

## Todays Content

1. Number System Basics
2. Binary to Decimal & Vice Versa
3. Adding 2 Binary Numbers
4. -ve numbers
5. Datatype range
6. When we multiply 2 numbers

$$S = 2^0 + 2^1 + 2^2 + \dots + 2^{N-2} + 2^{N-1} + 2^N = 2^{N+1} - 1$$

$$\begin{aligned} 2S &= 2^1 + 2^2 + 2^3 + \dots + 2^{N-1} + 2^N + 2^{N+1} \\ S &= 2^0 + 2^1 + 2^2 + \dots + 2^{N-2} + 2^{N-1} + 2^N \\ \hline S &= 2^{N+1} - 2^0 \Rightarrow 2^{N+1} - 1 \end{aligned}$$

$$S = 2^0 + 2^1 + 2^2 : 2^3 - 1$$

$$S = 2^0 + 2^1 + 2^2 + 2^3 = 2^4 - 1$$

## Decimal Number System

→ Each Digit: [0 1 ... 9]  
 ↳ Each power: [10]

$$10^3 \left[ \begin{array}{c} 10^2 \\ 3 \\ 10^1 \\ 4 \\ 10^0 \\ 2 \end{array} \right] = 3 * 10^2 + 4 * 10^1 + 2 * 10^0 = 342$$

$$2 \ 5 \ 6 \ 3 = 2 * 10^3 + 5 * 10^2 + 6 * 10^1 + 3 * 10^0 = 2563$$

## Binary Number System

→ Each Digit: [0 1]  
 ↳ Each power: [2]

$$\left[ \begin{array}{c} 2^5 \\ 1 \\ 2^4 \\ 0 \\ 2^3 \\ 0 \\ 2^2 \\ 1 \\ 2^1 \\ 0 \\ 2^0 \\ 1 \end{array} \right] = 2^5 * 1 + 2^4 * 0 + 2^3 * 1 + 2^2 * 0 + 2^1 * 1 + 2^0 * 1 = 37$$

$$\left[ \begin{array}{c} 2^4 \\ 1 \\ 2^3 \\ 0 \\ 2^2 \\ 0 \\ 2^1 \\ 1 \\ 2^0 \\ 1 \end{array} \right] = 2^4 * 1 + 2^3 * 0 + 2^2 * 1 + 2^1 * 1 + 2^0 * 1 = 19$$

$$\left[ \begin{array}{c} 1 \\ 1 \\ 0 \\ 0 \\ 1 \end{array} \right] = 2^3 * 1 + 2^2 * 0 + 2^1 * 1 + 2^0 * 1 = 5$$

$$\left[ \begin{array}{c} 2^6 \\ 1 \\ 2^5 \\ 0 \\ 2^4 \\ 1 \\ 2^3 \\ 1 \\ 2^2 \\ 0 \\ 2^1 \\ 0 \\ 2^0 \\ 1 \end{array} \right] = 2^6 * 1 + 2^5 * 0 + 2^4 * 1 + 2^3 * 1 + 2^2 * 0 + 2^1 * 1 + 2^0 * 1 = 90$$

## Decimal to Binary →

2	37	2	45
2	18 : 1	2	22 : 1
2	9 : 0	2	11 : 0
2	4 : 1	2	5 : 1
2	2 : 0	2	2 : 1
2	1 : 0	2	1 : 0
	0 : 1		0 : 1

$$37 : \left[ \begin{array}{c} 2^5 \\ 1 \\ 2^4 \\ 0 \\ 2^3 \\ 0 \\ 2^2 \\ 1 \\ 0 \\ 1 \end{array} \right] = 2^5 * 1 + 2^4 * 0 + 2^3 * 0 + 2^2 * 1 + 2^1 * 0 + 2^0 * 1 = 37$$

$$45 : \left[ \begin{array}{c} 2^5 \\ 1 \\ 2^4 \\ 0 \\ 2^3 \\ 1 \\ 2^2 \\ 1 \\ 1 \\ 0 \\ 1 \end{array} \right] = 2^5 * 1 + 2^4 * 0 + 2^3 * 1 + 2^2 * 1 + 2^1 * 1 + 2^0 * 1 = 45$$

Add 2 Decimal numbers:  $c = 5/10$   $d = 5\%/10$   $10 \rightarrow$  Decimal

$13/10$   $14/10$

$$\begin{array}{r} & 1 \\ & | \\ 9/10 & 7 \\ & | \\ 0 & 1 \\ \hline s = & 9/10 \quad 13/10 \quad 14/10 \end{array}$$

