

Discrete Assignment

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February 20, 2024

Problem Statement

Find the value of n so that $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ may be the geometric mean between a and b .

Solution

| Parameter | Value | Description |
|-----------|---|----------------|
| $x(0)$ | a | First term |
| $x(2)$ | b | Third term |
| $x(1)$ | $\sqrt{ab} = \frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ | Second term |
| r | $\sqrt{\frac{b}{a}}$ | Common ratio |
| n | - | Given variable |
| $x(k)$ | $ar^k u(k)$ | General term |

Table 1: Input parameters table

Consider a GP as in Table 1,

$$\therefore \frac{a^{n+1} + b^{n+1}}{a^n + b^n} = x(1) \quad (1)$$

$$\implies a^{n+1} + b^{n+1} = a^{n+\frac{1}{2}}b^{\frac{1}{2}} + a^{\frac{1}{2}}b^{n+\frac{1}{2}} \quad (2)$$

$$\implies a^{n+\frac{1}{2}}(a^{\frac{1}{2}} - b^{\frac{1}{2}}) = b^{n+\frac{1}{2}}(a^{\frac{1}{2}} - b^{\frac{1}{2}}) \quad (3)$$

$$\implies \left(\frac{a}{b}\right)^{n+\frac{1}{2}} = \left(\frac{a}{b}\right)^0 \quad (4)$$

$$\implies n = -\frac{1}{2} \quad (5)$$

From Table 1,

$$X(z) = \frac{a}{1 - \left(\sqrt{\frac{b}{a}}\right)z^{-1}} \quad |z| > \left|\sqrt{\frac{b}{a}}\right| \quad (6)$$