NCERT Question 11.9.3.9

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Question: Find the sum to indicated number of terms in the geometric progression:
$1, -a, a^2, -a^3, ...n$ terms (if $a \neq -1$).
Solution:

Input Parameters	Values	Description
$x(0)$	1	First term
r	$(-a)$	Common ratio
$x(n)$	$(-a)^n u(n)$	General term

TABLE 1
GIVEN INPUTS

Signal	Transform
$\frac{1}{1-z^{-1}}$	$u(n)$
$\frac{1}{1-az^{-1}}$	$(a)^n u(n)$

TABLE 2
Z TRANSFORM PAIRS

$$x(n) = (-a)^n u(n) \tag{1}$$

$$X(z) = \frac{1}{1+az^{-1}} \tag{2}$$

The ROC is $|z| > |a|$
From Table 1,

$$y(n) = (-a)^n u(n) * u(n) \tag{3}$$

$$Y(z) = X(z)\ U(z) \tag{4}$$

$$= \frac{1}{1+az^{-1}} \frac{1}{1-z^{-1}} \tag{5}$$

Using Z transform pairs to find the inverse Z-transform:

$$Y(z) = \frac{1}{a+1} \frac{1}{z^{-1}} \left[\frac{1}{1-z^{-1}} - \frac{1}{1+az^{-1}} \right] \tag{6}$$

$$y(n) = \left[\frac{1-(-a)^{n+1}}{1-(-a)} \right] u(n) \tag{7}$$

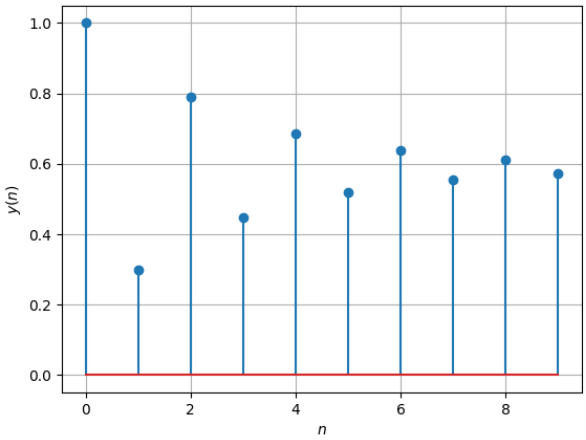

Fig. 1. Plot of $y(n)$ for $a = 0.7$