

## Today's Content

1. Number System Basics
2. Binary to Decimal & Vice Versa
3. Adding 2 Binary Numbers
4. -ve numbers
5. Datatype range
6. When we multiply 2 numbers

$$S = 2^0 + 2^1 + 2^2 + \dots + 2^{N-2} + 2^{N-1} + 2^N = 2^{N+1} - 1$$

$$2S = \cancel{2^1 + 2^2 + 2^3 + \dots + 2^{N-1} + 2^N + 2^{N+1}}$$

$$S = \cancel{2^0 + 2^1 + 2^2 + \dots + 2^{N-2} + 2^{N-1} + 2^N}$$

$$S = 2^{N+1} - 2^0 \Rightarrow 2^{N+1} - 1$$

$$S = 2^0 + 2^1 + 2^2 : 2^3 - 1$$

$$S = 2^0 + 2^1 + 2^2 + 2^3 = 2^4 - 1$$

Decimal Number System → Each Digit: [0 1 ... 9]  
→ Each power: [10]

$$\begin{array}{c} 10^3 \quad 10^2 \quad 10^1 \quad 10^0 \\ \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\ 3 \quad 4 \quad 2 \end{array} = 3 \times 10^2 + 4 \times 10^1 + 2 \times 10^0 = 342$$

$$2 \quad 5 \quad 6 \quad 3 = 2 \times 10^3 + 5 \times 10^2 + 6 \times 10^1 + 3 \times 10^0 = 2563$$

Binary Number System → Each Digit: [0 1]  
→ Each power: [2]

$$\begin{array}{c} 2^5 \quad 2^4 \quad 2^3 \quad 2^2 \quad 2^1 \quad 2^0 \\ \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\ 1 \quad 0 \quad 0 \quad 1 \quad 0 \quad 1 \end{array} = 2^5 \times 1 + 2^2 \times 1 + 2^0 \times 1 = 37$$

$$\begin{array}{c} 2^4 \quad 2^3 \quad 2^2 \quad 2^1 \quad 2^0 \\ \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\ 1 \quad 0 \quad 0 \quad 1 \quad 1 \end{array} = 2^4 \times 1 + 2^1 \times 1 + 2^0 \times 1 = 19$$

$$\begin{array}{c} 2^5 \quad 2^4 \quad 2^3 \quad 2^2 \quad 2^1 \quad 2^0 \\ \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\ 1 \quad 1 \quad 0 \quad 0 \quad 1 \end{array} = 25$$

$$\begin{array}{c} 2^6 \quad 2^5 \quad 2^4 \quad 2^3 \quad 2^2 \quad 2^1 \quad 2^0 \\ \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\ 1 \quad 0 \quad 1 \quad 1 \quad 0 \quad 1 \quad 0 \end{array} = 2^6 + 2^4 + 2^3 + 2^1 = 64 + 16 + 8 + 2 = 90$$

Decimal to Binary →

2	<u>37</u>	
2	<u>18 : 1</u>	↑
2	<u>9 : 0</u>	
2	<u>4 : 1</u>	
2	<u>2 : 0</u>	
2	<u>1 : 0</u>	
	0 : 1	

2	<u>45</u>	
2	<u>22 : 1</u>	↑
2	<u>11 : 0</u>	
2	<u>5 : 1</u>	
2	<u>2 : 1</u>	
2	<u>1 : 0</u>	
	0 : 1	

$$37 : \begin{array}{c} 2^5 \quad 2^4 \quad 2^3 \quad 2^2 \quad 2^1 \quad 2^0 \\ \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\ 1 \quad 0 \quad 0 \quad 1 \quad 0 \quad 1 \end{array} = 2^5 + 2^2 + 2^0 = 37$$

2	45	
2	22 : 1	↑
2	11 : 0	
2	5 : 1	
2	2 : 1	
2	1 : 0	
	0 : 1	

$$45 : \begin{array}{c} 2^5 \quad 2^4 \quad 2^3 \quad 2^2 \quad 2^1 \quad 2^0 \\ \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\ 1 \quad 0 \quad 1 \quad 1 \quad 0 \quad 1 \end{array} = 2^5 + 2^3 + 2^2 + 2^0 = 45$$

