

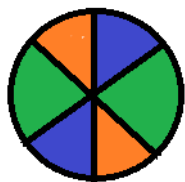
Question:1

Explain the concept of congruence of figures with the help of certain examples.

Solution:

Congruent objects or figures are exact copies of each other or we can say mirror images of each other. The relation of two objects being congruent is called congruence.

Consider Ball A and Ball B. These two balls are congruent.



Ball A



Ball B

Now consider the two stars below. Star A and Star B are exactly the same in size, colour and shape. These are congruent stars.



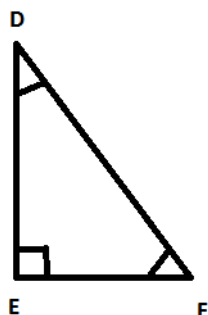
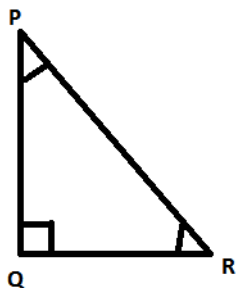
STAR A



STAR B

Let us look at the triangles below. Here we have triangle PQR and triangle DEF.

These two triangles have corresponding angles equal and corresponding sides equal. Thus these triangles are congruent to each other.

**Question:2**

Fill in the blanks:

- i Two line segments are congruent if
- ii Two angles are congruent if
- iii Two squares are congruent if
- iv Two rectangles are congruent if
- v Two circles are congruent if

Solution:

1) They have the same length, since they can superpose on each other.

2) Their measures are the same. On superposition, we can see that the angles are equal.

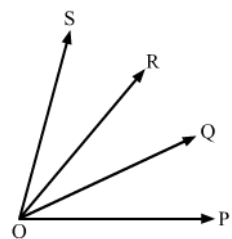
3) Their sides are equal. All the sides of a square are equal and if two squares have equal sides, then all their sides are of the same length. Also angles of a square are 90° which is also the same for both the squares.

4) Their lengths are equal and their breadths are also equal. The opposite sides of a rectangle are equal. So if two rectangles have lengths of the same size and breadths of the same size, then they are congruent to each other.

5) Their radii are of the same length. Then the circles will have the same diameter and thus will be congruent to each other.

Question:3

In Fig., $\angle POQ \cong \angle ROS$, can we say that $\angle POR \cong \angle QOS$



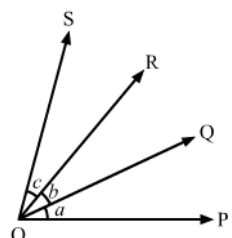
Solution:

We have,

$\angle POQ \cong \angle ROS$ (1) Also, $\angle ROQ \cong \angle ROQ$ (same angle) Therefore, adding $\angle ROQ$ to both sides of (1), we get: $\angle POQ + \angle ROQ \cong \angle ROS + \angle ROQ$ There

Question:4

In Fig., $a = b = c$, name the angle which is congruent to $\angle AOC$.



Solution:

We have,

$\angle AOB = \angle BOC = \angle COD$ Therefore, $\angle AOB = \angle COD$ Also, $\angle AOB + \angle BOC = \angle BOC + \angle COD$ $\angle AOC = \angle BOD$ Hence, $\angle AOC \cong \angle BOD$ $\angle BOD$ is congruent to

Question:5

Is it correct to say that any two right angles are congruent? Give reasons to justify your answer.

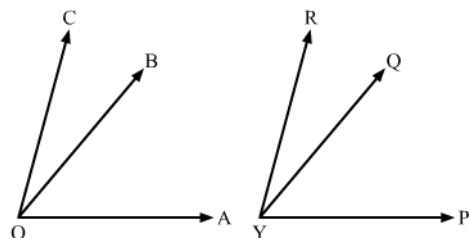
Solution:

Two right angles are congruent to each other because they both measure 90 degrees.

We know that two angles are congruent if they have the same measure.

Question:6

In Fig. 8, $\angle AOC \cong \angle PYR$ and $\angle BOC \cong \angle QYR$. Name the angle which is congruent to $\angle AOB$.



Solution:

$\angle AOC \cong \angle PYR$(i) Also, $\angle BOC \cong \angle QYR$ Now, $\angle AOC = \angle AOB + \angle BOC$ and $\angle PYR = \angle PYQ + \angle QYR$ By putting the value of $\angle AOC$ and

Question:7

Which of the following statements are true and which are false;

- i All squares are congruent.
- ii If two squares have equal areas, they are congruent.
- iii If two rectangles have equal area, they are congruent.
- iv If two triangles are equal in area, they are congruent.

