

Statistics and Probability - Detailed Notes

1. Introduction to Statistics

Statistics is the science of collecting, organizing, analyzing, and interpreting data to make decisions. It helps in understanding patterns and trends in data.

2. Sampling

Definition: Sampling is the process of selecting a subset of individuals from a population to estimate characteristics of the whole population.

Types of Sampling:

- **Random Sampling:** Each individual has an equal chance of being selected.
- **Stratified Sampling:** Population divided into groups (strata), then samples taken from each.
- **Systematic Sampling:** Every k th item is selected from a list.

Example:

If we have a population of 1000 students, and we randomly pick 100 students to analyze their performance, it is called sampling.

3. Central Tendencies

Central tendency refers to the middle or typical value in a dataset.

- **Mean (Average)** = Sum of all values / Total number of values
- **Median** = Middle value after sorting data
- **Mode** = Most frequent value

Example:

Given data: [2, 3, 5, 7, 7, 10]

- Mean = $(2+3+5+7+7+10)/6 = 5.67$
 - Median = $(5+7)/2 = 6$
 - Mode = 7
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4. Null Values

Definition: Null values represent missing or undefined data in a dataset.

Handling Techniques:

- Removing null rows/columns
- Imputing values using mean/median/mode

Example:

Name	Age
A	23
B	NaN

Impute with mean: Age = 23 (if only one value exists)

5. Duplicates

Definition: Duplicate records are repeated entries in a dataset.

Handling:

- Use `drop_duplicates()` in Python (Pandas) to remove them.

Example:

Name	Age
------	-----

A	23
---	----

A	23
---	----

After dropping duplicates, only one record remains.

6. Range

Definition: Range is the difference between the maximum and minimum values in a dataset.

Formula: Range = Max - Min

Example:

Data: [3, 7, 2, 9, 5] → Range = 9 - 2 = 7

7. Variance

Definition: Variance measures the spread of data points around the mean.

Formula:

$$\text{Variance} = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2$$

Example:

Data: [2, 4, 4, 4, 5, 5, 7, 9]

Mean = 5, Variance = 4

8. Standard Deviation (SD)

Definition: Standard deviation is the square root of variance. It indicates how data values spread around the mean.

Example:

If variance = 4, then SD = $\sqrt{4} = 2$

9. Percentile

Definition: A percentile indicates the value below which a given percentage of observations fall.

Example:

If you are in the 90th percentile in a test, you scored better than 90% of test takers.

10. Quantile

Definition: Quantiles divide the dataset into equal-sized intervals.

- Quartiles (4 parts), Deciles (10 parts), Percentiles (100 parts)

Example:

25th percentile = Q1 (first quartile), 50th percentile = median (Q2)

11. Outliers

Definition: Outliers are extreme values that differ significantly from other observations.

Detection Methods:

- IQR method:
 - $IQR = Q3 - Q1$
 - Lower bound = $Q1 - 1.5 * IQR$
 - Upper bound = $Q3 + 1.5 * IQR$

Example:

Data: [2, 3, 4, 5, 6, 100] → 100 is an outlier

12. Correlation

Definition: Correlation measures the relationship between two variables.

Range: -1 to 1

- +1: Perfect positive correlation
- -1: Perfect negative correlation
- 0: No correlation

Example:

Height and weight often show positive correlation.

13. Plots

Useful for visualizing data.

- **Histogram:** Distribution of numerical data
- **Boxplot:** Detects outliers and spread
- **Scatter plot:** Correlation between two variables
- **Bar chart:** Categorical data comparison

Example:

Use matplotlib/seaborn in Python:

```
import seaborn as sns
sns.boxplot(data=[2,4,4,4,5,5,7,9,100])
```

14. Probability

a. Joint Probability

Probability of two events happening together.

$$P(A \cap B) = P(A) \cdot P(B | A) \quad P(A \setminus \cap B) = P(A) \cdot P(B | A)$$

b. Marginal Probability

Probability of a single event irrespective of others.

c. Conditional Probability

Probability of A given B has occurred.

$$P(A | B) = \frac{P(A \cap B)}{P(B)} \quad P(A | B) = \frac{P(A \setminus \cap B)}{P(B)}$$

Example:

If 60% students play cricket and 30% of those also play football:

- Joint = $0.6 * 0.3 = 0.18$
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16. Skewness

Definition: Skewness measures the asymmetry of the distribution.

- Positive skew: Tail on right
- Negative skew: Tail on left

Example:

Income data often shows positive skew due to few very high incomes.

17. Kurtosis

Definition: Kurtosis measures the "tailedness" of a distribution.

- High kurtosis: heavy tails (outliers)
- Low kurtosis: light tails

Example:

Normal distribution has kurtosis = 3 (mesokurtic)

18. Label Encoding

Definition: Converts categorical labels into numerical form.

Example:

Colors: [Red, Green, Blue] \rightarrow [0, 1, 2]

In Python:

```
from sklearn.preprocessing import LabelEncoder  
le = LabelEncoder()  
le.fit_transform(['Red', 'Green', 'Blue'])
```

End of Notes