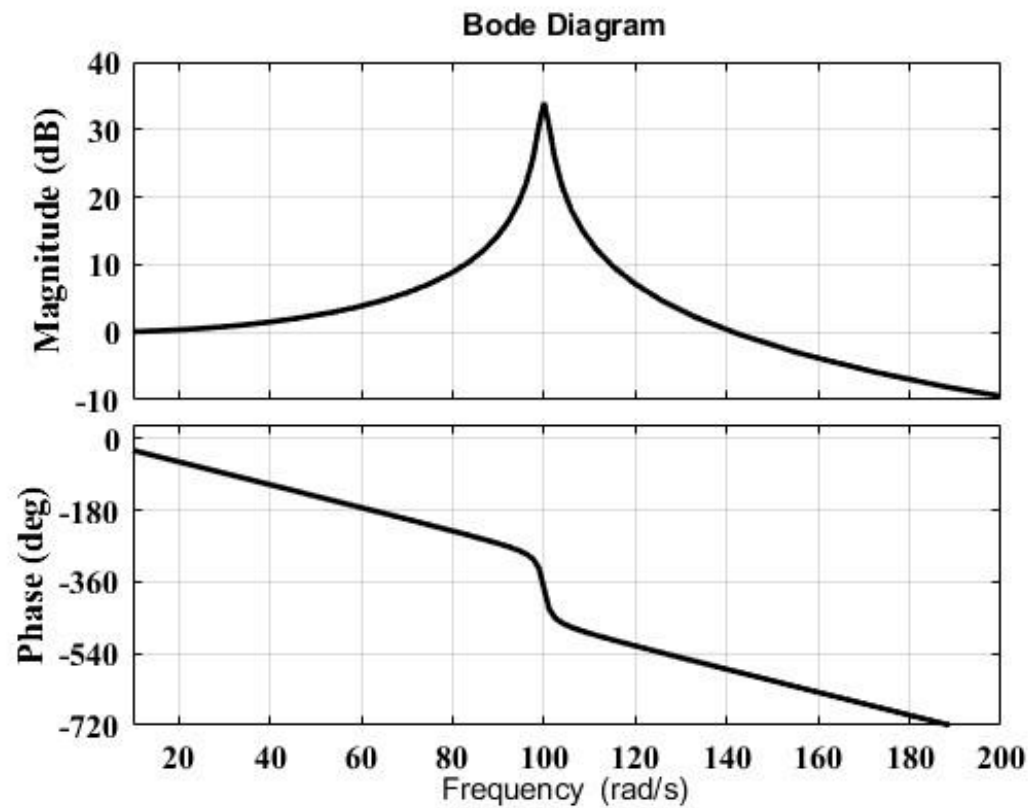
(GATE BM 2023)

**Solution:**

- 8.13 Given that  $\frac{dy}{dx} = 2x + y$  and  $y = 1$ , when  $x = 0$  Using Runge-Kutta fourth order method, the value of  $y$  at  $x = 0.2$  is (GATE 2023 AG 50)

**Solution:**

- 8.14 The magnitude and phase plots shown in the figure match with the transfer- function



(a)  $\frac{10000}{s^2 + 2s + 10000}$

(b)  $\frac{10000}{s^2 + 2s + 10000} e^{-0.05s}$



$$(c) \frac{10000}{s^2+2s+10000} e^{-0.5 \times 10^{-12} s}$$

$$(d) \frac{100}{s^2+2s+100}$$

(GATE IN 2023) **Solution:**

8.15 The Laplace transform of the continuous-time signal  $x(t) = e^{-3t}u(t-5)$  is \_\_\_\_\_, where  $u(t)$  denotes the continuous-time unit step signal.

A)  $\frac{e^{-5s}}{s+3}$ ,  $\text{Real}\{s\} > -3$

B)  $\frac{e^{-5(s-3)}}{s-3}$ ,  $\text{Real}\{s\} > 3$

C)  $\frac{e^{-5(s+3)}}{s+3}$ ,  $\text{Real}\{s\} > -3$

D)  $\frac{e^{-5(s-3)}}{s+3}$ ,  $\text{Real}\{s\} > -3$

**Solution:**

8.16 The solution  $x(t), t \geq 0$ , to the differential equation  $\ddot{x} = -k\dot{x}, k > 0$  with initial conditions  $x(0) = 1$  and  $\dot{x}(0) = 0$  is

**Solution:**

8.17 Consider the differential equation

$$x^2 \frac{d^2 y}{dx^2} + 4x \frac{dy}{dx} + 2y = 0 \quad \text{for } x \geq 1$$

with initial conditions  $y = 0$  and  $\frac{dy}{dx} = 1$  at  $x = 1$ . The value of  $y$  at  $x = 2$  is ?

