GATE-EE-Q14

EE23BTECH11015 - DHANUSH V NAYAK*

Question: Consider a unity-gain negative feedback system consisting of the plant $G(s)$ and a proportional-integral controller. Let the proportional gain and integral gain be 3 and 1, respectively. For a unit step reference input, the final values of the controller output and the plant output, respectively, are

$$G(s) = \frac{1}{(s-1)}$$

Solution:

Parameter	Description	Value
K_p	Proportional Gain	3
K_i	Integral Gain	1
$r(t)$	Reference Input	$u(t)$
$w(t)$	Controller Output	?
$y(t)$	Plant Output	?
$e(t)$	Error Input	$r(t)-y(t)$

TABLE 1
PARAMETER TABLE

From theFig. 1:

$$E(s) = U(s) - Y(s) \tag{1}$$

$$W(s) = 3E(s) + \frac{1}{s}E(s) \tag{2}$$

$$Y(s) = G(s)W(s) \tag{3}$$

Some results:

$$tx(t) \overset{\mathcal{L}}{\longleftrightarrow} -\frac{dX(s)}{ds} \tag{4}$$

$$e^{-at}x(t) \overset{\mathcal{L}}{\longleftrightarrow} X(s+a) \tag{5}$$

By using (4) and (5):

$$e^{-t}u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s+1}, Re(s) > -1 \tag{6}$$

$$te^{-t}u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{(s+1)^2}, Re(s) > -1 \tag{7}$$

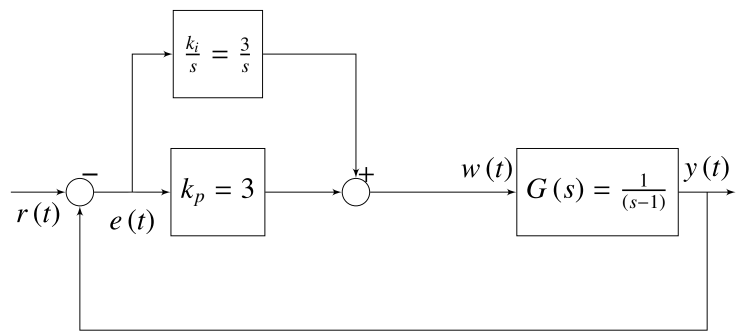

Fig. 1. Block Diagram of System

1) **Plant Output:**
From (1) , (2) and (3):

$$Y(s) = \frac{3s+1}{s(s+1)^2}, Re(s) > -1 \tag{8}$$

Final Value Theorem:

$$\lim_{t\to\infty} x(t) = \lim_{s\to 0} sX(s) \tag{9}$$

Using (9) on Y(s):

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) \tag{10}$$

$$= 1 \tag{11}$$

Taking partial fraction of (8) :

$$Y(s) = \frac{1}{s} + \frac{2}{(s+1)^2} - \frac{1}{s+1} \tag{12}$$

Using (6) and (7):

$$\therefore y(t) = u(t) + 2te^{-t}u(t) - e^{-t}u(t) \tag{13}$$

2) **Controller Output:**
From (2)

$$W(s) = \frac{3}{s} + \frac{1}{s^2} - Y(s)\left(3 + \frac{1}{s}\right) \tag{14}$$

Substituting (8)

$$W(s) = \frac{(s-1)(3s+1)}{s(s+1)^2}, Re(s) > -1 \tag{15}$$

Using (9) on W(s)

$$\lim_{t\to\infty} w(t) = \lim_{s\to 0} sW(s) \tag{16}$$

$$= -1 \tag{17}$$

Taking partial fraction of equation(15) :

$$W(s) = -\frac{1}{s} - \frac{4}{(s+1)^2} + \frac{4}{s+1} \tag{18}$$

Using equations (6) and (7) and taking inverse lapalace transform:

$$w(t) = -u(t) - 4te^{-t}u(t) + 4e^{-t}u(t) \tag{19}$$

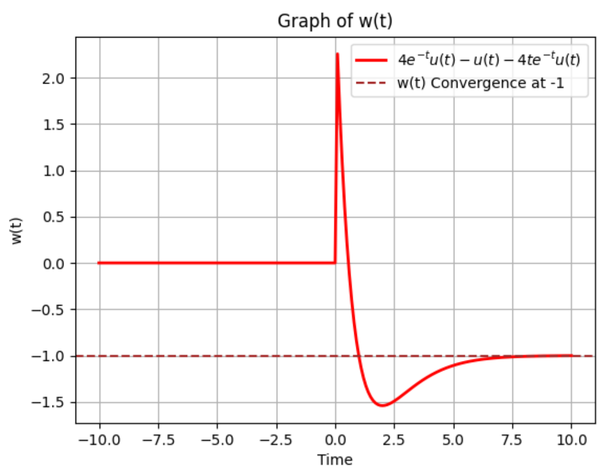

Fig. 2. $w(t)$ converges at -1.

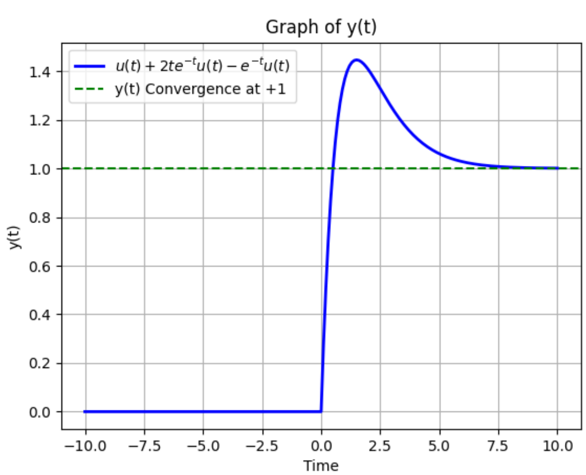

Fig. 3. $y(t)$ converges at +1