

## Today's Content

1. Searching  $\Theta(n)$
2. Search in sorted array
3. Search floor of  $x$  in sorted array
4. Search  $i^{\text{th}}$  occurrence of  $x$  in sorted array

## Searching Basics

Bro/Sis missing → Policy stating

Phish : whom to search: Target

Example:

Target  $\iff$  SearchSpace

Word { Dict / Books / News Paper }

PhoneNo { Contact / Phonebook }

Obs: If SearchSpace is Ordered, Searching becomes easier.

### Search Dog in Dictionary



How to where to land? mid?

N  $\boxed{\frac{N}{2} \quad \frac{N}{2}}$  : Always Discard =  $\frac{N}{2}$ ; always  $\frac{N}{2}$  better

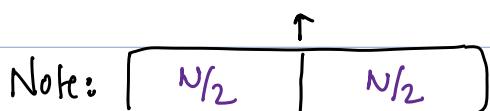
$\frac{N}{3}$  {wrrg.}

N  $\boxed{\frac{N}{3} \quad \frac{2N}{3}}$  : Discard left:  $\frac{N}{3}$  land at mid.

Discard right:  $\frac{2N}{3}$

When to apply BS:

After dividing search space into 2 parts, if we can discard 1 half of searchspace using some conditions, then we can apply binary search



Q) Given a sorted arr[N] search if k is present or not?

$$arr[10] = \{ 3, 6, 9, 12, 14, 19, 20, 23, 25, 27 \}$$

$$k = 12 : \text{Yes}$$

Idea1: Iterate on arr[] & search for k: (True/False) TC: O(N) SC: O(1)

Idea2: Target = k Search Space = In arr[]

$$== k$$



if (arr[mid] == k) { return True; }

$$\leftarrow \begin{matrix} k \\ k \end{matrix}$$



if (arr[mid] > k) { goto left; }

$$\leftarrow \begin{matrix} k \\ k \end{matrix}$$



if (arr[mid] < k) { goto right; }

Tracking Search Space:

We can use s & e to indicate search space

Ex1:  $arr[10] = \boxed{3, 6, 9, 12, 14, 19} \left[ \begin{matrix} 6 \\ 20 \end{matrix} \right] \boxed{23, 25, 27}$   $k = 20$

$$l \quad h \quad m : (l+h)/2$$

0 <= 9  $m = (0+9)/2 = 4$   $arr[m] < k$ : goto right  $l = m+1;$

5 <= 9  $m = (5+9)/2 = 7$   $arr[m] > k$ : goto left  $h = m-1;$

5 <= 6  $m = (5+6)/2 = 5$   $arr[m] < k$ : goto right  $l = m+1;$

6 <= 6  $m = (6+6)/2 = 6$   $arr[m] == k$ : return true;

0	1	2	3	4	$\left\{ \begin{array}{c} 5 \\ 6 \end{array} \right.$	7	8	9
3	6	9	12	14	19	20	23	25
$l$	$h$							

$$l \leftarrow h \quad m : (l+h)/2 \quad k = 15$$

$0 \leq 9 \quad m = (0+9)/2 = 4 \quad \text{arr}[m] < k : \text{goto right } l = m+1;$

$5 \leq 9 \quad m = (5+9)/2 = 7 \quad \text{arr}[m] > k : \text{goto left } h = m-1;$

$5 \leq 6 \quad m = (5+6)/2 = 5 \quad \text{arr}[m] > k : \text{goto left } h = m-1$

$5 \geq 4 \quad \# l > h \text{ stop process.}$

bool search( vector<int> &arr , int k ) { TC:  $O(\log_2 N)$  SC:  $O(1)$

int l=0, h=arr.size()-1;

while(  $l \leq h$  ) {

    int m = (l+h)/2;

    if( arr[m] == k ) { return true; }

    else if( arr[m] > k ) {

        h = m-1; # goto left

    else { # arr[m] < k    goto right }

        l = m+1;

        Goto right :  $l = m+1$

        Goto left :  $h = m-1$

}

3    return false;

TC: Initially Search Specie =  $N \xrightarrow{\text{1st}} N/2 \xrightarrow{\text{2nd}} N/4 \xrightarrow{\text{3rd}} N/8 \rightarrow \dots \dots \dots$  1 :  
 $\log_2 N$  iterations

28 Given a sorted arr[], find flow of a given num k

Floor: greater or  $i = k$

$$arr[] = \{ -5, 2, 3, 6, 9, 10, 11, 14, 18 \}$$

k else

5 : 3

Idea: Iterate in arr[] & update ans;

