

## Worksheet 1 — Sets and counting

1. (a) Write down any set  $A$  of size 5.  $A = \{a, b, c, d, e\}$   
 (b) What is the formal notation for all sequences of three elements from  $A$ ?  $A^3$   
 (c) How many such sequences are there, exactly?  $5^3$
2. How many binary sequences of length 500 are there?  $2^{500}$
3.  $A$  and  $B$  are sets with  $|A| = 3$  and  $|B| = 4$ .  
 (a) What is the largest size  $A \cup B$  could possibly have? 7  
 (b) What is the smallest size  $A \cup B$  could possibly have? 4  
 (c) Repeat for  $A \cap B$ .  $\max(|A \cap B|) = 3$  ~~max~~  $\min(|A \cap B|) = 0$
4. A donkey, an ox, a goat, and a tiger need to cross a river. They have a boat that can only hold one animal, so they need to go one at a time. How many different orderings are there?  $4!$
5. How many sequences of 5 English characters are there?  $26^5$
6. You have 10 good friends, and you want to choose 3 of them to accompany you on a trip. How many groups of three friends can you choose?  $10C3 = 120$
7. You have 10 different beer bottles, and you want to line 5 of them up on your mantelpiece. How many different arrangements can you make?  $10P5 = \frac{10!}{5!} = 30240$