(GATE AE 2023) **Solution:**

## Chapter 9

# Fourier transform

9.1 The discrete-time Fourier transform of a signal  $x[n]$  is  $X(\Omega) = (1 + \cos \Omega) e^{-j\Omega}$ .

Consider that  $x_p[n]$  is a periodic signal of period  $N = 5$  such that

$$x_p[n] = x[n], \text{ for } n = 0, 1, 2 \quad (9.1)$$

$$= 0, \text{ for } n = 3, 4 \quad (9.2)$$

Note that  $x_p[n] = \sum_{k=0}^{N-1} a_k e^{j\frac{2\pi}{N}kn}$ . The magnitude of the Fourier series coefficient  $a_3$  is \_\_\_\_\_ (Round off to 3 decimal places). (GATE EE 2023) **Solution:**

9.2 Let a frequency modulated (FM) signal :  $x(t) = A \cos(\omega_c t + k_f \int_{-\infty}^t m(\lambda) d\lambda)$  , where  $m(t)$  is a message signal of bandwidth  $W$ . It is passed through a non-linear system with output  $y(t) = 2x(t) + 5(x(t))^2$ . Let  $B_T$  denote the FM bandwidth. The minimum value of  $\omega_c$  required to recover  $x(t)$  from  $y(t)$  is:

(A)  $B_T + W$

(B)  $\frac{3}{2} B_T$

(C)  $2B_T + W$

(D)  $\frac{5}{2} B_T$

**Solution:**

- 9.3 Let an input  $x[n]$  having discrete-time Fourier transform  $X(e^{j\Omega}) = 1 - e^{-j\Omega} + 2e^{-3j\Omega}$  be passed through an LTI system. The frequency response of the LTI system is  $H(e^{j\Omega}) = 1 - \frac{1}{2}e^{-2j\Omega}$ . The output  $y[n]$  of the system is
- (GATE EC 2023) **Solution:**

9.4 The Fourier transform  $X(\omega)$  of  $x(t) = e^{-t^2}$  is

Note:  $\int_{-\infty}^{\infty} e^{-y^2} dy = \sqrt{\pi}$

A)  $\sqrt{\pi} e^{\frac{\omega^2}{2}}$

B)  $\frac{e^{-\frac{\omega^2}{4}}}{2\sqrt{\pi}}$

C)  $\sqrt{\pi} e^{-\frac{\omega^2}{4}}$

D)  $\sqrt{\pi} e^{-\frac{\omega^2}{2}}$

Gate 2023 EC Question 28



- 9.5 Let  $x(t) = 10 \cos(10.5\omega t)$  be passed through an LTI system with impulse response  $h(t) = \pi \left( \frac{\sin(\omega t)}{\pi t} \right)^2 \cos(10\omega t)$  . The output of the system is:  
(GATE EC 2023) **Solution:**

9.6 Q27) Let  $m(t)$  be a strictly band-limited signal with bandwidth  $B$  and energy  $E$ . Assuming  $\omega_0 = 10B$ , the energy in the signal  $m(t) \cos(\omega_0 t)$

(A)  $\frac{E}{4}$

(B)  $\frac{E}{2}$

(C)  $E$

(D)  $2E$

(GATE EC 2023)

**Solution:**

9.7 The following function is defined over the interval  $[-L, L]$  :

$$f(x) = px^4 + qx^5$$

It is expressed as a Fourier series,

$$f(x) = a_0 + \sum_{n=1}^{\infty} \left\{ a_n \sin\left(\frac{\pi x}{L}\right) + b_n \cos\left(\frac{\pi x}{L}\right) \right\}$$

which options amongst the following are true?

- (a)  $a_n, n = 1, 2, \dots, \infty$  depend on  $p$
- (b)  $a_n, n = 1, 2, \dots, \infty$  depend on  $q$
- (c)  $b_n, n = 1, 2, \dots, \infty$  depend on  $p$
- (d)  $b_n, n = 1, 2, \dots, \infty$  depend on  $q$

(GATE 2023 CE Question 25)

**Solution:**

9.8 A continuous real-valued signal  $x(t)$  has finite positive energy and  $x(t) = 0, \forall t < 0$ . From the list given below, select ALL the signals whose continuous-time Fourier transform is purely imaginary.

- (a)  $x(t) + x(-t)$
- (b)  $x(t) - x(-t)$
- (c)  $j(x(t) + x(-t))$
- (d)  $j(x(t) - x(-t))$

(GATE IN 2023)

**Solution:**

9.9 Let  $x_1(t) = u(t + 1.5) - u(t - 1.5)$  and  $x_2(t)$  is shown in the figure below. For  $y(t) = x_1(t) * x_2(t)$ , the  $\int_{-\infty}^{\infty} y(t) dt$  is \_\_\_\_\_.