$d = 0 \quad \underline{\underline{9 \quad 3 \quad 4}}$

Add 2 Binary Numbers  $c = 5/2$   $d = 5\%/2$   $2 \rightarrow$  Binary Number System

$$\begin{array}{r} 2^4 \quad 2^3 \quad 2^2 \quad 2^1 \quad 2^0 \\ 1/2 \quad 3/2 \quad 2/2 \quad 1/2 \\ c = 0 \quad 1 \quad 0 \quad 1 \quad 0 \\ 1/2 \quad 1 \quad 0 \quad 1 \quad 1 \\ 0 \quad 0 \quad 0 \quad 1 \quad 1 \\ \hline s = 1 \quad 1 \quad 1 \quad 0 \quad 1 \end{array}$$

$\rightarrow 22$   
 $\rightarrow 7$

$d = 0 \quad \underline{\underline{1 \quad 1 \quad 1 \quad 0 \quad 1}} \rightarrow 29$

$$\begin{array}{r} 3/2 \quad 2/2 \quad 2/2 \quad 3/2 \quad 2/2 \\ 1/2 \quad 1/2 \quad 1/2 \quad 1/2 \quad 1/2 \\ c = 1 \quad 1 \quad 1 \quad 1 \quad 1 \\ 0 \quad 1 \quad 1 \quad 0 \quad 1 \\ 0 \quad 1 \quad 0 \quad 1 \quad 1 \\ \hline s = 3/2 \quad 2/2 \quad 2/2 \quad 3/2 \quad 2/2 \\ \downarrow \\ d = 1 \quad 1 \quad 0 \quad 0 \quad 1 \quad 0 \end{array}$$

8 bit Numbers:

$$2^7 \ 2^6 \ 2^5 \ 2^4 \ 2^3 \ 2^2 \ 2^1 \ 2^0$$

$$13 : \underline{0} \ 0 \ 0 \ 0 \ 1 \ 1 \ 0 \ 1$$

$$-13 : \underline{1} \ 0 \ 0 \ 0 \ 1 \ 1 \ 0 \ 1 \text{ * wrong way to represent -ve?}$$

$$\left. \begin{array}{l} -13 : \underline{1} \ 0 \ 0 \ 0 \ 1 \ 1 \ 0 \ 1 \\ 1 : \underline{0} \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \end{array} \right\} \begin{array}{l} 2^7 \\ 2^6 \\ 2^5 \\ 2^4 \\ 2^3 \\ 2^2 \\ 2^1 \\ 2^0 \end{array}$$

$$-13 : \underline{1} \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 1 =$$

Q? Correct way to get negative:

$$-a = 2^8 a = \underbrace{\sim a + 1}_{1 \geq 0}$$

$$a = 13 : \underline{0} \ 0 \ 0 \ 0 \ 1 \ 1 \ 0 \ 1$$

$$\sim a : \underline{1} \ 1 \ 1 \ 1 \ 0 \ 0 \ 1 \ 0$$

$$1 : \underline{0} \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1$$

$$2^7 \ 2^6 \ 2^5 \ 2^4 \ 2^3 \ 2^2 \ 2^1 \ 2^0$$

$$-13 - a : \underline{1} \ 1 \ 1 \ 1 \ 0 \ 0 \ 1 \ 1 = 2^7 + 2^6 + 2^5 + 2^4 + 2^1 + 2^0 = 243$$

↳ LSB = MSB? > All bits combined.

= MSB base value = -ve

$$-2^7 \ 2^6 \ 2^5 \ 2^4 \ 2^3 \ 2^2 \ 2^1 \ 2^0$$

$$-a : \underline{1} \ 1 \ 1 \ 1 \ 0 \ 0 \ 1 \ 1 : -2^7 + 2^6 + 2^5 + 2^4 + 2^1 + 2^0$$

$$= -128 + 115 = -13$$

# Numbers

unsigned int a=10;

#bits

**Signed:** MSB base value is -ve

$$\begin{array}{c}
 \frac{1}{2^3} \frac{0}{2^2} \frac{0}{2^1} \frac{0}{2^0} = -2 \\
 \frac{0}{2^3} \frac{1}{2^2} \frac{1}{2^1} \frac{1}{2^0} \\
 \downarrow \\
 \frac{0}{2^3} \frac{1}{2^2} \frac{1}{2^1} \frac{1}{2^0} = 2-1
 \end{array}$$

$$\begin{array}{c}
 \frac{0}{2^3} \frac{0}{2^2} \frac{0}{2^1} \frac{0}{2^0} = 0 \\
 \frac{1}{2^3} \frac{1}{2^2} \frac{1}{2^1} \frac{1}{2^0} \\
 \downarrow \\
 \frac{1}{2^3} \frac{1}{2^2} \frac{1}{2^1} \frac{1}{2^0} = 2-1
 \end{array}$$

