Todays Content

1. Count of subsets with sum = k
2. Calculate & return count of Quadruplets with sum = k

Given ar[N] & k calculate &

Check if there exists a subset with sum = k

Constraints

$1 <= N <= 40$ $\qquad$ # $2^{10} = 1024 \approx 10^3$,

$1 <= ar[i] <= 10^7$ $\qquad$ # $(2^{10})^4 = (10^3)^4 = 10^{12}$

$1 <= k <= 10^9$

	0	1	2	3	4	5	6	
Ex: ar[] = {	3	7	2	4	2	5	4	} k = 10

Idea1: Generate all subsets:

For every subset, iterate & calculate sum == k

TC: $O(2^N * N) = O(N * 2^N)$ $\quad$ SC: $O(1)$

$N <= 40$, $N = 40$ $\quad 2^{40} * 40 >> 10^8$ TLE.

Idea2:

	0	1	2	3	4	5	
ar[] = {	3	7	4	9	2	5	} k = 18 $\quad 2^6$ Subsets

a[] = {3 7 4} : 2^3	b[] = {9 2 5}
{ }	{ }
{3}	{9}
{7}	{2}
{4}	{5}
{3 7}	{9 2}
{7 4}	{2 5}
{3 4}	{9 5}
{3 7 4}	{9 2 5}

#Obs1: If we combine 1 subset in a[] with 1 subset in b[] can get subset in c[].

#Obs2: Can get all subsets in c, if we combine 1 subset in a[] with 1 subset in b[]

0	1	2	3	4	5

arr[N] = {3 7 4 || 9 2 5} k = 18 2^6 Subsets

k = 15

a[N/2] = {3 7 4} : 2^3

b[N/2] = {9 2 5}

$2^{N/2}$ x

subset of a	sum
{ }	0
{3}	3
{7}	7
{4}	4
{3 7}	10
{7 4}	11
{3 4}	7
{3 7 4}	14

sum1[] = [0 3 4 7 7 ...]

sum2[] = [0 2 5 7 9 ...]

sum	subset of b
0	{ }
9	{9}
2	{2}
5	{5}
11	{9 2}
7	{2 5}
14	{9 5}
16	{9 2 5}

$2^{N/2}$ y

#Q: Check if there exists 2 elements x, y such that x + y = k

x is subset sums of a[] & y is subset sum of b[]

Idea1: Generate all pairs of x & y.

TC: $O(2^N)$

Idea2: Store all subset sums of b[] in array sums[]

Sort sums[]

Take each subset sum[] of a[] as x.

For every x, $y = k - x$, search in sorted array sums[]

TC: $O(2^{N/2} + N/2 * 2^{N/2} + 2^{N/2} * N/2)$ SC: $O(2^{N/2})$

TC: $O(N * 2^{N/2})$ SC: $O(2^{N/2})$

Idea3: Store all subset sums of a[] in suma[] & sort it

Store all subset sums of b[] in sumb[] & sort it

Given 2 sorted array, check if there exists a pair with sum = K.

↳ 2 pointer for pair sum.

TC: $O(N/2 * 2^{N/2} + N/2 * 2^{N/2} + 2 * 2^{N/2})$

TC: $O(N * 2^{N/2})$ SC = $O(2 * 2^{N/2}) = O(2^{N/2})$

Idea4: Store all subset sums of b[] in Hashset hs.

Take each subset sum[] of a[] as x.

For every x, $y = k - x$, search in hashset hs.

TC: $O(2^{N/2} * 1 + 2^{N/2} * 1)$ SC: $O(2^{N/2})$

TC: $O(2^{N/2})$ SC: $O(2^{N/2})$

#Note: Talk about when to apply meet in the middle

28

Given A[ ], B[ ], C[ ], D[ ] no: of quadraplets (i, j, k, l) such that

A[i] + B[j] + C[k] + D[l] = S

0 <= i, j, k, l < N;

k=15

	0	1	2	3	4	5
A[ ] :	3	4	2	7	3	9
B[ ] :	6	3	4	2	6	4
C[ ] :	2	7	6	4	3	5
D[ ] :	6	4	3	8	9	4

AB #N^2

CD # N^2

#N^4