

Today's Content

1. Modular Arithmetic

2. Problems based on %.

1. Modular Arithmetic Introduction.

$A \% M$ = Remainder when A divided by M

$$n \% 4 = \{0, 1, 2, 3\} \quad n \% 5 = \{0, 1, 2, 3, 4\}$$

Range: $[0 \dots M-1]$

Why do we need % To compress range to our use case

$$\left. \begin{matrix} -\infty \\ \infty \end{matrix} \right] \rightarrow \% M = \text{Min Max} \quad [0 \dots M-1]$$

Rules for % Arithmetic: $\{+, -, +, /\}$

$$\frac{a}{\{0, 2\}} + \frac{b}{\{0, 5\}} = \frac{a+b}{\{0, 7\}}$$

1. $(a+b)\%M = (a\%M + b\%M)\%M$

$$[0..M-1] + [0..M-1] = \{0, 2M-2\} \% M = \{0..M-1\}$$

$$\text{Ex: } a=9, b=8, M=5$$

$$(9+8)\%5 = (9\%5 + 8\%5)\%5$$

$$(17)\%5 = (4+3)\%5$$

$$2 = (7)\%5 = 2$$

2. $(a\%M)\%M = (a\%M)\%M = a\%M$

$$[0..M-1]\%M = [0..M-1]$$

3. $(a+m)\%M = (a\%M + m\%M)\%M$

$$= (a\%M + 0)\%M$$

$$= (a\%M)\%M$$

$(a+m)\%M = a\%M$

Note: if we have $\%M$ M outside, adding $+m$ inside bracket won't effect my value.

#Concepts..

$$\left. \begin{array}{c} -\infty \\ +\infty \end{array} \right\} \rightarrow \%_M = \{0..M-1\}$$

In C/C++/Java:

$\%M$ m-re numbers will give the remainders

In C/C++/Java:

↳ Latw:

Say $a < 0$:

$$a \% M = (a \% M + M) \% M$$

↓ the part should come for exapn
G11 By

$$\{0..M-1\} \quad \{-M..0\} + M = \{1..M\} \% M = \{0..M-1\}$$

In C/C++/Java

In Python

$$\text{Say } a < 0: \quad a \% M = \{0..M-1\}$$

-8%6
-10%7

$$5. \quad (a-b) \% M = (a \% M - b \% M + M) \% M$$

$$\{0..M-1\} \quad \{0..M-1\} - \{0..M-1\} = \underbrace{(-M)}_{[-M..0]} + M = \{1..M-1\} \% M = \{0..M-1\}$$

$$\text{Ex: } \frac{a}{[4 \ 10]} - \frac{b}{[6 \ 12]} = \frac{a-b}{[-8 \ 4]}$$

$$5. \quad (a+b) \% M = (a \% M + b \% M) \% M$$

$$\{0..M-1\} + \{0..M-1\} \rightarrow \{0..(M-1)^2\} \% M = \{0..M-1\}$$

$$6. \quad (a^2) \% M = (a \% M)^2 \% M$$

$$= (a \% M * a \% M) \% M$$

$$= (a \% M)^2 \% M$$

$$6. \quad (a^2) \% M = (a \% M)^2 \% M$$

$$7. \quad (ab) \% M = (a \% M)^b \% M$$

Properties.

$$(a+b)\%m = (a\%m + b\%m)\%m$$

$$(a*b)\%m = (a\%m * b\%m)\%m$$

$$(a+m)\%m = a\%m$$

$$(a\%m)\%m = a\%m$$

$$(a-b)\%m = (a\%m - b\%m + m)\%m$$

$$(a^b)\%m = (a\%m)^b \%m$$

$$\text{For re numbers } a\%m = (a\%m + m)\%m$$

Quizes:

$$Q1: (37^{103} - 1)\%12 = (37^{103}\%12 - 1\%12 + 12)\%12$$

$$= ((37\%12)^{103}\%12 - 1\%12 + 12)\%12$$

$$= (\underline{\underline{1}}^{103}\%12 - 1\%12 + 12)\%12$$

$$= (\cancel{1\%12} - \cancel{1\%12} + 12)\%12$$

$$= (0)$$

Divisibility rule : 3 : Sum of digit's divisible by 3. \Rightarrow Plus 9 added ~~861~~

$$(789)\%3 = (700 + 80 + 9)\%3$$

$$= (700\%3 + 80\%3 + 9\%3)\%3$$

$$= ((7*100)\%3 + (8*10)\%3 + 9\%3)\%3$$

$$= (7\%3 + \underline{100\%3})\%3 + (8\%3 + \underline{10\%3})\%3 + 9\%3\%3$$

$$= (7\%3 + 8\%3 + 9\%3)\%3$$

$$= (7+8+9)\%3$$

Q1: Given $\text{arr}[N]$ we find count of pairs (i, j) such that
 $(\text{arr}[i] + \text{arr}[j]) \% M = 0$ Note: $i \neq j$ and $\text{pair}(i, j)$ same as $\text{pair}(j, i)$

constraints

$$1 \leq N \leq 10^5$$

$$1 \leq \text{arr}[i] \leq 10^9$$



Ex: $\text{arr}[] = \{4, 3, 6, 3, 8, 12\}$ ans =

$$M=6 \quad (\text{arr}[i] + \text{arr}[j]) \% M = 0.$$

$$(\text{arr}[1] + \text{arr}[3]) \% 6 = (3 + 3) \% 6 = 0$$

$$(\text{arr}[0] + \text{arr}[4]) \% 6 = (4 + 8) \% 6 = 0$$

$$(\text{arr}[2] + \text{arr}[5]) \% 6 = (6 + 12) \% 6 = 0$$

Ideal: Generate all pairs & check if sum $\% M = 0$:

$c = 0$

TC: $O(N^2)$ SC: $O(1)$

$i = 0; i < N; i++ \{$

$j = i+1; j < N; j++ \{$

$\{ \text{if } [(\text{arr}[i] + \text{arr}[j]) \% M = 0] \{$

$c++;$

return c ;

Ideas: $(\text{ar}[i] + \text{ar}[j]) \% M \rightarrow (\underline{\text{ar}[i]} \% M + \underline{\text{ar}[j]} \% M) \% M$

Hint: Calculate $\text{ar}[i] \% M$ values

Ex:

$$M=6 \quad A[] = \{2 \ 5 \ 4 \ 8 \ 14 \ 13 \ 6 \ 12 \ 24 \ 16 \ 19\}$$

$$A[] = \{2 \ 5 \ 4 \ 2 \ 2 \ 1 \ 0 \ 0 \ 0 \ 4 \ 1\}$$

$$\text{clu} \% M = [0..M-1], M=6 \quad [0..5] \leftarrow$$

$$\text{obs: } (\underline{\text{ar}[i]} \% M + \underline{\text{ar}[j]} \% M) \% M = 0$$

$$\text{Ex } M=6: (2 + 5) \% 6 = 0$$

$$(1 + 5) \% 6 = 0$$

$$(3 + 3) \% 6 = 0$$

$$\boxed{\text{obs1: } (k + M-k) \% M = 0}$$

$$\text{Assume } M=6: (0 + 0, \cancel{K}) \% 6 = 0$$

$$\boxed{\text{obs2: } (0 + 0) \% M = 0}$$

Conclusion

$$\text{Case1: } \text{ar}[i] \% M = 0 \Leftrightarrow \text{ar}[j] \% M = 0$$

$$\text{Case2: } \text{ar}[i] \% M = k \Leftrightarrow \text{ar}[j] \% M = M-k$$

0 1 2 3 4 5 6 7 8 9 10

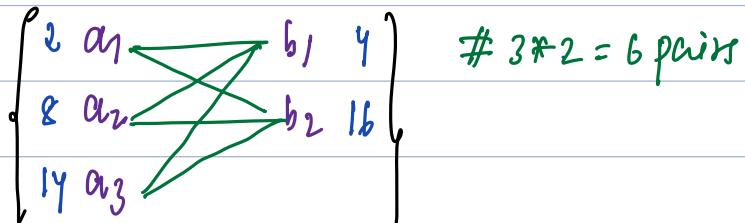
$$M=6 \quad A[] = \{2, 5, 4, 8, 14, 13, 6, 12, 24, 16, 19\}$$

$A[i] \% M$

$$A[] = \{2, 5, 4, 2, 2, 1, 0, 0, 0, 4, 1\}$$

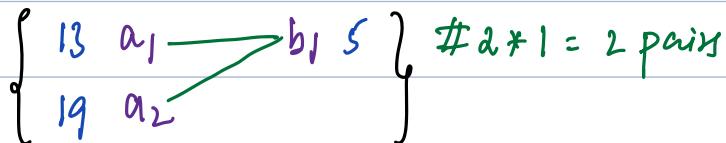
$$\text{Eqn } M=6 \quad \text{obs: } (\underline{\text{arr}[i]\%M} + \underline{\text{arr}[j]\%M}) \% M = 0$$

$$(2 \text{ AND } 4) \% 6 = 0$$



$$\text{Eqn } M=6 \quad \text{obs: } (\underline{\text{arr}[i]\%M} + \underline{\text{arr}[j]\%M}) \% M = 0$$

$$M=6 \quad (1 \quad \quad \quad 5) \% 6 = 0$$



hint: Sum count of all remainders, to get count of pairs

where?

l. $\text{cnt}[M]$ ↗ when we do $\% M = [0 \dots M-1]$ q, for each remainder
the count.

0 1 2 3 4 5 6 7 8 9 10

$$M=6 \quad A[] = \{ 2, 5, 4, 8, 14, 13, 6, 12, 24, 16, 19 \}$$

$A[i] \% M$

$$A[] = \{ 2, 5, 4, 2, 2, 1, 0, 0, 0, 4, 1 \}$$

$M=6$

$ar[i] \% M$

$ar[j] \% M$

$c=0$

0 1 2 3 4 5

Remainder

Pair for it

Count Frequency

$cnt[6] = \{ 3, 2, 3, 0, 2, 1 \}$

2

4

$C = C + 0$

5

1

$C = C + 0$

4

2

$C = C + 1$

2

4

$C = C + 1$

2

4

$C = C + 1$

1

5

$C = C + 1$

0

0

$C = C + 0$

0

0

$C = C + 1$

0

0

$C = C + 2$

$\rightarrow 4$

$\rightarrow 2$

$C = C + 3$

1

5

$C = C + 1$

return c_j

11

last diff

hashmap &

over

key in curr

probles

```
int pairs(vector<int> &ar, int M){ TC: O(N+N) = O(N) SC: O(M)
```

```
int N = ar.size();
for(int i=0; i < N; i++) {
    ar[i] = ar[i] % M
}
```

```
vector<int> cnt(m, 0);
```

```
int pair = 0;
for(int i=0; i < N; i++) {
```

```
    int t = M - ar[i];

```

```
    if(ar[i] == 0) {
```

```
        t = 0;
    }
```

```
    pair = pair + cnt[t]; # how many times target is present.
```

```
    cnt[ar[i]] += 1; # Insert array element
```

```
}
```

```
return pair;
```

```
}
```