Midterm: CS 7301
Spring 2016

The exam is closed book. You are allowed a one-page cheat sheet. Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, use an additional sheet (available from the instructor) and staple it to your exam.

- **NAME** _____
- **UTD-ID if known** _____

Question	Points	Score
Decision Trees	15	
Linear Classifiers and Regularization	10	
Neural Networks and Perceptrons	20	
Support Vector Machines	10	
KD-trees and Short questions	10	
Total:	65	

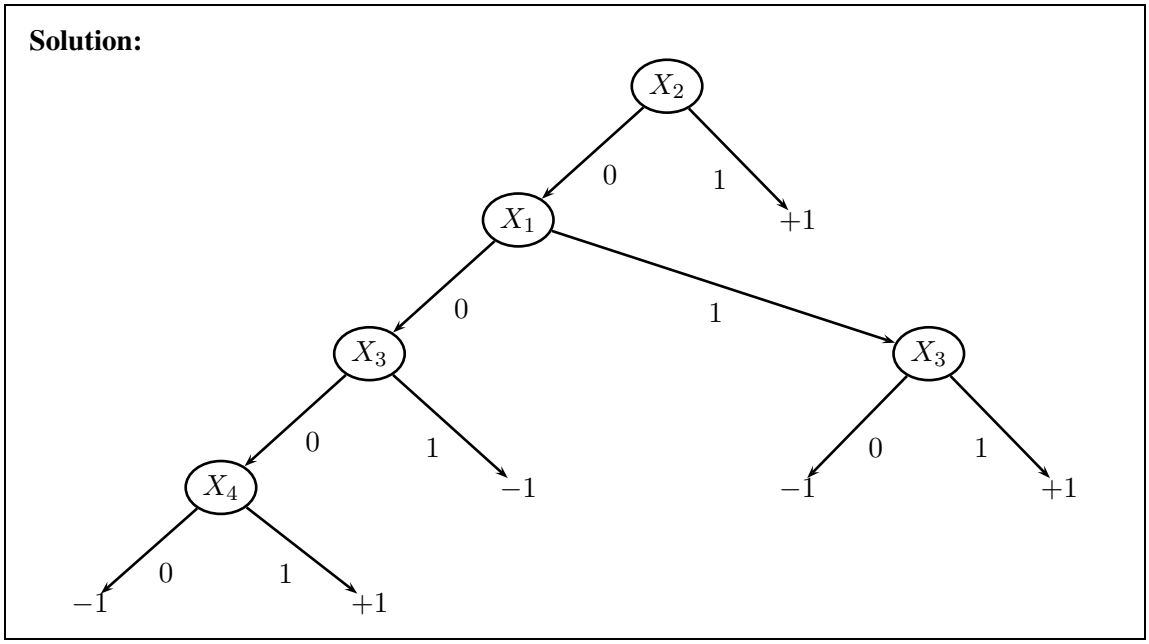
Question 1: Decision Trees (15 points)

Consider the training dataset given below. X_1, X_2, X_3 and X_4 are the attributes/features and Y is the class variable.

Y	X_1	X_2	X_3	X_4
+1	0	1	0	1
+1	1	0	1	0
+1	1	1	1	0
+1	0	0	0	1
+1	1	1	1	0
-1	0	0	1	1
-1	0	0	0	0
-1	0	0	1	0
-1	1	0	0	0
-1	0	0	1	1

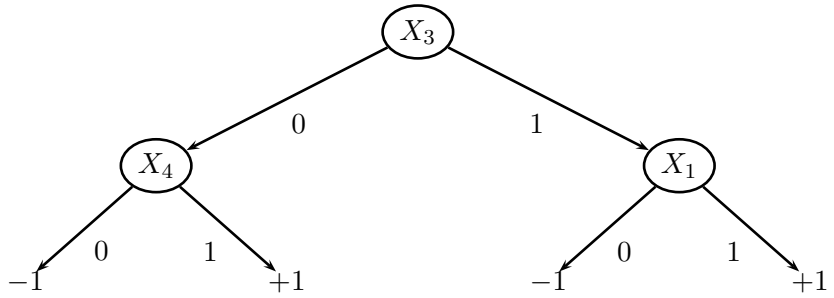
(a) (8 points) Greedily learn a decision tree using the ID3 (namely attributes selected using the information gain criteria) algorithm. Do the calculations on the rough sheets provided by the instructor and draw the tree below.

Solution:



(b) (5 points) Draw a decision tree having only 4 leaf nodes, 3 internal nodes and depth bounded by 2, that has 100% accuracy on the given dataset.

Solution:



(c) (2 points) Which decision tree will you prefer: (a) the ID3 tree you drew on the previous page or (b) the tree you drew on this page having 4 leaf nodes and 3 internal nodes. Also, explain in one or two sentences why you will prefer the tree (you said you will prefer)? No credit without a correct explanation.

Solution: I will prefer (b) because it has the fewest nodes, and is therefore simpler. Recall that our inductive bias was inducing simpler trees (Occam's Razor).

