NCERT Discrete - 11.9.1.8

EE23BTECH11045 - Palavelli Srija*

Question 11.9.1.8:

1) Find the seventh term of the sequence where the nth term is given by $a_n = \frac{n^2}{2^n}$

Solution:

$$x(n) = \frac{(n+1)^2}{2^{(n+1)}}u(n) \tag{1}$$

Parameter	Value
$x(n)$	$\frac{(n+1)^2}{2^{(n+1)}}u(n)$
$x(6)$	?

TABLE 1
INPUT PARAMETERS

$$x(6) = \frac{(6+1)^2}{2^{(6+1)}} \tag{2}$$

$$= \frac{49}{128} \tag{3}$$

1) Scaling property:

$$a^n u(n) \overset{\mathcal{Z}}{\leftrightarrow} \frac{1}{(1-az^{-1})}, \quad |z| > |a| \tag{4}$$

2) Differentiation property:

$$nu(n) \overset{\mathcal{Z}}{\leftrightarrow} (-z)\frac{dY(z)}{dz} \tag{5}$$

$$\implies nu(n) \overset{\mathcal{Z}}{\leftrightarrow} \frac{z^{-1}}{(1-z^{-1})^2}, \quad |z| > 1 \tag{6}$$

$$\implies n^2u(n) \overset{\mathcal{Z}}{\leftrightarrow} \frac{z^{-1}(1+z^{-1})}{(1-z^{-1})^3}, \quad |z| > 1 \tag{7}$$

3) Time shifting property:

$$y(n-k) \overset{\mathcal{Z}}{\leftrightarrow} z^{-k}Y(z) \tag{8}$$

The Z transform of $x(n)$ is given by:
from(4)

$$\frac{u(n)}{2^n} \overset{\mathcal{Z}}{\leftrightarrow} \frac{1}{(1-(2z)^{-1})}, \quad |z| > \frac{1}{2} \tag{9}$$

from(5)

$$\frac{n}{2^n}u(n) \overset{\mathcal{Z}}{\leftrightarrow} \frac{(2z)^{-1}}{(1-(2z)^{-1})^2}, \quad |z| > \frac{1}{2} \tag{10}$$

$$\frac{n^2}{2^n}u(n) \overset{\mathcal{Z}}{\leftrightarrow} \frac{(2z)^{-1}(1+(2z)^{-1})}{(1-(2z)^{-1})^3}, \quad |z| > \frac{1}{2} \tag{11}$$

from(8)

$$\frac{(n+1)^2}{2(n+1)}u(n) \overset{\mathcal{Z}}{\leftrightarrow} (z)\frac{(2z)^{-1}(1+(2z)^{-1})}{(1-(2z)^{-1})^3}, \quad |z| > \frac{1}{2} \tag{12}$$

$$X(z) = \frac{1+(2z)^{-1}}{2(1-(2z)^{-1})^3}, \quad |z| > \frac{1}{2} \tag{13}$$

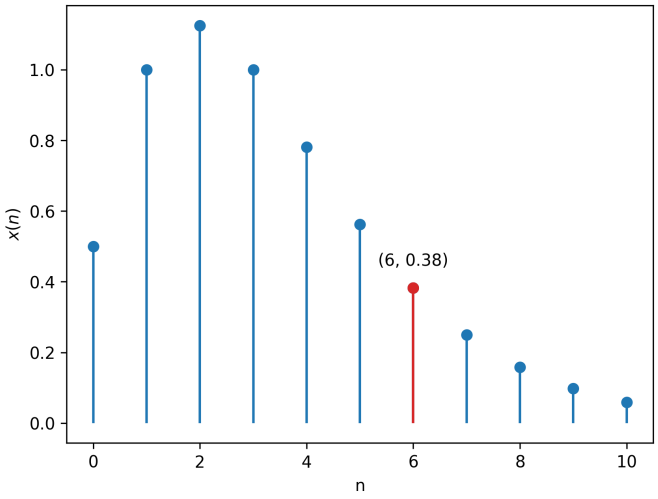

Fig. 3. Stem plot of $x(n)$