Bode Plot

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Lecture 1

Basics of Logarithms

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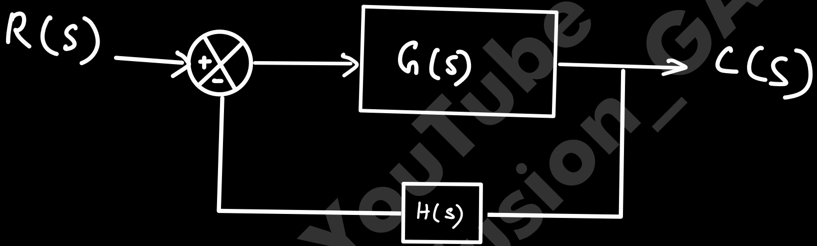
Basics of frequency Response



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↳ Let $G(s) = \frac{1}{s}$, $H(s) = 1$

From figure

$$T(s) = \frac{G(s)}{1 + G(s)H(s)} = \frac{1/s}{1 + 1/s} = \frac{1}{s+1}$$

$$T(j\omega) = \frac{1}{j\omega + 1}$$

$$|T(j\omega)| = \frac{1}{\sqrt{\omega^2 + 1}}$$

$$\angle T(j\omega) = -\tan^{-1}\omega$$

$$T(j\omega) = \frac{(1 - j\omega)}{1+\omega^2} = \underbrace{\frac{1}{1+\omega^2}}_{\text{Real}} - j\underbrace{\frac{\omega}{1+\omega^2}}_{\text{complex}}$$

$\downarrow$

Complex

$$T(j\omega) = |T(j\omega)| \angle T(j\omega)$$

- In Bode Plot we plot two graphs:—

$|T(j\omega)|$ vs ω → Magnitude Response

$\angle T(j\omega)$ vs ω → Phase Response

- **Basics of log**

- $\log_a x = y \rightarrow a^y = x$

- **Properties**

↳ $\log_a a = 1$ $\rightarrow a^y = a \Rightarrow y = 1$

↳ $\log_a 1 = 0$ $\rightarrow a^y = 1 \Rightarrow y = 0$

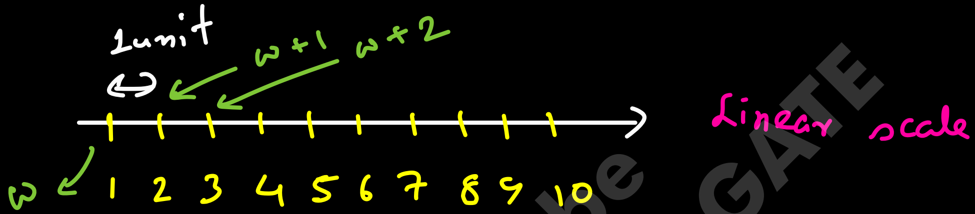
- **Properties**

↳ $\log_a(mn) = \log_a m + \log_a n$

↳ $\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$

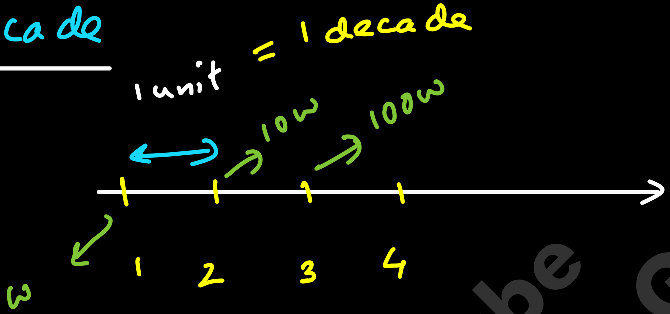
↳ $\log_a x^2 = 2\log_a x$

- Linear scale → Equidistant



- In linear scale, If I travel 1 unit distance then my current value gets increased by 1 of the previous value.

- Decade

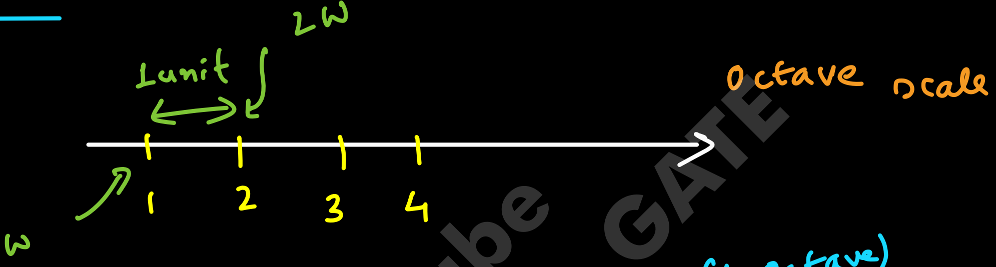


Decade scale

(1 decade)

- In decade scale, If I travel 1 unit distance then my current value becomes 10 times of the previous value.

• Octave



(1 octave)

• In Octave scale, If I travel 1 unit distance then my current value becomes 2 times of the previous value.

Conversion

↳ decimal to dBs

↓

$x \rightarrow 20\log_{10} x$ dB

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- # Understanding the Scale of x- axis

Let $f(\omega) = \begin{cases} 2\omega , & 0 \le \omega \le 1000 \\ 0 , & 0 \end{cases}$

① (i) Plot $f(\omega)$ v/s ω

(ii) Plot $f(\omega)$ v/s $\omega^{1/3}$

(iii) Plot $f(\omega)$ v/s $\log_{10}\omega$

} Change of scale.

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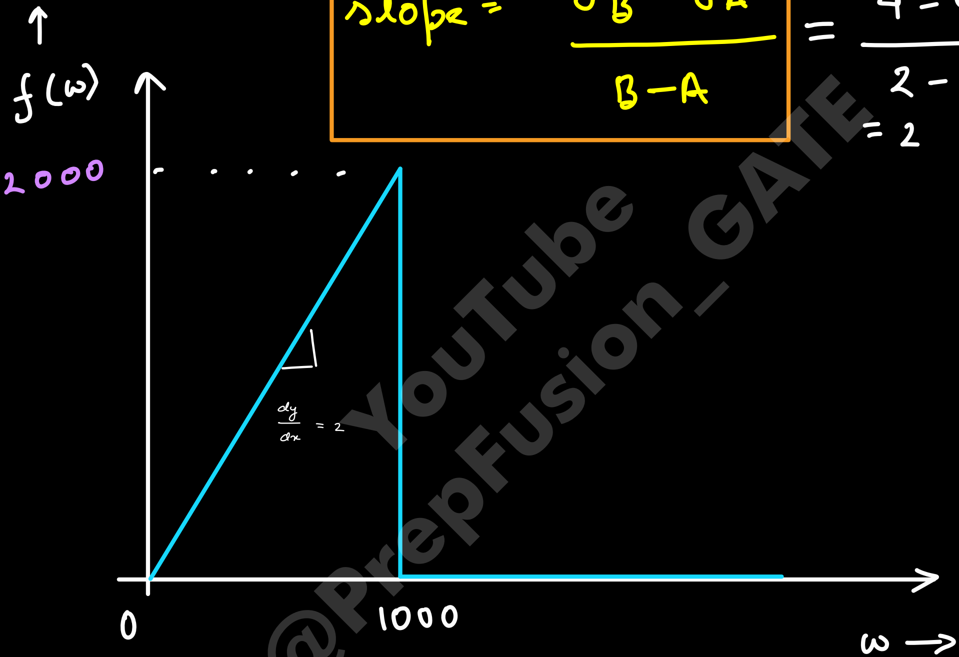
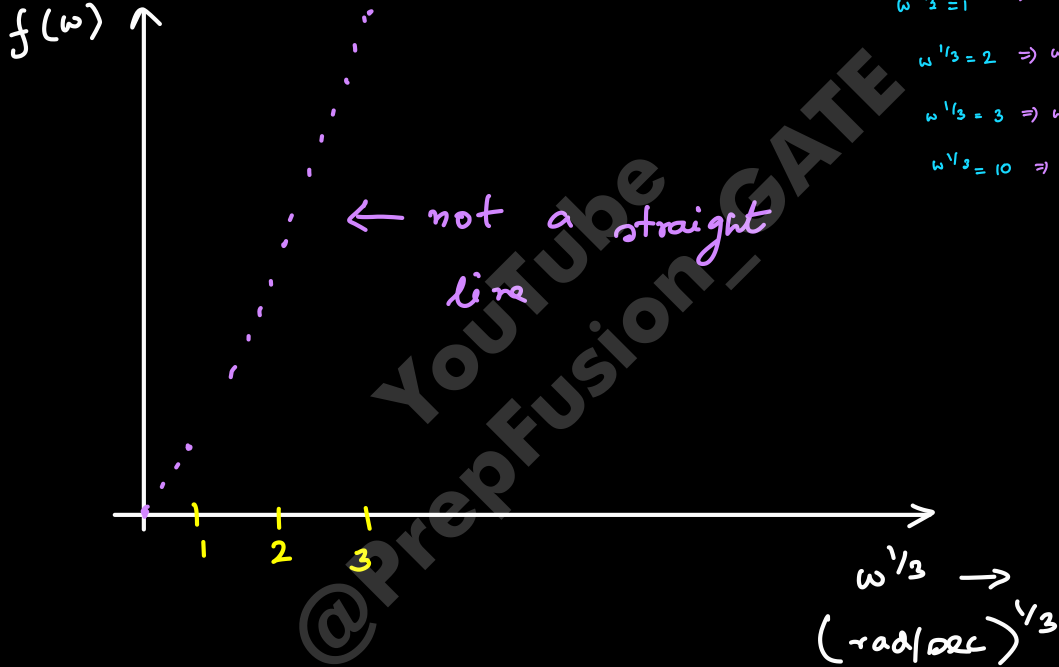
(i)

$$\boxed{\text{slope} = \frac{y_B - y_A}{B - A}} = \frac{4-0}{2-0} = 2$$



(ii)



$\omega^{1/3} = 0 \Rightarrow \omega = 0$

$\omega^{1/3} = 1 \Rightarrow \omega = 1$

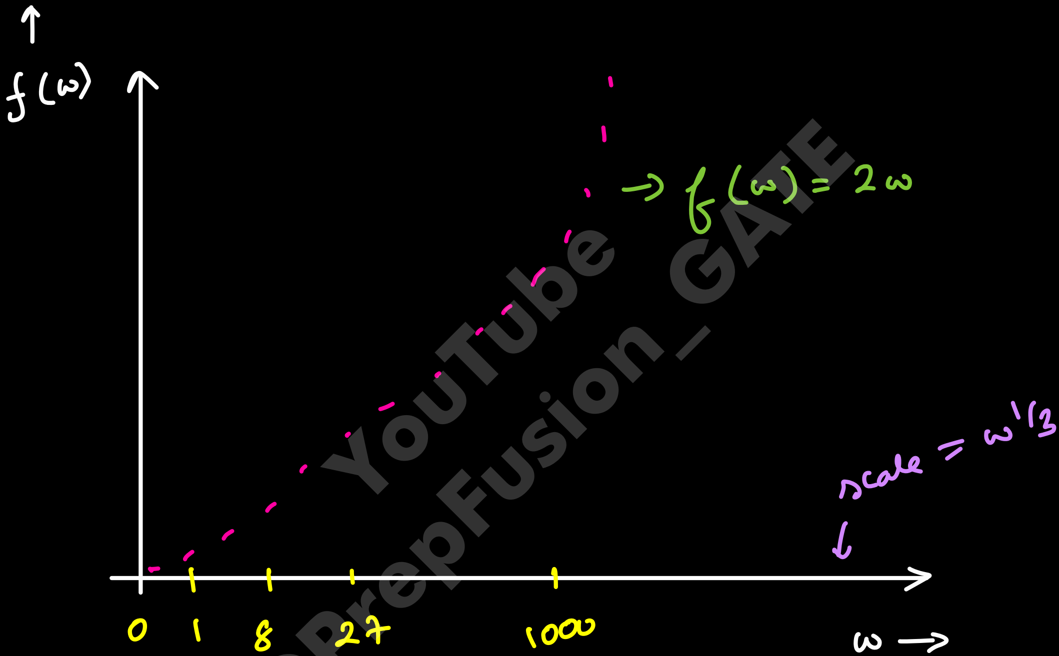
$\omega^{1/3} = 2 \Rightarrow \omega = 8$

$\omega^{1/3} = 3 \Rightarrow \omega = 27$

$\omega^{1/3} = 10 \Rightarrow \omega = 1000$

- $w^{1/3}$ is increasing linearly

But w is increasing non-linearly.



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(iii)

for one decade movement, ω gets multiplied by 10.

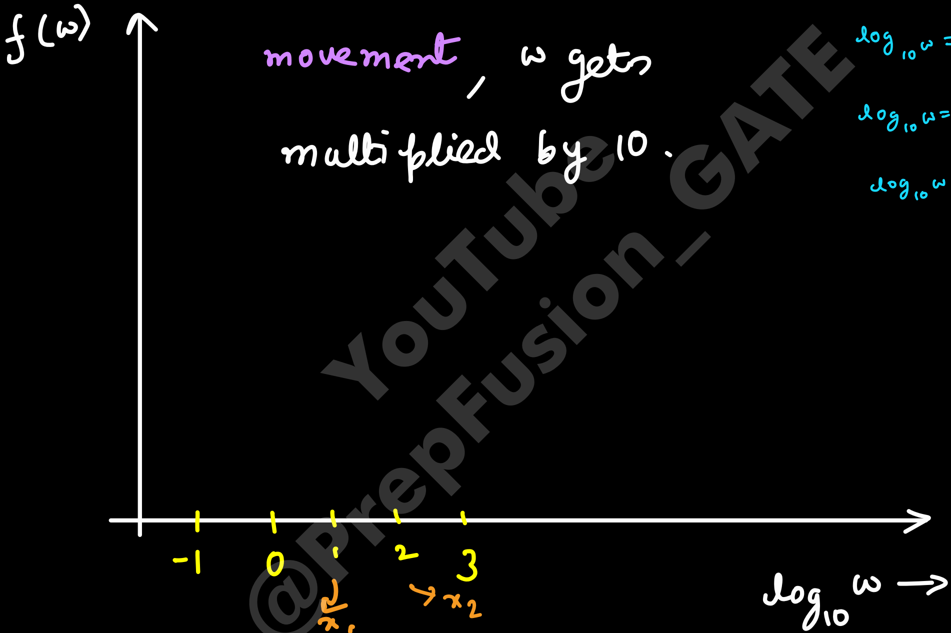
$\log_{10}\omega = -1 \Rightarrow \omega = 0.1$

$\log_{10}\omega = 0 \Rightarrow \omega = 1$

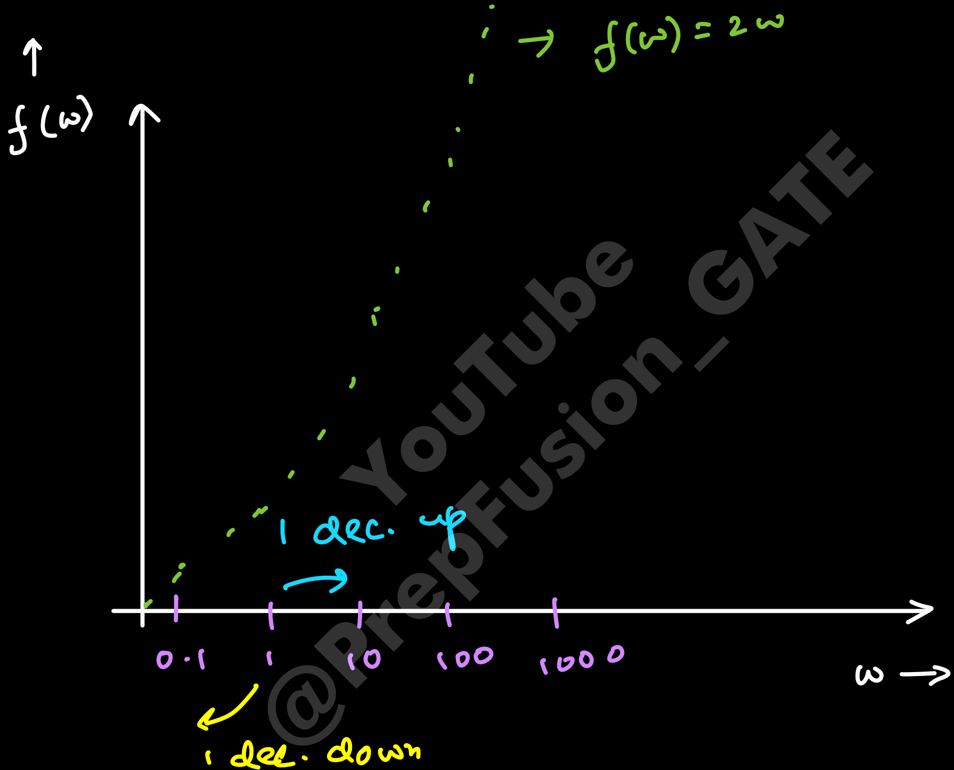
$\log_{10}\omega = 1 \Rightarrow \omega = 10$

$\log_{10}\omega = 2 \Rightarrow \omega = 100$

$\log_{10}\omega = 3 \Rightarrow \omega = 1000$



$[\log_{10}(rad/sec)]$



f(ω)
f(ω) = 2ω
1 dec. up
1 dec. down
0.1
1
10
100
1000
ω

• Decade & Octave scale

↳ 1 decade ⇒ multiples of 10's

↳ 1 octave ⇒ multiples of 2's

↳ 1 decade = $x_2 - x_1 = \log_{10}\omega_2 - \log_{10}\omega_1$

$$= \log_{10}\left(\frac{\omega_2}{\omega_1}\right)$$

↳ $1 = \log_{10}\left(\frac{\omega_2}{\omega_1}\right)$ $\quad \frac{\omega_2}{\omega_1} = 10^1$, $\boxed{\omega_2 = 10\omega_1}$

↳ 1 octave $= x_2 - x_1 = \log_2 \omega_2 - \log_2 \omega_1$

$$1 = \log_2\left(\frac{\omega_2}{\omega_1}\right)$$

$$2^1 = \frac{\omega_2}{\omega_1} \Rightarrow \boxed{\omega_2 = 2\omega_1}$$

- 1 octave is how much distance in decade scale. → $\omega_2 = 2\omega_1$

distance in decade $= \log_{10}\left(\frac{\omega_2}{\omega_1}\right)$ dec

$= \log_{10}(2)$

$$\boxed{1 \text{ oct} = 0.301 \text{ dec}}$$

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- 1 decade is how much distance in octave scale. $\rightarrow \omega_2 = 10\omega_1$

$$\text{distance in octave} = \log_2\left(\frac{\omega_2}{\omega_1}\right) \text{ oct}$$

$$= \log_2(10) \text{ oct}$$

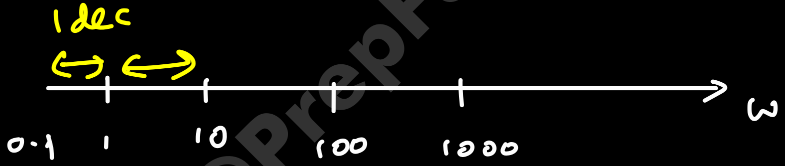
$$\boxed{1 \text{ dec} = 3.33 \text{ oct} \simeq 3 \text{ oct}}$$

Why Log scale in x-axis?

↳ To plot larger range of ω.

↳ Low frequency region can be expanded.

↳ We plot Bode Plots In Semi - Log Paper



Limitation of Log Scale

↳ In log scale we won't be able to plot ω=0

↳ $\log_{10} 0 = -\infty$

↳ for frequency Response → $\omega > 0$

$\sin(0t)$, $\cos(0t)$ → doesn't give me any information

$\sin(0t) \to 0$, $\cos(0t) \to 1$

The End

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Lecture 2

Exact Analysis
Building Blocks of Bode Plots
Concept of Corner Frequency

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Exact Frequency Analysis

Given $G(s) = \dfrac{K}{1+s\tau}$

① Put, $s = j\omega \rightarrow 0^+ < \omega < +\infty$

② $G(j\omega) = |G(j\omega)| e^{j\angle G(j\omega)}$

$\rightarrow$ Exact Phase Plot

$\angle G(j\omega)$ vs ω

Exact magnitude Plot

$\rightarrow |G(j\omega)|$ vs ω

Keypoints

linear, $y = mx + c$

↳ Exact plots are non-linear in shape in general.

$$|G(j\omega)| = \frac{K}{\sqrt{1+(\omega\tau)^2}} \rightarrow |G(j\omega)| \neq m\omega + c$$

$$\angle G(j\omega) = -\tan^{-1}(\omega\tau) \rightarrow \angle G(j\omega) \neq m\omega + c$$

non-linear

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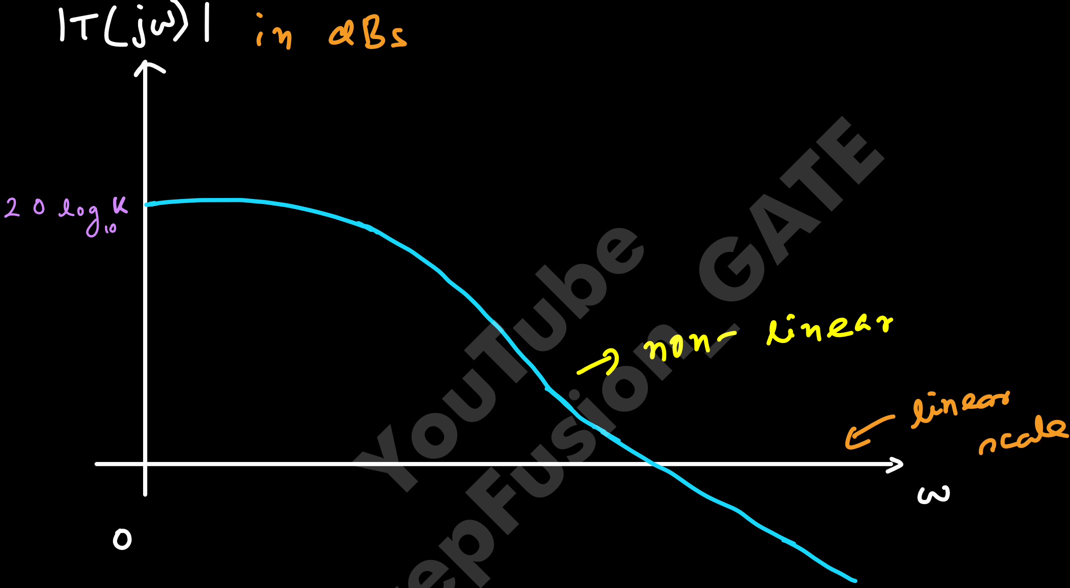
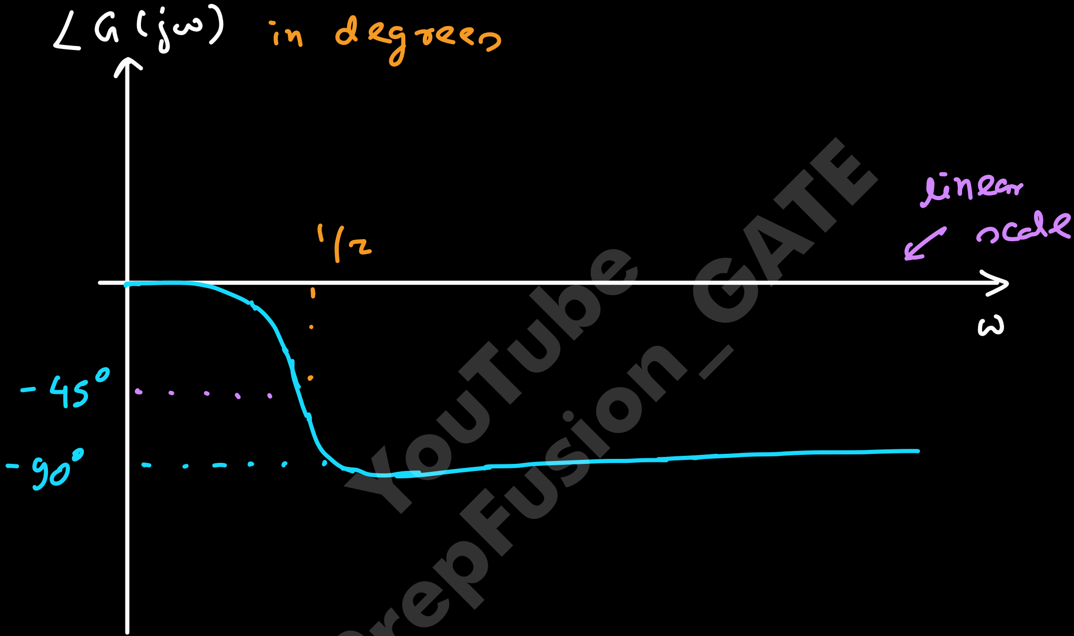


fig - Exact magnitude plot



∠G(jω) in degrees
1/2
linear scale
ω
-45°
-90°

Why dB scale in Y-axis of Magnitude Plot?

