

# 11.14-4

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**QUESTION:** Which of the following functions of time represent (a) simple harmonic, (b) periodic but not simple harmonic, and (c) non-periodic motion? Give period for each case of periodic motion ( $\omega$  is any positive constant):

- 1)  $\sin(\omega t) - \cos(\omega t)$
- 2)  $\sin^3(\omega t)$
- 3)  $3 \cos\left(\frac{\pi}{4} - 2\omega t\right)$
- 4)  $\cos(\omega t) + \cos(3\omega t) + \cos(5\omega t)$
- 5)  $\exp(-\omega^2 t^2)$
- 6)  $1 + \omega t + \omega^2 t^2$

**Solution: :**

- 1) Periodic function:

$$x(t + T) = x(t) \quad \forall x \in \mathbb{R} \quad (1)$$

where  $\min T$  s.t  $T > 0$  is time period

- 2) SHM:

For a function to be in shm it must satisfy

$$\frac{d^2 x(t)}{dt^2} = -(2\pi f_0)^2 x(t) \quad (2)$$

Variable	Description	formula
$x(t)$	Displacemen wrt mean position	none
$\omega$	Angular frequency	$2\pi f$
$T$	Time period	$\frac{1}{f}$
$\phi$	phase angle	none

TABLE 0

INPUT PARAMETERS

1)  $\sin(2\pi ft) - \cos(2\pi ft)$

The function can be rewritten as:

$$= \sin(2\pi ft) - \sin\left(\frac{\pi}{2} - 2\pi ft\right) \quad (3)$$

$$= 2 \cos\left(\frac{\pi}{4}\right) \sin\left(2\pi ft - \frac{\pi}{4}\right) \quad (4)$$

$$= \sqrt{2} \sin\left(2\pi ft - \frac{\pi}{4}\right) \quad (5)$$

$$\frac{d^2(\sin(2\pi ft) - \cos(2\pi ft))}{dt^2} = -(2\pi f)^2(\sin(2\pi ft) - \cos(2\pi ft)) \quad (6)$$

$$\frac{d^2x(t)}{dt^2} = -(2\pi f)^2x(t) \quad (7)$$

$$(8)$$

$\therefore$  SHM,  $T$  is  $\frac{1}{f}$  and  $\phi$  is  $\left(\frac{-\pi}{4}\right)$  or  $\left(\frac{7\pi}{4}\right)$

$$\sin\left(2\pi f\left(t + \frac{1}{f}\right)\right) - \cos\left(2\pi f\left(t + \frac{1}{f}\right)\right) = \sin(2\pi ft) - \cos(2\pi ft) \quad (9)$$

Graph of function is shown in (Fig. ??)

(3)  $3 \cos\left(\frac{\pi}{4} - 4\pi ft\right)$

This function can be rewritten as

$$= 3 \cos\left(4\pi ft - \frac{\pi}{4}\right) \quad (10)$$

$$\frac{d^2\left(3 \cos\left(\frac{\pi}{4} - 4\pi ft\right)\right)}{dt^2} = -3(4\pi f)^2 \left(\cos \frac{\pi}{4} - 4\pi ft\right) \quad (11)$$

$$\frac{d^2x(t)}{dt^2} = -(4\pi f)^2x(t) \quad (12)$$

$\therefore$  SHM,  $T$  is  $\frac{1}{2f}$  and  $\phi$  is  $\left(\frac{-\pi}{4}\right)$  or  $\left(\frac{7\pi}{4}\right)$

$$3 \cos\left(\frac{\pi}{4} - 4\pi f\left(t + \frac{1}{2f}\right)\right) = 3 \cos\left(\frac{\pi}{4} - 4\pi ft\right) \quad (13)$$

$$(14)$$

Graph of function is shown in (Fig. ??)

