

Today's Content

1. Searching $\Theta(n)$
2. Search in sorted array
3. Search floor of x in sorted array
4. Search i^{th} occurrence of x in sorted array

Searching Basics

Bro/Sis missing → Polin staining

- Details : whom to search: Target
- location : where to search: Search Space

Example:

Target \iff SearchSpan

Word { Dict / Books / News Paper }

PhoneNo {Contact/Phonebook}

Obs: If SearchSpace is Ordered, Searching becomes easier

Search Dog in Dictionary

[A B C D E F G H I J K L M N O ... X Y Z]

$$\begin{array}{c} \uparrow \\ y_1 \\ \downarrow \\ y_2 \end{array} \quad \begin{array}{c} \uparrow \\ 24 \\ \downarrow \end{array} \quad \begin{array}{c} \uparrow \\ 117 \\ \downarrow \end{array}$$

{ A B C D E F G H I J K } [L M N O ... X Y Z]

$$\begin{array}{c} \text{P} \\ \text{y} \\ \text{y} \\ \text{y} \end{array} \quad \begin{array}{c} \text{P} \\ \text{y} \\ \text{y} \\ \text{y} \end{array} \quad \begin{array}{c} \text{P} \\ \text{y} \\ \text{y} \\ \text{y} \end{array} \quad \begin{array}{c} \text{P} \\ \text{y} \\ \text{y} \\ \text{y} \end{array}$$

{ A B C D E F } G H I J K L M N O ... X Y Z]

$$\begin{array}{c} \uparrow \\ \gamma \\ \downarrow \end{array} \quad \begin{array}{c} \uparrow \\ \gamma \\ \downarrow \end{array} \quad \begin{array}{c} \uparrow \\ 2 \\ \downarrow \end{array} \quad \begin{array}{c} \uparrow \\ 1 \\ \downarrow \end{array}$$

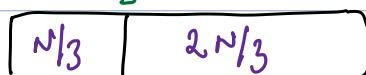
A B C { D E F } G H I J K L M N O ... X Y Z

$$\begin{array}{c} \uparrow \\ y^2 \\ \downarrow \end{array}$$

Binary Search: Divide search space into 2 parts & we search into 1 part of search space.

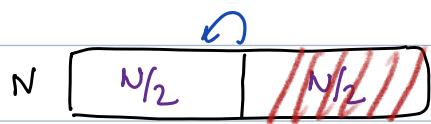
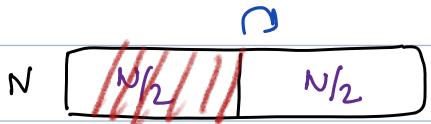
How to where to land?

N  : Always Discard = $N/2$ Search Space

N  : Discard left: $N/3$ Note: If we compare w.r.t above will always
Discard right: $2N/3$ Discard $N/3$.

When to apply BS:

After dividing search space into 2 parts, if we can discard 1 half of searchspace using some conditions, then we can apply binary search



Note:  :

If we cannot discard m side of searchspace, we cannot apply BS

Q) Given a sorted arr[N] search if k is present or not?

$$arr[10] = \{ 3, 6, 9, 12, 14, 19, 20, 23, 25, 27 \}$$

$k = 12$ Return True;

Idea1: Iterate on arr[] & search if k exists or not

TC: $O(N)$ SC: $O(1)$

Idea2: Target = k Search Space = In arr[]

$:= k$



if ($arr[mid] == k$) { return True }

$> k$



if ($arr[mid] > k$) { go to left }

$< k$



if ($arr[m] < k$) { go to right }

Tracking Search Space:

We can use l & h to indicate search space

Ex1: $arr[10] = \{ 3, 6, 9, 12, 14, 19, 20, 23, 25, 27 \}$ $k = 20$

$l_1 = h \quad m : (l+h)/2$

$0 \leq 9 \quad m = (0+9)/2 = 4 \quad arr[m] < k : \# \text{discard left go to right } l=m+1;$

$5 \leq 9 \quad m = (5+9)/2 = 7 \quad arr[m] > k : \# \text{discard right go to left } h=m-1;$

$5 \leq 6 \quad m = (5+6)/2 = 5 \quad arr[m] < k : \# \text{discard left go to right } l=m+1;$

$6 \leq 6 \quad m = (6+6)/2 = 6 \quad arr[m] == k : \text{return True};$

0	1	2	3	4	5	6	7	8	9
3	6	9	12	14	19	20	23	25	27
$l = h$	$m = \lfloor \frac{l+h}{2} \rfloor$	$k = 15$	$m:1$	$m:3$	l	$m:2$			

$$l = h \quad m : \lfloor (l+h)/2 \rfloor \quad k = 15$$

0 $\leftarrow 9 \quad m = (0+9)/2 = 4 \quad ar[m] > k : \text{go to right} \quad l = m+1;$

5 $\leftarrow 9 \quad m = (5+9)/2 = 7 \quad ar[m] > k : \text{go to left} \quad h = m-1;$

5 $\leftarrow 6 \quad m = (5+6)/2 = 5 \quad ar[m] > k : \text{go to left} \quad h = m-1; \quad h = 5-1 = 4$

