Discrete 11.9.2

EE:1205 Signals and System
Indian Institute of Technology, Hyderabad

Prashant Maurya
EE23BTECH11218

Question-2 : Find the sum of all natural numbers lying between 100 and 1000, which are multiples of 5.

Solution:

Parameter	Description	Value
$x(0)$	First Term	105
d	Common Difference	5
n	Total terms	179
$x(178)$	Last Term	995
m	No of poles	3

TABLE 1: Given Parameters

$$x(n) = (105 + 5n)(u(n)) \tag{1}$$

On taking Z transform

$$X(z) = \frac{x(0)}{(1 - z^{-1})} + \frac{dz^{-1}}{(1 - z^{-1})^2} \tag{2}$$
$$= \frac{105}{1 - z^{-1}} + \frac{5z^{-1}}{(1 - z^{-1})^2} \tag{3}$$
$$\implies X(z) = \frac{105 - 100z^{-1}}{(1 - z^{-1})^2} \quad |z| > 1 \tag{4}$$
$$y(n) = x(n) * u(n) \tag{5}$$
$$\implies Y(z) = X(z)U(z) \tag{6}$$
$$= \frac{105 - 100z^{-1}}{(1 - z^{-1})^2}\frac{1}{(1 - z^{-1})} \tag{7}$$
$$= \frac{105 - 100z^{-1}}{(1 - z^{-1})^3} \quad |z| > 1 \tag{8}$$

Using contour integration to find the inverse Z-transform:

$$\implies y(178) = \frac{1}{2\pi j}\oint_C Y(z)\ z^{177}\ dz \tag{9}$$
$$= \frac{1}{2\pi j}\oint_C \frac{\left(105 - 100z^{-1}\right)z^{177}}{(1 - z^{-1})^3}\ dz \tag{10}$$

We can observe that there is only a 3 times repeated pole at $z = 1$,

$$\implies R = \frac{1}{(m-1)!}\lim_{z\to a}\frac{d^{m-1}}{dz^{m-1}}\left((z-a)^m f(z)\right) \tag{11}$$

$$= \frac{1}{(2)!}\lim_{z\to 1}\frac{d^2}{dz^2}\left((z-1)^3\frac{\left(105 - 100z^{-1}\right)z^{180}}{(z-1)^3}\right) \tag{12}$$
$$= \frac{1}{2}\lim_{z\to 1}\frac{d^2}{dz^2}\left(105z^{180} - 100z^{179}\right) \tag{13}$$
$$= 98450 \tag{14}$$

$$\therefore y(178) = 98450 \tag{15}$$

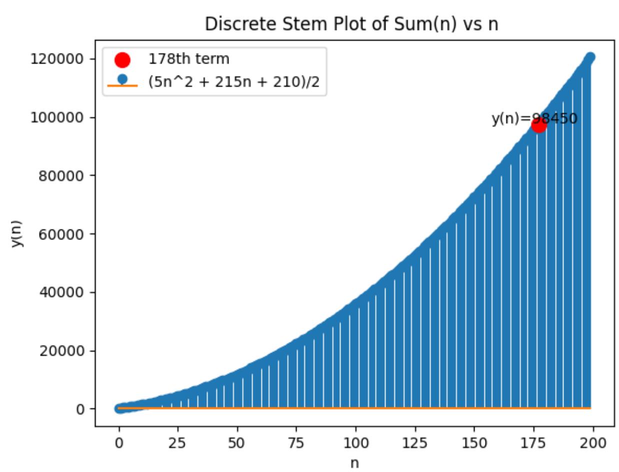

Fig. 1: Plot of $x(n)$ vs n