(4)  $\cos(2\pi ft) + \cos(6\pi ft) + \cos(10\pi ft)$

This function can be rewritten as

$$= \cos(2\pi ft) + \cos(10\pi ft) + \cos(6\pi ft) \quad (15)$$

$$= 2 \cos\left(\frac{2\pi ft + 10\pi ft}{2}\right) \cos\left(\frac{10\pi ft - 2\pi ft}{2}\right) + \cos(6\pi ft) \quad (16)$$

$$= 2 \cos(6\pi ft) \cos(2\pi ft) + \cos(6\pi ft) \quad (17)$$

$$= \cos(6\pi ft)(1 + 2 \cos(2\pi ft)) \quad (18)$$

$$\frac{d^2 \cos(2\pi ft) + \cos(6\pi ft) + \cos(10\pi ft)}{dt^2} = (2\pi f)^2 \cos(2\pi ft) + (6\pi f)^2 \cos(6\pi ft) + (10\pi f)^2 \cos(10\pi ft) \quad (19)$$

$$\frac{d^2 x(t)}{dt^2} \neq -(2\pi f)^2 x(t) \quad (20)$$

Period of  $\cos(6\pi ft)$  is  $\frac{1}{3f}$

Period of  $1 + 2 \cos(2\pi ft)$  is  $\frac{1}{f}$

Lcm is  $\frac{1}{f}$

$\therefore$  SHM,  $T$  is  $\frac{1}{f}$

$$\cos\left(2\pi f\left(t + \frac{1}{f}\right)\right) + \cos\left(6\pi f\left(t + \frac{1}{f}\right)\right) + \cos\left(10\pi f\left(t + \frac{1}{f}\right)\right) = \cos(2\pi ft) + \cos(6\pi ft) + \cos(10\pi ft) \quad (21)$$

Graph of function is shown in (Fig. ??)

$$(5) \exp(-(2\pi f)^2 t^2)$$

$$\text{As } T \rightarrow \infty \quad (22)$$

$$\exp(-(2\pi f)^2 t^2) \rightarrow \infty \quad (23)$$

$$\frac{d^2(\exp(-(2\pi f)^2 t^2))}{dt^2} = 2(2\pi f)^2 \exp(-(2\pi f)^2 t^2) + 2(2\pi f)^4 \exp(-(2\pi f)^2 t^2) \quad (24)$$

$$\frac{d^2 x(t)}{dt^2} \neq -(2\pi f)^2 x(t) \quad (25)$$

$\therefore$  This never repeats and non periodic

Graph of function is shown in (Fig. ??)

$$(6) 1 + 2\pi ft + (2\pi f)^2 t^2$$

$$\text{As } T \rightarrow \infty \quad (26)$$

$$1 + 2\pi ft + (2\pi f)^2 t^2 \rightarrow \infty \quad (27)$$

$$\frac{d^2(1 + 2\pi ft + (2\pi f)^2 t^2)}{dt^2} = 2(2\pi f)^2 \quad (28)$$

$$\frac{d^2 x(t)}{dt^2} \neq -(2\pi f)^2 x(t) \quad (29)$$

∴ This never repeats and non periodic  
Graph of function is shown in (Fig. ??)

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TABLE 1  
SUMMARY

	Function	Periodic	Simple harmonic motion	Non Periodic	T	$\phi$
(a)	$\sin(2\pi ft) - \cos(2\pi ft)$	Yes	Yes	No	$\frac{1}{f}$	$\left(\frac{-\pi}{4}\right)$
(b)	$\sin^3(2\pi ft)$	Yes	No	No	$\frac{1}{f}$	–
(c)	$3 \cos\left(\frac{\pi}{4} - 4\pi ft\right)$	Yes	Yes	No	$\frac{1}{2f}$	$\left(\frac{-\pi}{4}\right)$
(d)	$\cos(2\pi ft) + \cos(6\pi ft) + \cos(10\pi ft)$	Yes	No	No	$\frac{1}{f}$	–
(e)	$\exp\left(-(2\pi ft)^2\right)$	No	No	Yes	–	–
(f)	$1 + (2\pi f)t + (2\pi f)^2 t^2$	No	No	Yes	–	–