8

$$\begin{array}{c}
 \frac{1}{2^7} \frac{0}{2^6} \frac{0}{2^5} \dots \frac{0}{2^1} \frac{0}{2^0} = -2 \\
 \frac{0}{2^7} \frac{1}{2^6} \frac{1}{2^5} \dots \frac{1}{2^1} \frac{1}{2^0} \\
 \downarrow \\
 \frac{0}{2^7} \frac{1}{2^6} \frac{1}{2^5} \dots \frac{1}{2^1} \frac{1}{2^0} = 2-1
 \end{array}$$

$$\begin{array}{c}
 \frac{0}{2^7} \frac{0}{2^6} \frac{0}{2^5} \dots \frac{0}{2^1} \frac{0}{2^0} = 0 \\
 \frac{1}{2^7} \frac{1}{2^6} \frac{1}{2^5} \dots \frac{1}{2^1} \frac{1}{2^0} \\
 \downarrow \\
 \frac{1}{2^7} \frac{1}{2^6} \frac{1}{2^5} \dots \frac{1}{2^1} \frac{1}{2^0} = 2-1
 \end{array}$$

N

$$\begin{array}{c}
 \frac{1}{2^{N-1}} \frac{0}{2^{N-2}} \dots \frac{0}{2^1} \frac{0}{2^0} = -2 \\
 \frac{0}{2^{N-1}} \frac{1}{2^{N-2}} \dots \frac{1}{2^1} \frac{1}{2^0} \\
 \downarrow \\
 \frac{0}{2^{N-1}} \frac{1}{2^{N-2}} \dots \frac{1}{2^1} \frac{1}{2^0} = 2-1
 \end{array}$$

$$\begin{array}{c}
 \frac{1}{2^{N-1}} \frac{1}{2^{N-2}} \dots \frac{1}{2^1} \frac{1}{2^0} \\
 \downarrow \\
 \dots \frac{1}{2^1} \frac{1}{2^0} = 2-1
 \end{array}$$

## Datatype Ranges:

#datatype	#bits	Min	Max	Min	Max
	N	Signed : $-2^{N-1}$	$2^{N-1}-1$	Unsigned : 0	$2^N-1$
int	32	$-2^{31}$ $\{-2 \times 10^9\}$	$2^{31}-1$ $2 \times 10^9\}$	0	$2^{32}-1$ $4 \times 10^9\}$
long	64	$-2^{63}$ $\{-8 \times 10^{18}\}$	$2^{63}-1$ $8 \times 10^{18}\}$	0	$2^{64}-1$ $16 \times 10^{18}\}$

Approximations:  $2^{10} = 1024 \approx 1000 = 10^3$  #  $2^{10} \approx 10^3$

Care1:

$$2^{10} \approx 10^3$$

Cube m both Sides

$$(2^{10})^3 \approx (10^3)^3$$

$$2^{30} \approx 10^9$$

Multiply by 2

$$2^{31} \approx 2 \times 10^9$$

Multiply by 2

$$2^{32} \approx 4 \times 10^9$$

Care2:

$$2^{10} \approx 10^3$$

power 6 m both Sides

$$(2^{10})^6 \approx (10^3)^6$$

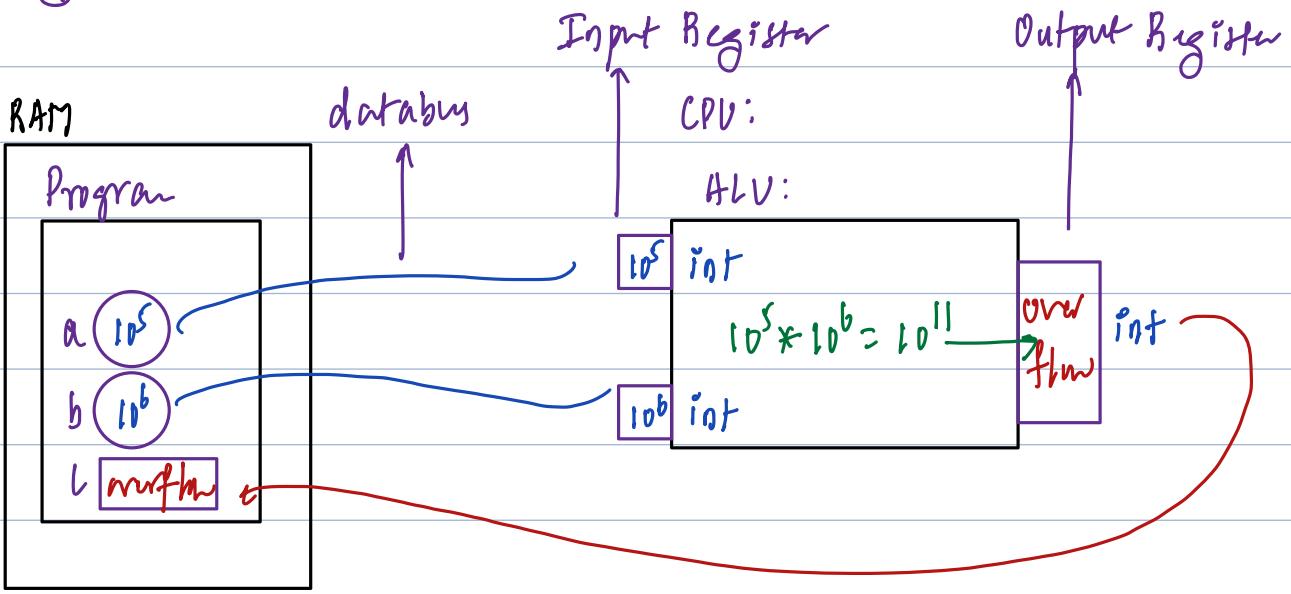
$$2^{60} \approx 10^{18}$$

Multiply by 2<sup>3</sup>

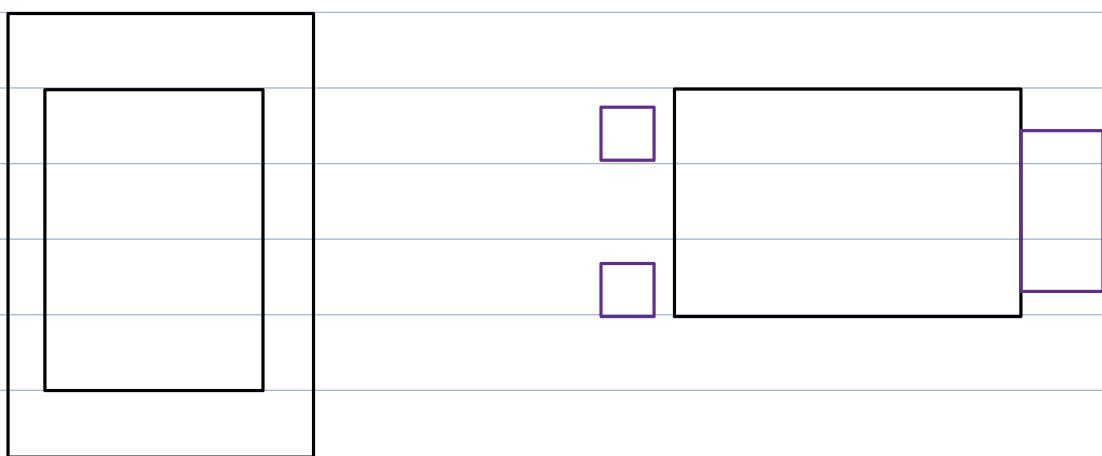
$$2^{63} \approx 8 \times 10^{18}$$

Q1:  $\text{int } a = 10^5, b = 10^6$   
 $\text{int } c = a * b; \# 10^5 * 10^6 = 10^{11}$   
 $\text{print}(c); \# \text{wrong}$

