Budapest University of Technology and Economics
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DPIM FINAL HW

Demand Planning and Inventory Management

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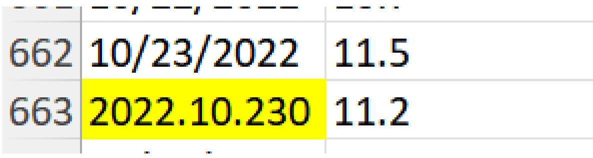
1. Data Cleaning

So, at first, all the extra data was deleted and the type of the task and types of data given were identified. The below decisions made for the data given:

- The value is in weight unit.
- The data is given in piece weight, hence only whole kg weight can be ordered. Meaning rounding is needed.

At first the missing data, data errors and dates that did not fall in the format was identified. Here is the wrongly formatted date.



662	10/23/2022	11.5
663	2022.10.230	11.2

Figure 1: Wrongly formatted data

But it was found that, if this date is used, the data already exists. Which means it was a double entry. So it was deleted. As for the missing data, they will be fixed in data cleaning.

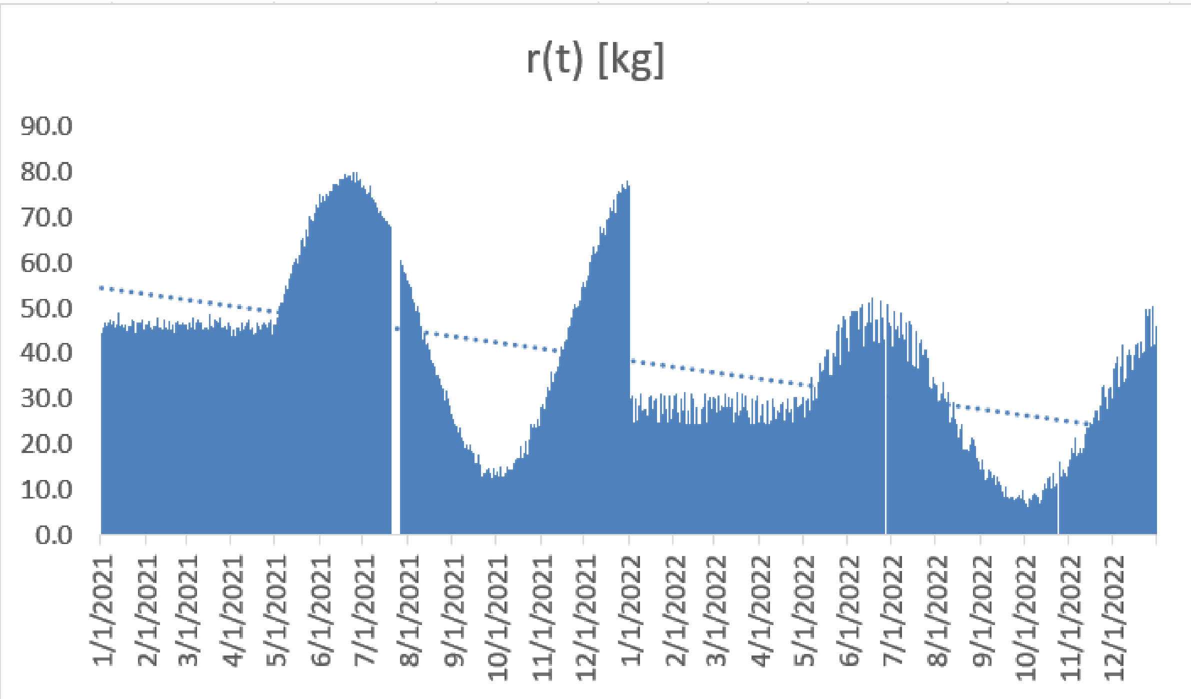


Figure 2: Data Before Cleaning

Then we inserted this chart from data, which clearly shows that there are missing data.

After that, all the blank and odd data in the r(t) column was highlighted.

Date	r(t) [kg]
7/21/2021	
7/22/2021	
7/23/2021	
7/24/2021	
7/25/2021	
6/27/2022	SKU02
10/6/2022	978020137902.0

Figure 3: Missing and Wrongly Formatted Data

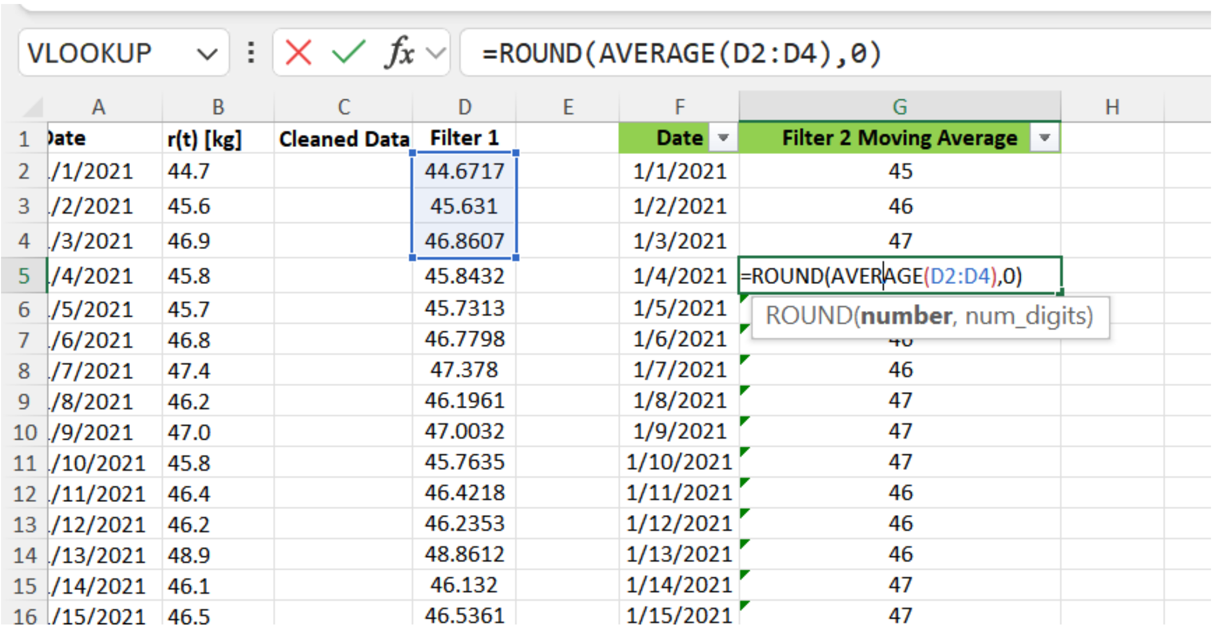
Then Data cleaning was done.

C203 =0.33*C200+0.33*C201+0.33*C202

A	B	C
7/15/2021	69.6	
7/16/2021	69.1	
7/17/2021	68.9	
7/18/2021	68.2	68.2
7/19/2021	67.7	67.7
7/20/2021	64.3	64.3
7/21/2021		66.07459

Figure 4: Data Cleaning

The data was further cleaned by using moving average method.



	A	B	C	D	E	F	G	H
1	)ate	r(t) [kg]	Cleaned Data	Filter 1		Date	Filter 2 Moving Average	
2	/1/2021	44.7		44.6717		1/1/2021	45	
3	/2/2021	45.6		45.631		1/2/2021	46	
4	/3/2021	46.9		46.8607		1/3/2021	47	
5	/4/2021	45.8		45.8432		1/4/2021	=ROUND(AVERAGE(D2:D4),0)	
6	/5/2021	45.7		45.7313		1/5/2021		
7	/6/2021	46.8		46.7798		1/6/2021	[illegible]	
8	/7/2021	47.4		47.378		1/7/2021	46	
9	/8/2021	46.2		46.1961		1/8/2021	47	
10	/9/2021	47.0		47.0032		1/9/2021	47	
11	/10/2021	45.8		45.7635		1/10/2021	47	
12	/11/2021	46.4		46.4218		1/11/2021	46	
13	/12/2021	46.2		46.2353		1/12/2021	46	
14	/13/2021	48.9		48.8612		1/13/2021	46	
15	/14/2021	46.1		46.132		1/14/2021	47	
16	/15/2021	46.5		46.5361		1/15/2021	47	

Figure 5: Data Cleaning With Moving Average

The cleaned data would look like below:

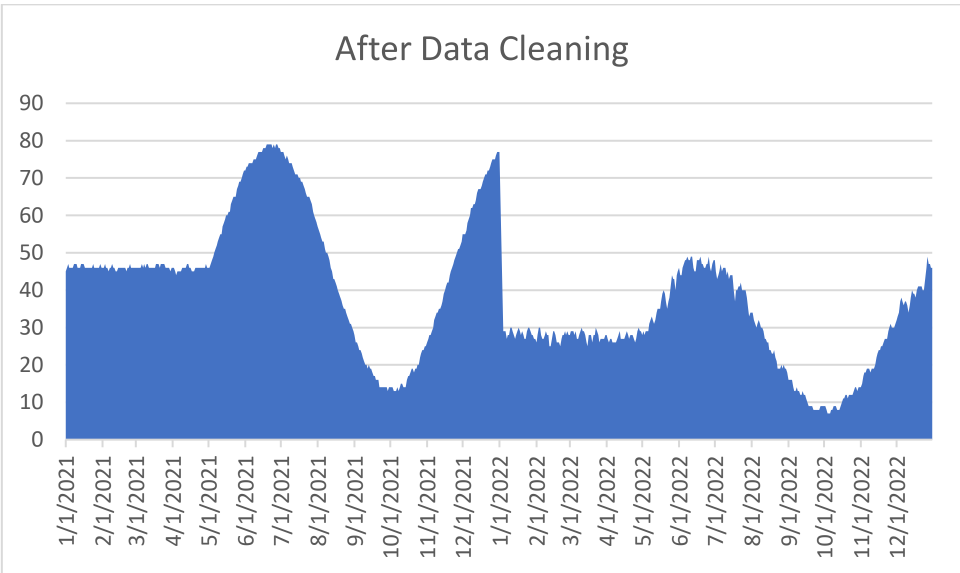


Figure 6: Data After Cleaning

2. Aggregation

As there is data for each day for 2 years here, , it is best to aggregate the data. As it can be seen that, the data. So it is absolutly vital to aggregate this to get insight.

Now the question is which kind of aggregation. If we look at the data we see the below characteriastics:

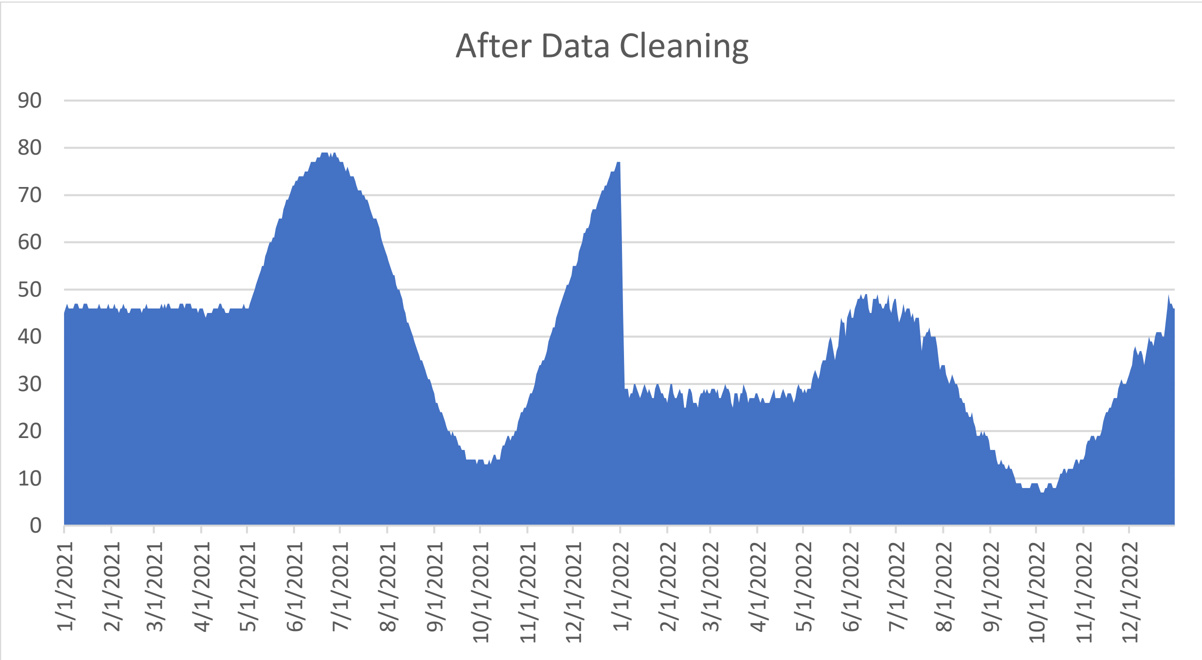


Figure 7: Data After Cleaning

2.1.Deciding the best Aggregation

- There is sporadic demand in the data, but the fluctuations in the demand is not that high
- There is a but of seasionality here, (e.g., higher demand in late spring and summer, lower in fall and winter).
- There is a noticeable upward trend in some parts and downward trends in others.