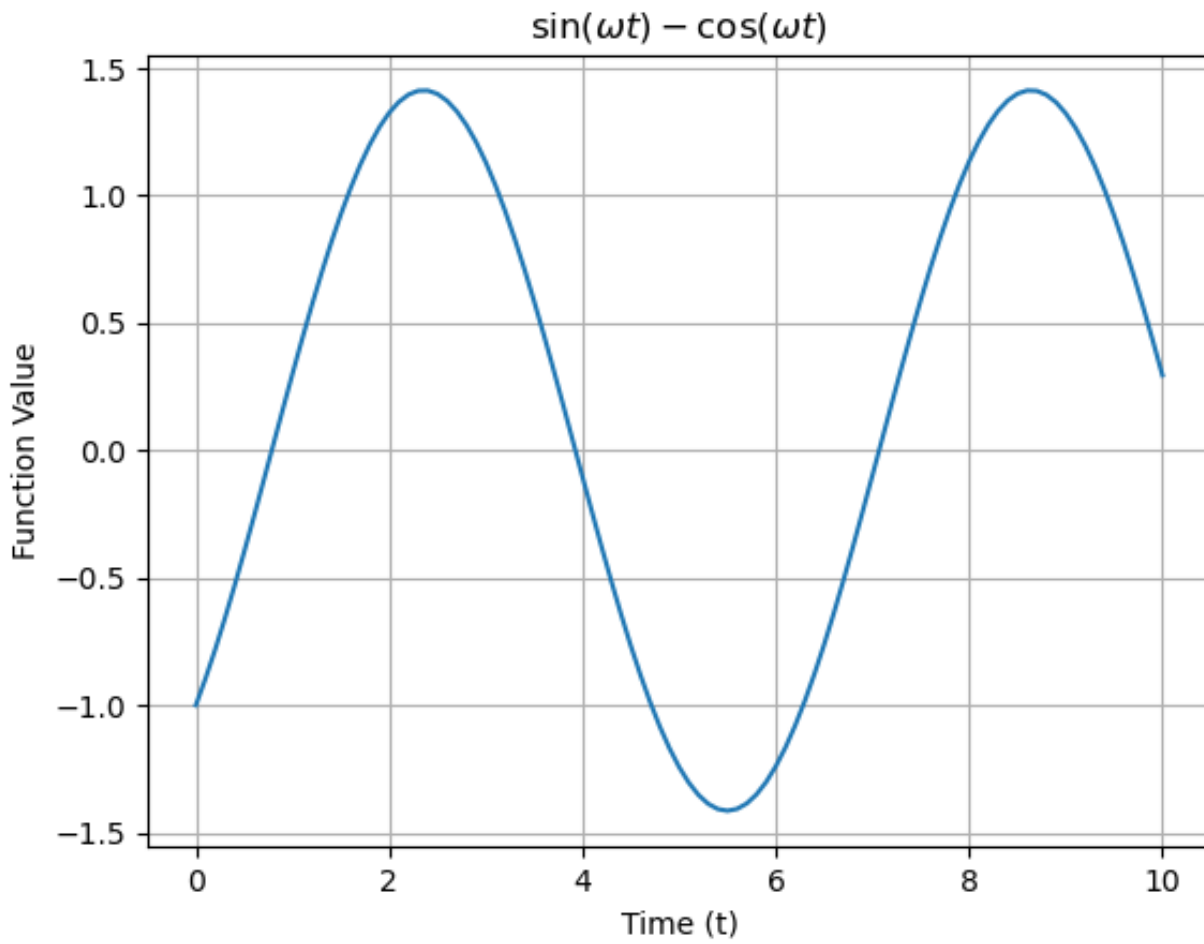


Fig. 1.  $\sin(2\pi ft) - \cos(2\pi ft)$