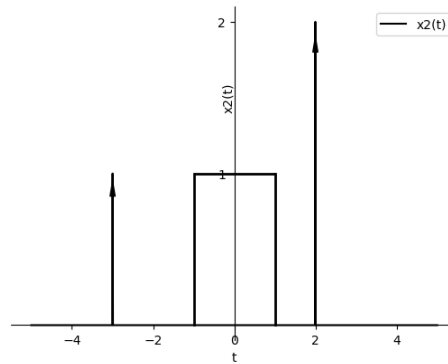


Figure 9.1: Figure

(GATE IN 2023)

**Solution:**

9.10 Consider a discrete-time signal with period  $N = 5$ . Let the discrete-time Fourier series (DTFS) representation be  $x[n] = \sum_{k=0}^4 a_k e^{\frac{jk2\pi n}{5}}$  where  $a_0 = 1$ ,  $a_1 = 3j$ ,  $a_2 = 2j$ ,  $a_3 = -2j$ ,  $a_4 = -3j$ . The value of the sum  $\sum_{n=0}^4 x[n] \sin\left(\frac{4\pi n}{5}\right)$  is

(A) -10

(B) 10

(C) -2

(D) 2

Gate 2023 EC 47 **Solution:**

9.11 A continuous time, band-limited signal  $x(t)$  has its Fourier transform described by:

$$X(f) = \begin{cases} 1 - \frac{|f|}{200} & \text{if } |f| \leq 200 \\ 0 & \text{if } |f| > 200 \end{cases}$$

The signal is uniformly sampled at a sampling rate of 600 Hz. The Fourier transform of the signal is  $X_s(f)$ . What is the value of  $\frac{X_s(600)}{X_s(500)}$ ?

(GATE 2023 BM) **Solution:**

9.12 The magnitude and phase plots of an LTI systems are shown in figure. Find the transfer function.

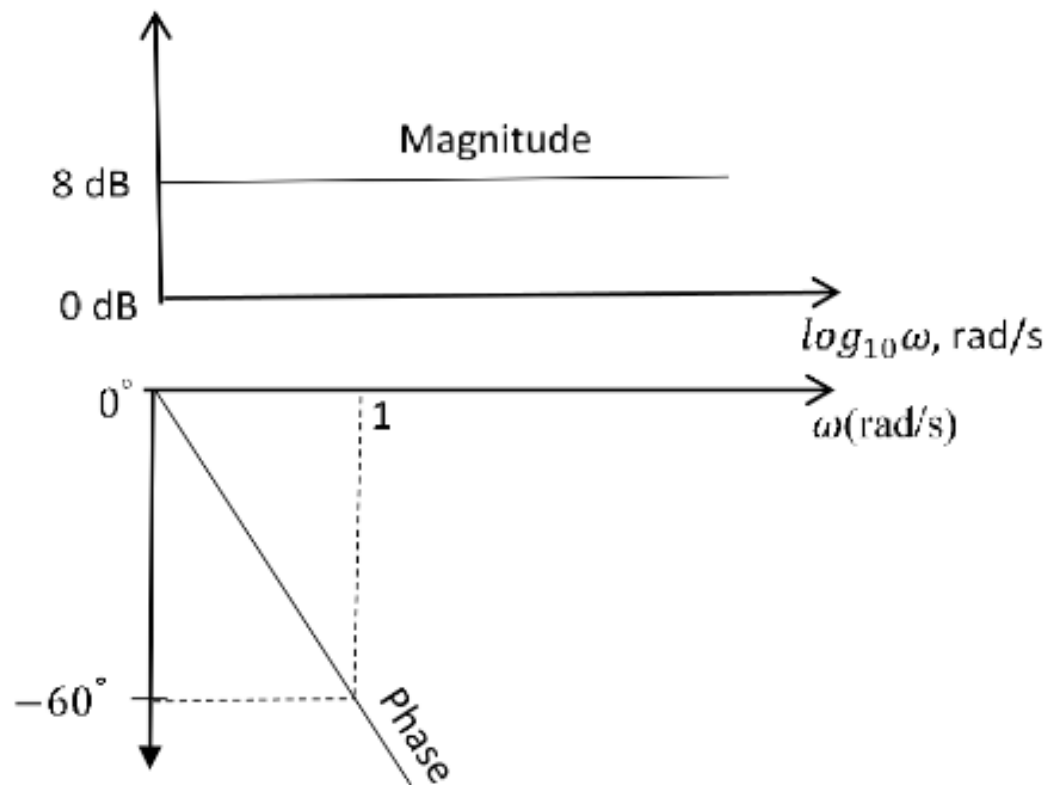


Figure 9.2:

(a)  $2.511e^{-0.0032s}$

(b)  $\frac{e^{-2.514s}}{s+1}$

(c)  $1.04e^{-2.514s}$

(d)  $2.511e^{-1.047s}$

(GATE EE 23)

**Solution:**



9.13 The value of the convolution of  $f(x) = 3\cos(2x)$  and  $g(x) = \frac{1}{3}\sin(2x)$  where  $x \in [0, 2\pi)$ , at  $x = \frac{\pi}{3}$ , is (Rounded off to 2 decimal places)  
(GATE 2023 GE)

**Solution:**

