COURSE NUMBER: DSE210 HW1 Worksheet 3
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PAGE: 1 of ____

1) $\Omega = \{H, T\}^3 \quad |\Omega| = 2^3 = 8$

$A = \{\text{Exactly 2 heads}\} = \{HHT, HTH, THH\}$

b) $B = \{\text{first outcome is tail}\} = \{TTT, THT, THH, TTH\}$

$A \cap B = \{THH\}$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{8}}{\frac{4}{8}} = \frac{1}{4}$$

a) $B = \{\text{first outcome is Head}\} = \{HHT, HHH, HTT, HTH\}$

$A \cap B = \{HHT, HTH\}$

$P(\text{Exact two H} / \text{First outcome is H}) = P(A/B)$

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{2}{8}}{4/8} = \frac{1}{2}$$

c) $B = \{\text{first two outcomes are H}\} = \{HHH, HHT\}$

$A \cap B = \{HHT\}$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{8}}{\frac{2}{8}} = \frac{1}{2}$$

d) $B = \{\text{first two outcomes are T}\} = \{TTT, TTH\}$

$A \cap B = \phi$

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{P(\phi)}{P(B)} = 0$$

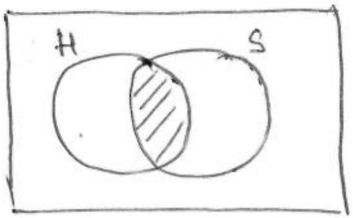
COURSE NUMBER: ________
STUDENT NAME: ________
PAGE: 2 of ____

1) e) $B = \{\text{first } H, \text{Third } T\} = \{HHT, HTT\}$

$A \cap B = \{HHT\}$

$$P(A/B) = \frac{P(A\cap B)}{P(B)} = \frac{\frac{1}{8}}{\frac{2}{8}} = \frac{1}{2}$$

3)



Given $P(H) = 0.6$, $P(S) = 0.8$ $P(H \cup S) = 0.9$

$P(H\cap S)$ = prob that both H & S pass the bill.

$$P(H \cup S) = P(H) + P(S) - P(H \cap S)$$

$$\therefore P(H\cap S) = 0.6 + 0.8 - 0.9 = 0.5$$

5) Total sample space $|\Omega| = 52$

$R = \{\text{card is red}\}$, $|R| = 26$ $P(R) = \frac{26}{52}$

$H = \{\text{card is Heart}\}$, $|H| = 13$ $P(H) = \frac{13}{52}$

$H \cap R = \{\text{card is red and heart}\}$, $|H\cap R| = 13$

a) $P(\text{Heart given it is red}) = P(H/R) = \frac{P(H\cap R)}{P(R)} = \frac{13/52}{26/52} = \frac{1}{2}$

b) $B = \{\text{card is higher than ten}\}$, $|B| = 16$

$B \cap H = \{\text{card is higher than ten and it is heart}\}$, $|B\cap H| = 4$

$P(B/H) = P(\text{card is higher than ten} / \text{it is heart}) = \frac{P(B\cap H)}{P(H)} = \frac{4/52}{13/52}$

$= 4/13$

COURSE NUMBER: ________
STUDENT NAME: ________________________________
PAGE: 3 of ____

5) c) $J = \{\text{card is Jack}\}$, $|J| = 4$

$J \cap B = \{\text{card is Jack and higher than ten}\}$; $|J \cap B| = 4$

$P(J/B) = P(\text{card is Jack} \mid \text{it is higher than ten})$

$$= \frac{P(J \cap B)}{P(B)} = \frac{4/52}{16/52} = \boxed{1/4}$$

7) Sample space $\Omega = \{(1,2,3,4,5,6)\}^2$, $|\Omega| = 36$

Let $A = \{\text{Sum is} > 7\}$, $|A| = 5+4+3+2+1+0 = 15$

a) $B = \{\text{first is } 4\}$, $|B| = 6$ ~~$A \cap B$~~

$A \cap B = \{\text{first is 4 and sum is} > 7\}$ $|A \cap B| = 3$

$$P(A/B) = P(\text{Sum} > 7 / \text{first is } 4) = \frac{P(A \cap B)}{P(B)} = \frac{3/36}{6/36} = \boxed{1/2}$$

b) $C = \{\text{first is } 1\}$, $|C| = 6$

$A \cap C = \{\text{Sum} > 7 \text{ \& first is } 1\} = \emptyset$ $|A \cap C| = 0$

