CH516: Programming Assignment 1 (Linearization and Time Series Modelling of MISO System) Computing tutorial for CSTR system

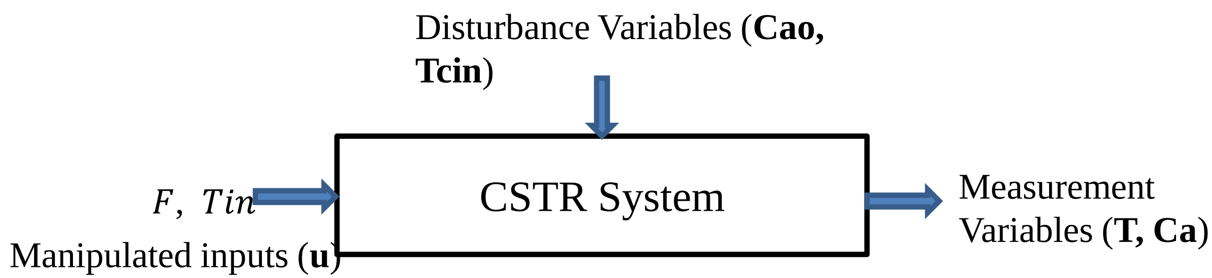
Process details

CSTR System

$$\frac{dCa}{dt} = \frac{F}{V}(Cao - Ca) - k_o exp^{\left(-\frac{E_a}{RT}\right)}$$

$$\frac{dT}{dt} = \frac{F}{V}(T_o - T) + \frac{(-\Delta H)}{(\rho C_p)} K C_a - \frac{Q}{V \rho C_p}$$

$$Q = \frac{a F_c{}^{b+1}}{F_c + \left(\frac{a F_c{}^{b}}{2 \rho_c C_{pc}}\right)} (T - T_{Cin})$$



Continuous time non-linear models

$$\frac{dCa}{dt} = f_1(Cao, Ca, F, V, k_o, E_a, R, T)$$

$$\frac{dT}{dt} = f_2(Cao, Ca, F, V, k_o, E_a, R, T)$$

Part 1 : Linearizing and developing discrete time state space models

- Steady state inputs :

$$\begin{bmatrix} 1(Fc, m^3/\sec) & 15(F, m^3/\sec) \end{bmatrix}$$

- Steady state disturbance

$$= 2(Cao, mol/m^3)$$

- Steady state operating point :

$$\begin{bmatrix} Ca & T \end{bmatrix} = \begin{bmatrix} 0.2646(mol/m^3) & 393.9521(K) \end{bmatrix}$$

- Continuous linearized model is obtained as :

$$A = \begin{bmatrix} -7.5594 & -0.0931 \\ 852.7187 & 5.7674 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 1.7354 \\ -6.0730 & -70.9521 \end{bmatrix}$$

$$H = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

- Eigen values of *A* matrix are given by

$$eig(A) = \begin{bmatrix} -0.8960 + 5.9184i & -0.8960 - 5.9184i \end{bmatrix}$$

- For a sampling interval of *0.1 min*, discrete state space model is obtained as