Add 2 Decimal numbers:  $c = s/10$   $d = s \% 10$   $10 \rightarrow$  Decimal

$13/10$   $14/10$

$\begin{array}{r} 17 \\ 9/10 \quad 7 \end{array} \quad \begin{array}{r} 17 \\ 8 \end{array} \quad 9$   
 $\begin{array}{r} 0 \\ 1 \end{array} \quad \begin{array}{r} 4 \end{array} \quad 5$   
 $s = \downarrow$   $9\%10$   $13\%10$   $14\%10$   
 $d = 0$  9 3 4

Add 2 Binary Numbers  $c = s/2$   $d = s \% 2$   $2 \rightarrow$  Binary Number System

$2^4$   $2^3$   $2^2$   $2^1$   $2^0$   
 $1/2$   $3/2$   $2/2$   $1/2$

$c =$   $\begin{array}{r} 0 \end{array} \quad \begin{array}{r} 1 \end{array} \quad \begin{array}{r} 1 \end{array} \quad \begin{array}{r} 0 \end{array}$   
 $1/2$   $\begin{array}{r} 1 \end{array} \quad \begin{array}{r} 0 \end{array} \quad \begin{array}{r} 1 \end{array} \quad \begin{array}{r} 1 \end{array} \quad 0 \rightarrow 22$   
 $0$   $\begin{array}{r} 0 \end{array} \quad \begin{array}{r} 0 \end{array} \quad \begin{array}{r} 1 \end{array} \quad \begin{array}{r} 1 \end{array} \quad 1 \rightarrow 7$   
 $s = \downarrow$   $1\%2$   $1\%2$   $3\%2$   $2\%2$   $1\%2$   
 $d = 0$  1 1 1 0 1  $\rightarrow 29$

$3/2$   $2/2$   $2/2$   $3/2$   $2/2$

$c =$   $1$   $\begin{array}{r} 1 \end{array} \quad \begin{array}{r} 1 \end{array} \quad \begin{array}{r} 1 \end{array} \quad \begin{array}{r} 1 \end{array}$   
 $0$   $\begin{array}{r} 1 \end{array} \quad \begin{array}{r} 1 \end{array} \quad \begin{array}{r} 0 \end{array} \quad \begin{array}{r} 1 \end{array} \quad \begin{array}{r} 1 \end{array}$   
 $0$   $\begin{array}{r} 1 \end{array} \quad \begin{array}{r} 0 \end{array} \quad \begin{array}{r} 1 \end{array} \quad \begin{array}{r} 1 \end{array} \quad \begin{array}{r} 1 \end{array}$   
 $s = \downarrow$   $3\%2$   $2\%2$   $2\%2$   $3\%2$   $2\%2$   
 $d =$  1 1 0 0 1 0

1

 $2^4 \quad 2^3 \quad 2^2 \quad 2^1 \quad 2^0$ 

✱

$$-a = 2^1 a = \underbrace{2}_{\geq 0} a + 1.$$
$$\sim a : \begin{array}{cccccc|c|c} & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 : & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}$$

↳ LSB : MSB ? > All bits combined.

$$-a: \begin{array}{cccccccc} -2^7 & 2^6 & 2^5 & 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 \end{array} : -2^7 + 2^6 + 2^5 + 2^4 + \dots + 2^1 + 2^0 = -128 + 115 = -13$$

Q:

## Numbers

unsigned int a=10;

#bits

Signed: MSB bare value is -ve

Unsigned: MSB base value is the

4

$$\begin{array}{cccc} \frac{0}{-2^3} & \frac{1}{2^2} & \frac{1}{2^1} & \frac{1}{2^0} \end{array} \rightarrow \begin{array}{cccc} \frac{1}{-2^3} & \frac{0}{2^2} & \frac{0}{2^1} & \frac{0}{2^0} \end{array} = -2^3$$

$$\begin{array}{cccc} \frac{0}{-2^3} & \frac{1}{2^2} & \frac{1}{2^1} & \frac{1}{2^0} \end{array} \rightarrow \begin{array}{cccc} \frac{0}{-2^3} & \frac{1}{2^2} & \frac{1}{2^1} & \frac{1}{2^0} \end{array} = 2^{-1}$$

$$\frac{1}{2^3} \quad \frac{1}{2^2} \quad \frac{1}{2^1} \quad \frac{1}{2^0} \quad \rightarrow \quad \frac{0}{2^3} \quad \frac{0}{2^2} \quad \frac{0}{2^1} \quad \frac{0}{2^0} = 0$$

$$\frac{1}{2^3} \quad \frac{1}{2^2} \quad \frac{1}{2^1} \quad \frac{1}{2^0} \quad \rightarrow \quad \frac{1}{2^3} \quad \frac{1}{2^2} \quad \frac{1}{2^1} \quad \frac{1}{2^0} = 2^4 - 1$$