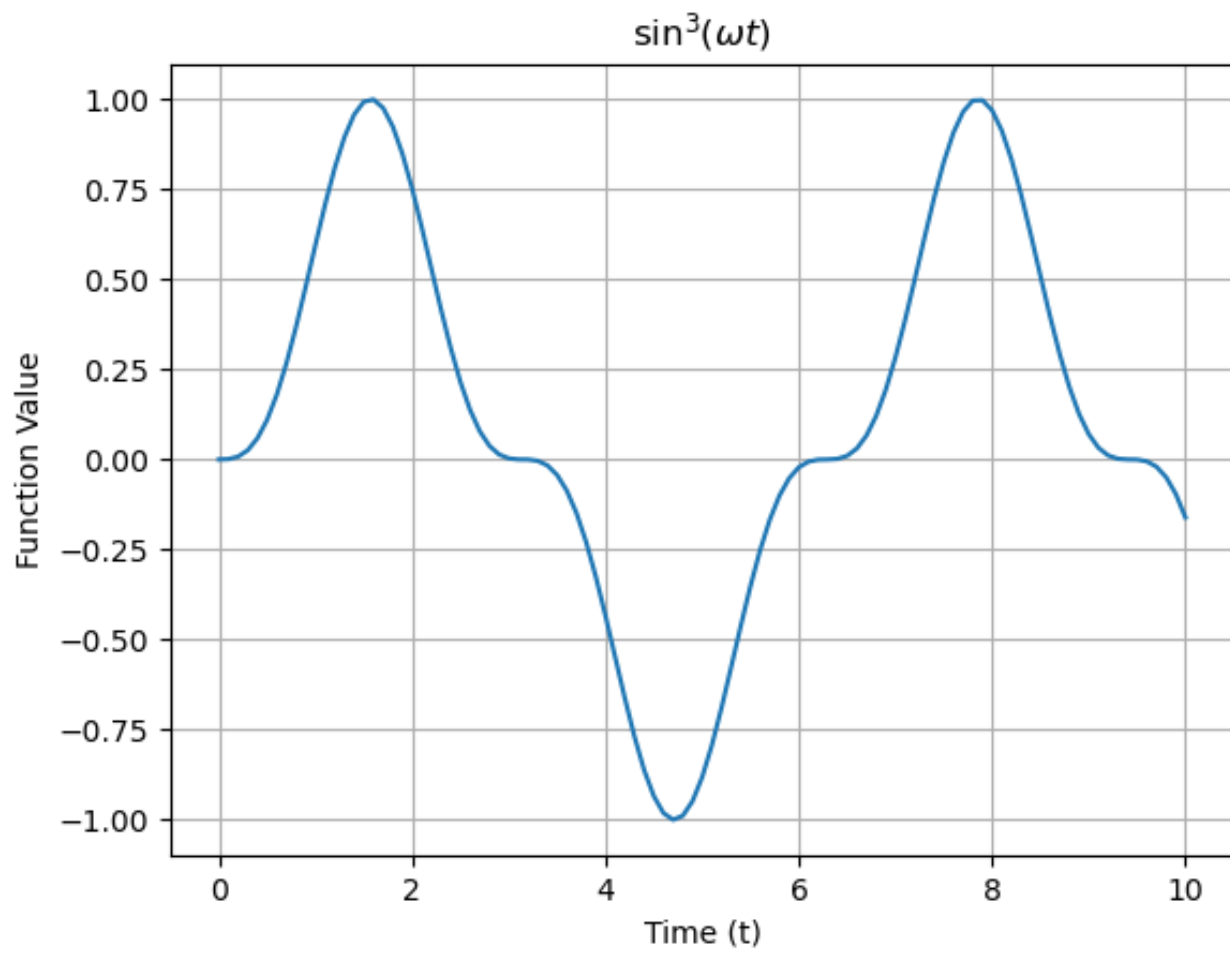


Fig. 1.  $\sin^3(2\pi ft)$