↳ To plot larger range of Magnitude.

↳ Can break the factors into simple algebric Sums.

→ Phase plot already algebric sum of $\tan^{-1}(\ )$

magnitude in multiplication & division.

log (magnitude) ⇒ Algebric sum.

Example -
$$T(s) = \frac{(1+3s)}{(1+2s)(1+s)}$$

write magnitude expression in dBs.

Soln - $$T(j\omega) = \frac{(1+j3\omega)}{(1+2j\omega)(1+j\omega)}$$

$$|T(j\omega)| = \frac{\sqrt{1+(3\omega)^2}}{\sqrt{1+(2\omega)^2}\sqrt{1+\omega^2}}$$

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$$20 \log |T(j\omega)|$$

$$= \underbrace{20 \log_{10}\left(\sqrt{1+(3\omega)^2}\right) - \left[20 \log_{10}\sqrt{1+(2\omega)^2} + 20 \log_{10}\left(\sqrt{1+\omega^2}\right)\right]}$$

Algebric Sum

Building Blocks of Bode Plot

Question

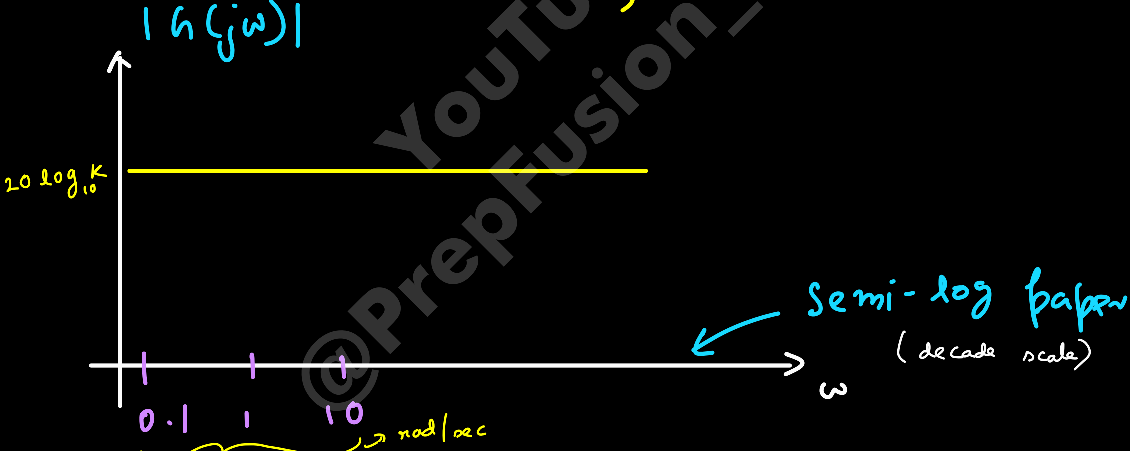
Plot bode plots for the following functions

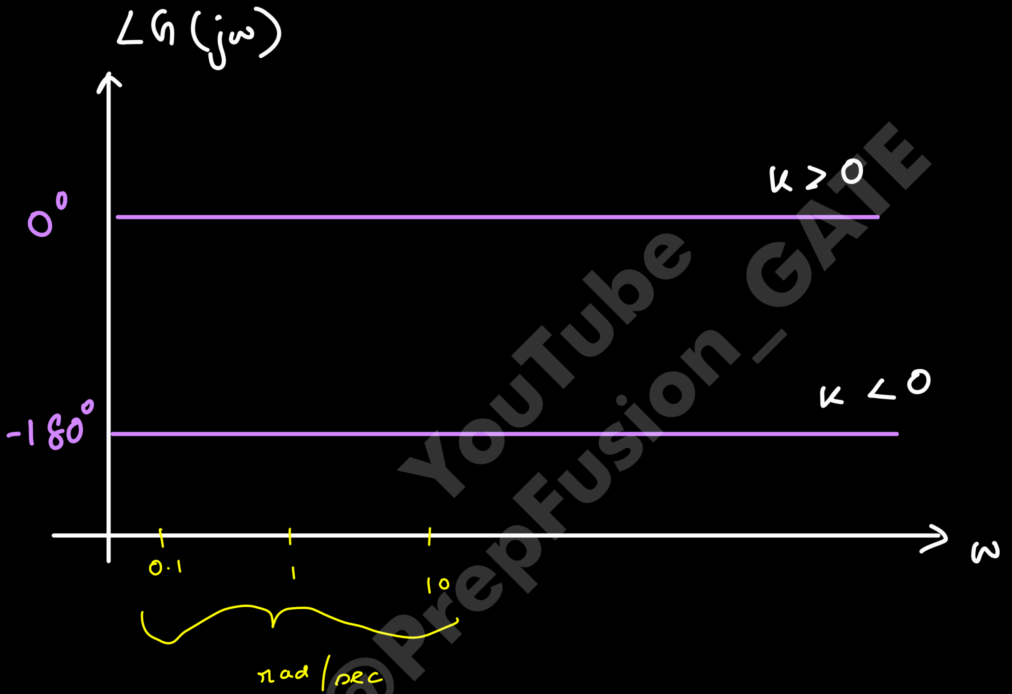
① $G(s) = K$ ② $G(s) = \frac{1}{s}$ ③ $G(s) = s$

$K \in \mathbb{R}$

1) $G(j\omega) = K$, $|G(j\omega)| = |K| = K$

$$\angle G(j\omega) = \begin{cases} 0^\circ & ; K > 0 \\ -180^\circ & ; K < 0 \end{cases}$$




∠G(jω)
0°
-180°
K > 0
K < 0
ω
0.1
1
10
rad/sec

- If $G(j\omega) = k$

- Bode magnitude plots shifts up/ down by $20\log k$ dB

- Bode phase plot doesn't change, $k > 0$

  shifts down by -180°, $k < 0$

② $G(s) = \frac{1}{s} = \frac{1}{j\omega} \rightarrow |G(j\omega)| = \frac{1}{\omega}$

$\angle G(j\omega) = -90^\circ$

$|G(j\omega)|$ in dB $= 20\log_{10}\left(\frac{1}{\omega}\right) = 20\log_{10} 1$

$- 20\log_{10}\omega$

$|G(j\omega)| = -20\log_{10}\omega \rightarrow$ decades

$\downarrow$

dB

slope $= -20$ dB/dec

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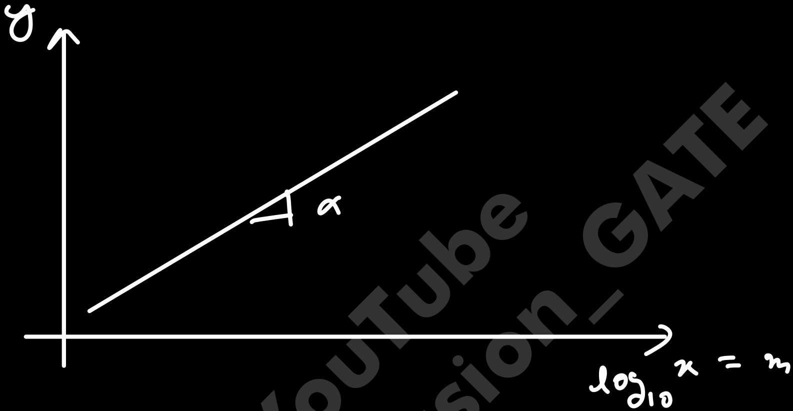
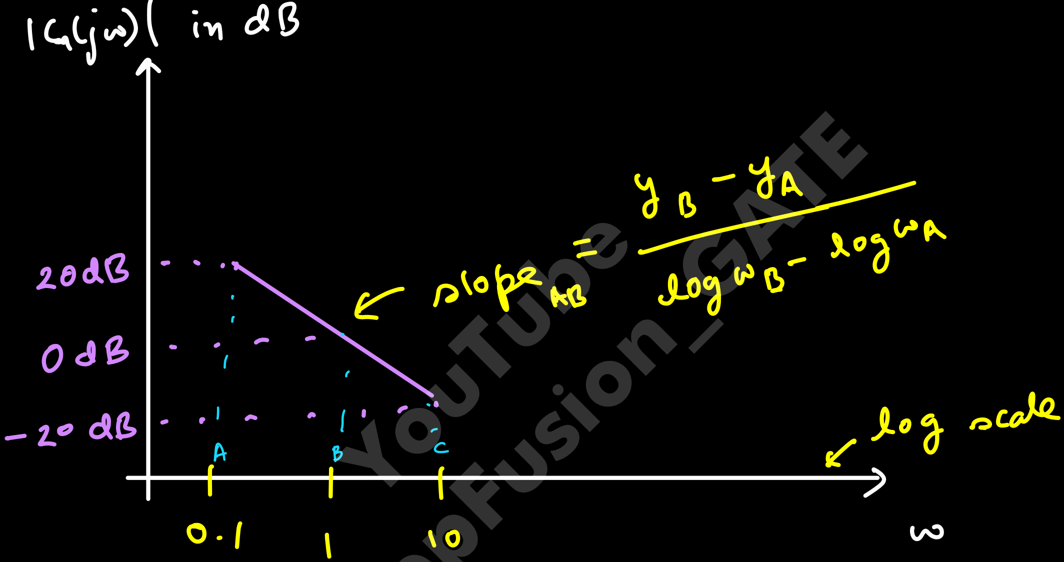


$$y = \alpha m = \alpha \log_{10} x$$



$$\text{slope}_{AB} = \frac{y_B - y_A}{\log \omega_B - \log \omega_A}$$

- As we move every decade up, our magnitude decreases by 20 dB

↳ hence slope = -20 dB/dec

$$\boxed{slope_{AB} = \frac{y_B - y_A}{\log \omega_B - \log \omega_A} = \frac{y_B - y_A}{\log\left(\frac{\omega_B}{\omega_A}\right)} \text{ dBs/dec}}$$

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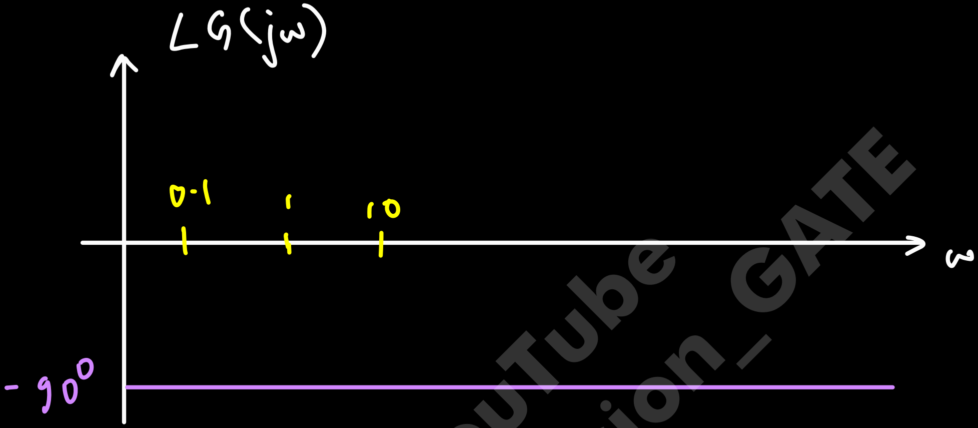
∠G(jω)
0.1
1
10
ω
-90°

- If $G(j\omega) = \frac{1}{j\omega}$

- Slope of Bode magnitude plots changes by -20 dB/dec.

- Bode phase plot shifts down by $90°$.

③ $G(s) = s = j\omega \rightarrow |G(j\omega)| = \omega$

$\angle G(j\omega) = +90^\circ$

$|G(j\omega)|$ in dB $= \underset{m}{20} \underbrace{\log_{10}(\omega)}_{x}$

slope = $+20$ dB/dec

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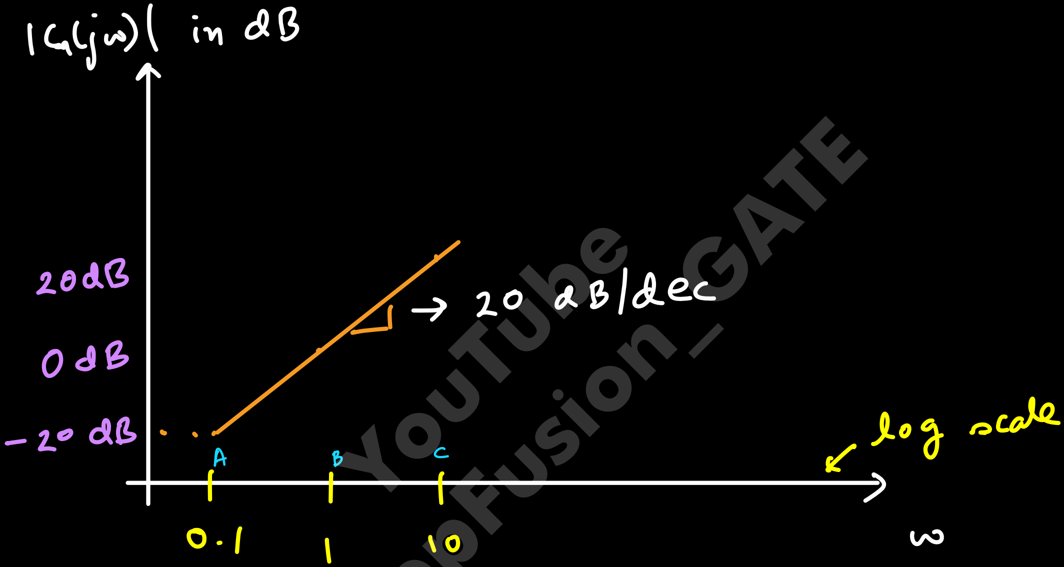
|G(jω)| in dB
20dB
0 dB
-20 dB
→ 20 dB/dec
A
B
C
0.1
1
10
log scale
ω

- As we move every decade up, our magnitude increases by 20 dB

↳ hence slope = +20 dB/dec

$$\boxed{\text{slope}_{AB} = \frac{y_B - y_A}{\log \omega_B - \log \omega_A} = \frac{y_B - y_A}{\log\left(\frac{\omega_B}{\omega_A}\right)} \text{ dBs/dec}}$$

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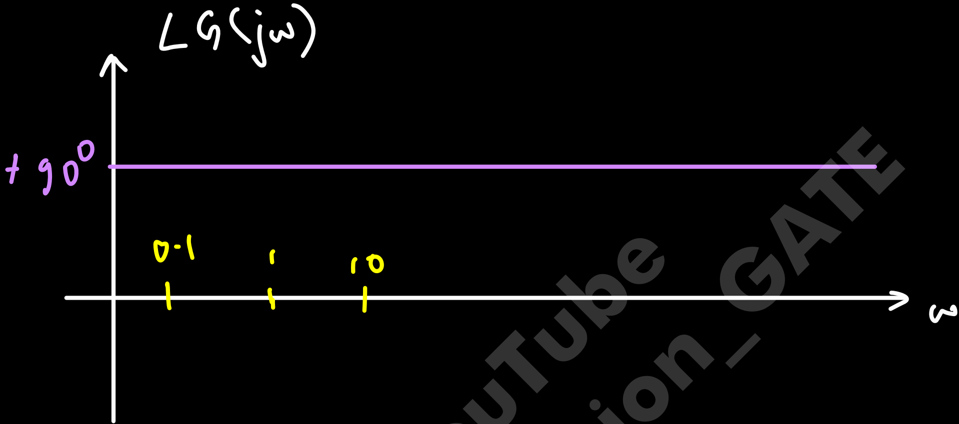

∠G(jω)
+90°
0.1
1
10
ω

- If $G(j\omega) = j\omega$

- Slope of Bode magnitude plots changes by $+20$ dB/dec.

- Bode phase plot shifts up by 90°.

Question

Plot bode plots for the following functions

④ $G(s) = \frac{1}{1+s\tau}$

⑤ $G(s) = 1 + s\tau$

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(4) $|G(j\omega)| = \dfrac{1}{\sqrt{1+(\omega\tau)^2}}$, $\angle G(j\omega) = -\tan^{-1}(\omega\tau)$

Approximation

- At low freq^n, $\omega << \dfrac{1}{\tau} \Rightarrow \omega\tau << 1$

(LFR)

$\rightarrow |G(j\omega)| \simeq 1 \Rightarrow |G(j\omega)|$ in dB $\simeq 0$

low frequency Asymptote

(don't know from where it started)

$\angle G(j\omega) \simeq 0^\circ$

- At high freq^n, $\omega >> \frac{1}{\tau}$
(HFR)

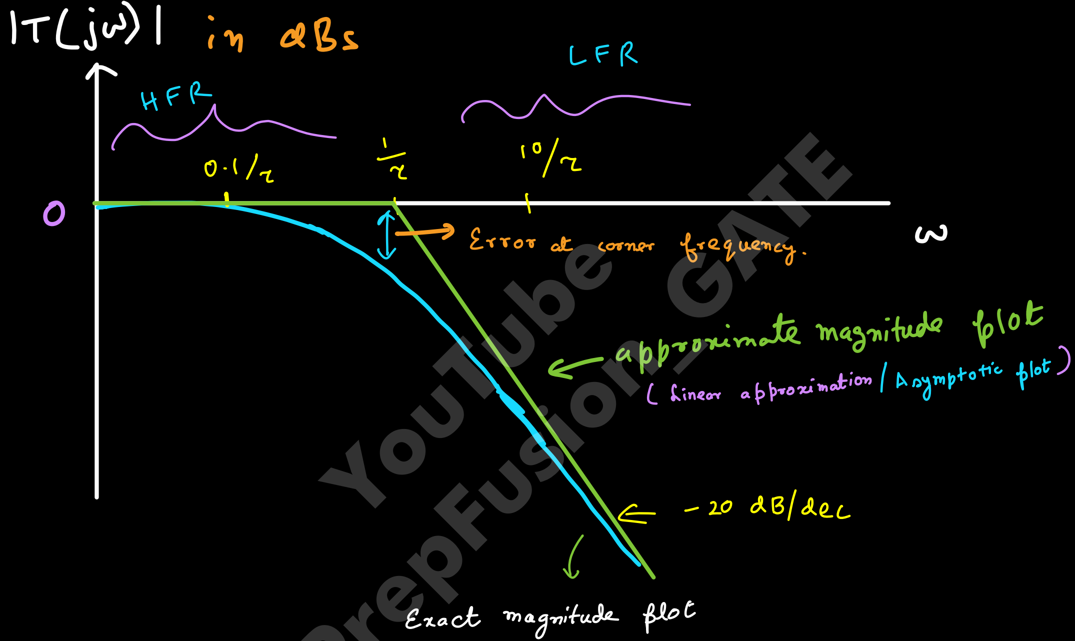
$\tau\omega >> 1 \Rightarrow |G(j\omega)| \approx \frac{1}{\omega\tau}$

$\Rightarrow G(j\omega)$ in dB $= -20\log(\omega\tau)$

High frequency Asymptote

$\angle G(j\omega) \approx 90^\circ$

⇒ The point of Intersection of low frequency Asymptote & high frequency Asymptote is known as Corner frequency.



