

## Today's Content

1. Aggressive Cows

2. When to apply BS.

a. When can we discard search space : Monotonic



3. Smallest subarray with  $\text{sum} \geq k$ .

2Q) Given  $N$  Cows &  $M$  stalls, all stalls are on  $x$ -axis at different locations, Place all  $N$  cows in such a way that min distance between any 2 cows is maximized. Maximize min distance

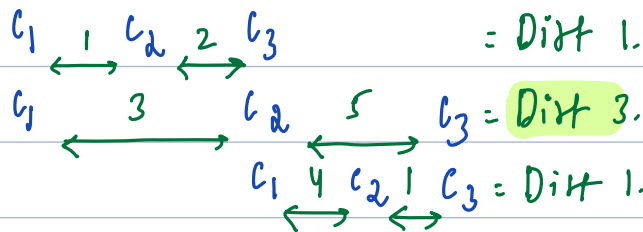
#Note1 In a stall only 1 cow can be present

#Note2 All cows have to placed, stalls  $M > N$  cows

#Note3 All stall positions are sorted, if not sorted we can sort them.

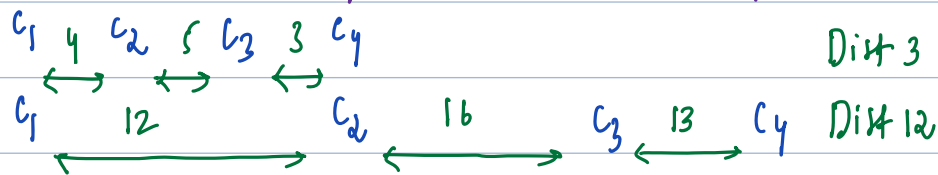
#Ex1:  $x$ -axis[]

#Stalls = 5      0      1      2      3      4  
#Cows = 3      { 1      2      4      8      9 } Inc Min Dist Between cows.



#Ex2:  $x$ -axis[]

#Stalls = 9      0      1      2      3      4      5      6      7      8  
#Cows = 4      { 2      6      11      14      19      25      30      39      43 }



Search:

Target: Maximum, minimum distance between 2 cows

Search Space: # It should be in distance

#Note: Search space, make sure ans is in space.

$[l \dots h]$   $l$  = smallest value  $h$  = greatest value

# Min distance among cows  $l$  = Min diff among adj elements,

# Max distance among cows  $h$  =  $ar[N-1] - ar[0]$ ; #last - first

# Final distance among  $M$  cows should be in search space  $[l \dots h]$

Discard:

#En2: n-cows()

#Stalls = 9      0      1      2      3      4      5      6      7      8  
#Cows = 4      { 2      6      11      14      19      25      30      39      43 }

Dist = 21 \*       $c_1$             $c_2$             $c_3$   $c_4$  \*

Dist = 10 ✓       $c_1$             $c_2$             $c_3$             $c_4$

#Search Space: {  $l$ ,  $h$  }

$l$        $h$        $m$  :      - - - - - T T T T T F F F F F F F F F

1      41      21 : Check if min dist among cows is atleast  $\geq 21$ .  
If we cannot keep them at 21 dist apart.

21      22      23 ...

We cannot keep them at 22 23... apart as well  
#go to left;  $h = m - 1$ ;

1      20      10 : Check if min dist among cows is atleast  $\geq 10$   
If we can keep them at 10 dist apart

7      8      9      10

We can keep them at 9, 8, 7... apart as well  
#update ans,  $l = m + 1$

# Dry Run:

# En2: n-cows()

# Stalls = 9      0      1      2      3      4      5      6      7      8  
# Cows = 4      { 2      6      11      14      19      25      30      39      43 }

d=13	c <sub>1</sub>				c <sub>2</sub>			c <sub>3</sub>	#C=3 < 4 *
d=12	c <sub>1</sub>	12		c <sub>2</sub>	16		c <sub>3</sub>	13	c <sub>4</sub> #C=4=4 ✓
d=15	c <sub>1</sub>				c <sub>2</sub>			c <sub>3</sub>	#C=3 < 4 *
d=10	c <sub>1</sub>			c <sub>2</sub>		c <sub>3</sub>		c <sub>4</sub>	#C=4 ✓
d=21	c <sub>1</sub>					c <sub>2</sub>			#C=2 < 4 *

d    h    m: dist h/2    Can we cows at m distance apart

1	41	21	Can we keep all cows at 21 dist apart *	h = m-1;
5	20	10	Can we keep all cows at 10 dist apart ✓	ans=10, d=m+1
11	20	15	Can we keep all cows at 15 dist apart *	h = m-1;
11	14	12	Can we keep all cows at 12 dist apart ✓	ans=12, d=m+1
13	14	13	Can we keep all cows at 13 dist apart *	h = m-1
13	12		# Stop process & return ans=12.	

