

## Today's Content

1. Searching Intro
2. Search in sorted array
3. Search floor of  $x$  in sorted array
4. Search  $n^{\text{th}}$  element of  $x$  in sorted array

## Searching Basics

Bro/Sis missing → Police stating

- Phon : Whom to search: Target
- Locatin : Where to search: Search space / Answer space

Example:

Example:

Target  $\rightleftharpoons$  Search Space

Word { Dict / Books / News Paper }

PhoneNo {Contact/Phonebook}

Obs: If Searchspace is Ordered, Searching becomes easier.

## Search Dog in Dictionary

$[A \ B \ C \ D \ E \ F \ G \ H \ I \ J \ K \ L \ M \ N \ O \dots X \ Y \ Z]$   
 $C < D$     $\uparrow$   $J \rightarrow y$     $\uparrow$   $K \rightarrow y$     $\uparrow$   $L \rightarrow y$     $\uparrow$   $M \rightarrow y$     $\uparrow$   $N \rightarrow y$     $\uparrow$   $O \rightarrow y$     $\uparrow$   $P \rightarrow y$     $\uparrow$   $Q \rightarrow y$     $\uparrow$   $R \rightarrow y$     $\uparrow$   $S \rightarrow y$     $\uparrow$   $T \rightarrow y$     $\uparrow$   $U \rightarrow y$     $\uparrow$   $V \rightarrow y$     $\uparrow$   $W \rightarrow y$     $\uparrow$   $X \rightarrow y$     $\uparrow$   $Y \rightarrow y$     $\uparrow$   $Z \rightarrow y$

How to where to land? mid?

$N$ 

$N/2$	$N/2$
-------	-------

 : Always Discard =  $N/2$  ; always  $N/2$  better  $N/3$  [wrr].

N 

$N/3$	$2N/3$
-------	--------

 : Discard left:  $N/3$     load out mid.  
Discard right:  $2N/3$

### When to apply BS:

After dividing search space into 2 parts, if we can discard 1 half of search space using some conditions, then we can apply binary search

Diagram illustrating the rotation of two boxes. Each box is labeled  $N$  and contains two compartments, each labeled  $N/2$ . Arrows indicate a clockwise rotation of the boxes.

Note: 

$N/2$	$N/2$
-------	-------

Q) Given a sorted  $ar[n]$  search if  $k$  is present or not?

$ar[10] = \{ \begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 3 & 6 & 9 & 12 & 14 & 19 & 20 & 23 & 25 & 27 \end{matrix} \}$

$k = 12 : 94$

Idea1: Iterate on  $ar[]$  & search for  $k$ : (True/False) TC:  $O(N)$  SC:  $O(1)$

Idea2: Target =  $k$  Search Space =  $\ln ar[]$



if  $(ar[mid] == k) \{ \text{return True} \}$

$\swarrow < k$



if  $(ar[mid] > k) \{ \text{goto left} \}$

$\swarrow < k$



if  $(ar[mid] < k) \{ \text{goto right} \}$

Tracking Search Space:

We can use  $s$  &  $e$  to indicate search space

Ex1:  $ar[10] = \{ \begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 3 & 6 & 9 & 12 & 14 & 19 & 20 & 23 & 25 & 27 \end{matrix} \}$   $k = 20$

$\begin{matrix} \uparrow m \\ \downarrow l \end{matrix}$

$l \quad h \quad m : (l+h)/2$

$0 < 9 \quad m = (0+9)/2 = 4 \quad ar[m] < k : \text{goto right } l = m+1;$

$5 < 9 \quad m = (5+9)/2 = 7 \quad ar[m] > k : \text{goto left } h = m-1;$

$5 < 6 \quad m = (5+6)/2 = 5 \quad ar[m] < k : \text{goto right } l = m+1$

$6 < 6 \quad m = (6+6)/2 = 6 \quad ar[m] == k : \text{return true};$

Ex:  $arr[10] =$

0	1	2	3	4	5	6	7	8	9
3	6	9	12	14	19	20	23	25	27
				$h$	$l$				

$l = h$     $m = (l+h)/2$     $k = 15$

$0 \leq 9$     $m = (0+9)/2 = 4$     $arr[m] < k$  : goto right  $l = m+1$ ;

$5 \leq 9$     $m = (5+9)/2 = 7$     $arr[m] > k$  : goto left  $h = m-1$ ;

$5 \leq 6$     $m = (5+6)/2 = 5$     $arr[m] > k$  : goto left  $h = m-1$

$5 \leq 4$  #  $l > h$  stop process.

bool search(vector<int> &arr, int k) { TC:  $O(\log_2 N)$  SC:  $O(1)$

int  $l = 0$ ,  $h = arr.size() - 1$ ;

while( $l \leq h$ ) {

int  $m = (l+h)/2$ ;

if( $arr[m] == k$ ) { return true; }

else if( $arr[m] > k$ ) {

h = m-1; # goto left

else { #  $arr[m] < k$  goto right

l = m+1;

