Assignment

11.9.5 - 22

EE23BTECH11220 - R.V.S.S Varun

Question

Find the 20th term in this series.

$$2\times4+4\times6+6\times8\cdots+n\,terms$$

Solution:

PARAMETER	VALUE	DESCRIPTION
$x(0)$	8	First term
$x(n)$	$4(n+1)n+2u(n)$	General term of the series

TABLE 0
Table of parameters

Using Z- transform,

$$n^2u(n)\overset{Z}{\leftrightarrow}\frac{z^{-1}\left(z^{-1}+1\right)}{\left(1-z^{-1}\right)^3},|z|>1 \tag{1}$$

$$nu(n)\overset{Z}{\leftrightarrow}\frac{z^{-1}}{\left(1-z^{-1}\right)^2},|z|>1 \tag{2}$$

$$u(n)\overset{Z}{\leftrightarrow}\frac{1}{\left(1-z^{-1}1\right)},|z|>1 \tag{3}$$

$$X(z)=\sum_{n=-\infty}^{n=\infty}4(n+1)(n+2)u(n)z^{-n} \tag{4}$$

$$X(z)=\sum_{n=-\infty}^{n=\infty}4\left(n^2+3n+2\right)u(n)z^{-n} \tag{5}$$

$$X(z)=\frac{8}{\left(1-z^{-1}\right)^3},|z|>1 \tag{6}$$

$$y(n)=x(n)*u(n) \tag{7}$$

$$\implies Y(z)=X(z)U(z) \tag{8}$$

$$Y(z)=\left(\frac{8}{\left(1-z^{-1}\right)^3}\right)\left(\frac{1}{1-z^{-1}}\right) \tag{9}$$

$$=\frac{8}{\left(1-z^{-1}\right)^4},\quad |z|>1 \tag{10}$$

Using Contour Integration to find the inverse Z-transform,

$$y(19)=\frac{1}{2\pi j}\oint_C Y(z)\,z^{18}\,dz \tag{11}$$

$$=\frac{1}{2\pi j}\oint_C\frac{8z^{18}}{\left(1-z^{-1}\right)^4}\,dz \tag{12}$$

We can observe that the pole is repeated 4 times and thus $m=4$,

$$R=\frac{1}{(m-1)!}\lim_{z\to a}\frac{d^{m-1}}{dz^{m-1}}\left((z-a)^m f(z)\right) \tag{13}$$

$$=\frac{1}{(3)!}\lim_{z\to1}\frac{d^3}{dz^3}\left((z-1)^4\frac{8z^{22}}{(z-1)^4}\right) \tag{14}$$

$$=\frac{4}{3}\lim_{z\to1}\frac{d^3}{dz^3}(z^{22}) \tag{15}$$

$$=12320 \tag{16}$$

$$\therefore\ \boxed{y(19)=12320} \tag{17}$$