Todays Content

1. Addition & Multiplication Rule
2. Permutation Basics
3. Combination Basics
4. Pascal Triangle

Addition Rule: OR +

Ex1: Given 4 boys & 3 girls how many ways we can pick a single person

7 : Boy OR Girl :

$4 + 3 = 7$

Multiplication Rule: AND

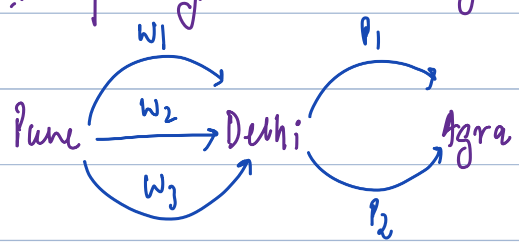
Ex1: Given 4 boys & 3 girls how many different pairs can be informed

Boys:	Girls
B_1	G_1
B_2	G_2
B_3	G_3
B_4	

Boy and Girl

$4 * 3 = 12$

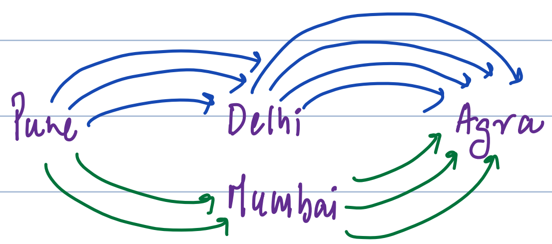
Ex2: No: of ways to reach Agra from Pune via Delhi



Pune → Delhi & # Delhi → Agra

$3 * 2 = 6$ ways

Q: Ways to reach Pune to Agra



Pune → Agra via Delhi or # Pune → Agra via Mumbai

Pune → Delhi & Delhi → Agra + Pune → Mumbai & Mumbai → Agra

$3 * 4 + 2 * 3$

$12 + 6 = 18$

Permutation: #ways to arrange {order matter}
Pair(i, j) (j, i) : Different

#ways to arrange: P_1 P_2 P_3

P_1	P_2	P_3
P_1	P_3	P_2
P_2	P_1	P_3
P_2	P_3	P_1
P_3	P_1	P_2
P_3	P_2	P_1

$\rightarrow$

$\cancel{P_1}$ $\cancel{P_2}$ P_3

$\underline{3} * \underline{2} * \underline{1} = 6$ ways

Keep: person in 1^{st} pos and
Keep: person in 2^{nd} pos and
Keep: person in 3 pos

#ways to arrange: 4 people $\longrightarrow$

P_1	P_2	P_3	P_4	=	6
P_2	P_1	P_3	P_4	=	6
P_3	P_1	P_2	P_4	=	6
P_4	P_1	P_2	P_3	=	6
					24

$\underline{4} * \underline{3} * \underline{2} * \underline{1} = 4! = 24$ ways

Q2: Ways to arrange: N people

$\underline{N} * \underline{N-1} * \underline{N-2} * \ldots\ldots * \underline{1} = N!$

#Ways to arrange 2 from 4 people: $\underline{4} * \underline{3} = 12$

P_1 P_2	P_1 P_3	P_1 P_4	P_2 P_1
P_2 P_3	P_2 P_4	P_3 P_1	P_3 P_2
P_3 P_4	P_4 P_1	P_4 P_2	P_4 P_3

#Way to arrange 3 from 5 people:

$\underline{5} * \underline{4} * \underline{3} = 60$ ways.

$^{N}P_{R}$: Permutation : (Arrangement)

\# ways to arrange r from N people.

$$\underset{1^{st}}{\underline{N}} * \underset{2^{nd}}{\underline{N-1}} * \underset{3^{rd}}{\underline{N-2}} * \underset{4^{th}}{\underline{N-3}} * \underset{5^{th}}{\underline{N-4}} \ldots\ldots \underset{r^{th}}{\underline{N-r+1}}$$

$$\text{ways} = \frac{N * N-1 * N-2 * N-3 * \ldots N-r+1 \quad N-r * N-r-1 * N-r-2 * \ldots)}{N-r * N-r-1 * N-r-2 * \ldots)}$$

$$\boxed{\begin{array}{l|l} ^{N}P_{R} = \dfrac{N!}{(N-R)!} & \text{\# ways to arrange :} \\ R <= N. & \text{P: Permutation / Arrangement / order Yes} \end{array}}$$

Permutation: Arrangement : Order

Combination: Selection : Order doesn't matter

Combination: #ways to select {order won't matter}

{(i,j) → (j,i)} Both are same

Say 4 people how many ways we can select 2 people.

$P_1 \quad P_2 \quad P_3 \quad P_4$

P_1	P_2	P_2	P_3
P_1	P_3	P_2	P_4
P_1	P_4	P_3	P_4

Order doesn't matter only selection matters.

Say 4 people how many ways we can select 3 people.

$P_1 \quad P_2 \quad P_3 \quad P_4$:

$P_1\ P_2\ P_3$	$P_1\ P_2\ P_4$	$P_1\ P_3\ P_4$	$P_2\ P_3\ P_4$
_ _ _			
_ _ _			
_ _ _			
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_ _ _			

Corelation:

	Arangement	Selection
N=4, arrange 3	3!	1
$^{N}P_{R} = \frac{4!}{1!}$	4!	n = 4

$$n * 3! = 4!$$

$$n = \frac{4!}{3!} = \frac{24}{6} = 4$$

^NC_R : Number of ways to select {order*} r from N.

