

Today's Content

1. Equilibrium index
2. ABAB pairs
3. Triplet pairs.

Carry forward Idea:

When we calculate same data from $L \rightarrow R$ or $R \rightarrow L$ multiple times we carry forward & calculate data in only 1 iteration.

Note: We iterate from direction we are calculating

Equilibrium Index:

Given an array elements count no. of equilibrium indices.

An element $arr[i]$ is said to be equilibrium

if Sum of elements on left $[0..i-1]$ = right $[i+1..N-1]$ sum of elements on right

Note: if $i \geq 0$: No elements on left can assume = 0

if $i = N-1$: No elements on right can assume = 0

Sum $[0..i-1]$	i	Sum $[i+1..N-1]$
----------------	---	------------------

Constraints:

$$1 \leq N \leq 10^5$$

$$-10^9 \leq arr[i] \leq 10^9$$

Ex1: $arr[4] = \begin{matrix} & 0 & 1 & 2 & 3 \\ \begin{Bmatrix} -3 & 2 & 4 & -1 \end{Bmatrix} \end{matrix}$ ans = 1

$$\text{leftSum} = \begin{matrix} & 0 & -3 & -1 & 3 \end{matrix}$$

$$\text{RightSum} = \begin{matrix} 5 & 3 & -1 & 0 \end{matrix}$$

Ex2: $arr[7] = \begin{matrix} & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ \begin{Bmatrix} -7 & 1 & 5 & 2 & -4 & 3 & 0 \end{Bmatrix} \end{matrix}$ ans = 2

$$\text{leftSum} = \begin{matrix} 0 & -7 & -6 & -1 & 1 & -3 & 0 \end{matrix}$$

$$\text{RightSum} = \begin{matrix} 7 & 6 & 1 & -1 & 3 & 0 & 0 \end{matrix}$$

Idea1: for every $arr[i]$:

Estimated Tc: $O(N^2)$

check if equilibrium.

Iterate & calculate $lsum = [0..i-1]$

Iterate & calculate $rsum = [i+1..N-1]$

if $lsum == rsum$ {

$arr[i]$ is equilibrium

```
int equilibrium(int arr[], int n) {
```

```
    int c = 0;
```

```
    for (int i = 0; i < n; i++) {  $T.C: O(N^2)$   $S.C: O(1)$ 
```

```
        int lsum = 0, rsum = 0;
```

```
        for (int j = 0; j < i; j++) {  $\rightarrow [0..i-1] = i$ 
```

```
            lsum = lsum + arr[j];
```

```
        for (int j = i+1; j < n; j++) {  $\rightarrow [i+1..N-1] = N-i-1$ 
```

```
            rsum = rsum + arr[j];
```

```
        if (lsum == rsum) {
```

```
            c = i;
```

```
        }
    }
    return c;
```

```
}
```

$= N-1 \approx N$

$a \quad b$

$b-a+1$

$N-1-(i+1)+1 = N-1-i-x+x$

Trailing:

arr[] = { 0, 1, 2, 3, 4, 5, 6 }
 arr[] = { -7, 1, 5, 2, -4, 3, 0 }

Lsum: \rightarrow 0, -7, -6, -1, 1, -3, 0

Rsum: 7, 6, 1, -1, 3, 0, 0

Hint: For every $a \in \mathbb{Z}$:

check if equilibrium.

1. Iterate & calculate $\text{rtn} = [0..i-1]$ // Carry forward Left \rightarrow Right
2. Doubt: How to calculate $\text{rtn} =$

$$0 \ 1 \ 2 \dots i-1 \quad \boxed{i} \quad i+1 \ i+2 \dots N-1$$

$$lsun = [0..i-1]$$

$$rsum = [i+1..N-1] = \text{TotalSum} - \text{lsun} - ar[i]$$

obs:

1. Keep a loop and calculate Total sum
2. Lsum carry forward
3. Rsum calculate using Lsum

Dry Run :

un: 0 1 2 3 4 5 6
 arr[7] = { -7 1 5 2 -4 3 0 }
 Total = 0
 LSum = 0 0 -7 -6 -1 1 -3 0
 RSum = 7 6 1 -1 3 0 0

Total-Lbm-ar(i)

2. Count ABA Pairs:

Given a `char[]`, calculate no. of triplet indices = i, j, k such that $i < j < k$ & $ch[i] == 'a'$ & $ch[j] == 'b'$ & $ch[k] == 'a'$

Constraints:

$1 \leq N \leq 10^5$

$'a' \leq ch[i] \leq 'z'$

Ex: 0 1 2 3 4 5 6 7

`ch[] =` d a b a d b a c

Triplets: $i < j < k$ $ch[i] == 'a'$ & $ch[j] == 'b'$ & $ch[k] == 'a'$

1 2 3

1 2 6

3 5 6

1 5 6

6 5 3

Idea 1: Generate all triplets & check if they form valid pair.