8

$$\begin{array}{ccccccc} \frac{1}{-2^8} & \frac{0}{2^6} & \frac{0}{2^5} & \dots & \frac{0}{2^1} & \frac{0}{2^0} & = -2^{-7} \end{array}$$
  

$$\begin{array}{ccccccc} \frac{0}{-2^8} & \frac{1}{2^6} & \frac{1}{2^5} & \dots & \frac{1}{2^1} & \frac{1}{2^0} & \end{array}$$
  

$$\begin{array}{ccccccc} \frac{0}{-2^8} & \frac{1}{2^6} & \frac{1}{2^5} & \dots & \frac{1}{2^1} & \frac{1}{2^0} & = 2^{-7} - 1 \end{array}$$

$$\frac{0}{2^7} + \frac{0}{2^6} + \frac{0}{2^5} + \dots + \frac{0}{2^1} + \frac{0}{2^0} = 0$$

N

$$\begin{array}{ccccccc} \frac{1}{2^{N-1}} & \frac{0}{2^{N-2}} & \frac{0}{2^{N-3}} & \dots & \frac{0}{2^1} & \frac{0}{2^0} & -2^{N-1} \\ \swarrow & & & & & & \\ -2^{N-1} & 2^{N-2} & & & & & \end{array}$$

Diagram illustrating a butterfly network structure for  $N=8$ . The nodes are arranged in two rows. The top row nodes are labeled  $0, 1, 2, 3, 4, 5, 6, 7$ . The bottom row nodes are labeled  $2^N-1, 2^N-2, \dots, 2^1, 2^0$ . Arrows indicate the flow of data from the top row to the bottom row, showing a butterfly pattern of connections.

## Datatype Ranges:

#datatype	#bits N	Min Signed: $-2^{N-1}$	Max $2^{N-1}-1$	Min Unsigned: 0	Max $2^N-1$
int	32	$-2^{31}$ $\{-2 \times 10^9\}$	$2^{31}-1$ $2 \times 10^9\}$	0 $\{0$	$2^{32}-1$ $4 \times 10^9\}$
long	64	$-2^{63}$ $\{-8 \times 10^{18}\}$	$2^{63}-1$ $8 \times 10^{18}\}$	0 $\{0$	$2^{64}-1$ $16 \times 10^{18}\}$

Approximations:  $2^{10} = 1024 \approx 1000 = 10^3$  #  $2^{10} \approx 10^3$

Case 1:

$$2^{10} \approx 10^3$$

Cube on both side

$$(2^{10})^3 \approx (10^3)^3$$

$$2^{30} \approx 10^9$$

Multiply by 2

$$2^{31} \approx 2 \times 10^9$$

Multiply by 2

$$2^{32} \approx 4 \times 10^9$$

Case 2:

$$2^{10} \approx 10^3$$

power 6 on both side

$$(2^{10})^6 \approx (10^3)^6$$

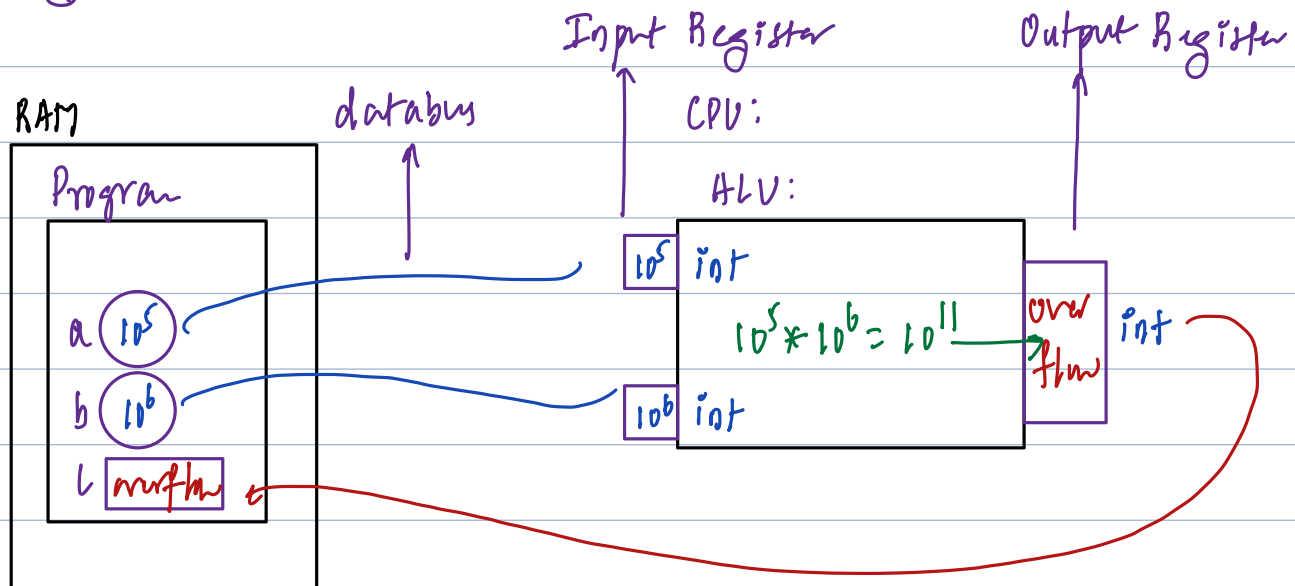
$$2^{60} \approx 10^{18}$$

Multiply by  $2^3$

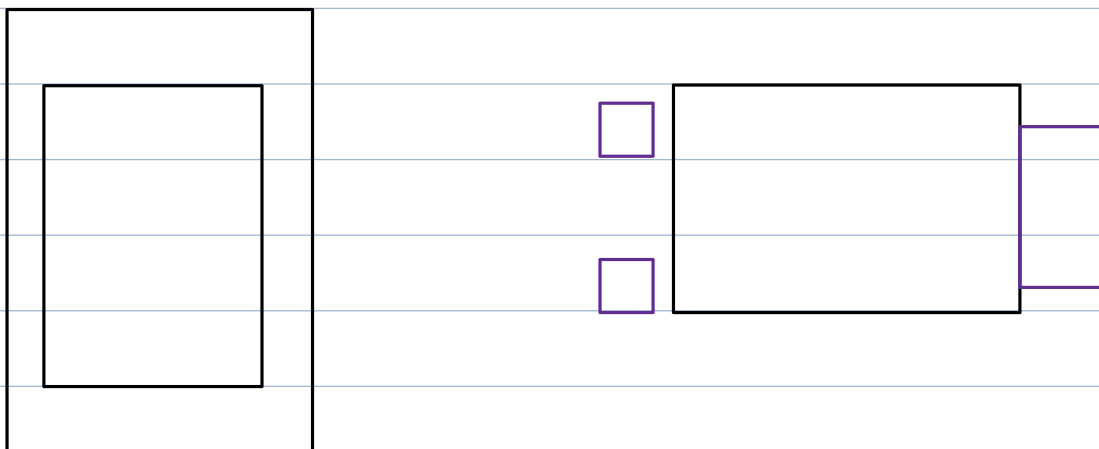
$$2^{63} \approx 8 \times 10^{18}$$

Q1:  $\text{int } a = 10^5, b = 10^6$   
 $\text{int } c = a * b; \# 10^5 * 10^6 = 10^{11}$   
 $\text{print}(c); \# \text{wrong}$