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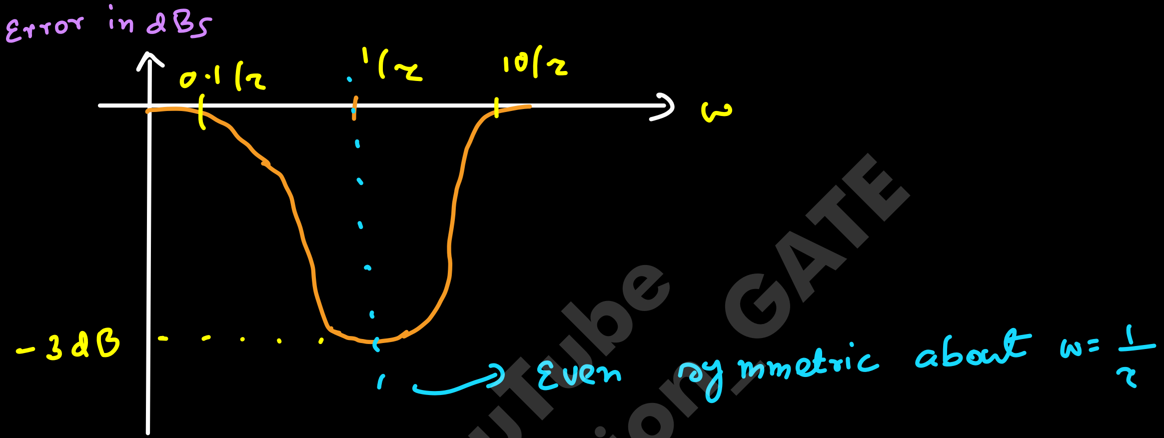

$$|G(j\omega)|_{\omega=\frac{1}{\tau}} = \frac{1}{\sqrt{1+\left(\frac{1}{\tau}\times\tau\right)^2}} = \frac{1}{\sqrt{2}}$$

$$= -20\log_{10}\sqrt{2}$$

$$= -3.01\ dB$$

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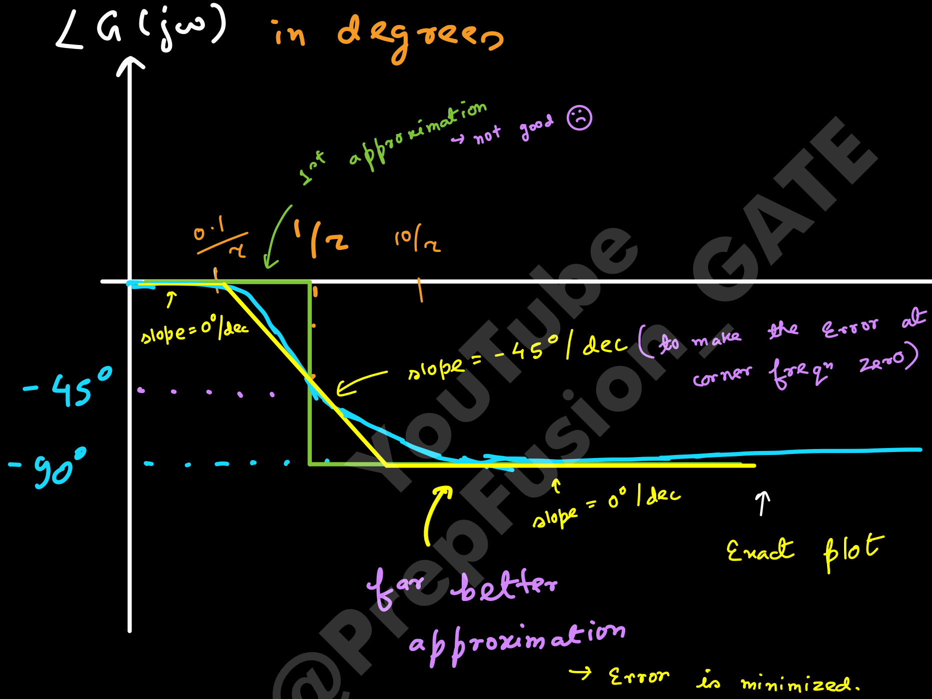
KeyPoints about Corner Frequency

↳ The frequency at which the slope of asymptote of Bode magnitude plot will change is known as corner frequency.

↳ The frequency at which error in Bode magnitude plot will be maximum is known as corner frequency.

↳ Corner freq^n ⇒ |Pole location| &

|zero location|



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⑤ $G(s) = 1 + s\tau \Rightarrow G(j\omega) = 1 + j\omega\tau$

$|G(j\omega)| = \sqrt{1+(\omega\tau)^2} \Rightarrow 20\log\left(\sqrt{1+(\omega\tau)^2}\right)$

$\angle G(j\omega) = \tan^{-1}(\omega\tau)$

LFR: $\omega << \frac{1}{\tau} \Rightarrow \omega\tau << 1$

$|G(j\omega)| \approx 0$

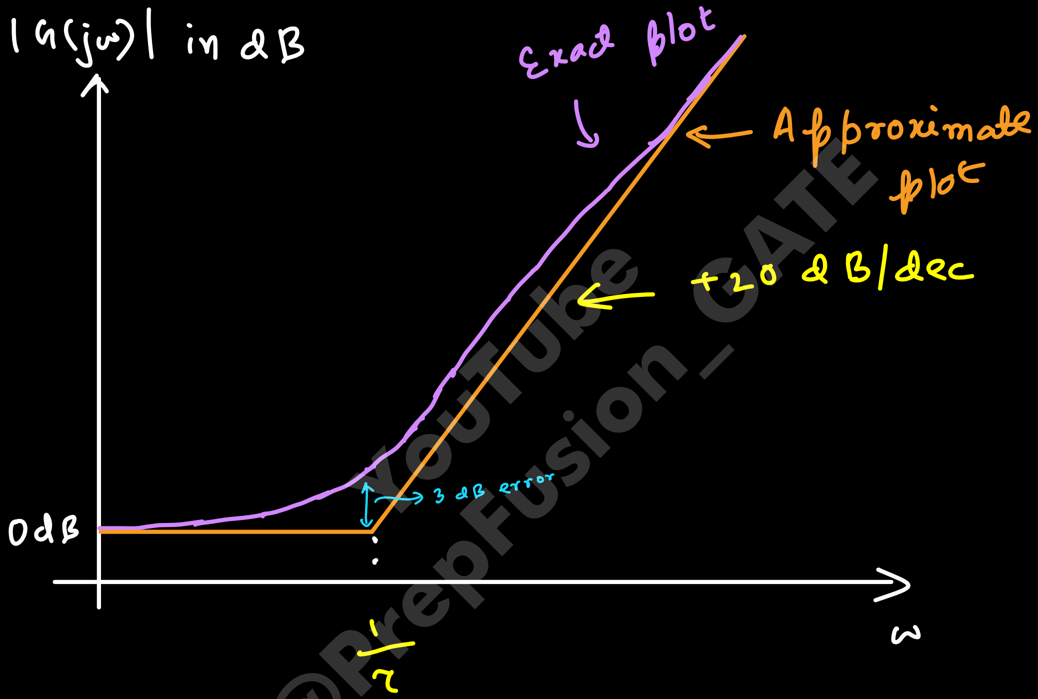
$\angle G(j\omega) \approx 0^\circ$

HFR: $\omega >> \frac{1}{\tau} \Rightarrow \omega\tau >> 1 \Rightarrow |G(j\omega)| = 20\log_{10}(\omega\tau)$

$\downarrow$

slope $= +20\,dB/dec$

$\angle G(j\omega) \approx 90^\circ$



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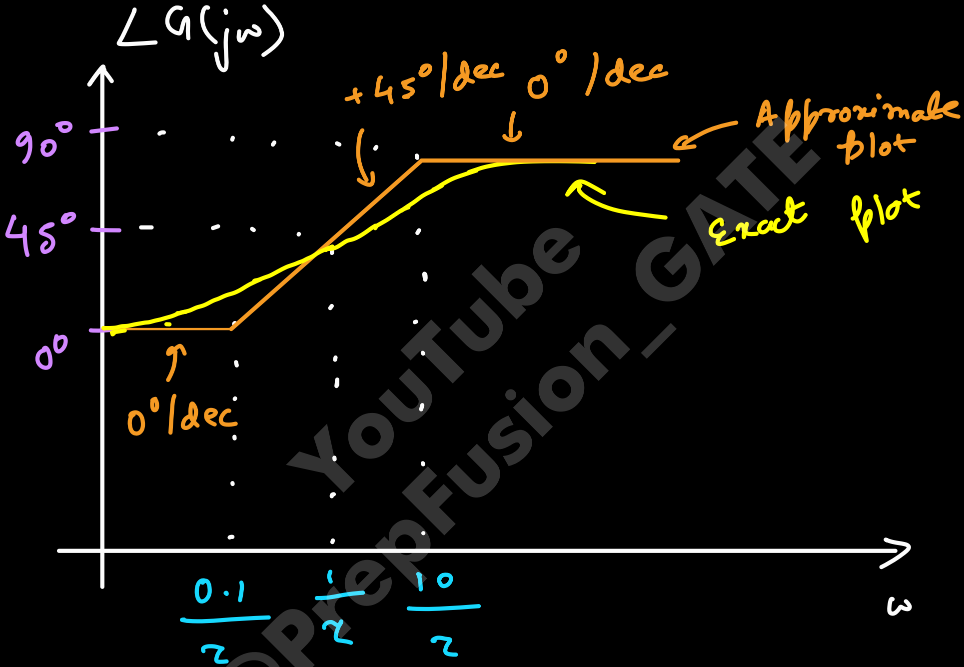

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Lecture 3

Bode gain & Phase Plots

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Types of Question asked in Bode Plot

① Bode plot given

Find : Transfer function

90%

② Transfer function given

Find : Bode plot

10%

Keypoints about Bode Gain Plot

↳ A pole gives a slope of -20 dB/dec → $\omega \geq \omega_p$

↳ A zero gives a slope of +20 dB/dec → $\omega \geq \omega_z$

↳ Error will be maximum at corner frequency w.r.t exact plot → 3 dB

↳ The slope of the magnitude plot of G(jω) would tend to -20(n-m) dB/decade as ω tends to ∞

n = no. of poles

m = no. of zeros

Keypoints about Bode Phase Plot

↳ A pole gives a slope of -45 deg/dec → $0.1\omega_p < \omega < 10\omega_p$

- Angle contributed due to pole

For $\omega < 0.1\omega_p \Rightarrow 0^\circ$ (const) → slope = 0

For $\omega > 10\omega_p \Rightarrow -90^\circ$ (const) → slope = 0

↳ A zero gives a slope of +45 deg/dec → $0.1\omega_z < \omega < 10\omega_z$

- Angle contributed due to zero

For $\omega < 0.1\omega_z \Rightarrow 0^\circ$ (const) → slope = 0

For $\omega > 10\omega_z \Rightarrow 90^\circ$ (const) → slope = 0

Keypoints about Bode Phase Plot

↳ Error will be minimum at corner frequency w.r.t exact plot. → 0°

↳ The phase of G(jω) would tend to -90(n-m)° as ω tends to ∞

n = no. of poles

m = no. of zeros

only valid

for (Non-minimum phase system.) ***

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Question

Plot bode plots for the following function

6 $G(s) = \dfrac{(s+1)}{(s+0.1)(s+100)}$

$n = 2$, $m = 1$

$\text{slope}\big|_{\omega=\infty} = -20(2-1) = -20\ \text{dB/dec}$

$\angle G(j\omega)\big|_{\omega=\infty} = -90(2-1) = -90^\circ$

Convert $G(s)$ into std. time constant format :—

$$G(s) = \frac{(s+1)}{(0.1)\left(\frac{s}{0.1}+1\right)100\left(\frac{s}{100}+1\right)}$$

ω_{c_1} ω_{c_2}

$$G(s) = \frac{0.1(s+1)}{(10s+1)(0.01s+1)} = (0.1)(s+1)\left(\frac{1}{10s+1}\right)\left(\frac{1}{0.01s+1}\right)$$

(a) (b) (c) (d)

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(a) 0.1 → Constant Magnitude → $20\log(0.1)$

$= -20\text{ dB}$

Phase = $0°$

(b) $(s+1)$ → Zero → Corner freq^n = 1 rad/sec

Magnitude → slope = +20 dB/dec

$\omega > 1$

Phase → slope = $+45°$/dec

$0.1 < \omega < 10$

slope = $0°$/dec

$\omega > 10$ & $\omega < 0.1$

(c) $\frac{1}{(10s+1)}$ → Pole → Corner freq^n = 0.1 rad/sec

Magnitude → slope = −20 dB/dec

$\omega > 0.1$

Phase → slope = -45°/dec

$0.01 < \omega < 1$

slope = 0° dec

$\omega < 0.01$ & $\omega > 1$

(d) $\dfrac{1}{(0.01s+1)}$ → Pole → Corner freq^n = 100 rad/sec

Magnitude → slope = −20 dB/dec

$\omega > 100$

Phase → slope = -45°/dec

$10 < \omega < 1000$

slope = 0° dec

$\omega < 10$ & $\omega > 1000$

Table of slopes

ω (rad/sec)	Overall slope in dB/dec
0.1	−20 (Pole)
1	0 (Pole + zero)
100	−20 (Pole + zero + Pole)

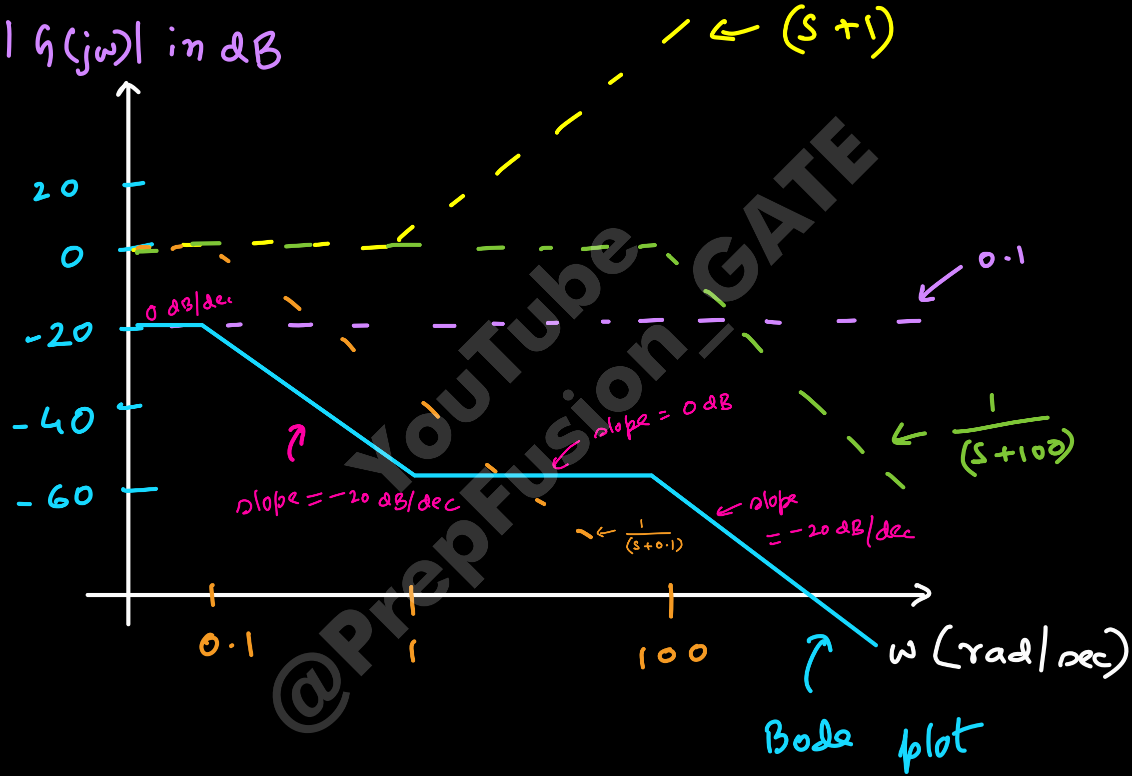

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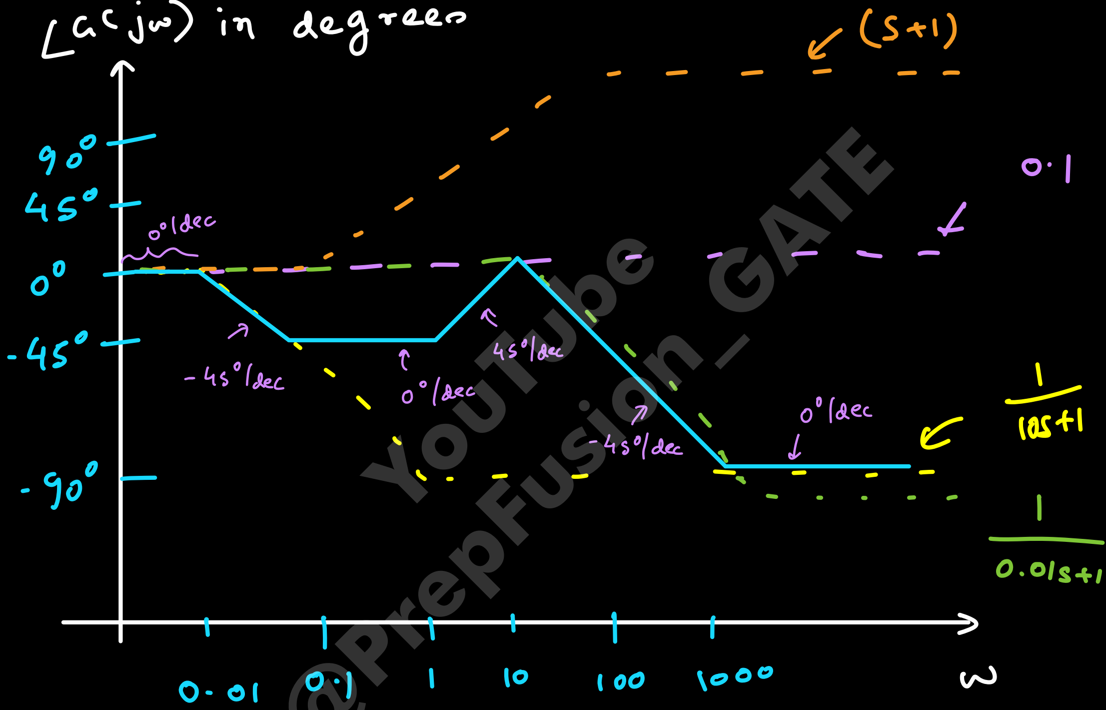
Table of slopes

ω (rad/sec)	Overall slope in deg/dec
$\underbrace{0.01 - 0.1}$ ↳ 1 angle contribution Pole	−45 (Pole)
$\underbrace{0.1 - 1}$ ↳ 2 angle contribution Pole + zero	0 (Pole + zero)
$\underbrace{1 - 10}$ ↳ 1 angle contribution zero	+45 (zero)
$10 - 1000$ ↳ 1 angle contribution Pole	−45 (Pole)



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Keypoints about Bode Plots

↳ In control systems, Bode plots are used to describe the frequency response of open loop transfer function.

ω_{gc}, ω_{pc}, GM, PM

↳ Bode plot describes the relative stability of closed loop system by using the frequency response of open loop system.

↳ Bode plot is a combination of two individual plots approximate magnitude plot and approximate phase plot.

↳ Effect of addition of poles and zeros in OLTF can be easily analysed by using Bode plots .

↳ Bode Plots is analysed only for minimum phase system.

The End

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Lecture 4

MPS & NMPS

Bode Plots of different functions

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MPS & NMPS

↳ MPS = Minimum Phase System

↳ NMPS = Non - Minimum Phase System

↳ For a Minimum Phase System all the Poles & Zeros should lie on the left hand side of jw axis.

Def^n of a minimum phase system.

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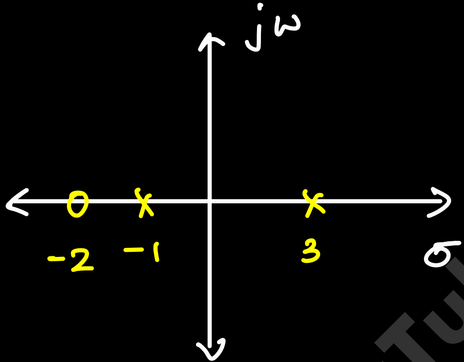
Example - From the following functions, identify MPS & NMPS.

(i) $G_1(s) = \dfrac{(s+2)}{(s+1)(s-3)}$

(ii) $G_2(s) = \dfrac{s+2}{(s+1)(s+3)}$

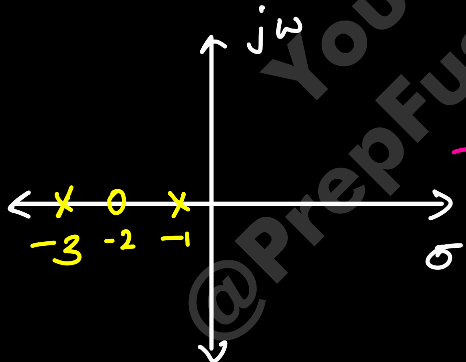
(iii) $G_3(s) = \dfrac{(s-2)}{(s+1)(s+3)}$

(i)



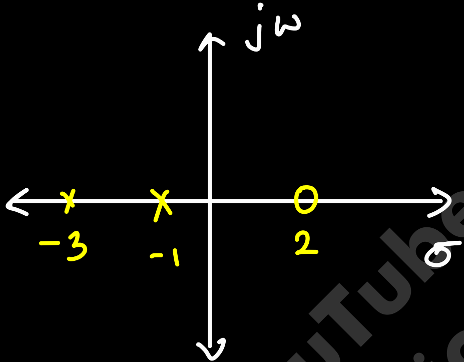
→ NMPS → Unstable

(ii)



→ MPS → Stable

(iii)



→ NMPS

↳ stable

- If CLTF of system is given & it is MPS then the system will be stable.

But converse may not be always true.

Why is it called minimum phase system?