$$phy = \begin{bmatrix} 01845 & -0.0080 \\ 73.4917 & 1.3331 \end{bmatrix}$$

$$gama\_u = \begin{bmatrix} 0.0026 & 0.1340 \\ -0.7335 & -1.7969 \end{bmatrix}$$

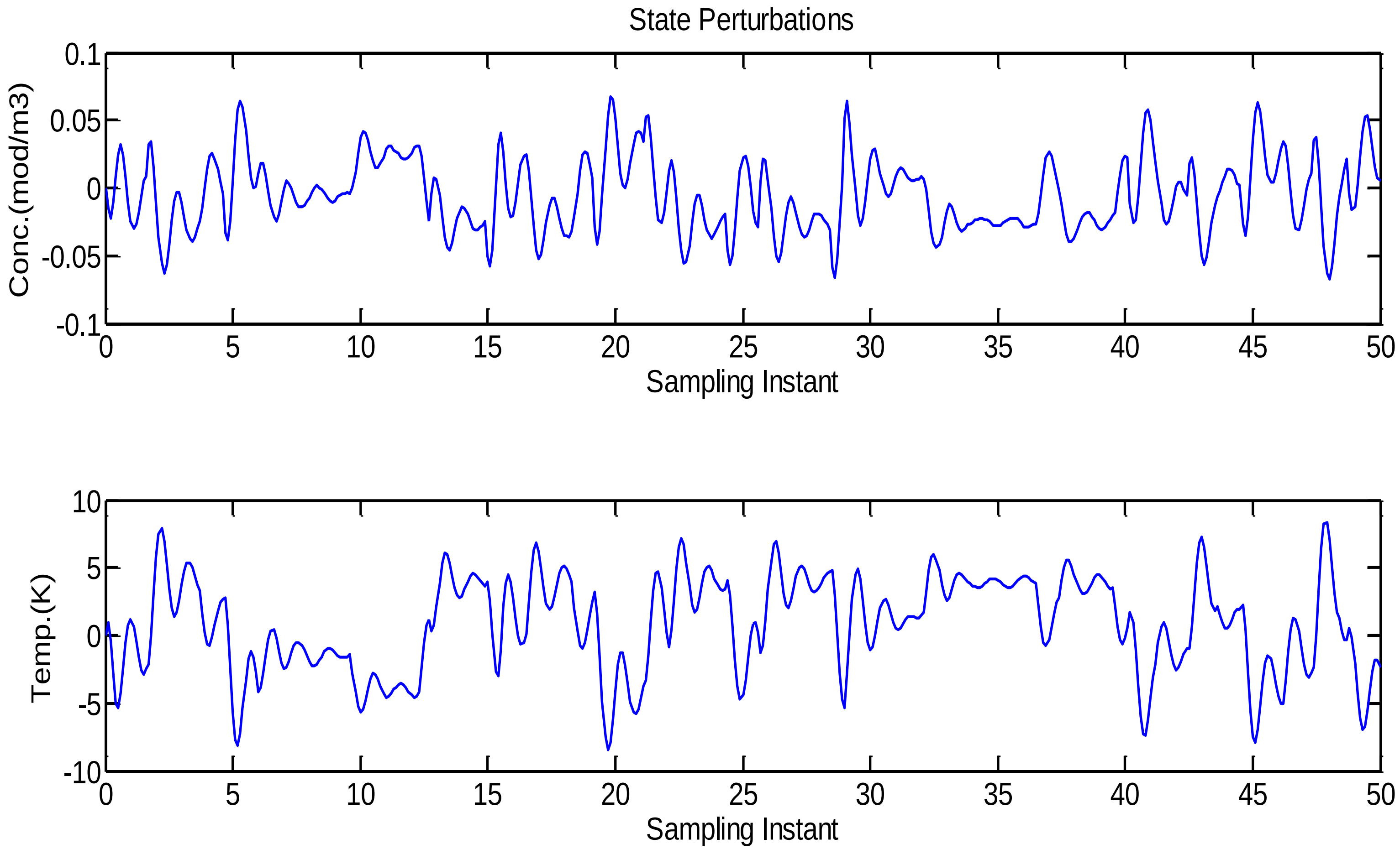
$$gama\_d = \begin{bmatrix} 0.0598 \\ 3.9028 \end{bmatrix}$$

- Eigen values and spectral radius of *phy* matrix are obtained as

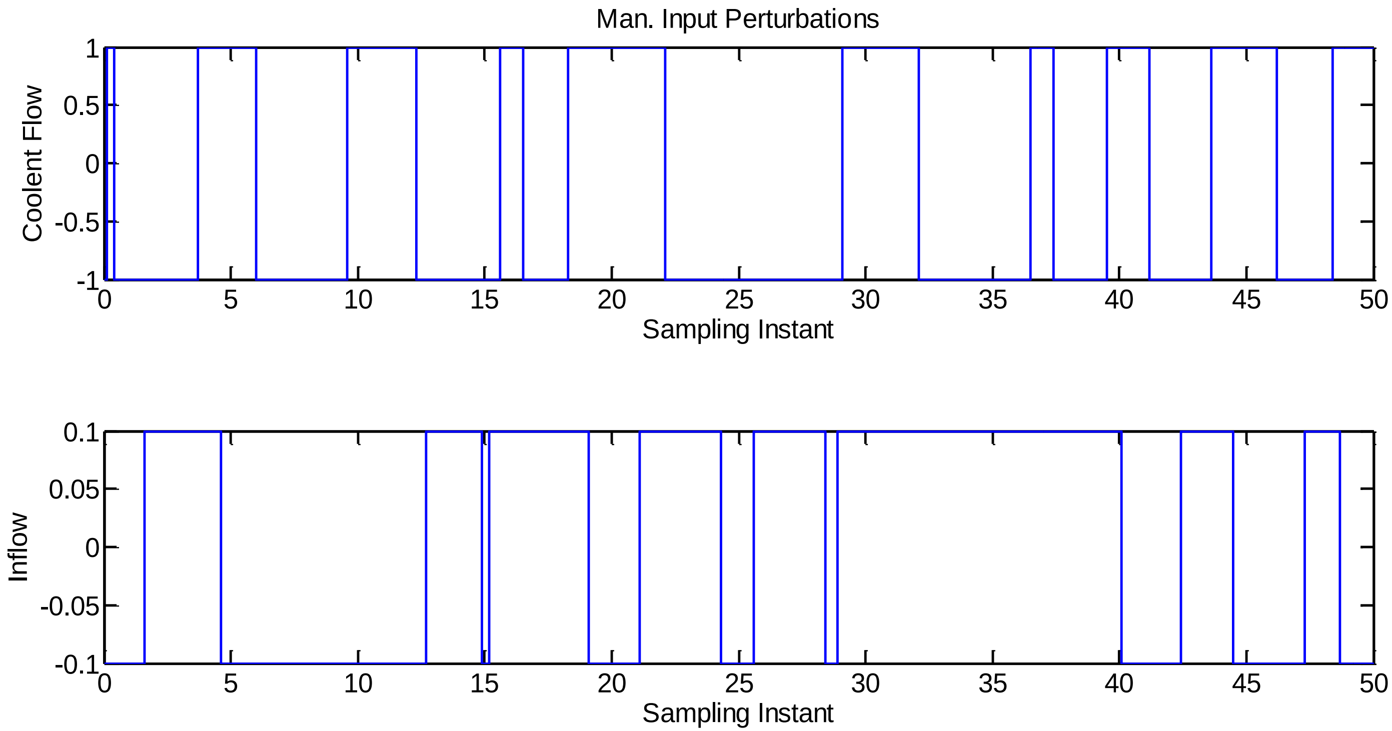
$$eig(phy) = \begin{bmatrix} 0.7588 + 0.5101i & 0.7588 - 0.5101i \end{bmatrix}$$
$$\rho(phy) = 0.914300$$

