
SIGNAL PROCESSING FUNDAMENTALS Through NCERT

G. V. V. Sharma



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Introduction

This book introduces some concepts in signal processing through maths and physics problems in NCERT textbooks.

Chapter 1

Analog

1.1. Harmonics

1.1.1 A charged particle oscillates about its mean equilibrium position with a frequency of $10^9 Hz$. What is the frequency of the electromagnetic waves produced by the oscillator?

Solution:

Symbol	Value	Description
$y(t)$	$\cos(2\pi f_c t)$	Wave equation of electro-magnetic wave
f_c	10^9	Frequency of electromagnetic wave
t	seconds	Time

Table 1.1: Variable description

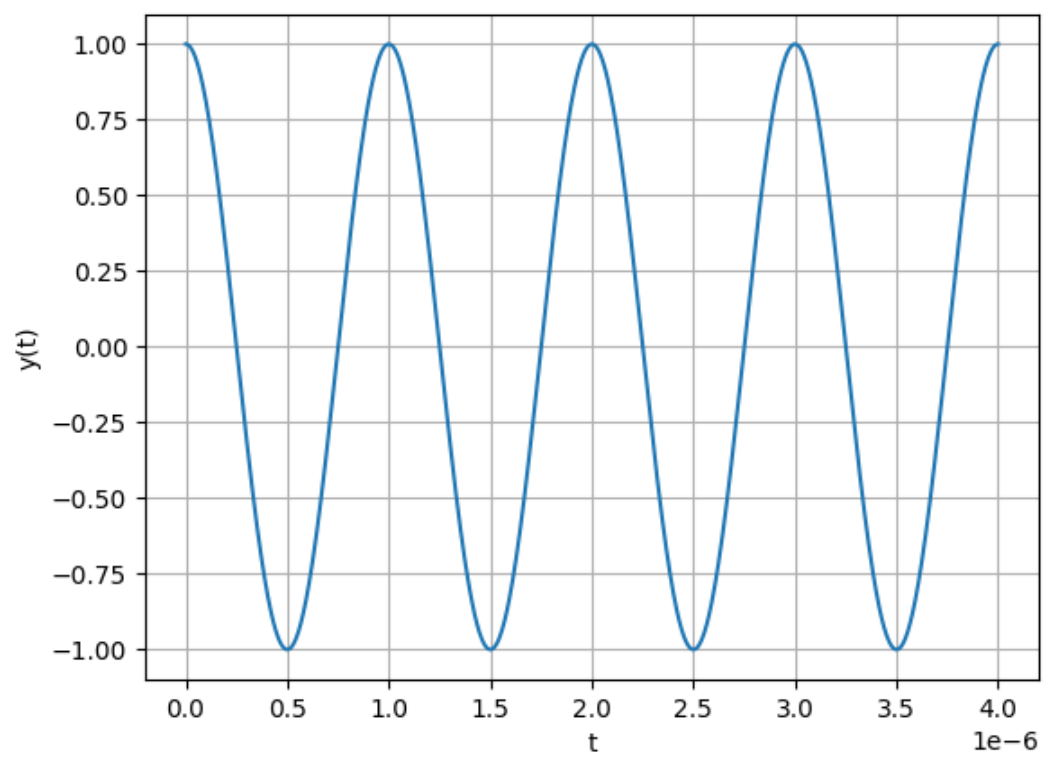


Figure 1.1: $y(t) = \cos(2\pi \times 10^9 t)$

Chapter 2

Discrete

2.1. Z-transform

2.1.1 Write the five terms at $n = 1, 2, 3, 4, 5$ of the sequence and obtain the Z-transform of the series

$$x(n) = -1, \quad n = 0 \quad (2.1)$$

$$= \frac{x(n-1)}{n}, \quad n > 0 \quad (2.2)$$

$$= 0, \quad n < 0 \quad (2.3)$$

Solution:

$$x(1) = \frac{x(0)}{1} = -1 \quad (2.4)$$

$$x(2) = \frac{x(1)}{2} = -\frac{1}{2} \quad (2.5)$$

$$x(3) = \frac{x(2)}{3} = -\frac{1}{(2)(3)} = -\frac{1}{6} \quad (2.6)$$

$$x(4) = \frac{x(3)}{4} = -\frac{1}{(2)(3)(4)} = -\frac{1}{24} \quad (2.7)$$

$$x(5) = \frac{x(4)}{5} = -\frac{1}{(2)(3)(4)(5)} = -\frac{1}{120} \quad (2.8)$$

$$x(n) = \frac{-1}{n!} (u(n)) \quad (2.9)$$

$$x(n) \xleftrightarrow{\mathcal{Z}} X(z) \quad (2.10)$$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} \quad (2.11)$$

using (2.9),

$$= \sum_{n=-\infty}^{\infty} \frac{-1}{n!} u(n) z^{-n} \quad (2.12)$$

$$= \sum_{n=0}^{\infty} \frac{-1}{n!} z^{-n} \quad (2.13)$$

$$= -e^{z^{-1}} \quad \{z \in \mathbb{C} : z \neq 0\} \quad (2.14)$$

Symbol	Value	Description
$x(n)$	$\frac{-1}{n!}$	general term of the series
$X(z)$	$-e^{z^{-1}}$	Z-transform of x(n)
$u(n)$		unit step function

Table 2.1: Parameters

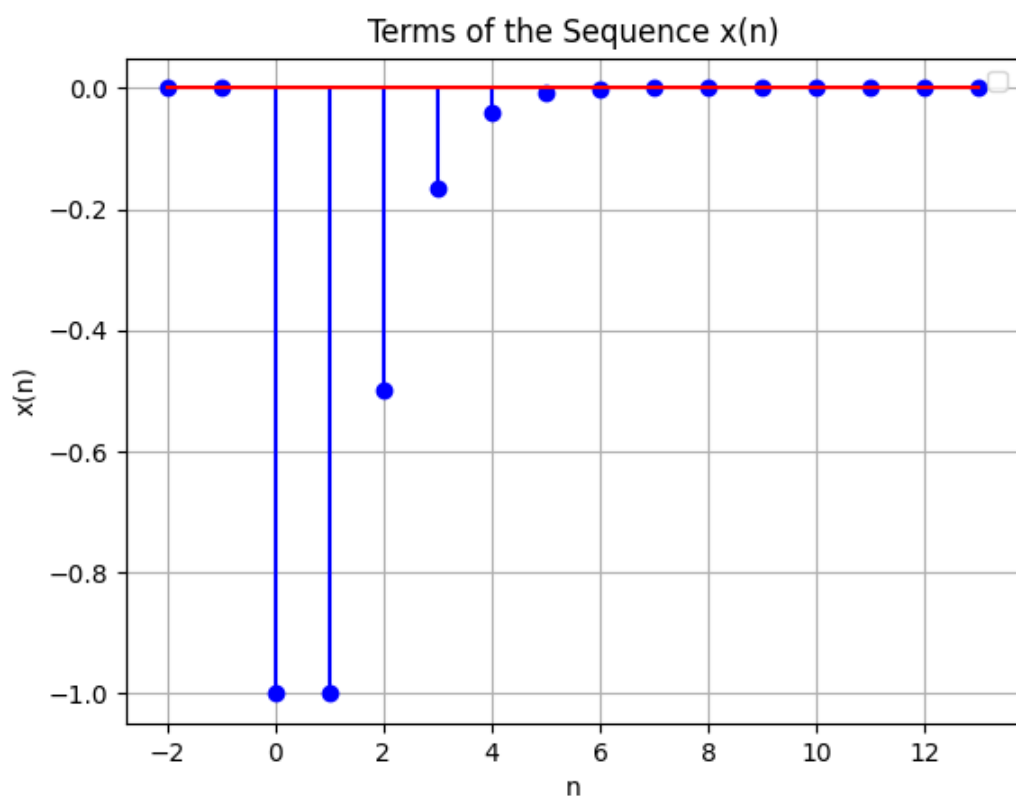


Figure 2.1: Plot of $x(n)$ vs n

Appendix A

Z-transform

A.1 The Z-transform of $p(n)$ is defined as

$$P(z) = \sum_{n=-\infty}^{\infty} p(n)z^{-n} \quad (\text{A.1.1})$$

A.2 If

$$p(n) = p_1(n) * p_2(n), \quad (\text{A.2.1})$$

$$P(z) = P_1(z)P_2(z) \quad (\text{A.2.2})$$

The above property follows from Fourier analysis and is fundamental to signal processing.

