DSE 210: Probability and statistics **Winter 2016**

Worksheet 3 — Multiple events, conditioning, and independence

1. A coin is tossed three times. What is the probability that there are exactly two heads, given that:
   (a) the first outcome is a head?
   (b) the first outcome is a tail?
   (c) the first two outcomes are both heads?
   (d) the first two outcomes are both tails?
   (e) the first outcome is a head and the third outcome is a tail?
2. A student must choose exactly two of the following three electives: art, French, or mathematics. The probability that he chooses art is 5/8, the probability he chooses French is 5/8, and the probability that he chooses both art and French is 1/4.
   (a) What is the probability that he chooses mathematics?
   (b) What is the probability that he chooses either art or French?
3. For a bill to come before the president of the United States, it must be passed by both the House of Representatives and the Senate. Assume that, of the bills presented to the two bodies, 60% pass the House, 80% pass the Senate, and 90% pass at least one of the two. Calculate the probability that the next bill presented to the two groups will come before the president.
4. In a fierce battle, not less than 70% of the soldiers lost one eye, not less than 75% lost one ear, not less than 80% lost one hand, and not less than 85% lost one leg. What is the minimal possible percentage of those who simultaneously lost one ear, one eye, one hand, and one leg?
5. A card is drawn at random from a standard deck. What is the probability that:
   (a) it is a heart, given that it is red?
   (b) it is higher than a ten, given that it is a heart (interpret J, Q, K, A as having numeric value $11, 12, 13, 14$)?
   (c) it is a jack, given that it is higher than a 10?
6. If $\Pr(B^c) = 1/4$ and $\Pr(A|B) = 1/2$, what is $\Pr(A \cap B)$?
7. A die is rolled twice. What is the probability that the sum of the two rolls is > 7, given that:
   (a) the first roll is a 4?
   (b) the first roll is a 1?
   (c) the first roll is > 3?
   (d) the first roll is < 5?
8. Two cards are drawn successively from a deck of 52 cards.

(a) Find the probability that the second card is equal in rank to the first card. (Rank is defined according to the following ordering: $2, 3, \ldots, 10, J, Q, K, A$. The suit is irrelevant.)

(b) Find the probability that the second card is higher in rank than the first card.

9. A particular car manufacturer has three factories F_1, F_2, F_3 making 25%, 35%, and 40%, respectively, of its cars. Of their output, 5%, 4%, and 2%, respectively, are defective. A car is chosen at random from the manufacturer's supply.

   (a) What is the probability that the car is defective?

   (b) Given that it is defective, what is the probability that it came from factory F_1?

10. Suppose that there are equal numbers of men and women in the world, and that 5% of men are colorblind whereas only 1% of women are colorblind. A person is chosen at random and found to be colorblind. What is the probability that the person is male?

11. A doctor assumes that his patients has one of the three diseases d_1, d_2, or d_3, each with probability 1/3. He carries out a test that will be positive with probability 0.8 if the patient has d_1, with probability 0.6 if the patient has d_2, and with probability 0.4 if the patient has d_3.

    (a) What is the probability that the test will be positive?

    (b) Suppose that the outcome of the test is positive. What probabilities should the doctor now assign to the three possible diseases?

12. One coin in a collection of 65 coins has two heads; the rest of the coins are fair. If a coin, chosen at random from the lot and then tossed, turns up heads six times in a row, what is the probability that it is the two-headed coin?

13. A scientist discovers a fossil fragment that he believes is either some kind of tiger (with probability 1/3) or mammoth (with probability 2/3). To shed further light on this question, he conducts a test which has the property that for tigers, it will come out positive with probability 5/6 whereas for mammoths it will come out positive with probability just 1/3. Suppose the test comes out negative. What is the probability, given the outcome of the test, that the fossil comes from a tiger?

14. Sherlock Holmes finds paw prints at the scene of a murder, and thinks that they are either from a dog, with probability 3/4, or from a small bear, with probability 1/4. He then discovers some unusual scratches on a nearby tree. The probability that a dog would produce these scratches is 1/10, while the probability that a bear would is 3/5. What is the probability, given the presence of scratches, that the animal is a bear?

15. A coin is tossed three times. Consider the following five events:

    - A: Heads on the first toss
    - B: Tails on the second toss
    - C: Heads on the third toss
    - D: All three outcomes the same
    - E: Exactly one head

    (a) Which of the following pairs of events are independent?

    (1) A, B

    (2) A, D

(3) A, E
(4) D, E

(b) Which of the following triples of events are independent?

(1) A, B, C
(2) A, B, D
(3) C, D, E

16. You randomly shuffle a standard deck and deal two cards. Which of the following pairs of events are independent?

(1) $A = \{\text{first card is a heart}\}, B = \{\text{second card is a heart}\}$
(2) $A = \{\text{first card is a heart}\}, B = \{\text{first card is a 10}\}$
(3) $A = \{\text{first card is a 10}\}, B = \{\text{second card is a 9}\}$
(4) $A = \{\text{first card is a heart}\}, B = \{\text{second card is a 10}\}$

17. A student applies to UCLA and UCSD. He estimates that he has a probability of 0.5 of being accepted at UCLA and a probability of 0.3 of being accepted at UCSD. He further estimates that the probability that he will be accepted by both is 0.2.

(a) What is the probability that he is accepted at UCSD if he is accepted at UCLA?
(b) Is the event "accepted at UCLA" independent of the event "accepted at UCSD"?