SEQUENCE AND SERIES

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Q: Find the sum to n terms of $3 \times 8 + 6 \times 11 + 9 \times 14 + ...$

Solution:

Variable	Description	Value
$x(n)$	n^{th} term of sequence	$(3n+3)(3n+8)u(n)$

TABLE 0

INPUT PARAMETERS

Sum of n terms of AP is given by

$$x(n) = (3n+3)(3n+8)u(n) \tag{1}$$

$$y(n) = x(n) * u(n) \tag{2}$$

$$u(n) \overset{\mathcal{Z}}{\longleftrightarrow} \frac{1}{(1-z^{-1})} \quad |z| > 1 \tag{3}$$

$$nu(n) \overset{\mathcal{Z}}{\longleftrightarrow} \frac{z^{-1}}{(1-z^{-1})^2} \quad |z| > 1 \tag{4}$$

$$n^2u(n) \overset{\mathcal{Z}}{\longleftrightarrow} \frac{z^{-1}(1+z^{-1})}{(1-z^{-1})^3} \quad |z| > 1 \tag{5}$$

$$n^3u(n) \overset{\mathcal{Z}}{\longleftrightarrow} \frac{z^{-1}\left(1+4z^{-1}+z^{-2}\right)}{(1-z^{-1})^4} \quad |z| > 1 \tag{6}$$

$$\implies X(z) = 9z^{-1}\frac{\left(1+z^{-1}\right)}{(1-z^{-1})^3} + \frac{33\left(z^{-1}\right)}{(1-z^{-1})^2} + \frac{24}{(1-z^{-1})} \, |z| > 1 \tag{7}$$

$$Y(z) = X(z)U(z) \tag{8}$$

$$\implies Y(z) = 9z^{-1}\frac{\left(1+z^{-1}\right)}{(1-z^{-1})^4} + \frac{33\left(z^{-1}\right)}{(1-z^{-1})^3} + \frac{24}{(1-z^{-1})^2} \, |z| > 1 \tag{9}$$

Now from (3), (4), (5), (6), (9) By using inverse Z-transform pairs,

$$y(n) = \left(\frac{9n(n+1)(2n+1)}{6} + \frac{33n(n+1)}{2} + 24(n+1)\right)u(n) \tag{10}$$

$\therefore$ Sum of n terms of the series whose n^{th} term is given by $(3n+3)(3n+8)u(n)$ is $(\frac{9n(n+1)(2n+1)}{6} + \frac{33n(n+1)}{2} + 24(n+1))$u($n$)

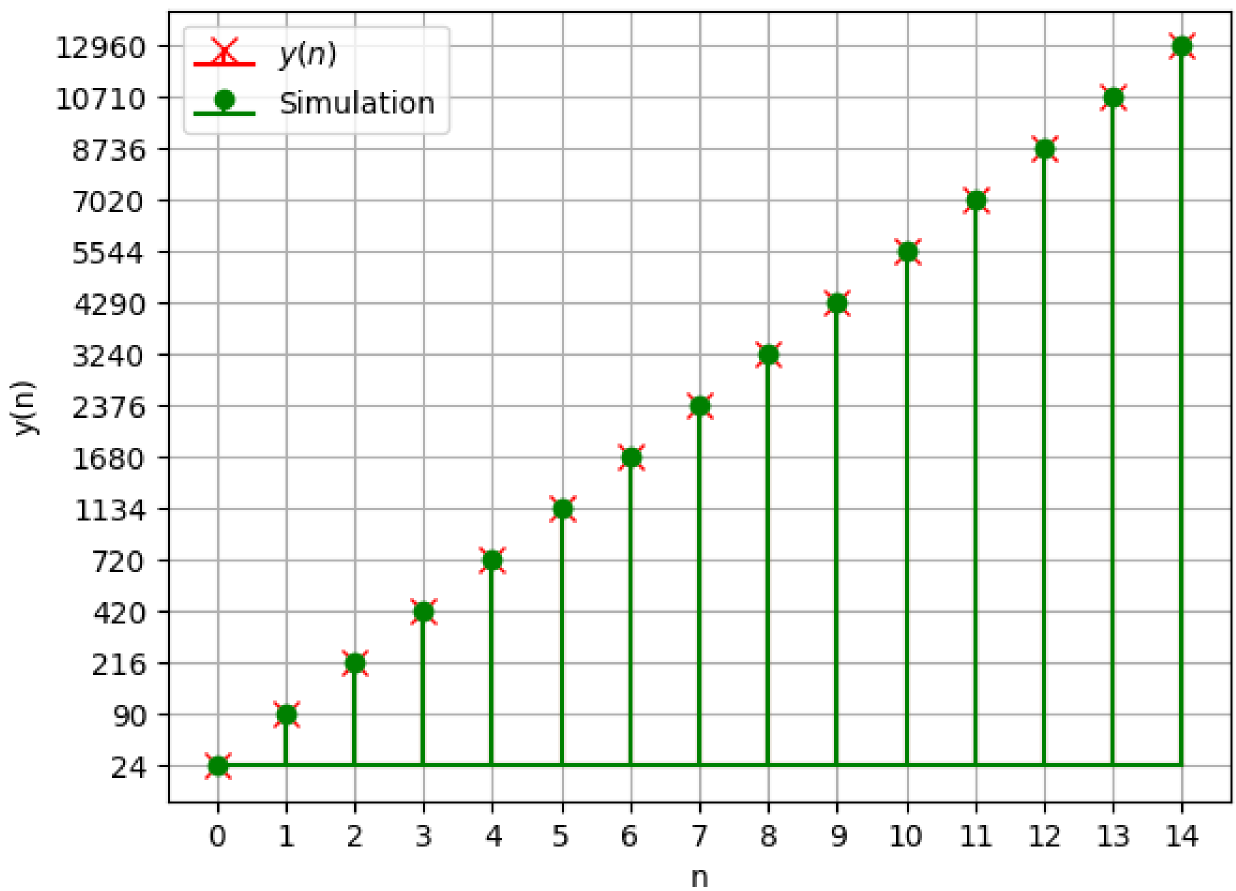

Fig. 0. Theory vs Simulation