There is seasonal patterns here, also the data is given on daily basis. So, monthly or weekly aggregation can be the best way to do this. Here monthly, quarterly and weekly aggregation is done. The below is what it looks like:

Table 1: Querterly aggregation

Quarterly Aggregation	
Row Labels	**Moving Average**
2021	**17487**
Qtr1	**4157**
Qtr2	**5504**
Qtr3	**4029**
Qtr4	**3797**
2022	**10607**
Qtr1	**2617**
Qtr2	**3338**
Qtr3	**2420**
Qtr4	**2232**
Grand Total	**28094**

And the chart for it would look like below:

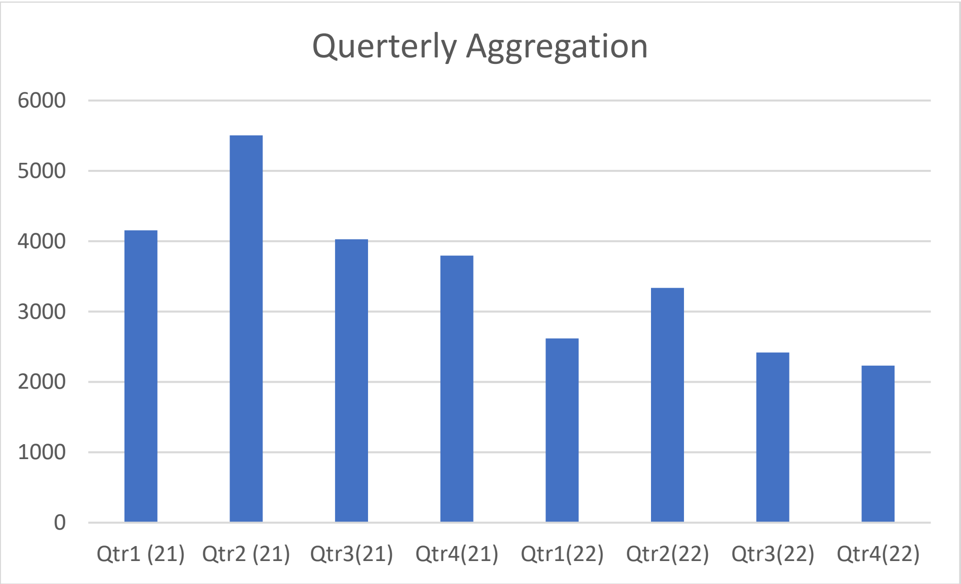


Figure 8: Querterly aggregation

Table 2: Monthly Aggregation

Monthly Aggregation	
Row Labels	**Moving Average**
2021	
Jan	1434
Feb	1287
Mar	1436
Apr	1373
May	1830
Jun	2301
Jul	2152
Aug	1316
Sep	561
Oct	539
Nov	1184
Dec	2074
2022	
Jan	980
Feb	772
Mar	865
Apr	824
May	1104
Jun	1410
Jul	1300
Aug	783
Sep	337
Oct	308
Nov	695
Dec	1229
Grand Total	**28094**

The chart of Monthly Aggregation would look like below:

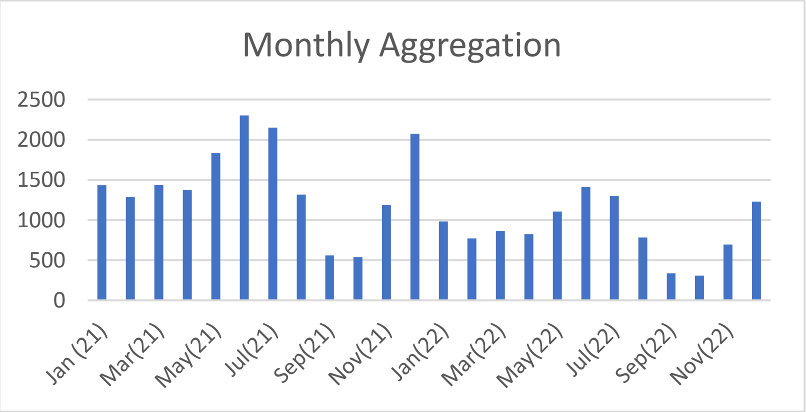


Figure 9: Monthly Aggregation

Then there is weekly aggregation. Since, there are two years, it was done in two parts. See Table 3 in the Appendix for the weekly aggregation of 2021 and see Table 4 on the Appendix for the weekly aggregation of 2022.

The chart of Aggregation for Year 2021 can be seen below:

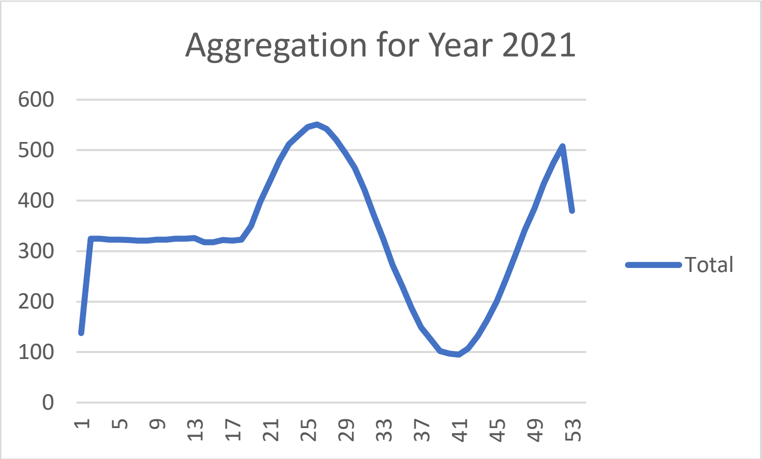


Figure 10: Aggregation for Year 2021

The chart of Weekly Aggregation for Year 2022 can be seen below:

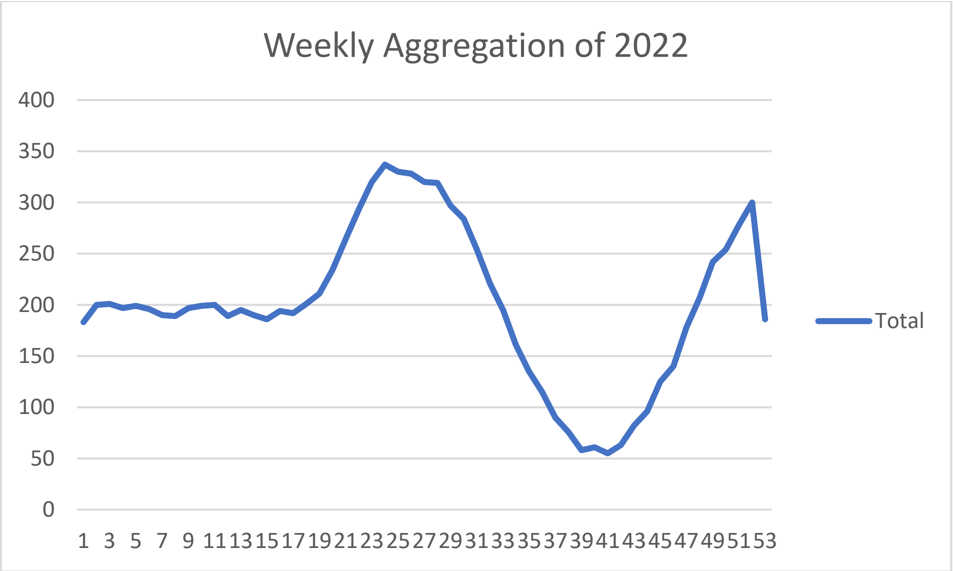


Figure 11: Weekly Aggregation for Year 2022

3. Autocorelation

Here, at first D0d0, D0d1, D0d2, D1d0 and D1d1 was determined and the output looks like this:

Table 3: Data Transformation

Month	Value/ D0d0	D0d1	D0d2	D1d0	D1d1
Jan (21)	1434				
Feb(21)	1287	-147			
Mar(21)	1436	149	296		
Apr(21)	1373	-63	-212		
May(21)	1830	457	520		
Jun(21)	2301	471	14		
Jul(21)	2152	-149	-620		
Aug(21)	1316	-836	-687		
Sep(21)	561	-755	81		
Oct(21)	539	-22	733		
Nov(21)	1184	645	667		
Dec(21)	2074	890	245		
Jan(22)	980	-1094	-1984	-454	
Feb(22)	772	-208	886	-515	-61
Mar(22)	865	93	301	-571	-56
Apr(22)	824	-41	-134	-549	22
May(22)	1104	280	321	-726	-177
Jun(22)	1410	306	26	-891	-165
Jul(22)	1300	-110	-416	-852	39
Aug(22)	783	-517	-407	-533	319
Sep(22)	337	-446	71	-224	309
Oct(22)	308	-29	417	-231	-7
Nov(22)	695	387	416	-489	-258
Dec(22)	1229	534	147	-845	-356

These Values were compared in a chart like below:

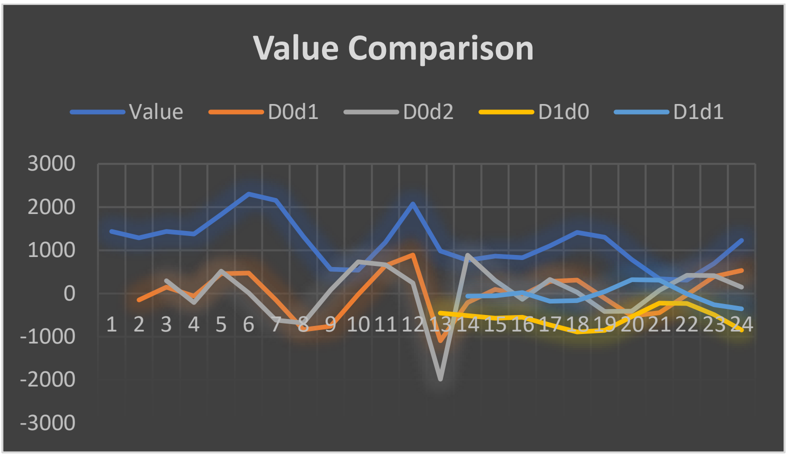


Figure 12: Transformation Value Comparison

From this, at first the Normal and Barret Errors were calculated for all of these transformation. They have been compared with the Wessa Autocorelation in the next chapter.

