

Today's Content

1. No: of unique pairs
2. No: of right triangle
3. No: of rectangle { If Time permits }

28 Given N , 2d points calculate no. of distinct points?

Inputs: 2 arrays $x[N]$ & $y[N]$ given where i^{th} point = $x[i]$ and $y[i]$

Ex: $\# \text{Distinct} = 4$

$x[6] = \{2, 1, 3, 2, 3, 2\}$

$y[6] = \{3, 1, 2, 5, 2, 3\}$

$i^{\text{th}} \text{ point} = (x[i], y[i]);$

Idea 1: Insert each point in hashmap

#Way 1: $(x, y) : \text{hm}[x] = y$

points: $(2, 3) (1, 1) (2, 5) (2, 3)$

$\text{hm}[2] = 3$

$\text{hm}[1] = 1$

$\text{hm}[2] = 5 ; (2, 3) \Rightarrow (2, 5)$

$\text{hm}[2] = 3 ; (2, 5) \Rightarrow (2, 3)$

hm
 $\begin{bmatrix} (2, 3) & (1, 1) \\ & 5 \end{bmatrix}$

Issue: Will not work, will just override existing value

#Way 2: Make each point as pair & insert in hashset.

hashset $\langle \text{pair}(\text{int}, \text{int}) \rangle$ & $\text{hs};$

Only primitive type & pointers are possible

Possible: Override hashfunction, Next semester

#Way 3: Combine point (x, y) into string & Insert in hashset $\langle \text{String} \rangle$

$P_1 = (2, 5) = \# \text{Comb}_1 = 25$ # We cannot differentiate

$P_2 = (2, 3) = \# \text{Comb}_2 = 23$ both P_1 & P_2 .

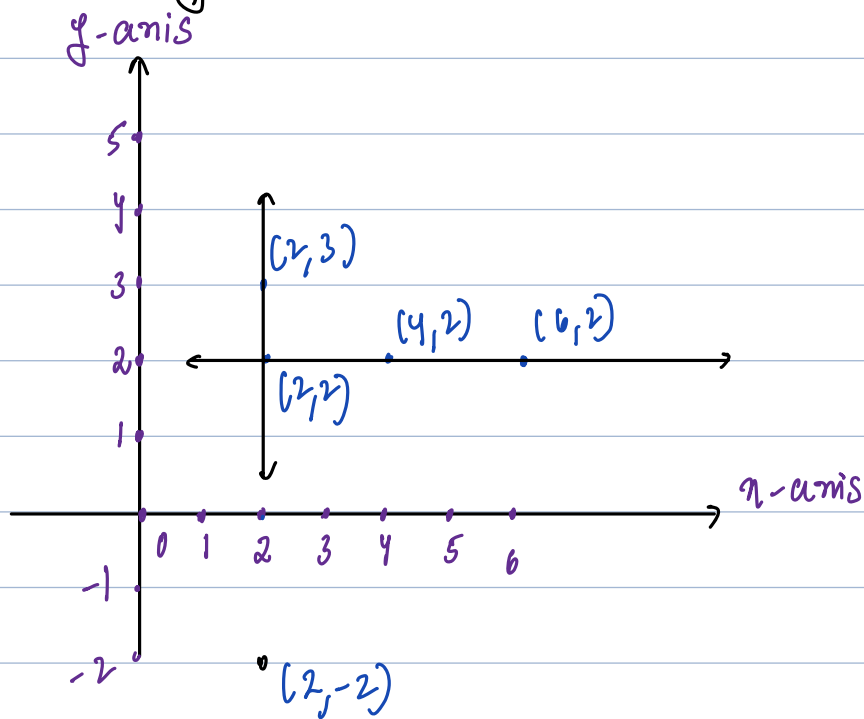
#Note: When we combine P_1 & P_2 keep a separator.

$P_1 = (2, 5) = \# \text{Comb}_1 = 23@5$

$P_2 = (2, 3) = \# \text{Comb}_2 = 2@35$

Return $\text{hs.size}()$, it will give no. of distinct points

// Geometry Basis



Q

1. line parallel to x -axis: x changes y fixed
2. line parallel to y -axis: x fixed y change

