

Today's Content

1. Kadane's

2. Sliding Window

Q: # No: of subarrays of len = k

arr = { a_0 a_1 a_2 a_3 a_4 a_5 }

k = 4, ans = 3

arr = { a_0 a_1 a_2 a_3 a_4 a_5 a_6 }

k = 3, ans = 5

arr = { a_0 a_1 a_2 a_3 a_4 a_5 a_6 a_7 }

k = 5, ans = 4

Given arr(n) no: of subarrays of len = k

$$\text{ans} = N - k + 1$$

10 Given $arr[N]$ return Max Subarray Sums of len = k

Constraints:

Consider subarray of len = k

$$1 \leq N \leq 10^5$$

$$1 \leq k \leq N$$

$$-10^6 \leq arr[i] \leq 10^6$$

$$\# \text{ sum} = \{-10^{11} \text{ --- } 10^{11}\} = 10^{25}$$

$$arr[10^5] = \{-10^6 -10^6 \dots -10^6\} \quad arr[10^5] = \{10^6 10^6 \dots 10^6\}$$

Ex: $arr[10] : \{ -3 \ 4 \ -2 \ 5 \ 3 \ -2 \ 8 \ 2 \ -1 \ 4 \}$

$k = 5$

s e sum = 16

[0 4]	7
[1 5]	8
[2 6]	12
[3 7]	16
[4 8]	10
[5 9]	11

[6 10] *

Idea: Generate all subarrays of len = k:

Iterate & calculate sum & get overall max.

Tc: $O(N-k+1) * O(k)$ Sc: $O(1)$

$$k=1 \quad O(N-1+1) * O(1) = O(N * 1) = O(N)$$

$$k=N \quad O(N-N+1) * O(N) = O(1 * N) = O(N)$$

$$k=N/2 \quad O(N-N/2+1) * O(N/2) = O(N/2 * N/2) = O(N^2)$$

Code:

0 1 2 3 4 5 6 7 8
Ex: ar[9] : { -3 4 -2 5 3 -2 8 2 -1 }
k = 5

s e sum

[0 4] iterate from s..e & get sum = 7; s++; e++;

[1 5] iterate from s..e & get sum = 8; s++; e++;

[2 6] iterate from s..e & get sum = 12; s++; e++;

[3 7] iterate from s..e & get sum = 16; s++; e++;

[4 8] iterate from s..e & get sum = 16; s++; e++;

[5 9]: if e outside stop process & return max sum; 16.

```
long maxSum(vector<int> &ar) {
```

```
    int N = ar.size();
```

```
    long max = INT_MIN;
```

```
    int s = 0, e = k-1;
```

a b b-a+1
1st subarray : [0..k-1]; k-1-0+1 = k;

```
    while(e < N) {
```

```
        long sum = 0;
```

```
        for(int i = s; i <= e; i++) { → 1. Iterate & calculate sum
```

```
            sum = sum + ar[i];
```

```
        } if (sum > max) { → 2. Comp with max
```

```
            max = sum;
```

```
            s++; e++; → 3. Go to next subarray.
```

```
        }
```

```
    return max;
```

```
}
```

Idea: For every subarray of len=k, calculate sum using pf() & get overall max
Tc: $O((N + (N-k+1) * O(1))) = O(N + N-k+1) \approx O(N)$ & $O(N)$

Code:

Ex: arr[9]: { -3 4 -2 5 3 -2 8 2 -1 }

k=5

pf[9]: { -3 1 -1 4 7 5 13 15 14 }

pf()

s e sum:

[0 4] = pf[4] = 7

[1 5] = pf[5] - pf[0] = 5 - (-3) = 8

[2 6] = pf[6] - pf[1] = 13 - 1 = 12

[3 7] = pf[7] - pf[2] = 15 - (-1) = 16

[4 8] = pf[8] - pf[3] = 14 - 4 = 10

[5 9]: ✗

sum[s..e]: if (s==0) { // [0..e]

pf[e]

else {

pf[e] - pf[s-1]

```
long maxSum(vector<int> &arr) { TC:  $O(N)$  SC:  $O(N)$ 
```

```
int N = arr.size();
```

```
long pf[N], sum = 0;
```

```
for(int i = 0; i < N; i++) {
```

```
    sum = sum + arr[i]; // 1. update sum
```

```
    pf[i] = sum;
```

```
}
```

```
long man = INT_MIN;
```

```
int s = 0, e = k-1;
```

```
while(e < N) {
```

```
    long sum = 0;
```

```
    if(s == 0) {
```

```
        sum = pf[e];
```

```
    } else {
```

```
        sum = pf[e] - pf[s-1];
```

```
    if(sum > man) {
```

```
        man = sum;
```

```
        s++; e++;
```

```
}
```

```
return man;
```

```
}
```

a b b-a+1

1st subarray : [0..k-1]; k-1-0+1 = k;

→ 1. # Sum [s..e] get using pf[]

→ 2. Compare with man

→ 3. Go to next subarray.