	Arrangement	Selection
:	$r!$	1
Arrange r from N :	^NP_r	x

$$x * r! = {}^NP_r$$

$$x = \frac{^NP_r}{r!} = \frac{N!}{(N-r)! * r!}$$

$$\boxed{^NC_R : \{\text{\#ways to select } R \text{ from } N\} = \frac{N!}{(N-R)! * R!}}$$

C: Combination/Selection/Order Not

Properties:

1. #ways to select 0 from N : $^NC_0 = \frac{N!}{(N-0)! * 0!} = \frac{N!}{N!} = 1$

2. #ways to select N from N : $^NC_N = \frac{N!}{(N-N)! * N!} = \frac{\cancel{N!}}{0! * \cancel{N!}} = 1.$

3. #ways to select N-R from N :

$$^NC_R = {}^NC_{N-R}$$

Special Property:

\# ways to select R from N P_1 _ P_2 _ P_3 _ .... P_{N-1} _ P_N _ : N people.

$$\boxed{{}^{N}C_{R} = {}^{N-1}C_{R-1} + {}^{N-1}C_{R}}$$

Ex: N=6 R=4

$${}^{6}C_{4} = {}^{5}C_{3} + {}^{5}C_{4}$$

$$15 = 10 + 5$$

obs:

$$N! = N * (N-1)!$$

$$R! = R * (R-1)!$$

$$(N-R)! = (N-R)(N-R-1)!$$

Maths:

$$\frac{N!}{(N-R)! * R!} = \frac{(N-1)!}{(N-R-1)! * R!} + \frac{(N-1)!}{(N-R)!\,(R-1)!}$$

$$= \frac{(N-1)!}{(N-R-1)!\; R * (R-1)!} + \frac{(N-1)!}{(N-R)(N-R-1)! * (R-1)!}$$

$$= \frac{(N-1)!}{(N-R-1)! * (R-1)!}\left[\frac{1}{R} + \frac{1}{N-R}\right]$$

$$= \frac{(N-1)!}{(N-R-1)! * (R-1)!}\left[\frac{N-R}{R*(N-R)} + \frac{R}{R*N-R}\right]$$

$$= \frac{(N-1)!}{(N-R-1)! * (R-1)!}\left[\frac{N-\cancel{R}+\cancel{R}}{R*(N-R)}\right]$$

$$= \frac{(N-1)!}{(N-R-1)! * (R-1)!} * \frac{N}{R*(N-R)} \longrightarrow \frac{N!}{R! * (N-R)!} = {}^{N}C_{R}$$

Pascal Triangle:

Given N, return mat[N][N]

if $i >= j$: mat[i][j] = ${}^{i}C_{j}$

if $i < j$: mat[i][j] = 0;

Ex: N = 5 mat[5][5]

i \ j	0	1	2	3	4
0	1	0	0	0	0
1	1	1	0	0	0
2	1	2	1	0	0
3	1	3	3	1	0
4	1	4	6	4	1

	0	1	2	3	4
0	${}^{0}C_{0}$	0	0	0	0
1	${}^{1}C_{0}$	${}^{1}C_{1}$	0	0	0
2	${}^{2}C_{0}$	${}^{2}C_{1}$	${}^{2}C_{2}$	0	0
3	${}^{3}C_{0}$	${}^{3}C_{1}$	${}^{3}C_{2}$	${}^{3}C_{3}$	0
4	${}^{4}C_{0}$	${}^{4}C_{1}$	${}^{4}C_{2}$	${}^{4}C_{3}$	${}^{4}C_{4}$

Constraints: $1 <= N <= 22$

```
int[][] Pascal(int N){
    int mat[N][N];
    for(int i=0; i<N; i++){
        for(int j=0; j<N; j++){
            if(i<j){ mat[i][j] = 0;}
            else{
                mat[i][j] = iCj
            }
        }
    }
    return mat;
}
```

$\# {}^{N}C_{2} = \frac{(N)(N-1)}{2}$

Issue:

Ex: N = 22, mat[22][22];

Calculate mat[21][2] $= {}^{21}C_{2} = \frac{21!}{19! * 2!} = \frac{21*20*19*18*\ldots 1}{19! * 2!}$ overflow

$= 210$

Idea2:

$$^{N}C_{R} = {}^{N-1}C_{R} + {}^{N-1}C_{R-1}$$

N : i R : j

$$^{i}C_{j} = {}^{i-1}C_{j} + {}^{i-1}C_{j-1}$$

Mat[i][j] = Mat[i-1][j] + Mat[i-1][j-1]

if i ≠ 0 & j ≠ 0

	j-1	j
i-1	Mat[i-1][j-1]	Mat[i-1][j]
i		Mat[i][j]

Trace: N=5 : mat[5][5]

	j : 0	1	2	3	4
i : 0	1	0	0	0	0
i : 1	1	1	0	0	0
i : 2	1	2	1	0	0
i : 3	1	3	3	1	0
i : 4	1	4	6	4	1

int[][] Pascal(int N){ TC: $O(N^2 * 1) = O(N^2)$ SC: $O(1)$

```
int[][] Pascal(int N){
    int mat[N][N];
    for(int i=0; i<N; i++){
        for(int j=0; j<N; j++){
            if(i<j){ mat[i][j] = 0; }
            else if(j==0 || i==0){ mat[i][j] = 1; }
            else{ mat[i][j] = mat[i-1][j] + mat[i-1][j-1] }
        }
    }
    return mat;
}
```