Todays Content

1. Modular Arthmetic
2. Problems based on %

1. Modular Arithmetic Introduction.

$A \% M$ = Remainder when A divided by M

$x \% 4 = \{0\ 1\ 2\ 3\}$ $x \% 5 = \{0\ 1\ 2\ 3\ 4\}$

Range : $[0 \dots M-1]$

Why do we need %. To compress range to our use case

$$\left.\begin{matrix} -\infty \\ \\ \infty \end{matrix}\right] \longrightarrow \% M = \underset{\text{Min}\quad\text{Max}}{[0 \quad M-1]}$$

Rules for % Arithmetic : $\{+, -, *, /\}$

$$\underset{\{0\ 2\}}{a} + \underset{\{0\ 5\}}{b} = \underset{\{0\ 7\}}{a+b}$$

1. $\boxed{(a+b) \% m = (a \% m + b \% M) \% M}$

$[0..M-1]$ $\quad [0..M-1] + [0..M-1] = \{0 \quad 2M-2\} \% M = \{0..M-1\}$

Ex: $a = 9,\ b = 8,\ m = 5$

$(9+8) \% 5 = (9 \% 5 + 8 \% 5) \% 5$

$(17) \% 5 = (4 + 3) \% 5$

$2 = (7) \% 5 = 2$

2. $\boxed{(a \% m) \% M = (a \% m) \% m = a \% m}$

$[0..M-1] \% M = [0..M-1]$

3. $(a + m) \% m = (a \% m + m \% m) \% m$

$= (a \% m + 0) \% m$

$= (a \% m) \% m$

$\boxed{(a + m) \% m = a \% M}$

Note: if we have %m m outside, adding +m inside bracket won't effect my value.

Conceptua..

$$\left.\begin{matrix}-\infty \\ +\infty\end{matrix}\right\} \rightarrow \% M = [0..M-1]$$

In C/C++/Java:

%M on -ve numbers will give -ve remainders

↳ Later:

In C/C++/Java/

Say $a < 0$:

$$\boxed{a \% M = (a \% M + M) \% M}$$

This part should come for except

C++ Python

$\{0..M-1\}$ $\{-(M-1)..0\} + M = \{1..M\} \% M = \{0..M-1\}$

In C/C++/Java

-8 % 6

-10 % 7

In Python

Say $a < 0$: $a \% M = [0..M-1]$

5 $\boxed{(a-b) \% m = (a \% m - b \% m + m) \% m}$

$[0..M-1]$ $[0..M-1] - [0..M-1] = (-(M-1) \quad M-1) + M = (1 \quad 2M-1) \% M = [0..M-1]$

Ex:

$$\frac{a}{[4 \quad 10]} - \frac{b}{[6 \quad 12]} = \frac{a-b}{[-8 \quad 4]}$$

5. $(a * b) \% m = (a \% m * b \% m) \% m$

$[0..M-1]$ $[0..M-1] * [0..M-1] \rightarrow \{0..(M-1)^2\} \% M = \{0..M-1\}$

6. $(a^2) \% m = (a * a) \% m$

$= (a \% m * a \% m) \% m$

$= (a \% m)^2 \% m$

6. $\boxed{(a^2) \% m = (a \% m)^2 \% m}$

7. $\boxed{(a^b) \% m = (a \% m)^b \% m}$

Properties.

$(a+b)\%m = (a\%m + b\%m)\%m$

$(a*b)\%m = (a\%m * b\%m)\%m$

$(a+m)\%m = a\%m$

$(a\%m)\%m = a\%m$

$(a-b)\%m = (a\%m - b\%m + m)\%m$

$(a^b)\%m = (a\%m)^b\%m$

For -ve numbers $a\%m = (a\%m + m)\%m$

Quizes:

Q1: $(37^{103} - 1)\%12 = (37^{103}\%12 - 1\%12 + 12)\%12$

$= ((37\%12)^{103}\%12 - 1\%12 + 12)\%12$

$= ((1)^{103}\%12 - 1\%12 + 12)\%12$

$= (\cancel{1\%12} - \cancel{1\%12} + 12)\%12$

$= (0)$

Divisibility rule : 3 : Sum of digits divisible by 3. $\Rightarrow$ Prove Induction HW)

$(789)\%3 = (700 + 80 + 9)\%3$

$= (700\%3 + 80\%3 + 9\%3)\%3$

$= ((7*100)\%3 + (8*10)\%3 + 9\%3)\%3$

$= ((7\%3 * 100\%3)\%3 + (8\%3 * 10\%3)\%3 + 9\%3)\%3)$

$= ((7\%3)\%3 + (8\%3)\%3 + 9\%3)\%3$

$= (7\%3 + 8\%3 + 9\%3)\%3$

$= (7+8+9)\%3$

Q1: Given ar[N] ele find count of pairs (i, j) such that

(arr[i]+arr[j]) % M = 0 Note: i != j and pair(i, j) same as pair(j, i)

Constraints

$1 <= N <= 10^5$

$1 <= arr[i] <= 10^9$

0 1 2 3 4 5

Ex: ar[] = { 4 3 6 3 8 12 } ans =

M = 6 (ar[i] + ar[j]) % M = 0.