A.3 For a Geometric progression

$$x(n) = x(0) r^n u(n), \quad (\text{A.3.1})$$

$$\Rightarrow X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} = \sum_{n=0}^{\infty} x(0) r^n z^{-n} \quad (\text{A.3.2})$$

$$= \sum_{n=0}^{\infty} x(0) (rz^{-1})^n \quad (\text{A.3.3})$$

$$= \frac{x(0)}{1 - rz^{-1}}, \quad |z| > |r| \quad (\text{A.3.4})$$

A.4 Let

$$u(n) = \begin{cases} 1 & n \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad (\text{A.4.1})$$

Substituting $r = 1$ in (A.3.4),

$$u(n) \xleftrightarrow{\mathcal{Z}} U(z) = \frac{1}{1 - z^{-1}}, \quad |z| > 1 \quad (\text{A.4.2})$$

A.5 From (A.1.1) and (A.4.2),

$$U(z) = \sum_{n=-\infty}^{\infty} u(n) z^{-n} \quad (\text{A.5.1})$$

$$\Rightarrow \frac{dU(z)}{dz} = -z^{-1} \sum_{n=-\infty}^{\infty} nu(n) z^{-n} \quad (\text{A.5.2})$$

$$\therefore nu(n) \xleftrightarrow{\mathcal{Z}} \frac{z^{-1}}{(1 - z^{-1})^2}, \quad |z| > 1 \quad (\text{A.5.3})$$

A.6 For an AP,

$$x(n) = [x(0) + nd] u(n) = x(0)u(n) + dnu(n) \quad (\text{A.6.1})$$

$$\Rightarrow X(z) = \frac{x(0)}{1 - z^{-1}} + \frac{dz^{-1}}{(1 - z^{-1})^2}, \quad |z| > 1 \quad (\text{A.6.2})$$

upon substituting from (A.4.2) and (A.5.3).

A.7 General form of GP

$$x(n) = x(0) r^n u(n) \quad (\text{A.7.1})$$

Let

$$y(n) = x(n) * u(n) \quad (\text{A.7.2})$$

$$= \sum_{k=-\infty}^{\infty} x(k) u(n-k) \quad (\text{A.7.3})$$

on substituting (A.7.1) in The convolution sum

$$y(n) = \sum_{k=-\infty}^{\infty} x(0) r^n u(n) u(n-k) \quad (\text{A.7.4})$$

$$= \sum_{k=0}^n x(0) r^n \quad (\text{A.7.5})$$

Which gives us sum of GP, So On Taking Z-Transform on (A.7.2)

$$Y(z) = X(z) U(z) \quad (\text{A.7.6})$$

From Z-Transform of a term in GP

$$X(z) = \frac{x(0)}{1 - rz^{-1}} \quad |z| > |r| \quad (\text{A.7.7})$$

and Z-Transform of unit step is

$$U(z) = \frac{1}{1 - z^{-1}} \quad |z^{-1}| < 1 \quad (\text{A.7.8})$$

$$\Rightarrow Y(z) = \left(\frac{x(0)}{1 - rz^{-1}} \right) \left(\frac{1}{1 - z^{-1}} \right) \quad |z| > |r| \quad \text{and} \quad |z| > |1| \quad (\text{A.7.9})$$

By using Partial Fractions

$$Y(z) = x(0) \left(\frac{A}{1 - rz^{-1}} + \frac{B}{1 - z^{-1}} \right) \quad (\text{A.7.10})$$

$$A = \frac{r}{r - 1} \quad (\text{A.7.11})$$

$$B = \frac{-1}{r - 1} \quad (\text{A.7.12})$$

Using (A.7.7) and Taking inverse Z-transform

$$y(n) = \frac{x(0)}{r - 1} (r(r^n) - 1) u(n) \quad (\text{A.7.13})$$

$$= x(0) \left(\frac{r^{n+1} - 1}{r - 1} \right) u(n) \quad (\text{A.7.14})$$