

## Todays Content

1. Majority Element
2. Min swaps  $\leq B$  elements

## #Majority Element

Given  $\text{ar}[n]$  elements, return majority element.

An element is said to be majority, if its frequency  $> n/2$

#If no majority return -1.

Ex1:      0 1 2

$$\text{ar}[3] = \{2, 1, 4\} \quad \# \text{No majority return -1};$$

Ex2:      0 1 2 3 4 5 6

$$\text{ar}[7] = \{3, 4, 3, 2, 4, 4, 4\} \quad \text{freq}(4) > 7/2 \\ 4 > 3 \quad \Rightarrow$$

Ex3:      0 1 2 3 4 5 6 7

$$\text{ar}[8] = \{3, 3, 4, 2, 4, 4, 2, 4\} \quad \text{freq}(4) > 8/2 \\ 4 > 4 \quad *$$

#No majority return -1;

Ex4:      0 1 2 3 4 5 6 7 8 9 10

$$\text{ar}[11] = \{3, 4, 3, 6, 1, 3, 2, 5, 3, 3, 3\} \quad \text{freq}(3) > 11/2 \\ 6 > 5$$

Ex5:      0 1 2 3 4 5 6 7 8 9

$$\text{ar}[10] = \{4, 6, 5, 3, 4, 5, 6, 4, 4, 4\} \quad \text{freq}(4) > 10/2 \\ 5 > 5$$

#No majority return -1;

Q At max how many diff majority elements we can have? 1.

$$\text{ar}[N] = \boxed{>n/2} \quad \boxed{\leq n/2}$$

↳ Thus, we cannot have another majority.

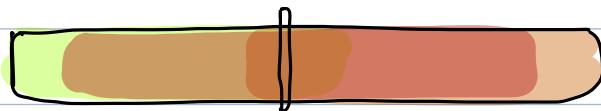
Ideas: For every element  $ar[i]$ :

Iterate on  $ar[]$  & calculate freq of  $ar[i] = c$ .  
if ( $c > n/2$ ) { return  $ar[i]$ }  
3

return -1;

TC:  $\Theta(N \times N) = \Theta(N^2)$  SC:  $\Theta(1)$

Ideas: Sort  $ar[]$  & Take cent ele =  $ar[n/2]$



Iterate on  $ar[]$  & get freq of  $ar[n/2] = c$ .

if ( $c > n/2$ ) {  
} return  $ar[n/2]$   
else {  
} return -1;

TC:  $\Theta(n \log n + n)$

↳ Sort ↳ Check middle element

Ideas: Insert all elements in hashmap

For every element  $ar[i]$ :

Get freq of  $ar[i]$  from hashmap =  $c$ .

if ( $c > n/2$ ) { return  $ar[i]$ }  
3

return -1;

TC:  $\Theta(N + N * 1) = \Theta(N)$  SC:  $\Theta(N)$

hashmap

# Ideay : election GQA 13 Seats

Ex1:

Manash : ♀ ♂ ♂ ♂ ♂ ♂ ♂ ♂ ♂ ♂ ♂ ♂ ♂ ♂

Nazneen : ♀ ♂ ♂ ♂

Munaf : ♂ ♂

$N = 13$

# Seats(Manash) > 13/2 ♀ ♀  $N = N - 2$

# Seats(Manash) > 11/2 ♀ ♀  $N = N - 2$

# Seats(Manash) > 9/2 ♀ ♀  $N = N - 2$

# Seats(Manash) > 7/2

Obs: By deleting 2 diff items, majority will not change.

Ex2: election Andaman 7 Seats

Manash : ♀ ♂ ♂ ♂ ♂ ♂ ♂

Nazneen : ♀ ♀

Munaf : ♂  $N = 7$

# Seats(Manash) > 7/2 ♀ ♀  $N = N - 2$

# Seats(Manash) > 5/2 # majority lost

Obs: By deleting 2 same items, we might lose majority

# Ideay:

1. keep deleting 2 distinct elements, until we have a 1 unique element

2. iterate & check if 1 unique ele is majority.