↳ consider 2 stable systems whose transfer function are :-

$$G_1(s) = \frac{1+\tau_1 s}{1+\tau_2 s} \qquad G_2(s) = \frac{1-\tau_1 s}{1+\tau_2 s}$$

$\tau_1 > 0, \ \tau_2 > 0$

$$G_1(j\omega) = \frac{1 + j\tau_1\omega}{1 + j\tau_2\omega} \quad , \quad G_2(j\omega) = \frac{1 - j\tau_1\omega}{1 + j\tau_2\omega}$$

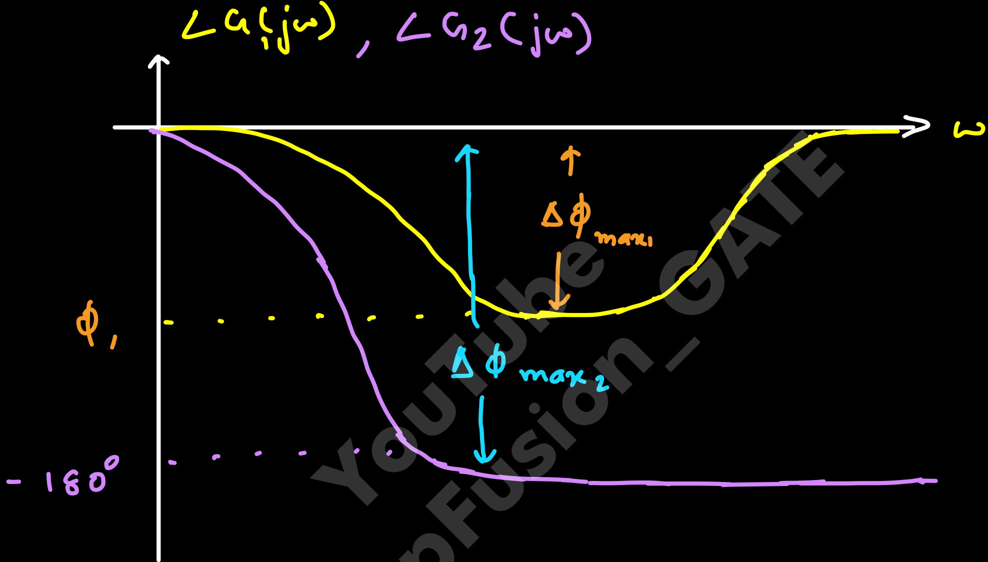
$$|G_1(j\omega)| = \frac{\sqrt{1 + (\tau_1\omega)^2}}{\sqrt{1 + (\tau_2\omega)^2}} \quad , \quad |G_2(j\omega)| = \frac{\sqrt{1 + (\tau_1\omega)^2}}{\sqrt{1 + (\tau_2\omega)^2}}$$

$$\angle G_1(j\omega) = \tan^{-1}(\tau_1\omega) - \tan^{-1}(\tau_2\omega) \quad , \quad \angle G_2(j\omega) = -\tan^{-1}(\tau_1\omega) - \tan^{-1}(\tau_2\omega)$$

$\omega = 0$ $\qquad$ $\omega = \infty$

$\angle G_1(j\omega)\big|_{\omega=0} = 0^\circ \; ; \; \angle G_1(j\omega)\big|_{\omega=\infty} = 0^\circ$

$\angle G_2(j\omega)\big|_{\omega=0} = 0^\circ \; ; \; \angle G_2(j\omega)\big|_{\omega=\infty} = -180^\circ$



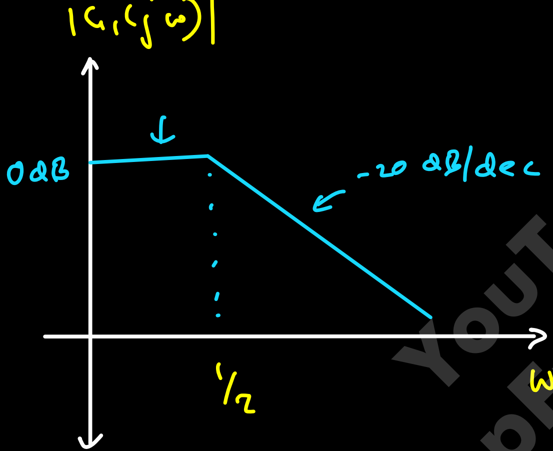
- As maximum distance is less of $G_1(j\omega)$ hence it is called MPS.

Why we only draw Bode Plot for MPS?

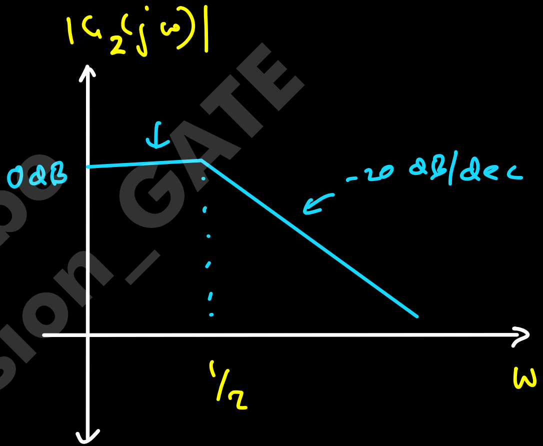
Eg - (i) $G_1(s) = \dfrac{1}{1+s\tau}$ (ii) $G_2(s) = \dfrac{1}{1-s\tau}$

$|G_1(j\omega)| = \dfrac{1}{\sqrt{1+(\omega\tau)^2}}$; $|G_2(j\omega)| = \dfrac{1}{\sqrt{1+(\omega\tau)^2}}$

$= -20\log\left(\sqrt{1+(\omega\tau)^2}\right)$ $= -20\log\left(\sqrt{1+(\omega\tau)^2}\right)$

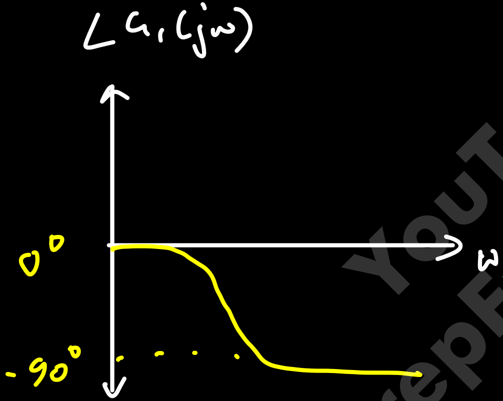


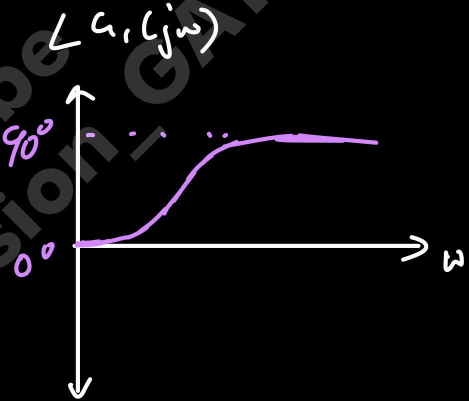
|G1(jω)|
0dB
-20 dB/dec
1/2
ω



|G2(jω)|
0dB
-20 dB/dec
1/2
ω

$\angle G_1(j\omega) = -\tan^{-1}(\omega\tau)$; $\angle G_2(j\omega) = +\tan^{-1}(\omega\tau)$





MPS & NMPS

- For Minimum Phase System we can determine transfer function from Bode Plot either by using magnitude plot or by phase plot.

- For Non - Minimum Phase System we need both magnitude & phase plot to determine transfer function.

How to draw Bode Magnitude Plot

Convert it into General time Constant format.

Replace S by Jω for the given function to convert it into frequency domain.

Find the DC gain.

Identify the corner frequencies (Poles & Zeros)

Do the algebraic sum of slopes at each corner frequencies depending on numbers of poles/zeros

steps

The General time constant form of a control system:

$$G(s) = \frac{K(1+s\tau_1')(1+s\tau_2')\ldots}{(1+s\tau_1)(1+s\tau_2)\ldots}$$

Time constant $\rightarrow$ $\tau_1, \tau_2, \tau_3 \ldots$

Corner freq^n $\rightarrow$ $1/\tau_1, 1/\tau_2 \ldots$ & $\frac{1}{\tau_1'}, \frac{1}{\tau_2'} \ldots$

Questions

Draw the bode plot for the following function.

(7) $G(s) = \dfrac{15}{s+3}$

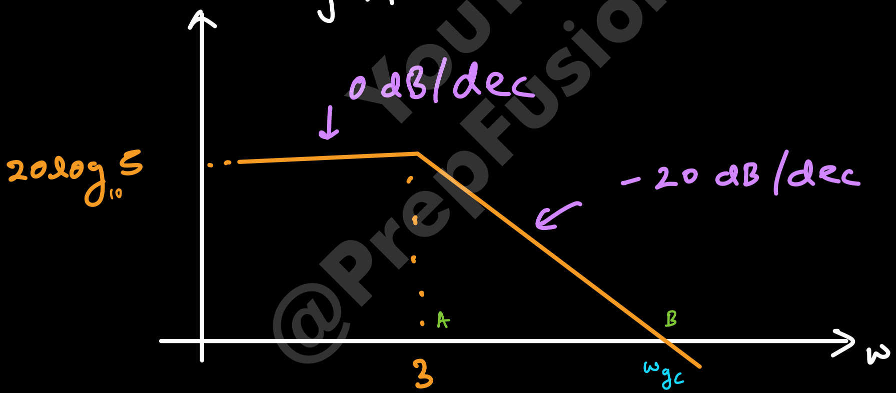
(8) $G(s) = \dfrac{(s+2)}{(s+3)(s+1)}$

(9) $G(s) = \dfrac{(s+1)}{(s+2)^2(s+3)}$

(7) $G(s) = \dfrac{15}{s+3} \rightarrow \dfrac{15}{3\left(s/3+1\right)} = \dfrac{5}{\left(\frac{s}{3}+1\right)}$

DC gain $= 5 \rightarrow 20\log_{10} 5$

$\omega_c = 3$ rad/sec

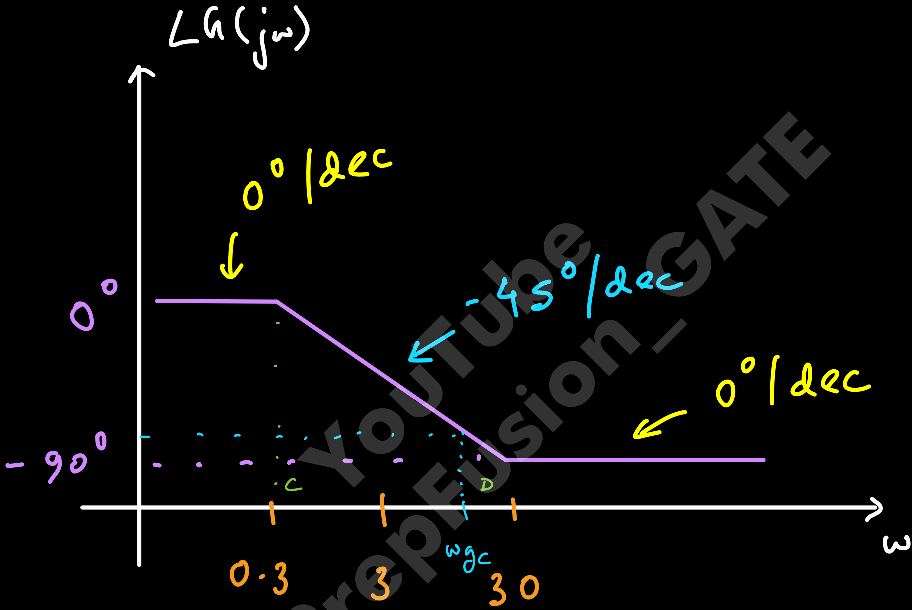


$$\text{slope}_{AB} = \frac{y_B - y_A}{\log_{10} \frac{\omega_B}{\omega_A}}$$

$$-20 = \frac{0 - 20\log_{10} 5}{\log_{10}\left(\frac{\omega_{gc}}{3}\right)}$$

$$\log_{10}\left(\frac{\omega_{gc}}{3}\right) = \log_{10} 5 \Rightarrow \frac{\omega_{gc}}{3} = 5$$

$$\omega_{gc} = 15 \text{ rad/sec}$$



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$$PM = 180^\circ + \angle G(j\omega)\Big|_{\omega=\omega_{gc}}$$

$$\text{slope}_{CD} = \frac{y_D - y_C}{\log_{10}\left(\frac{\omega_{gc}}{0.3}\right)}$$

$$-45 = \frac{y_D - 0}{\log_{10}\left(\frac{15}{0.3}\right)}$$

$$\gamma_D = -45\log_{10}(50)$$

$$PM = 180^\circ - 45\log_{10}(50)$$

(8) $G(s) = \dfrac{(s+2)}{(s+3)(s+1)} = \dfrac{\frac{2}{3}\left(\frac{s}{2}+1\right)}{\left(\frac{s}{3}+1\right)(s+1)}$

$\omega_{p_1} = 1$ rad/sec

$\omega_{p_2} = 3$ rad/sec

$\omega_{z_1} = 2$ rad/sec

Table of slopes

ω (rad/sec)	Overall slope in dB/dec
$\omega < 1$	0
$1 < \omega < 2$	-20 (Pole)
$2 < \omega < 3$	0 (Pole + zero)
$\omega > 3$	-20 (Pole + zero + Pole)

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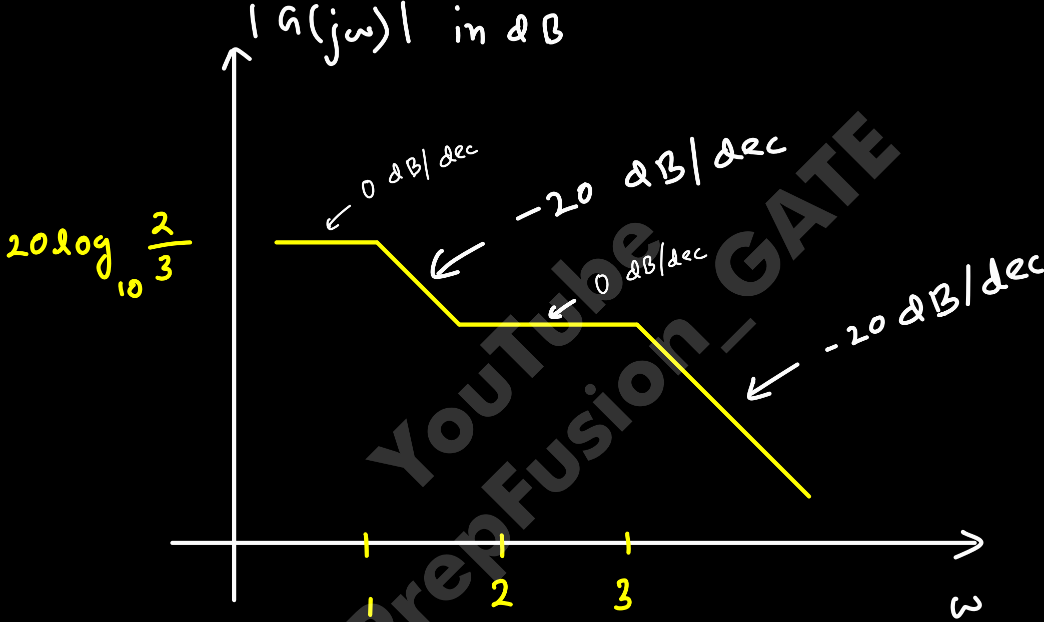

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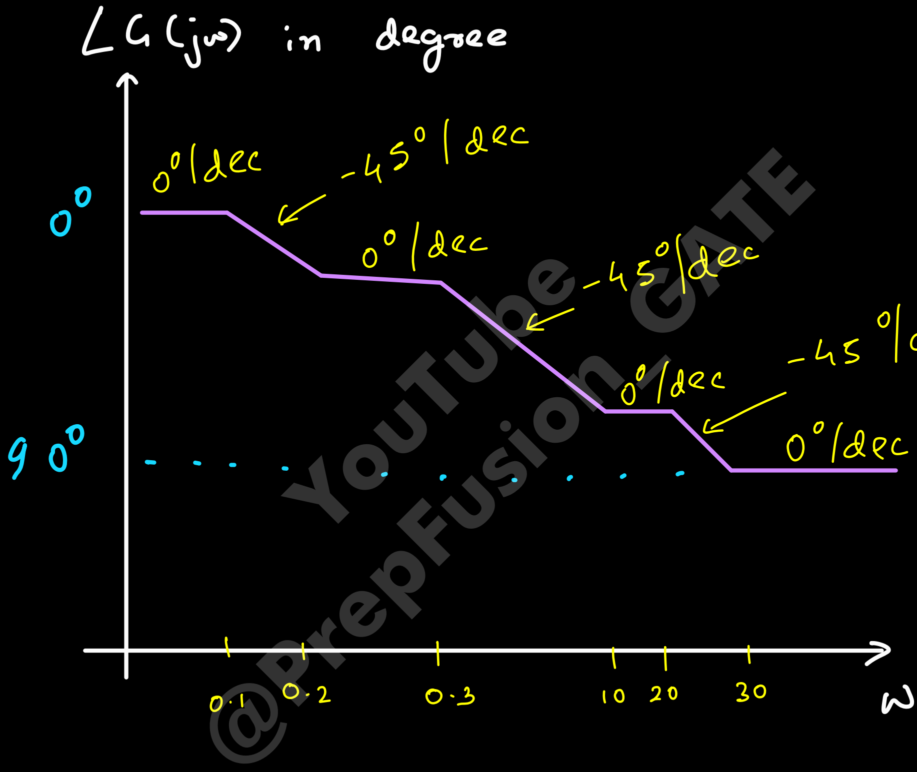
Effects of poles/Zeros

$0.2 < \omega_{z_1} < 20$, $0.1 < \omega_{p_1} < 10$, $0.3 < \omega_{p_2} < 30$

Table of slopes

ω (rad/sec)	Overall slope in deg/dec
$\omega < 0.1$	0
$0.1 < \omega < 0.2$	-45 (ω_{p_1})
$0.2 < \omega < 0.3$	0 (ω_{p_1} & ω_{z_1})
$0.3 < \omega < 10$	-45 ($\omega_{p_1}, \omega_{p_2}$ & ω_{z_1})
$10 < \omega < 20$	0 (ω_{p_2} & ω_{z_1})
$20 < \omega < 30$	-45 (ω_{p_2})
$\omega > 30$	0


∠G(jω) in degree
0°
90°
0°/dec
-45°/dec
0°/dec
-45°/dec
0°/dec
-45°/dec
0°/dec
0.1
0.2
0.3
10
20
30
ω
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The End

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Lecture 5

Bode Plots of Repeated poles and zeros Questions

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Questions

Draw the bode plot for the following function.

(10) $G(s) = \frac{K}{s^2}$

(11) $G(s) = \frac{K}{s^3}$

(12) $G(s) = \frac{K}{s^n}$; n = natural number

(10) $G(s) = \frac{K}{s^2} \rightarrow \frac{K}{(j\omega)^2} = -\frac{K}{\omega^2} \rightarrow \angle G(j\omega) = -90 \times 2$

$$|G(j\omega)| = \frac{K}{\omega^2} \Rightarrow 20\log_{10}K - 20\log_{10}\omega^2$$

ω	$\lvert G(j\omega)\rvert$ in dB
0.1	$20\log_{10}K + 40$
1	$20\log_{10}K$
10	$20\log_{10}K - 40$

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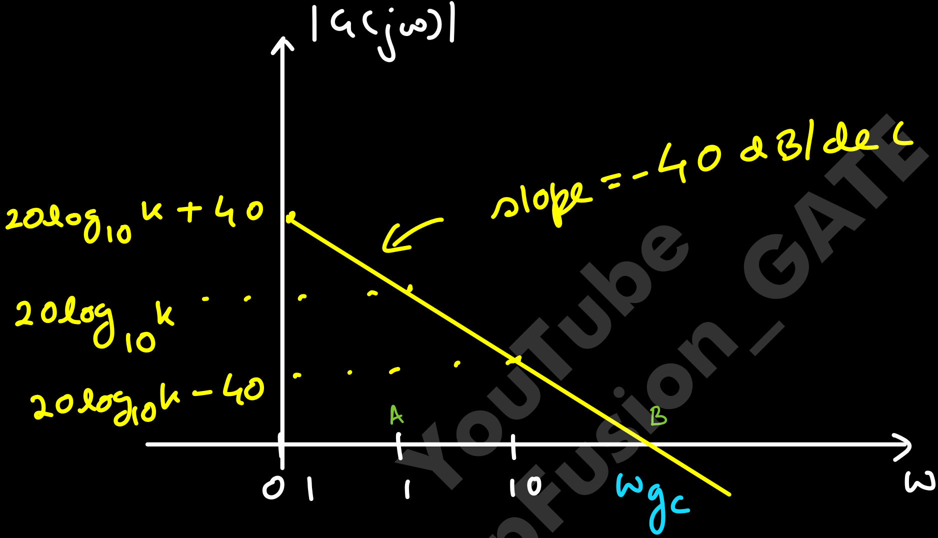


2 poles → slope = − 40 dB/dec

$$|G(j\omega)|_{\omega=\omega_{gc}} = 1$$

$$\frac{K}{\omega_{gc}^2} = 1$$

$$\boxed{\omega_{gc} = K^{1/2}}$$

$$\text{slope}_{AB} = \frac{y_B - y_A}{\log\left(\frac{\omega_{gc}}{1}\right)} = \frac{0 - 20\log_{10}K}{\log_{10}(\omega_{gc})}$$

$$\overset{2}{\cancel{-40}} = \frac{-\cancel{20}\log_{10} k}{\log_{10}(\omega_{gc})}$$

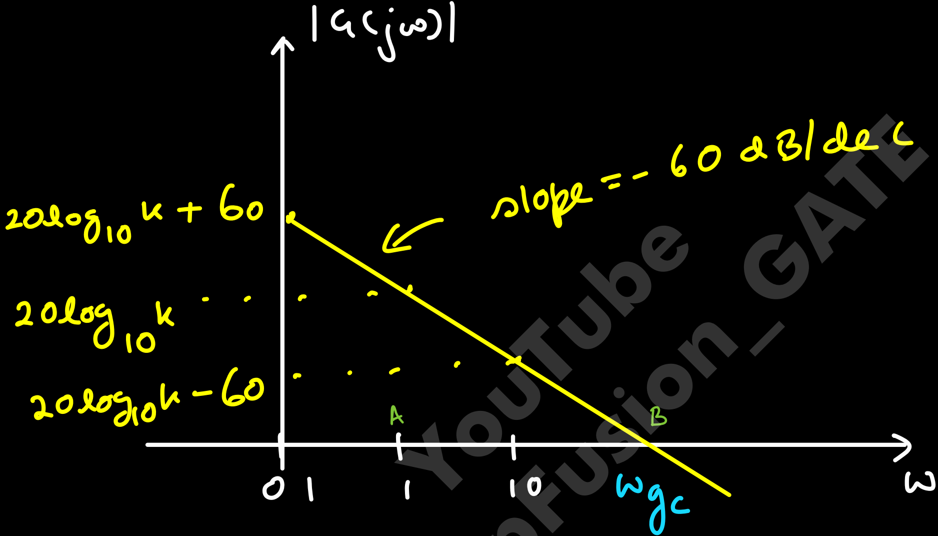
$$\boxed{\omega_{gc} = k^{1/2}}$$

(10) $G(s) = \dfrac{K}{s^3} \Rightarrow \dfrac{K}{(j\omega)^3} = \dfrac{K}{-j\omega} = j\dfrac{K}{\omega^3}$

$\angle G(s) = -90^\circ \times 3$

$|G(j\omega)| = \dfrac{K}{\omega^3} \Rightarrow 20\log_{10}K - 20\log_{10}\omega^3$

ω	$\lvert G(j\omega)\rvert$ in dB
0.1	$20\log_{10}K + 60$
1	$20\log_{10}K$
10	$20\log_{10}K - 60$



2 poles → slope = − 60 dB /dec

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$$|G(j\omega)|_{\omega=\omega_{gc}} = 1$$

$$\frac{K}{\omega_{gc}^{3}} = 1$$

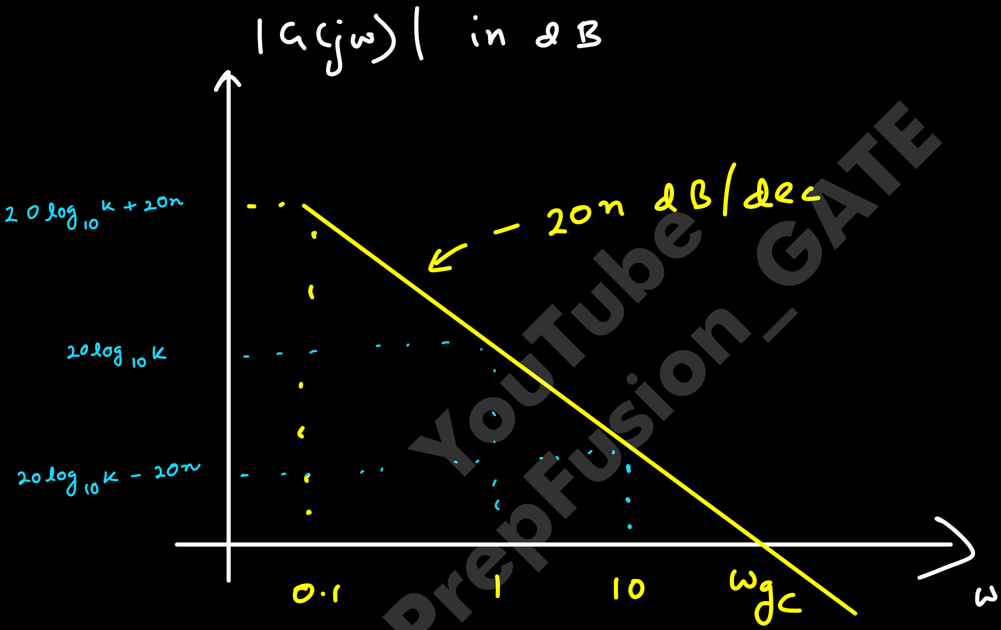
$$\boxed{\omega_{gc} = K^{1/3}}$$

(12) $G(s) = \frac{K}{s^n}$; n = natural number

$$|G(j\omega)| = 20\log_{10} K - 20\log_{10} \omega^n$$

$$= 20\log_{10} K - \underbrace{20n}_{\text{slope}} \log_{10} \omega$$

$$\angle G(j\omega) = -90^\circ \times n$$



$\omega_{gc} = K^{1/n}$

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Questions

Draw the bode plot for the following function.

