

Today's Content:

1. Issues with Iterative to compare
2. Asymptotic Analysis
 - a. Big O
 - b. Theta
 - c. Omega
3. How to calculate Big O
4. Issues with Big O
5. Why TLE occurs?
6. Idea from constraints

Con: We are iterating two compare 2 Algo's

Q1: Sort $arr[N]$ input elements

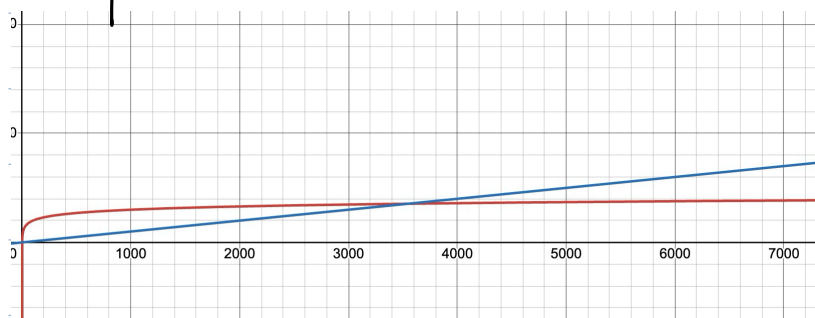
S_1 : DSort

S_2 : MSort

Code1: $100 \log_2 N$

Code2: $N/\log N$

Graph:



obs1:

$N < 3500$ DSort more iterative : MSort is efficient

$N > 3500$ MSort more iterative : DSort is efficient

Con: For input size increases: Prefer DSort.

Issue: Comparing with iterative is hard, to resolve it

Asymptotic Analysis: Tracking performance of algorithm for large inputs

1. Big O 2. Θ Theta 3. Ω Omega.

Steps to Calculate Big O : What? Why? How?

1. Calculate iterations of program based on input ~
2. Neglect lower order terms or Consider only highest order term
3. Neglect constant coefficients.

According to Big O

Code 1: $100 \log_2 N \rightarrow O(\log_2 N)$ } $\log_2 N$ is much more efficient.
 Code 2: $N/10 \rightarrow O(N)$

Order of Terms

$\log_2 N < \sqrt{N} < N < N \log N < N\sqrt{N} < N^2 < N^3 < \underline{2^N} < N! < N^N$

$N=64$

$\log_2^{64} 6$ $\sqrt{64}$ 64 $64 \log_2^{64} 64 \times 6$ $64 \sqrt{64}$ 64^2 64^3 2^{64} $\underline{64!}$ 64^{64}

	Highest Order	Big O
Q1 $F(N) = 4N + 3N \log N + 1$	$3N \log N$	$O(N \log N)$
Q2 $F(N) = 4N \log N + 3N\sqrt{N} + 10^3$	$3N\sqrt{N}$	$O(N\sqrt{N})$
Q3 $F(N) = 4N^2 + 5N \log N + 10^2$	$4N^2$	$O(N^2)$

Why consider only higher order terms?

$N \rightarrow$ Input

Q. Say $f(n) = N^2 + 10N$: Iterations

Input Size:

Total iterations

% lower order terms

iterations in Total

$$N: 10 \rightarrow N^2 + 10N = 10^2 + 10 \times 10 = 200$$

$$\frac{100}{200} \times 100\% = 50\%$$

$$N: 10^2 \rightarrow N^2 + 10N = (10^2)^2 + 10 \times 10^2 = 10^4 + 10^3$$

$$\frac{10^3}{10^4 + 10^3} \times 100\% \approx 10\%$$

$$N: 10^4 \rightarrow N^2 + 10N = (10^4)^2 + 10 \times 10^4 = 10^8 + 10^5$$

$$\frac{10^5}{10^8 + 10^5} \times 100\% \approx 0.1\%$$

obs: if we inc input \uparrow , contribution of lower order term decreases, hence it won't effect comparison, so we can neglect it.

Neglect Constant Coefficient

Algo1

Algo2

for larger inputs

efficient algo

Q1

$10 \log_2 N$

N

Algo1

Q2

$100 \log_2 N$

N

Algo1

Q3

$10^3 \log_2 N$

$N/10$

Algo1

Q4

$10N$

$N^2/10$

Algo1

obs2: Constant coefficient won't effect your comparison hence we neglect them.

Issue 1: Q: Algo1 Algo2

Iterations: $10N$ N^2

$N=10$:	100	100	} Con: Algo1 is better than Algo2 $N \geq 10$
$N=11$:	10×11	11×11	
$N=12$:	10×12	12×12	

BigO: $O(N)$ $O(N^2)$

Con: With BigO we can only claim Algo1 is better than Algo2 for large inputs

Issue: If 2 BigO notations are same go back to iterations.

Q: Algo1 Algo2

Iterations: $\underline{10N^2 + N}$	$\underline{3N^2 + 10N}$	} Algo2 is better.
BigO: $O(N^2)$	$O(N^2)$	

Note: For BigO: We consider worst case scenario for calculating iterations.

Q: Search k in arr[]

```
bool search(int arr[], int N, int k) {
```

```
    for(int i=0; i<N; i++) {
        if(arr[i] == k) { return true; }
    }
    return false;
}
```

	Best	Worst
Iterations =	1	N .
BigO =	$O(N)$	

Note2:

Q: Given an arr[] return max of first 4 elements of array.

```
int max4(int arr[], int N) { // N > 4
```

```
    int max = INT_MIN;
    for(int i=0; i<4; i++) {
        if(arr[i] > max) {
            max = arr[i];
        }
    }
```

```
    return max;
}
```

Iterations = 4, \rightarrow constant iterations, Independent of input
BigO = $O(1)$ \rightarrow constant iterations

TLE: Time limit Exceeded

Online Editor:

In general $\text{time limit} = 1 \text{ sec}$

processing speed = $1 \text{ GHz} = 10^9 \text{ instructions/sec}$

```
int countfactor(int n) {
```

```
    int c = 0; +1
```

```
    for( int i = 1; i <= n; i++) { +1 +1 +1
```

```
        if( n % i == 0 ) { +1 +1
```

```
            c++; +1
```

```
    } +1
```

```
    return c; +1
```

Total Instructions = $5N + 3$

1 iteration ≈ 5 instructions

Assumption:

1 iteration ≈ 10 instructions.

Online editor = $10^9 \text{ instructions/sec}$

$= 10^8 * 10 \text{ instructions/sec}$

Online editor = $10^8 \text{ iteration/sec}$