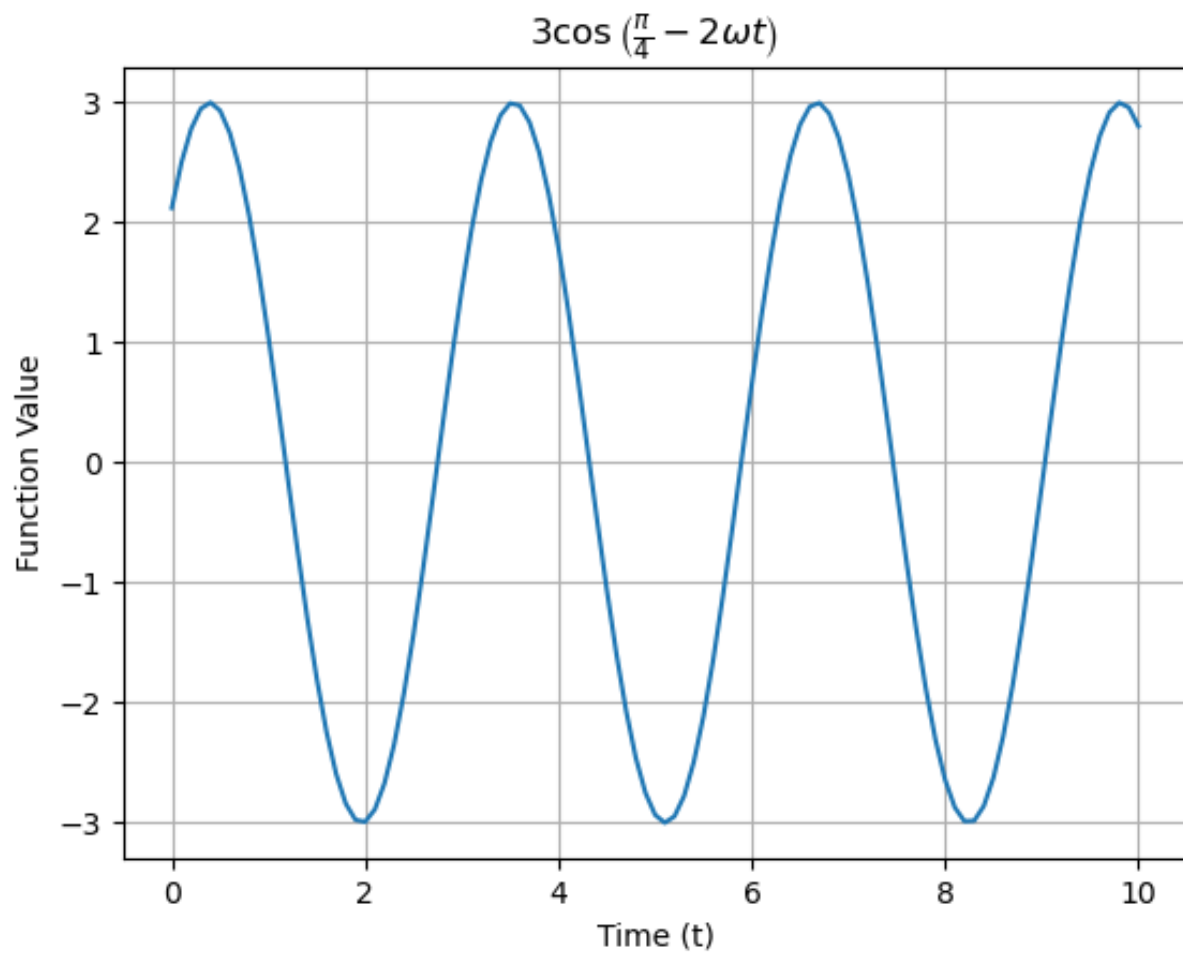


Fig. 1.  $3\cos\left(\frac{\pi}{4} - 4\pi ft\right)$

