

## Today's Content

1. SQRAT ✓

2. Painters Partition.

Q: Given  $n$  find  $\text{sqrt}(n)$

Find greatest  $i$  such that  $i^2 \leq n$

$\text{sqrt}(25)$  : return 5

$\text{sqrt}(37)$  :  $5^2 \leq 37$ ,  $6^2 \leq 37$ ,  $7^2 \leq 37$  return 6.

$N = 30$

$i$	$i^2 \leq N$	ans =
1	$1^2 \leq 30$	ans = 1 ✓
2	$2^2 \leq 30$	ans = 2 ✓
3	$3^2 \leq 30$	ans = 3 ✓
4	$4^2 \leq 30$	ans = 4 ✓
5	$5^2 \leq 30$	ans = 5 ✓
6	$6^2 \leq 30$	return ans = 5

int sqrt(int N) { TC:  $O(\sqrt{N})$  SC:  $O(1)$

```

int i=1, ans=0;
while(i*i <= N) {
    ans = i; # update ans
    i++; # look for better ans
}
return ans;

```

$i^2 \leq N \Rightarrow i \leq \sqrt{N}$

Idea:

1. Target: Greatest  $i$ , with  $i^2 \leq N$ .

2. Search space: space where we search for target, it can be imaginary

$S_1$  :  $[1.. \sqrt{N}] \rightarrow$  We don't know  $\sqrt{N}$  &  $h$

$S_2$  :  $[1.. N/2]$ , if  $N=1$ , search space  $[1.. 0]$  \*

$S_3$  :  $[1.. N]$ , if  $N=1$ , search space  $[1.. 1]$  ✓

Note: In your search space it should contain target for every case.

3. Discard

$N=30$  Search space  $[1.. N]$  # Imaginary: No need to store

$l$	$h$	$m$		$l$	$m$	$h$	
1	30	15	$m^2 \leq 30$	{ 1	15 16 17 ... 30 }		$h = m-1$
1	14	7	$m^2 \leq 30$	{ 1	7 8 9 ... 14 }		$h = m-1$
1	6	3	$m^2 \leq 30$	{ 1	3	6 }	ans = 3, $l = m+1$
4	6	5	$m^2 \leq 30$	{ 4	5	6 }	ans = 5, $l = m+1$
6	6	6	$m^2 \leq 30$	{ 6	6	6 }	$h = m-1$
6	5		return ans = 5.				

Constraints:

$$1 \leq N \leq 10^{18}$$

$$N = 1$$

$$10^{18}$$

$\rightarrow \text{sqrt} = [1, 10^9]$  # If we take  $1..10^9$ , product of  $m \times m$  will be in long range

```
int Sqrt(int N){
```

```
    int l = 1, h = N 109, min(N, 109), ans = 0;    Note: Talk about datatype
```

```
    while(l <= h){
```

```
        long m = (l+h)/2;
```

```
        if(m*m <= N){
```

```
            ans = m;
```

```
            l = m+1;
```

```
        } else { // m*m > N go to left
```

```
            h = m-1;
```

```
    } return ans;
```

Note: # Search Space Size  $\{1..N\}$  : # N Elements

T.C:  $O(\log_2 N)$  SC:  $O(1)$

Q1:

We have to paint  $N$  boards of length  $\{c_0, c_1, c_2, \dots, c_{N-1}\}$

There are  $W$  painters available and each of them takes  $1$  unit of time to paint  $1$  unit of board

Calculate & return the minimum time required to paint all boards

Notes:

1. Two painters cannot share a board to paint, A board cannot be painted partially by 1 painter & partially by another.

2. A painter can only paint contiguous boards

3. All painters start work at same time.

$l = 10$   $h = 71$

Ex:	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	
N=15	3	5	1	7	8	2	5	3	10	1	4	7	5	4	6	<u>Time Taken</u>
W=3	W <sub>1</sub> =34								W <sub>2</sub> =15			W <sub>3</sub> =22				34.
	W <sub>1</sub> =31							W <sub>2</sub> =18				W <sub>3</sub> =22				31.
	W <sub>1</sub> =24					W <sub>2</sub> =25						W <sub>3</sub> =22				25

