

Today's Content

1. Degree problems

2. Peak

Q: Given $arr[N]$ return length of smallest subarray with sum $\geq k$.

Constraints:

$$1 \leq N \leq 10^5$$

$$-10^9 \leq arr[i] \leq 10^9$$

$$-10^{11} \leq K \leq 10^{11}$$

Ex: $arr = \{ 4, -2, 3, 1, 4, 2, -3, 6 \}$ $k = 10$ $ans = 4$.

Idea: Generate all subarrays:

for all subarrays, iterate & calculate sum

if $sum \geq k$:

$ans = \min(ans, \text{subarray length})$

TC: $O(N^2 \times N) = O(N^3)$ SC: $O(1)$

Idea 2:

arr[] = { 4, -2, 3, 1, 4, 2, -3, 6 } k=10 ans=4.

s e len

{0 4} #sum >= k 5

{1 7} #sum >= k 7

{2 5} #sum >= k 4

ans = ∞;

for (i=0; i < N; i++) {

 # start subarray at i, iterate till we get sum >= k;

 sum = 0;

 for (j=i; j < N; j++) {

 sum = sum + arr[j];

 if (sum >= k) { # [i..j] {i... j j+1

 ans = min(ans, j-i+1);

 } break;

 }

}

return ans;

TC: O(N²) SC: O(1)

3. Binary Search.

Target: Min subarray len with $\text{sum} \geq k$

Search space: $l=1$ $r=N$

Discard? $m = (l+r)/2$

Say There exists a subarray of m length with $\text{sum} \geq k$

m $m+1$ $m+2$...

T T F T T F

Just because m is possible we cannot claim, everything m might be True, few might not be possible

Say There doesn't exist a subarray of m length with $\text{sum} \geq k$

m

T F T T F

We cannot directly discard left side, because for subarray lengths might be valid on left side.

4. 2 pointer: Not possible.

1. If in a technique BS is not possible, 2 pointer is not possible.

2. If subarray $\{i \dots j\}$ is possible:

$\{i \dots j\}$ $j+1$ $j+2$... $n-1$: We cannot claim outside are valid
 $\{i \dots j-2$ $j-1$ $j\}$: We cannot claim inside are valid

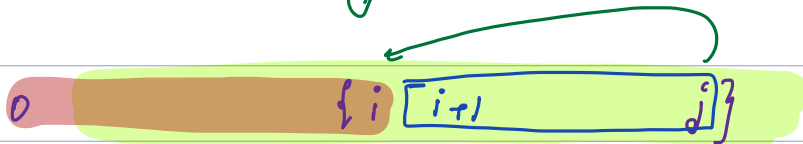
Idea 3: Subarray sums related problems start with pfc() sums

0 1 2 3 4 5 6 7 8 9 10 11
arr[] = { 3 2 1 2 3 1 -3 2 7 6 -10 6 } k = 10

0 ↘
↓
pfc[] = { -1 0 3 5 6 8 11 12 9 11 18 24 14 20 }

When dealing with subarray sums, there will always be a 0 at start.

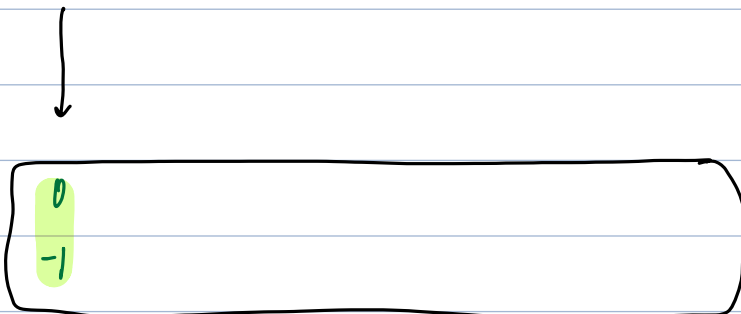
Obs: if # (i, j) = if $pfc[j] - pfc[i] \geq k$
subarray sum $[i+1 - j] \geq k$ len = $j - i$



Dry Run:


Step 1:

pfc[] = { -1 0 3 5 6 8 11 12 9 11 18 24 14 20 }



Steps 2:

pf[] = { -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 }
pf[] = { 0, 3, 5, 6, 8, 11, 12, 9, 11, 18, 24, 14, 20 }



| | |
|----|---|
| 0 | 3 |
| -1 | 0 |

Steps:


pf[] = { -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 }
pf[] = { 0, 3, 5, 6, 8, 11, 12, 9, 11, 18, 24, 14, 20 }



| | | | |
|------|----|---|---|
| val: | 0 | 3 | 5 |
| ind: | -1 | 0 | 1 |

Step 4:

pf[] = { -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 }
pf[] = { 0, 3, 5, 6, 8, 11, 12, 9, 11, 18, 24, 14, 20 }



| | | | | |
|------|----|---|---|---|
| val: | 0 | 3 | 5 | 6 |
| ind: | -1 | 0 | 1 | 2 |

