SIGNAL PROCESSING FUNDAMENTALS Through NCERT

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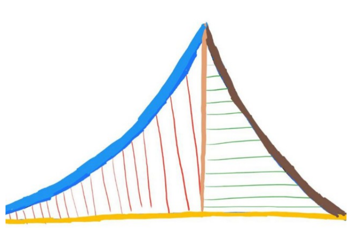

Contents

Introduction

This book introduces some concepts in signal processing through maths and physics problems in NCERT textbooks.

Chapter 1

Analog

1.1. Harmonics

1.1.1 A charged particle oscillates about its mean equilibrium position with a frequency of $10^9 Hz$. What is the frequency of the electromagnetic waves produced by the oscillator?

Solution:

Symbol	Value	Description
$y(t)$	$\cos(2\pi f_c t)$	Wave equation of electro-magnetic wave
f_c	10^9	Frequency of electromagnetic wave
t	seconds	Time

Table 1.1: Variable description

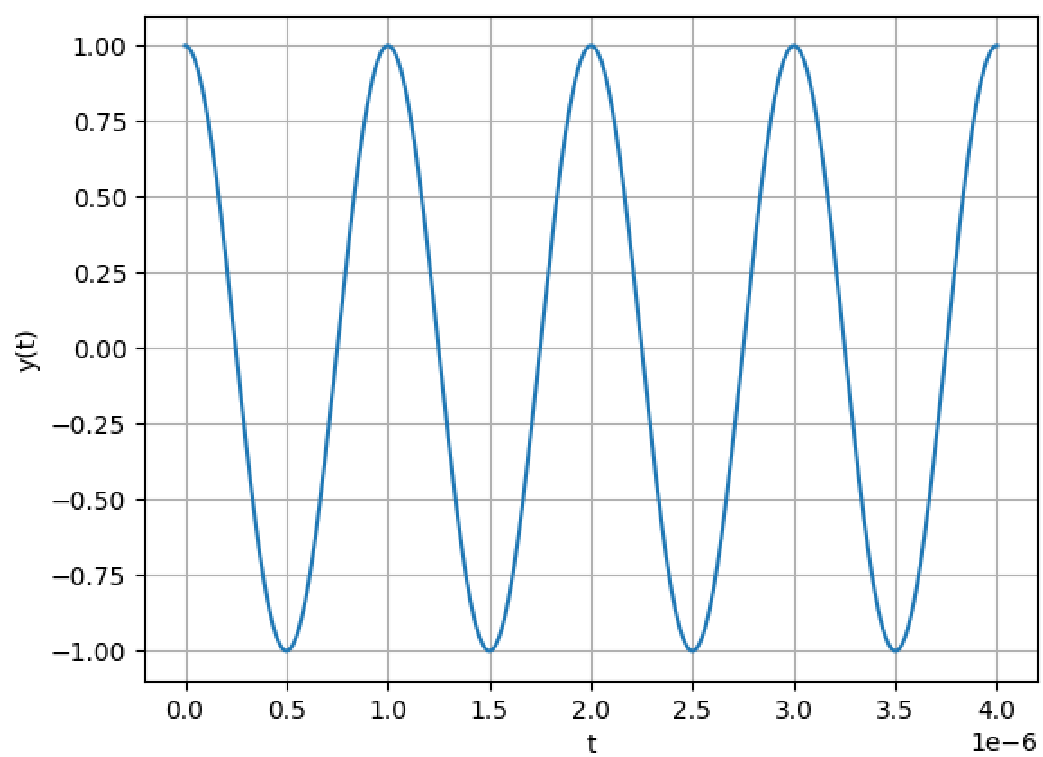

Figure 1.1: $y(t) = \cos\left(2\pi \times 10^9 t\right)$

Chapter 2

Discrete

2.1. Z-transform

2.1.1 Write the five terms at n = 1, 2, 3, 4, 5 of the sequence and obtain the Z-transform of the series

$$x(n) = -1, \qquad n = 0 \tag{2.1}$$

$$= \frac{x(n-1)}{n}, \qquad n > 0 \tag{2.2}$$

$$= 0, \qquad n < 0 \tag{2.3}$$

Solution:

