

## Todays Content

1. Deque problems
2. Peak

IQ: Given  $arr[n]$  return length of smallest subarray with sum  $\geq k$ .

Constraints:

$$1 \leq N \leq 10^5$$

$$-10^9 \leq arr[i] \leq 10^9$$

$$-10^{11} \leq k \leq 10^{11}$$

Ex:

$$arr = \{ 4, -2, 3, 1, 4, 8, -3, 6 \} \quad k = 10 \quad ans = 4.$$

Ideas: Generate all subarrays:

for all subarrays, iterate & calculate sum

if  $sum \geq k$ :

$$ans = \min(ans, \text{subarray length})$$

$$TC: O(N^2 * N) = O(N^3) \quad SC: O(1)$$

Idea2:

$$arr[] = \{ 0, 1, 2, 3, 4, 5, 6, 7 \} \quad k=10 \quad arr = 4.$$

# s e sum

{0 4} #sum >= k 5

{1 7} #sum >= k 7

{2 5} #sum >= k 9

ans = ∞;

i=0; i < N; i++ {

# start subarray at i, iterate till we get sum >= k;

sum = 0;

j=i; j < N; j++ {

sum = sum + arr[j];

if (sum >= k) { i..j }

{ i .. j .. j+1 }

ans = min(ans, j-i+1);

} break;

}

return ans;

TC: O(N^2) SC: O(1)

### 3. Binary Search.

Target : Min subarray len with  $\text{sum} \geq k$

Search Space:  $i=1 \quad h=N$

Discard?  $m = (l+r)/2;$

Say There exists a subarray of  $m$  length with  $\text{sum} \geq k$

$m \quad m+1 \quad m+2 \dots$

T T F T T F

Just because  $m$  is possible we cannot claim, everything  $m$  right is True, few might not be possible

Say There doesn't exist a subarray of  $m$  length with  $\text{sum} \geq k$

$m$

T F T T F

We cannot directly discard left side, because for subarray lengths might be valid on left side.

### 4. 2 pointer: Not possible.

1. If  $m$  a technique BS is not possible, 2 pointer to is not possible.

2. If Subarray  $\{i \dots j\}$  is possible:

$\{i \dots j\} \ j+1 \ j+2 \dots n-1$  : We cannot claim outside are valid  
 $\{i \dots j-2 \ j-1 \ j\}$  : We cannot claim inside are valid

Ideas: Subarray sums related problems start with  $\text{Pf}(i)$  sums

$$\text{ar}[i] = \{ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 \}$$
$$\text{ar}[i] = \{ 3, 2, 1, 2, 3, 1, -3, 2, 7, 6, -10, 6 \} \quad k=10$$

0 ↗ ↓

$$\text{Pf}[i] = \{ -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 \}$$
$$\text{Pf}[i] = \{ 0, 3, 5, 6, 8, 11, 12, 9, 11, 18, 24, 14, 20 \}$$

/

# When dealing with subarray sums, there will always be a 0 at start.

# Obs: if  $\#(i, j) = \text{if } \text{Pf}(j) - \text{Pf}(i) > k$

subarray sum  $[i+1-j] \geq k \quad \text{len} = j-i$

0 

# Dry Run:

Steps:

$$\text{Pf}[i] = \{ -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 \}$$
$$\text{Pf}[i] = \{ 0, 3, 5, 6, 8, 11, 12, 9, 11, 18, 24, 14, 20 \}$$

↓



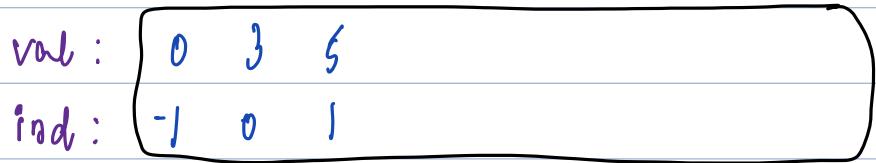
Step 2:

	-1	0	1	2	3	4	5	6	7	8	9	10	11
$f(x) = \{ 0 \ 3 \ 5 \ 6 \ 8 \ 11 \ 12 \ 9 \ 11 \ 18 \ 24 \ 14 \ 20 \}$													