Solution:

i) False. All the sides of a square are of equal length. However, different squares can have sides of different lengths. Hence all squares are not congruent.

ii) True

Area of a square = side \times side

Therefore, two squares that have the same area will have sides of the same lengths. Hence they will be congruent.

iii) False

Area of a rectangle = length \times breadth

Two rectangles can have the same area. However, the lengths of their sides can vary and hence they are not congruent.

Example: Suppose rectangle 1 has sides 8 m and 8 m and area 64 metre square.

Rectangle 2 has sides 16 m and 4 m and area 64 metre square.

Then rectangle 1 and 2 are not congruent.

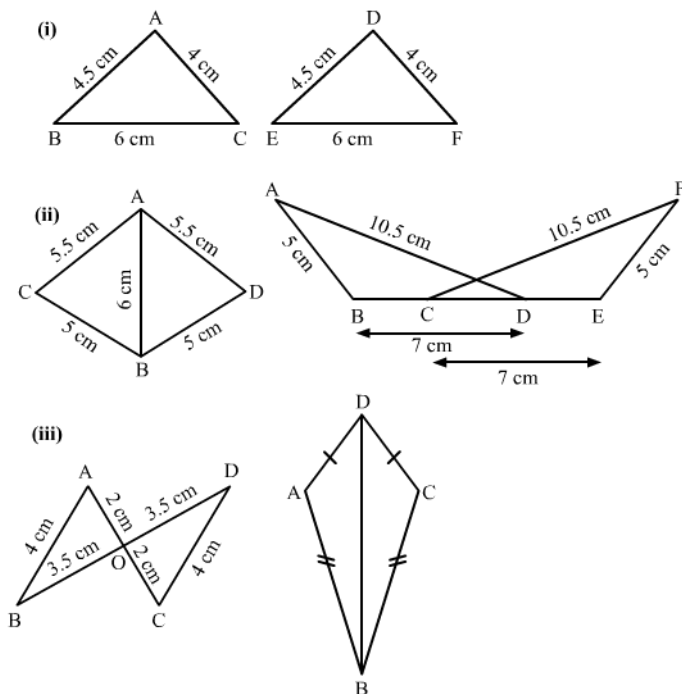
iv) False

Area of a triangle = $\frac{1}{2} \times$ base \times height

Two triangles can have the same area but the lengths of their sides can vary and hence they cannot be congruent.

Question:8

In the following pairs of triangles *Fig. 12 to 15*, the lengths of the sides are indicated along sides. By applying SSS condition, determine which are congruent. State the result in symbolic form.



Solution:

1) In $\triangle ABC$ and $\triangle DEF$

$AB = DE = 4.5$ cm Side

$BC = EF = 6$ cm Side

and $AC = DF = 4$ cm Side

Therefore, by SSS criterion of congruence, $\triangle ABC \cong \triangle DEF$.

2)

In $\triangle ACB$ and $\triangle ADB$ $AC = AD$ (Side) $BC = BD$ (Side) and $AB = AB$ (Side)

Therefore, by SSS criterion of congruence, $\triangle ACB \cong \triangle ADB$.

3)

In $\triangle ABD$ and $\triangle BDC$, $AB = BC$ (Side) $AD = DC$ (Side) $BD = BD$ (Side)

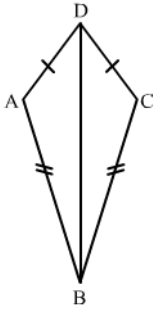
Therefore, by SSS criterion of congruence, $\triangle ABD \cong \triangle BDC$.

Question:9

In Fig. 16, $AD = DC$ and $AB = BC$.

i) Is $\triangle ABD \cong \triangle CBD$?

ii) State the three parts of matching pairs you have used to answer i.



Solution:

Yes $\triangle ABD \cong \triangle CBD$ by the SSS criterion.

We have used the three conditions in the SSS criterion as follows:

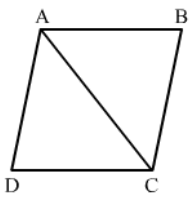
$AD = DC$

$AB = BC$

and $DB = BD$

Question:10

In Fig. 17, $AB = DC$ and $BC = AD$.



i Is $\triangle ABC \cong \triangle CDA$?

ii What congruence condition have you used?

iii You have used some fact, not given in the question, what is that?

Solution:

We have $AB = DC$

$BC = AD$

and $AC = AC$

Therefore by SSS $\triangle ABC \cong \triangle CDA$.

We have used Side Side Side congruence condition with one side common in both the triangles.

Yes, we have used the fact that $AC = CA$.

Question:11

If $\triangle PQR \cong \triangle EFD$,

i Which side of $\triangle PQR$ equals ED ?