$$x(1) = \frac{x(0)}{1} = -1 \tag{2.4}$$

$$x(2) = \frac{x(1)}{2} = -\frac{1}{2} \tag{2.5}$$

$$x(3) = \frac{x(2)}{3} = -\frac{1}{(2)(3)} = -\frac{1}{6} \tag{2.6}$$

$$x(4) = \frac{x(3)}{4} = -\frac{1}{(2)(3)(4)} = -\frac{1}{24} \tag{2.7}$$

$$x(5) = \frac{x(4)}{5} = -\frac{1}{(2)(3)(4)(5)} = -\frac{1}{120} \tag{2.8}$$

$$x(n) = \frac{-1}{n!}(u(n)) \tag{2.9}$$

$$x(n) \overset{\mathcal{Z}}{\longleftrightarrow} X(z) \tag{2.10}$$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} \tag{2.11}$$

using (2.9),

$$= \sum_{n=-\infty}^{\infty} \frac{-1}{n!} u(n) z^{-n} \tag{2.12}$$

$$= \sum_{n=0}^{\infty} \frac{-1}{n!} z^{-n} \tag{2.13}$$

$$= -e^{z^{-1}} \qquad \{z \in \mathbb{C} : z \neq 0\} \tag{2.14}$$

Symbol	**Value**	**Description**
$x(n)$	$\frac{-1}{n!}$	general term of the series
$X(z)$	$-e^{z^{-1}}$	Z-transform of x(n)
$u(n)$		unit step function

Table 2.1: Parameters

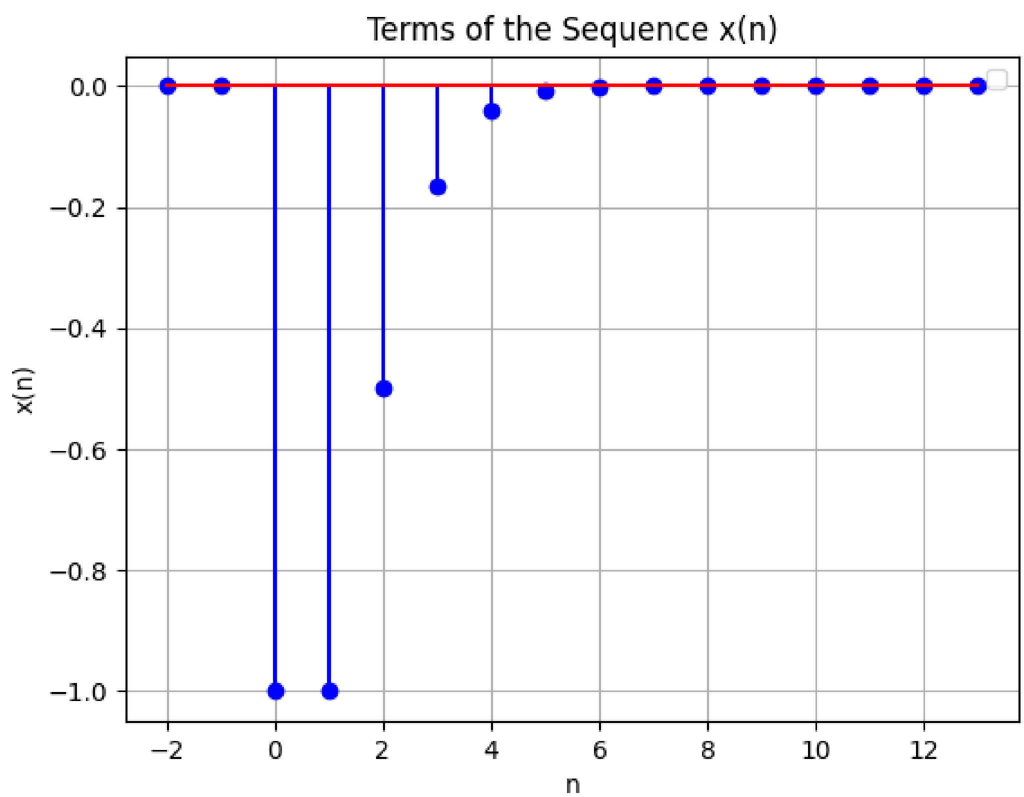

Figure 2.1: Plot of x(n) vs n

Appendix A

Convolution

A.1 The convolution sum is defined as

$$y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k) \tag{A.1.1}$$

A.2 The unit step function is defined as

$$u(n) = \begin{cases} 1 & n \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{A.2.1}$$