ACF and PACF for this time series was also calculated from Wessa,

Table 4: Wessa ACF for D0d1

Time lag k	ACF(k)	T-STAT	P-value
1	0.221901	1.0642	0.149142
2	-0.39217	-1.8808	0.036362
3	-0.50192	-2.4071	0.012251
4	-0.22611	-1.0844	0.144711
5	0.206842	0.992	0.165765
6	0.357341	1.7137	0.050012
7	0.154889	0.7428	0.232554
8	-0.14562	-0.6984	0.245968
9	-0.2124	-1.0186	0.159491
10	-0.24714	-1.1853	0.124009
11	0.078023	0.3742	0.355848
12	0.34609	1.6598	0.055266
13	0.162049	0.7772	0.222493
14	-0.09083	-0.4356	0.333595
15	-0.2185	-1.0479	0.152785
16	-0.14112	-0.6768	0.252648
17	0.011661	0.0559	0.477942
18	0.084179	0.4037	0.345078
19	0.054343	0.2606	0.398352
20	0.006763	0.0324	0.487204
21	0.005847	0.028	0.488936
22	-0.01411	-0.0677	0.473315

The PACF values can be found for D0d1 below:

Table 5: PACF values can be found for D0d1

Time lag k	PACF(k)	T-STAT	P-value
1	0.221901	1.0642	0.149142
2	-0.46427	-2.2266	0.018027
3	-0.36399	-1.7456	0.04711
4	-0.31339	-1.503	0.073227
5	-0.11281	-0.541	0.296845
6	-0.06026	-0.289	0.387585
7	-0.04898	-0.2349	0.408184
8	-0.07005	-0.336	0.369973
9	0.008075	0.0387	0.484721
10	-0.28586	-1.3709	0.091813
11	-0.00037	-0.0018	0.499299
12	0.083192	0.399	0.346796
13	-0.04693	-0.2251	0.411962
14	0.027603	0.1324	0.447919
15	0.051135	0.2452	0.404224
16	0.042739	0.205	0.4197
17	-0.05176	-0.2482	0.403074
18	-0.15594	-0.7479	0.231061
19	-0.11874	-0.5695	0.287282
20	-0.17035	-0.8169	0.211171
21	-0.10088	-0.4838	0.316546
22	-0.02419	-0.116	0.45432

For the D0d1, the comparison between the original time series and the working time series is shown below:

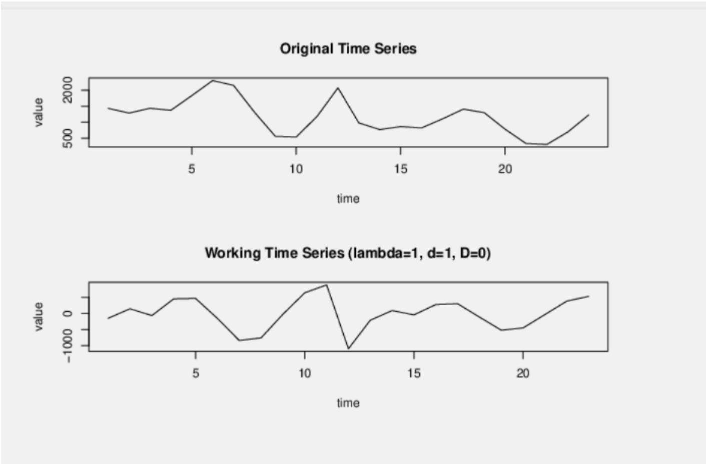


Figure 13: The comparison between the original time series and the working time series

The PACF can be found below:

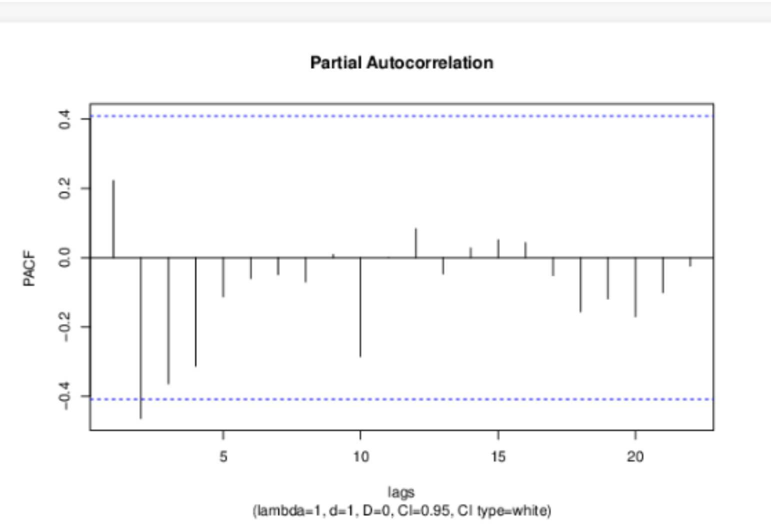


Figure 14: PACF for D0d1

Similarly, the Autocorelation of D1d0 from Wessa is the below table:

Table 6: ACF Values for D1d0

Time lag k	ACF(k)	T-STAT	P-value
1	0.496155	1.7187	0.055665
2	-0.22835	-0.791	0.222143
3	-0.58601	-2.03	0.032564
4	-0.4299	-1.4892	0.081118
5	-0.09027	-0.3127	0.379932
6	0.09535	0.3303	0.373434
7	0.128286	0.4444	0.332332
8	0.101946	0.3532	0.36505
9	0.083297	0.2886	0.388925
10	-0.01067	-0.037	0.485558
11	-0.05983	-0.2072	0.419647

The PACF values for D1d0 looks like the below one:

Table 7: PACF Values for D1do

Time lag k	PACF(k)	T-STAT	P-value
1	0.496155	1.7187	0.055665
2	-0.62948	-2.1806	0.024922
3	-0.19573	-0.678	0.255307
4	-0.08869	-0.3072	0.381972
5	-0.20862	-0.7227	0.241859
6	-0.25378	-0.8791	0.198296
7	-0.10705	-0.3708	0.358615
8	-0.12703	-0.44	0.333867
9	-0.10342	-0.3582	0.36319
10	-0.25987	-0.9002	0.192856
11	-0.05144	-0.1782	0.430777

The comparison between The original Time series and the Working Time Series for D0d1 is given below:

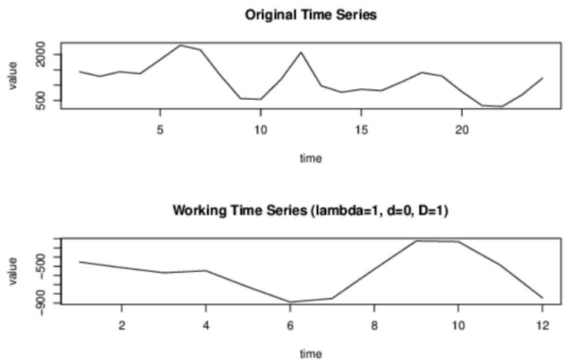


Figure 15: The comparison between The original Time series and the Working Time Series for D0d1

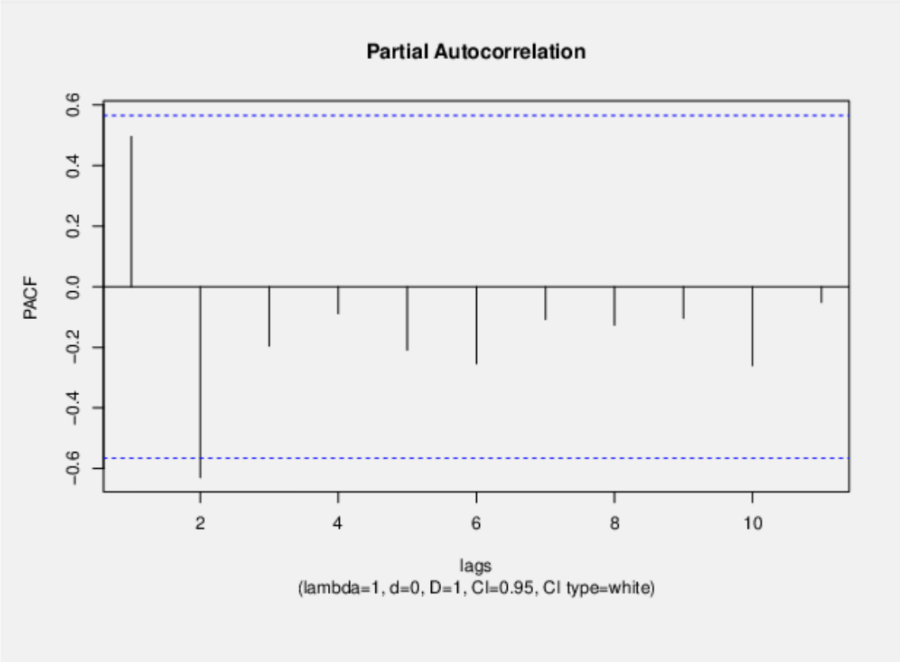


Figure 16: PACF for D1d0

As for D1d1 the Wessa ACF values looks like below:

Table 8: For D1d1 the Wessa ACF

Time lag k	ACF(k)	T-STAT	P-value
1	0.502633	1.667	0.061846
2	-0.25321	-0.8398	0.209451
3	-0.61104	-2.0266	0.033822
4	-0.36131	-1.1983	0.127988
5	0.026005	0.0862	0.46641
6	0.131774	0.437	0.335268
7	0.052182	0.1731	0.432871
8	-0.03291	-0.1092	0.45752
9	0.02751	0.0912	0.464472
10	0.018367	0.0609	0.476259

As for D1d1 the Wessa PACF values looks like below:

Table 9: For D1d1 the Wessa PACF values

Time lag k	PACF(k)	T-STAT	P-value
1	0.502633	1.667	0.061846
2	-0.67685	-2.2448	0.023153
3	-0.1418	-0.4703	0.323662
4	0.063539	0.2107	0.418474
5	-0.25218	-0.8364	0.210372
6	-0.2677	-0.8879	0.196807
7	0.030793	0.1021	0.460246
8	-0.17309	-0.5741	0.288733
9	-0.02745	-0.091	0.464551
10	-0.27597	-0.9153	0.189834

The Original Time series vs The Working Time Series for D1d1 can be shown from the below image from Wessa:

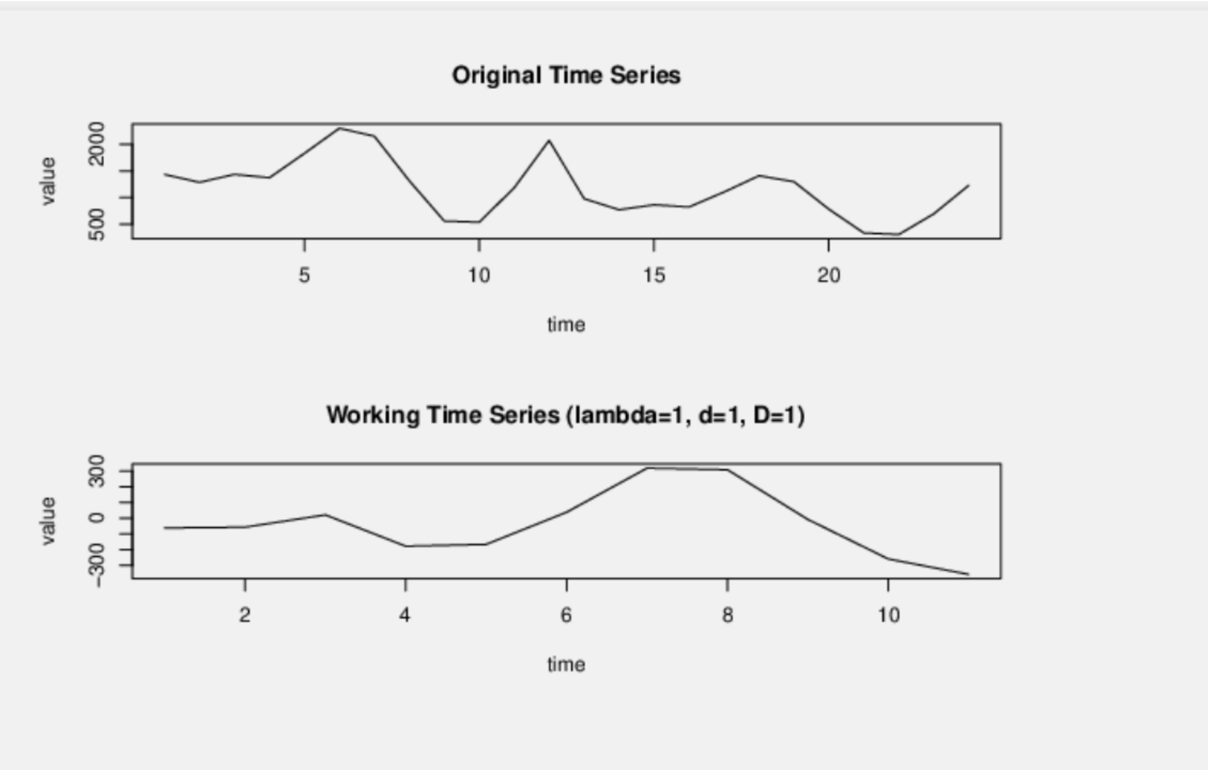


Figure 17: The Original Time series vs The Working Time Series

The PACF for D1d1 from Wessa:

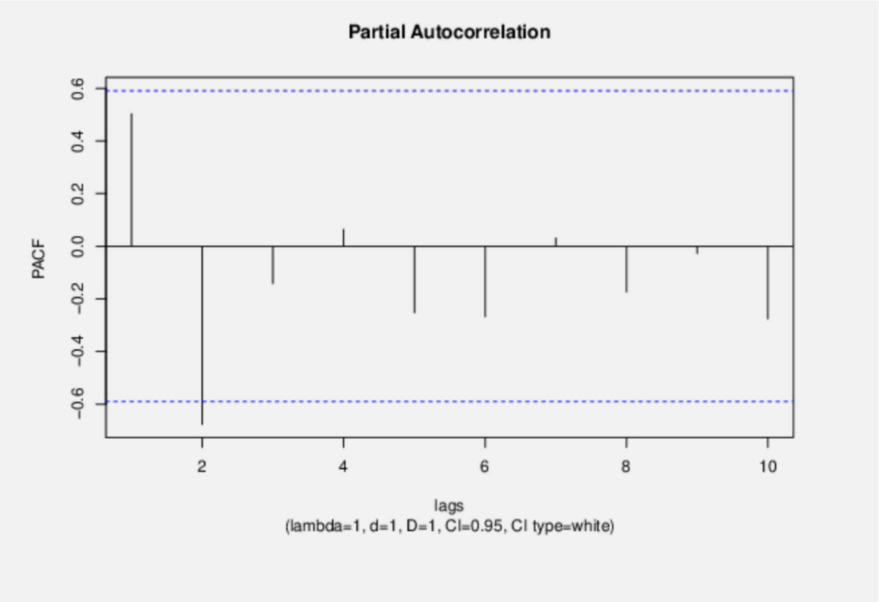


Figure 18: The PACF for D1d1 from Wessa

Now, for D0d0, the ACF chart from Wessa can be found in the appendix in This table.

