

Today's Content

1. Count Set Bits
2. Set n y^{th} bit
3. Continuous n set bits y unset bits
4. Unset i^{th} Bit

Revision:

```
int Set(int N, int i) { Tc: 061) Sl: 061)
```

```
    N = N | (1 << i)    # Set  $i^{\text{th}}$  bit in N
```

```
    return N;
```

3

```
boolean checkBit(int N, int i) { Tc: 061) Sl: 061)
```

```
    return (N >> i) & 1 == 1;
```

3

Count Set Bits:

Given N , return no. of set bits in N .

Ex1: $2^5 \ 2^4 \ 2^3 \ 2^2 \ 2^1 \ 2^0$

$N=21$ 0 1 0 1 0 1

return 3

$2^5 \ 2^4 \ 2^3 \ 2^2 \ 2^1 \ 2^0$

$N=45$ 1 0 1 1 0 1

return 4

Idea1: Iterate on all bits from 0..31 & if bit is set = $c++$

int $c=0$;

for (int $i=0$; $i<32$; $i++$) { # iterations = 32 $TC=O(1)$

if ($(N \gg i) \& 1 == 1$) {

$c++$;

}
return c ;

Idea2:

$2^5 \ 2^4 \ 2^3 \ 2^2 \ 2^1 \ 2^0$ $N=1$ $c=0$

$N=45$ 1 0 1 1 0 1 ✓ $c=1$ ✓

$N \gg 1$ 1 0 1 1 0 * ✓

$N \gg 1$ 1 0 1 1 ✓ $c=2$ ✓

1 0 1 ✓ $c=3$ ✓

1 0 * ✓

1 ✓ $c=4$

0 Stop return $c=4$.

int countSet(int N) {

int $c=0$;

while ($N > 0$) {

 if ($(N \& 1) == 1$) {

$c++$;

$N = N \gg 1$;

}

return c ;

Issue:

$N=10$;

print($N \gg 1$) ; 20

print(N) ; 10

$N=10$;

print($N \gg 1$) 5

print(N) 10

We need to use assignment operators to update value N .

3rd Approach:

N	N-1	N & (N-1)
$2^6 \ 2^5 \ 2^4 \ 2^3 \ 2^2 \ 2^1 \ 2^0$ $N=49$ <u>0</u> <u>1</u> <u>1</u> <u>0</u> <u>0</u> <u>0</u> <u>*</u>	$2^6 \ 2^5 \ 2^4 \ 2^3 \ 2^2 \ 2^1 \ 2^0$ $N=48$ <u>0</u> <u>1</u> <u>1</u> <u>0</u> <u>0</u> <u>0</u> <u>0</u>	$2^6 \ 2^5 \ 2^4 \ 2^3 \ 2^2 \ 2^1 \ 2^0$ <u>0</u> <u>1</u> <u>1</u> <u>0</u> <u>0</u> <u>0</u> <u>0</u>
$N=24$ <u>0</u> <u>0</u> <u>1</u> <u>*</u> <u>0</u> <u>0</u> <u>0</u>	$N=23$ <u>0</u> <u>0</u> <u>1</u> <u>0</u> <u>1</u> <u>1</u> <u>1</u>	<u>0</u> <u>0</u> <u>1</u> <u>0</u> <u>0</u> <u>0</u> <u>0</u>
$N=8$ <u>0</u> <u>0</u> <u>0</u> <u>*</u> <u>0</u> <u>0</u> <u>0</u>	$N=7$ <u>0</u> <u>0</u> <u>0</u> <u>0</u> <u>1</u> <u>1</u> <u>1</u>	<u>0</u> <u>0</u> <u>0</u> <u>0</u> <u>0</u> <u>0</u> <u>0</u>
$N=52$ <u>0</u> <u>1</u> <u>1</u> <u>0</u> <u>*</u> <u>0</u> <u>0</u>	$N=51$ <u>0</u> <u>1</u> <u>1</u> <u>0</u> <u>0</u> <u>1</u> <u>1</u>	<u>0</u> <u>1</u> <u>1</u> <u>0</u> <u>0</u> <u>0</u> <u>0</u>
$N=42$ <u>0</u> <u>1</u> <u>0</u> <u>1</u> <u>0</u> <u>*</u> <u>0</u>	$N=41$ <u>0</u> <u>1</u> <u>0</u> <u>1</u> <u>0</u> <u>0</u> <u>1</u>	<u>0</u> <u>1</u> <u>0</u> <u>1</u> <u>0</u> <u>0</u> <u>0</u>

#obs: $N \& (N-1)$: Will unset right most set bit.

int c=0; # iterations = No. of set bits

while(N>0)

$N = N \& (N-1)$; # In each iteration right most bit becomes unset

c++;

}

return c;

$2^5 \ 2^4 \ 2^3 \ 2^2 \ 2^1 \ 2^0$
 $N=45$
1 0 1 1 0 1
 $N = N \& (N-1)$
1 0 1 1 0 0
c++;
 $N = N \& (N-1)$
1 0 1 0 0 0
c++;
 $N = N \& (N-1)$
1 0 0 0 0 0
c++;
 $N = N \& (N-1)$
0 0 0 0 0 0
c++;
 $N==0$; return c=4;

Q8 Given n & y : Set n^{th} & y^{th} Bit in 0.

