SIGNAL PROCESSING FUNDAMENTALS Through NCERT

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Introduction

This book introduces some concepts in signal processing through maths and physics problems in NCERT textbooks.

Chapter 1

Analog

1.1. Harmonics

1.1.1 A charged particle oscillates about its mean equilibrium position with a frequency of $10^9 Hz$. What is the frequency of the electromagnetic waves produced by the oscillator?

Solution:

Symbol	Value	Description
$y(t)$	$\cos(2\pi f_c t)$	Wave equation of electro-magnetic wave
f_c	10^9	Frequency of electromagnetic wave
t	seconds	Time

Table 1.1: Variable description

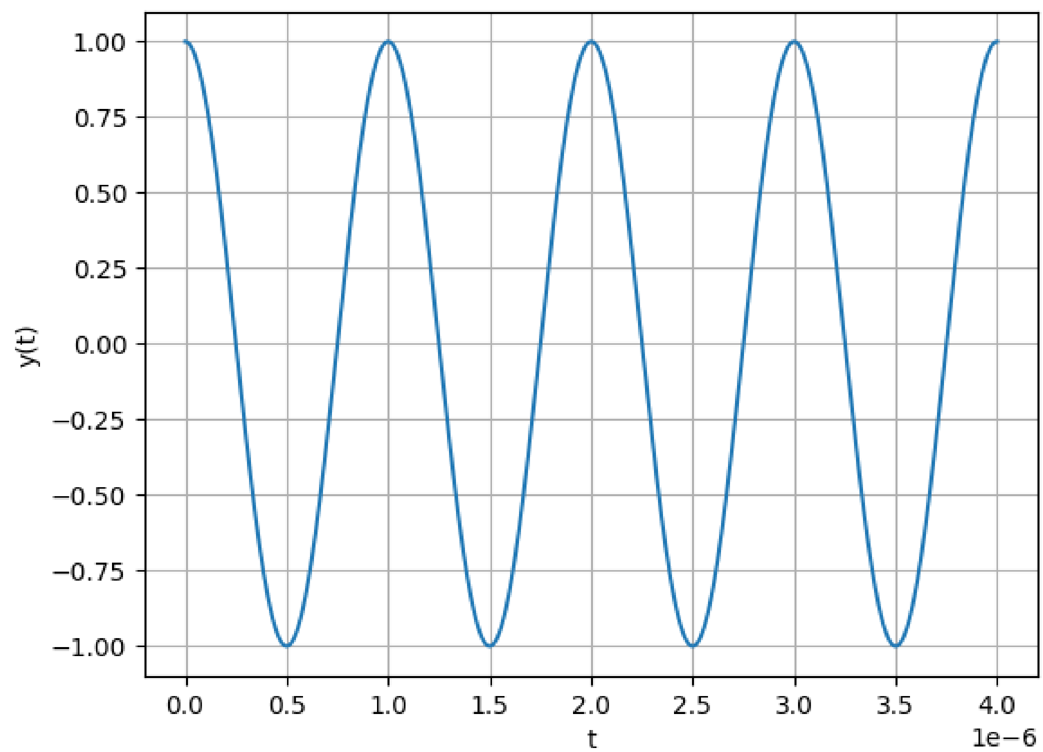

Figure 1.1: $y(t) = \cos\left(2\pi \times 10^9 t\right)$

Chapter 2

Discrete

Appendix A

Axioms

Appendix B

Z-transform

B.1 The Z-transform of $p(n)$ is defined as

$$P(z) = \sum_{n=-\infty}^{\infty} p(n) z^{-n} \tag{B.1.1}$$

B.2 If

$$p(n) = p_1(n) * p_2(n), \tag{B.2.1}$$

$$P(z) = P_1(z) P_2(z) \tag{B.2.2}$$

The above property follows from Fourier analysis and is fundamental to signal processing.

B.3 For a Geometric progression defined as follows

$$x\,(n) = x\,(0)\, r^n u\,(n) \tag{B.3.1}$$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} \tag{B.3.2}$$

$$= \sum_{n=0}^{\infty} x(0) r^n z^{-n} \tag{B.3.3}$$

$$= \sum_{n=0}^{\infty} x(0) \left(rz^{-1}\right)^n \tag{B.3.4}$$

$$= \frac{x(0)}{1 - rz^{-1}} \qquad \left|rz^{-1}\right| < 1 \tag{B.3.5}$$

$$ROC \implies |z| > |r| \tag{B.3.6}$$

B.0.1 Write the five terms at n = 1, 2, 3, 4, 5 of the sequence and obtain the Z-transform of the series