4 : 3

$k=8$  ans =  $-\infty$  TC:  $O(N)$  SC:  $O(1)$

10 : 10

i	arr[i]	ans =
---	--------	-------

24 : 18

0  $-5 \leftarrow 8$  ans =  $-5$ ; update & look for better

-7 :  $-\infty$ ;

1  $2 \leftarrow 8$  ans =  $2$ ; update & look for better

if flow doesn't

exist return

$-\infty$  // INT-MIN;

2  $3 \leftarrow 8$  ans =  $3$ ; update & look for better

3  $6 \leftarrow 8$  ans =  $6$ ; update & look for better

4  $9 \leftarrow 8$  9 not possible, Right or not possible.

return ans = 6

Idea2:

Target = greater or  $i = k$  Search Space = In arr[] ans =  $-\infty$

$= k$

	mid
--	-----

if  $arr[mid] == k$ : return  $k$

$\nwarrow$

sk	sk	mid
----	----	-----

if  $arr[mid] < k$ :

ans =  $arr[mid]$ ;

goto right

$\curvearrowright$

	mid	$>k>k$
--	-----	--------

if  $arr[mid] > k$ :

goto left

Ex:  $arr[] = \{ 2, 4, 8, 12, 15, 22, 24, 26 \}$

$k=20$

l h m

0 7 3  $arr[m] < k$ : ans =  $arr[m]$  goto right

4 7 5  $arr[m] > k$ : goto left

4 4 4  $arr[m] < k$ : ans =  $arr[m]$  goto right

5 4 # stop & return ans = 15.

int floorIndex(vector<int> &arr, int k) { TC:  $O(\log_2 N)$  SC:  $O(1)$

int l = 0, h = arr.size() - 1, ans = -∞;  
while (l <= h) {  
 int m = (l + h) / 2;  
 if (arr[m] == k) {  
 return k;  
 } else if (arr[m] < k) {  
 ans = arr[m];  
 l = m + 1; // goto right;  
 } else if (arr[m] > k) {  
 h = m - 1; // goto left;  
 }  
}  
return ans;

TODD: ceil(n): Smaller ele  $>= k$ .

38) Given an sorted arr[], find the first occurrence index of given element

arr[] = { 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16  
          -5 -5 -3 0 0 1 1 5 5 5 5 5 5 8 10 10 15 }  
k:                       $P_1 = 12 - 7 + 1 = 6$      $P_2 = \#occ = [P_1, P_2] = P_2 - P_1 + 1$

5: ?      Idea: Iterate in arr[] & get 1<sup>st</sup> occurance of k.

-5: 0      TC: O(N)    SC: O(1)

20: -1 if element doesn't exist

Idea2:

Target = 1<sup>st</sup> occurance index of k    Search Space = In arr[]    ans = -1;

$\nearrow k$



if  $arr[mid] > k$ : goto left

$\nwarrow k$



if  $arr[mid] < k$ : goto right

$\curvearrowleft m$



if  $arr[mid] == k$ :

ans = m; #update ans

goto left; #backward

Example:

Ex: arr[] = { 0 1 2 3 4 5 6 7  
          2 5 5 6 5 5 8 10 }

$k = 5$

0 1 2 3 4 5 6 7

2 5 5 6 5 5 8 10

↑ ↑  
h l

$l = h - m$

0  $l = 7 - 3 = 4$      $arr[m] == k$ :  $ans = 3$ ; goto left  $h = m - 1$

0  $l = 2 - 1 = 1$      $arr[m] == k$ :  $ans = 1$ ; goto left  $h = m - 1$

0  $l = 0 - 0 = 0$      $arr[m] < k$ : goto right  $l = m + 1$

1  $l = 0 - 1 = -1$     Stop & return ans = 1;

int firstOccurrence( vector<int> arr, int k) { TODO }

TODO: 1<sup>st</sup> occurrence of k in sorted arr[] ✓ TC: O(log<sub>2</sub><sup>N</sup>)

TODO: last occurrence of k in sorted arr[] ✓ TC: O(log<sub>2</sub><sup>N</sup>)

TODO: Frequency of k in sorted arr[] { p<sub>2</sub>[last\_occ\_index] - p<sub>1</sub>[1<sup>st</sup>\_occ\_index] }

$$TC: O(\log_2^N + \log_2^N) = O(\log_2^N)$$