$\therefore P(A/C) = P(\text{Sum} > 7 / \text{first is } 1) = \boxed{0}$

c) $D = \{\text{first is} > 3\}$, $|D| = 3 \times 6 = 18$

$A \cap D = \{\text{Sum} > 7 \text{ and first is} > 3\}$, $|A \cap D| = 3+4+5 = 12$

$$P(A/D) = P(\text{Sum} > 7 / \text{first} > 3) = \frac{P(A \cap D)}{P(D)} = \frac{12/36}{18/36} = \boxed{2/3}$$

d) $E = \{\text{first is} < 5\}$ $|E| = 4 \times 6 = 24$

$A \cap E = \{\text{Sum} > 7 \text{ and first} < 5\}$, $|A \cap E| = 3+2+1+0 = 6$

$$P(A/E) = P(\text{Sum} > 7 / \text{first} < 5) = \frac{P(A \cap E)}{P(E)} = \frac{6}{24} = \boxed{\frac{1}{4}}$$

COURSE NUMBER: ________
STUDENT NAME: ________________________
PAGE: 4 of ____

9)

$$P(D) = P(\text{defective car}) = \frac{|\text{defective cars from each factory}|}{\text{total num of cars}}$$

a) $$P(D) = \frac{\frac{5}{100} \times 25 + \frac{4}{100} \times 35 + \frac{2}{100} \times 40}{100} = \frac{345}{100,00} = 0.0345$$

b)

$$P(F_1 \cap D) = P(\text{car is from } F_1 \text{ and defective})$$

$$= \frac{\frac{5}{100} \times 25}{100} = \frac{5}{400} = \frac{1}{80}$$

$$P(F_1 / D) = P(\text{car is from } F_1 / \text{car is defective}) = \frac{P(F_1 \cap D)}{P(D)}$$

$$= \frac{\frac{1}{80}}{0.0345} = 0.362$$

11) Given $P(d_1) = P(d_2) = P(d_3) = 1/3$

$P(\text{pos} / d_1) = 0.8$

$P(\text{pos} / d_2) = 0.6$

$P(\text{pos} / d_3) = 0.4$

a) $$P(\text{pos}) = P(\text{Pos} / d_1) \times P(d_1) + P(\text{Pos} / d_2) * P(d_2) + P(\text{Pos} / d_3) * P(d_3)$$

$$= 0.8 \times \frac{1}{3} + 0.6 \times \frac{1}{3} + 0.4 \times \frac{1}{3} = 0.6$$

$$\boxed{P(\text{pos}) = 0.6}$$

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11) b)

Using Bayes theorem

$$P(d_1/pos) = \frac{P(pos/d_1)\cdot P(d_1)}{P(pos)} = \frac{0.8 \times \frac{1}{3}}{0.6}$$

$$= \underline{\underline{0.4444}}$$

$$P(d_2/pos) = \frac{P(pos/d_2)\cdot P(d_2)}{P(pos)} = \frac{0.6 \times \frac{1}{3}}{0.6} = \underline{\underline{1/3}}$$

$$P(d_3/pos) = \frac{P(pos/d_3)\cdot P(d_3)}{P(pos)} = \frac{0.4 \times \frac{1}{3}}{0.6} = \underline{\underline{0.2222}}$$