5 $\leftarrow 4 \quad l > h : \text{Stop proum search span is completed.} \quad \# \text{return false.}$

bool search(vector<int> &ar, int k) { TC: $O(\log_2 N)$ SC: $O(1)$

int l=0, h=N-1;

while(l <= h) {

 int m = (l+h)/2;

 if(ar[m] == k) { return true; }

 else if(ar[m] > k) { #got left

 h = m-1;

 else { #ar[m] < k go to right

 l = m+1;

 go to right $l = m+1;$
 go to left $h = m-1;$

}

return false;

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TC: Initial Search Spec: $N \xrightarrow{1} N/2 \xrightarrow{2} N/4 \xrightarrow{3} N/8 \rightarrow \dots \xrightarrow{\log_2 N}$;

Note: In BS, there are $\log_2 N$ iterations # $N = \text{Size of Search Spec}$

2Q Given a sorted arr[], find floor of a given ele k.
Floor: greatest ele $i = k$ in arr[]

$$arr[9] = \{ -5, 2, 3, 6, 9, 10, 11, 14, 18 \}$$

k ele

5 : 3

4 : 3

10 : 10

24 : 18

-7 : -∞

Idea: Iterate & calculate floor & return ans.

$k=8$ ans = -∞ TC: O(N) SC: O(1)

i	arr[i]	ans =
0	-5 $\leftarrow 8$	ans = -5 update & look for better
1	2 $\leftarrow 8$	ans = 2 \leftarrow update & look for better
2	3 $\leftarrow 8$	ans = 3 \leftarrow update & look for better
3	6 $\leftarrow 8$	ans = 6 \leftarrow update & look for better
4	9 $\leftarrow 8$	Stop & return ans = 6.

Idea2:

Target = greatest ele $i = k$ Search Space = In arr[] ans = -∞;

$\approx k$

	mid	
--	-----	--

if $arr[mid] == k$: return $arr[mid]$

$\approx k$	$\approx k$	mid	
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if $arr[mid] < k$:

ans = $arr[mid]$

goto right $l = m + 1$

0	1	2	3	4	5	6	7
2	4	8	12	15	22	24	26

$k = 20$

$l = h = m$ $arr[m] \approx k$ ans = -1;

0 7 3 $arr[3] \approx 20$ ans = 12, $l = m + 1$;

4 7 5 $arr[5] \approx 20$ $h = m - 1$;

4 7 4 $arr[4] \approx 20$ ans = 15; $l = m + 1$;

5 7 4 #stop prun return ans = 15.

if $arr[mid] > k$:

goto left $h = m - 1$;

	mid	$\approx k$	$\approx k$
--	-----	-------------	-------------

int floorValue(vector<int> arr, int k) { TC: O(log₂N) SC: O(1)

int l=0, h=N-1, ans = -∞;

while(l <= h) {

int m = (l+h)/2;

if(arr[m] == k) { return k; }

else if(arr[m] < k) {

ans = arr[m];

l = m+1;

} else { # arr[m] > k

h = m-1;

}

return ans;

TODO: ceil(n): Smallest element $\geq n$

If ceil doesn't exist return +∞

38 Given an sorted arr[], find the first occurrence index of given ele

$$arr[] = \{ -5, -5, -3, 0, 0, 1, 1, 7, 7, 9, 10, 11, 12, 13, 14, 15, 16 \}$$

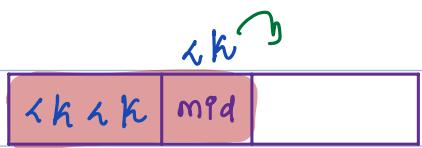
$\underbrace{_f}^f \quad \underbrace{_l}^l$ # $c = \{ f..l \} = l - f + 1 = 6.$

$f = f$ Ideal: Iterate q search for k q return ith occurrence index.

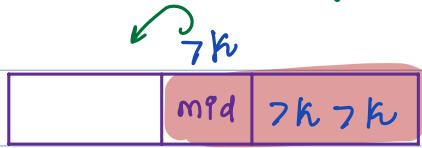
$-5 = 0 \quad TC: O(N) \quad SC: O(1)$

$20 = -1$

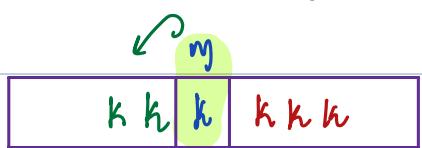
Target = 1st occurrence index: k Search Space = In arr[] ans = -1;



if $arr[\text{mid}] < k$: goto right



if $arr[\text{mid}] = k$: goto left



if $arr[\text{mid}] = k$:

ans = m ;

goto left;

Example $0, 1, 2, 3, 4, 5, 6, 7$
In: $arr[] = \{ 2, \underbrace{5, 5, 5, 5, 5, 8, 10 \}_{m} \}$
 $k = 5 \quad l = 6$

ans = -1;

$l \quad b \quad m$

$0 \quad 7 \quad 3 \quad arr[m] == k : \text{ans} = 3; \quad b = m - 1$

$0 \quad 2 \quad 1 \quad arr[m] == k : \text{ans} = 1; \quad b = m - 1$

$0 \quad 0 \quad 0 \quad arr[m] < k : \quad l = m + 1$

$1 \quad 0 \quad \# \text{Stop q return ans} = 1.$

TODO: 1st occurrence code

TODO: last occurrence code q idea

TODO: In sorted arr[] find frequency of k :

$f = \text{first occurrence of } k$

$l = \text{last occurrence of } k$

$c = l - f + 1$

int firstOccurrence(const string& s, int k) {