0 1 2 3 4 5 6 7 8 9 10

Ex1:  $ar[11] = \{ \underline{\textcolor{red}{1}} \ \underline{\textcolor{red}{1}} \ \underline{\textcolor{red}{3}} \ \underline{\textcolor{red}{3}} \ \underline{\textcolor{red}{6}} \ \{ \underline{\textcolor{red}{3}} \ \underline{\textcolor{red}{3}} \ \underline{\textcolor{red}{8}} \ \underline{\textcolor{red}{8}} \ \underline{\textcolor{red}{3}} \ \underline{\textcolor{red}{3}} \ \underline{\textcolor{red}{3}} \}$

0 1 2 3 4 5 6 7 8 9 10

Ex2:  $ar[11] = \{ \underline{\textcolor{red}{1}} \ \underline{\textcolor{red}{1}} \ \underline{\textcolor{red}{3}} \ \underline{\textcolor{red}{3}} \ \underline{\textcolor{red}{4}} \ \underline{\textcolor{red}{6}} \ \underline{\textcolor{red}{6}} \ \underline{\textcolor{red}{4}} \ \underline{\textcolor{red}{4}} \ \underline{\textcolor{red}{1}} \ \underline{\textcolor{red}{1}} \}$

# Moore's Voting Algo:

$arr[11] = \{3, 3, 4, 6, 1, 5, 2, 5, 3, 3, 3\}$

#ele = 4  $\neq 3$

freq = 0 1 2 1 0 1 0 1 0 1 2 3

Iterate in arr[]:

if freq of 3 = 6 > 11/2 : return 3.

$arr[11] = \{4, 6, 5, 3, 4, 5, 6, 4, 4, 4, 10\}$

#ele = 4  $\neq 6$

freq = 0 1 0 1 0 1 0 1 0 1 2 1

Iterate in arr[]:

if freq of 9 = 5 > 11/2 : return -1; # No Majority.



```
int majorityElement(vector<int> &arr) {
```

```
    int ele = 0, f = 0;
```

```
    int n = arr.size();
```

```
    for (int i = 0; i < n; i++) {
```

```
        # Compare 2 ele # {ele, arr[i]}
```

```
        if (f == 0) {
```

```
            ele = arr[i]; f = 1;
```

```
        } else if (ele == arr[i]) {
```

```
            f++;
```

```
        } else { # ele != arr[i]
```

```
            f--;
```

```
}
```

```
    int l = 0;
```

```
    for (int i = 0; i < n; i++) {
```

```
        if (arr[i] == ele) {
```

```
            c++;
```

```
        if (c > n / 2) { return ele; }
```

```
    } else { return -1; }
```

```
}
```

Variation: If's majority of its  $> n/3$

Hint: Delete 3 distinct elements majority won't change

{ ele1 ele2, arr[i] }

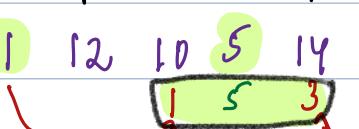
f1 f2

[ TODO: 5 conditions ]

2.0 Min swaps to bring all elements  $i=B$  together.

0 1 2 3 4 5 6 ↳ continuous / Subarray

Ex1:  $\text{arr}[] = \{1, 12, 10, 5, 14, 10, 3\}$   $B=8$  #swaps=2



Ex2:  $\text{arr}[] = \{3, 7, 6, 13, 2, 15\}$   $B=7$  #swaps=1



Ex3:  $\text{arr}[] = \{25, 30, 2, 18, 7, 6, 9, 50, 3\}$   $B=10$  #swap=1



#Ideas

Hint1: subarray of fix length.

0 1 2 3 4 5 6 7 8

$\text{arr}[] = \{19, 11, 3, 9, 7, 25, 6, 20, 4\}$   $B=10$



#subarray  $k=5$   $i=B$   $> B$

s..e	#good ele	#bad ele	#swaps	
0 .. 4	3	2	2 swaps	
1 .. 5	3	2	2 swaps	
2 .. 6	4	1	1 swap	
3 .. 7	3	2	2 swaps	
4 .. 8	3	2	2 swaps	

ans = min swaps = 1

#obs For every subarray of len=k  $TC: O(N-k+1) + O(k) = O(N^2)$  so:  $O(1)$

Iterate & calculate no: of bad elements = no: of swaps

$$\text{ans} = \min(\text{ans}, \text{swaps})$$

return ans;

## # Idea 2 Optimize with sliding window

We slide no. of bad elements forward.

	0	1	2	3	4	5	6	7	8
$\text{arr}[] = \{ 11, 19, 11, 9, 7, 25, 6, 20, 4 \}$									

## # subarray

s .. e	remove	add	#bad ele	#swaps
0 .. 4			2	2
1 .. 5	19	25	2	2
2 .. 6	11	6	1	1
3 .. 7	3	20	2	2
4 .. 8	9	4	2	2

Note:

If we remove bad element decrease count by 1

If we add bad element increase count by 1

int minSwaps(vector<int> &arr, int B) { TC:  $O(N + N) = O(N)$  SC:  $O(1)$

↳ # Sliding window of len = k  
 ↳ # Count ele  $= B$ , say = k