And the PACF chart can be found in This table.

The Original Time Series Vs the Working Time series for D0d0 is below:

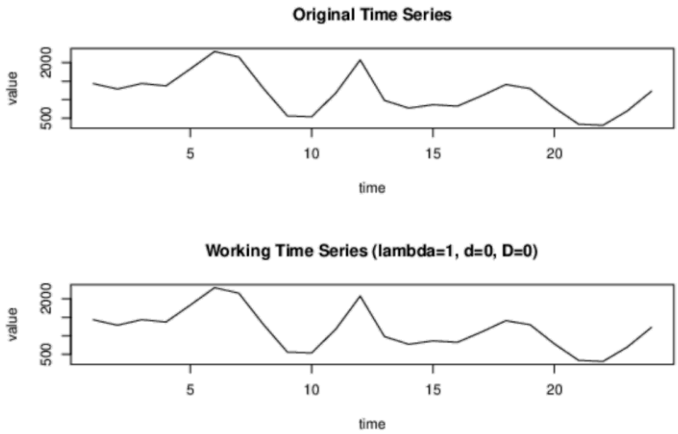


Figure 19: The Original Time Series Vs the Working Time series for D0d0

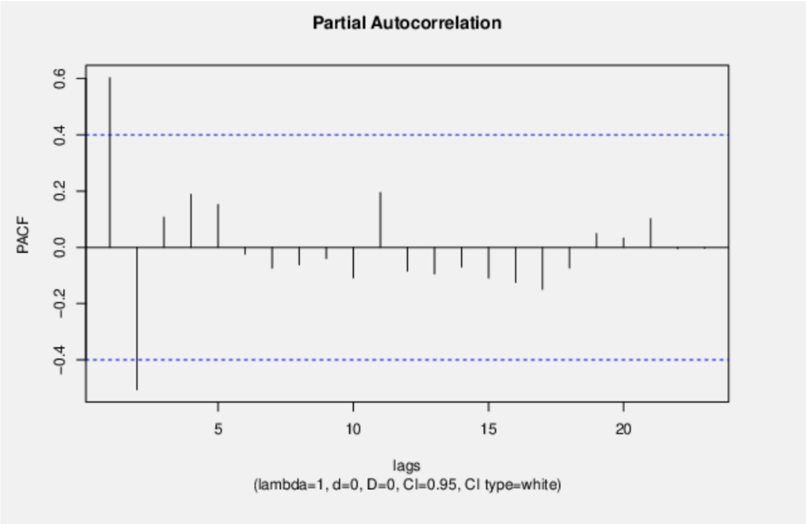


Figure 20: The PACF chart from D0d0

3.1.The Significance of the Errors with Relation to the lags and transformations:

1. Early Lag Significance (k=1,2,3) : The autocorrelation values AC(k) at early lags across all charts exceed both Normal bounds (UB, LB) and Bartlett bounds (UB BA, LB BA). These significant values indicate strong short-term dependencies within the time series. The series exhibits a memory effect, where current values are highly influenced by immediately preceding values. In particular, D0d0 and D0d1 have AC(1) close to or at 1, indicating near-perfect correlation between adjacent time points, suggesting a predictable short-term pattern or trend.

2. Bartlett Bounds for Intermediate Lags (k=4 to 8): As k increases, AC(k) begins to taper off but still occasionally breaches the Normal bounds while remaining within the Bartlett bounds. As the lag increases (from $k = 4$k=4 to $k = 8$k=8), the autocorrelation values gradually decrease but still show some significance within Bartlett's wider bounds, which provide a more realistic way of interpreting these patterns. For example, in D0d0, the values at $k = 4$k=4 and $k = 5$k=5 suggest that there's still some lingering connection between the data points, possibly due to trends or cycles in the series. In contrast, in D1d1 and D1d0, the correlations fade much faster, showing that these series have less structure or are more random at these intermediate lags.

3. At higher lags ($k > 8$): The autocorrelation values $AC(k)$ fall well within both the Normal and Bartlett bounds across all charts, indicating no significant long-term dependencies. This implies that the data points are effectively uncorrelated as the

separation between them increases. As a result, the time series becomes unpredictable or increasingly dominated by noise at these higher lags.

4. The Comparison of The Excel Values and the Wessa Values

4.1.The Comparison of the Error Values

The data was inserted into wessa to calculate the ACF. The data is put into excel. The values of Partial Autocorelation Functions were also obtained.

For D0d0

Data Comparison of AC(K) in Wessa And Excel

Lag	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
AC(k) (Excel)	0.602929407	0.041595421	-0.21657	-0.08087	0.20891	0.312588	0.14021	-0.11873	-0.23165	-0.17741	0.045475	0.169344	0.05653	-0.16706	-0.31817	-0.29728	-0.18316	-0.09647	-0.07161	-0.06403	-0.03937	-0.01746	0.002268
AC(k) (Wessa)	0.602929	0.041595	-0.21657	-0.08087	0.20891	0.312588	0.14021	-0.11873	-0.23165	-0.17741	0.045475	0.169344	0.05653	-0.16706	-0.31817	-0.29728	-0.18316	-0.09647	-0.07161	-0.06403	-0.03937	-0.01746	0.002268
Difference (up till 5 decimal points)	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

For D0d1

Lag	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
AC(k) (Excel)	0.221901194	-0.392168485	-0.50192	-0.22611	0.206842	0.357341	0.154889	-0.14562	-0.2124	-0.24714	0.078023	0.34609	0.162049	-0.09083	-0.2185	-0.14112	0.011661	0.084179	0.054343	0.006763	0.005847	-0.01411
AC(k) (Wessa)	0.221901	-0.392168	-0.50192	-0.22611	0.206842	0.357341	0.154889	-0.14562	-0.2124	-0.24714	0.078023	0.34609	0.162049	-0.09083	-0.2185	-0.14112	0.011661	0.084179	0.054343	0.006763	0.005847	-0.01411
Difference (up till 5 decimal points)	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

For D1d0

Lag	1	2	3	4	5	6	7	8	9	10	11
AC(k) (Excel)	0.496154559	-0.228348121	-0.58601	-0.4299	-0.09027	0.09535	0.128286	0.101946	0.083297	-0.01067	-0.05983
AC(k) (Wessa)	0.496155	-0.228348	-0.58601	-0.4299	-0.09027	0.09535	0.128286	0.101946	0.083297	-0.01067	-0.05983
Difference (up till 5 decimal points)	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

For D1d1

Lag	1	2	3	4	5	6	7	8	9	10
AC(k) (Excel)	0.502633001	-0.253207148	-0.61104	-0.3613	0.026005	0.131774	0.052182	-0.03291	0.02751	0.018367
AC(k) (Wessa)	0.502633	-0.253207	-0.61104	-0.36131	0.026005	0.131774	0.052182	-0.03291	0.02751	0.018367
Difference (up till 5 decimal points)	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

Figure 21: Comparison of the Autocorelation Values with Wessa

As it can be seen from the image above and the excel file, there are no difference between the excel data and the Wessa data, which means that the calculations done in excel was correct. The data is too big to put here as a table, hence it was put as an image.

4.1 The Comparison of Excel Errors with Wessa Autocorelation

For D0d0, the Normal error and Barret Errors can be seen from below:

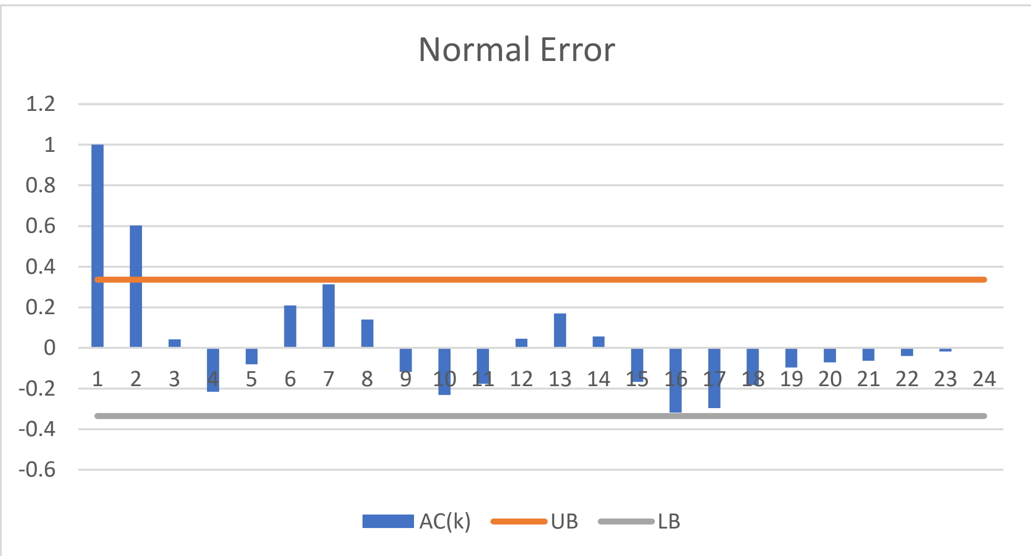


Figure 22: The Normal Error for D0d0

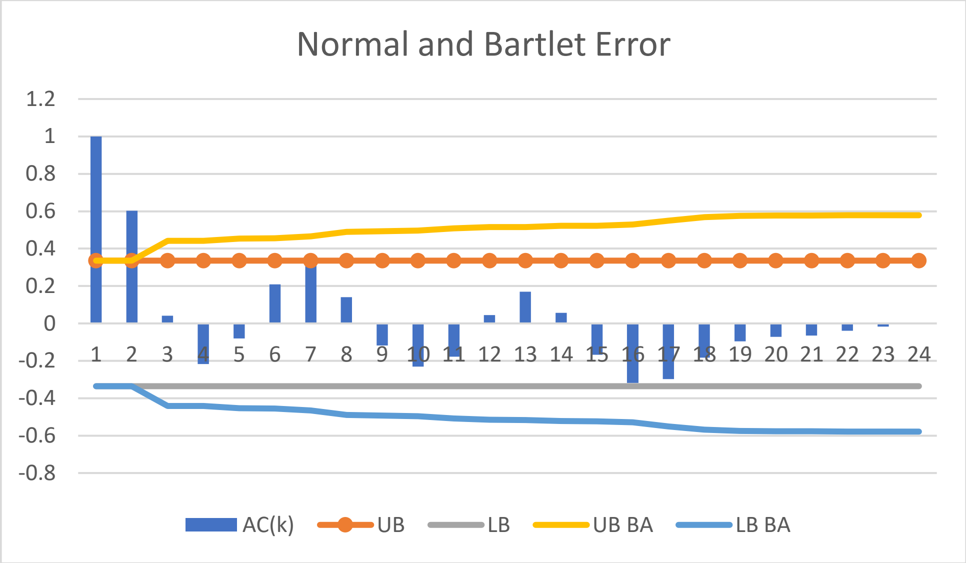


Figure 23: The Normal and Bartlett Error for D0d0

The Autocorelation chart from Wessa is below for easy comparison:

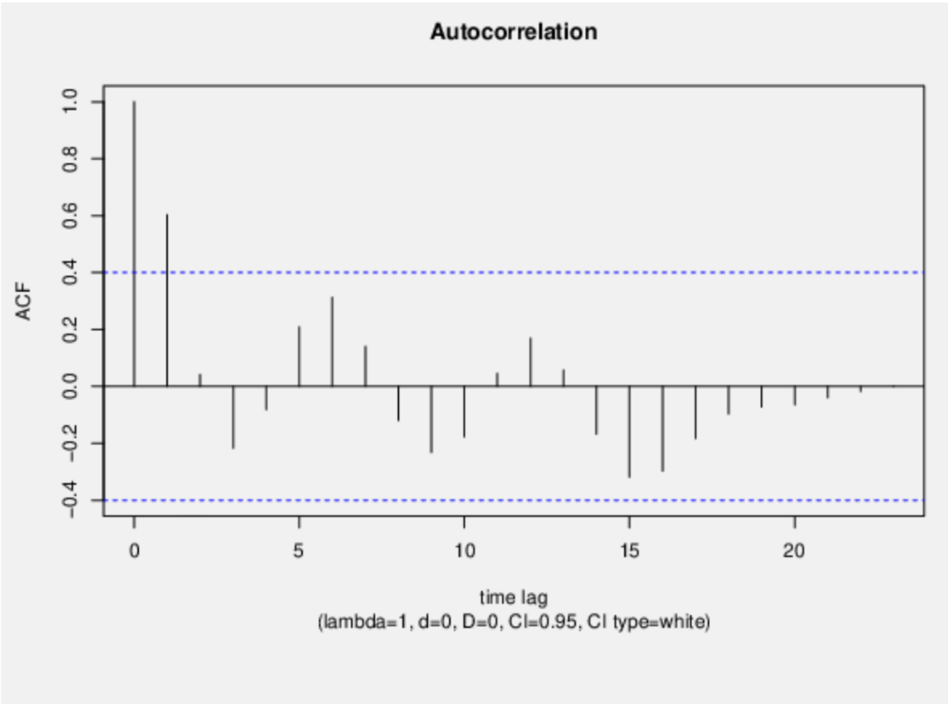


Figure 24: The Autocorelation Chart for D0d0

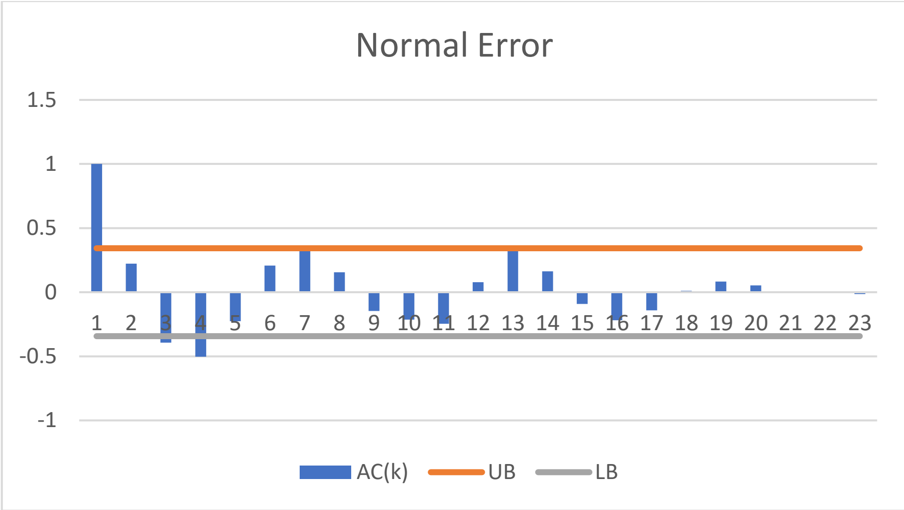


Figure 25: Normal Error D0d1

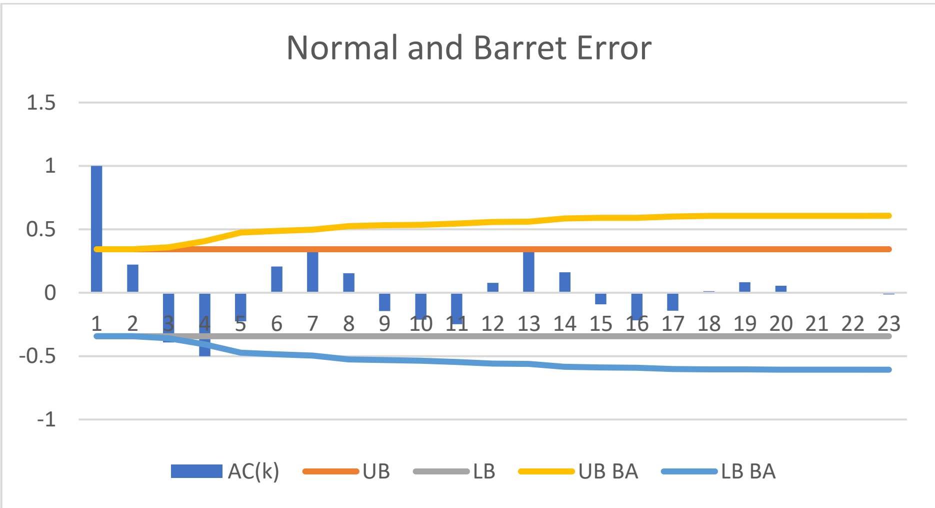


Figure 26: Normal and Barret Error D0d1

The Autocorelation chart from Wessa is inserted below for easy comparison.

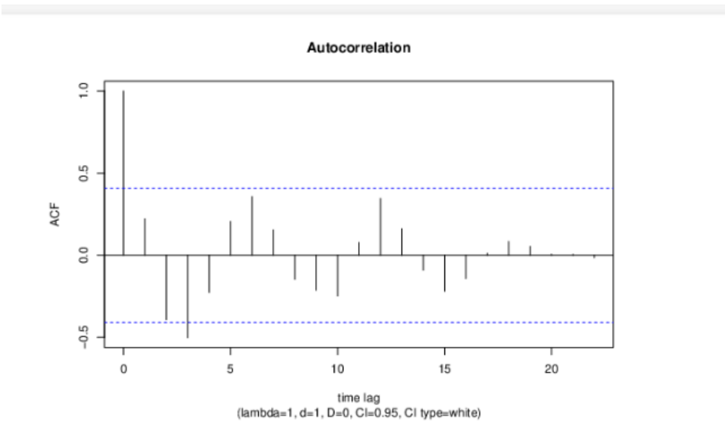


Figure 27: The Autocorelation for D0d1

As for D1d0, the Normal Error here would be:

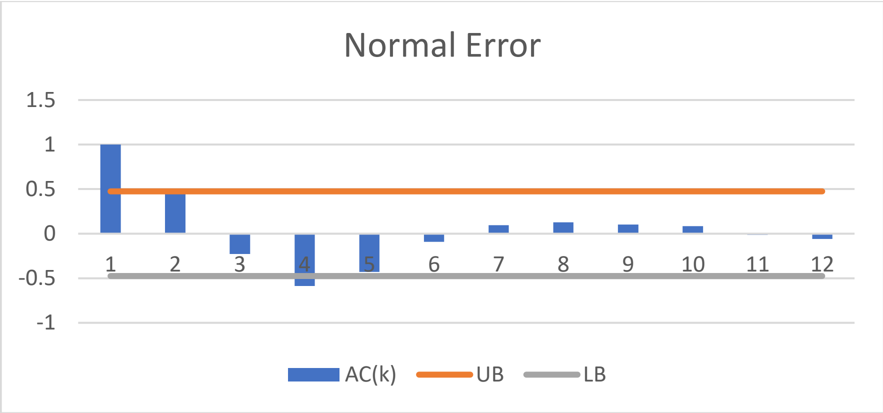


Figure 28: The Normal Error for D1d0

As for D1d0, the Normal Error and Barret Error here would be:

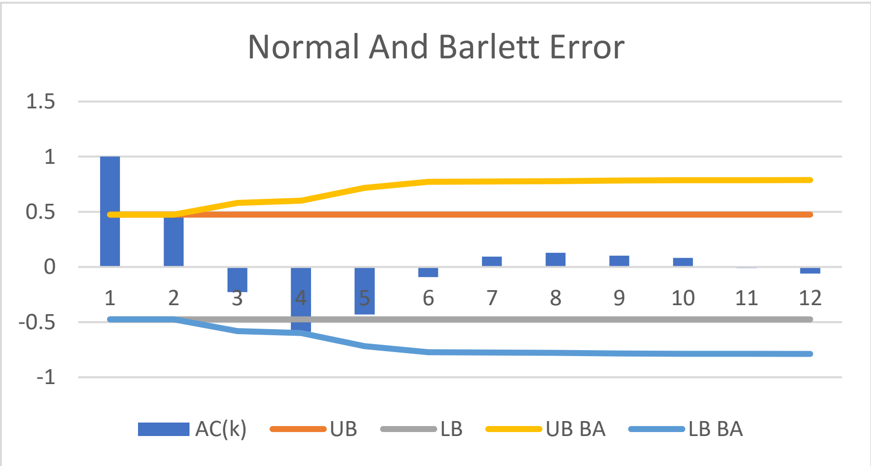


Figure 29: The Normal and Barlett Error for D1d0

The Autocorelation value from D1d0 is inserted below for easy comparison.

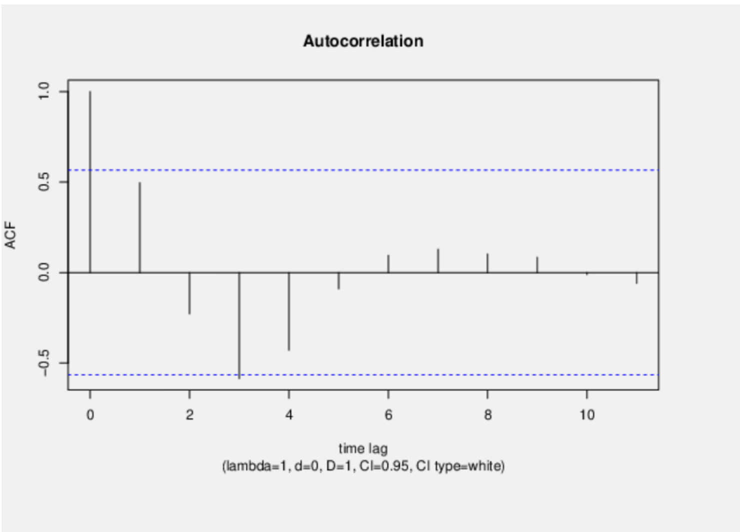


Figure 30: The Autocorelation of D1d0

For D1d1 the Normal Error Chart is inserted below:

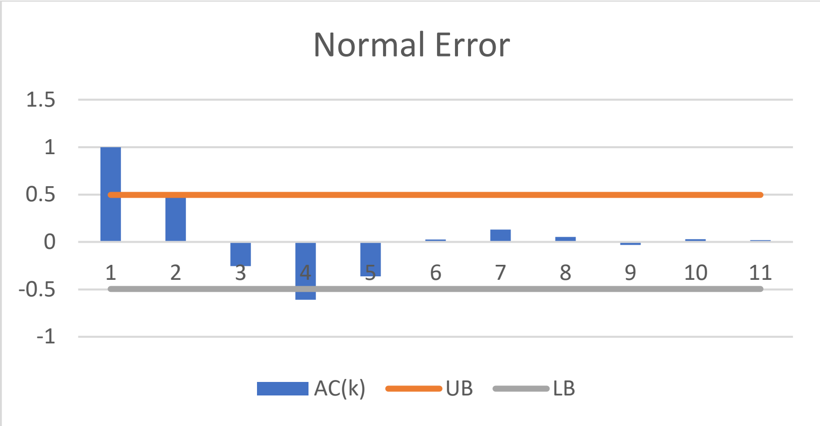


Figure 31: For D1d1 the Normal Error Chart

The normal and Bartlett Erros is inserted below:

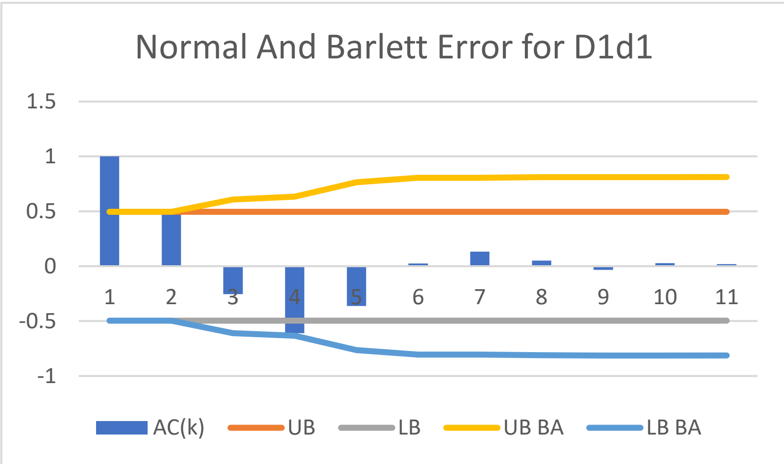


Figure 32: Normal And Barlett Error for D1d1

The ACF for D1d1 from Wessa:

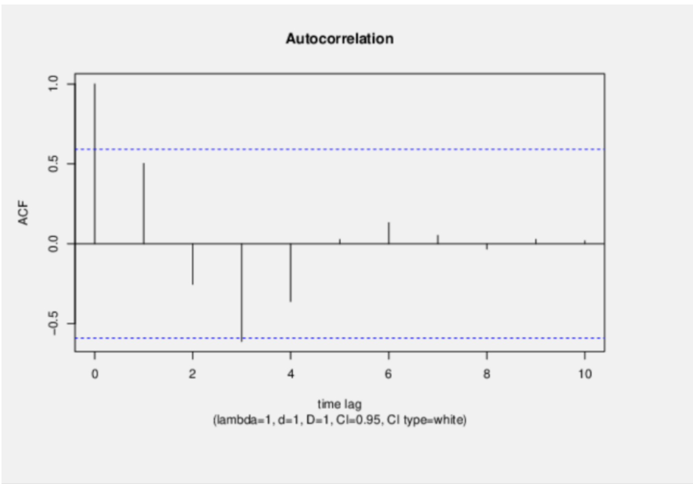


Figure 33: The Autocorelation for D1d1 from Wessa

4.2.Finding the Significance of PACF Values

Now all the PACF values received from Wessa was put into excel together in the table in the next page. Pls find the table in the next page.