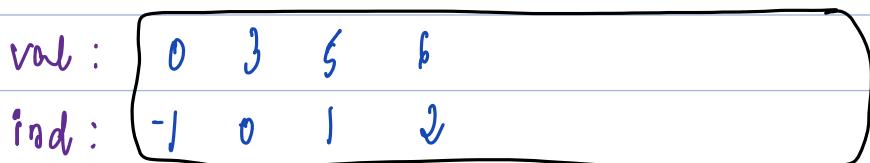
Step 3:

	-1	0	1	2	3	4	5	6	7	8	9	10	11
$f(x) = \{ 0 \ 3 \ 5 \ 6 \ 8 \ 11 \ 12 \ 9 \ 11 \ 18 \ 24 \ 14 \ 20 \}$													



Step 4:

	-1	0	1	2	3	4	5	6	7	8	9	10	11
$f(x) = \{ 0 \ 3 \ 5 \ 6 \ 8 \ 11 \ 12 \ 9 \ 11 \ 18 \ 24 \ 14 \ 20 \}$													



Step 5:

$$pf[] = \{ 0, 3, 5, 6, 8, 11, 12, 9, 11, 18, 24, 14, 20 \}$$

val :  $\boxed{0, 3, 5, 6, 8}$

ind :  $\boxed{-1, 0, 1, 2, 3}$



Step 6:

$$pf[] = \{ 0, 3, 5, 6, 8, 11, 12, 9, 11, 18, 24, 14, 20 \}$$

val :  $\boxed{0, 3, 5, 6, 8}$

ind :  $\boxed{-1, 0, 1, 2, 3}$

len =  $\cancel{5}$

11  
4 l=5

Step 7:

$$pf[] = \{ 0, 3, 5, 6, 8, 11, 12, 9, 11, 18, 24, 14, 20 \}$$

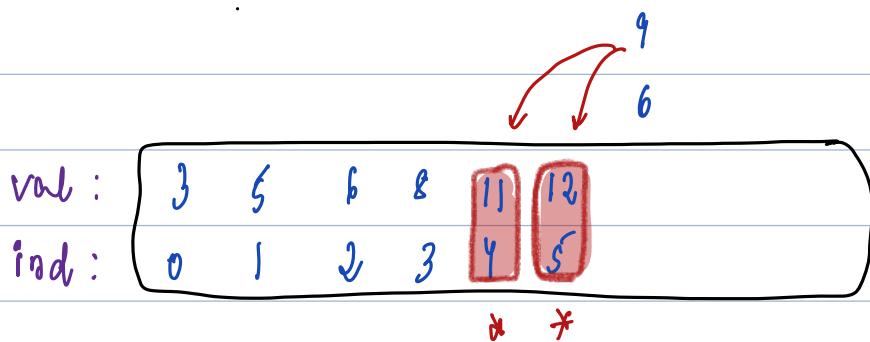
val :  $\boxed{3, 5, 6, 8, 11, 12}$

ind :  $\boxed{0, 1, 2, 3, 4, 5}$



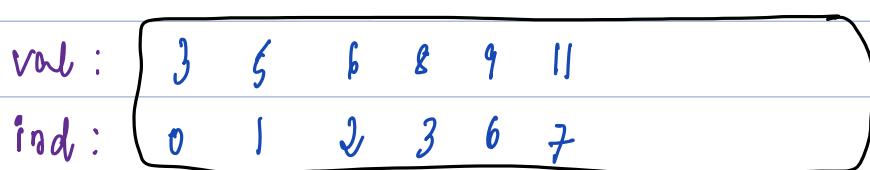
Step 8:

$$-1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 \quad 10 \quad 11$$
$$Pf[] = \{ 0 \quad 3 \quad 5 \quad 6 \quad 8 \quad 11 \quad 12 \quad 9 \quad 11 \quad 18 \quad 24 \quad 14 \quad 20 \}$$



