EE23BTECH11212 - MANUGUNTA MEGHANA SAI*

Question: A circuit containing a $80mH$ inductor and a $60\mu F$ capacitor in series is connected to a $230V$, $50Hz$ supply. A resistance of 15Ω is connected in series. Obtain the average power transferred to each element of the circuit, and the total power absorbed.

Solution:

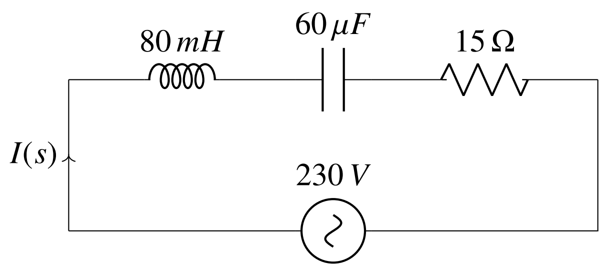

Fig. 1. LCR Circuit

In Fig. 1 the following information is provided:

Symbol	Value	Description
L	$80mH$	Inductance
C	$60\,\mu F$	Capacitance
R	$15\,\Omega$	Resistance
V_{rms}	$230\,V$	Voltage
f	$50\,Hz$	Frequency
ω	$2\pi f = 100\pi$	Angular Frequency
ϕ	$-$	Phase difference between current and voltage
I_{rms}	$-$	rms value of current
V_m	$-$	Maximum voltage
I_m	$-$	Maximum current
P_m	$-$	Maximum Power

TABLE I
GIVEN PARAMETERS

Applying Kirchoff's Voltage Law in the Fig. 2

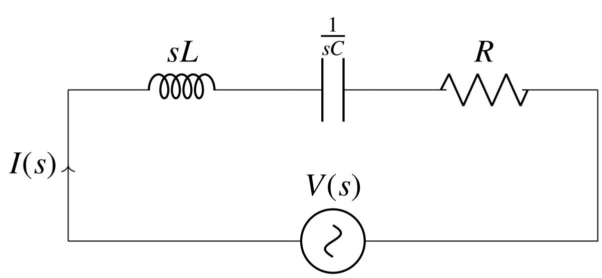

Fig. 2. s domain circuit

$$V(s) = RI(s) + sLI(s) + \frac{1}{sC}I(s) \tag{1}$$

$$= I(s)\left(R + sL + \frac{1}{sC}\right) \tag{2}$$

$$I(s) = \frac{V(s)}{\left(R + sL + \frac{1}{sC}\right)} \tag{3}$$

$$H(s) = \frac{V(s)}{I(s)} \tag{4}$$

$$H(s) = R + sL + \frac{1}{sC} \tag{5}$$

Substituting s with $j\omega$

$$H(j\omega) = R + j\omega L + \frac{1}{j\omega C} \tag{6}$$

$$\Rightarrow |H(j\omega)| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \tag{7}$$

Let the input voltage be:

$$V = V_m sin\,(\omega t) \tag{8}$$

Let the current at a given instant be:

$$I = I_m sin\,(\omega t - \phi) \tag{9}$$

Instantaneous power is given by:

$$P = VI \tag{10}$$

$$P = V_m sin\,(\omega t) \times I_m sin\,(\omega t - \phi) \tag{11}$$

Average power is given by:

$$P_{av} = \frac{W}{T} \tag{12}$$

$$dW = Pdt \tag{13}$$

Integrating on both sides

$$W = V_m I_m \int_0^T \sin(\omega t)\sin(\omega t - \phi)\; dt \tag{14}$$

$$= V_m I_m \int_0^T \sin(\omega t)\left(\sin(\omega t)\cos(\phi) - \cos(\omega t)\sin(\phi)\right)\; dt \tag{15}$$

$$= V_m I_m \int_0^T (\sin(\omega t))^2 \cos(\phi)\, dt - V_m I_m \int_0^T \sin(\omega t)\cos(\omega t)\sin(\phi)\, dt \tag{16}$$

$$= V_m I_m \int_0^T \frac{1 - \cos(2\omega t)}{2}\cos(\phi)\, dt - V_m I_m \int_0^T \sin(2\omega t)\sin(\phi)\, dt \tag{17}$$

After solving the integral we get,

$$W = \frac{1}{2} V_m I_m T cos\phi \tag{18}$$

Relation between V_{rms} and V_m:

$$V_{\mathrm{rms}} = \frac{V_m}{\sqrt{2}} \tag{19}$$

Relation between I_{rms} and I_m:

$$I_{\text{rms}} = \frac{I_m}{\sqrt{2}} \tag{20}$$

a) The average power dissipated in a RLC circuit is given by :

$$P = V_{rms} I_{rms} cos(\phi) \tag{21}$$

The phase difference is given by:

$$tan\,(\phi) = \frac{\frac{1}{\omega C} - \omega L}{R} \tag{22}$$

After substituting the values from Table I:

$$tan\,(\phi) = 1.86 \tag{23}$$

Rms value of current I_{rms} is given by :

$$I_{rms} = \frac{V_{rms}}{R} = \frac{230}{15} = 15.33A \tag{24}$$

Now, susbtituting the value of ϕ, I_{rms} and values from Table I in (21) we obtain the total power :

