NCERT 9.1 Q.14

EE23BTECH11203 - Adarsh A*

Question : The Fibonacci sequence is defined by $1 = a_1 = a_2$ and

$a_n = a_{n-1} + a_{n-2}\ , n > 2$

Find $\frac{a_{n+1}}{a_n}$, for $n = 1, 2, 3, 4, 5$

Answer :

Parameter	Value	Description
n	≥ 0	Non negative Integer
$x(n)$	$x(n) = x(n-1) + x(n-2) + u(-n)$	$(n+1)^{th}$ term
$y(n)$	$\frac{x(n+1)}{x(n)}$	Required function
$x(0)$	1	1^{st} term
$x(1)$	1	2^{nd} term

Input Table

Here, $a_1 = 1, a_2 = 1$

$$a_n = a_{n-1} + a_{n-2}\ , n > 2 \tag{1}$$

Applying z transform,

$$X(z) = z^{-1}X(z) + z^{-2}X(z) + z^{-0} \tag{2}$$

$$= \frac{1}{1 - z^{-1} - z^{-2}} \tag{3}$$

$$= \frac{1}{(1 - \alpha z^{-1})(1 - \beta z^{-1})}\ ,\quad |z| > |\alpha| \tag{4}$$

Where, $\alpha = \frac{1 + \sqrt{5}}{2}$ and $\beta = \frac{1 - \sqrt{5}}{2}$

Using partial fractions,

$$X(z) = \frac{\alpha}{(\alpha - \beta)} \frac{1}{(1 - \alpha z^{-1})} - \frac{\beta}{(\alpha - \beta)} \frac{1}{(1 - \beta z^{-1})} \tag{5}$$

$$a^n u(n) \overset{z}{\longleftrightarrow} \frac{1}{1 - az^{-1}} \quad |z| > |a|$$

Substituting this result,

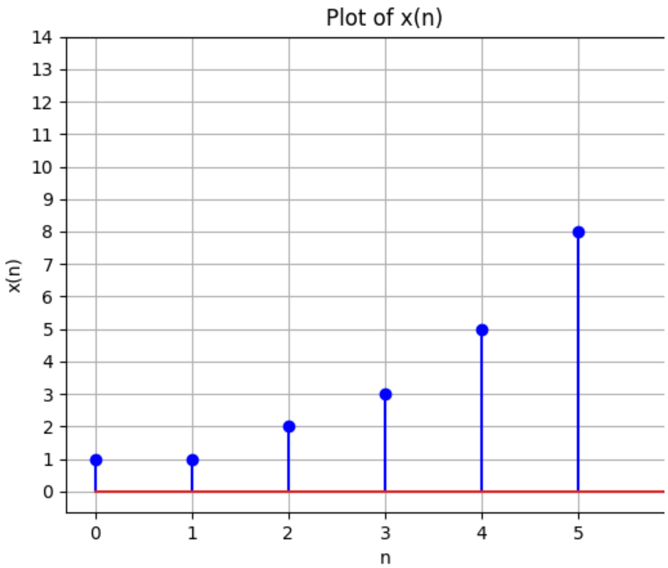
$$x(n) = \frac{\alpha}{(\alpha - \beta)}(\alpha^n u(n)) - \frac{\beta}{(\alpha - \beta)}(\beta^n u(n)) \tag{6}$$

$$x(n) = \frac{\alpha^{n+1} - \beta^{n+1}}{\alpha - \beta}\ u(n) \tag{7}$$

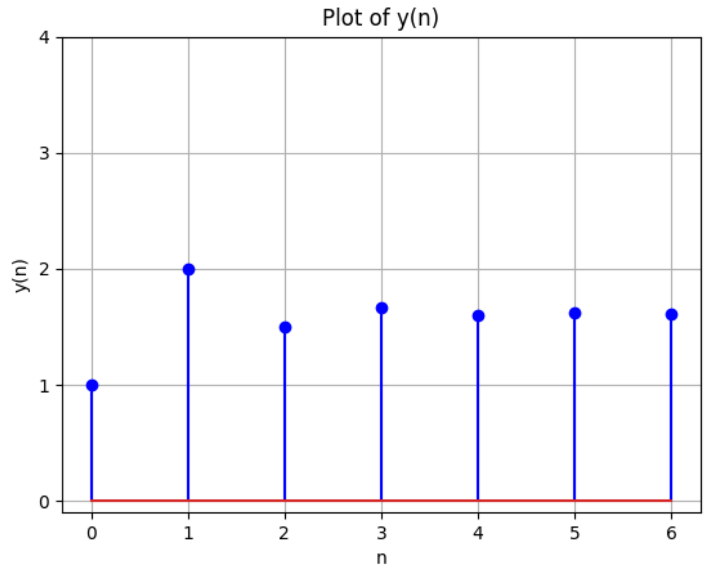
$$x(n) = \frac{(1 + \sqrt{5})^{n+1} - (1 - \sqrt{5})^{n+1}}{2^{n+1}\sqrt{5}}\ u(n) \tag{8}$$

$$y(n) = \frac{x(n+1)}{x(n)} \tag{9}$$

$$y(n) = \frac{1}{2}\left[\frac{(1 + \sqrt{5})^{n+2} - (1 - \sqrt{5})^{n+2}}{(1 + \sqrt{5})^{n+1} - (1 - \sqrt{5})^{n+1}}\right] \tag{10}$$



(a) Plot of $x(n)$ vs n



(b) Plot of $y(n)$ vs n