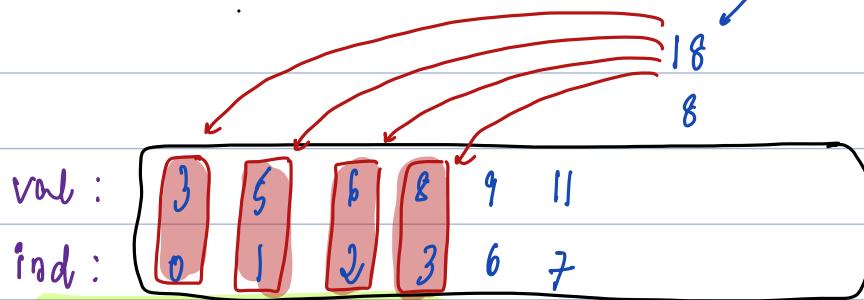
Step 9:

$$-1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 \quad 10 \quad 11$$
$$Pf[] = \{ 0 \quad 3 \quad 5 \quad 6 \quad 8 \quad 11 \quad 12 \quad 9 \quad 11 \quad 18 \quad 24 \quad 14 \quad 20 \}$$



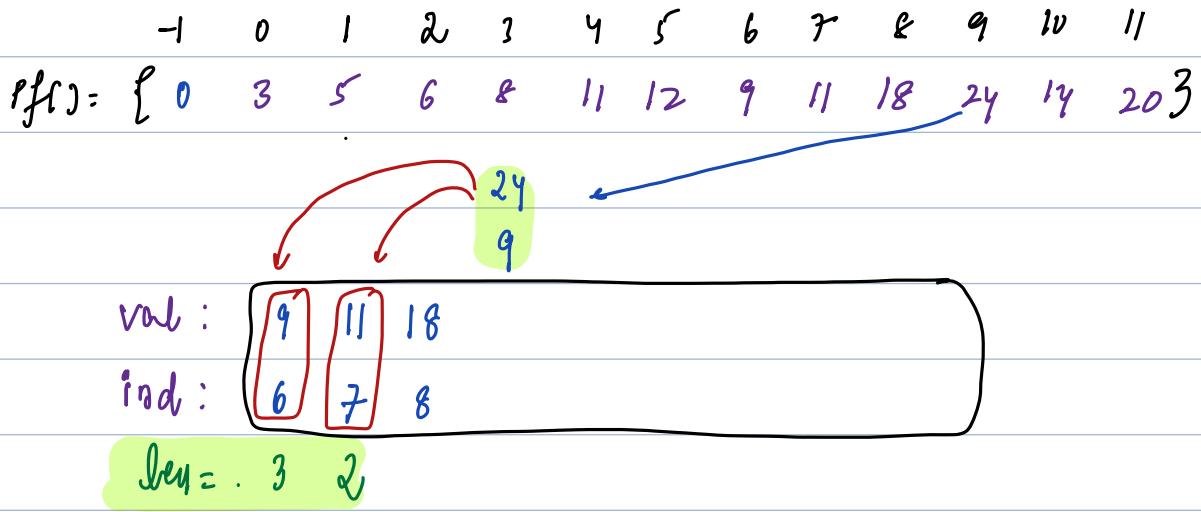
Step 10:

$$-1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 \quad 10 \quad 11$$
$$Pf[] = \{ 0 \quad 3 \quad 5 \quad 6 \quad 8 \quad 11 \quad 12 \quad 9 \quad 11 \quad 18 \quad 24 \quad 14 \quad 20 \}$$