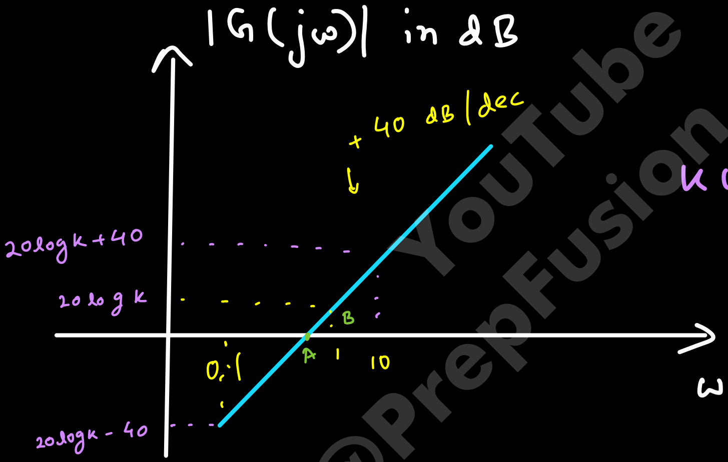
(13) $G(s) = K s^2$

(14) $G(s) = K s^n$; $n =$ natural number

(13) $G(s) = k s^2 \Rightarrow G(j\omega) = k\omega^2$

$= 20\log_{10} k + 40\log_{10}\omega$

$G(j\omega) = k(j\omega)^2 = -k\omega^2$



$\angle G(j\omega) = +90 \times 2$

$k\omega_{gc}^2 = 1$

$$\boxed{\omega_{gc} = k^{-1/2}}$$

$$\text{slope}_{AB} = \frac{20\log_{10} k - 0}{\log_{10}\left(\frac{1}{\omega_{gc}}\right)}$$

$$2\ \cancel{40} = \frac{\cancel{20}\log_{10} k}{-\log_{10}(\omega_{gc})}$$

$$\log_{10}(\omega_{gc}) = -\frac{1}{2}\log_{10} k$$

$$\boxed{\omega_{gc} = k^{-1/2}}$$

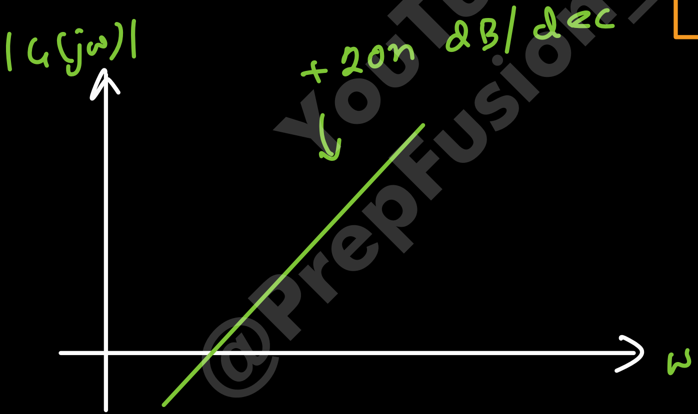
(14) $G(s) = K s^n$

$\rightarrow |G(j\omega)| = K\omega^n \rightarrow 20\log_{10} K + 20n\log_{10} \omega$

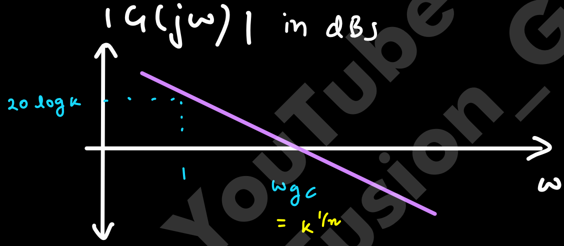
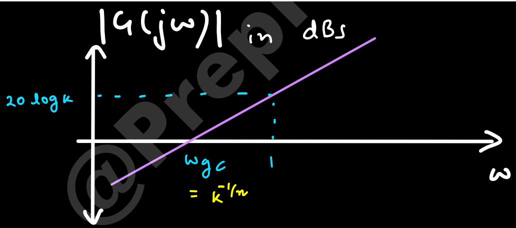
slope $= +20n$ dB/dec

$\angle G(j\omega) = +90^\circ \times n$

$$\boxed{\omega_{gc} = K^{-1/n}}$$



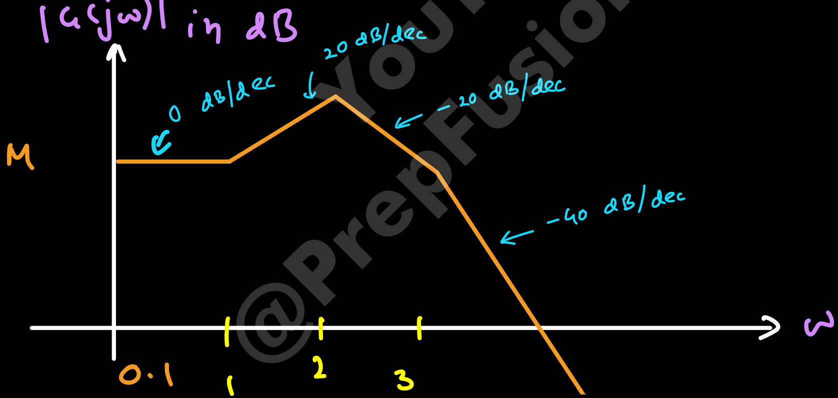
Conclusion :-

G(s)	Bode Magnitude Plot	Remarks
$G(s) = \frac{K}{s^n}$		Slope = −20n dB/dec $\omega_{gc} > 1$ $(K > 1)$ $\omega_{gc} < 1$ $(K < 1)$ $\angle G(j\omega) = -90^\circ \times n \rightarrow$ Const.
$G(s) = Ks^n$		Slope = +20n dB/dec $\omega_{gc} > 1$ $(K < 1)$ $\omega_{gc} < 1$ $(K > 1)$ $\angle G(j\omega) = +90^\circ \times n \rightarrow$ Const.

(9) $G(s) = \dfrac{(s+1)}{(s+2)^2(s+3)} = \dfrac{(s+1)}{2^2\left(\frac{s}{2}+1\right)^2\left(\frac{s}{3}+1\right)\times 3}$

$$G(s) = \frac{1}{12\left(\frac{s}{2}+1\right)^2\left(\frac{s}{3}+1\right)}$$

$$M = 20\log_{10}\frac{1}{12}$$



$$\angle G(j\omega) = \tan^{-1}(\omega) - \left[\underbrace{2}\tan^{-1}\left(\frac{\omega}{2}\right) + \tan^{-1}\left(\frac{\omega}{3}\right)\right]$$

$\downarrow$

slope is getting doubled.

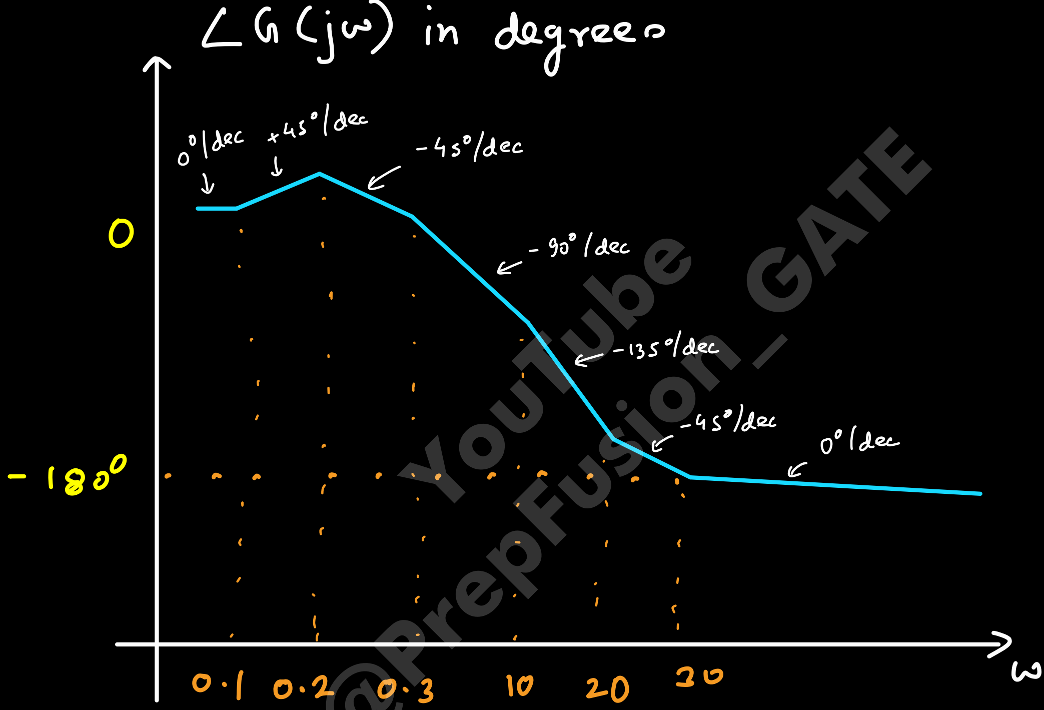
Effects of poles/zeros

$0.1 < \omega_{z_1} < 10$, $0.2 < \omega_{p_1} < 20$, $0.3 < \omega_{p_2} < 30$

$\times 2$

Table of slopes

ω (rad/sec)	Overall slope in deg/dec
$\omega < 0.1$	0
$0.1 < \omega < 0.2$	$+45$ (ω_{z_1})
$0.2 < \omega < 0.3$	-45 ($2\times\omega_{p_1}$ & ω_{z_1})
$0.3 < \omega < 10$	-90 ($2\times\omega_{p_1}$, ω_{p_2} & ω_{z_1})
$10 < \omega < 20$	-135 ($2\times\omega_{p_1}$ & ω_{p_2})
$20 < \omega < 30$	-45 (ω_{p_2})
$\omega > 30$	0



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Summary of Repeated Poles & Zeros

n - Real Poles	m - Real Zeros
$GH(s) = \dfrac{1}{(1+s\tau)^n}$	$GH(s) = (1+s\tau)^m$
$\lvert GH(j\omega)\rvert_{dB} = -20n\log_{10}\left(\sqrt{1+(\omega\tau)^2}\right)$	$\lvert GH(j\omega)\rvert_{dB} = 20m\log_{10}\left(\sqrt{1+(\omega\tau)^2}\right)$
slope $= -20n$ dB/dec	slope $= +20m$ dB/dec
$\angle GH(j\omega) = -n\tan^{-1}(\omega\tau)$	$\angle GH(j\omega) = m\tan^{-1}(\omega\tau)$
slope $= -45n\ ^\circ$/dec	slope $= +45m\ ^\circ$/dec
Corner frequency = |pole location| $\Rightarrow \omega_c = \frac{1}{\tau}$	Corner frequency = |zero location| $\Rightarrow \omega_c = \frac{1}{\tau}$

Question

Draw bode magnitude plot for the following function

(15) $$GH(s) = \frac{(s+4)^2}{s(s+2)(s+8)}$$

(16) $$GH(s) = \frac{s(s+10)(s+400)^2(s+1000)}{(s+2)^2(s+20)(s+100)^3}$$

(15) $GH(s) = \dfrac{(s+4)^2}{s(s+2)(s+8)}$

$$= \frac{4^2\left(\frac{s}{4}+1\right)^2}{s\left(\frac{s}{2}+1\right)\left(\frac{s}{8}+1\right) \times 8 \times 2}$$

$$= \frac{\left(\frac{s}{4}+1\right)^2}{s\left(\frac{s}{2}+1\right)\left(\frac{s}{8}+1\right)}$$

$\rightarrow \omega_z = 4$ rad/sec $\times 2$

$\omega_{p_0} = 0$

$\omega_{p_1} = 2$ rad/s

$\rightarrow \omega_{p_2} = 8$ rad/s

- magnitude due to all the corner freq^n can be ignored as 0.1 << corner freq^{n}s

$$\frac{\left(\frac{s}{4}+1\right)^2}{s\left(\frac{s}{2}+1\right)\left(\frac{s}{8}+1\right)}$$

This can be ignored.

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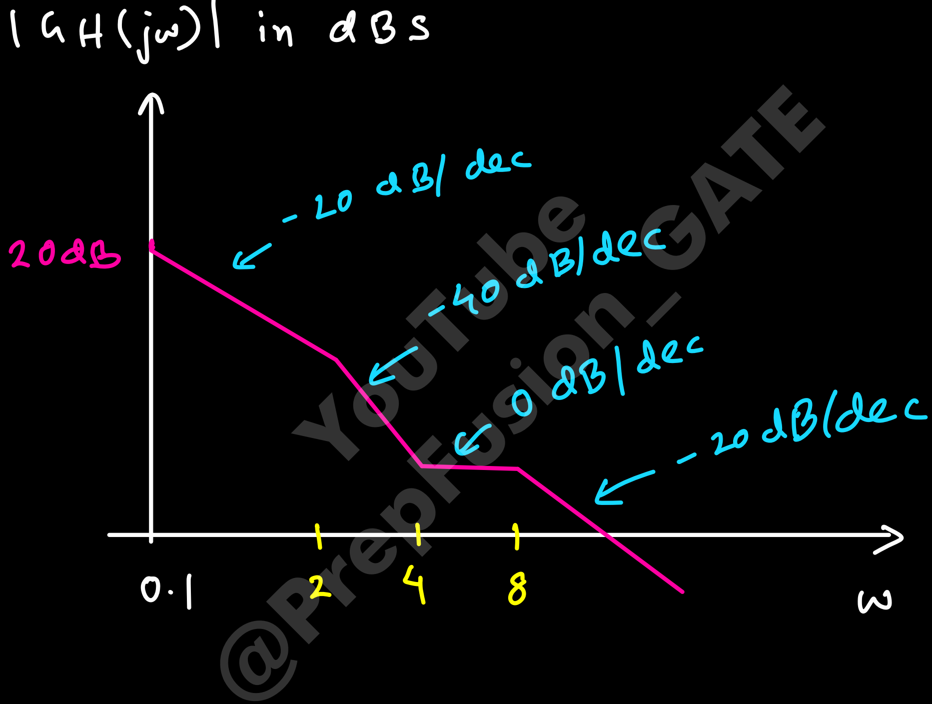
$$\left. |GH(j\omega)| \right|_{\omega=0.1} = \frac{1}{0.1} = 10$$

$\hookrightarrow$ 20 dB



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(16) $$GH(s) = \frac{s\,(s+10)\,(s+400)^2\,(s+1000)}{(s+2)^2\,(s+20)\,(s+100)^3}$$

$10^8 \times 2^4$

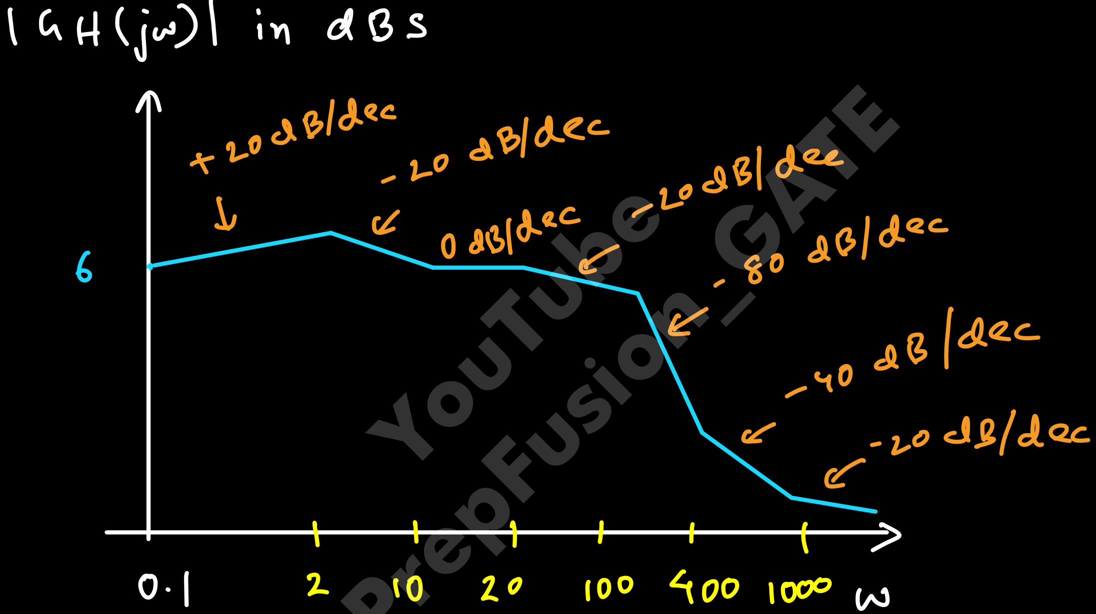
$$GH(s) = \frac{s \times 10\left(\frac{s}{10}+1\right) \times (400)^2\left(\frac{s}{400}+1\right)^2 1000\left(\frac{s}{1000}+1\right)}{2^2\left(\frac{s}{2}+1\right)^2 \times 20\left(\frac{s}{20}+1\right) \times (100)^3\left(\frac{s}{100}+1\right)^3}$$

$2^3 \times 10^7$

$$GH(s) = \frac{20\,s\left(\frac{s}{10}+1\right)\left(\frac{s}{400}+1\right)^2\left(\frac{s}{1000}+1\right)}{\left(\frac{s}{2}+1\right)^2\left(\frac{s}{20}+1\right)\left(\frac{s}{100}+1\right)^3}$$

$$|GH(j\omega)| = 20 \times 0.1 = 2 \rightarrow 6\ \text{dB}$$

$$s = j0.1$$



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The End

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Lecture 6

Recovery of Transfer function from Asymptotic Bode Magnitude Plot

Most Important

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Question

17 Find the DC gain of the following transfer function.

(i) $G_1(s) = \dfrac{100\,(s+1)}{\left(\frac{s}{2}+1\right)\left(\frac{s}{3}+1\right)}$

(ii) $G_2(s) = \dfrac{(s+1)}{s^2\left(\frac{s}{2}+1\right)\left(\frac{s}{3}+1\right)}$

DC gain $\Rightarrow G(s)\Big|_{s=0}$

(i) $G_1(s) = \dfrac{100\,(s+1)}{\left(\frac{s}{2}+1\right)\left(\frac{s}{3}+1\right)}$

DC gain of $G_1(s) = \dfrac{100\,(0+1)}{(0+1)\,(0+1)} = 100$

(ii) $G_2(s) \rightarrow$ DC gain $\lim_{s \to 0} s^2 G(s) = 1$

$\downarrow$

DC gain $= \infty$

$\downarrow$

defn for acceleration error coefficient.

in dB $\rightarrow$ $20\log_{10}\infty = \infty$

Extra

(iii) $G_3(s) = \dfrac{s(s+1)}{(s+2)(s+3)} \rightarrow$ DC gain $= 0$

$20\log_{10} 0 = -\infty$

Question

18 The asymptotic approximation of the log magnitude versus frequency plot of a minimum phase system with real poles and one zero is shown in figure its transfer functions is

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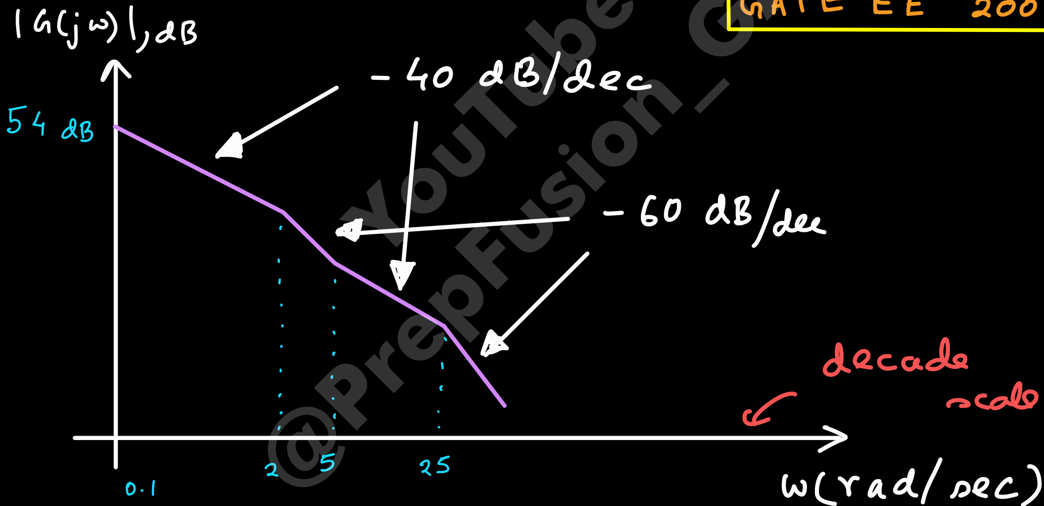

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(A) $\dfrac{20(s+5)}{s(s+2)(s+25)}$ ✗

(B) $\dfrac{10(s+5)}{s(s+2)^2(s+25)}$ ✗

(C) $\dfrac{20(s+5)}{s^2(s+2)(s+25)}$ ✓

(D) $\dfrac{50(s+5)}{s^2(s+2)(s+25)}$ ✓

$$GH(s) = \frac{K\left(\frac{s}{5}+1\right)}{s^2\left(\frac{s}{2}+1\right)\left(\frac{s}{25}+1\right)}$$

$$|GH(j\omega)|_{\omega=0.1} = \frac{K}{\omega^2} = \frac{K}{(0.1)^2} = x$$

$54\ dB \rightarrow 20 \log x = 54$

$$x = 10^{2.7} = 500$$

$$K = (0.1)^2 \times 500$$

$$K = 5$$

$$GH(s) = \frac{5\left(\frac{s+5}{5}\right)}{s^2\left(\frac{s+2}{2}\right)\left(\frac{s+25}{25}\right)}$$

$$GH(s) = \frac{50(s+5)}{s^2(s+2)(s+25)}$$

(D)

Identification of Transfer function from Bode Plot

1. Identify the corner frequencies (Poles & Zeros)
2. Observe initial slope. It gives the number of poles or zeros at the origin.
3. Determine the change in slope at each corner frequency

Δ slope $= -20n$ dB/dec or $-6n$ dB/oct

pole of order n at that freq^n

Δ slope $= 20m$ dB/dec or $6m$ dB/oct

zero of order m at that freq^n

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$$1 \text{ dec} = \frac{10}{3} \text{ oct}$$

$$\Rightarrow \quad 20 \text{ dB/dec} = \frac{20 \text{ dB}}{1 \text{ dec}} = \frac{20 \text{ dB}}{\frac{10}{3} \text{ oct}}$$

$$20 \text{ dB/dec} = 6 \text{ dB/oct}$$

Identification of Transfer function from Bode Plot

4. Find the value of K.

↳ if gain at any freq^n is given in magnitude plot

Equate the approximated TF at that freq & gain from the plot

↳ if intersection of a line with "ω-axis" is given

Put the value of ω in the approximated transfer function, Equate it "1", Find K from Equation.