Question 2: Linear Classifiers and Regularization (10 points)

(a) (4 points) Recall that a linear threshold function or a linear classifier is given by:

If $(w_0 + \sum_i w_i x_i) > 0$ then class is positive, otherwise it is negative. Assume that 1 is true and 0 is false.

Consider a function over n Binary features, defined as follows. If at least k variables are false, where $k <= n$ is a constant, then the class is **positive**, otherwise the class is **negative**. Can you represent this function using a linear threshold function. If your answer is **YES**, then give a precise numerical setting of the weights. Otherwise, clearly explain, why this function cannot be represented using a linear threshold function.

Solution:

Yes. Use the following weights: $w_i = -1;$ for $i = 1, \ldots, n$ and $w_0 = n - k + 0.5$.

How did I derive this.

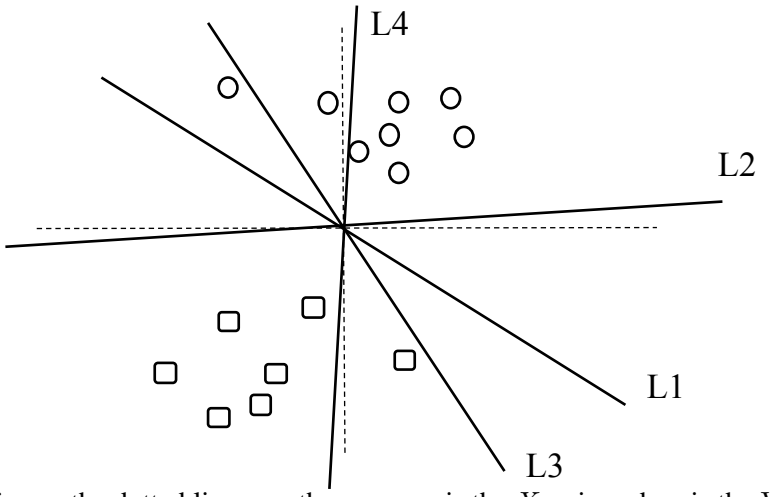
$$\#zeros \geq k \Rightarrow \#ones < n - k \Rightarrow n - k - \#ones > 0$$

To count the number of ones, we can use $w_i = 1$. However, since we want to $-\#ones$, we will use $w_i = -1$. In this case, $w_0 = n - k + 0.5$.

In this problem, we will refer to the binary classification task depicted in the figure given below. Consider the following **logistic regression (LR)** model:

$$P(y = 1|x_1, x_2, w_1, w_2) = \frac{1}{1 + \exp(w_1 x_1 + w_2 x_2)}$$

Notice that the model is assuming that the bias term w_0 equals 0, namely the induced classifiers will pass through the origin.



In the figure, the dotted lines are the axes. x_1 is the X-axis and x_2 is the Y-axis.

Let $L1$ be the solution (line) output by a gradient ascent algorithm using the maximum likelihood estimation (MLE) criteria. In the following, you will answer how regularization will impact the solution.

(b) (6 points) Consider a regularization approach where we try to maximize:

$$\sum_{i=1}^{n} \log\{P(y_i|x_{i,1}, x_{i,2}, w_1, w_2)\} - \frac{C}{2}(w_1)^2$$

for large C. Note that only w_1 is penalized. We'd like to know which of the lines in the figure above could arise as a result of such regularization. For each potential line L2, L3 or L4 determine whether it can result from regularizing w_1. If not, explain very briefly why not.

- L2 (answer Yes/NO and briefly explain why):

  Solution: Yes. This is because large value of C will force w_1 towards zero, which means that the line will be horizontal.

- L3 (answer Yes/NO and briefly explain why):

  Solution: No. The line is more vertical than L1. Can't happen because L3's w_1 value is larger than L1.

- L4 (answer Yes/NO and briefly explain why):

Solution: No. The line is more vertical than L1. Can't happen because L4's w_1 value is larger than L1.

Question 3: Neural Networks and Perceptrons (20 points)

(a) (10 points) Draw a neural network that represents the following function. You can only use two types of units: linear units and sign units. Recall that the linear unit takes as input weights and attribute values and outputs $w_0 + \sum_i w_i x_i$, while the sign unit outputs $+1$ if $(w_0 + \sum_i w_i x_i) > 0$ and -1 otherwise.

y_1 and y_2 are outputs and x_1 and x_2 are inputs. Therefore, your neural network will have two output nodes.

x_1	x_2	y_1	y_2
0	0	0	0
0	1	1	0
1	0	1	1
1	1	0	0

Note that to get full credit, you have to write down the precise numeric weights (e.g., -1, -0.5, $+1$, etc.) as well as the precise units used at each hidden and output node.

Solution: Easy (Model y_1 and y_2 separately using the description in the appendix available here:
`http://www.hlt.utdallas.edu/ vgogate/ml/exams/solutions/spring2015-midterm-solutions.pdf`
Skipped.