Estimated: $O(N^3)$

int triplets(`char s[]`, int N) { T.C: $O(N^3)$ S.C: $O(1)$

int c = 0;

for (int i = 0; i < N; i++) {

for (int j = i + 1; j < N; j++) {

for (int k = j + 1; k < N; k++) {

if (`s[i] == 'a'` & `s[j] == 'b'` & `s[k] == 'a'`) {
 c++;
}

return c;

}

Idea2: In Triplet based Questions:

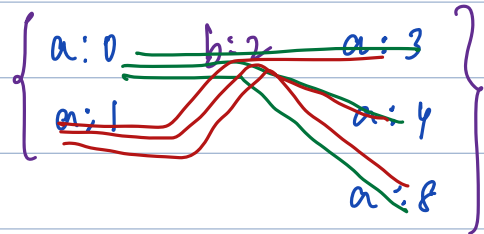
Try to find center element of the problem.

$i < j < k$
 $a \quad b \quad a$

Dry Run:

	j	j	j	j	j	j	j	j	
idx:	0	1	2	3	4	5	6	7	8
ch[8] =	a	a	b	a	a	b	e	b	a

	↓		↓		↓
la=2		la=4		la=4	
ra=3		ra=1		ra=1	
lr=6		lr=4		lr=4	



Steps: Iterate on ch[i]

if ch[j] == 'b'

1. Iterate on left & calculate a's : la
2. Iterate on right & calculate a's : ra
3. ans += la * ra

int pairs(char ch[], int n) { Tc: $O(N^2)$ Sc: $O(1)$

int ans = 0;

for(int j = 0; j < n; j++) {

if (ch[j] == 'b') {

int la = 0, ra = 0;

for(int i = 0; i < j; i++) {

if (ch[i] == 'a') { la++; }

for(int k = j+1; k < n; k++) {

if (ch[k] == 'a') { ra++; }

ans = ans + la * ra;

}

return ans;

[0.. j-1] j [j+1.. n-1]

Idea:

Iterate on $ch[]$

if $ch[j] == 'b'$

0. Iterate & calculate Total a's

1. Iterate on left & calculate a's : la // optimize with carry forward

2. How to calculate a's on right?

$$ra = \text{Total a's} - la's$$

$$3. ans += la * ra$$

Dry Run:

	0	1	2	3	4	5	6	7	8
$ch[8] =$	a	a	b	a	a	b	e	b	a
Total a's									
$la = 0 \rightarrow$	1	2	2	3	4	4	4	4	5
$ra = Ta - la$			3			1		1	
Triplets			6			4		4	

$$= 14.$$

int pairs(char ch[], int n) { T.C: $O(N+N) = O(N)$ S.C: $O(1)$

int ans = 0, la = 0, totala = 0, ra = 0;

for(int i = 0; i < N; i++) {

if(ch[i] == 'a') {

totala++;

}

for(int i = 0; i < N; i++) {

if(ch[i] == 'b') {

ra = totala - la;

ans = ans + la * ra;

else if (ch[i] == 'a') {

la++;

}

return ans;

la ra
b

Q3

Count of Triplets

Given $ar[N]$, calculate no: of triplets i, j, k s.t. $ar[i] < ar[j] < ar[k]$

Constraints:

$$1 \leq N \leq 10^3$$

Ex: $ar[5] = \{ 2 \ 6 \ 9 \ 4 \ 10 \}$

$i \quad j \quad k \quad ar[i] < ar[j] < ar[k]$

Idea:

Idea:

	0	1	2	3	4	5	6	7
arr:	3	2	6	4	7	10	9	12

$i \quad j \quad k \quad arr[i] < arr[j] < arr[k]$

	0	1	2	3	4	5	6	7
arr:	3	2	6	4	7	10	9	12

Count less :

Count more :