Constraints

$$0 \leq n, y \leq 30$$

Ex1:

$$n=3 \quad y=5 \quad \text{ans}=40$$

2^6	2^5	2^4	2^3	2^2	2^1	2^0
0	1	0	1	0	0	0

Ex2:

$$n=2 \quad y=4 \quad \text{ans}=20$$

2^6	2^5	2^4	2^3	2^2	2^1	2^0
0	0	1	0	1	0	0

Ex3:

$$n=3 \quad y=1 \quad \text{ans}=10$$

2^6	2^5	2^4	2^3	2^2	2^1	2^0
0	0	0	1	0	1	0

int setBits(int n, int y) {

return (1 < n) | (1 < y) ✓

return (1 < n) ^ (1 < y) # if $n=y$ we get zero

}

3Q Given n, y set consecutive n bits & y unset bits.

Ex:

$$n=3 \quad y=2 \quad \text{ans}=28$$

$$n=4 \quad y=3 \quad \text{ans}=120$$

$$n=2 \quad y=5 \quad \text{ans}=96$$

$$2^6 \ 2^5 \ 2^4 \ 2^3 \ 2^2 \ 2^1 \ 2^0$$

$$2^6 \ 2^5 \ 2^4 \ 2^3 \ 2^2 \ 2^1 \ 2^0$$

$$2^6 \ 2^5 \ 2^4 \ 2^3 \ 2^2 \ 2^1 \ 2^0$$

$$\underline{0 \ 0 \ 1 \ 1 \ 1 \ 0 \ 0}$$

$$\underline{1 \ 1 \ 1 \ 1 \ 0 \ 0 \ 0}$$

$$\underline{1 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0}$$

Constraints $0 \leq n+y \leq 30$

Ideas

1. Solving with loops: TODO

2. Without loops

Hint: $2^0 + 2^1 + 2^2 + \dots + 2^N = 2^{N+1} - 1$; $\# \ 2^0 + 2^1 + 2^2 + 2^3 = 2^4 - 1 = 15$

Ex 1:

1. $n=3 \quad y=2$

$$2^6 \ 2^5 \ 2^4 \ 2^3 \ 2^2 \ 2^1 \ 2^0$$

$$\underline{1 \ 1 \ 1} = 2^0 + 2^1 + 2^2 = 2^3 - 1$$

$$(2^3 - 1) \ll 2$$

$$\underline{1 \ 1 \ 1 \ 0 \ 0}$$

2. $n=4 \quad y=3$

$$2^6 \ 2^5 \ 2^4 \ 2^3 \ 2^2 \ 2^1 \ 2^0$$

$$\underline{1 \ 1 \ 1 \ 1} = 2^0 + 2^1 + 2^2 + 2^3 = 2^4 - 1$$

$$(2^4 - 1) \ll 3$$

$$\underline{1 \ 1 \ 1 \ 1 \ 0 \ 0 \ 0}$$

3. $n=2 \quad y=5$

$$2^6 \ 2^5 \ 2^4 \ 2^3 \ 2^2 \ 2^1 \ 2^0$$

$$\underline{1 \ 1} = 2^0 + 2^1 = 2^2 - 1$$

$$(2^2 - 1) \ll 5$$

$$\underline{1 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0}$$

$$\# a \ll n = a * 2^n$$

Given n, y : $(2^n - 1) \ll y \rightarrow (1 \ll n - 1) \ll y$

$$a \ll y \rightarrow a * 2^y = (2^n - 1) * 2^y = 2^{n+y} - 2^y =$$

$$= 1 \ll (n+y) - 1 \ll y$$

Q8: Given N & i: #if i^{th} Bit in N is already Unset: Leave it
 #if i^{th} Bit in N is Set: Unset

Ex1:

N=45 i=3

	2^6	2^5	2^4	2^3	2^2	2^1	2^0
Unset i=3	0	1	0	1	1	0	1
Ans=37	0	1	0	0	1	0	1

N=57 i=3

	2^6	2^5	2^4	2^3	2^2	2^1	2^0
Unset i=3	0	1	1	1	0	0	1
Ans=49	0	1	1	0	0	0	1

#Ideas: $1 \oplus 0 = 1$ $0 \oplus 0 = 0$

N=45 i=3

	2^6	2^5	2^4	2^3	2^2	2^1	2^0
Unset i=3	0	1	0	1	1	0	1
(1<4<3)	0	0	0	1	0	0	0
$\sim(1<4<3)$	1	1	1	0	1	1	1
$N \& \sim(1<4<3)$	0	1	0	0	1	0	1

N=57 i=4

	2^6	2^5	2^4	2^3	2^2	2^1	2^0
Unset i=3	0	1	1	1	0	0	1
(1<4<4)	0	0	1	0	0	0	0
$\sim(1<4<4)$	1	1	0	1	1	1	1
$N \& \sim(1<4<4)$	0	1	0	1	0	0	1

```
int Unset(int N, int i) {
```

```
    N = N & ~ (1 << i)
```

```
    return N;
```

```
}
```