Ex2:	0	1	2	3	4	5	
$arr[6] =$	1	1	1	1	1	101	Time Taken
$W=2$	$W_1=5$					$W_2=101$	101

Idea: Average Time will not work, as seen in above example

Search:

Target: Min time taken to paint all boards

Search Space: Should be based on time:

$\{l, \dots, h\}$

#smallest time to paint all boards  $l$ :  $\text{Max of } arr[]$

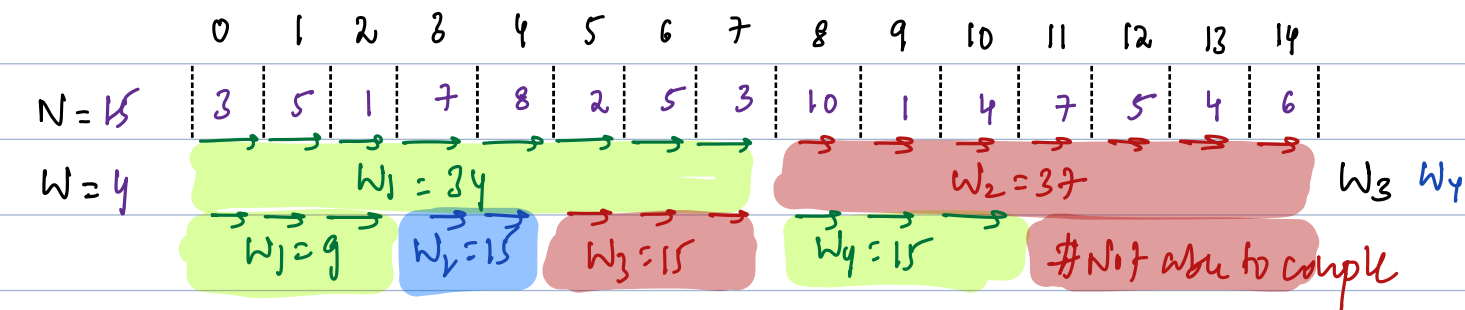
#greatest time to paint all boards  $h$ :  $\text{Sum of } arr[]$

$l$  = smallest val pos

$h$  = greatest val pos.

So that for every input as in search space

Discard?



$l$     $h$     $m$

10   71   40 : Check if we can paint all boards  $\leq 40$  mins ✓  
 $ans = 40$ ; goto left

[  $l$     $m$     $m+1$     $m+2$    ...    $h$  ]

if we can do task in  $m$  time:  
 We can do task in  $m+1, m+2, \dots$   
 update  $m$  & goto better ans  $m$  left.