Idea3: Sliding Window: Fixed length subarrays.

arr[10] = { 0 1 2 3 4 5 6 7 8 9 }
 { 3 4 -2 5 3 -2 8 2 1 4 }

k = 6

s e

[0 5] sum = 11 : 1st window iterate & calculate

For remaining windows calculate using sliding window.

[1 6] sum = sum - arr[0] + arr[6] = 11 - 3 + 8 = 16

[2 7] sum = sum - arr[1] + arr[7] = 16 - 4 + 2 = 14

[3 8] sum = sum - arr[2] + arr[8] = 14 - (-2) + 1 = 17

[4 9] sum = sum - arr[3] + arr[9] = 17 - 5 + 4 = 16

[5 10] Stop & return max sum = 17

s-1 s s+1 s+2 . . . e-1 e

[s, e] sum = sum - arr[s-1] + arr[e]

Remove arr[s-1]

Adding arr[e]

int subarray(vector<int> &arr, int k) { TC: $O(N)$ SC: $O(1)$

Step 1: For 1st window iterate & get it. & 1st subarray [0.. k-1]

long sum = 0, max = INT_MIN;

2nd subarray [1.. k]

for (int i = 0, i < k; i++) { # [0.. k-1] = k iterations

sum = sum + arr[i];

}

if (sum > max) {

max = sum;

}

Step 2: For remaining subarrays, slide & calculate sum.

int s = 1, e = k;

while (e < N) { # (N-k) iterations

get sum from [s.. e] using sliding.

sum = sum - arr[s-1] + arr[e];

if (sum > max) {

max = sum;

s++; e++;

}

return max;

}

Max Subarray Sum:

→ {Continuous part of an array}

Given $arr(N)$ return max subarray sum.

Ex1: $arr[] = \begin{matrix} & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ -2 & 3 & 4 & -1 & 5 & -10 & 7 \end{matrix}$ ans =

Ex2: $arr[] = \begin{matrix} & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ -3 & 4 & 6 & 8 & -10 & 2 & 7 \end{matrix}$ ans =

Q1: $arr[] = \begin{matrix} & 0 & 1 & 2 & 3 & 4 \\ 4 & 5 & 2 & 1 & 6 \end{matrix}$ ans =

Q2: $arr[] = \begin{matrix} & 0 & 1 & 2 & 3 & 4 \\ -4 & -3 & -6 & -9 & -2 \end{matrix}$ ans = -

Idea1:



Optimisation: Kadane's Algo: Max Subarray Sum.

Case 1: If all the elements in the array are positive.:

$$\begin{array}{cccccc} & 0 & 1 & 2 & 3 & 4 \\ \text{arr}[] = \{ & 4 & 2 & 1 & 6 & 7 \} \end{array} \text{ ans} =$$

Case 2: If all the elements in the array are negative

$$\begin{array}{cccccc} & 0 & 1 & 2 & 3 & 4 \\ \text{arr}[] = \{ & -4 & -8 & -3 & -10 & -5 \} \end{array} \text{ ans} =$$

Case 3: If positives are present in between.

$$\begin{array}{cccccccccc} & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ \text{arr}[] = \{ & -3 & -5 & -3 & 4 & 3 & 2 & 10 & -5 & -7 \} \end{array} \text{ ans} =$$

Case 4: Say we have max subarray sum

$$4.1 \text{ arr}[] = \{ -3 \quad -5 \quad -3 \quad \boxed{} \quad -5 \quad -7 \}$$

$$4.2 \text{ arr}[] = \{ -3 \quad 5 \quad -3 \quad \boxed{} \quad -5 \quad -7 \}$$

Idea:

Tran:

$$\begin{array}{cccccccccc} & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \text{arr}[] = \{ & 3 & 4 & -6 & 8 & -10 & 12 & 10 & -3 & 7 & -5 \} \end{array}$$

Sum =

0 1 2 3 4 5 6
arr[] = { -2 3 4 -1 5 -10 7 }

sum = 0

max =

int maxSubKadane's(int arr[]) { Tc: 0 Sc: O(1)