(ar[1] + ar[3]) % 6 = (3+3) % 6 = 0

(ar[0] + ar[4]) % 6 = (4+8) % 6 = 0

(ar[2] + ar[5]) % 6 = (6+12) % 6 = 0

Idea1: Generate all pairs & check if sum % M = 0;

```
c=0;                    TC: O(N^2)   SC: O(1)
i=0; i < N; i++) {
    j = i+1; j < N; j++) {
        if((ar[i] + ar[j]) % M == 0) {
            c++;
        }
    }
}
return c;
```

Ideas: $(arr[i] + arr[j]) \% M \rightarrow (\underline{arr[i] \% M} + \underline{arr[j] \% M}) \% M$

Hint: Calculate $arr[i] \% M$ values

Eg:

		0	1	2	3	4	5	6	7	8	9	10
M = 6	A[ ] =	2	5	4	8	14	13	6	12	24	16	19
	A[ ] =	2	5	4	2	2	1	0	0	0	4	1

el % M = [0 .. M-1], M = 6, [0 .. 5]

obs: $(\underline{arr[i] \% M} + \underline{arr[j] \% M}) \% M = 0$

Eg M = 6: (2 + 4) % 6 = 0

: (1 + 5) % 6 = 0

: (3 + 3) % 6 = 0

$\boxed{\text{obs1: } (k + M-k) \% M = 0}$

Also M = 6: (0 + 0, ~~6~~) % 6 = 0

$\boxed{\text{obs2: } (0 + 0) \% M = 0}$

Conclusion

Case 1: $arr[i] \% M = 0 \rightleftarrows arr[j] \% M = 0$

Case 1: $arr[i] \% M = k \rightleftarrows arr[j] \% M = M-k$

	0	1	2	3	4	5	6	7	8	9	10
$M=6$ $A[\,] =$	2	5	4	8	14	13	6	12	24	16	19

$A[i] \% M$

$A[\,] = \{2\ 5\ 4\ 2\ 2\ 1\ 0\ 0\ 0\ 4\ 1\}$

Eg $M=6$ obs: $(\underline{arr[i]\%M} + \underline{arr[j]\%M}) \% M = 0$

$(\quad 2 \quad$ AND $\quad 4 \quad) \ \% \ = 0$

$\begin{Bmatrix} 2 & a_1 \\ 8 & a_2 \\ 14 & a_3 \end{Bmatrix} \quad \begin{Bmatrix} b_1 & 4 \\ b_2 & 16 \end{Bmatrix}$ # $3*2 = 6$ pairs

Eg $M=6$ obs: $(\underline{arr[i]\%M} + \underline{arr[j]\%M}) \% M = 0$

$M=6$ $(\quad 1 \qquad\qquad 5 \quad) \ \%6 = 0$

$\begin{Bmatrix} 13 & a_1 \\ 19 & a_2 \end{Bmatrix} \quad \begin{Bmatrix} b_1 & 5 \end{Bmatrix}$ # $2*1 = 2$ pairs

Hint: Store count of all remainders, to get count of pairs

where?

1. cnt[M] when we do %M : [0 1 .. M-1] & for each remainder store count.

	0	1	2	3	4	5	6	7	8	9	10
M = 6 A[ ] =	2	5	4	8	14	13	6	12	24	16	19

A[i] % M

A[ ] = {2 5 4 2 2 1 0 0 0 4 1}

Very diff!!
hashmap &
tower
less in cour
problem

M = 6

ar[i] % M   ar[j] % M   c = 0;

Remainder	Pair for it	Count Frequency
2	4	c = c + 0
5	1	c = c + 0
4	2	c = c + 1
2	4	c = c + 1
2	4	c = c + 1
1	5	c = c + 1
0	0	c = c + 0
0	0	c = c + 1
0	0	c = c + 2
→ 4	→ 2	c = c + 3
1	5	c = c + 1

	0	1	2	3	4	5
cnt[6] =	3	2	3	0	2	1

return c;   11

```
int pairs(vector<int> &arr, int M){   TC: O(N+N) = O(N)    SC: O(M)

    int N = arr.size();
    for(int i=0; i<N; i++){
        arr[i] = arr[i] % M
    }

    vector<int> cnt(M,0);
    int pair=0;
    for(int i=0; i<N; i++){

        int t = M-ar[i];
        if(ar[i]==0){
            t=0;
        }

        pair = pair + cnt[t];  # how many times target is present.
        cnt[ar[i]]++;  # Insert array element
    }
    return pair;
}
```