}

}

return false;

TC: Initially Search Space =  $N \xrightarrow{1^{st}} N/2 \xrightarrow{2^{nd}} N/4 \xrightarrow{3^{rd}} N/8 \rightarrow \dots \rightarrow 1$  :  
 $\log_2 N$  iterations

goto right :  $l = m+1$

goto left :  $h = m-1$

28 Given a sorted arr[], find floor of a given num k

Floor: greatest  $x \leq k$

arr[] = { -5, 2, 3, 6, 9, 10, 11, 14, 18 }

k = 8

5: 3

4: 3

10: 10

24: 18

-7:  $-\infty$

If floor doesn't

exist return

$-\infty$  // INT\_MIN;

Idea: Iterate over arr[] & update ans;

k = 8 ans =  $-\infty$  T.C:  $O(N)$  S.C:  $O(1)$

i	arr[i]	ans =
---	--------	-------

0	-5 $\leq 8$	ans = -5; update & look for better
---	-------------	------------------------------------

1	2 $\leq 8$	ans = 2; update & look for better
---	------------	-----------------------------------

2	3 $\leq 8$	ans = 3; update & look for better
---	------------	-----------------------------------

3	6 $\leq 8$	ans = 6; update & look for better
---	------------	-----------------------------------

4	9 $\leq 8$	9 not possible, Right element not possible.
---	------------	---

return ans = 6

Idea2:

Target = greatest  $x \leq k$  Search Space = ln arr[] ans =  $-\infty$

$x \leq k$

	mid	
--	-----	--

If arr[mid] == k: return k;

$x < k$

$x < k$	mid	
---------	-----	--

If arr[mid] < k:

ans = arr[mid];

goto right

↩

	mid	$x > k$
--	-----	---------

If arr[mid] > k:

goto left

Ex: arr[] = { 2, 4, 8, 12, 15, 22, 24, 26 }

k = 20

l h m

0 7 3 arr[m] < k: ans = arr[m] goto right

4 7 5 arr[m] > k: goto left

4 4 4 arr[m] < k: ans = arr[m] goto right

5 4 # stop & return ans = 15.

int floorIndex(vector<int> arr, int k) { TC:  $O(\log_2 N)$  SC:  $O(1)$

```
int l = 0, h = arr.size() - 1, ans = -1;
while (l <= h) {
    int m = (l + h) / 2;
    if (arr[m] == k) {
        return k;
    }
    else if (arr[m] < k) {
        l = m + 1; # go to right;
    }
    else { # arr[m] > k
        h = m - 1; # go to left
    }
}
return ans;
```

TODO: Ceil(n): Smallest ele  $\geq k$ .

3Q) Given an sorted arr[], find the first occurrence index of given element

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16  
 arr[] = { -5 -5 -3 0 0 1 1 5 5 5 5 5 5 8 10 10 15 }

k:  $P_1 \quad 12 - 7 + 1 = 6 \quad P_2 \quad \#occ = [P_1, P_2] = P_2 - P_1 + 1$

5: 7      Idea1: Iterate in arr[] & get 1<sup>st</sup> occurrence of k.

-5: 0      TC:  $O(N)$  SC:  $O(1)$

20: -1; if elemnt doesn't exist

Idea2:

Target = 1<sup>st</sup> occurrence index of k    Search Space = ln arr[]    ans = -1;

$> k$   

	mid	$> k > k$
--	-----	-----------

if arr[mid] > k: goto left

$< k$   

$< k < k$	mid	
-----------	-----	--

if arr[mid] < k: goto right

$= k$   

kk	k	kkk
----	---	-----

if arr[mid] == k:

ans = m; #update ans

goto left; #better ans

Example:

0 1 2 3 4 5 6 7  
 arr[] = { 2 5 5 5 5 5 8 10 }

k = 5

l = h    m

0 l = 7    3    arr[m] == k: ans = 3; goto left h = m - 1;

0 l = 2    1    arr[m] == k: ans = 1; goto left h = m - 1;

0 l = 0    0    arr[m] < k: goto right l = m + 1;

1 l = 0    \*    Stop & return ans = 1;

int firstOccurrence(vector<int> &arr, int k) { TODO

TODO: 1<sup>st</sup> occurrence of k in sorted arr() ✓ TC:  $O(\log_2 N)$

TODO: last occurrence of k in sorted arr() ✓ TC:  $O(\log_2 N)$

TODO: Frequency of k in sorted arr() ✓  $\{ P_2[\text{last\_occ\_index}] - P_1[\text{1<sup>st</sup>\_occ\_index} + 1] \}$   
TC:  $O(\log_2 N + \log_2 N) = O(\log_2 N)$