Table 10: Comparing the PACF Values

The PACF Comparison				
	D0d0	D0d1	D1d0	D1d1
Time Lag k	PACF(k)	PACF(k)	PACF(k)	PACF(k)
1	0.602929	0.221901	0.496155	0.502633
2	-0.505798	-0.464269	-0.629475	-0.676845
3	0.106879	-0.363992	-0.195731	-0.141802
4	0.188153	-0.313391	-0.088686	0.063539
5	0.152153	-0.112812	-0.208623	-0.252175
6	-0.023536	-0.060261	-0.253784	-0.267701
7	-0.073543	-0.048979	-0.107049	0.030793
8	-0.062122	-0.070053	-0.127026	-0.173090
9	-0.039455	0.008075	-0.103417	-0.027448
10	-0.108234	-0.285860	-0.259872	-0.275969
11	0.194659	-0.000370	-0.051436	
12	-0.083874	0.083192		
13	-0.094455	-0.046927		
14	-0.069917	0.027603		
15	-0.108845	0.051135		
16	-0.124200	0.042739		
17	-0.148852	-0.051762		
18	-0.072978	-0.155940		
19	0.048875	-0.118740		
20	0.032589	-0.170345		
21	0.101830	-0.100882		
22	-0.004589	-0.024193		
23	-0.002272			

Drawing conclusions about the significant values of the partial autocorrelation functions:

- Short-Term Dependence: The high PACF at Lag 1 across all series suggests the immediate past strongly influences the current value. As it can be seen, the PACF at lag one is the only positive value here (pls see below). Meaning that the demand data of the previous month has a larger affect on the demand than the other months.

Time Lag k	PACF(k)	PACF(k)	PACF(k)	PACF(k)
1	0.602929	0.221901	0.496155	0.502633

- Balancing Corrections: The negative values at Lag 2 indicate that the system tries to balance or counteract earlier fluctuations. Looking at Lag 2, it can be seen that the value here is negative. Meaning that the value two months ago is inversely related to the value in the current month (t). Thus, it tries to balance the system.

- Diminishing Influence: After 2 or 3 lags, the PACF values drop close to zero, meaning additional lags don't provide much new information.

5. Finding the expected demand for the next year and choosing the best model

5.1.Forecasting with Soft models

For this purpuse, all the below models must be used to calculate the forcast and then the errors for each of them must be determined. Then the best methode can be found out by comparing the error values with each other. For the linear regression and HOLT methods, the “Solver” function is utilized to optimize the parameters. The below methods are used for forecasting

1. Moving Average
2. Weighted Moving Average
3. Linear Regression
4. HOLT

The original time series graphs Y(t) and F(t) graphs has been given below one by one for easy comparison

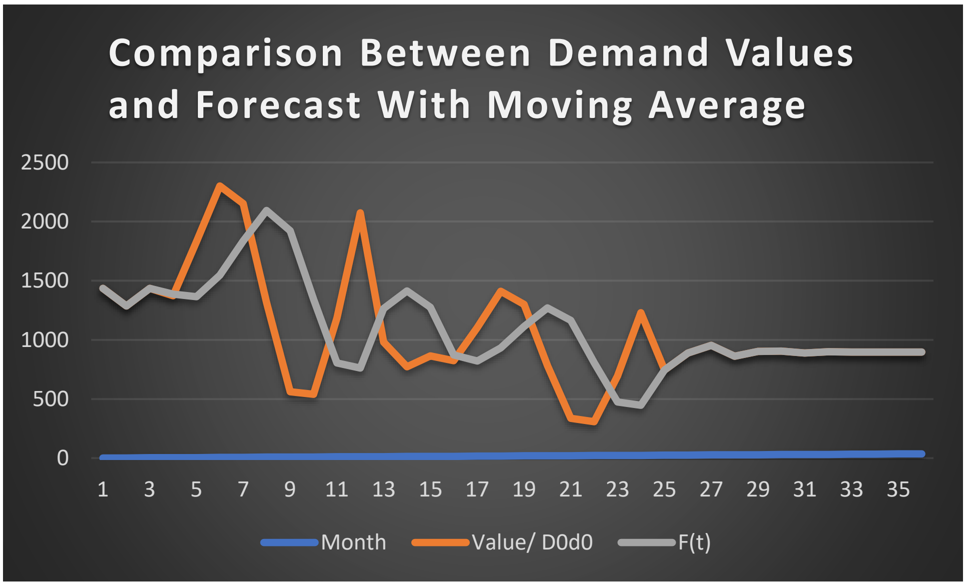


Figure 34: Graph showing the monthly aggregation Y(t) values and Forecast Values F(t) in Moving Average Method

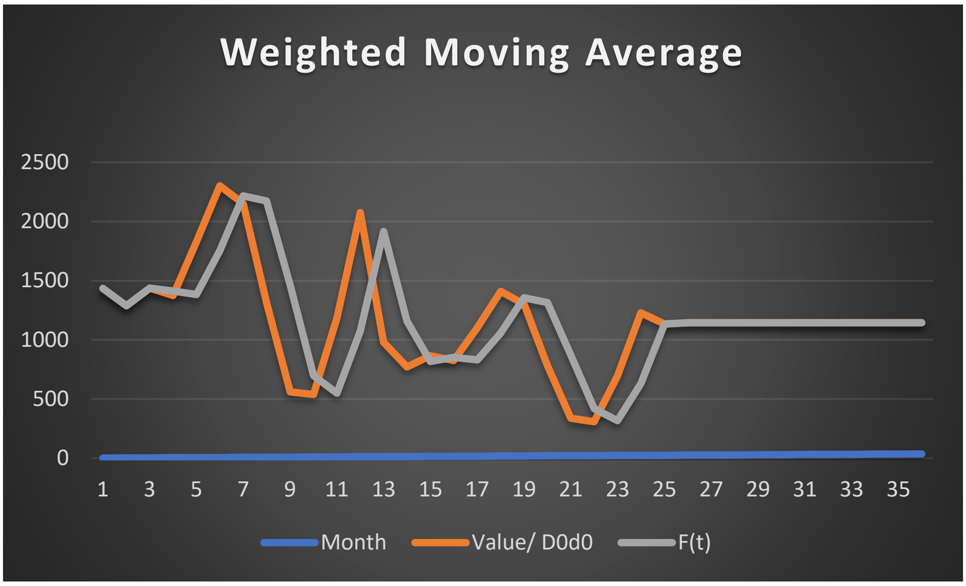


Figure 35: Graph showing the monthly aggregation Y(t) values and Forecast Values F(t) in Weighted Moving Average Method

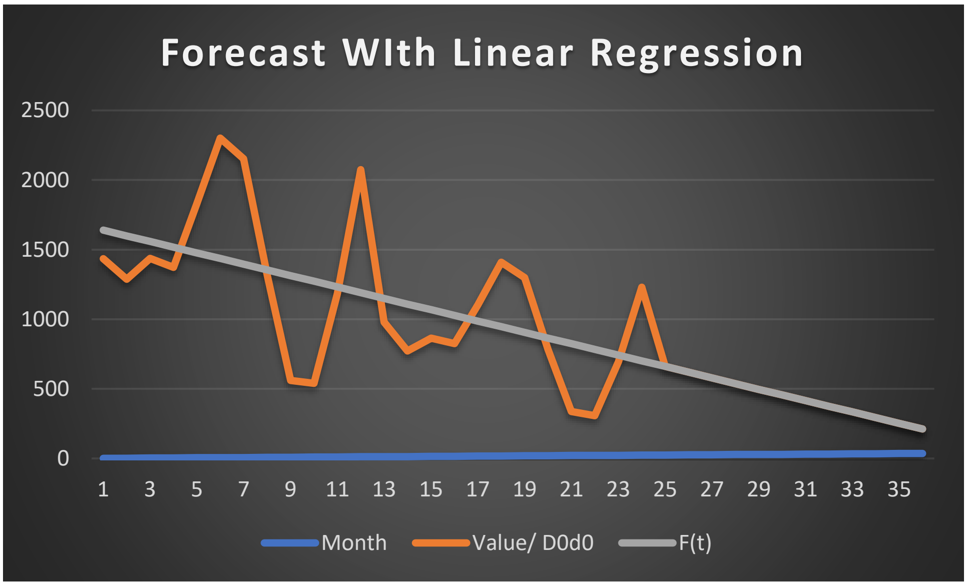


Figure 36: Graph showing the monthly aggregation Y(t) values and Forecast Values F(t) in Linear Regression

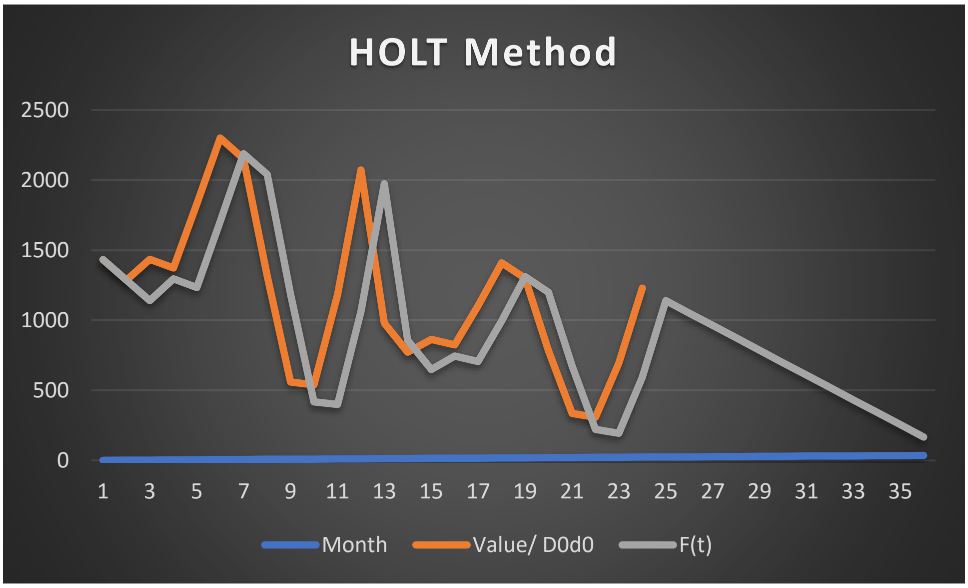


Figure 37: Graph showing the monthly aggregation Y(t) values and Forecast Values F(t) in HOLT Method

5.1.1. Comparing the Soft models

After all the Soft Model forecasting is done, then the values are put together in a table and the values are compared in a Graph.