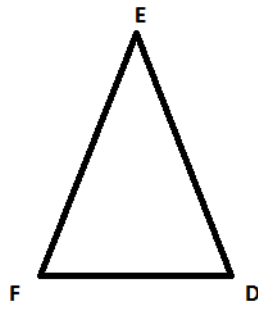
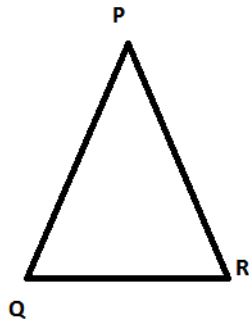
ii Which angle of $\triangle PQR$ equals $\angle E$?

Solution:

$$\triangle PQR \cong \triangle EDF$$

1) Therefore $PR = ED$ since the corresponding sides of congruent triangles are equal.

2) $\angle QPR = \angle FED$ since the corresponding angles of congruent triangles are equal.

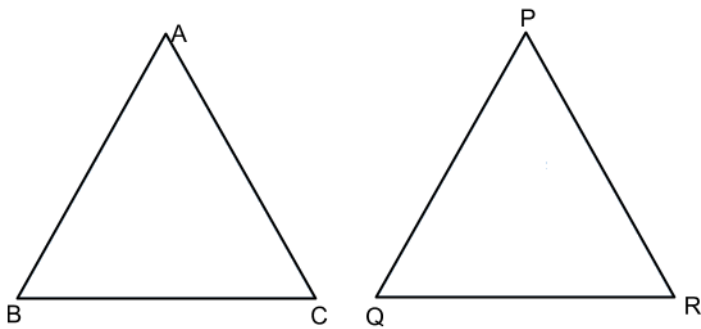


Question:12

Triangles ABC and PQR are both isosceles with $AB = AC$ and $PQ = PR$ respectively. If also, $AB = PQ$ and $BC = QR$, are the two triangles congruent? Which condition do you use?

If $\angle B = 50^\circ$, what is the measure of $\angle R$?

Solution:



We have $AB = AC$ in isosceles $\triangle ABC$

and $PQ = PR$ in isosceles $\triangle PQR$.

Also, we are given that $AB = PQ$ and $QR = BC$.

Therefore, $AC = PR$ $AB = AC$, $PQ = PR$ and $AB = PQ$

Hence, $\triangle ABC \cong \triangle PQR$.

Now

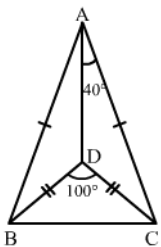
$\angle ABC = \angle PQR$ (Since triangles are congruent) However, $\triangle PQR$ is isosceles. Therefore, $\angle PRQ = \angle PQR = \angle ABC = 50^\circ$

Question:13

ABC and DBC are both isosceles triangles on a common base BC such that A and D lie on the same side of BC . Are triangles ADB and ADC congruent?

Which condition do you use? If $\angle BAC = 40^\circ$ and $\angle BDC = 100^\circ$; then find $\angle ADB$.

Solution:



YES $\triangle ADB \cong \triangle ADC$ (By SSS)

$AB = AC$, $DB = DC$ AND $AD = DA$

$$\angle BAD = \angle CAD \text{ (c.p.c.t.) } \angle BAD + \angle CAD = 40^\circ \quad 2\angle BAD = 40^\circ \quad \angle BAD = \frac{40^\circ}{2} = 20^\circ$$

$\angle ABC + \angle BCA + \angle BAC = 180^\circ$ (Angle sum property) Since $\triangle ABC$ is an isosceles triangle, $\angle ABC = \angle BCA$
 $\angle ABC + \angle ABC + 40^\circ = 180^\circ$
 $2\angle ABC = 180^\circ - 40^\circ$
 $2\angle ABC = 140^\circ$
 $\angle ABC = 70^\circ$

$\angle DBC + \angle BCD + \angle BDC = 180^\circ$ (Angle sum property) Since $\triangle ABC$ is an isosceles triangle, $\angle DBC = \angle BCD$
 $\angle DBC + \angle DBC + 100^\circ = 180^\circ$
 $2\angle DBC = 180^\circ - 100^\circ$
 $2\angle DBC = 80^\circ$
 $\angle DBC = 40^\circ$

In $\triangle BAD$, $\angle ABD + \angle BAD + \angle ADB = 180^\circ$ (Angle sum property)
 $30^\circ + 20^\circ + \angle ADB = 180^\circ$ ($\angle ABD = \angle ABC - \angle DBC$)
 $\angle ADB = 180^\circ - 20^\circ - 30^\circ$
 $\angle ADB = 130^\circ$