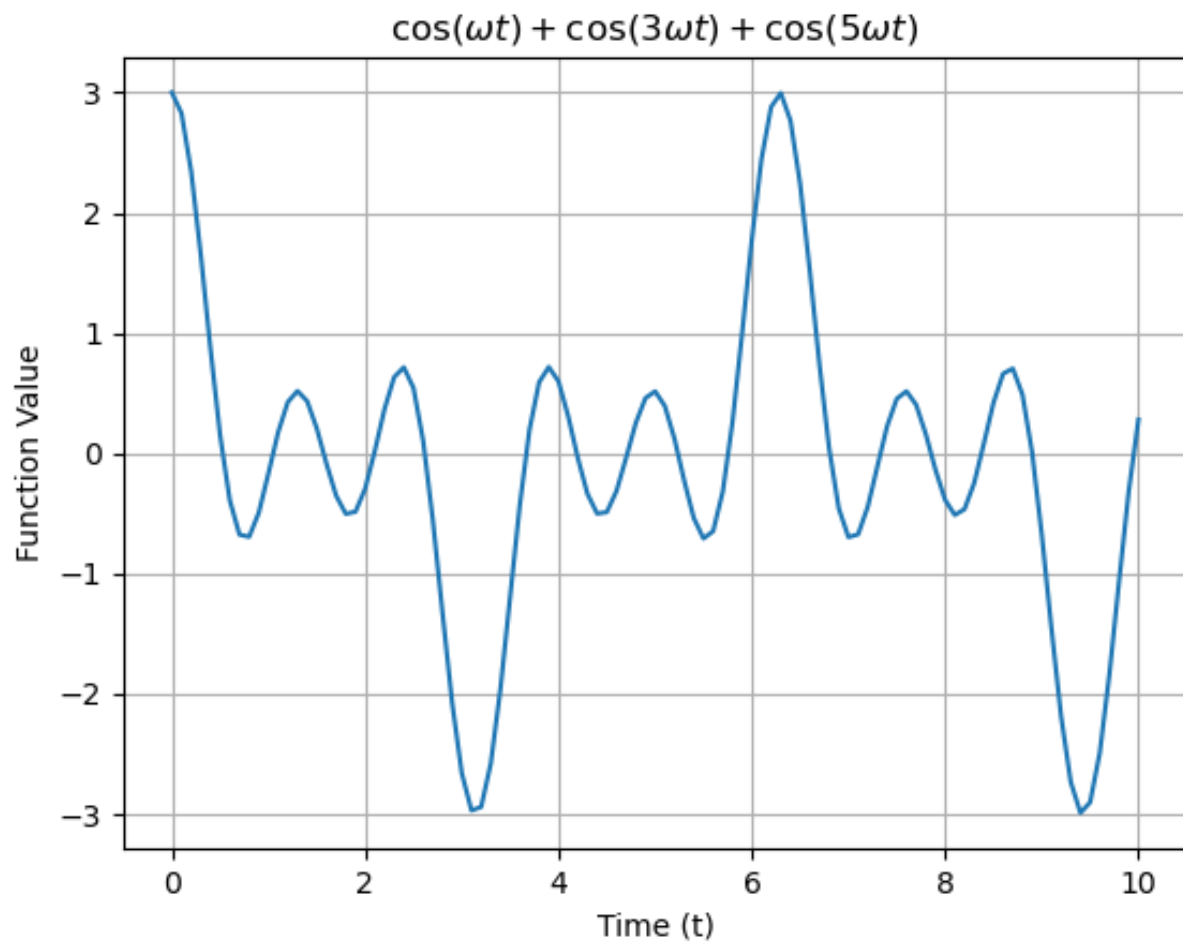


Fig. 1.  $\cos(2\pi ft) + \cos(6\pi