Step 5:

pf[]: { -1 0 1 2 3 4 5 6 7 8 9 10 11
0 3 5 6 8 11 12 9 11 18 24 17 20 }



val: [0 3 5 6 8]
ind: [-1 0 1 2 3]

Step 6:

pf[]: { -1 0 1 2 3 4 5 6 7 8 9 10 11
0 3 5 6 8 11 12 9 11 18 24 17 20 }

11
4 d=5

val: [0 3 5 6 8]
ind: [-1 0 1 2 3]
len = 5

Step 7:

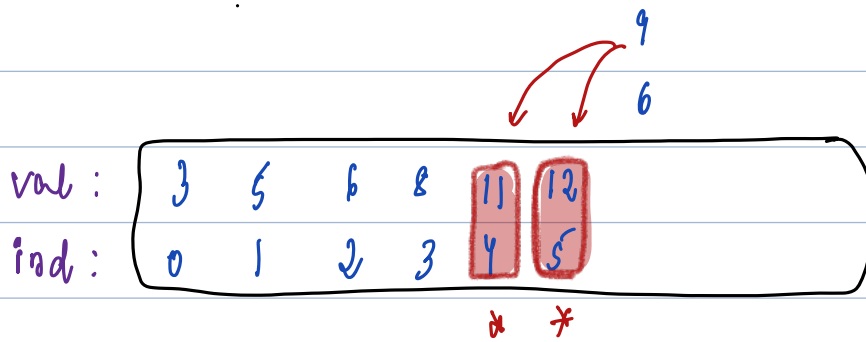
pf[]: { -1 0 1 2 3 4 5 6 7 8 9 10 11
0 3 5 6 8 11 12 9 11 18 24 17 20 }



val: [3 5 6 8 11 12]
ind: [0 1 2 3 4 5]

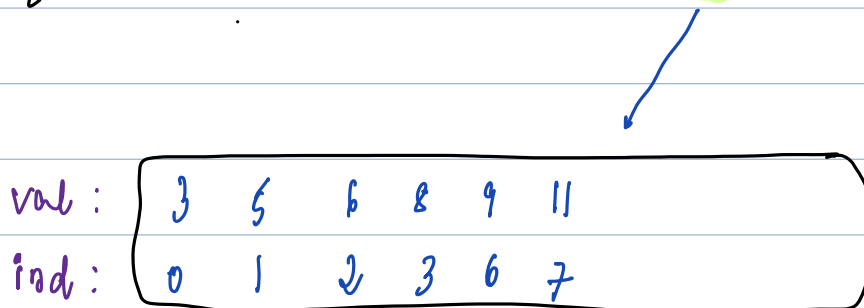
Step 8:

-1 0 1 2 3 4 5 6 7 8 9 10 11
pf[] = { 0 3 5 6 8 11 12 9 11 18 24 14 20 }



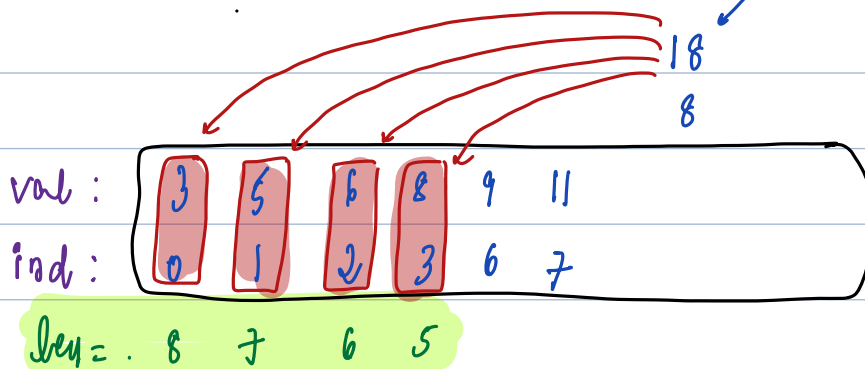
Step 9:

-1 0 1 2 3 4 5 6 7 8 9 10 11
pf[] = { 0 3 5 6 8 11 12 9 11 18 24 14 20 }



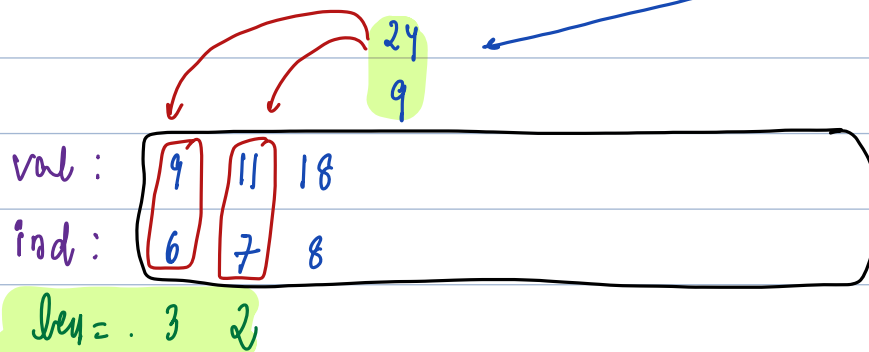
Step 10:

-1 0 1 2 3 4 5 6 7 8 9 10 11
pf[] = { 0 3 5 6 8 11 12 9 11 18 24 14 20 }



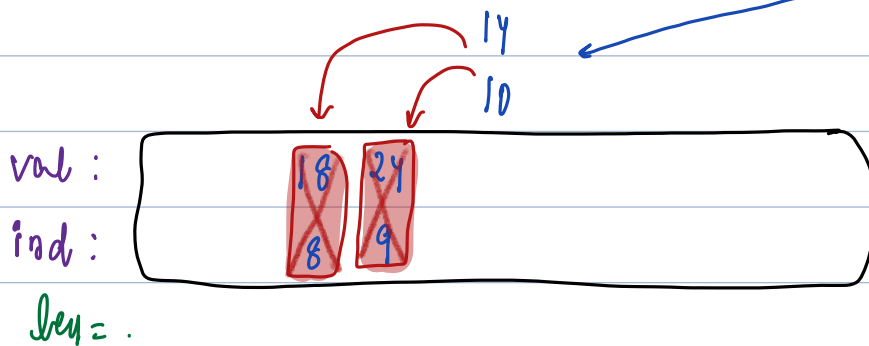
Step 11:

-1 0 1 2 3 4 5 6 7 8 9 10 11
pf[] = { 0 3 5 6 8 11 12 9 11 18 24 14 20 }



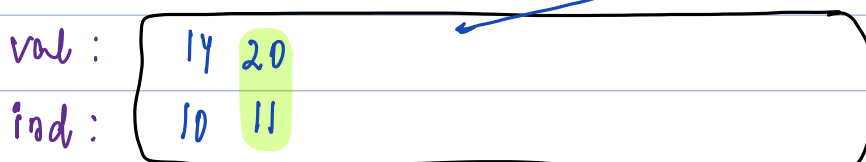
Step 12:

-1 0 1 2 3 4 5 6 7 8 9 10 11
pf[] = { 0 3 5 6 8 11 12 9 11 18 24 14 20 }



Step 13:

-1 0 1 2 3 4 5 6 7 8 9 10 11
pf[] = { 0 3 5 6 8 11 12 9 11 18 24 14 20 }



Con: Keep data in the order in Deque & $u = \text{index}$

For iterate $Pf[R]$:

TC: $O(N \times N) = O(N^2)$ SC: $O(N)$

Iterate at start of Deque:

if $Pf[R] - \text{start of Deque} \geq k$:

update ans;

remove start;

else

break

Iterate at back of Deque:

if $Pf[R] \leq \text{end of Deque}$:

remove end

else

insert $Pf[R]$ at end.

```
int length(vector<int> row) {
```

```
    long sum = 0;
```

```
    vector<long> pf(0, av.size());
```

```
    long sum = 0;
```

```
    for (int i = 0; i < av.size(); i++) {
```

```
        sum = sum + av[i];
```

```
        pf[i] = sum;
```

```
    }
```

```
deque<pair<long, int>> dq;
```

```
dq.push_front({0, -1});
```

```
int ans = 0;
```

```
for (int i = 0; i < av.size(); i++) {
```

```
    // for pf[i] first chunk, if we can have ans;
```

```
    while (dq.size() > 0 && pf[i] - dq.front().first >= k) {
```

```
        ans = min(ans, i - dq.front().second);
```

```
        dq.pop_front();
```

```
    }
```

```
    // Before inserting pf[i] check if we can delete elements
```

```
    while (dq.size() > 0 && pf[i] < dq.front()) {
```

```
        dq.pop_back();
```

```
    }
```

```
    dq.push_back({pf[i], i});
```

```
return ans;
```