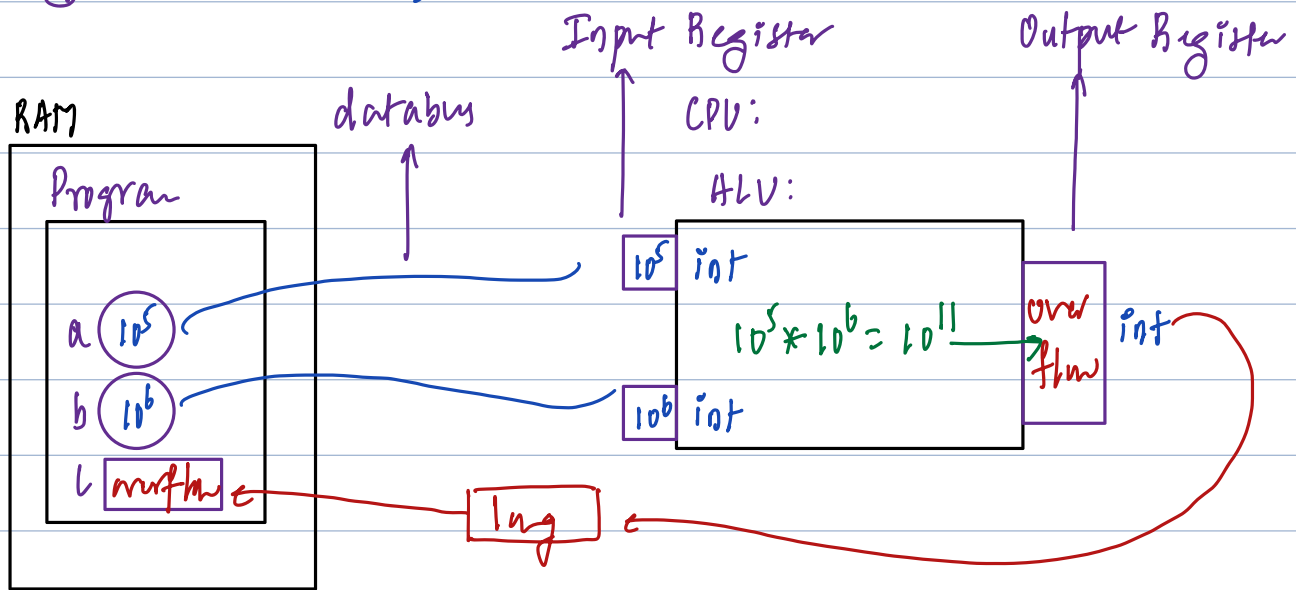
Q2:  $\text{int } a = 10^5, b = 10^6 \checkmark$   
 $\text{long } c = a * b; \checkmark$



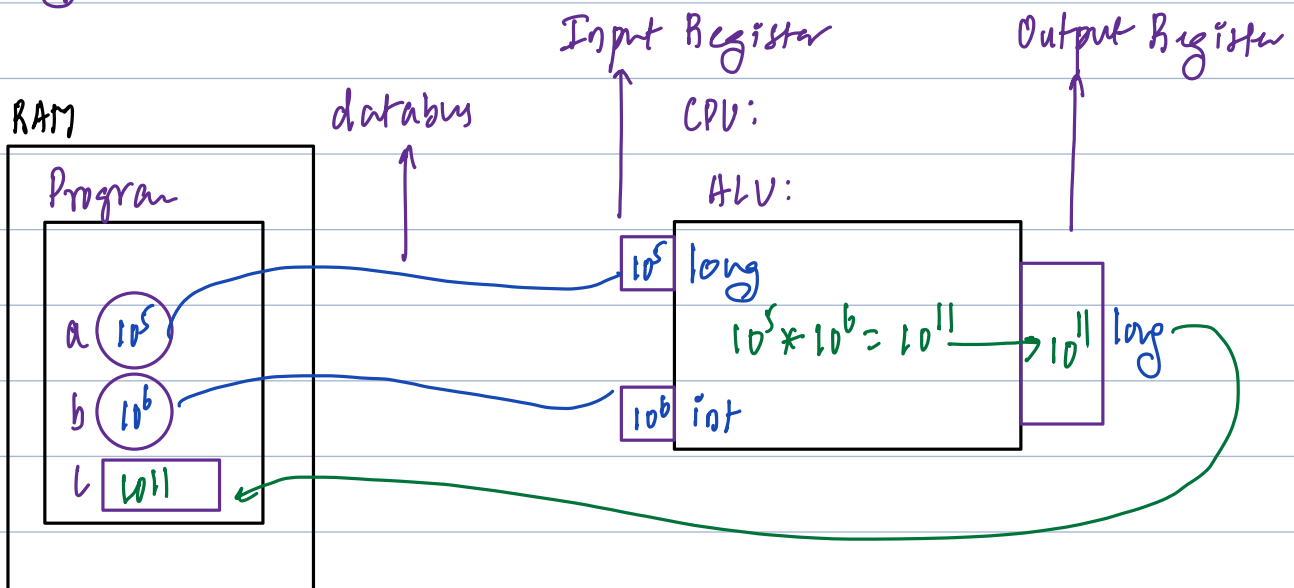
Q3:  $\text{int } a = 10^5, b = 10^6$   
 $\text{long } c = (\text{long}) a * b;$



Q2:  $\text{int } a = 10^5, b = 10^6$  ✓  
 $\text{long } c = (\text{long}) a * b;$



Q2:  $\text{int } a = 10^5, b = 10^6$  ✓  
 $\text{long } c = (\text{long}) a * b;$





long  $a = 10^5$ ,  $b = 10^6$  ✓

long  $c = a * b$ ;

