

Today's Content

1. Sum of xor of all pairs
- 2.

pow a^n using bit manipulation

1Q Given an $arr[N]$ find sum of $n \times n$ of all pairs

Ex:

$arr[3] = \{4 \ 2 \ 5 \ 3\}$ return 56

#pairs

$(4^1 4)$	$(4^1 2)$	$(4^1 5)$	$(4^1 3)$
$(2^1 4)$	$(2^1 2)$	$(2^1 5)$	$(2^1 3)$
$(5^1 4)$	$(5^1 2)$	$(5^1 5)$	$(5^1 3)$
$(3^1 4)$	$(3^1 2)$	$(3^1 5)$	$(3^1 3)$

#Idea1: Generate all $n \times n$ pairs & calculate $n \times n$ for each pair and add in sum;

Given $arr[N]$; TC: $O(N^2)$ SC: $O(1)$

int sum = 0;

for (int i = 0; i < N; i++) {

for (int j = 0; j < N; j++) {

sum = sum + $arr[i] \times arr[j]$

return sum;

#Idea2: Iterate in lower or upper triangle calculate sum of $n \times n$ of pairs & return $sum \times 2$;

$arr[3] = \{4 \ 2 \ 5 \ 3\}$ return 56

i \ j:	0	1	2	3	Upper
<u>0</u>	$(4^1 4)$	$(4^1 2)$	$(4^1 5)$	$(4^1 3)$	$i=0, j=1$
<u>1</u>	$(2^1 4)$	$(2^1 2)$	$(2^1 5)$	$(2^1 3)$	$i=1, j=2$
<u>2</u>	$(5^1 4)$	$(5^1 2)$	$(5^1 5)$	$(5^1 3)$	$i=2, j=3$
<u>3</u>	$(3^1 4)$	$(3^1 2)$	$(3^1 5)$	$(3^1 3)$	

Given $arr[N]$; TC: $O(N^2/2)$ SC: $O(1)$

int sum = 0;

for (int i = 0; i < N; i++) {

for (int j = i + 1; j < N; j++) {

sum = sum + $arr[i] \times arr[j]$

return $sum \times 2$;

Idea 1: Contribution: Add contribution of each element.

Update: Add contribution of each bit.

Ex: arr[] = { 15 24 11 19 28 9 }

↓

Dry Run:

	2^4	2^3	2^2	2^1	2^0	
	4	3	2	1	0	
15 =	0	1	1	1	1	$1^m: 1$
24 =	1	1	0	0	0	$1^m: 0$
11 =	0	1	0	1	1	
19 =	1	0	0	1	1	
28 =	1	1	1	0	0	
9 =	0	1	0	0	1	

# Bits:	# Set	# UnSet	# Pairs	# Contribution
0	{15 11 19 9} : 4	{24 30} = 2	$4 * 2 * 2 = 16$	$16 * 2^0$
1	{15 11 19} : 3	{24 28 9} = 3	$3 * 3 * 2 = 18$	$18 * 2^1$
2	{15 28} : 2	{24 11 19 9} = 4	$2 * 4 * 2 = 16$	$16 * 2^2$
3	{15 24 11 28 9} : 5	{19} = 1	$5 * 1 * 2 = 10$	$10 * 2^3$
4	{24 19 28} : 3	{15 11 9} = 3	$3 * 3 * 2 = 18$	$18 * 2^4$

Add contribution of each bit Final ans = 984

Idea 2:

Iterate on each bit: [0 31] = i

Calculate contribution of bit position i.

Iterate on arr[] & calculate no. of elements with i^{th} bit set c.

Set = c, # Unset = N - c # Pairs = $c * (N - c) * 2$

Contribution of i^{th} bit = Pairs * 2^i

```
long pairSum (vector<int> &arr)
```

```
long sum = 0;
```

```
int N = arr.size();
```

```
for (int i = 0; i < 32; i++) { # i = bit position
```

```
    # For i, calculate no. of elements with ith bit set.
```

```
    int c = 0;
```

```
    for (int j = 0; j < N; j++) { # j: arr element
```

```
        if ((arr[j] >> i) & 1 == 1) {
```

```
            c++;
```

```
    # Set = c, # Unset = N - c, # Pairs = 2 * c * (N - c)
```

```
    # Contribution =  $2 * c * (N - c) * 2^i$ 
```

```
    sum = sum +  $2 * c * (N - c) * (1 \ll i)$ 
```

```
return sum;
```

TODD: Sum of OR of all pairs ✓ Medium

TODD: Sum of AND of all pairs ✓ Simple

28 Given a, n calculate & return a^n

Ex: $a \quad n$

3 5

2 4

#Ideas:

long power(int a, int n) {

3

28 Given $arr[N]$ it contains all elements from $1..N$.

1 element from 1 to N repeats

1 element from 1 to N missing

Return both repeat & missing element

Note: No Extra space, No modifying array.

Constraints:

$$1 \leq N \leq 10^6$$

$$1 \leq arr[i] \leq N.$$

Ex:

$arr[5] = \{ 2, 2, 1, 4, 5 \}$

missing

3

repeat

2

$arr[7] = \{ 1, 3, 6, 5, 4, 6, 7 \}$

2

6

Idea: With $arr()$ elements consider all elements from $1..N$

If we consider entire data:

Every $arr[i]$ element repeats twice except 2 elements

These 2 elements are repeating & missing elements: TODO

Given $arr()$

Expected Data

$arr[7] = \{ \cancel{1}, \cancel{2}, 6, \cancel{3}, \cancel{4}, \cancel{5}, \cancel{6}, \cancel{7} \} \quad \{ \cancel{1}, 2, \cancel{3}, \cancel{4}, \cancel{5}, \cancel{6}, \cancel{7} \}$