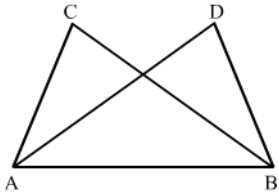
Question:14

$\triangle ABC$ and $\triangle ABD$ are on a common base AB , and $AC = BD$ and $BC = AD$ as shown in Fig. 18. Which of the following statements is true?

i $\triangle ABC \cong \triangle ABD$

ii $\triangle ABC \cong \triangle ADB$

iii $\triangle ABC \cong \triangle BAD$



Solution:

In $\triangle ABC$ and $\triangle BAD$ we have,

$AC = BD$ given

$BC = AD$ given

and $AB = BA$ common

Therefore by SSS criterion of congruency, $\triangle ABC \cong \triangle BAD$.

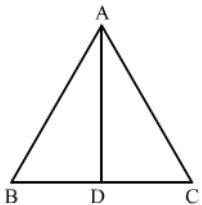
There option iii is true.

Question:15

In Fig. 19, $\triangle ABC$ is isosceles with $AB = AC$, D is the mid-point of base BC .

i Is $\triangle ADB \cong \triangle ADC$?

ii State the three pairs of matching parts you use to arrive at your answer.



Solution:

We have $AB = AC$.

Also since D is the midpoint of BC , $BD = DC$.

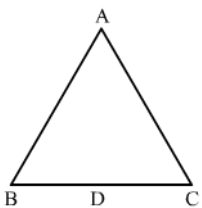
And $AD = DA$.

Therefore by SSS condition, $\triangle ABD \cong \triangle ADC$.

We have used $AB, AC : BD, DC$ and AD, DA .

Question:16

In Fig. 20, $\triangle ABC$ is isosceles with $AB = AC$. State if $\triangle ABC \cong \triangle ACB$. If yes, state three relations that you use to arrive at your answer.



Solution:

Yes $\triangle ABC \cong \triangle ACB$ by SSS condition.

Since ABC is an isosceles triangle, $AB = AC$, $BC = CB$ and $AC = AB$.

Question:17

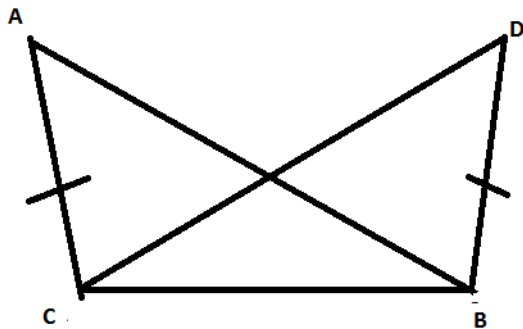
Triangles ABC and DBC have side BC common, $AB = BD$ and $AC = CD$. Are the two triangles congruent? State in symbolic form. Which congruence condition do you use? Does $\angle ABD$ equal $\angle ACD$? Why or why not?

Solution:

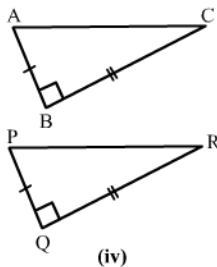
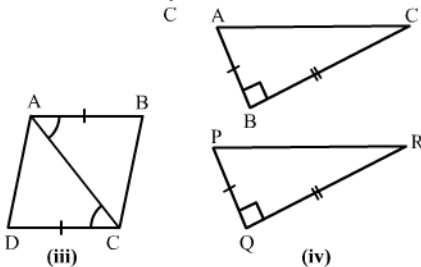
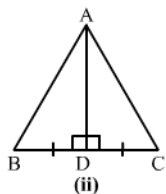
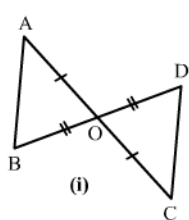
Yes.

In $\triangle ABC$ and $\triangle DBC$ $AB = DB$ (Given) $AC = DC$ (Given) $BC = BC$ (Common) By SSS criterion of congruency, $\triangle ABC \cong \triangle DBC$

No, $\angle ABD$ and $\angle ACD$ are not equal
because $AB \neq AC$.

**Question:18**

By applying SAS congruence condition, state which of the following pairs *Fig. 28* of triangles are congruent. State the result in symbolic form

**Solution:**

1) We have $OA = OC$ and $OB = OD$ and $\angle AOB = \angle COD$ which are vertically opposite angles.
Therefore by SAS condition, $\triangle AOC \cong \triangle BOD$.

2) We have $BD = DC$
 $\angle ADB = \angle ADC = 90^\circ$
and $AD = AD$
Therefore by SAS condition, $\triangle ADB \cong \triangle ADC$.

