

# NCERT 9.5 Q.27

EE23BTECH11203 - Adarsh A\*

**Question :** A farmer buys a used tractor for Rs 12000. He pays Rs 6000 in cash and agrees to pay the balance in annual installments of Rs 500 plus 12 % interest on the unpaid amount. How much will the tractor cost him?

**Solution:**

Parameter	Value	Description
$n + 1$	-	Number of years
$a$	6000	Amount paid
$r$	6000	Remaining Amount
$x(n)$	$(1220 - 60n) u(n)$	Amount to be paid at $(n + 1)^{th}$ year
$y(n)$	$(1220 + 1190n - 30n^2) u(n)$	Total amount after $(n + 1)$ yrs

Input Table

Number of years taken to pay the remaining amount,

$$n + 1 = \frac{r}{500} \quad (1)$$

$$n = 11 \quad (2)$$

The amount to be paid by the farmer after  $(n + 1)$  year/s is,

$$x(n) = 500 + 0.12 (6000 - 500n) \quad (3)$$

$$x(n) = (1220 - 60n) u(n) \quad (4)$$

Some results,

$$x(n) \xleftrightarrow{z} X(z) \quad (5)$$

$$u(n) \xleftrightarrow{z} \frac{1}{1 - z^{-1}} : U(z) \quad (6)$$

$$n.u(n) \xleftrightarrow{z} -z \frac{d}{dz} \frac{1}{1 - z^{-1}} = \frac{z^{-1}}{(1 - z^{-1})^2} \quad (7)$$

By taking  $z$  transform,

$$X(z) = \frac{1220}{1 - z^{-1}} - \frac{60z^{-1}}{(1 - z^{-1})^2} \quad (8)$$

$$X(z) = \frac{1220 - 1280z^{-1}}{(1 - z^{-1})^2}, \quad |z| > 1 \quad (9)$$

Convolution in time domain is multiplication in  $z$  domain :

$$y(n) = x(n) * u(n) \quad (10)$$

$$Y(z) = X(z) \cdot U(z) \quad (11)$$

$$Y(z) = \frac{1220 - 1280z^{-1}}{(1 - z^{-1})^2} \cdot \frac{1}{1 - z^{-1}} \quad (12)$$

$$Y(z) = \frac{1220 - 1280z^{-1}}{(1 - z^{-1})^3}, \quad |z| > 1 \quad (13)$$

Using Partial fractions,

$$Y(z) = \frac{Az}{z-1} + \frac{Bz}{(z-1)^2} + \frac{Cz}{(z-1)^3} \quad (14)$$

$$A = 1220 \quad (15)$$

$$B = 1160 \quad (16)$$

$$C = -60 \quad (17)$$

$$\frac{n(n-1)(n-2)\dots(n-k-2)}{k!} u(n) \xleftrightarrow{z} \frac{z}{(z-1)^k} \quad (18)$$

Using this result,

$$y(n) = 1220 u(n) + 1160 n u(n) - 60 \frac{n(n-1)}{2} u(n) \quad (19)$$

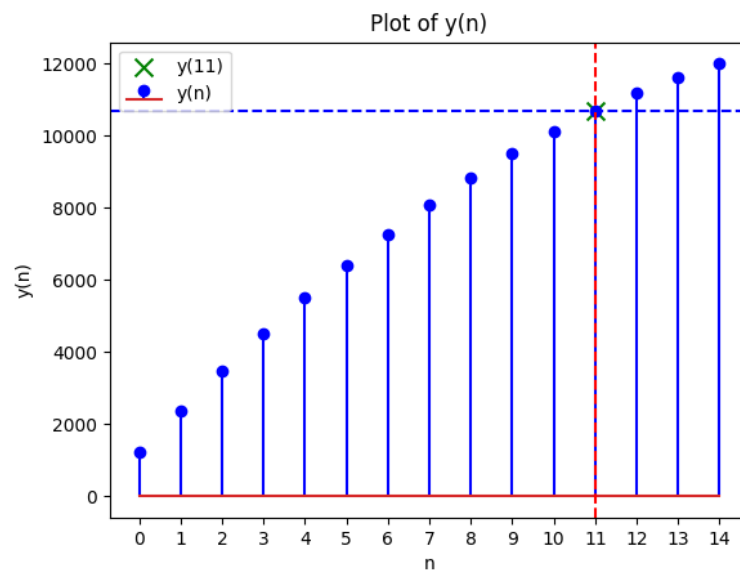
$$y(n) = (1220 + 1190n - 30n^2) u(n) \quad (20)$$

$$y(11) = 10680 \quad (21)$$

The total tractor cost to be paid,

$$= a + y(11) \quad (22)$$

$$= 16,680 \quad (23)$$



(a) Plot of  $y(n)$  vs  $n$