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# SIGNAL PROCESSING

## Through GATE

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# Introduction

This book provides solutions to signal processing problems in GATE.



## Chapter 1

# Harmonics





## Chapter 2

# Filters



## Chapter 3

# Z-transform



## Chapter 4

# Systems

- 4.1 Consider a unity-gain negative feedback system consisting of the plant  $G(s)$  and a proportional-integral controller. Let the proportional gain and integral gain be 3 and 1, respectively. For a unit step reference input, the final values of the controller output and the plant output, respectively, are

$$G(s) = \frac{1}{(s-1)}$$

(GATE EE 2023)

**Solution:**

Parameter	Description	Value
$K_p$	Proportional Gain	3
$K_i$	Integral Gain	1
$r(t)$	Reference Input	$u(t)$
$w(t)$	Controller Output	?
$y(t)$	Plant Output	?
$e(t)$	Error Input	$r(t) - y(t)$

Table 1: Parameter Table

From the Fig. 4.1:

$$E(s) = U(s) - Y(s) \quad (4.1)$$

$$W(s) = 3E(s) + \frac{1}{s}E(s) \quad (4.2)$$

$$Y(s) = G(s)W(s) \quad (4.3)$$

Some results:

$$tx(t) \xleftrightarrow{\mathcal{L}} -\frac{dX(s)}{ds} \quad (4.4)$$

$$e^{-at}x(t) \xleftrightarrow{\mathcal{L}} X(s+a) \quad (4.5)$$

By using (4.4) and (4.5):

$$e^{-t}u(t) \xleftrightarrow{\mathcal{L}} \frac{1}{s+1}, \text{Re}(s) > -1 \quad (4.6)$$

$$te^{-t}u(t) \xleftrightarrow{\mathcal{L}} \frac{1}{(s+1)^2}, \text{Re}(s) > -1 \quad (4.7)$$



Figure 4.1: Block Diagram of System

(a) **Plant Output:**

From (4.1) , (4.2) and (4.3):

$$Y(s) = \frac{3s+1}{s(s+1)^2}, \text{Re}(s) > -1 \quad (4.8)$$

Final Value Theorem:

$$\lim_{t \rightarrow \infty} x(t) = \lim_{s \rightarrow 0} sX(s) \quad (4.9)$$

Using (4.9) on Y(s):

$$\lim_{t \rightarrow \infty} y(t) = \lim_{s \rightarrow 0} sY(s) \quad (4.10)$$

$$= 1 \quad (4.11)$$

Taking partial fraction of (4.8) :

$$Y(s) = \frac{1}{s} + \frac{2}{(s+1)^2} - \frac{1}{s+1} \quad (4.12)$$

Using (4.6) and (4.7):

$$\therefore y(t) = u(t) + 2te^{-t}u(t) - e^{-t}u(t) \quad (4.13)$$

(b) **Controller Output:**

From (4.2)

$$W(s) = \frac{3}{s} + \frac{1}{s^2} - Y(s) \left( 3 + \frac{1}{s} \right) \quad (4.14)$$

Substituting (4.8)

$$W(s) = \frac{(s-1)(3s+1)}{s(s+1)^2}, \operatorname{Re}(s) > -1 \quad (4.15)$$

Using (4.9) on  $W(s)$

$$\lim_{t \rightarrow \infty} w(t) = \lim_{s \rightarrow 0} sW(s) \quad (4.16)$$

$$= -1 \quad (4.17)$$

Taking partial fraction of equation(4.15) :

$$W(s) = -\frac{1}{s} - \frac{4}{(s+1)^2} + \frac{4}{s+1} \quad (4.18)$$

Using equations (4.6) and (4.7) and taking inverse lapalace transform:

$$w(t) = -u(t) - 4te^{-t}u(t) + 4e^{-t}u(t) \quad (4.19)$$





Figure 4.2:  $w(t)$  converges at -1.