$$x(n) = -1, \qquad n = 0 \tag{B.0.1.1}$$

$$= \frac{x(n-1)}{n}, \qquad n > 0 \tag{B.0.1.2}$$

$$= 0, \qquad n < 0 \tag{B.0.1.3}$$

Solution:

$$x(1) = \frac{x(0)}{1} = -1 \tag{B.0.1.4}$$

$$x(2) = \frac{x(1)}{2} = -\frac{1}{2} \tag{B.0.1.5}$$

$$x(3) = \frac{x(2)}{3} = -\frac{1}{(2)(3)} = -\frac{1}{6} \tag{B.0.1.6}$$

$$x(4) = \frac{x(3)}{4} = -\frac{1}{(2)(3)(4)} = -\frac{1}{24} \tag{B.0.1.7}$$

$$x(5) = \frac{x(4)}{5} = -\frac{1}{(2)(3)(4)(5)} = -\frac{1}{120} \tag{B.0.1.8}$$

$$x(n) = \frac{-1}{n!}(u(n)) \tag{B.0.1.9}$$

$$x(n) \xleftrightarrow{\mathcal{Z}} X(z) \tag{B.0.1.10}$$

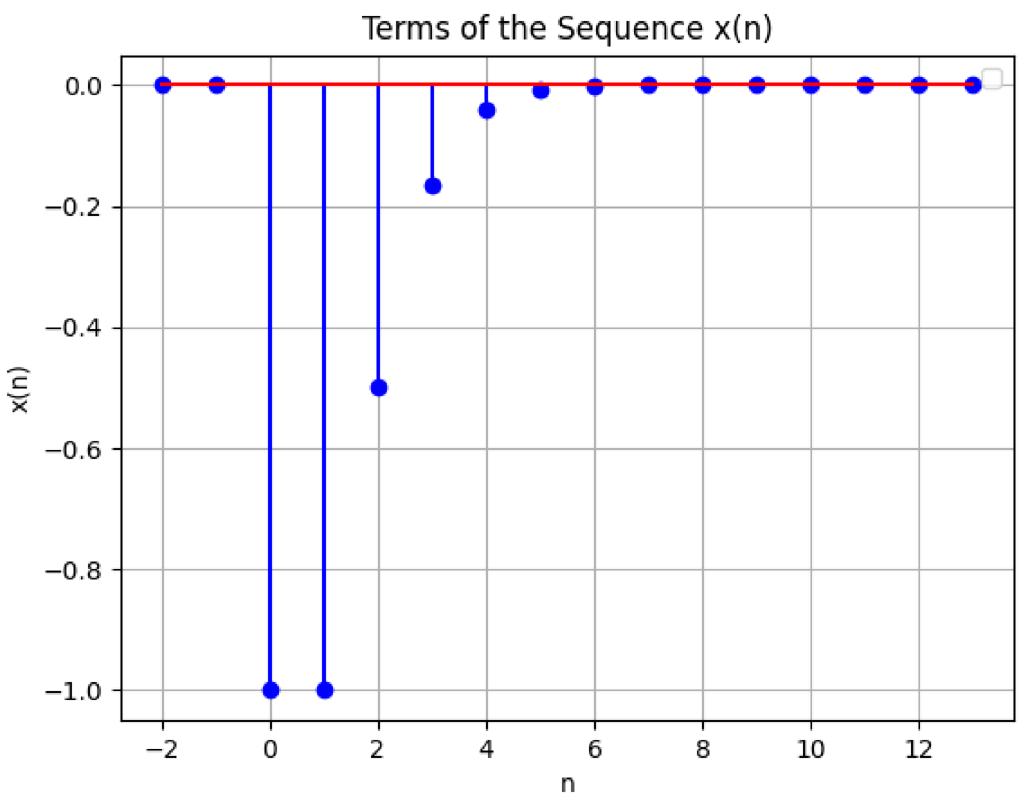

Figure B.0.1.1: Plot of x(n) vs n

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} \tag{B.0.1.11}$$

using (B.0.1.9),

$$= \sum_{n=-\infty}^{\infty} \frac{-1}{n!} u(n) z^{-n} \tag{B.0.1.12}$$

$$= \sum_{n=0}^{\infty} \frac{-1}{n!} z^{-n} \tag{B.0.1.13}$$

$$= -e^{z^{-1}} \qquad \{z \in \mathbb{C} : z \neq 0\} \tag{B.0.1.14}$$

Symbol	**Value**	**Description**
$x(n)$	$\frac{-1}{n!}$	general term of the series
$X(z)$	$-e^{z^{-1}}$	Z-transform of x(n)
$u(n)$		unit step function

Table B.0.1.1: Parameters