Part 2a: Identification of ARMAX model by using linear plant model

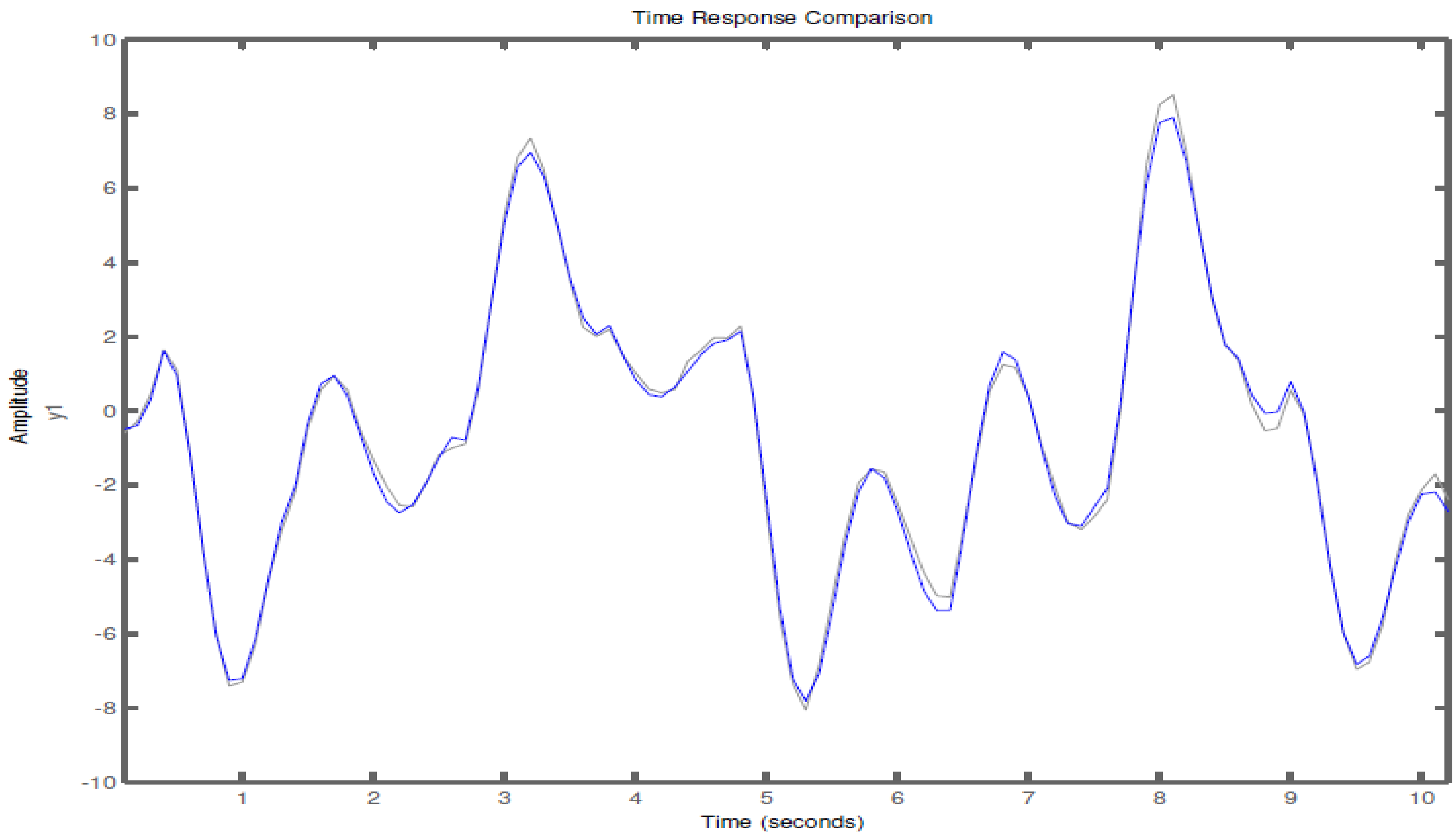
- State perturbations obtained from linear plant model



