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# SIGNAL PROCESSING FUNDAMENTALS Through NCERT

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# Introduction

This book introduces some concepts in signal processing through maths and physics problems in NCERT textbooks.



# Chapter 1

## Analog

### 1.1. Harmonics

1.1.1 A charged particle oscillates about its mean equilibrium position with a frequency of  $10^9 Hz$ . What is the frequency of the electromagnetic waves produced by the oscillator?

**Solution:**

Symbol	Value	Description
$y(t)$	$\cos(2\pi f_c t)$	Wave equation of electro-magnetic wave
$f_c$	$10^9$	Frequency of electromagnetic wave
$t$	seconds	Time

Table 1.1: Variable description

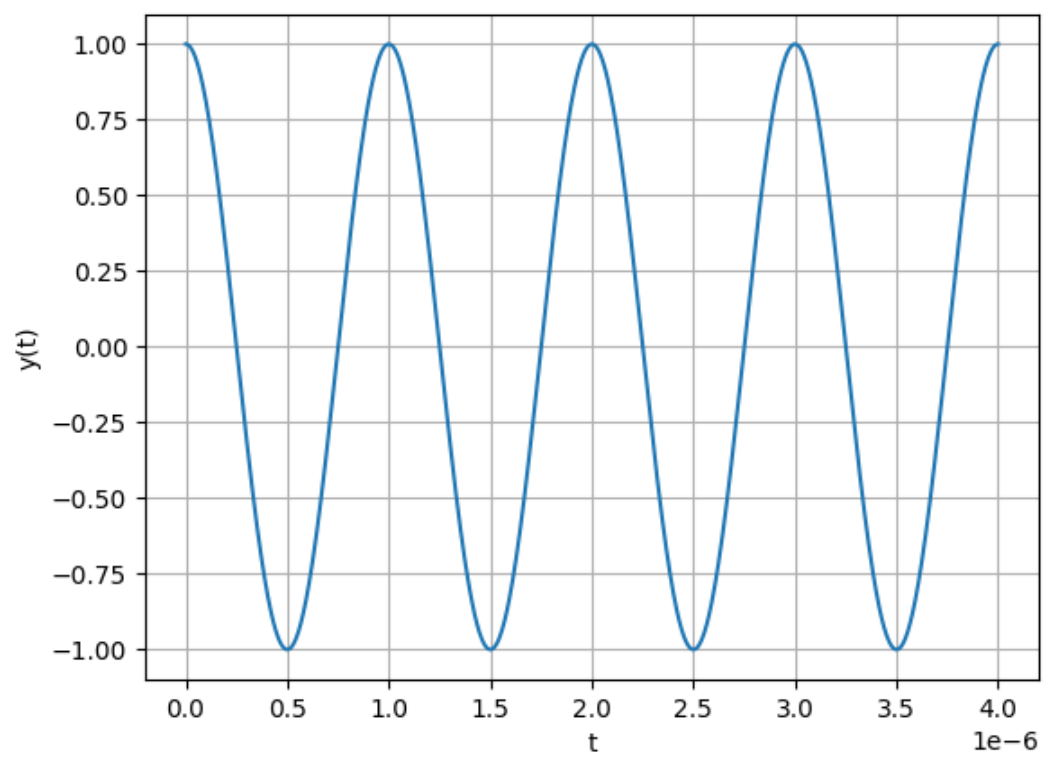


Figure 1.1:  $y(t) = \cos(2\pi \times 10^9 t)$



## Chapter 2

# Discrete

## 2.1. Z-transform

2.1.1 Write the five terms at  $n = 1, 2, 3, 4, 5$  of the sequence and obtain the Z-transform of the series

$$x(n) = -1, \quad n = 0 \quad (2.1)$$

$$= \frac{x(n-1)}{n}, \quad n > 0 \quad (2.2)$$

$$= 0, \quad n < 0 \quad (2.3)$$

**Solution:**

$$x(1) = \frac{x(0)}{1} = -1 \quad (2.4)$$

$$x(2) = \frac{x(1)}{2} = -\frac{1}{2} \quad (2.5)$$

$$x(3) = \frac{x(2)}{3} = -\frac{1}{(2)(3)} = -\frac{1}{6} \quad (2.6)$$

$$x(4) = \frac{x(3)}{4} = -\frac{1}{(2)(3)(4)} = -\frac{1}{24} \quad (2.7)$$

$$x(5) = \frac{x(4)}{5} = -\frac{1}{(2)(3)(4)(5)} = -\frac{1}{120} \quad (2.8)$$

$$x(n) = \frac{-1}{n!} (u(n)) \quad (2.9)$$

$$x(n) \xleftrightarrow{\mathcal{Z}} X(z) \quad (2.10)$$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} \quad (2.11)$$