Identification of Transfer function from Bode Plot

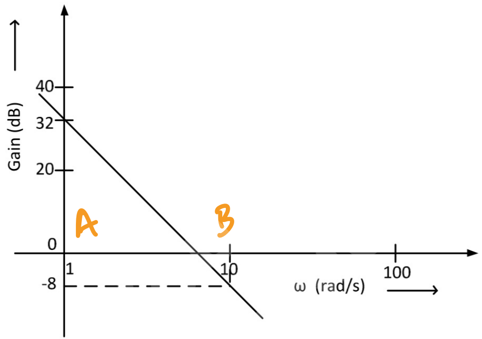
5. Write transfer function with all poles & zeros in time constant form and DC gain K.

$$G(s) = \frac{K(1+s\tau_1')(1+s\tau_2')\cdots}{(1+s\tau_1)(1+s\tau_2)\cdots}$$

Question

19

The Bode plot of a transfer function $G(s)$ is shown in the figure below.



The gain $(20 \log|G(s)|)$ is 32 dB and -8 dB at 1 rad/s and 10 rad/s respectively. The phase is negative for all ω. Then $G(s)$ is

(A) $\frac{39.8}{s}$ ✗

(B) $\frac{39.8}{s^2}$ ✓

(C) $\frac{32}{s}$ ✗

(D) $\frac{32}{s^2}$ ✓ ✗

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M-1

$$\text{slope}_{A-B} = \frac{y_B - y_A}{\log_{10}\left(\frac{\omega_B}{\omega_A}\right)} = \frac{-8-32}{\log_{10}\left(\frac{10}{1}\right)}$$

$$= -40 \text{ dB/dec}$$

$$G(S) = \frac{K}{s^2} \qquad |G(j\omega)| = \frac{K}{\omega^2}$$

$$|G(j\omega)| \text{ in dB} = 20\log K - 20\log \omega^2$$

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$$32\ dB = 20\log_{10}K - 0$$

$$K = 10^{1.6} = 39.81$$

M-2

$\rightarrow$ $32\ dB \Rightarrow 10^{1.6} = \frac{K}{(1)^2} \Rightarrow 39.81$

$\rightarrow$ $-8\ dB \Rightarrow 10^{-0.4} = \frac{K}{(10)^2}$

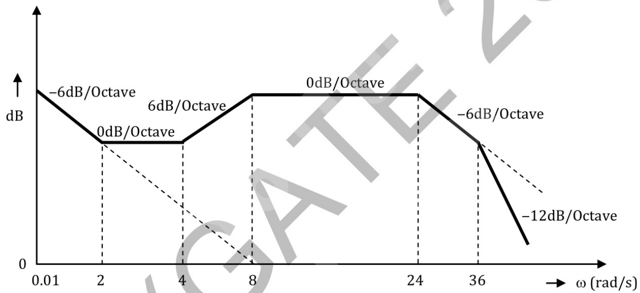
$$K = 39.81$$

Question

20

The Bode magnitude plot of the transfer function $G(s) = \dfrac{K(1+0.5s)(1+as)}{s\left(1+\frac{s}{8}\right)(1+bs)\left(1+\frac{s}{36}\right)}$ is shown below:

Note that –6 dB/octave = –20 dB/decade. The value of $\dfrac{a}{bK}$ is ______________.



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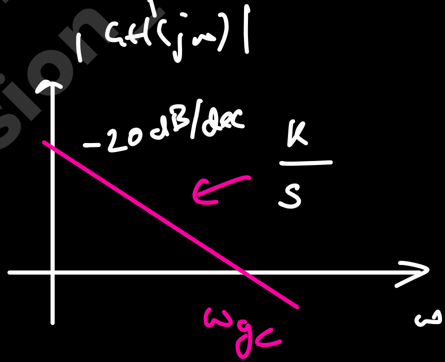
$$GH(s) = \frac{K\left(\frac{s}{2}+1\right)\left(\frac{s}{4}+1\right)}{s\left(\frac{s}{8}+1\right)\left(\frac{s}{24}+1\right)\left(\frac{s}{36}+1\right)}$$

$$a = \frac{1}{4}, \quad b = \frac{1}{24}$$

$$\boxed{\omega_{gc} = K^{1/n}}$$



$$|GH'(j\omega)| = \frac{K}{\omega} = 1$$

$K = \omega_{gc} = 8$

$\checkmark$

$$20\log_{10} K - 20\log_{10} \omega = 0$$

$K = \omega$

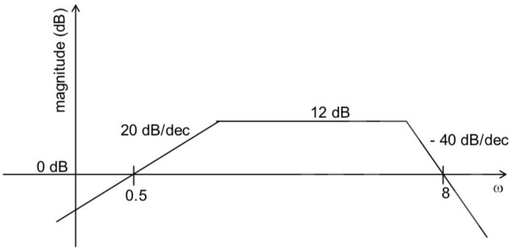
$K = 8$

$$\frac{a}{bk} = \frac{1/4}{\frac{1}{24} \times 8} = \frac{3}{4} = 0.75 \text{ (ans)}$$

Question

20

Consider the following asymptotic Bode magnitude plot (ω is in rad/s).



Which one of the following transfer functions is best represented by the above Bode magnitude plot?

(A) $\dfrac{2s}{(1+0.5s)(1+0.25s)^2}$ ✓

(B) $\dfrac{4(1+0.5s)}{s(1+0.25s)}$ ✗

(C) $\dfrac{2s}{(1+2s)(1+4s)}$ ✗

(D) $\dfrac{4s}{(1+2s)(1+4s)^2}$ ✗

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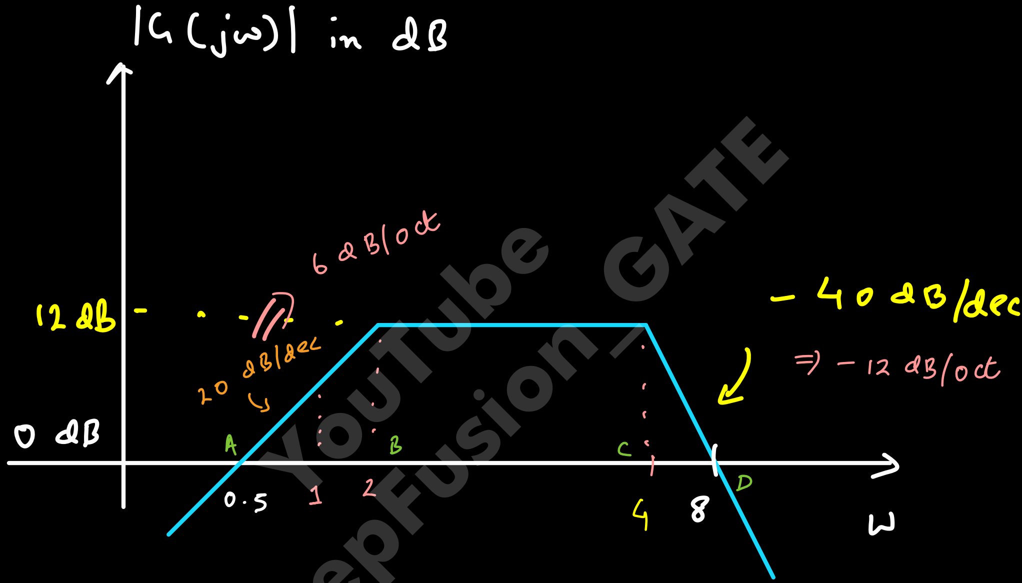
Octave Scale



$K(\omega_{gc}) = 1 \qquad K = 2$

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$$GH(s) = \frac{2(s)}{\left(\frac{s}{2}+1\right)\left(\frac{s}{4}+1\right)^2}$$

$$\text{slope}_{AB} = \frac{12-0}{\log_{10}\left(\frac{\omega_{P_1}}{0.5}\right)}$$

$$\boxed{10^{0.3} \simeq 2}$$

$$\log_{10}\left(\frac{\omega_{P_1}}{0.5}\right) = \frac{12}{20}$$

$$\frac{\omega_{P_1}}{0.5} = 10^{0.6} = 4$$

$\omega_{p_1} = 2 \text{ rad/s}$

$$\text{slope}_{C-D} = \frac{0-12}{\log_{10}\left(\frac{8}{\omega_{p_2}}\right)}$$

$$-40 = \frac{-12}{\log_{10}\left(\frac{8}{\omega_{p_2}}\right)}$$

$$\log_{10}\left(\frac{8}{\omega_{p_2}}\right) = 3/10$$

$$\frac{8}{\omega_{p_2}} = 10^{0.3}$$

$$\omega_{p_2} = 8/2 = 2 \text{ rad/sec}$$

The End

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Lecture 7

Error in Asymptotic Bode Magnitude Plot
Analysis of Relative Stability Parameters from Bode Plots
Analysis of Steady - State Error from Asymptotic Magnitude Plot

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Error in Magnitude at corner frequency

(i) Error in $dB_s \Big|_{\omega = \omega_{CF}}$ → ω_{CF} : Corner freq^n

$$= \left\{ \text{Actual Value of}\,(|G(j\omega)|) - \text{Approximate Value of}\,(|G(j\omega)|) \right\} \Big|_{\omega=\omega_{CF}}$$

- For n- Real Repeated poles

Let $G(s) = \dfrac{K}{(1+sz)^n}$, $\omega_{CF} = \dfrac{1}{z}$

$$|G(j\omega)| = 20\log_{10} K - 20n\log_{10}\left(\sqrt{1+(\omega z)^2}\right)$$

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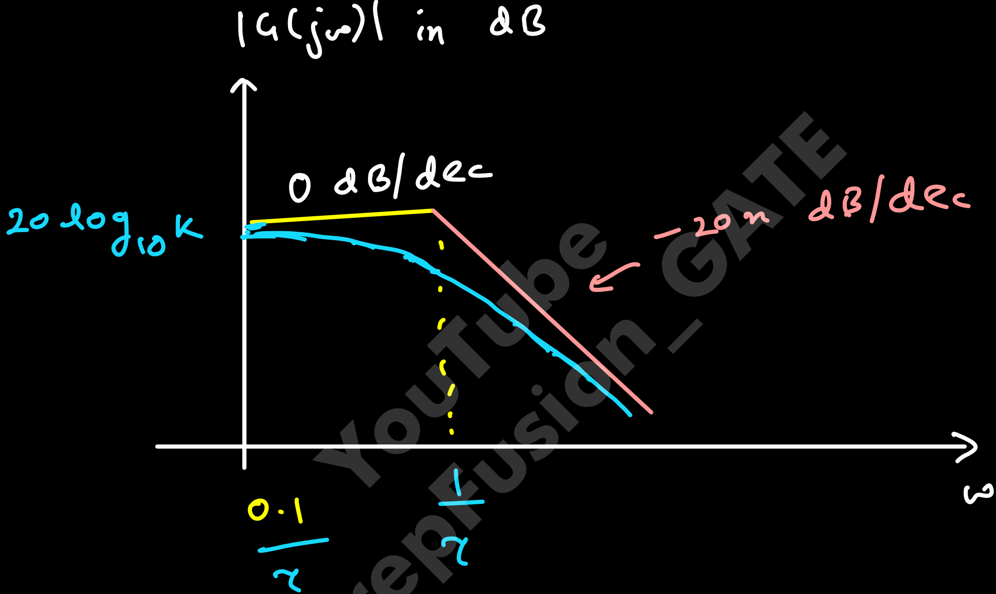

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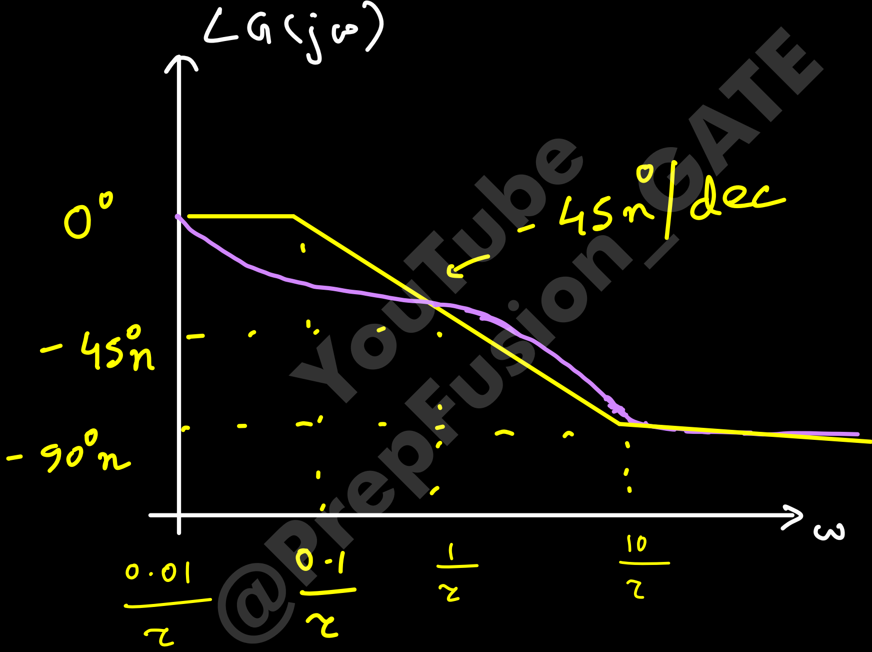
$$\angle G(j\omega) = -n\tan^{-1}(\omega\tau)$$



Actual Value of $\left.\left(|G(j\omega)|\right)\right|_{\omega=\frac{1}{\tau}} = 20\log_{10}K - 20n\log_{10}\left(\sqrt{2}\right)$

$= 20\log_{10}K - 10n \times 0.3$

$= 20\log_{10}K - 3n$

Approximate Value of $\left.\left(|G(j\omega)|\right)\right|_{\omega=\frac{1}{\tau}} = 20\log_{10}K$

$$\boxed{\left.\text{Error}\right|_{\omega=\omega_{CF}} = -3n \text{ dB}} \rightarrow \text{Bode Gain plot}$$

Actual Value of $\angle G(j\omega)\Big|_{\omega=\omega_{cf}} = -n\tan^{-1}\left(\frac{1}{2}\times 2\right)$

$= -n\times 45 = -45n^\circ$

Approximate Value of $\angle G(j\omega)\Big|_{\omega=\omega_{cf}} = -45^\circ n$

$$\boxed{\text{Error}\Big|_{\omega=\omega_{cf}} = 0^\circ}$$

$\rightarrow$ Bode phase plot

- Conclusion

Error in dBs

n - Real & Repeated Poles : $-3n$

m - Real & Repeated Zeros : $+3m$

Error in gain plot is always even symmetric about the corner frequency.

Error in phase plot is always odd symmetric about corner frequency

Relative Stability Parameters from Bode Plots

↳ unity Negative Feedback system.

(i) Gain cross – Over Frequency (ω_g) :

↳ The frequency at which the magnitude of the OLTF becomes 1.

(ii) Phase cross – Over Frequency (ω_p) :

↳ The frequency at which the phase of the OLTF becomes -180.

(iii) Gain Margin (K_g / GM):

↳ It is the reciprocal of $|G(j\omega)H(j\omega)|$ at phase crossover frequency.

→ additional amount of margin left for it to become unstable.

$$K_g = \frac{1}{|G(j\omega_p)H(j\omega_p)|} \rightarrow |G(j\omega_p)H(j\omega_p)|^{-1}$$

$$GM = K_g = -20\log\left(|G(j\omega_g)H(j\omega_g)|\right) \text{ in dB}$$

$$GM \neq f(\omega_{gc})$$

(iv) Phase Margin (γ / PM) :

clockwise direction

↳ The additional amount of phase lag that needs to be added to the phase of the system at ω = ωg to bring the system to the verge of instability.

↳ gain crossover frequency.

$$PM = \gamma = 180^\circ + \angle G(j\omega_g)H(j\omega_g)$$

$$PM \neq f(\omega_{pc})$$

Question

21

The phase margin in degrees of $G(s) = \frac{10}{(s+0.1)(s+1)(s+10)}$ calculated using the asymptotic Bode plot is ________.