# Say  $m = 15$ .

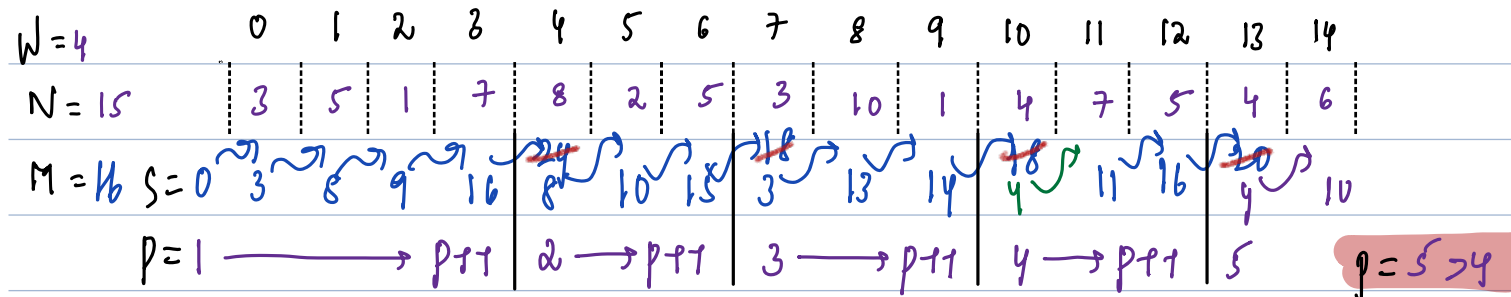
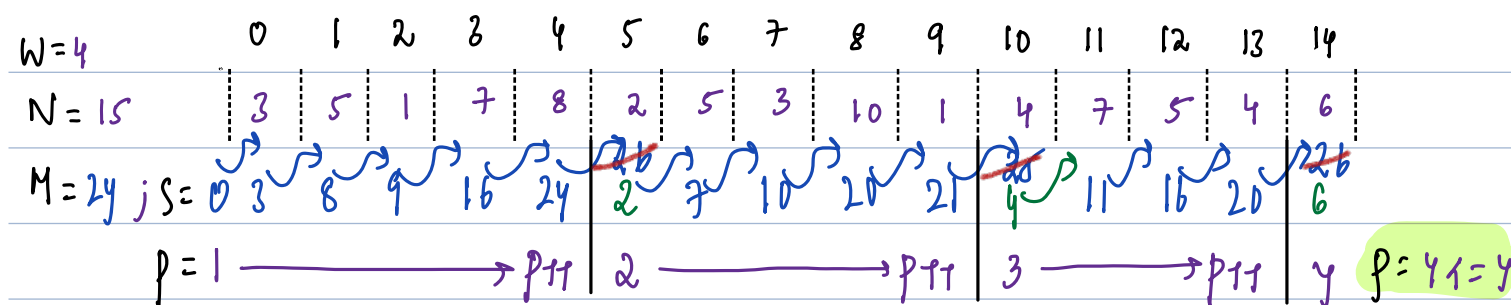
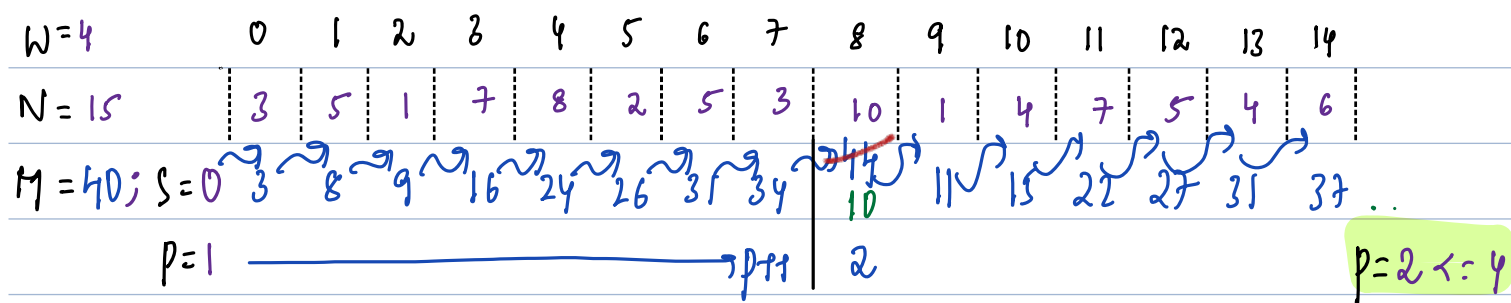
$m$

15 : Check if we can paint all boards  $\leq 15$  mins

[  $l$    ...    $m-1$     $m$     $m+1$    ...    $h$  ]

if we cannot do task in  $m$  time:  
 We cannot do task in  $m-1, m-2, \dots$   
 goto right

## Check function Idea:



l h m:  $l+h/2$  Can we finish tasks in m time update

10 71 40: Can we finish tasks in 40m  $ans = 40; h = m - 1;$

10 89 27: Can we finish tasks in 27m  $ans = 27; h = m - 1;$

10 23 16: Can we finish tasks in 16m  $l = m + 1;$

```

int minTime(vector<int> b, int n, int w){
    int l = max of b, h = sum of b, ans = 0;
    while(l <= h){
        int m = (l+h)/2;
        if( check(b, m, w) ){
            ans = m;
            h = m-1;
        }
        else{
            l = m+1;
        }
    }
    return ans;
}

```

```

bool check(vector<int> &b, int m, int w){
    int p=1, s=0;
    for(int i=0; i<b.size(); i++){
        s = s+b[i];
        if( s > m ){
            p++;
            s = b[i];
        }
    }
    return p <= w;
}

```

# i<sup>th</sup> task allocate new person

# p = person and w = workers allocated