ft) + \cos(10\pi ft)$

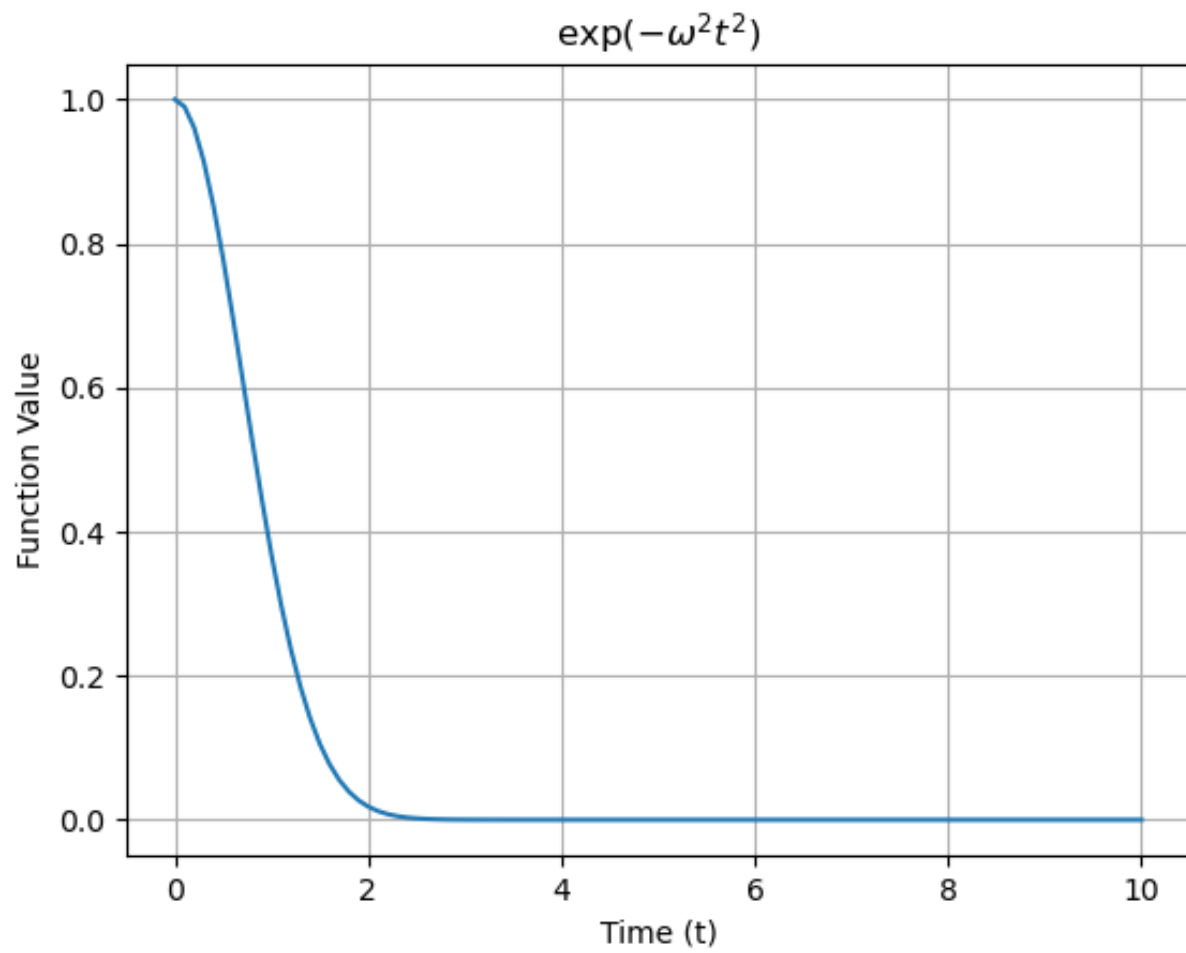


Fig. 1.  $\exp(-2\pi f t^2)$



