EE23BTECH11054 - Sai Krishna Shanigarapu*

EXERCISE 9.2

13. If the sum of n terms of an A.P. is $3n^2 + 5n$ and its mth term is 164, find the value of m.

Solution:

$$Y(z) = \sum_{n=0}^{\infty} y(n) z^{-n} \qquad (1)$$

$$= \frac{2(4 - z^{-1})}{(1 - z^{-1})^3}, \qquad |z| > 1 \qquad (2)$$

$$U(z) = \frac{1}{1 - z^{-1}}, \qquad |z| > 1 \qquad (3)$$

$$X(z) = \frac{Y(z)}{U(z)} \qquad (4)$$

$$= 2\left(\frac{1}{1 - z^{-1}}\right) + 6\left(\frac{1}{(1 - z^{-1})^2}\right) \qquad (5)$$

$$= \frac{8z^2 - 2z}{(z - 1)^2} \qquad (6)$$

Using Contour Integration to find the inverse Z-transform,

$$x(n) = \frac{1}{2\pi j} \oint_C X(z) z^{n-1} dz \qquad (7)$$

$$= \frac{1}{2\pi j} \oint_C \frac{(8z^{n+1} - 2z^n) dz}{(z - 1)^2} \qquad (8)$$

$$= \frac{1}{(m-1)!} \lim_{z \to a} \frac{d^{m-1}}{dz^{m-1}} ((z - a)^m f(z)) \qquad (9)$$

$$= \lim_{z \to 1} \frac{d}{dz} \left((z - 1)^2 \frac{8z^{n+1} - 2z^n}{(z - 1)^2}\right) \qquad (10)$$

$$= \lim_{z \to 1} \left(8(n+1) z^n - 2nz^{n-1}\right) \qquad (11)$$

$$\implies x(n) = (6n + 8)(u(n)) \qquad (12)$$

$$164 = (6m + 8)(u(m)) \qquad (13)$$

$$\implies m = 26 \qquad (14)$$

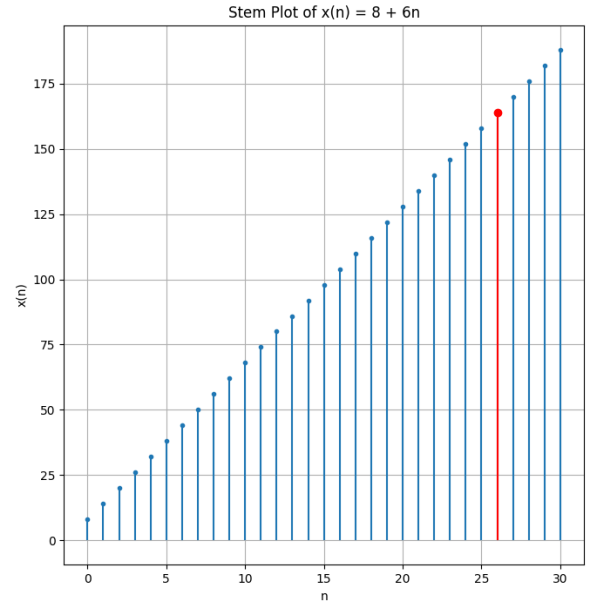| Symbol	Remarks
$y(n) = (3n^2 + 11n + 8)(u(n))$	Sum of n terms
$x(m - 1)$	164
$y(n)$	$x(n) * u(n)$

TABLE I
PARAMETERS

Fig. 1. Plot of x(n) vs n