A.3 If

$$x(n) = 0, \quad n < 0, \tag{A.3.1}$$

from (A.1.1),

$$x(n) * u(n) = \sum_{k=0}^{n} x(k) \tag{A.3.2}$$

Appendix B

Z-transform

B.1 The Z-transform of $p(n)$ is defined as

$$P(z) = \sum_{n=-\infty}^{\infty} p(n)z^{-n} \tag{B.1.1}$$

B.2 If

$$p(n) = p_1(n) * p_2(n), \tag{B.2.1}$$

$$P(z) = P_1(z)P_2(z) \tag{B.2.2}$$

B.3 For a Geometric progression

$$x(n) = x(0)\, r^n u(n), \tag{B.3.1}$$

$$\implies X(z) = \sum_{n=-\infty}^{\infty} x(n)\, z^{-n} = \sum_{n=0}^{\infty} x(0)\, r^n z^{-n} \tag{B.3.2}$$

$$= \sum_{n=0}^{\infty} x(0) \left(rz^{-1}\right)^n \tag{B.3.3}$$

$$= \frac{x(0)}{1 - rz^{-1}}, \quad |z| > |r| \tag{B.3.4}$$

B.4 Substituting $r = 1$ in (B.3.4),

$$u(n) \overset{\mathcal{Z}}{\longleftrightarrow} U(z) = \frac{1}{1 - z^{-1}}, \quad |z| > 1 \tag{B.4.1}$$

B.5 From (B.1.1) and (B.4.1),

$$U(z) = \sum_{n=-\infty}^{\infty} u(n) z^{-n} \tag{B.5.1}$$

$$\implies \frac{dU(z)}{dz} = -z^{-1} \sum_{n=-\infty}^{\infty} n u(n) z^{-n} \tag{B.5.2}$$

$$\therefore n u(n) \overset{\mathcal{Z}}{\longleftrightarrow} \frac{z^{-1}}{(1 - z^{-1})^2}, \quad |z| > 1 \tag{B.5.3}$$

B.6 For an AP,

$$x(n) = [x(0) + nd]\, u(n) = x(0)u(n) + dnu(n) \tag{B.6.1}$$

$$\implies X(z) = \frac{x(0)}{1 - z^{-1}} + \frac{dz^{-1}}{(1 - z^{-1})^2}, \quad |z| > 1 \tag{B.6.2}$$

upon substituting from (B.4.1) and (B.5.3).

B.7 From (A.3.2), the sum to n terms of a GP can be expressed as

$$y(n) = x(n) * u(n) \tag{B.7.1}$$

where $x(n)$ is defined in (B.3.1). From (B.2.2), (B.3.4) and (B.4.1),

$$
\begin{aligned}
Y(z) &= X(z)\,U(z) && \text{(B.7.2)}\\
&= \left(\frac{x(0)}{1-rz^{-1}}\right)\left(\frac{1}{1-z^{-1}}\right) \quad |z|>|r| \cap |z|>|1| && \text{(B.7.3)}\\
&= \frac{x(0)}{\left(1-rz^{-1}\right)\left(1-z^{-1}\right)} \quad |z|>|r| && \text{(B.7.4)}
\end{aligned}
$$

which can be expressed as

$$
Y(z) = \frac{x(0)}{r-1}\left(\frac{r}{1-rz^{-1}} - \frac{1}{1-z^{-1}}\right) \tag{B.7.5}
$$

using partial fractions. Again, from (B.3.4) and (B.4.1), the inverse of the above can be expressed as

$$
y(n) = x(0)\left(\frac{r^{n+1}-1}{r-1}\right)u(n) \tag{B.7.6}
$$