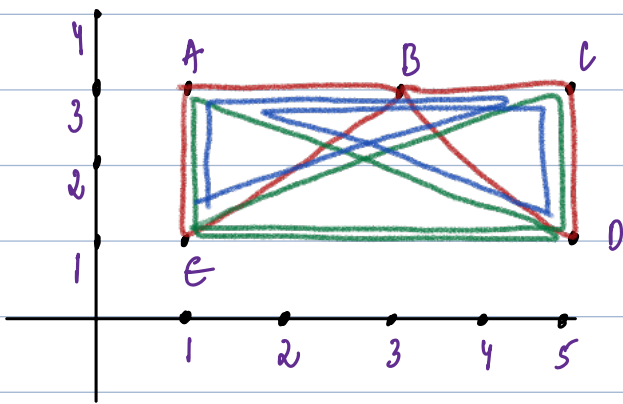
Given N distinct points, calculate how many Right angle Triangles are formed such that shorter sides are parallel to x -axis & y -axis

Inputs: 2 arrays $x[N]$ & $y[N]$ given where i^{th} point = $x[i]$ and $y[i]$

Points:

	0	1	2	3	4	
$x[5] = \{$	1	3	5	5	1	$\}$
$y[5] = \{$	3	3	3	1	1	$\}$

Ex:



Right Angle Triangles

EAB EAC

AED BCD

AED CED

Idea:

For 3 points

Generate all triplets:

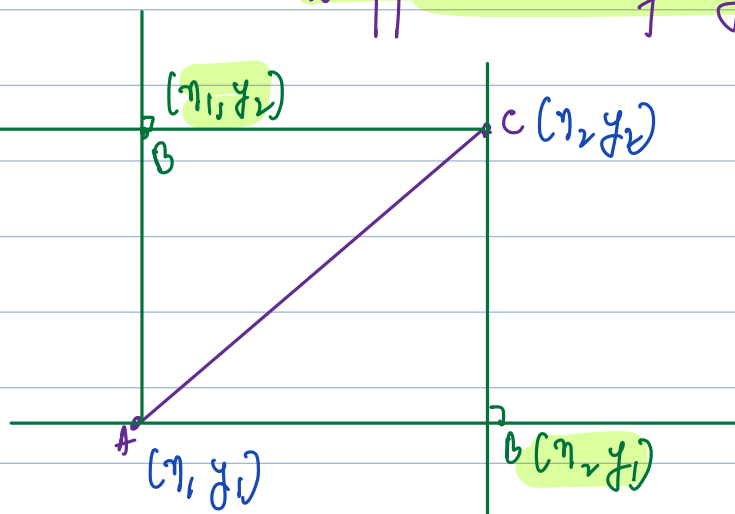
For all triplets, check if we can form right angled triangles parallel to x -axis & y -axis

TC: $O(N^3) \times O(1)$ SC: $O(1)$

↳ To check if it forms right angled triangle parallel to x -axis & y -axis

Hint 2: Find 2 points

#Case 1: Find 2 opposite sides of hypothesis



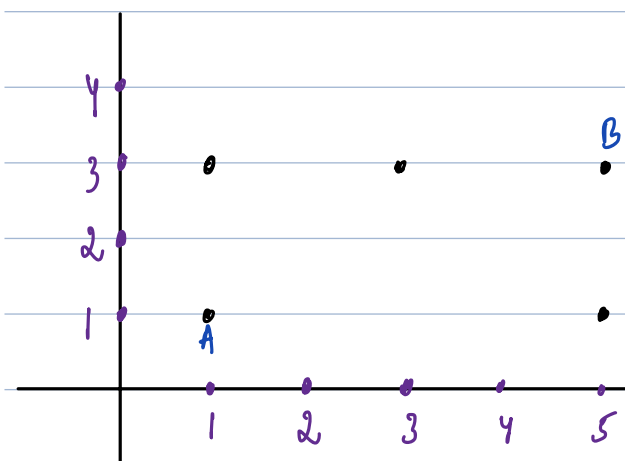
#obs1: When we find 2 opposite sides of hypothesis say $(x_1, y_1) \neq (x_2, y_2)$

if (x_1, y_1) exist $cr=1$

if (x_2, y_2) exist $cr=1$

#obs2: hypothesis cannot be parallel to x-axis & y-axis.

#Dry Run: $(1, 3)$ $(3, 3)$ $(5, 3)$ $(1, 1)$ $(5, 1)$



#Find 2 points

x_1, y_1	x_2, y_2	x_2, y_1	x_1, y_2
$(1, 3)$	$(3, 3)$	* parallel to x	
$(1, 3)$	$(5, 3)$	* parallel to x	
$(1, 3)$	$(1, 1)$	* parallel to y	
$(1, 3)$	$(5, 1)$	$(5, 3)$	$(1, 1)$
$(3, 3)$	$(5, 3)$	* parallel to x	
$(3, 3)$	$(1, 1)$	$(1, 3)$	$(3, 1)$
$(3, 3)$	$(5, 1)$	$(5, 3)$	$(3, 1)$
$(5, 3)$	$(1, 1)$	$(1, 3)$	$(5, 1)$
$(5, 3)$	$(5, 1)$	* parallel to y	
$(1, 1)$	$(5, 1)$	* parallel to x	

