Q: If $a\left(\frac{1}{b}+\frac{1}{c}\right)$, $b\left(\frac{1}{c}+\frac{1}{a}\right)$, $c\left(\frac{1}{a}+\frac{1}{b}\right)$ are in arithmetic progression (AP), prove that a, b, c are also in AP.

Solution: Common difference can be written as:

$$b\left(\frac{1}{c}+\frac{1}{a}\right)-a\left(\frac{1}{b}+\frac{1}{c}\right)=c\left(\frac{1}{a}+\frac{1}{b}\right)-b\left(\frac{1}{c}+\frac{1}{a}\right) \tag{1}$$

$$\implies (b-a)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)=(c-b)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) \tag{2}$$

$$\implies b-a=c-b \tag{3}$$

Hence proved that a, b, c are in AP.

parameter	value	description
$x(0)$	$a\left(\frac{1}{b}+\frac{1}{c}\right)$	First Term of given AP
d	(b - a)$\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$	Common Difference of given AP
$x(n)$	$(x(0)+nd)u(n)$	General Term of given AP

TABLE I
INPUT PARAMETER TABLE

From table Table I

$$X(z)=x(0)\left(\frac{1}{1-z^{-1}}\right)+d\left(\frac{z^{-1}}{(1-z^{-1})^2}\right) \tag{4}$$

$$=a\left(\frac{1}{b}+\frac{1}{c}\right)\left(\frac{1}{1-z^{-1}}\right)+(b-a)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\left(\frac{z^{-1}}{(1-z^{-1})^2}\right)\quad |z|>1 \tag{5}$$

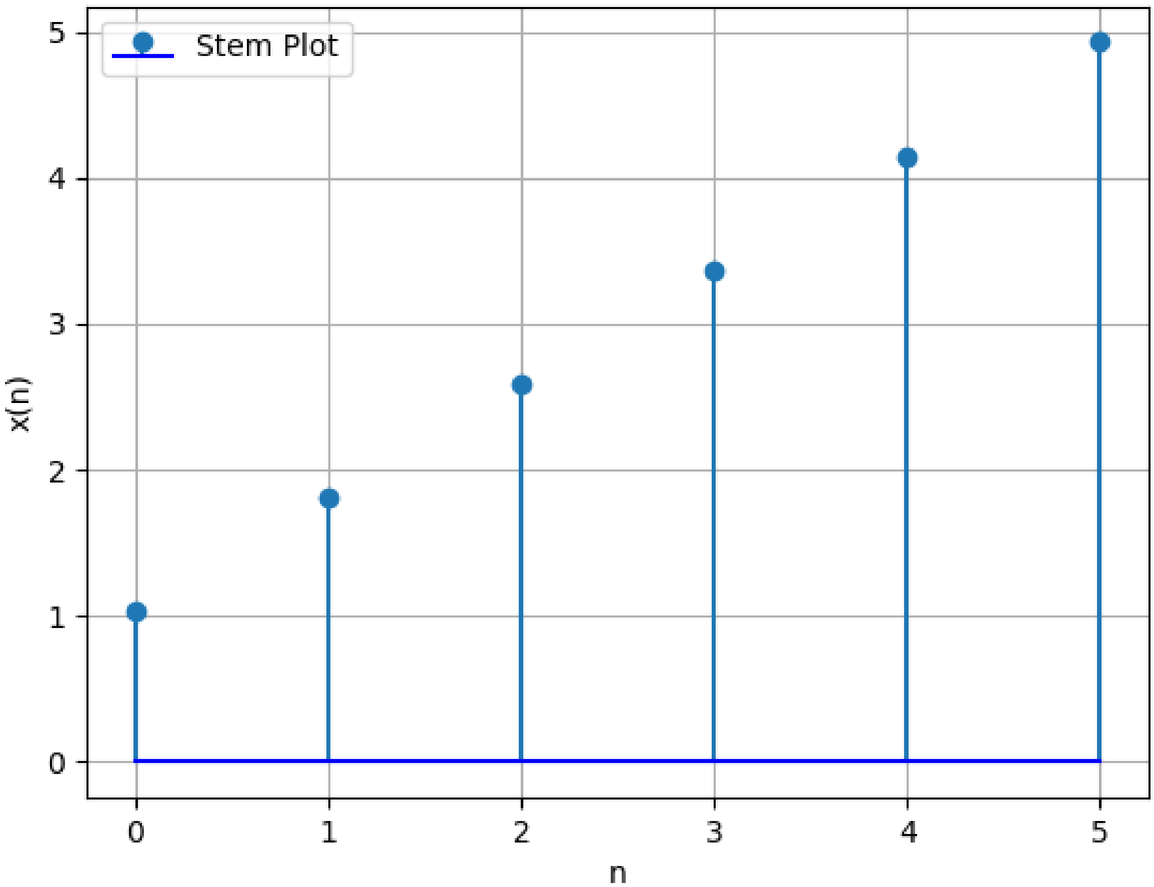

Fig. 1. graph with value of $a = 3, b = 5, c = 7$