3) We have $AB = DC$
 $\angle ABD = \angle CDB$ and $BD = DB$
Therefore by SAS condition, $\triangle ABD \cong \triangle CBD$.

4) We have $BC = QR$

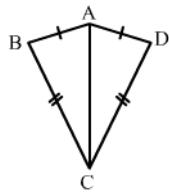
$\angle ABC = \angle PQR = 90^\circ$

and $AB = PQ$

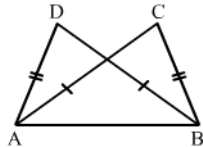
Therefore by SAS condition, $\triangle ABC \cong \triangle PQR$.

Question:19

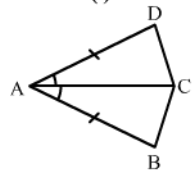
State the condition by which the following pairs of triangles are congruent.



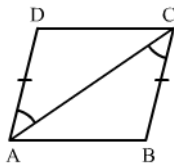
(i)



(ii)



(iii)



(iv)

Solution:

1) $AB = AD$

$BC = CD$

and $AC = CA$

Therefore by SSS condition, $\triangle ABC \cong \triangle ADC$.

2) $AC = BD$

$AD = BC$ and $AB = BA$

Therefore by SSS condition, $\triangle ABD \cong \triangle BAC$.

3) $AB = AD$

$\angle BAC = \angle DAC$

and $AC = AC$

Therefore by SAS condition, $\triangle BAC \cong \triangle DAC$.

4) $AD = BC$

$\angle DAC = \angle BCA$

and $AC = CA$

Therefore by SAS condition, $\triangle ABC \cong \triangle ADC$.

Question:20

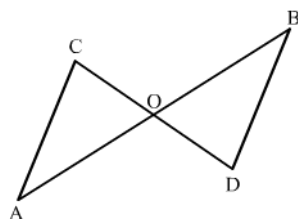
In Fig. 30, line segments AB and CD bisect each other at O . Which of the following statements is true?

i) $\triangle AOC \cong \triangle DOB$

ii) $\triangle AOC \cong \triangle BOD$

iii) $\triangle AOC \cong \triangle ODB$.

State the three pairs of matching parts, you have used to arrive at the answer.



Solution:

We have $AO = OB$.

And $CO = OD$

Also $\angle AOC = \angle BOD$

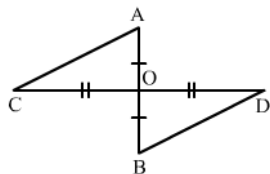
Therefore by SAS condition, $\triangle AOC \cong \triangle BOD$.

Therefore, statement ii is true.

Question:21

Line-segments AB and CD bisect each other at O . AC and BD are joined forming triangles AOC and BOD . State the three equality relations between the parts of the two triangles, that are given or otherwise known. Are the two triangles congruent? State in symbolic form. Which congruence condition do you use?

Solution:



We have $AO = OB$ and $CO = OD$ since AB and CD bisect each other at O .

Also $\angle AOC = \angle BOD$ since they are opposite angles on the same vertex.

Therefore by SAS congruence condition, $\triangle AOC \cong \triangle BOD$.

Question:22

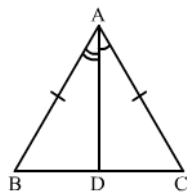
$\triangle ABC$ is isosceles with $AB = AC$. Line segment AD bisects $\angle A$ and meets the base BC in D .

i Is $\triangle ADB \cong \triangle ADC$?

ii State the three pairs of matching parts used to answer *i*.

iii Is it true to say that $BD = DC$?

Solution:



i We have $AB = AC$ *given*

$\angle BAD = \angle CAD$ (AD bisects $\angle BAC$)

and $AD = AD$ *common*

Therefore by SAS condition of congruence, $\triangle ABD \cong \triangle ACD$.

ii We have used $AB, AC; \angle BAD = \angle CAD; AD, DA$.

iii Now $\triangle ABD \cong \triangle ACD$ therefore by c.p.c.t $BD = DC$.

Question:23

In Fig. 31, $AB = AD$ and $\angle BAC = \angle DAC$.

