Assignment

11.9.5 - 22

EE23BTECH11220 - R.V.S.S Varun

QUESTION

Find the 20th term in this series.

$$2 \times 4 + 4 \times 6 + 6 \times 8 \cdots + n \text{ terms}$$

Solution:

PARAMETER	VALUE	DESCRIPTION
x (0)	8	First term
x(n)	4(n+1)n + 2u(n)	General term of the series

TABLE 0
Table of parameters

Using Z- transform,

$$n^2 u(n) \stackrel{\mathcal{Z}}{\longleftrightarrow} \frac{z^{-1} \left(z^{-1} + 1\right)}{\left(1 - z^{-1}\right)^3}, |z| > 1$$
 (1)

$$nu(n) \stackrel{\mathcal{Z}}{\longleftrightarrow} \frac{z^{-1}}{(1-z^{-1})^2}, |z| > 1$$
 (2)

$$u(n) \stackrel{\mathcal{Z}}{\longleftrightarrow} \frac{1}{(1-z^{-1})}, |z| > 1$$
 (3)

$$X(z) = \sum_{n=-\infty}^{\infty} 4(n+1)(n+2)u(n)z^{-n}$$
 (4)

$$X(z) = \sum_{n=-\infty}^{\infty} 4(n^2 + 3n + 2)u(n)z^{-n}$$
 (5)

$$X(z) = \frac{8}{(1 - z^{-1})^3}, |z| > 1$$
 (6)

$$y(n) = x(n) * u(n)$$
 (7)

$$\implies Y(z) = X(z) U(z)$$
 (8)

$$Y(z) = \left(\frac{8}{(1-z^{-1})^3}\right) \left(\frac{1}{1-z^{-1}}\right) \quad (9)$$

$$= \frac{8}{(1-z^{-1})^4}, \quad |z| > 1 \tag{10}$$

Using Contour Integration to find the inverse Z-transform,

$$y(19) = \frac{1}{2\pi i} \oint_C Y(z) z^{18} dz$$
 (11)

$$= \frac{1}{2\pi j} \oint_C \frac{8z^{18}}{(1-z^{-1})^4} dz \tag{12}$$

We can observe that the pole is repeated 4 times and thus m = 4

$$R = \frac{1}{(m-1)!} \lim_{z \to a} \frac{d^{m-1}}{dz^{m-1}} \left((z-a)^m f(z) \right)$$
 (13)

$$= \frac{1}{(3)!} \lim_{z \to 1} \frac{d^3}{dz^3} \left((z - 1)^4 \frac{8z^{22}}{(z - 1)^4} \right)$$
 (14)

$$= \frac{4}{3} \lim_{z \to 1} \frac{d^3}{dz^3} (z^{22}) \tag{15}$$

$$= 12320$$
 (16)

$$\therefore y(19) = 12320 \tag{17}$$