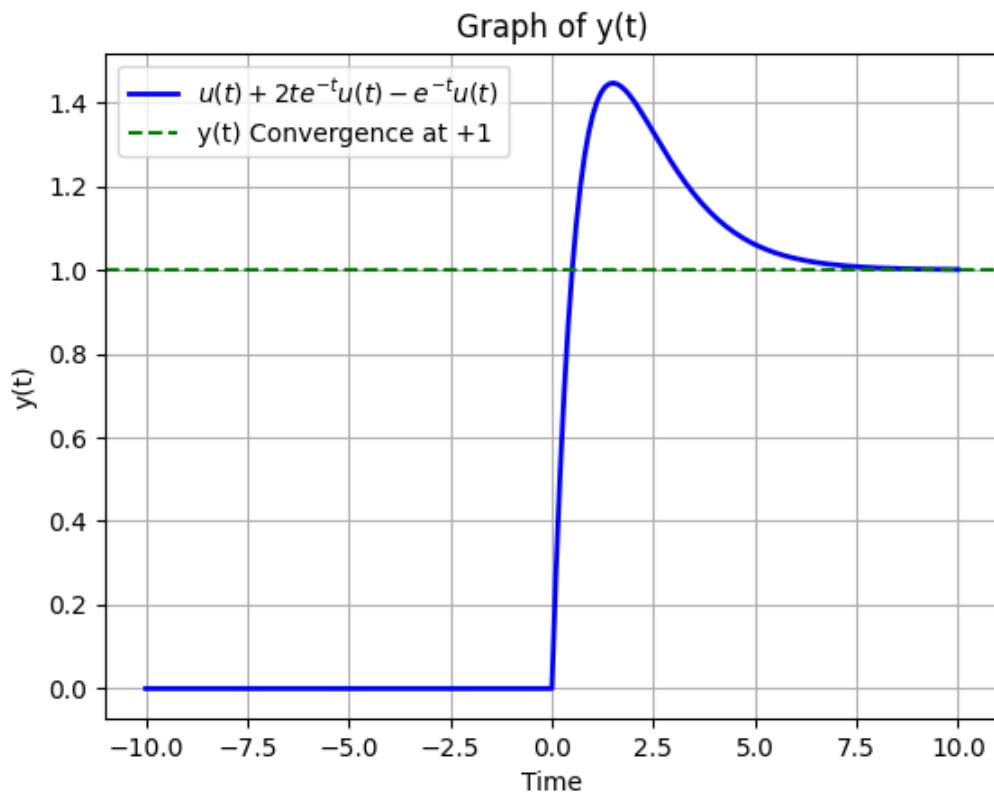


Figure 4.3:  $y(t)$  converges at  $+1$

## Chapter 5

# Sequences

5.1 Consider the discrete time signal  $x[n] = u[-n + 5] - u[n + 3]$ , where

$$u[n] = \begin{cases} 1; n \geq 0 \\ 0; n < 0 \end{cases}$$

The smallest  $n$  for which  $x[n] = 0$  is?