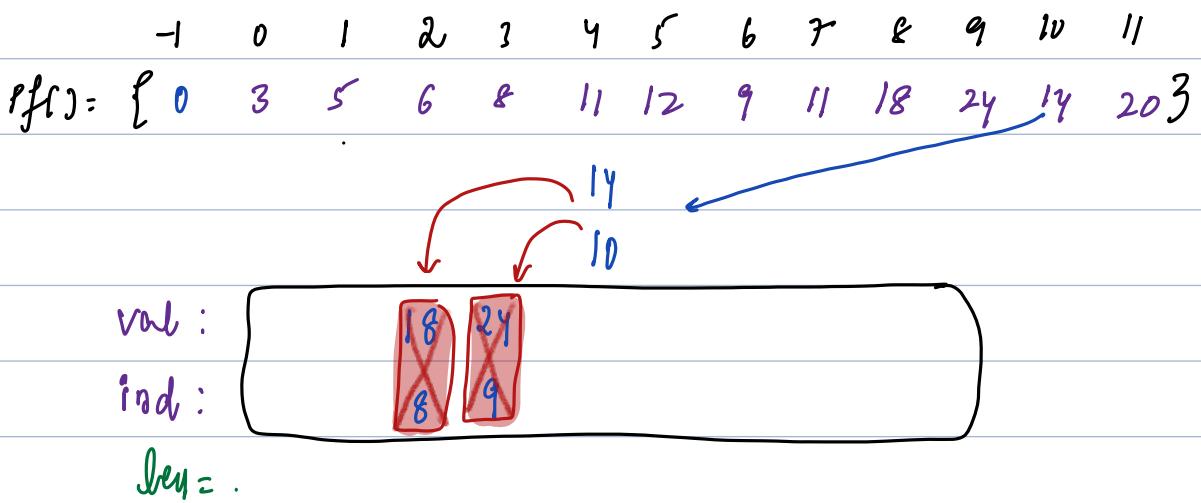


key = . 8 7 6 5

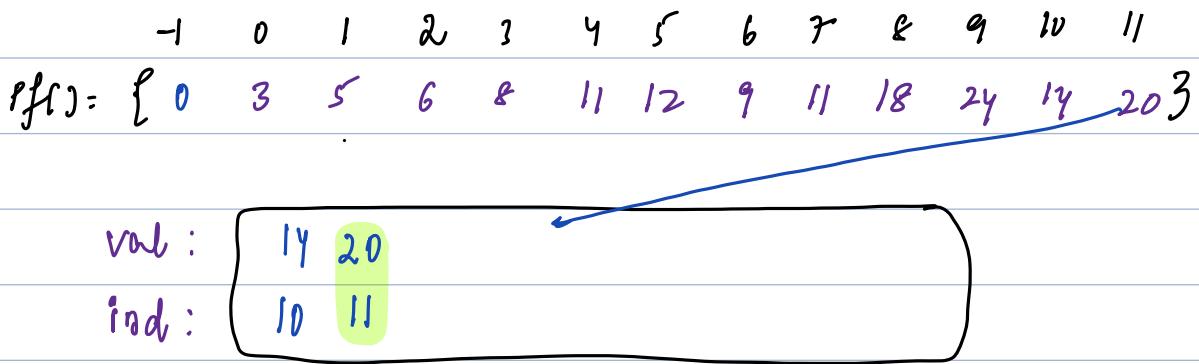
Step 11:



Step 12:



Step 13:



Con: Copy data in the order in Degree & cell: index

For iterate Pf[R]:

TC:  $\Theta(N \cdot n) = O(N) \ SC: O(n)$

Iterate at start of Degree:

if  $Pf[R] - start \text{ of } Degree \geq k$ :

update ans;

remove start;

else

break

Iterate at back of Degree:

if  $Pf[R] \leq end \text{ of } Degree$ :

remove end

else

insert  $Pf[R]$  at end.

```
int length (vector<int> arr) {
```

```
    long sum = 0;
```

```
    vector<long> pf(0, arr.size());
```

```
    long sum = 0;
```

```
    for (int i = 0; i < arr.size(); i++) {
```

```
        sum = sum + arr[i];
```

```
        pf[i] = sum;
```

```
}
```

```
dequeue pair<long, int> dq;
```

```
dq.push_front({0, -1});
```

```
int ans = 0;
```

```
for (int i = 0; i < arr.size(); i++) {
```

```
# for pf[i] first chunk, if we can have ans;
```

```
while (dq.size() > 0 && pf[i] - dq.front().first >= k) {
```

```
    ans = min(ans, i - dq.front().second);
```

```
dq.pop_front();
```

```
# Before inserting pf[i] check if we can delete elements
```

```
while (dq.size() > 0 && pf[i] < dq.front()) {
```

```
dq.pop_back();
```

```
dq.push_back({pf[i], i});
```

```
} return ans;
```