(b) (10 points) Derive a gradient descent training algorithm for the following "special" unit and evaluation function. The "special" unit takes as input a vector $(x_1, \ldots, x_n)$ of feature values and outputs o, where o is given by the following equation:

$$o = w_0 + \sum_{j=1}^{n} w_j (x_j + x_j^{1.5}) \tag{1}$$

Here, $w_0, w_1, \ldots, w_n$ are the parameters which you have to learn from the training dataset D having m examples. Use the following evaluation (error) function:

$$E = \frac{1}{3} \sum_{i=1}^{m} (y_i - o_i)^3$$

where y_i is the value of the i-th example that your algorithm will predict and o_i is given in equation (1). Use the batch gradient descent approach. Use the following notation: $x_{i,d}$ denotes the value of the i-th attribute (feature) in the d-th example in the training set D.

Solution: The gradient w.r.t. w_i:

$$-\sum_{j=1}^{m} (y_j - o_j)^2 (x_{i,j} + x_{i,j}^{1.5})$$

The gradient w.r.t. w_0 is:

$$-\sum_{j=1}^{m} (y_j - o_j)^2$$

Batch Algorithm pseudo code:
Input: Data and learning rate η

- Initialize w_i's and w_0 to random values.
- Until Convergence do
  - $\Delta w_0 = 0$
  - $\Delta w_i = 0$ for $i = 1$ to n
  - For each training example indexed by j do
    - Compute $o_j = w_0 + \sum_{i=1}^{n} w_i (x_i + x_i^{1.5})$
  - For each training example indexed by j do
    - $\Delta w_0 = \Delta w_0 + (y_j - o_j)^2$
    - $\Delta w_i = \Delta w_i + (y_j - o_j)^2 (x_{i,j} + x_{i,j}^{1.5})$ for $i = 1$ to n
  - $w_0 = w_0 + \eta \Delta w_0$
  - $w_i = w_i + \eta \Delta w_i$ for $i = 1$ to n

Question 4: Support Vector Machines (10 points)

Consider the dataset given below (x_1, x_2) are the attributes and y is the class variable):

x_1	x_2	y
0	0	$+1$
-2	1	-1
2	1	-1

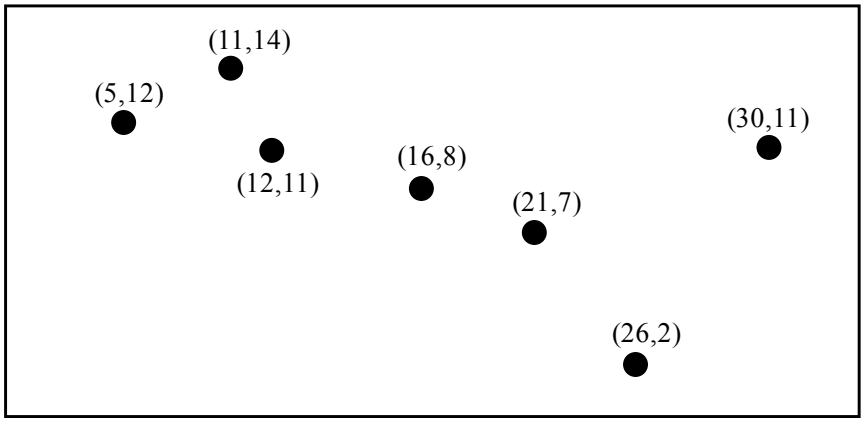
(a) (10 points) Find the linear SVM classifier for the dataset given above. Do your optimization either using the primal problem or the dual problem. Provide a precise setting of the weights **w** and the bias term b. What is the size of the margin? (Hint: The primal problem seems to be easier than the dual).

Solution:

Primal problem:

For each data point, we have a lagrange multiplier λ_i. This gives us the following problem: $L(\mathbf{w}, \lambda) = \frac{1}{2}||\mathbf{w}||_2^2 + \sum_{i=1}^{3} \lambda_i(y_i(\mathbf{w}^T\phi(x_i) + b) - 1)$

Question 5: KD-trees and Short questions (10 points)



Consider the set of 2-D points along with the co-ordinates given above:

(a) (3 points) Build a balanced kd-tree for the points given above. Draw the tree below and the separating planes in the figure.

Solution: Skipped. Easy.

(b) (3 points) Make another copy of the tree and highlight the edges traversed when looking for the closest point to $(5, 8)$.

Solution: Skipped. Easy.

(a) (2 points) Yes/No. For linearly separable data, can a small slack penalty hurt the training accuracy when using a linear SVM (no kernel)? If so, explain how. If not, why not?

Solution: See Previous years' midterm and final for a solution.

(b) (2 points) Provide a reasonable approach for determining the value of K in the K-nearest neighbors algorithm.

Solution: Several approaches possible here. Choose k using Cross Validation. Try different ones and pick the one having the smallest cross validation error.