ANALOG

EE23BTECH11006 - Ameen Aazam*

Question : You are riding in an automobile of mass 3000 kg. Assuming that you are examining the oscillation characteristics of its suspension system. The suspension sags 15 cm when the entire automobile is placed on it. Also, the amplitude of oscillation decreases by 50% during one complete oscillation. Estimate the values of

(a) The spring constant K
(b) The damping constant b for the spring and shock absorber system of one wheel, assuming that each wheel supports 750 kg.

Solution: The parameters are :

Parameter	Value(SI)	Description
x_0	0.15	Initial spring compression
m	750	Mass
g	9.8	Gravitational acc
k	mg/x_0	Spring constant
b		Damping constant

TABLE 2
INPUT PARAMETERS

Parameter	Value(SI)	Description
x		Spring Extension
F_1	kx	Spring Force
F_2	$b\frac{dx}{dt}$	Damping Force
s		Complex Frequency
s_1, s_2		Values of s

TABLE 2
INTERMEDIATE PARAMETERS

Initially the automobile is in rest, so we can use,

$$mg = kx_0 \tag{1}$$

$$\implies k = \frac{mg}{x_0} \tag{2}$$

Now, as the oscillation begins, from the Fig. 2 we net force on the mass as,

$$F = F_1 + F_2 + mgu(t) \tag{3}$$

$$\implies -m\frac{d^2x(t)}{dt^2} = kx(t) + b\frac{dx(t)}{dt} + mgu(t) \tag{4}$$

$$\implies \frac{d^2x(t)}{dt^2} + \left(\frac{b}{m}\right)\frac{dx(t)}{dt} + \left(\frac{k}{m}\right)x(t) = -gu(t) \tag{5}$$

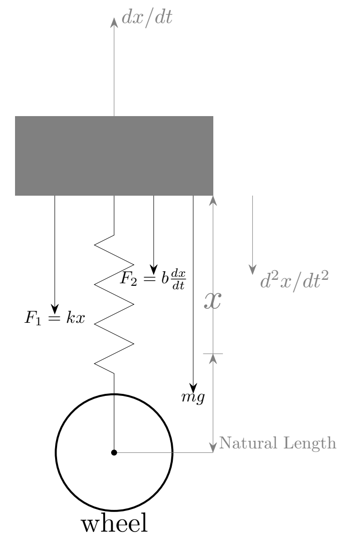

Fig. 2. FBD of the damped oscillation system

Now, taking the Laplace transform on both sides,

$$s^2X(s) + \frac{b}{m}sX(s) + \frac{k}{m}X(s) = -\frac{g}{s} \tag{6}$$

$$\implies X(s) = -\frac{g}{s\left((s^2 + \frac{b}{m}s + \frac{k}{m}\right)} \tag{7}$$

$$\implies X(s) = -\frac{g}{s(s-s_1)(s-s_2)} \tag{8}$$

Where

$$s_1 = -\frac{b}{2m} + \sqrt{\left(\frac{b}{2m}\right)^2 - \frac{k}{m}} \tag{9}$$

$$s_2 = -\frac{b}{2m} - \sqrt{\left(\frac{b}{2m}\right)^2 - \frac{k}{m}} \tag{10}$$

From (8) we get,

$$\implies X(s) = \frac{g}{(s_1 - s_2)}\left[\frac{1}{s_2(s-s_2)} - \frac{1}{s_1(s-s_1)}\right] + \frac{g}{s_1 s_2}\left(\frac{1}{s}\right) \tag{11}$$

Now again taking the inverse Laplace transform we have,

$$x(t) = \frac{g}{s_1 s_2} u(t) + \frac{g}{(s_1 - s_2)} \left[\frac{1}{s_2} e^{s_2 t} - \frac{1}{s_1} e^{s_1 t} \right] u(t) \tag{12}$$

$$\begin{aligned} \implies x(t) = & \sqrt{\left(\frac{mg}{k}\right)^2 + \left(\frac{gb}{2mk}\right)^2} e^{-bt/2m} u(t) \\ & \sin\left(\sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2} t + \tan^{-1}\left(\frac{2mg\sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}}{gb} \right) \right) \\ & + \frac{mg}{k} u(t) \end{aligned} \tag{13}$$

(Substituting the values of s_1 and s_2 from (9) and (10))

From (13) we have the amplitude after one time period T,

$$\begin{aligned} & \frac{1}{2} \sqrt{\left(\frac{mg}{k}\right)^2 + \left(\frac{gb}{2mk}\right)^2} = \\ & \sqrt{\left(\frac{mg}{k}\right)^2 + \left(\frac{gb}{2mk}\right)^2} e^{-bT/2m} \end{aligned} \tag{14}$$

$$\implies e^{\pi b/\sqrt{mk}} = 2 \tag{15}$$

$$\implies b = \frac{\sqrt{mk} \ln 2}{\pi} \tag{16}$$

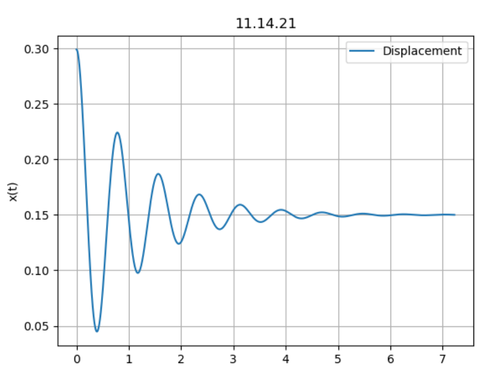

Fig. 2. Displacement Vs. Time Graph