

Today's Content

1. Kadane's

2. Sliding Window

Ques # No. of subarrays of len = k

$$arr[6] = \{ \underline{a_0} \underline{a_1} \underline{a_2} \underline{a_3} \underline{a_4} \underline{a_5} \}$$

$$k=4, ans = 3$$

$$arr[7] = \{ \underline{a_0} \underline{a_1} \underline{a_2} \underline{a_3} \underline{a_4} \underline{a_5} \underline{a_6} \}$$

$$k=3, ans = 5$$

$$arr[8] = \{ \underline{a_0} \underline{a_1} \underline{a_2} \underline{a_3} \underline{a_4} \underline{a_5} \underline{a_6} \underline{a_7} \}$$

$$k=5, ans = 4$$

Given arr[n] no. of subarrays of len = k

$$ans = N - k + 1$$

Q Given $\text{arr}[N]$ return Max Subarray Sums of len = k

Constraints:

Consider subarray of len = k

$$1 \leq N \leq 10^5$$

$$1 \leq k \leq N$$

$$-10^6 \leq \text{arr}[i] \leq 10^6$$

$$\# \text{sum} = \{-10^{11} \dots 10^{11}\} = \underline{\text{long}}$$
$$\text{arr}[10^5] = \{-10^6 \dots 10^6\}$$
$$\text{arr}[10^5] = \{10^6 \dots 10^6\}$$

Ex: $\text{arr}[10] : \{-3, 4, -2, 5, 3, -2, 8, 2, -1, 4\}$

$$k=5$$

s e sum = 16

[0 4]	7
[1 5]	8
[2 6]	12
[3 7]	16
[4 8]	10
[5 9]	11
[6 10]	*

Idea: Generate all subarrays of len = k:

Iterate & calculate sum & get overall max.

$$TC: \underline{O(N-k+1) * O(k)} \quad SC: O(1)$$

$$k=1 \quad O(N-1+1) * O(1) = O(N*1) = O(N)$$

$$k=N \quad O(N-N+1) * O(N) = O(1*N) = O(N)$$

$$k=\frac{N}{2} \quad O(N-\frac{N}{2}+1) * O(\frac{N}{2}) = O(\frac{N}{2} * \frac{N}{2}) = \underline{O(N^2)}$$

Code:

0 1 2 3 4 5 6 7 8

Ex: arr[9] : { -3 4 -2 5 3 -2 8 2 -1 }

k = 5

S

e

s e sum

[0 4] iterate from S..e q get sum = 7; s++; e++;

[1 5] iterate from S..e q get sum = 8; s++; e++;

[2 6] iterate from S..e q get sum = 12; s++; e++;

[3 7] iterate from S..e q get sum = 16; s++; e++;

[4 8] iterate from S..e q get sum = 16; s++; e++;

[5 9]: if e outside loop process by return max sum; 16.

long maxSum(vector<int> &arr) {

int N = arr.size();

long max = INT_MIN;

int s = 0, e = k-1;

while(e < N) {

long sum = 0;

for(int i=s; i<=e; i++) { → 1. Iterate q calculate sum

} sum = sum + arr[i];

if (sum > max) {

max = sum;

s++; e++;

a b b-a+1
1st subarray : [0..k-1]; k-1-0+1 = k;

→ 2. Cmp with max

→ 3. Goto next subarray.

return max;

3

Intuition: For every subarray of len=k, calculate sum using pf() & get overall max

$$TC: O(N + (N-k+1) * O(1)) = O(N + N - k + 1) \approx O(N) \quad SC = \underline{O(N)}$$

Code:

0 1 2 3 4 5 6 7 8

PF(T)

Ex: arr[9] : { -3 4 -2 5 3 -2 8 2 -1 }



pf[9] : { -3 | -1 4 7 5 13 15 14 }

s e sum:

$$[0 \ 4] = Pf[4] = 7$$

$$[1 \ 5] = Pf[5] - Pf[0] = 5 - (-3) = 8$$

$$[2 \ 6] = Pf[6] - Pf[1] = 13 - 1 = 12$$

$$[3 \ 7] = Pf[7] - Pf[2] = 15 - (-1) = 16$$

$$[4 \ 8] = Pf[8] - Pf[3] = 14 - 4 = 10$$

[5 9]: *

sum[s..e] : if [s==0] { // [0..e]

} Pf[e]

else {

Pf[e] - Pf[s-1]

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long maxSum(restraints arr){ TC: O(N) SC: O(N)}
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int N = arr.size();  
long pf[N], sum = 0;  
for(int i=0; i < N; i++){  
    sum = sum + arr[i]; // 1. update sum  
    pf[i] = sum;  
}
```