int mindist (vector<int> &ar, int c) { # Search space size:

int N = ar.size();

$[l, h] = h - l + 1$

int l = 1, h = ar[N-1] - ar[0], ans = 0;

# Binary Search Iterative

while (l <= h) {

$= \log_2^{h-l+1}$

int m = (l+h)/2;

# Check function

if (check(ar, c, m)) {

$= N$

ans = m;

Tc:  $O(N \log_2^{h-l+1})$

l = m+1;

Sc:  $O(1)$

}

else {

h = m-1;

}

}

return ans;

bool check (vector<int> &ar, int m, int c) { Tc:  $O(N)$

int last\_c = ar[0], cows = 1;

for (int i = 1; i < ar.size(); i++) {

if (ar[i] - last\_c >= m) { # Can place cow at i<sup>th</sup> stall

cows++;

last\_c = ar[i];

}

}

if (cows < c) { return false; }

else { return true; }

For what type of problems we can try binary search?

Quesim:

1. Finding some target
2. Finding largest/smallest/Big/Max
3. Finding smallest/lr/smallest/Min.

When can we apply BS?

1. Target, 2. Search space 3. When we can discard search space.

When we can discard search space?

# Search space

F F F F T T T T..

If search space can be written as above or below pattern using idea in that search space we can discard.

T T T T + + + + ..

n

If using some idea : En: peak

# Monotonicity:

Increasing or Decreasing

Assume T=1, F=0

F F F F T T T T.. = 0 0 0 0 0 1 1 1 1

T T T T F F F F... = 1 1 1 1 1 0 0 0 0

Note: If search space follows, above pattern in your check function we can apply BS.

28 Given  $arr[N]$ , calculate length of smallest subarray with  $sum \geq k$

Constraints

$$1 \leq N \leq 10^6$$

$$1 \leq arr[i] \leq 10^9$$

#Ex1:

	0	1	2	3	4	5	6	7	8	
$arr[10] = \{$	3	2	5	7	4	8	9	2	6	$\} \quad k = 18$

#Ex2:

	0	1	2	3	4	5	6	7	8	9	
$arr[10] = \{$	4	5	7	3	6	9	8	3	2	4	$\} \quad k = 20$

#Idea1:

#Idea3:

Search:

Target:

Search Space: # Search  $m$  of subarray

# Note: Search space, make sure ans is in space.

$[l \dots h]$   $l$  = smallest value  $h$  = greatest value

# smallest subarray length  $l = 1$ .

# greatest subarray length  $h = N$

# length of subarray with  $sum \geq k$  will be in  $[l..h]$

#Ex2:

0 1 2 3 4 5 6 7 8 9  
ar[10] = { 4 5 7 3 6 9 8 3 2 4 } k = 20

Search Space: { }

l h m

1 10 5 ;

#check if there exists a subarray of len = 5 with sum >= 20 ✓

if subarray of len = 5, with sum >= 20 exists.

subarray of len > 5 with sum >= 20 exists

5 6 7 8 ...

$$a_1 + a_2 + a_3 + a_4 + a_5 \geq k$$

if we add + n

$$a_1 + a_2 + a_3 + a_4 + a_5 + n \geq k$$

$$m = 5; h = m - 1;$$

1 4 2 ;

#check if there exists a subarray of len = 2 with sum >= 20 ✓

if subarray of len = 2 with sum >= 20 doesn't exist

subarray of len < k with sum >= 20 doesn't exist

1 2

$$l = m + 1;$$



# Dry Run.

# Ex2:

	0	1	2	3	4	5	6	7	8	9	
arr[10] = {	4	5	7	3	6	9	8	3	2	4	}

k = 20

# Check if there exists a subarray of len  $m$  with sum  $\geq k$

$l$

$h$

$m$

int smallest(vector<int> &arr, int k) {

}

bool isSub(vector<int> &arr, int m, int k) {

}