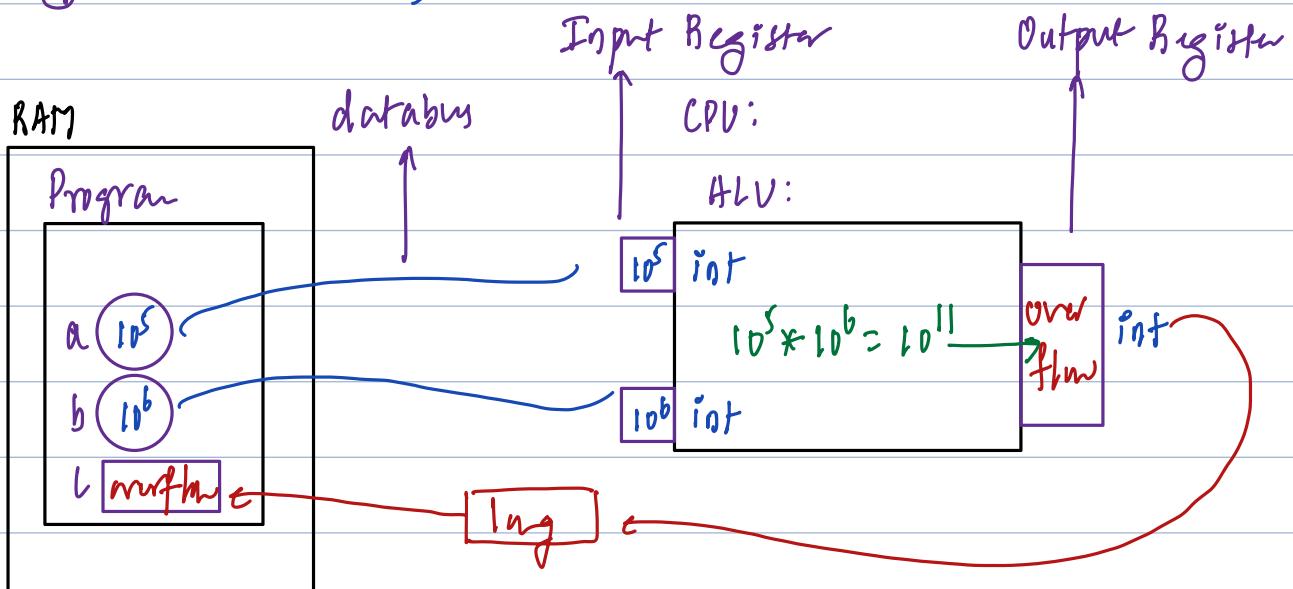
Q2:  $\text{int } a = 10^5, b = 10^6$   
 $\text{long } c = a * b;$



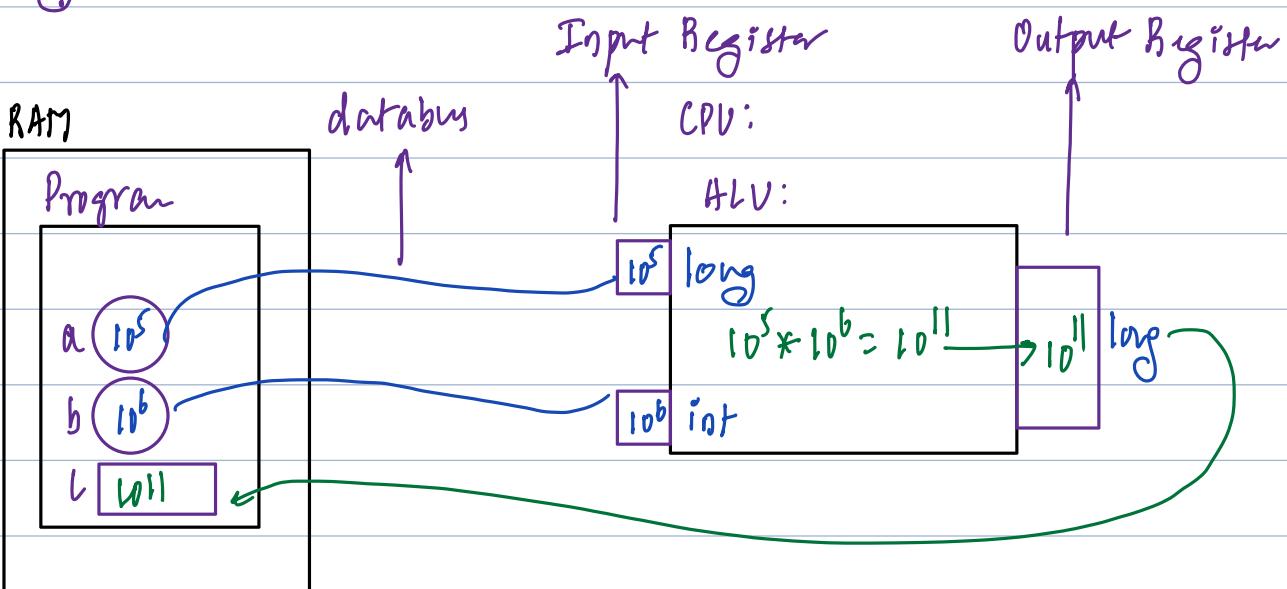
Q3:  $\text{int } a = 10^5, b = 10^6$   
 $\text{long } c = (\text{long}) a * b;$



Q2:  $\text{int } a = 10^5, b = 10^6;$   
 $\text{long } c = (\text{long})(a * b);$



Q2:  $\text{int } a = 10^5, b = 10^6;$   
 $\text{long } c = (\text{long})a * b;$



long a =  $10^5$ , b =  $10^6$

long c = a \* b;