13) Given $P(\text{Tiger}) = 1/3$, $P(\text{mamoth}) = 2/3$

$P(pos/\text{Tiger}) = 5/6$ $P(pos/\text{mamoth}) = 1/3$

$\therefore P(\text{Neg}/\text{Tiger}) = 1 - \frac{5}{6} = \frac{1}{6}$

$P(\text{Neg}/\text{mamoth}) = 1 - \frac{1}{3} = \frac{2}{3}$

$$P(\text{Neg}) = P(\text{Neg}/\text{Tiger})\cdot P(\text{Tiger}) + P(\text{Neg}/\text{mamoth})\cdot P(\text{mamoth})$$

$$= \frac{1}{6} \times \frac{1}{3} + \frac{2}{3} \times \frac{2}{3} = \frac{1}{2}$$

$$P(\text{Tiger}/\text{Neg}) = \frac{P(\text{Neg}/\text{Tiger})\cdot P(\text{Tiger})}{P(\text{Neg})} = \frac{\frac{1}{6} \times \frac{1}{3}}{\frac{1}{2}} = \boxed{\frac{1}{9}}$$

COURSE NUMBER: ________
STUDENT NAME: ________________________
PAGE: 6 of ____

15)

a) $\Omega = \{H, T\}^3 = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$ $|\Omega| = 8$

1)

$P(A) = P(\text{H on first toss}) = \frac{4}{8} = \frac{1}{2}$

$P(B) = P(\text{T on second toss}) = \frac{4}{8} = \frac{1}{2}$

$P(A \cap B) = P(\text{head on first \& T on second}) = \frac{2}{8} = \frac{1}{4}$

$P(A \cap B) = P(A) \cdot P(B)$.

$\therefore$ $\boxed{\text{A \& B are Independent}}$

2)

$P(D) = P(\text{identical o/p}) = \frac{2}{8} = \frac{1}{4}$

$P(A \cap D) = P(\text{H on first and identical o/p}) = \frac{1}{8}$

$P(A \cap D) = P(A) \cdot P(D)$

$\boxed{\therefore \text{A \& D are Independent}}$

a 3) $P(E) = P(\text{Exactly One H}) = \frac{3}{8}$

$P(A \cap E) = P(\text{H on first and Exactly One H}) = \frac{1}{8}$

$P(A \cap E) \neq P(A) \cdot P(B) = \frac{1}{2} \times \frac{3}{8} = \frac{3}{16}$

$\boxed{\therefore \text{A \& E are Not Independent}}$

a 4) $P(D \cap E) = P(\text{Identical \& Exactly 1 H}) = 0$

$\therefore P(D \cap E) \neq P(D) \cdot P(E) = \frac{1}{4} \times \frac{3}{8}$

$\boxed{\text{D \& E are not Independent}}$

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COURSE NUMBER: ________
STUDENT NAME: ________________________________
PAGE: 7 of ____

15) b1) $P(C) = P(\text{H on third toss}) = \frac{1}{2}$

$P(A \cap B \cap C) = P(\text{H on first \& T on second \& H on third}) = \frac{1}{8}$

$P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

$\boxed{\therefore \text{ A \& B \& C are Independent}}$

b2)

$P(A \cap B \cap D) = P(\text{H on first \& Tail on second \& 3 identical}) = 0$

$P(A \cap B \cap D) \neq P(A) \cdot P(B) \cdot P(D)$

$\boxed{\therefore \text{ A, B, D are not Independent}}$

b3)

$P(C \cap D \cap E) = P(\text{H on third, All identical, Exactly one H}) = 0$

$\because P(C \cap D \cap E) \neq P(C) \cdot P(D) \cdot P(E)$

$\boxed{\text{C, D, E are not Independent}}$

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COURSE NUMBER: ________
STUDENT NAME: ________
PAGE: 8 of ____

17)

Given: $P(UCLA) = 0.5$

$P(UCSD) = 0.3$

$P(UCSD \cap UCLA) = 0.2$

a) $P(UCSD \mid UCLA) = \dfrac{P(UCSD \cap UCLA)}{P(UCLA)} = \dfrac{0.2}{0.5}$

$\boxed{= \dfrac{2}{5}}$

b) $P(UCSD) \cdot P(UCLA) = 0.5 \times 0.3 = 0.15$

$P(UCSD) \cdot P(UCLA) \neq P(UCSD \cap UCLA)$

$\boxed{\therefore \text{The two events are not independent}}$

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