#Note: Since all points has a string in a hashmap, so that searching for a point in hashmap becomes $O(1)$

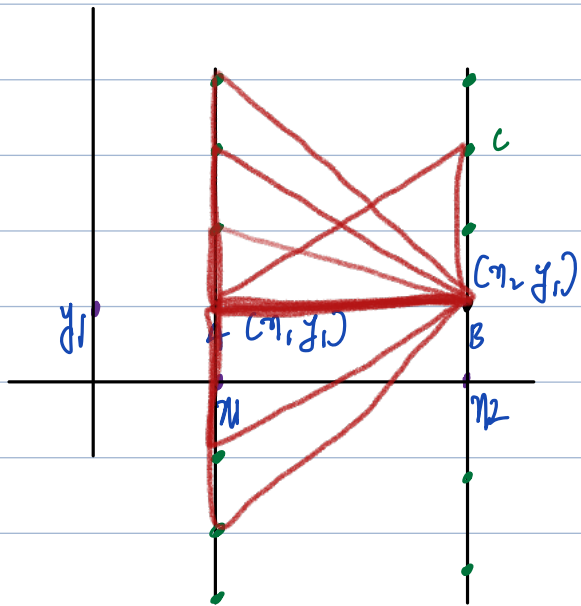
TC: $O(N^2)$

SC: $O(N)$

↳ #Store all points in hashmap

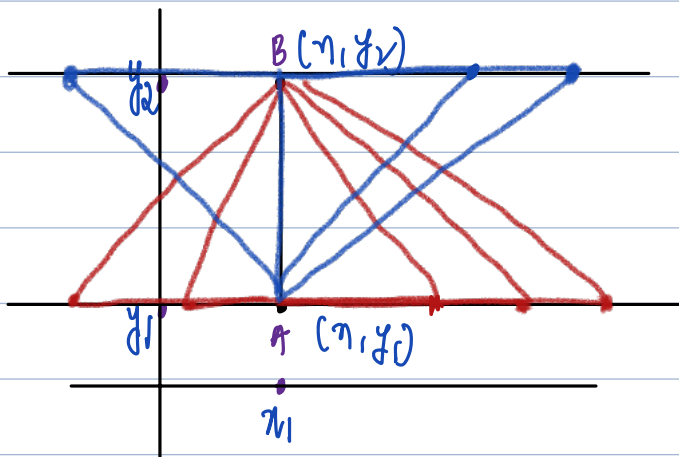
↳ #All pairs, make sure 2 points not parallel to x & y axis.

#Case 2:



#obs2: If we find 2 points parallel to x-axis, it's complicated to find 3rd point

#Case 3:



#obs3: If we find 2 points parallel to y-axis, it's complicated to find 3rd point

```
int Triangle(int x[], int y[]) { TC
```

```
unordered_set<string> hs;
```

```
for (int i=0; i<N; i++) {
```

```
    hs.insert(x[i] + " " + y[i]);
```

```
int c=0;
```

```
for (int i=0; i<N; i++) {
```

```
    for (int j=i+1; j<N; j++) {
```

```
        x1 = x[i], y1 = y[i], x2 = x[j], y2 = y[j];
```

```
        # (x1, y1) & (x2, y2)
```

```
        if (x1 != x2 && y1 != y2) {
```

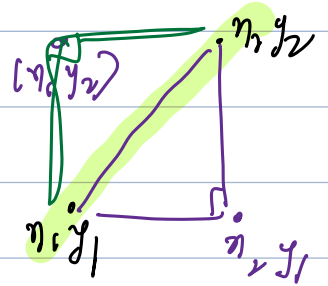
```
            # 3 point (x1, y2) (x2, y1)
```

```
            if (hs.search(x2 + " " + y1)) {
```

```
                c++;
```

```
            if (hs.search(x1 + " " + y2)) {
```

```
                c++;
```




```
return c;
```