using (2.9),

$$= \sum_{n=-\infty}^{\infty} \frac{-1}{n!} u(n) z^{-n} \quad (2.12)$$

$$= \sum_{n=0}^{\infty} \frac{-1}{n!} z^{-n} \quad (2.13)$$

$$= -e^{z^{-1}} \quad \{z \in \mathbb{C} : z \neq 0\} \quad (2.14)$$

Symbol	Value	Description
$x(n)$	$\frac{-1}{n!}$	general term of the series
$X(z)$	$-e^{z^{-1}}$	Z-transform of x(n)
$u(n)$		unit step function

Table 2.1: Parameters

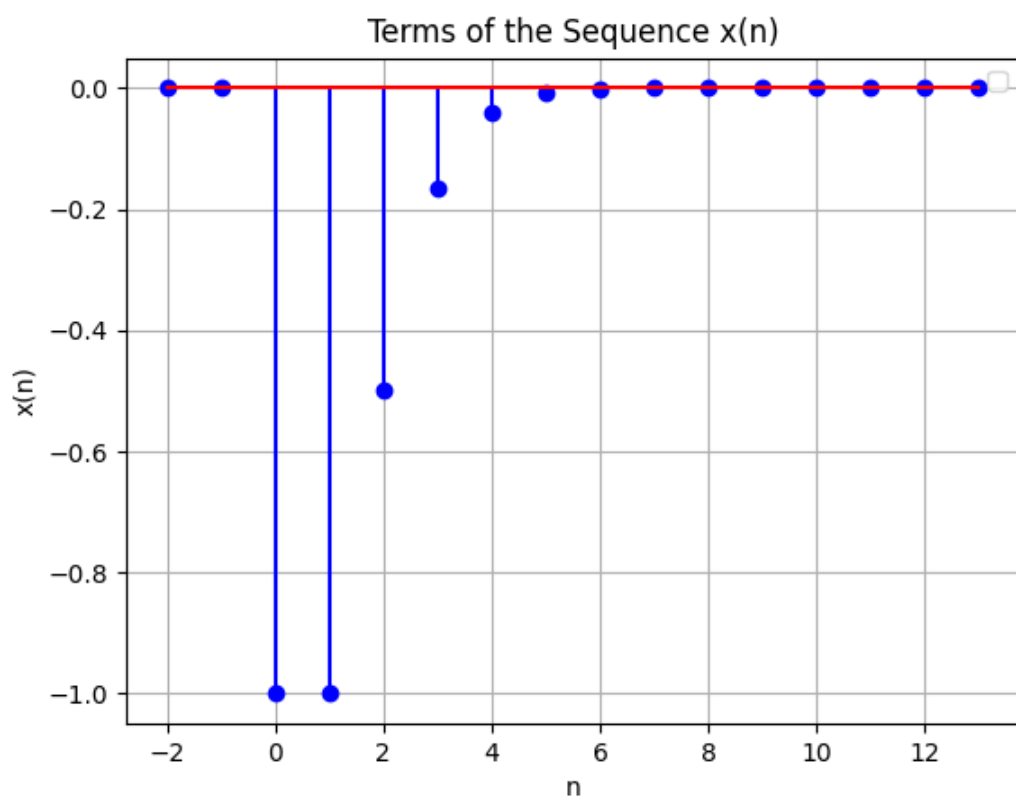


Figure 2.1: Plot of  $x(n)$  vs  $n$



## Appendix A

### Z-transform

A.1 The Z-transform of  $p(n)$  is defined as

$$P(z) = \sum_{n=-\infty}^{\infty} p(n)z^{-n} \quad (\text{A.1.1})$$

A.2 If

$$p(n) = p_1(n) * p_2(n), \quad (\text{A.2.1})$$

$$P(z) = P_1(z)P_2(z) \quad (\text{A.2.2})$$

The above property follows from Fourier analysis and is fundamental to signal processing.

### A.3 For a Geometric progression

$$x(n) = x(0) r^n u(n), \quad (\text{A.3.1})$$

$$\Rightarrow X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} = \sum_{n=0}^{\infty} x(0) r^n z^{-n} \quad (\text{A.3.2})$$

$$= \sum_{n=0}^{\infty} x(0) (rz^{-1})^n \quad (\text{A.3.3})$$

$$= \frac{x(0)}{1 - rz^{-1}}, \quad |z| > |r| \quad (\text{A.3.4})$$