45°

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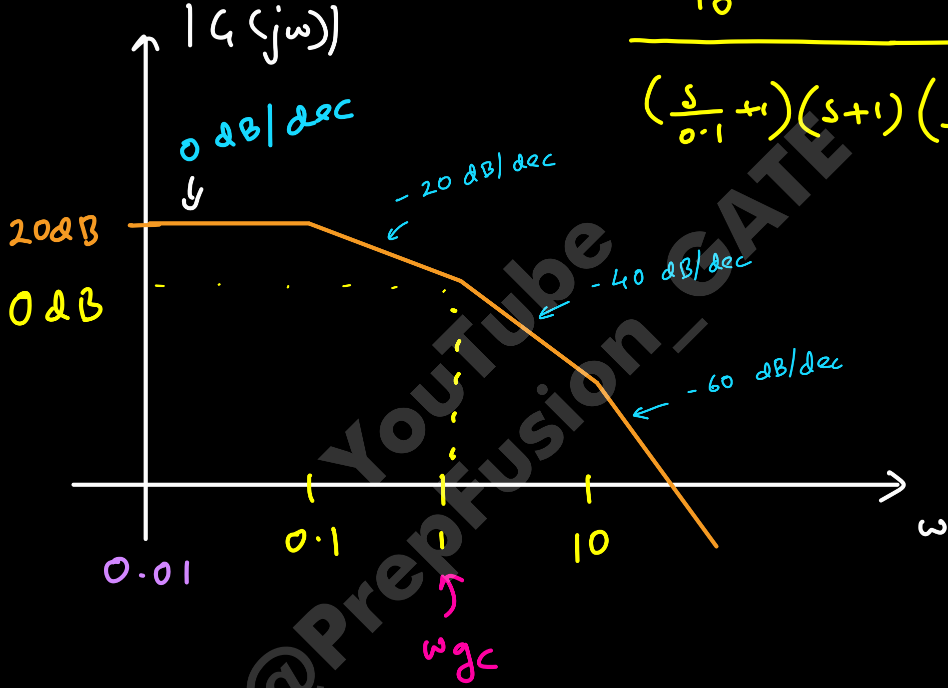
- Bode plots, help us find Relative stability parameters of higher order systems with less calculation than conventional method

↳ Conditional ⇒ as this is approximate analysis, we don't get exact values.

$$\frac{10}{\left(\frac{s}{0.1}+1\right)(s+1)\left(\frac{s}{10}+1\right)}$$



Actual value $|G(j\omega)|_{\omega=1}$

$$= \frac{10}{\sqrt{\left(\left(\frac{1}{0.1}\right)^2+1^2\right)\left(1^2+1^2\right)\left[\left(\frac{1}{10}\right)^2+1^2\right]}}$$

$$= \frac{10}{\sqrt{2\times 100}} = \frac{1}{\sqrt{2}}$$

$$= 0.707$$

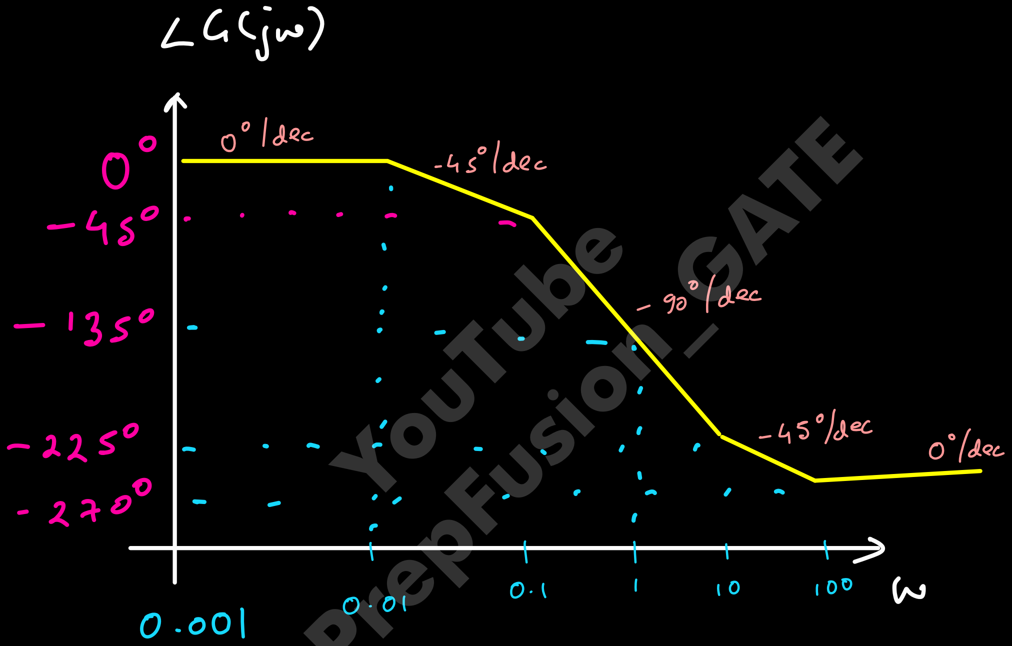
$$= -3\ dB$$

Effects of poles

$0.01 < \omega_{p_1} < 1$, $0.1 < \omega_{p_2} < 10$, $1 < \omega_{p_3} < 100$

Table of slopes

ω (rad/sec)	Overall slope in deg/dec
$\omega < 0.01$	0
$0.01 < \omega < 0.1$	-45 (ω_{p_1})
$0.1 < \omega < 1$	-90 (ω_{p_1} & ω_{p_2})
$1 < \omega < 10$	-90 (ω_{p_2} & ω_{p_3})
$10 < \omega < 100$	-45 (ω_{p_3})
$\omega > 100$	0



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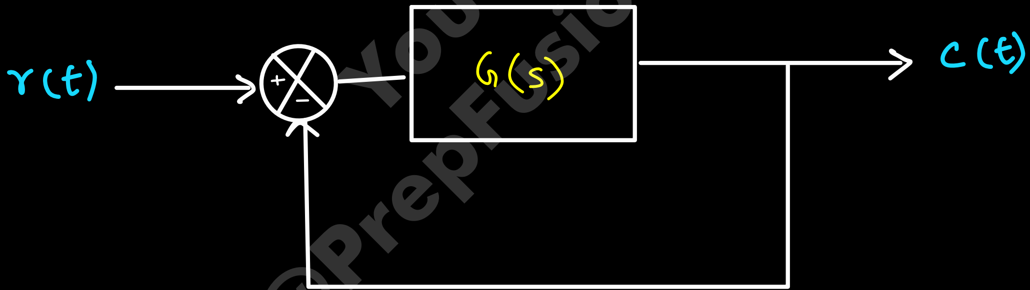
$$\angle G(j\omega)\Big|_{\omega=\omega_{gc}} = -135^\circ$$

$$PM = 180^\circ + (-135^\circ) = 45^\circ$$

Steady State Error

↳ Only valid if Bode plot of OLTF is given for a unity Negative Feedback system.



$$e(t) = r(t) - c(t)$$

Positional Error Coefficient

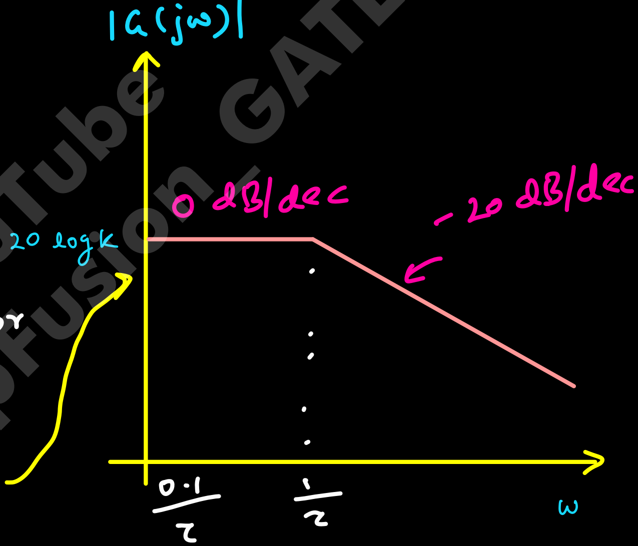
↳ Type - 0 system

$$G(s) = \frac{K}{(1+s\tau)}$$

K_p = positional Error coefficient

$= \lim_{s \to 0} G(s) = K$

= DC gain in OLTF



• If Initial slope = 0 $\Rightarrow$ we can find k_p.

$$20 \log k = 20 \log k_p$$

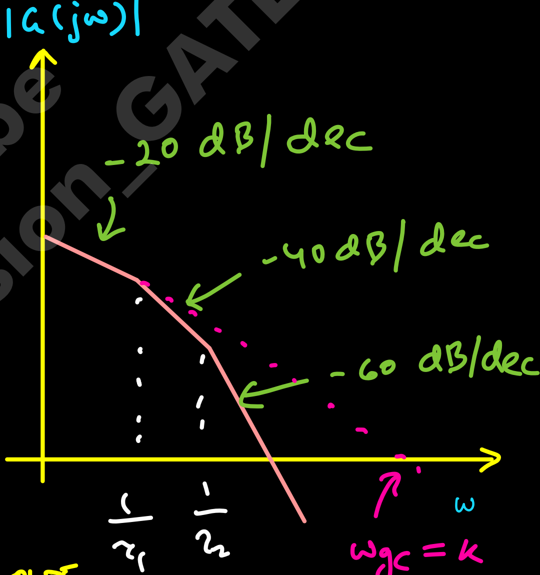
$$e_{ss} = \frac{A}{1+k_p} \; ; \; \underset{(I/P)}{R(s)} = \frac{A}{s} \Rightarrow r(t) = A\,u(t)$$

Velocity Error Coefficient

$\tau_1 > \tau_2$

↳ Type - 1 system

$$G(s) = \frac{K}{s(1+s\tau_1)(1+s\tau_2)}$$



K_v = velocity error coefficient

$$= \lim_{s \to 0} s\,G(s) = K$$

≠ DC gain of OLTF

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• value of ω where the initial line cuts the 0 dB axis is equal to K_v

LFR → $G(s) \simeq \frac{K}{s}$

$|G(j\omega)| = \frac{K}{\omega} = 1$

$\omega = \omega_{gc}$

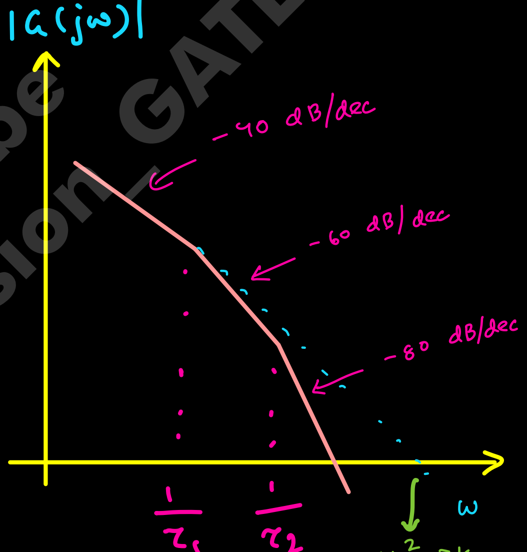
$K = \omega_{gc} = K_v$

$e_{ss} = \frac{A}{K_v}$; $r(t) = A\,t\,u(t)$

Acceleration Error Coefficient

↳ Type-2 system

$$G(s) = \frac{K}{s^2(1+s\tau_1)(1+s\tau_2)}$$



$\omega_{gc}^2 = K$

$\omega_{gc} = \sqrt{K_a}$

K_a = Acceleration error coefficient.

$$= \lim_{s\to 0} s^2 G(s) = K$$

≠ DC gain of OLTF

- value of ω where the initial line cuts the 0 dB axis is equal to square root of K_a

L.F.R $\Rightarrow$ $G(s) = \frac{K}{s^2}$, $|G(j\omega)| = \frac{K}{\omega^2} = 1$

$\omega = \omega_{gc}$

$K = \omega_{gc}^2$; $K_a = \omega_{gc}^2$

$e_{ss} = \frac{A}{K_a}$; $r(t) = \frac{A}{2} t^2 u(t)$

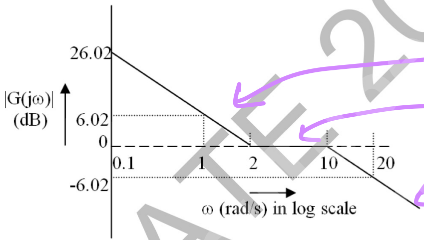
Conclusion

Initial line slope	Type	Error Coefficient	I/P	e_{ss}
0 dB/decade	0	Amplitude = $20 \log_{10} K_p$	$r(t) = A u(t)$	$\frac{A}{1+K_p}$
−20 dB/decade	1	K_v = freq^n of 0 dB axis Intersection	$r(t) = A t u(t)$	$\frac{A}{K_v}$
−40 dB/decade	2	$\sqrt{K_a}$ = freq^n of 0 dB axis Intersection	$r(t) = \frac{A t^2 u(t)}{2}$	$\frac{A}{K_a}$

Question

22

The Bode asymptotic magnitude plot of a minimum phase system is shown in the figure.



-20 dB/dec

0 dB/dec

-20 dB/dec

If the system is connected in a unity negative feedback configuration, the steady state error of the closed loop system, to a unit ramp input, is_________.

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Observations

↳ Initial slope $= -20$ dB/dec → Type - 1 system

$\omega_{z_1} = 2$ rad/sec

$\omega_{p_2} = 10$ rad/s

$$G(s) = \frac{K\left(\frac{s}{2}+1\right)}{s\left(\frac{s}{10}+1\right)}$$

M-1 $\left|G(j\omega)\right|_{\omega=0.1} = 26.02 \text{ dB}$

$\rightarrow 20\log_{10}(|G(j\omega)|) = 26.02$

$|G(j\omega)| = 10^{1.3} = 10^1 * 10^{0.3}$

$|G(j\omega)| = 20$

$\left|G(j\omega)\right|_{\omega=0.1} \simeq \frac{K}{\omega} = 20 \qquad K = 2$

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M-2

$k = \omega_{gc} = 2$

$$G(s) = \frac{2\left(\frac{s}{2}+1\right)}{s\left(\frac{s}{10}+1\right)}$$

M-1

$$\lim_{s\to 0} s\,E(s) = \lim_{s\to 0} s\left[R(s) - C(s)\right]$$

$$= \lim_{s\to 0} s\,R(s)\left[1 - \frac{G(s)}{1+G(s)}\right]$$

$$= \lim_{s\to 0} s\,R(s)\left[\frac{1}{1+G(s)}\right]$$

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$$= \lim_{s \to 0} s \times \frac{1}{s^2} \left[\frac{1}{1 + \frac{2}{s} \left(\frac{s/2 + 1}{s/10 + 1} \right)} \right]$$

$$= \lim_{s \to 0} \left[\frac{1}{s + 2 \left(\frac{0+1}{0+1} \right)} \right]$$

$$e_{ss} = \frac{1}{2} = 0.5$$

M-2 → velocity Error coefficient

$$K_v = \lim_{s \to 0} s\,G(s) = K = 2$$

$$e_{ss} = \frac{1}{K_v} = \frac{1}{2} = 0.5$$

The End

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Lecture 8

Bode Plots of Std. 2nd Order Systems → underdamped systems

Questions

$$\hookrightarrow \frac{1}{(s\tau_1+1)(s\tau_2+1)} \rightarrow \text{Real roots}$$

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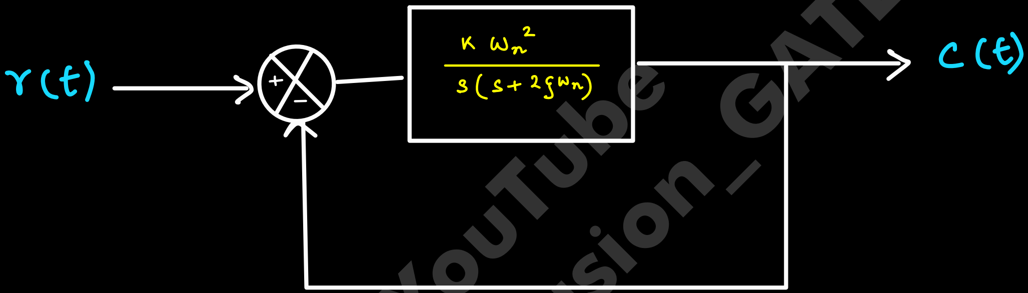
Bode Plot of Standard 2nd Order System



$$G(s) = \frac{K\omega_n^2}{s(s+2\zeta\omega_n)}$$

Type − 1

Initial slope = −20 dB/dec

Order − 2

Ending slope = −40 dB/dec

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Bode Plot of Standard 2nd Order System

↳ ξ - based system

$(0 < \xi < 1)$

CLTF

$$T(S) = \frac{K\omega_n^2}{S^2 + 2\xi\omega_n S + \omega_n^2}$$

Freq Response $\Rightarrow s = j\omega$

$$T(j\omega) = \frac{K\omega_n^2}{-\omega^2 + 2\xi\omega_n j\omega + \omega_n^2}$$

$$T(j\omega) = \frac{K}{1 - \left(\frac{\omega}{\omega_n}\right)^2 + j2\xi\frac{\omega}{\omega_n}}$$

$$|T(j\omega)| = \frac{K}{\sqrt{\left\{1 - \left(\frac{\omega}{\omega_n}\right)^2\right\}^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}}$$

$0 < \xi < 1$

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$$\angle T(j\omega) = -\tan^{-1}\left[\frac{2\xi\,\omega/\omega_n}{\left|1-\left(\frac{\omega}{\omega_n}\right)^2\right|}\right] \quad ; \frac{\omega}{\omega_n} < 1$$

$$= -180^\circ + \tan^{-1}\left[\frac{2\xi\,\omega/\omega_n}{\left|1-\left(\frac{\omega}{\omega_n}\right)^2\right|}\right] \quad ; \frac{\omega}{\omega_n} > 1$$

- **Approximations**

LFR $\rightarrow$ $\omega << \omega_n$ $\rightarrow$ $\frac{\omega}{\omega_n} << 1$

$$|T(j\omega)| = \frac{K}{\sqrt{\left\{1 - \underbrace{\left(\frac{\omega}{\omega_n}\right)^2}_{0}\right\}^2 + \underbrace{\left(2\xi\frac{\omega}{\omega_n}\right)^2}_{0}}}$$

$$= \frac{K}{1} = K$$

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$|T(j\omega)|$ in dB $\simeq 20\log_{10}K$, if $K=1$

$\rightarrow$ 0 dB

$\angle T(j\omega) \simeq 0$

High freqn Region

$\omega >> \omega_n, \quad \frac{\omega}{\omega_n} >> 1$

$$|T(j\omega)| = \frac{K}{\sqrt{\left(-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}}$$

$$= \frac{K}{\left(\frac{\omega}{\omega_n}\right)^2}$$

$$|T(j\omega)| = \left(\frac{\omega_n}{\omega}\right)^2 K$$

$$|T(j\omega)| = 20\log_{10}(K\omega_n^2) - 20\log_{10}\omega^2$$

if $K=1$

$$= 20\log_{10}(\omega_n^2) - 40\log_{10}\omega$$

slope $= -40$ dB/dec

$$\angle T(j\omega) = -180^\circ + \tan^{-1}\left[\underbrace{\frac{2\xi\downarrow}{\left(\frac{\omega}{\omega_n}\right)\uparrow\uparrow}}_{0^\circ}\right]$$

$$\angle T(j\omega) \simeq -180^\circ$$

- In Both HFR & LFR, ξ is getting neglected.

• Magnitude & Phase of 2^{nd} Order T/F

LFR: $|T(j\omega)| = 20\log K$; $\angle T(j\omega) = 0^\circ$

$(\omega << \omega_n)$

HFR: $|T(j\omega)| = 20\log K + 20\log \omega_n^2 - 40\log \omega$

$(\omega >> \omega_n)$

slope = -40 dB/dec

Corner frequency → where my Asymptotes meet.

The corner frequency for underdamped system = ω_n

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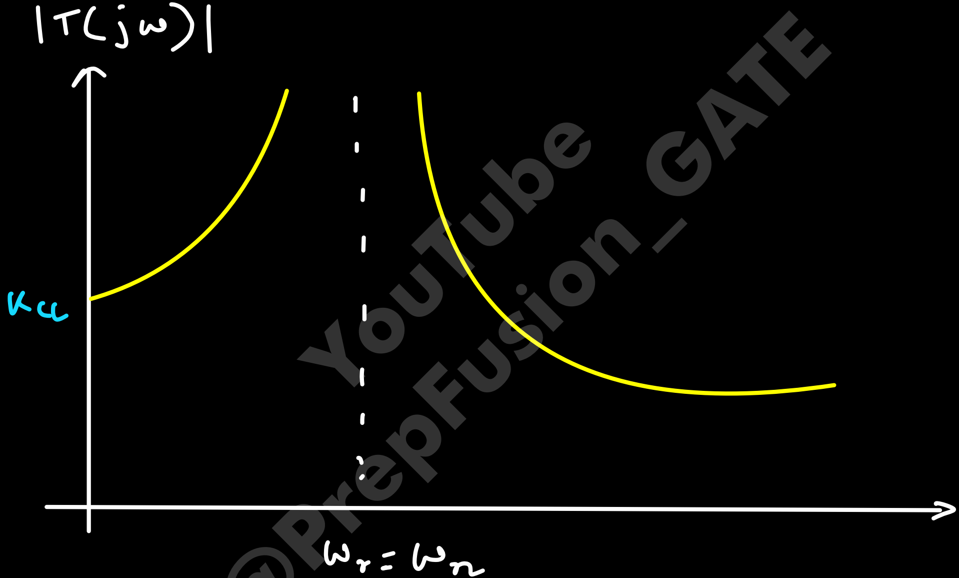
① $\xi = 0$, undamped system



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$$T(s) = \frac{K\omega_n^2}{s^2 + \omega_n^2}$$

$$T(j\omega) = \frac{K\omega_n^2}{-\omega^2 + \omega_n^2} \rightarrow \omega = \omega_n \Rightarrow \text{Blows up!!}$$

☹

↳ LFR

↳ $\omega << \omega_n$

$T(j\omega) \simeq K \rightarrow 20\log K$ in dB

$\angle T(j\omega) = 0^\circ$

HFR $\rightarrow \omega >> \omega_n$

$$T(j\omega) = \frac{K\omega_n^2}{-\omega^2} = -K\omega_n^2\left(\frac{1}{\omega^2}\right)$$

$$|T(j\omega)| = \frac{K\omega_n^2}{\omega^2} \rightarrow 20\log K\omega_n^2 \underbrace{-40\log\omega}_{-40\,dB}$$

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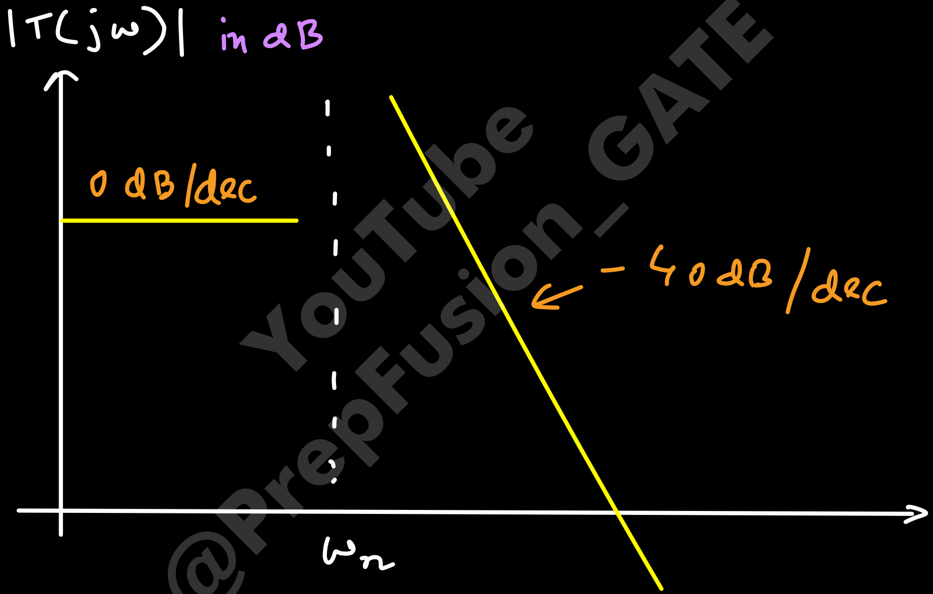
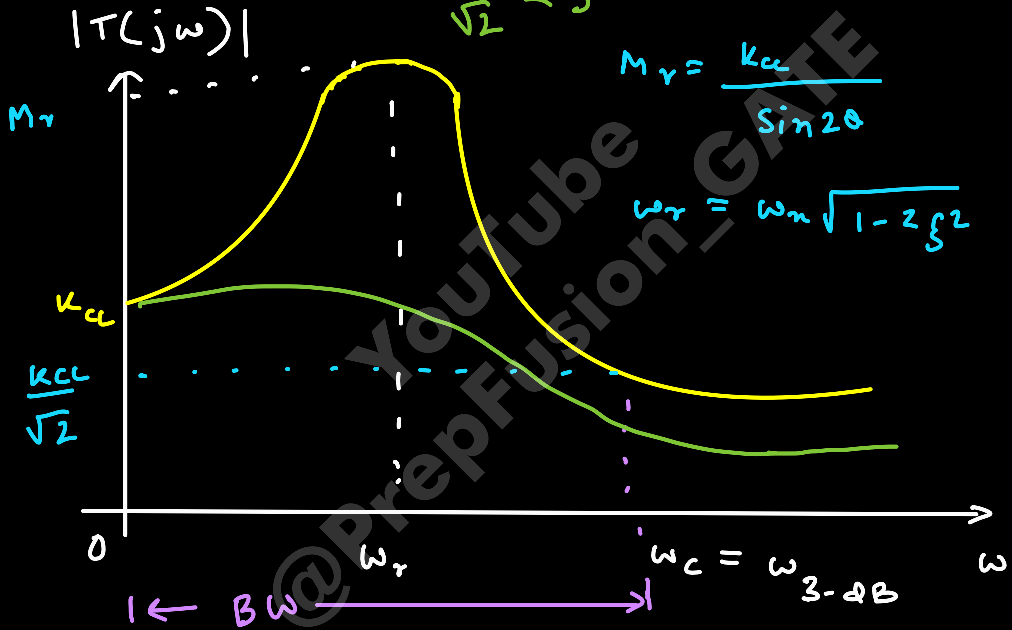
① Bode magnitude Plot



② $0 < \xi < \frac{1}{\sqrt{2}}$, underdamped system

$\frac{1}{\sqrt{2}} < \xi < 1$



$$M_r = \frac{K_{cc}}{\sin 2\theta}$$

$$\omega_r = \omega_n \sqrt{1 - 2\xi^2}$$

BW → Def^n of Bandwidth is up to the designer

here as my Resonant peak varies with

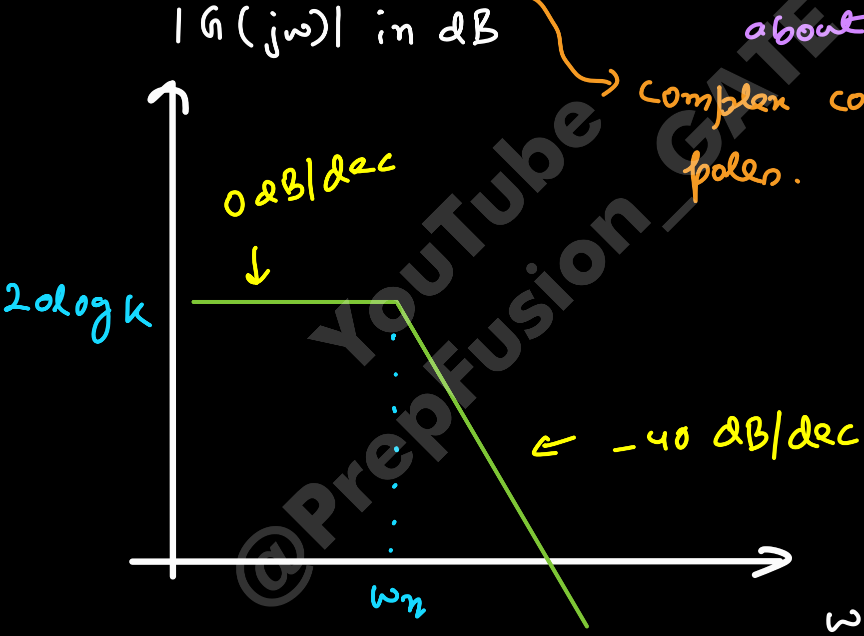
ξ, hence we define bandwidth w.r.t K_{cc}

↓

not changing

② Bode magnitude Plot

→ $0 < \xi < 1$ → we lose information about complex conjugate poles.