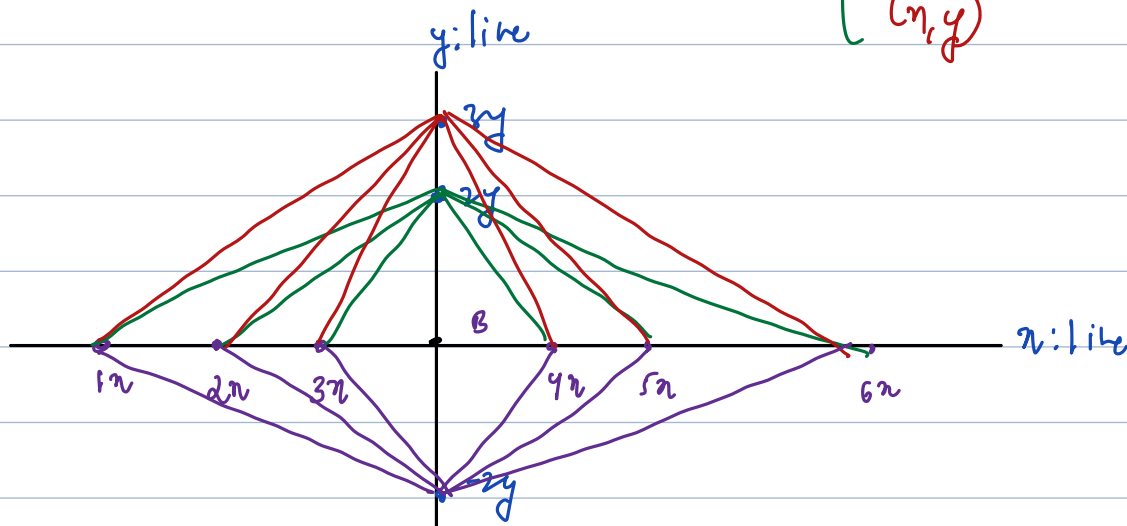
```
}
```

Hints: fin 1 point, which is right angled.

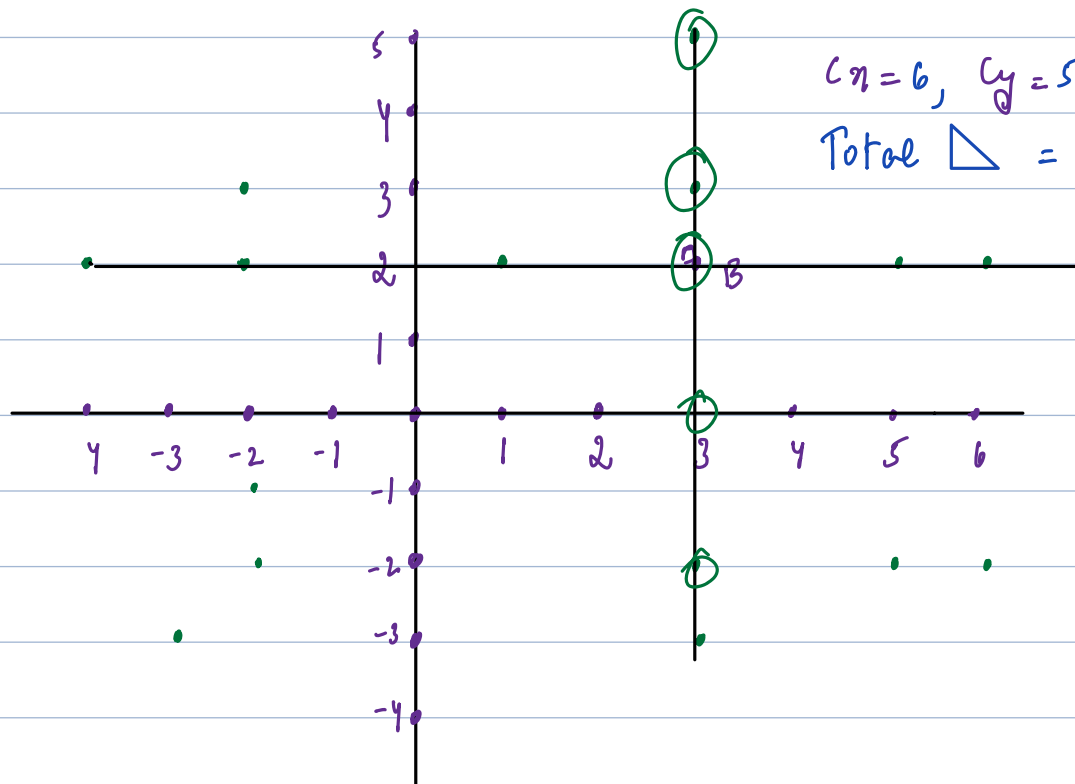
2 smaller sides should be parallel to x-axis & y-axis.

