

EE23BTECH11047 - Deepakreddy P

17 If a, b, c, d are in G.P, prove that $(a^n + b^n), (b^n + c^n), (c^n + d^n)$ are in G.P

Solution:

TABLE I
INPUT PARAMETERS

Symbol	Remarks
$x(0)$	a
$x(1)$	b
$x(2)$	c
$x(3)$	d
r	ratio of G.P a, b, c, \dots
r_1	ratio of G.P $a^n + b^n, b^n + c^n, \dots$
$X(z)$	z transform of G.P a, b, c, \dots
$Y(z)$	z transform of G.P $a^n + b^n, b^n + c^n, \dots$

From Table I

$$r = \frac{b}{a} = \frac{c}{b} = \frac{d}{c} \quad (1)$$

From eq (1)

$$\frac{b^n + c^n}{a^n + b^n} = \frac{(ar)^n + (ar^2)^n}{(a)^n + (ar)^n} \quad (2)$$

$$= \frac{a^n r^n (1 + r^n)}{a^n (1 + r^n)} \quad (3)$$

$$= r^n \quad (4)$$

$$\frac{c^n + d^n}{b^n + c^n} = \frac{(ar^2)^n + (ar^3)^n}{(ar)^n + (ar^2)^n} \quad (5)$$

$$= \frac{a^n r^{2n} (1 + r^n)}{a^n r^n (1 + r^n)} \quad (6)$$

$$= r^n \quad (7)$$

$$\frac{b^n + c^n}{a^n + b^n} = \frac{c^n + d^n}{b^n + c^n} \quad (8)$$

Hence proved they are in G.P

$$x(n) = a \left(\frac{b}{a} \right)^n u(n) \quad (9)$$

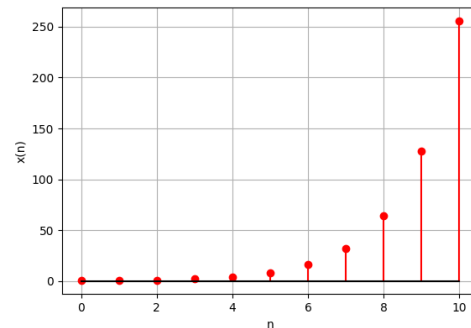
$$X(z) = \frac{a}{1 - \left(\frac{b}{a} \right) z^{-1}}, \quad |z| > \left| \frac{b}{a} \right| \quad (10)$$

$$r_1 = \frac{b^n + c^n}{a^n + b^n} = \frac{c^n + d^n}{b^n + c^n} \quad (11)$$

From eq (11)

$$y(n) = (a^n + b^n) \left(\frac{b^n + c^n}{a^n + b^n} \right)^n u(n) \quad (12)$$

$$Y(z) = \frac{a^n + b^n}{1 - \left(\frac{b^n + c^n}{a^n + b^n} \right) z^{-1}}, \quad |z| > \left| \frac{b^n + c^n}{a^n + b^n} \right| \quad (13)$$



(2) Fig. 1. Stem Plot of $x(n) = (0.25)2^n u(n)$, $a = 0.25, r = 2$

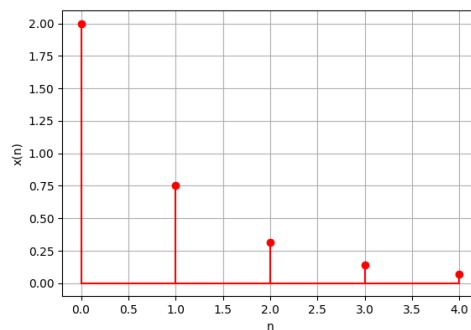


Fig. 2. Stem Plot of $x(n) = (0.25^n + 0.5^n) u(n)$, $a = 0.25, b = 0.5$