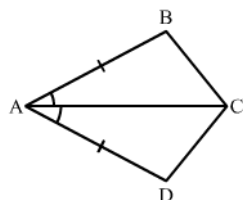
i State in symbolic form the congruence of two triangles ABC and ADC that is true.

ii Complete each of the following, so as to make it true:

a $\angle ABC = \dots\dots\dots$

b $\angle ACD = \dots\dots\dots$

c Line segment AC bisects $\dots\dots$ and $\dots\dots$



Solution:

i) $AB = AD$ *given*

$\angle BAC = \angle DAC$ *given*

$AC = CA$ *common*

Therefore by SAS condition of congruency, $\triangle ABC \cong \triangle ADC$.

ii) $\angle ABC = \angle ADC$ *c.p.c.t*

$\angle ACD = \angle ACB$ *c.p.c.t*

Question:24

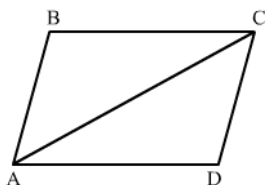
In Fig. 32, $AB \parallel DC$ and $AB = DC$.

i Is $\triangle ACD \cong \triangle CAB$?

ii State the three pairs of matching parts used to answer i.

iii Which angle is equal to $\angle CAD$?

iv Does it follow from iii that $AD \parallel BC$?

**Solution:**

i Yes $\triangle ACD \cong \triangle CAB$ by SAS condition of congruency.

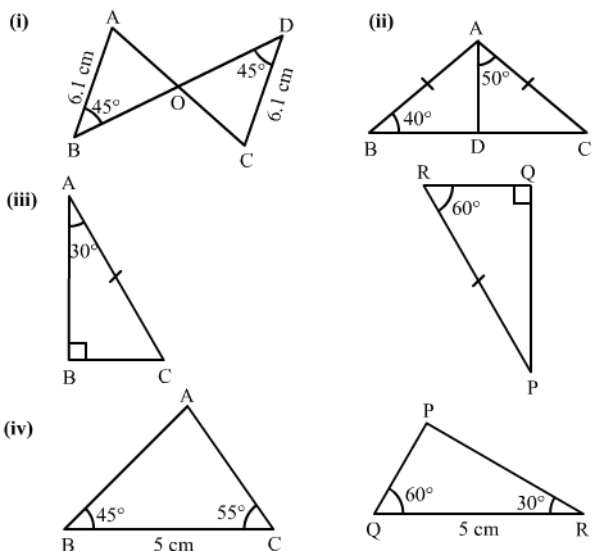
ii We have used $AB = DC$, $AC = CA$ and $\angle DCA = \angle BAC$.

iii $\angle CAD = \angle ACB$ since the two triangles are congruent.

iv Yes, this follows from $AD \parallel BC$ as alternate angles are equal. If alternate angles are equal the lines are parallel.

Question:25

Which of the following pairs of triangles are congruent by ASA condition?

**Solution:**

1) We have

Since $\angle ABO = \angle CDO = 45^\circ$ and both are alternate angles, $AB \parallel DC$. $\angle BAO = \angle DCO$ (alternate angle, $AB \parallel DC$ and AC is a transversal line). $\angle ABC$

2)

In $\triangle ABC$, Now $AB = AC$ (Given) $\angle ABD = \angle ACD = 40^\circ$ (Angles opposite to equal sides) $\angle ABD + \angle ACD + \angle BAC = 180^\circ$ (Angle sum property) $40^\circ + 40^\circ$

3)

In $\triangle ABC$, $\angle A + \angle B + \angle C = 180^\circ$ (Angle sum property) $\angle C = 180^\circ - \angle A - \angle B = 180^\circ - 30^\circ - 90^\circ = 60^\circ$ In $\triangle PQR$, $\angle P + \angle Q + \angle R = 180^\circ$ (Angle sum property) $\angle P = 180^\circ - 60^\circ - 30^\circ = 90^\circ$

4)

We have only $BC = QR$ but none of the angles of $\triangle ABC$ and $\triangle PQR$ are equal. Therefore, $\triangle ABC \not\cong \triangle PQR$

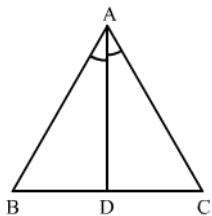
Question:26

In Fig. 37, AD bisects $\angle A$ and $AD \perp BC$.

i Is $\triangle ADB \cong \triangle ADC$?

ii State the three pairs of matching parts you have used in i.

iii Is it true to say that $BD = DC$?



Solution:

(i) Yes, $\triangle ADB \cong \triangle ADC$, by ASA criterion of congruency (ii) We have used $\angle BAD = \angle CAD$ $\angle ADB = \angle ADC = 90^\circ$ since $AD \perp BC$ and $AD = DA$ (iii) Yes

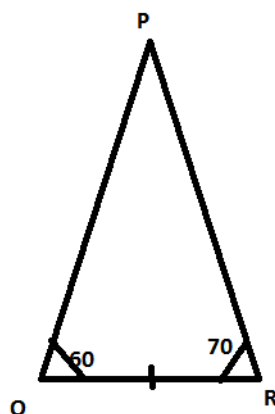
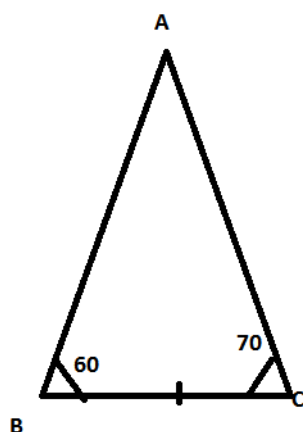
Question:27

Draw any triangle ABC . Use ASA condition to construct another triangle congruent to it.