- <u>Error at corner frequency</u>

$$T(j\omega) = \frac{K}{1 - \left(\frac{\omega}{\omega_n}\right)^2 + j2\xi\frac{\omega}{\omega_n}}$$

$$|T(j\omega)|_{\omega=\omega_n} = \frac{K}{2\xi} \Rightarrow (20\log K - 20\log 2\xi)\ \text{dB}$$

$|\hat{T}(j\omega)| = 20\log k$

$$\text{Error}_{\text{in dBs}} = \text{Actual} - \text{Approximate}$$

$$= 20\log k - 20\log 2\xi - 20\log k$$

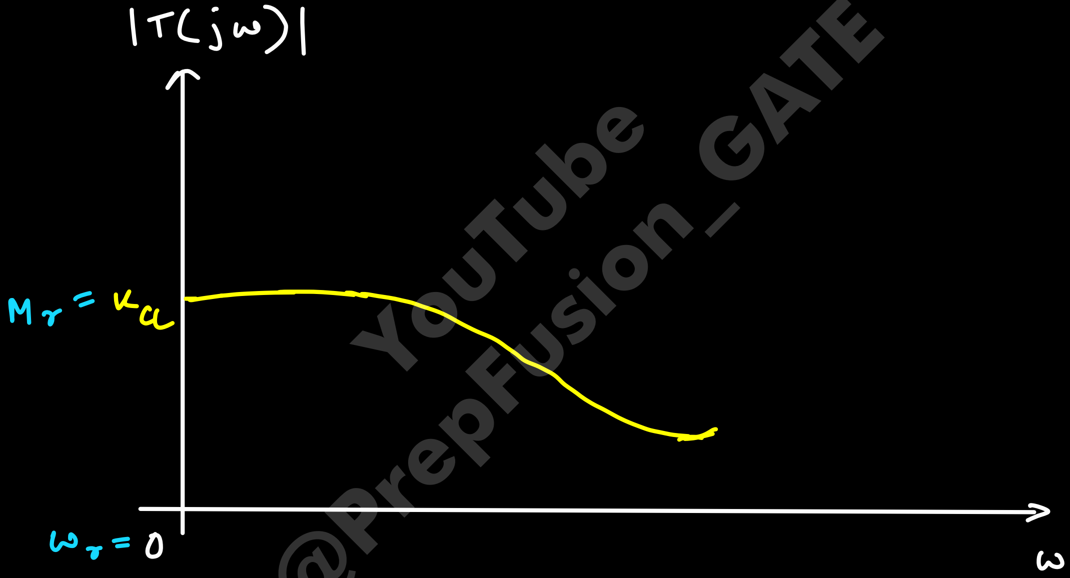
$$\boxed{\text{Error}_{\text{in dBs}} = -20\log 2\xi}$$

↳ 2^{nd} order underdamped system

- While calculating HFR & LFR the information of ξ is lost hence we get huge error.

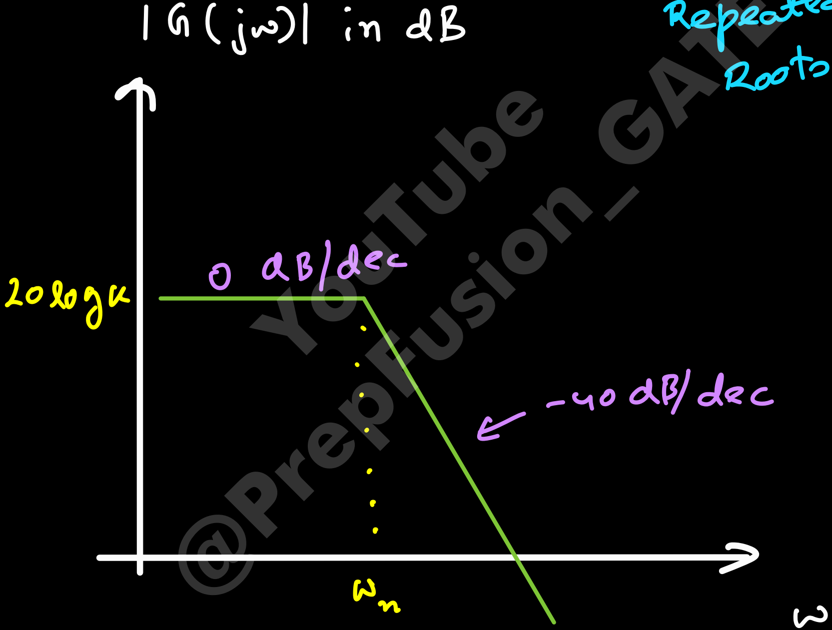
- Just by seeing the Bode plot we won't be able tell about damping.

③ $\xi = 1 \rightarrow$ critical damping $\xi = 1$



③ Bode magnitude Plot

→ $\zeta = 1$ → -ve Real Repeated Roots



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$$T(s) = \frac{K\omega_n^2}{s^2 + 2\omega_n s + \omega_n^2} = \frac{K\omega_n^2}{(s+\omega_n)^2}$$

Two repeated poles @ ω_n

$$T(j\omega) = \frac{K}{\left(j\frac{\omega}{\omega_n} + 1\right)^2}$$

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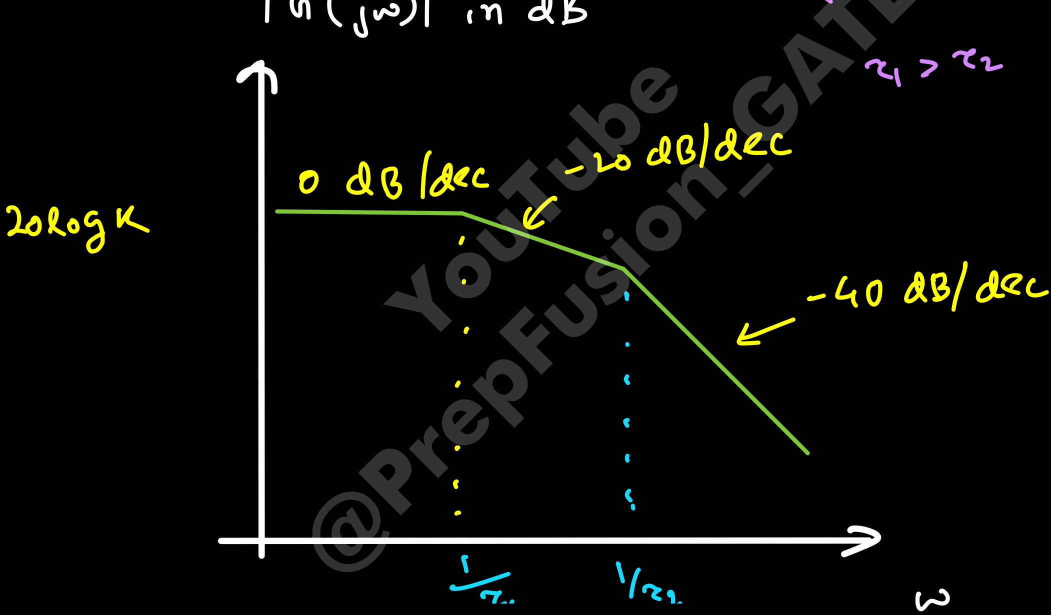
④ Overdamped, $\xi > 1$ → $-ve$ unequal Real Roots

4 Bode magnitude Plot

$$\rightarrow \frac{K\omega_n^2}{(s\tau_1+1)(s\tau_2+1)}$$

$\tau_1 > \tau_2$



Question

23

For the transfer function

$$G(s) = \frac{5(s+4)}{s(s+0.25)(s^2+4s+25)}$$

The values of the constant gain term and the highest corner frequency of the Bode plot respectively are

(A) 3.2, 5.0 (B) 16.0, 4.0 (C) 3.2, 4.0 (D) 16.0, 5.0

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$K \neq 5 \rightarrow$ Transfer function not given in time constant format.

↳ for finding corner freq^n check location of poles.

↳ $s^2 + 4s + 25 = 0$

↳ $D = \sqrt{b^2 - 4ac} = \sqrt{16 - 100} = \sqrt{-84}$

Complex conjugate roots

$$s^2 + 2\xi\omega_n s + \omega_n^2$$ → Assume corner freq @ ω_n

slope after ω_n $= -40$ dB/dec

$\omega_n^2 = 25$

$\omega_n = 5$ rad/sec

$2\xi\omega_n = 4 \Rightarrow \xi = 0.4$ → complex conjugate poles.

Convert G(s) into std. time constant format:-

$$= \frac{5\times 4\left(\frac{s}{4}+1\right)}{s\times 0.25\left(\frac{s}{0.25}+1\right)25\left(\frac{s^2}{25}+\frac{4}{25}+1\right)}$$

$$= \frac{\underbrace{3.2}_{=K}\left(\frac{s}{4}+1\right)}{\left(\frac{s}{0.25}+1\right)\left(\frac{s^2}{25}+\frac{4}{25}+1\right)}$$

The End

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Lecture 9

Questions on Bode Plots

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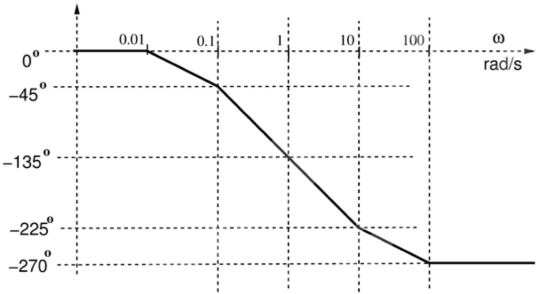
Question

24

The asymptotic Bode phase plot of $G(s) = \dfrac{k}{(s+0.1)(s+10)(s+p_1)}$, with k and p_1 both positive, is shown below.



The value of p_1 is 1

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Observations from the Graph

$0.01 < \omega_{p_2} < 1$ $\quad\quad$ $1 < \omega_{p_3} < 100$

$\downarrow$ $\quad\quad$ $\downarrow$

$-45^\circ/dec$ $\quad\quad$ $-45^\circ/dec$

pole will lie 10 times after this.

$0.01 < \omega < 0.1 \rightarrow 0.1 < \omega < 10$

$\downarrow$ $\quad\quad$ $\downarrow$

$-45^\circ/dec$ $\quad\quad$ $-90^\circ/dec$

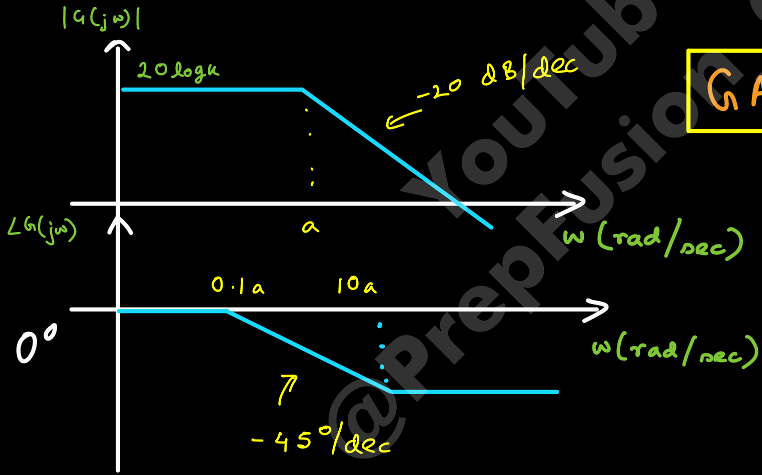
- After 0.1 rad/sec the slope reduces further by $-45°/dec$ → Their in a pole @ $0.1 * 10$ $= 1$ rad/sec

- After 10 rad/sec the slope increases further by $+45°/dec$ → Their in a pole @ $\frac{10}{10} = 1$ rad/s

$P_1 = 1$

Question

25 The asymptotic Bode plot of the transfer function K/[1+(s/a)] is given in figure. The error in phase angle and dB gain at a frequency of ω = 0.5a are respectively.



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$\omega = 0.5a \rightarrow$ not corner frequency.

Error in

$$|G(j\omega)|_{\omega=0.5a} = (\text{Actual} - \text{Approximate})$$

$$|G(j\omega)|\Big|_{\omega=0.5a} = \frac{K}{\sqrt{\left(\frac{\omega}{a}\right)^2+1}} = \frac{K}{\sqrt{0.25+1}}$$

$$= 20\log K - 20\log\left(\sqrt{1.25}\right)$$

$$= 20\log K - 0.97$$

$$\left|\hat{G}(j\omega)\right|_{\omega=0.5a} = 20\log K$$

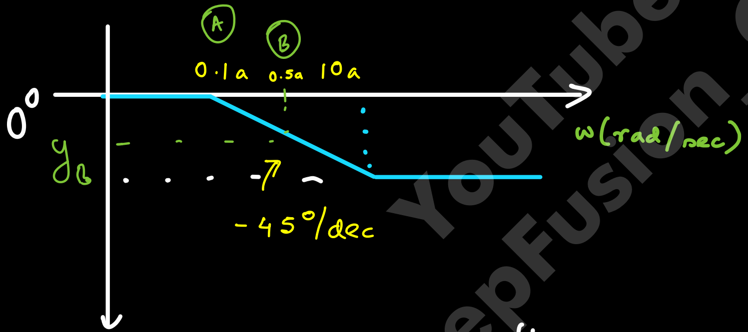
$$\text{Error}_{\text{in dB}} = 20\log K - 0.97 - 20\log K$$

$$\text{Error} = -0.97\ \text{dB.}\ (\text{Ans})$$

$$\angle G(j\omega)\Big| = -\tan^{-1}\left(\frac{\omega}{a}\right) = -\tan^{-1}(0.5)$$

$$\omega = 0.5a$$

$$= -26.56^{\circ}$$



$$\text{slope}_{A-B} = \frac{y_B - y_A}{\log\left(\frac{\omega_B}{\omega_A}\right)} = \frac{y_B - 0}{\log_{10}(5)}$$

$$-45^\circ \log_{10}(5) = y_B = \angle \hat{G}(j\omega)\Big|_{\omega=0.5a}$$

$$\angle \hat{G}(j\omega)\Big|_{\omega=0.5a} = -31.4536^\circ$$

$$\text{Error} = -26.56^\circ - (-31.4536^\circ)$$

$$= 4.9^\circ \text{ (Ans)}$$

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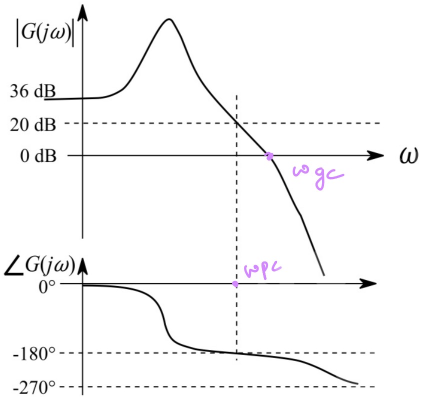
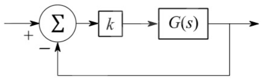
Question

26

The figure below shows the Bode magnitude and phase plots of a stable transfer function $G(s) = \dfrac{n_0}{s^3+d_2s^2+d_1s+d_0}$.





Consider the negative unity feedback configuration with gain k in the feedforward path. The closed loop is stable for $k < k_0$. The maximum value of k_0 is ______.

$\omega_{pc} > \omega_{gc}$

↓

for stability

0.1 (Ans)

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↳ Information from graph

↳ $|G(j\omega)|_{\omega=\omega_{pc}} = 20\ dB$

$\rightarrow 10^{\frac{20}{20}} = 10$

↳ LFR gain → DC gain ⇒ $\frac{n_0}{d_0} = 36\ dB$

$\downarrow$

$10^{1.8}$

$\frac{n_0}{d_0} = 10^{1.8} = 63.1$

→ DB scale

$$GM = \frac{1}{|G(j\omega)|_{\omega_{pc}}} = -|G(j\omega)|_{\omega=\omega_{pc}} \text{ in dBs}$$

$$GM = -20\text{ dB}$$

for stability ⇒ $GM > 0$ → currently unstable

$$GM = \frac{1}{|KG(j\omega)|_{\omega=\omega_{pc}}} = -20\log K \underbrace{-20}_{\text{from graph}}$$

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for stability :-

$$-20\log k - 20 > 0$$

$$-20 \log k > 20$$

$\times -\frac{1}{20} \rightarrow$ sign of inequality will change

$$\log_{10} k < -1$$

$$k < 10^{-1}$$

$k < 0.1 \Rightarrow k_0$

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Absolute scale

$$|G(j\omega)|_{\omega=\omega_p} = 10$$

$$|KG(j\omega)|_{\omega=\omega_{pc}} = 10K$$

$$GM = \frac{1}{|KG(j\omega)|_{\omega=\omega_{pc}}} = \frac{0.1}{K} > 1$$

for stability

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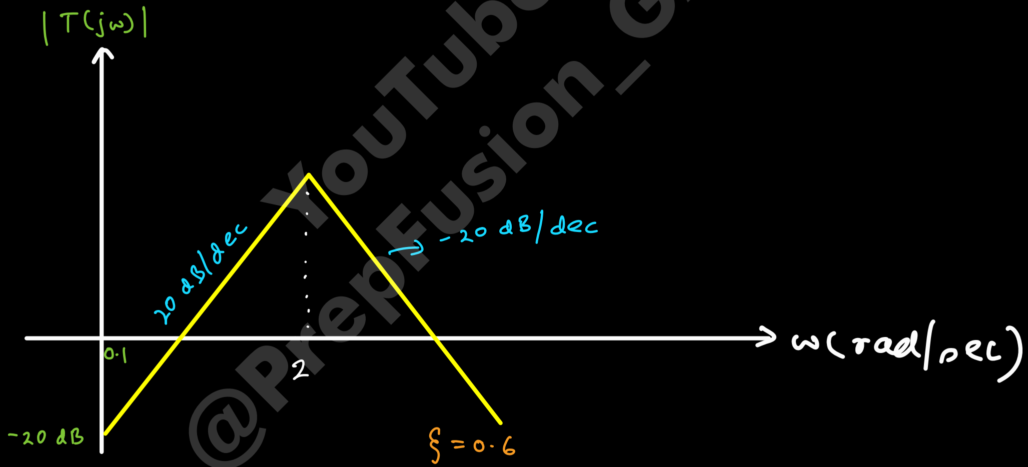
$$\boxed{u < 0.1}$$

Question

(27) Find the transfer function from the given Bode magnitude Plot.

(28) Find the error at the corner frequency in dBs



27 — Initial slope ⇒ +20 dB/dec → KS

Corner freqn ⇒ 2 rad/sec

→ $\omega_n = 2$

Δ slope = −40 dB/dec

$\xi = 0.6$

$s^2 + 2 \times (0.6) \times 2s + 4 = 0$

$(s^2 + 2.4s + 4) = 0 \rightarrow \left(\frac{s^2}{4} + 0.6s + 1\right) = 0$

$$T(s) = \frac{ks}{\left(\frac{s^2}{4} + 0.6s + 1\right)}$$

$-20dB \rightarrow 10^{-1}$

$$|T(j\omega)|_{\omega=0.1} = \frac{1}{10}$$

$$\Rightarrow \frac{k \times 0.1}{1} = 0.1$$

$$\Rightarrow k = 1$$

$$T(s) = \frac{s}{\left(\frac{s^2}{4} + 0.6s + 1\right)}$$ (Ans)

$$\text{Error} = -20\log(2\xi)$$

$$= -20\log(1.2) = -1.58\ \text{dB}$$ (Ans)

The End

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