```
long man = INT_MIN;
```

```
int s = 0, e = k-1;
```

```
while(e < N){
```

```
    long sum = 0;  
    if(s == 0){  
        sum = pf[e];  
    }
```

```
    else{  
        sum = pf[e] - pf[s-1];  
    }
```

```
    if(sum > man){  
        man = sum;  
    }
```

```
    s++; e++;
```

a b b-a+1
1st Subarray : [0..k-1]; k-1-0+1 = k;

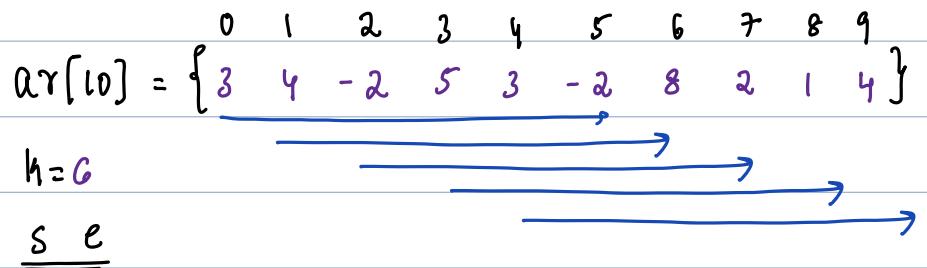
→ 1. # Sum [s..e] get using pf[]

→ 2. Comp with man

→ 3. Go to next subarray.