Solution:

We have drawn

$\triangle ABC$ with $\angle ABC = 60^\circ$ and $\angle ACB = 70^\circ$ We now construct $\triangle PQR \cong \triangle ABC$ $\triangle PQR$ has $\angle PQR = 60^\circ$ and $\angle PRQ = 70^\circ$ Also we construct



Question:28

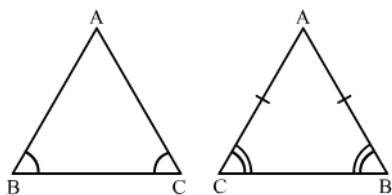
In $\triangle ABC$, it is known that $\angle B = \angle C$. Imagine you have another copy of $\triangle ABC$

i Is $\triangle ABC \cong \triangle ACB$?

ii State the three pairs of matching parts you have used to answer i.

iii Is it true to say that $AB = AC$?

Solution:



i Yes $\triangle ABC \cong \triangle ACB$.

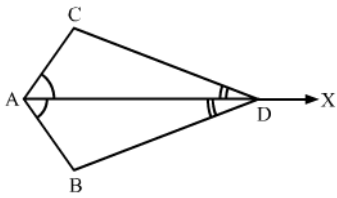
ii We have used $\angle ABC = \angle ACB$ and $\angle ACB = \angle ABC$ again.

Also $BC = CB$

iii Yes, it is true to say that $AB = AC$ since $\angle ABC = \angle ACB$.

Question:29

In Fig. 38, AX bisects $\angle BAC$ as well as $\angle BDC$. State the three facts needed to ensure that $\triangle ABD \cong \triangle ACD$.

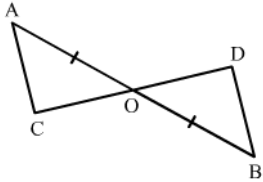


Solution:

As per the given conditions, $\angle CAD = \angle BAD$ and $\angle CDA = \angle BDA$ (because AX bisects $\angle BAC$) $AD = DA$ (common) Therefore, by ASA, $\triangle ACD \cong \triangle ABD$

Question:30

In Fig. 39, $AO = OB$ and $\angle A = \angle B$.



i) Is $\triangle AOC \cong \triangle BOD$?

ii) State the matching pair you have used, which is not given in the question.

iii) Is it true to say that $\angle ACO = \angle BDO$?

Solution:

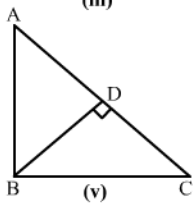
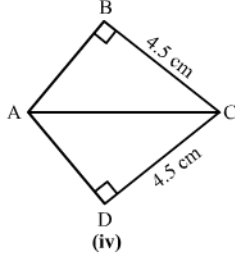
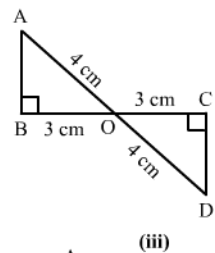
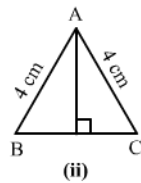
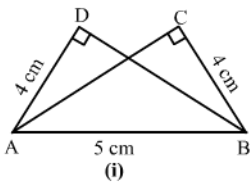
We have

$\angle OAC = \angle OBD$, $AO = OB$ Also, $\angle AOC = \angle BOD$ (Opposite angles on same vertex)

Therefore, by ASA $\triangle AOC \cong \triangle BOD$

Question:31

In each of the following pairs of right triangles, the measures of some parts are indicated along side. State by the application of RHS congruence condition which are congruent. State each result in symbolic form. Fig. 46



Solution:

i) $\angle ADC = \angle BCA = 90^\circ$ $AD = BC$ and hyp $AB = \text{hyp } AB$ Therefore, by RHS, $\triangle ADB \cong \triangle ACB$.

ii)