**Solution:** From Fig. 1, the minimum value of  $n$  is given as

$$n = -3 \tag{5.1}$$

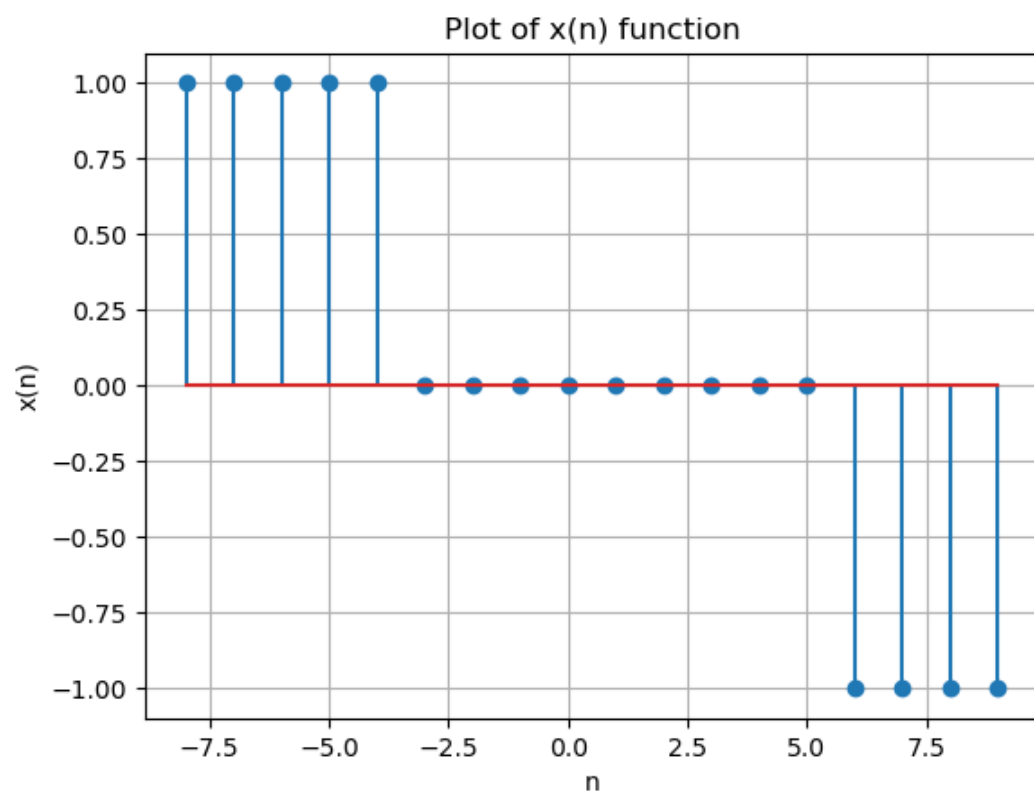


Figure 1: Plot of function  $x(n)$  taken from python3

## Chapter 6

# Contour Integration



## Chapter 7

# Laplace Transform

7.1 The number of zeroes of the polynomial  $P(s) = s^3 + 2s^2 + 5s + 80$  in the right side of the plane? (GATE IN 2023)

**Solution:** The table below shows the Routh array of the  $n^{th}$ - order characteristic polynomial :

$$a_0s^n + a_1s^{n-1} + \dots + a_{n-1}s^1 + a_ns^0 \quad (7.1)$$

$s^n$	$a_0$	$a_2$	$a_4$	...
$s^{n-1}$	$a_1$	$a_3$	$a_5$	...
$s^{n-2}$	$b_1 = \frac{a_1a_2 - a_3a_0}{a_1}$	$b_2 = \frac{a_1a_4 - a_5a_0}{a_1}$	...	..
$s^{n-3}$	$c_1 = \frac{b_1a_3 - b_2a_1}{b_1}$	$\vdots$		
$\vdots$	$\vdots$	$\vdots$		
$s^1$	$\vdots$	$\vdots$		
$s^0$	$a_n$			

Table 7.1: Routh Array

Characteristic Equation:

$$s^3 + 2s^2 + 5s + 80 = 0 \quad (7.2)$$

From Table 7.1:

$s^3$	1	5
$s^2$	2	80
$s^1$	$\frac{2 \times 5 - 80 \times 1}{2} = -35$	
$s^0$	$\frac{-35 \times 80}{-35} = 80$	

Table 7.2:

From Table 7.2:

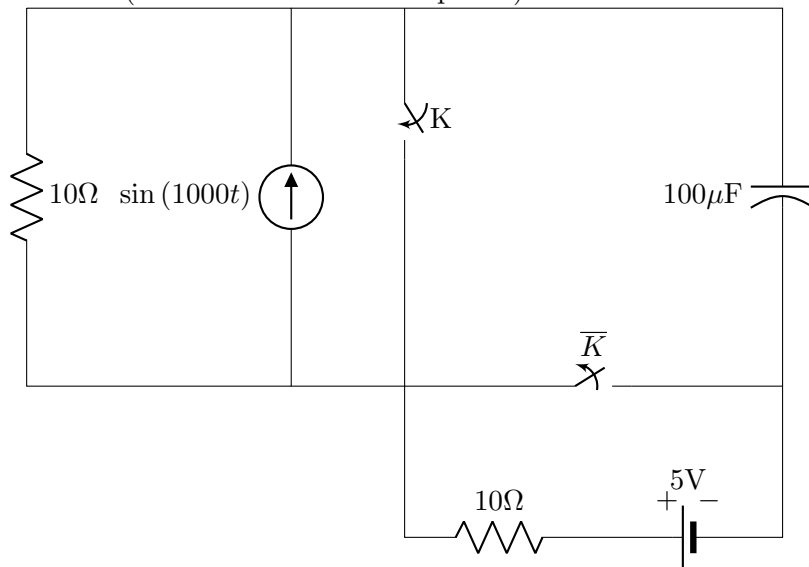
Since there are 2 sign changes in the first column of the Routh tabulation. So, the number of zeros in the right half of the s-plane will be 2.





Figure 7.1:

7.2 The circuit shown in the figure is initially in the steady state with the switch K in open condition and  $\overline{K}$  in closed condition. The switch K is closed and  $\overline{K}$  is opened simultaneously at the instant  $t = t_1$ , where  $t_1 > 0$ . The minimum value of  $t_1$  in milliseconds such that there is no transient in the voltage across the  $100\ \mu\text{F}$  capacitor, is \_\_\_\_ (Round off to 2 decimal places) (GATE EE 2023)



7.3  $y = e^{mx} + e^{-mx}$  is the solution of which differential equation?

1.  $\frac{dy}{dx} - my = 0$
2.  $\frac{dy}{dx} + my = 0$
3.  $\frac{d^2y}{dx^2} + m^2y = 0$
4.  $\frac{d^2y}{dx^2} - m^2y = 0$

(GATE AG 2023) **Solution:**