$$P_{av} = 789.62W \tag{25}$$

b) Average power transferred to the capacitor, P_C:
For a capacitor the phase angle is:

$$\phi = \frac{\pi}{2} \tag{26}$$

$$cos(\phi) = 0 \tag{27}$$

$$P_C = 0 \tag{28}$$

c) Average power transferred to the inductor, P_L:
For an inductor the phase angle is:

$$\phi = -\frac{\pi}{2} \tag{29}$$

$$cos(\phi) = 0 \tag{30}$$

$$P_L = 0 \tag{31}$$

d) Average Power transferred to the resistor, P_R:

$$P_{avg} = P_R + P_C + P_L \tag{32}$$

$$P_R = P_{avg} - P_C - P_L \tag{33}$$

$$P_R = 789.62 - 0 - 0 \tag{34}$$

$$P_R = 789.62W \tag{35}$$

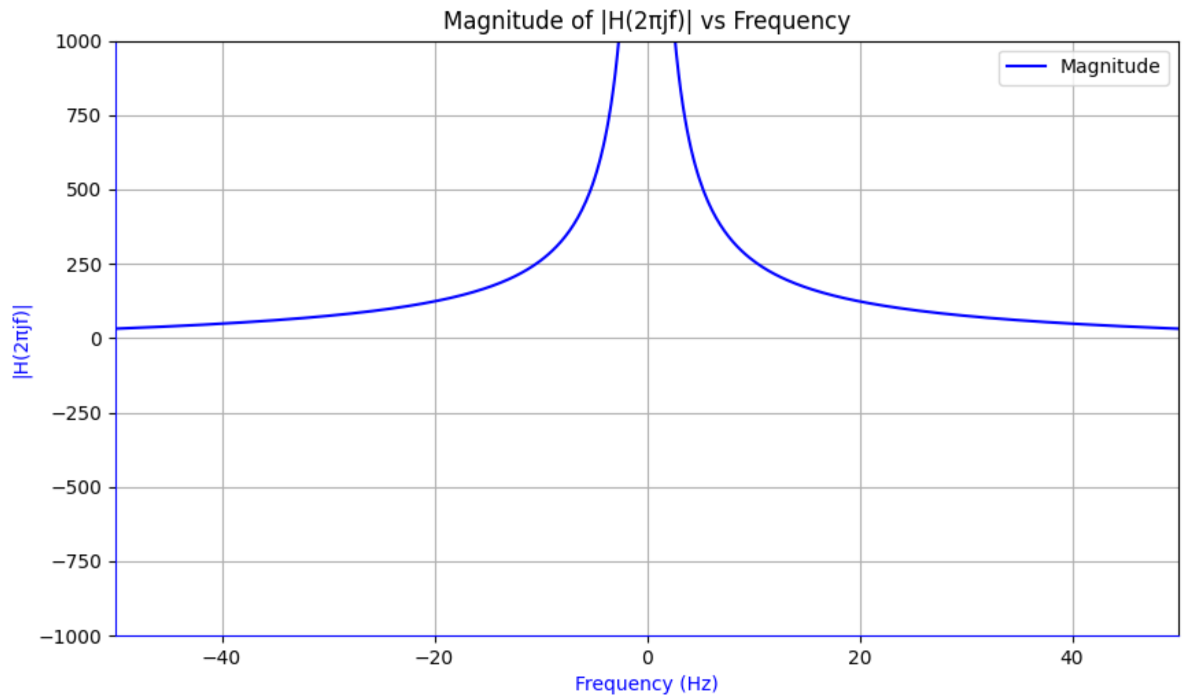

Fig. 3. $|H(j/omega)|$ vs ω

Bandwidth is defined as the range of frequencies, where power ranges from its maximium value to half of its maximum value.

$$I_{rms} = \frac{V_{rms}}{|H(j\omega)|} \tag{36}$$

At maximum power, $|H(j\omega)|$ will be minimum,

$$|H(j\omega)| = R \tag{37}$$

$$I_m = \frac{V_{rms}}{R} \tag{38}$$

when, power is half of the maximum value of power

$$P = \frac{P_m}{2} \tag{39}$$

$$I_{rms} = \frac{I_m}{\sqrt{2}} \tag{40}$$

$$|H(j\omega)| = \sqrt{2}R \tag{41}$$

$$\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = \sqrt{2}R \tag{42}$$

$$\left(\omega L - \frac{1}{\omega C}\right)^2 = R^2 \tag{43}$$

This equation has 2 roots, ω_1 and ω_2:

$$\omega_1 = -\frac{R}{2L} + \sqrt{\frac{R^2}{4R} + \frac{1}{LC}} \tag{44}$$

$$\omega_2 = \frac{R}{2L} + \sqrt{\frac{R^2}{4R} + \frac{1}{LC}} \tag{45}$$

Thus Bandwidth of circuit is :

$$\omega_2 - \omega_1 = \frac{R}{L} = 187.5 \tag{46}$$

$$f = \frac{\omega_2 - \omega_1}{2\pi} = 29.85 \tag{47}$$