$AD = AD$ (Common) hyp $AC = \text{hyp } AB$ (Given) $\angle ADB + \angle ADC = 180^\circ$ (Linear pair) $\angle ADB + 90^\circ = 180^\circ$ $\angle ADB = 180^\circ - 90^\circ = 90^\circ$ $\angle ADB = \angle ADC$

iii)

hyp $AO = \text{hyp } DO$ $BO = CO$ $\angle B = \angle C = 90^\circ$ Therefore, by RHS, $\triangle AOB \cong \triangle DOC$

iv)

Hyp $AC = \text{Hyp } CB$ $AB = DC$ $\angle ABC = \angle DCB = 90^\circ$ Therefore, by RHS, $\triangle ABC \cong \triangle DCB$

v)

$BD = BD$ Hyp $AB = \text{Hyp } BC$, as per the given figure. $\angle BDA + \angle BDC = 180^\circ$ $\angle BDA + 90^\circ = 180^\circ$ $\angle BDA = 180^\circ - 90^\circ = 90^\circ$ $\angle BDA = \angle BDC = 90^\circ$

Question:32

$\triangle ABC$ is isosceles with $AB = AC$. AD is the altitude from A on BC .

i Is $\triangle ABD \cong \triangle ACD$?

ii State the pairs of matching parts you have used to answer i.

iii Is it true to say that $BD = DC$?

Solution:

i Yes, $\triangle ABD \cong \triangle ACD$ by RHS congruence condition.

ii We have used Hyp $AB = AC$

$AD = DA$

and $\angle ADB = \angle ADC = 90^\circ$ ($AD \perp BC$ at point D)

iii Yes, it is true to say that $BD = DC$ c. p. c. t since we have already proved that the two triangles are congruent.

Question:33

$\triangle ABC$ is isosceles with $AB = AC$. Also, $AD \perp BC$ meeting BC in D . Are the two triangles ABD and ACD congruent? State in symbolic form. Which congruence condition do you use? Which side of $\triangle ADC$ equals BD ? Which angle of $\triangle ADC$ equals $\angle B$?

Solution:

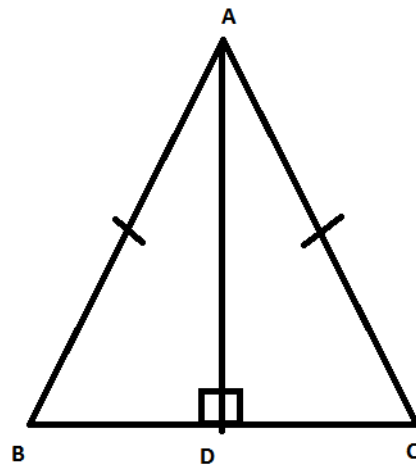
We have $AB = AC$ 1

$AD = DA$ common.....2

and $\angle ADC = \angle ADB$ ($AD \perp BC$ at point D).....3

Therefore from 1, 2 and 3, by RHS congruence condition,

$\triangle ABD \cong \triangle ACD$ Now, the triangles are congruent . Therefore, $BD = CD$. And $\angle ABD = \angle ACD$ (c.p.c.t).

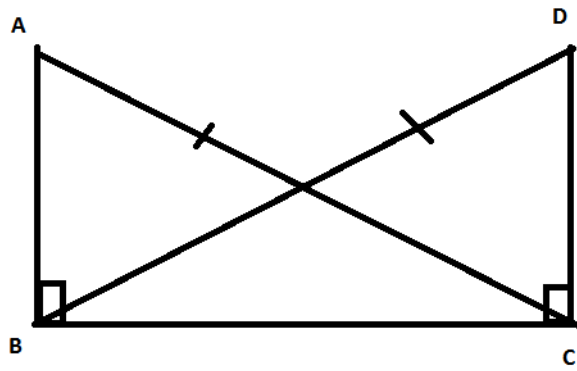
**Question:34**

Draw a right triangle ABC . Use RHS condition to construct another triangle congruent to it.

Solution:

Consider

$\triangle ABC$ with $\angle B$ as right angle. We now construct another right triangle on base BC , such that $\angle C$ is a right angle and $AB = DC$. Also, $BC = CB$. The

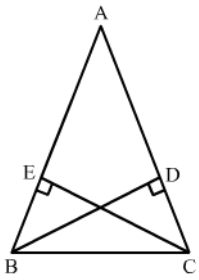


Question:35

In Fig. 47, BD and CE are altitudes of $\triangle ABC$ and $BD = CE$.

i Is $\triangle BCD \cong \triangle CBE$?

ii State the three pairs of matching parts you have used to answer *i*.



Solution:

i Yes, $\triangle BCD \cong \triangle CBE$ by RHS congruence condition.

ii We have used hyp $BC = hyp CB$

$BD = CE$ *given in question*

and $\angle BDC = \angle CEB = 90^\circ$.