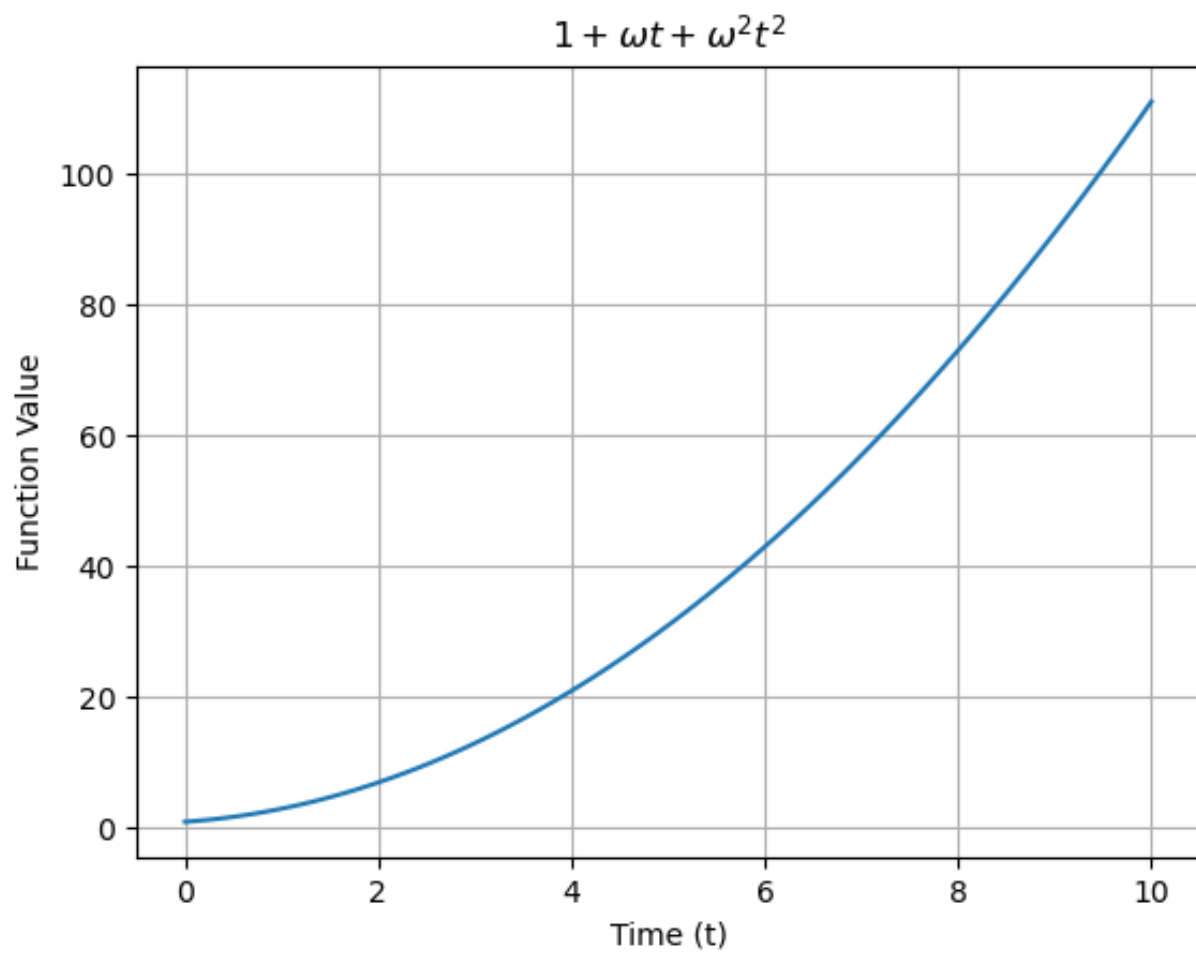


Fig. 1.  $1 + 2\pi ft + (2\pi ft)^2$