```
return man;
```

Ideas: Sliding Window: Fixed length subarrays.



[0 5] sum = 11 : 1st window iterate & calculate

For remaining windows calculate using sliding window.

$$\{ [1 6] \text{ sum} = \text{sum} - ar[0] + ar[6] = 11 - 3 + 8 = 16 \}$$

$$\{ [2 7] \text{ sum} = \text{sum} - ar[1] + ar[7] = 16 - 4 + 2 = 14 \}$$

$$\{ [3 8] \text{ sum} = \text{sum} - ar[2] + ar[8] = 14 - (-2) + 1 = 17 \}$$

$$\{ [4 9] \text{ sum} = \text{sum} - ar[3] + ar[9] = 17 - 5 + 4 = 16 \}$$

[5 10] Stop & return max sum = 17

s-1 s s+1 s+2 ... e-1 e

$$[s, e] \text{ sum} = \text{sum} - ar[s-1] + ar[e]$$

Remove $ar[s-1]$

Adding $ar[e]$

int subarray (vector<int> &arr, int k) { Tc: O(N) Sc: O(1)

Step 1: For 1st window iterate q & set it q 1st subarray [0.. k-1]

long sum=0, max=INT_MIN;

2nd subarray [1.. k]

for (int i=0, i<k; i++) { # [0.. k-1] = k iterations

sum = sum + arr[i];

3

if (sum > max) {

max = sum;

3

Step 2: For remaining subarrays, slide by calculate sum.

int s=1, e=k;

while (e < N) { # (N-k) iterating

get sum from [s.. e] using sliding. ?

sum = sum - arr[s-1] + arr[e];

if (sum > max) {

max = sum;

s++; e++;

3

return max;

3

Man Subarray Sum: \rightarrow {Continuous part of an array}

Given $ar(N)$ return man subarray sum.

0 1 2 3 4 5 6

Ex1: $ar[] = \{ -2 3 4 -1 5 -10 7 \}$ ans =

0 1 2 3 4 5 6

Ex2: $ar[] = \{ -3 4 6 8 -10 2 7 \}$ ans =

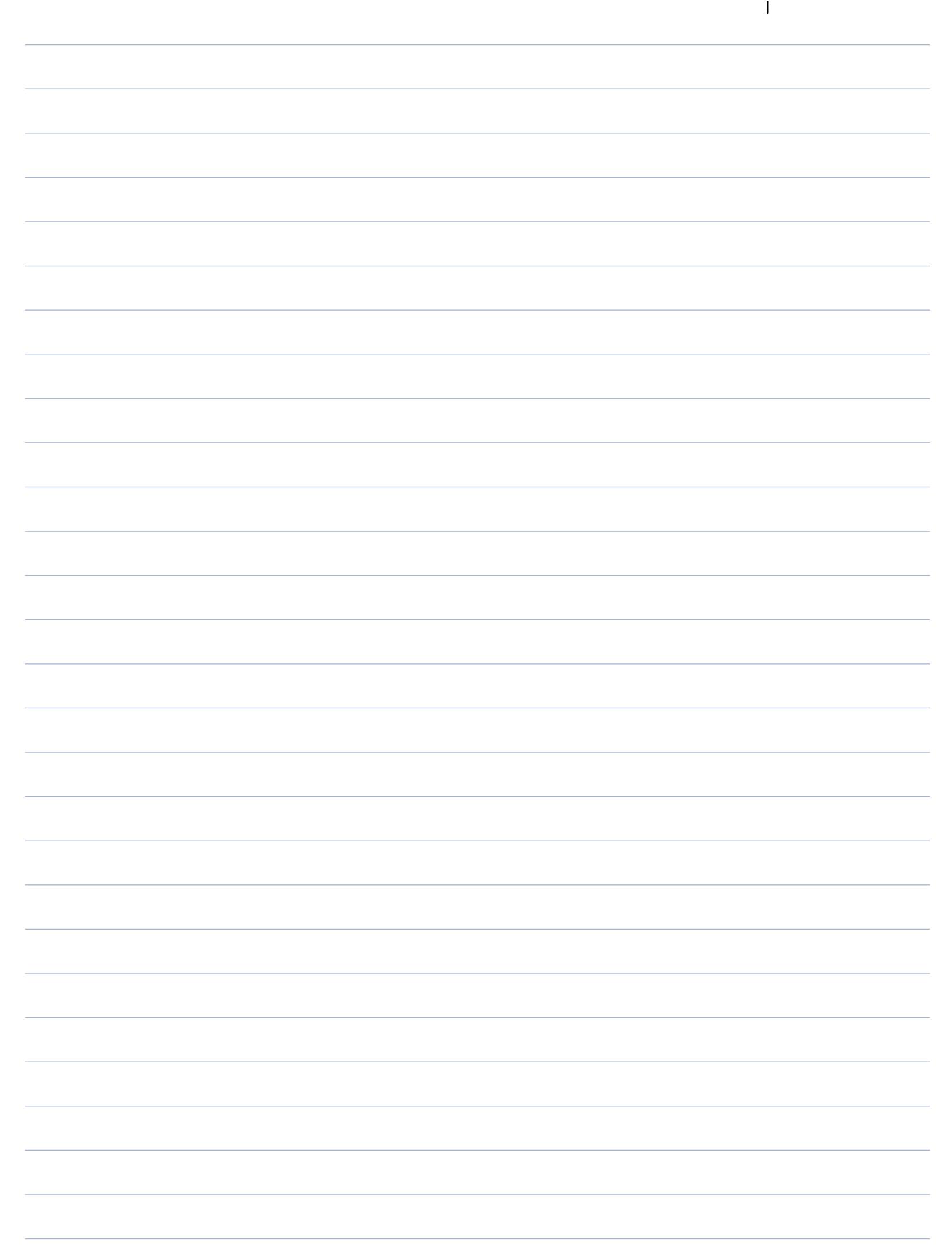
0 1 2 3 4

Q1: $ar[] = \{ 4 5 2 1 6 \}$ ans =

0 1 2 3 4

Q2: $ar[] = \{ -4 -3 -6 -9 -2 \}$ ans = -

Idea1:



Optimisation: Kadane's Algo: Max Subarray Sum.

Case 1: If all the elements in the array are positive. :

$$\text{arr}[] = \{ \begin{matrix} 0 & 1 & 2 & 3 & 4 \\ 4 & 2 & 1 & 6 & 7 \end{matrix} \} \text{ ans} =$$

Case 2: If all the elements in the array are negative

$$\text{arr}[] = \{ \begin{matrix} 0 & 1 & 2 & 3 & 4 \\ -4 & -8 & -3 & -10 & -5 \end{matrix} \} \text{ ans} =$$

Case 3: If positives are present in between.

$$\text{arr}[] = \{ \begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ -3 & -5 & -3 & 4 & 3 & 2 & 10 & -5 & -7 \end{matrix} \} \text{ ans} =$$

Case 4: Say we have max subarray sum

4.1 $\text{arr}[] = \{ -3 \text{ } -5 \text{ } -3 \text{ } \boxed{-5 \text{ } -7} \}$

4.2 $\text{arr}[] = \{ -3 \text{ } 5 \text{ } -3 \text{ } \boxed{-5 \text{ } -7} \}$

Idea:

True:

$$\text{arr}[] = \{ \begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 3 & 4 & -6 & 8 & -10 & 12 & 10 & -3 & 7 & -5 \end{matrix} \}$$

Sum =

0 1 2 3 4 5 6
ar[] = { -2 3 4 -1 5 -10 7 }

sum = 0

max =

int maxSubKadane's (int ar[]) { Tc: O(n) Sc: O(1) }

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