Worksheet 2 — Probability spaces

1. Give a possible sample space Ω for each of the following experiments.

   (a) An election decides between two candidates A and B.

   (b) A two-sided coin is tossed.

   (c) A student is asked for the month and day-of-week on which her birthday falls.

   (d) A student is chosen at random from a class of ten students.

   (e) You choose the color of your new car's exterior (choices: red, black, silver, green) and interior (choices: black, beige).

2. In each of the following situations, define the sample space Ω.

   (a) A fair coin is tossed 200 times in a row.

   (b) You count the number of people who enter a department store on a particular Sunday.

   (c) You open up *Hamlet* and pick a word at random.

3. Let A, B, and C be events defined on a particular sample space Ω. Write expressions for the following combinations of events:

   (a) All three events occur.

   (b) At least one of the events occurs.

   (c) A and B occur, but not C.

4. Consider a sample space $\Omega = \{a, b, c\}$ with probabilities $\Pr(a) = 1/2$ and $\Pr(b) = 1/3$.

   (a) What is $\Pr(c)$?

   (b) How many distinct events can be defined on this space?

   (c) Find the probabilities of each of these possible events.

5. A fair coin is tossed three times in succession. Describe in words each of the following events on sample space $\{H, T\}^3$.

   (a) $E_1 = \{HHH, HHT, HTH, HTT\}$

   (b) $E_2 = \{HHH, TTT\}$

   (c) $E_3 = \{HHT, HTH, THH\}$

   What are the probabilities of each of these events?

6. Let A and B be events defined on a sample space Ω such that $\Pr(A \cap B) = 1/4$, $\Pr(A^c) = 1/3$, and $\Pr(B) = 1/2$. Here $A^c = \Omega \setminus A$ is the event that A *doesn't* happen. What is $\Pr(A \cup B)$?

7. A pair of dice are rolled. What is the probability that they show the same value?

8. In Morse code, each letter is formed by a succession of dashes and dots. For instance, the letter S is represented by three dots and the letter O is represented by three dashes. Suppose a child types a sequence of 9 dots/dashes at random (each position is equally likely to be a dot or a dash). What is the probability that it spells out SOS?

9. A die is loaded in such a way that the probability of each face turning up is proportional to the number of dots on that face (for instance, a six is three times as probable as a two). What is the probability of getting an even number in one throw?

10. A certain lottery has the following rules: you buy a ticket, choose 3 different numbers from 1 to 100, and write them on the ticket. The lottery has a box with 100 balls numbered 1 to 100. Three (different) balls are chosen. If any of the balls has one of the numbers you have chosen, you win. What is the probability of winning?

11. Five people of different heights are lined up against a wall in random order. What is the probability that they just happen to be in increasing order of height (left-to-right)?

12. Five people get on an elevator that stops at five floors. Assuming that each person has an equal probability of going to any one floor, find the probability that they all get off at different floors.

13. You are dealt five cards from a standard deck. What is the probability that the first four are aces and the fifth is a king?

14. A barrel contains 90 good apples and 10 rotten apples. If ten of the apples are chosen at random, what is the probability that they are all good?

15. Four women check their hats at a concert, but when each woman returns after the performance, she gets a hat chosen randomly from those remaining. What is the probability that each woman gets her own hat back?

16. Assume that whenever a child is born, it is equally likely to be a girl or boy, independent of any earlier children. What is the probability that a randomly-chosen family with six children has exactly three girls and three boys?

17. Snow White asks three of the seven dwarfs, chosen at random, to accompany her on a trip.

    (a) What is the probability that Dopey is in this group?
    (b) What is the probability that both Dopey and Sneezy are in the group?
    (c) What is the probability that neither Dopey nor Sneezy are in the group?