

Today's Content

1. Searching Intro
2. Search in sorted array
3. Search floor of x in sorted array
4. Search n^{th} element of x in sorted array

Searching Basics

Bro/Sis missing \rightarrow Policing $\left\{ \begin{array}{l} \text{Details : Whom to search: Target} \\ \text{Location : Where to search: SearchSpace} \end{array} \right.$

Example:

Target \rightleftharpoons SearchSpace

Word $\{ \text{Dict/Books/News Paper} \}$

PhoneNo $\{ \text{Contact/Phonebook} \}$

Obs: If SearchSpace is Ordered, Searching becomes easier

Search Done in Dictionary

[A B C D E F G H I J K L M N O ... X Y Z]
 \uparrow \uparrow \uparrow \uparrow
 1st 2nd 2nd 1st

{ A B C D E F G H I J K } [L M N O ... X Y Z]
 \uparrow \uparrow \uparrow \uparrow
 1st 2nd 2nd 1st

{ A B C D E F } [G H I J K] [L M N O ... X Y Z]
 \uparrow \uparrow \uparrow \uparrow
 1st 2nd 2nd 1st

[A B C] [D E F] [G H I J K] [L M N O ... X Y Z]
 \uparrow \uparrow \uparrow
 1st 2nd 1st

Binary Search: Divide search space into 2 parts & we search into 1 part of search space.

How to where to land?

N

| | |
|-------|-------|
| $N/2$ | $N/2$ |
|-------|-------|

 : Always Discard = $N/2$ Search space

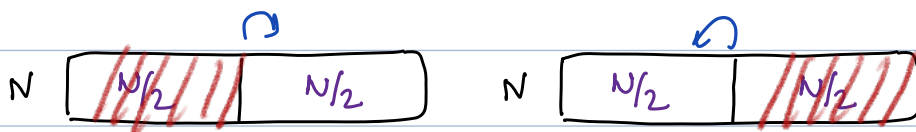
N

| | |
|-------|--------|
| $N/3$ | $2N/3$ |
|-------|--------|

 : Discard left: $N/3$ Discard right: $2N/3$ Note: If we compare with above will always discard $N/2$.

When to apply BS:

After dividing search space into 2 parts, if we can discard 1 half of search space using some conditions, then we can apply binary search



Note:

| | |
|-------|-------|
| $N/2$ | $N/2$ |
|-------|-------|

 :

If we cannot discard on side of search space, we cannot apply BS

Q) Given a sorted arr[n] search if k is present or not?

arr[10] = {
0 1 2 3 4 5 6 7 8 9
3 6 9 12 14 19 20 23 25 27}

$k=12$ Return True;

Idea1: Iterate on arr[] & search if k exists or not
TC: $O(N)$ SC: $O(1)$

Idea2: Target = k Search Space = ln arr[]

$= k$

| | | |
|--|-----|--|
| | mid | |
|--|-----|--|

if (arr[mid] == k) { return True }

$> k$

| | | |
|--|-----|-----------|
| | mid | $> k > k$ |
|--|-----|-----------|

if (arr[mid] > k) { goto left }

$< k$

| | | |
|-----------|-----|--|
| $< k < k$ | mid | |
|-----------|-----|--|

if (arr[m] < k) { goto right }

Tracking Search Space:

We can use s & e or l & h to indicate search space

Ex1: arr[10] =
0 1 2 3 4 5 6 7 8 9
3 6 9 12 14 19 20 23 25 27 $k=20$
 l h m l h m
 1 9 5 6 24

$l \leq h$ $m = (l+h)/2$

$0 \leq 9$ $m = (0+9)/2 = 4$ arr[m] < k: #discard left goto right $l = m+1$;

$5 \leq 9$ $m = (5+9)/2 = 7$ arr[m] > k: #discard right goto left $h = m-1$;

$5 \leq 6$ $m = (5+6)/2 = 5$ arr[m] < k: #discard left goto right $l = m+1$;

$6 \leq 6$ $m = (6+6)/2 = 6$ arr[m] == k: return True;

Ex 1: $arr[10] =$

| | | | | | | | | | |
|---|---|---|----|--------------|--------------|----|-------|----|----|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 3 | 6 | 9 | 12 | 14 | 19 | 20 | 23 | 25 | 27 |
| | | | | $m:1$ h | $m:3$ l | | $m:2$ | | |

$l \leq h$ $m = (l+h)/2$ $k=15$

$0 \leq 9$ $m = (0+9)/2 = 4$ $arr[m] < k$: goto right $l = m+1$;