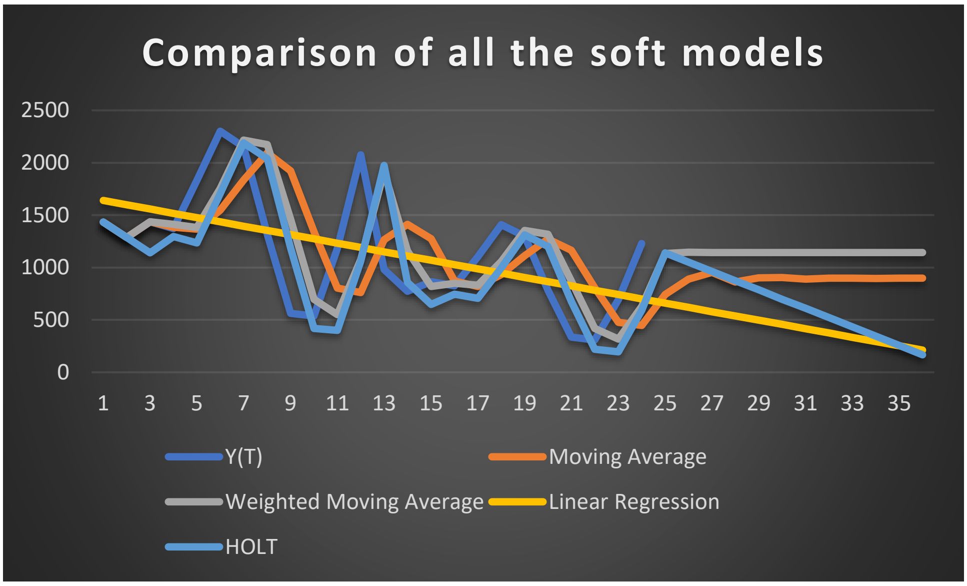


Figure 38: Comparison of all the soft models

The values were put in a table like below:

Table 11: Soft Model Comparison

Mont h	Y(T)	Moving Average	Weighted Moving Average	Linear Regression	HOLT
1	1434	1434	1434	1640	1434
2	1287	1287	1287	1599	1287
3	1436	1436	1436	1558	1140
4	1373	1386	1412	1517	1296
5	1830	1365	1382	1477	1234
6	2301	1546	1753	1436	1705
7	2152	1835	2216	1395	2189
8	1316	2094	2173	1354	2040
9	561	1923	1460	1313	1187
10	539	1343	698	1273	418
11	1184	805	550	1232	399
12	2074	761	1075	1191	1062
13	980	1266	1916	1150	1974
14	772	1413	1157	1109	858
15	865	1275	818	1069	648
16	824	872	851	1028	746
17	1104	820	830	987	707
18	1410	931	1057	946	996
19	1300	1113	1355	905	1311
20	783	1271	1316	865	1201
21	337	1164	872	824	674
22	308	807	418	783	221
23	695	476	317	742	194
24	1229	447	630	701	592
25		744	1134	661	1141
26		889	1145	620	1052
27		954	1144	579	964
28		862	1144	538	875
29		902	1144	497	787
30		906	1144	457	698
31		890	1144	416	610
32		899	1144	375	521
33		898	1144	334	433
34		896	1144	293	344
35		898	1144	253	256
36		897	1144	212	167

6. Box-Jenkins forecasting models

After the soft forecasting models, Box- Jenkins models were used to calculate forecasts too. The monthly aggregated data previously has been used in this analysis. The "Solver" function has been utilized to optimize the parameters of each ARIMA model.

The below models has been used here:

1. ARIMA (1,0,1)
2. ARIMA (1,1,1)
3. ARIMA (2,2,1)

The graphs for these are put one by one below, for easy comparison:

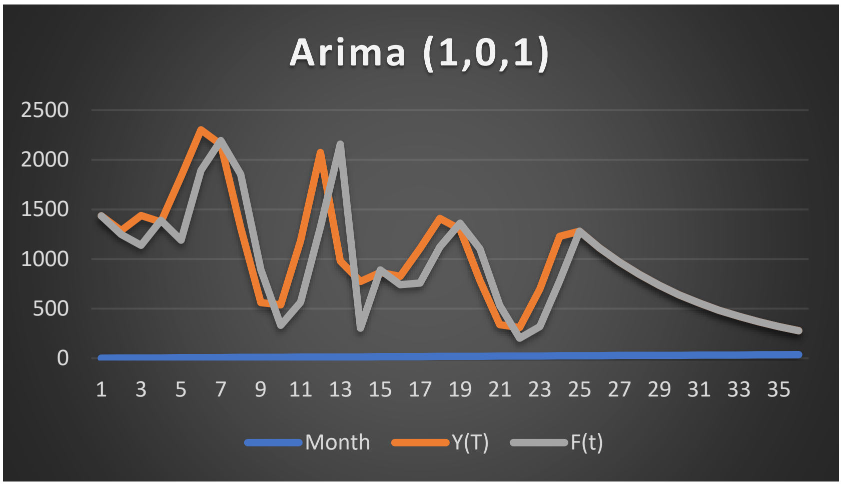


Figure 39: Graph showing the monthly aggregation Y(t) values and Forecast Values F(t) in Arima (1,0,1)

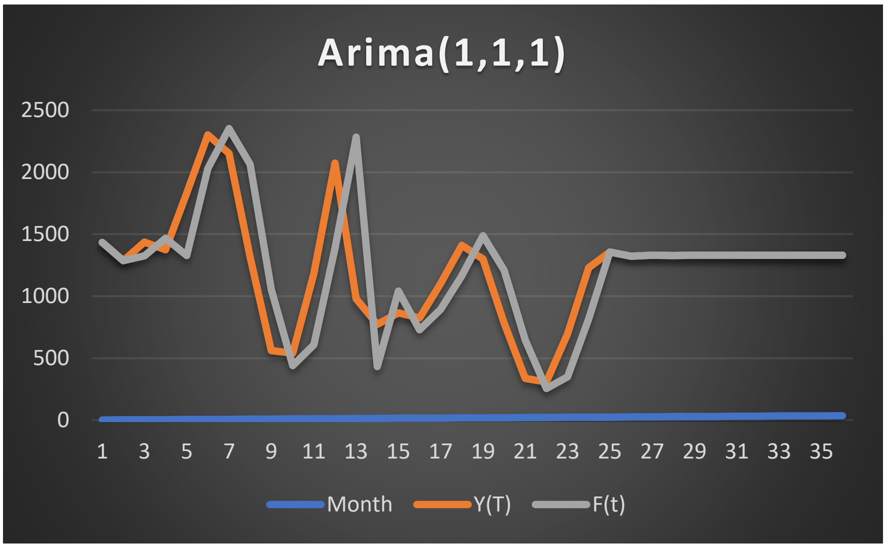


Figure 40: Graph showing the monthly aggregation Y(t) values and Forecast Values F(t) in Arima(1,1,1)

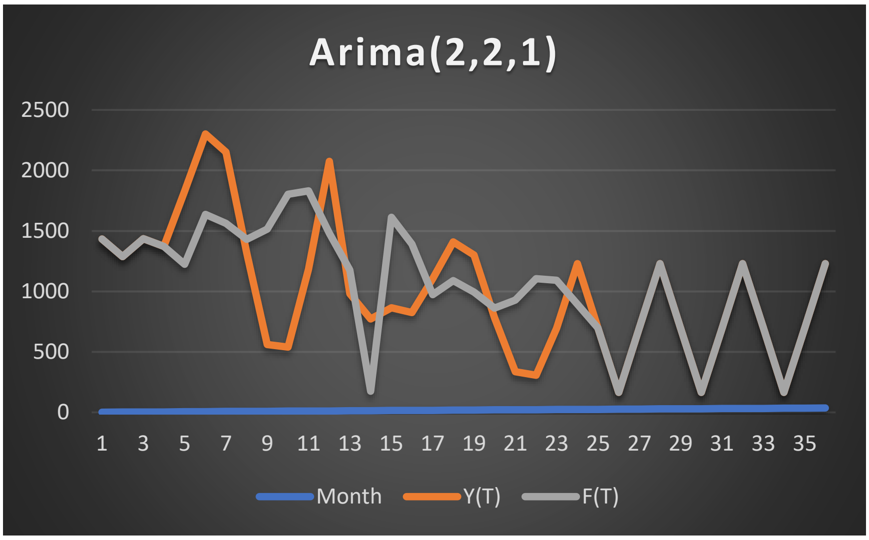


Figure 41: Graph showing the monthly aggregation Y(t) values and Forecast Values F(t) in Arima(2,2,1)

6.1.Calculation of ARIMA (1,1,1) by Wessa

As per the requirement of the Homework, ARIMA (1,1,1) was analysed witht the help of Wessa.

Table 12: ARIMA (1,1,1) Table Done by Wessa

time	Y[t]	F[t]
12	2074	-
13	980	-
14	772	-
15	865	-
16	824	-
17	1104	-
18	1410	-
19	1300	-
20	783	-
21	337	-
22	308	-
23	695	-
24	1229	-
25	1355	1354.4585
26	1323	1328.1859
27	1331	1333.6877
28	1329	1332.5356
29	1330	1332.7768
30	1330	1332.7263
31	1330	1332.7369
32	1330	1332.7347
33	1330	1332.7351
34	1330	1332.735
35	1330	1332.7351
36	1330	1332.7351

The Graph of it can be seen from the image below:

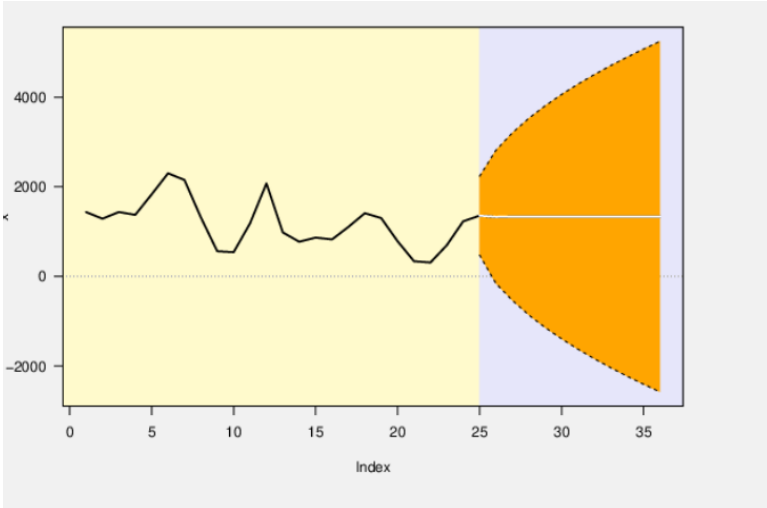


Figure 42: Graph depicting the output of (1,1,1) ARIMA by Wessa

7. Error Calculation and Best Model Determination

7.1. Error Calculation

At first, all the MSE, RMSE, MAE, MdAE, MAPE, MdAPE, RMSPE, and RMdSPE errors were calculated from forecasts. All the errors were put into a single table like the one below:

Table 13: Cobining all the Errors in one Table

	MSE	RMSE	MAE	MdAE	MAPE	MdAPE	RMSPE	RMdSPE
Moving Average	414284.2	643.6	539.8	479.0	0.678	0.340	0.974	0.340
Weighted Moving Average	275646.7	525.0	422.8	385.0	0.469	0.357	0.642	0.357
Linear Regression	224549.4	473.9	385.0	353.0	0.459	0.304	0.668	0.304
HOLT	267578.8	517.3	416.7	414.0	0.423	0.326	0.532	0.326
ARIMA (1,0,1)	204219.9	451.9	354.9	333.0	0.361	0.364	0.449	0.364
ARIMA (1,1,1)	216776.4	465.6	370.0	308.0	0.384	0.274	0.498	0.274
ARIMA (2,2,1)	342778.0	585.5	500.1	590.0	0.679	0.289	1.004	0.289

7.2. Best Model Selection

For the convenience of calculation, the best model selection has been done with two different weights of the erros. One where all the errors have the same weight, and another one with the errors having different weights.

Below is the best model selection by errors having same weights:

Table 14: Best Model Selection Using Same Weights for Errors

	MSE	RMSE	MAE	MdAE	MAPE	MdAPE	RMSPE	RMdSPE		
Weight	0.125	0.125	0.125	0.125	0.125	0.125	0.125	0.125	Sumproduct	Ranking
Moving Average	1.0	1.0	1.0	0.8	1.0	0.9	1.0	0.9	0.96	7
Weighted Moving Average	0.7	0.8	0.8	0.7	0.7	1.0	0.6	1.0	0.78	5
Linear Regression	0.5	0.7	0.7	0.6	0.7	0.8	0.7	0.8	0.70	3
HOLT	0.6	0.8	0.8	0.7	0.6	0.9	0.5	0.9	0.73	4
ARIMA (1,0,1)	0.5	0.7	0.7	0.6	0.5	1.0	0.4	1.0	0.67	2
ARIMA (1,1,1)	0.5	0.7	0.7	0.5	0.6	0.8	0.5	0.8	0.63	1
ARIMA (2,2,1)	0.8	0.9	0.9	1.0	1.0	0.8	1.0	0.8	0.91	6

The best model has been marked marked by green at right. Here the best model is ARIMA(1,1,1).