#obs: Any point along x line & Any point along y line will form 

#con: Calculate no. of points along x line = C_x
 Calculate no. of points along y line = C_y $\left\{ \begin{array}{l} \text{Total } \triangle = (C_x - 1)(C_y - 1) \\ -1, \text{ because we neglect } (C_x, C_y) \end{array} \right.$



Q Calculate how many right angled triangles are there with $(3, 2)$ as 90° vertex & shorter sides are parallel to x-axis & y-axis



$$C_x = 6, C_y = 5$$

$$\text{Total } \triangle = (C_x - 1) * (C_y - 1) = 20$$

#idea3:

Create 2 hashmaps h_{mx} & h_{my} .

Insert all points x in h_{mx}

Insert all points y in h_{my}

Iterate on all points;

Consider each point as 90° vertex.

Calculate no. of points along x line = C_x

Calculate no. of points along y line = C_y

ans = ans + $(C_x - 1) * (C_y - 1)$

```
int Triangles(vector<int> &x, vector<int> &y) { Tc: O(N) Sc: O(N)
```

```
    unordered_map<int, int> hx;
```

```
    unordered_map<int, int> hy;
```

```
    for(int i=0; i<N; i++) {
```

```
        # (x[i], y[i])
```

```
        hx[x[i]]++;
```

```
        hy[y[i]]++;
```

```
    }
```

```
    int c=0;
```

```
    for(int i=0; i<N; i++) {
```

```
        # (x[i], y[i]) is 90° angle
```

```
        int cx = hx[x[i]]
```

```
        int cy = hy[y[i]]
```

```
    }    c = c + (cx-1) * (cy-1);
```

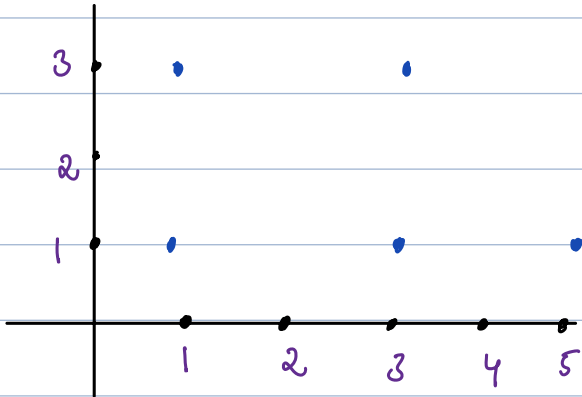
```
    return c;
```

```
}
```

38 Given N distinct points, calculate no: of rectangles are formed such that sides are parallel to x -axis & y -axis

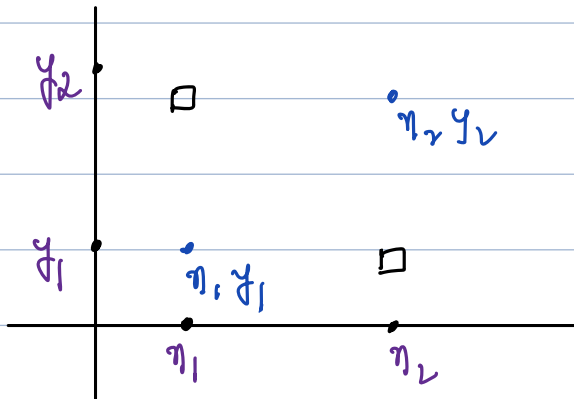
Expected TC: $O(N^2)$

Ex: $N=6$ $(3,1)$ $(1,1)$ $(3,3)$ $(1,3)$ $(3,5)$ $(1,5)$



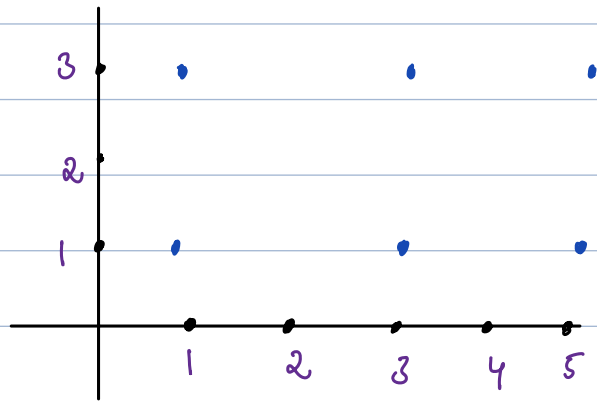
Idea 1:

Idea 2:



Dry Run:

	0	1	2	3	4	5
$N=6$	$(3,1)$	$(1,1)$	$(3,3)$	$(1,3)$	$(5,1)$	$(5,3)$



Fin 2 points

x_1	y_1	x_2	y_2	x_2, y_1	x_1, y_2
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int Rectangle(rectr<int> &x, rectr<int> &y){