$5 \leq 9$ $m = (5+9)/2 = 7$ $arr[m] > k$: goto left $h = m-1$;

$5 \leq 6$ $m = (5+6)/2 = 5$ $arr[m] > k$: goto left $h = m-1$; $h = 5-1 = 4$

$5 \leq 4$ $l > h$: Stop process search space is completed. $\#$ return false.

bool search(vector<int> &arr, int k) { $TC: O(\log_2 N)$ $SC: O(1)$

int $l=0$, $h = n-1$;

while($l \leq h$) {

int $m = (l+h)/2$;

if($arr[m] == k$) { return true; }

else if($arr[m] > k$) { $\#$ goto left

{ $h = m-1$;

else { $\# arr[m] < k$ goto right

{ $l = m+1$;

}

} return false;

goto right $l = m+1$;
goto left $h = m-1$;

TC: Initial Search Space: $N \xrightarrow{1} N/2 \xrightarrow{2} N/4 \xrightarrow{3} N/8 \rightarrow \dots \xrightarrow{\log N} 1$

Note: In BS, there are $\log_2 N$ iterations $\# N = \text{Size of Search Space}$

28 Given a sorted arr[], find floor of a given ele k.

Floor: greatest ele $\leq k$ in arr[]

arr[] = { -5, 2, 3, 6, 9, 10, 11, 14, 18 }

k ele

5 : 3

4 : 3

10 : 10

24 : 18

-7 : $-\infty$

Idea: Iterate & calculate floor & return ans.

k=8 ans = $-\infty$ Tc: $O(N)$ Sc: $O(1)$

| i | arr[i] | ans = |
|---|-------------|------------------------------------|
| 0 | -5 ≤ 8 | ans = -5; update & look for better |
| 1 | 2 ≤ 8 | ans = 2; update & look for better |
| 2 | 3 ≤ 8 | ans = 3; update & look for better |
| 3 | 6 ≤ 8 | ans = 6; update & look for better |
| 4 | 9 ≤ 8 | Stop & return ans = 6. |

Idea2:

Target = greatest ele $\leq k$ Search Space = In arr[] ans = $-\infty$;

$\leq k$

mid

if arr[mid] $\leq k$: return arr[mid]

$\leq k$

mid

if arr[mid] $\leq k$:

ans = arr[mid]

goto right $l = m + 1$

$> k$

mid $> k > k$

if arr[mid] $> k$:

goto left $h = m - 1$

arr[] = { 2, 4, 8, 12, 15, 22, 24, 26 }

k = 20

l h m arr[m] $\leq k$ ans = -1;

0 7 3 arr[3] ≤ 20 ans = 12, $l = m + 1$;

4 7 5 arr[5] ≤ 20 $h = m - 1$;

4 4 4 arr[4] ≤ 20 ans = 15; $l = m + 1$;

5 > 4 # stop process return ans = 15.

int floorValue (vector<int> arr, int k) { TC: $O(\log N)$ SC: $O(1)$

int l=0, h=N-1, ans=-∞;

while (l<=h) {

int m = (l+h)/2;

if (arr[m] == k) { return k; }

else if (arr[m] < k) {

ans = arr[m];

l = m+1;

else {

h = m-1;

}

return ans;

TODO: ceil(n): Smallest element $\geq n$

if ceil doesn't exist return -∞

3Q Given an sorted arr(), find the first occurrence index of given ele

arr() = { -5 -5 -3 0 0 1 1 5 5 5 5 5 5 8 10 10 15 }

k: 5

f: 7 l: 12 # c = {f..l} = l - f + 1 = 6.

Idea: Iterate & search for k & return 1st occurrence index.

TC: O(N) SC: O(1)

20: -1

Target = 1st occurrence index: k SearchSpace = ln arr() ans = -1;

arr() < k

mid

if arr[mid] < k: goto right

arr() > k

mid

if arr[mid] > k: goto left

arr() k k k k k

mid

if arr[mid] == k:

ans = mid;

goto left;

Example

arr() = { 2 5 5 5 5 5 8 10 }

k = 5

l = 1 h = 6

ans = -1;

l h m

0 7 3 arr[m] == k: ans = 3; h = m - 1

0 2 1 arr[m] == k: ans = 1, h = m - 1;

0 0 0 arr[m] < k: l = m + 1

1 0 # stop & return ans = 1.

TODO: 1st occurrence code

TODO: last occurrence code & idea

TODO: In sorted arr() find frequency of k:

f = first occurrence of k

l = last occurrence of k

c = l - f + 1;

int firstOccurrence(vector<int> &arr, int k) {