

Gate.in.21

EE22BTECH11008 - Annapureddy Siva Meenakshi*

Q: A system has transfer function

$$\frac{Y(s)}{X(s)} = \frac{s - \pi}{s + \pi}$$

let $u(t)$ be the unit step function. The input $x(t)$ that results in a steady-state output $y(t) = \sin(\pi t)$ is ____.

Solution:

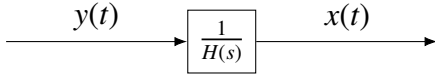


Fig. 0: Block diagram of the inverse system

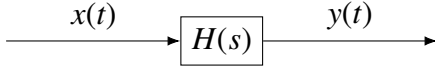


Fig. 0: Block diagram of the system

Variable	Description	Value
$x(t)$	input function	none
$y(t)$	output function	$\sin(\pi t)$
$H(s)$	Transfer-function	$\frac{s-\pi}{s+\pi}$

TABLE 0: Input parameters

$$H(s) = \frac{s - \pi}{s + \pi} \quad (1)$$

from Fig. 0

$$\frac{1}{H(s)} = \frac{s + \pi}{s - \pi} \quad (2)$$

Converting transfer function to frequency response, we get

$$\frac{1}{H(j\omega)} = \frac{j\omega + \pi}{j\omega - \pi} \quad (3)$$

from Table 0 $\omega = \pi$

$$\frac{1}{H(j\pi)} = \frac{j + 1}{j - 1} = -j = e^{-j\frac{\pi}{2}} \quad (4)$$

from (4)

$$\left| \frac{1}{H(j\pi)} \right| = 1 \quad \text{and} \quad \arg\left(\frac{1}{H(j\pi)}\right) = -90^\circ \quad (5)$$

$$y(t) = \sin(\pi t) \quad (6)$$

$$\sin(\pi t) \xrightarrow{\frac{1}{H(j\omega)}} \left| \frac{1}{H(j\omega)} \right| \sin\left(\pi t + \arg\left(\frac{1}{H(j\omega)}\right)\right) \quad (7)$$

Therefore, by (6) and (7), we get

$$x(t) = \sin\left(\pi t - \frac{\pi}{2}\right) \quad (8)$$

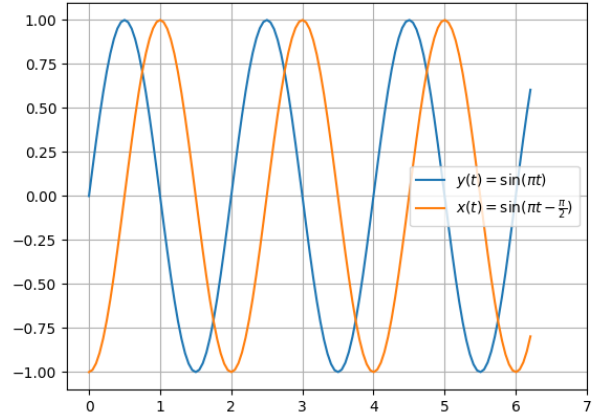


Fig. 0: Plot of $x(t)$ and $y(t)$ taken from Python

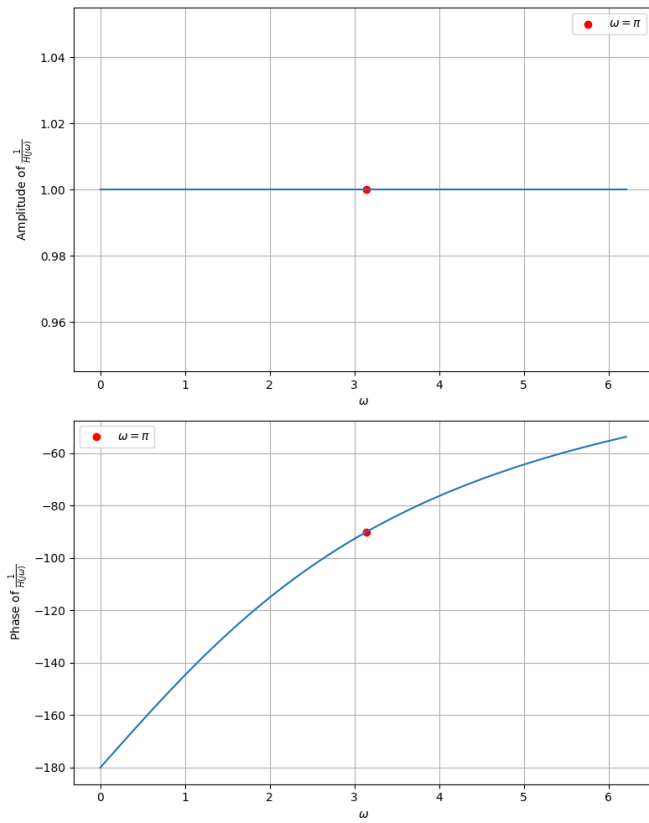


Fig. 0: Amplitude and phase of $\frac{1}{H(j\omega)}$