Below is the best model selection by errors having different weights:

Table 15: Best Model Selection Using Different Weights for Errors

	MSE	RMSE	MAE	MdAE	MAPE	MdAPE	RMSPE	RMdSPE		
Weight	3	1	2	1	2	1	2	1		
	0.230769231	0.076923	0.153846	0.076923	0.153846	0.076923	0.153846	0.076923	Sumproduct	Ranking
Moving Average	1	1	1	0.811864	0.999527	0.934445	0.969362	0.934445	0.970656346	7
Weighted Moving Average	0.665356585	0.815694	0.783257	0.652542	0.690734	0.98238	0.638994	0.98238	0.742694965	5
Linear Regression	0.542017856	0.736219	0.713126	0.598305	0.676637	0.835778	0.664795	0.835778	0.672403976	3
HOLT	0.645882261	0.803668	0.771965	0.701695	0.623552	0.895844	0.529208	0.895844	0.698780717	4
ARIMA (1,0,1)	0.492946296	0.702101	0.657463	0.564407	0.531232	1	0.447195	1	0.61670212	2
ARIMA (1,1,1)	0.523255373	0.723364	0.685515	0.522034	0.565671	0.75305	0.495661	0.75305	0.601150761	1
ARIMA (2,2,1)	0.827398331	0.909614	0.926517	1	1	0.79615	1	0.79615	0.910549568	6

The best model has been marked marked by green at right. Here the best model is ARIMA(1,1,1).

8. Inventory Control Strategy

Our next step is to refine the current inventory management approach, specifically by looking at how much to order (Q) and when to reorder (R). We'll do this by reviewing historical data and calculating the Economic Order Quantity (EOQ) for two scenarios: one where backorders are allowed and one where they aren't.

Table 16: Inventory Control Parameters: Without Backlog

c_1	4.197	EUR/order
c_2	0.004	EUR/kg*day
c_3	0.021	EUR/kg*day
c_b	3.259	EUR/kg
M(LT)	1	day
D(LT)	0.5	day
EOT	7.203971	day
EOQ	277.6246	pcs
EOT*	7	day
EOQ*	270	pcs

C1	440.6818	EUR
C2	38.15045	EUR
C3	28374.87	EUR
Cb	92378.48	EUR
C	121232.2	EUR

The table shows that the Economic Order Time (EOT) is just 7 days, meaning the data need to be placed 7 days apart to keep inventory levels at their best.

The data can be seen in the graph below:

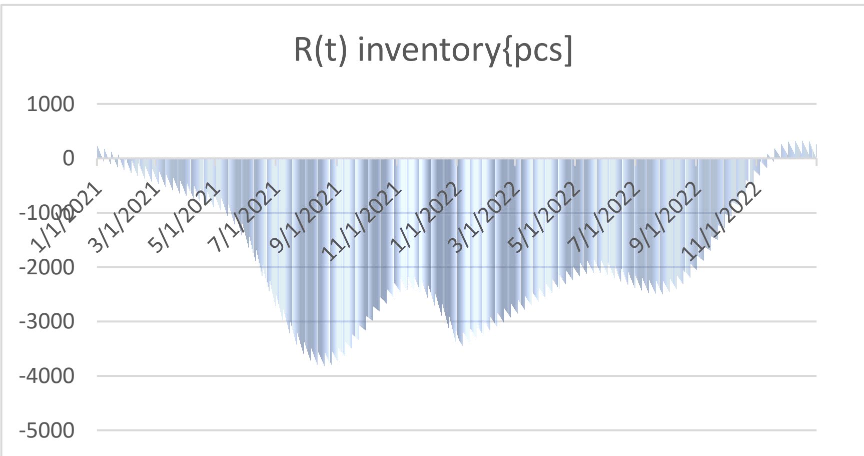


Figure 43: Inventory results without backlog

Table 17: Inventory Control Parameters: With Backlog

c_1	4.197	EUR/order
c_2	0.004	EUR/kg*day
c_3	0.021	EUR/kg*day
c_b	3.259	EUR/kg
M(LT)	1	day
D(LT)	0.5	day
EOT	54.47656	day
EOQ	2099.403	pcs
EOT*	54	day
EOQ*	2081	pcs
C1	58.75758	EUR
C2	472.1591	EUR
C3	21900.44	EUR
Cb	93802.44	EUR
C	116233.8	EUR

The table shows that the Economic Order Time (EOT) is just 54 days, meaning the data need to be placed 54 days apart to keep inventory levels at their best.

The data can be seen in the graph below:

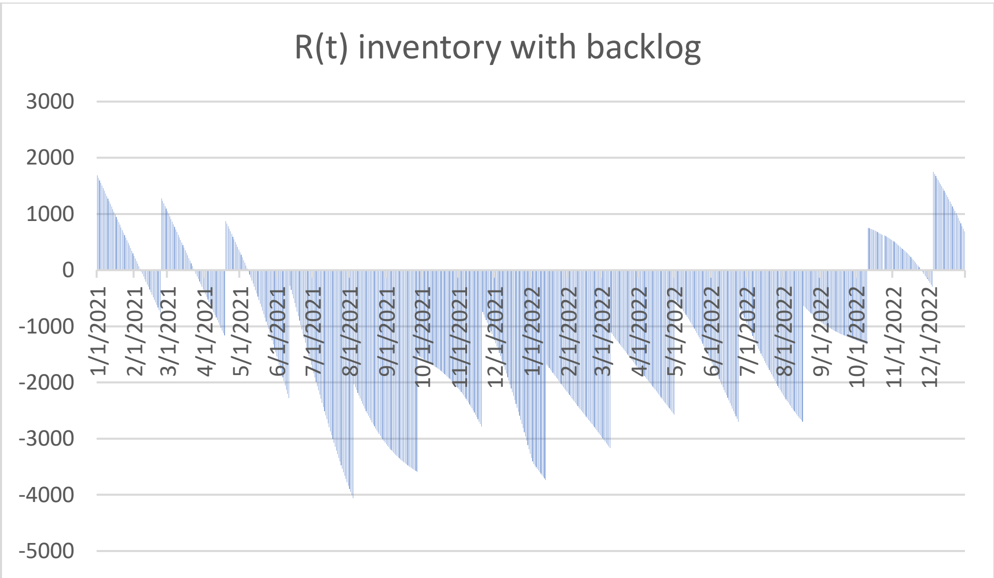


Figure 44: Inventory results with backlog

9. The [s, q] Inventory Strategy

The feasibility of adopting an (s, Q) inventory control strategy will be assessed next. This evaluation will take into account factors such as lead time variability and demand distribution. To optimize the strategy, the optimal safety stock (SS) and reorder point (ROP) will be calculated. The SS will address uncertainties in demand and supply, while the ROP will specify when a new order should be triggered. By fine-tuning these parameters, stockouts and overstock can be minimized, leading to improved inventory efficiency. The findings from this analysis will help determine whether the (s, Q) strategy better suits the company's needs compared to the (t, Q) model.

Table 18: Values of ROP

c1	4.197	EUR/order
c2	0.004	EUR/kg*day
c3	0.021	EUR/kg*day
cb	3.259	EUR/kg
M(LT)	1	day
D(LT)	0.5	day
EOT	54.47656	day
EOQ	2099.403	pcs
EOT*	54	day
EOQ*	2081	pcs
Reliability	95%	
C1	440.6818	EUR
C2	38.15045	EUR
C3	28374.87	EUR
Cb	92378.48	EUR
C	121232.2	EUR
M(DEM)	38.53772	
D(DEM)	17.97974	
Time period	730	days
Udem	1.644854	
Expected sum demand R	28132.54	
LTS	39	Pcs
SS	44	Pcs
ROP	83	Pcs

Based on an analysis of historical data and consideration of potential backlogs, the optimal parameters for the (s, Q) inventory control strategy have been identified. The primary parameters are as follows:

- Safety Stocks – 44 Pcs
- Lead Time Supply- 39 Pcs
- Reorder Point- 83 Pcs

To optimize inventory levels, minimize stockouts, and maximize customer satisfaction, key parameters must be carefully configured. The safety stock provides a buffer to manage demand fluctuations during lead time. The reorder point ensures a new order is triggered when inventory reaches a specific threshold. Meanwhile, lead time supply represents the inventory required to meet demand while waiting for replenishment.

10. Appendix

The Weekly Aggregation for 2021 can be found below:

Table 19: The Weekly Aggregation for 2021

Week Number	Sum of Filter 2 Moving Average
1	138
2	325
3	325
4	323
5	323
6	322
7	321
8	321
9	323
10	323
11	325
12	325
13	326
14	318
15	318
16	322
17	321
18	323
19	350
20	398
21	439
22	479
23	512
24	529
25	546
26	551
27	542
28	521
29	494
30	465
31	422
32	372
33	324
34	273
35	231
36	188
37	149
38	126
39	102

40	97
41	95
42	107
43	132
44	163
45	200
46	244
47	293
48	342
49	384
50	433
51	474
52	508
53	380
Grand Total	**17487**

The Weekly Aggregation for 2022 can be found below:

Table 20: The Weekly Aggregation for 2022

Row Labels	Sum of Filter 2 Moving Average
1	183
2	200
3	201
4	197
5	199
6	196
7	190
8	189
9	197
10	199
11	200
12	189
13	195
14	190
15	186
16	194
17	192
18	201
19	211
20	234
21	264
22	293
23	320

24	337
25	330
26	328
27	320
28	319
29	297
30	284
31	254
32	221
33	195
34	161
35	135
36	115
37	90
38	76
39	58
40	61
41	55
42	63
43	82
44	96
45	125
46	140
47	178
48	207
49	242
50	254
51	278
52	300
53	186
Grand Total	**10607**

Table 21: For D0d0, the ACF chart from Wessa

Time lag k	ACF(k)	T-STAT	P-value
1	0.602929	2.9537	0.003462
2	0.041595	0.2038	0.420124
3	-0.21657	-1.061	0.149627
4	-0.08087	-0.3962	0.34774
5	0.20891	1.0234	0.158152
6	0.312588	1.5314	0.069378
7	0.14021	0.6869	0.249369
8	-0.11873	-0.5817	0.283108
9	-0.23165	-1.1349	0.133821
10	-0.17741	-0.8691	0.196691
11	0.045475	0.2228	0.412796
12	0.169344	0.8296	0.207467
13	0.05653	0.2769	0.392098
14	-0.16706	-0.8184	0.210582
15	-0.31817	-1.5587	0.066078
16	-0.29728	-1.4564	0.079125
17	-0.18316	-0.8973	0.189242
18	-0.09647	-0.4726	0.320385
19	-0.07161	-0.3508	0.364389
20	-0.06403	-0.3137	0.378232
21	-0.03937	-0.1929	0.424344
22	-0.01746	-0.0856	0.466265

23	0.002268	0.0111	0.495613

Table 22: For D0d0, the PACF chart from Wessa

Time lag k	PACF(k)	T-STAT	P-value
1	0.602929	2.9537	0.003462
2	-0.5058	-2.4779	0.010322
3	0.106879	0.5236	0.30268
4	0.188153	0.9218	0.182914
5	0.152153	0.7454	0.231636
6	-0.02354	-0.1153	0.454583
7	-0.07354	-0.3603	0.360892
8	-0.06212	-0.3043	0.381749
9	-0.03946	-0.1933	0.424179
10	-0.10823	-0.5302	0.300409
11	0.194659	0.9536	0.174889
12	-0.08387	-0.4109	0.342397
13	-0.09446	-0.4627	0.323862
14	-0.06992	-0.3425	0.367469
15	-0.10885	-0.5332	0.299389
16	-0.1242	-0.6085	0.274301
17	-0.14885	-0.7292	0.236461
18	-0.07298	-0.3575	0.361914
19	0.048875	0.2394	0.406399
20	0.032589	0.1597	0.437246
21	0.10183	0.4989	0.311209
22	-0.00459	-0.0225	0.491125
23	-0.00227	-0.0111	0.495605
24	0.602929	2.9537	0.003462

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