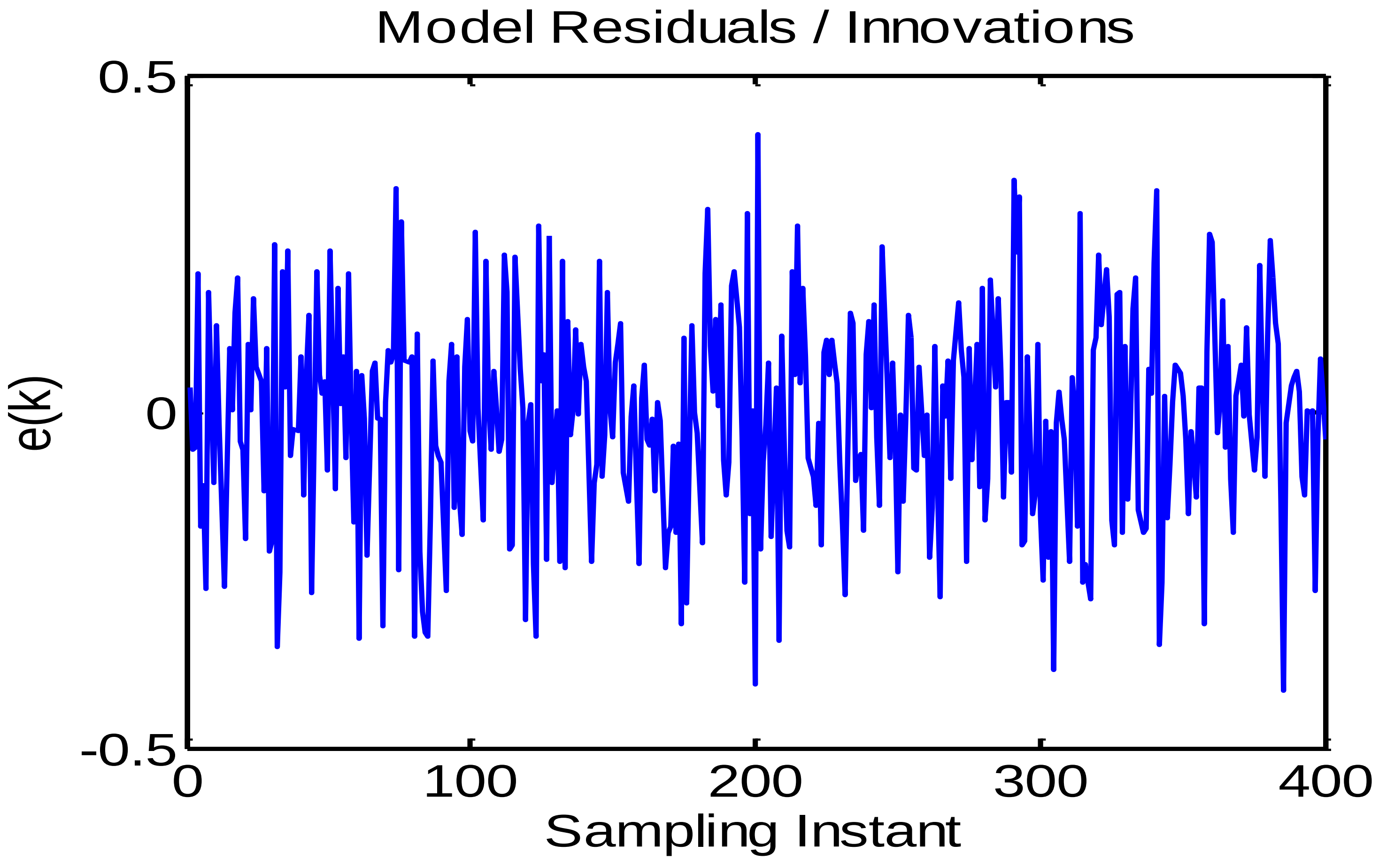
- Manipulated input perturbations given to linear plant model



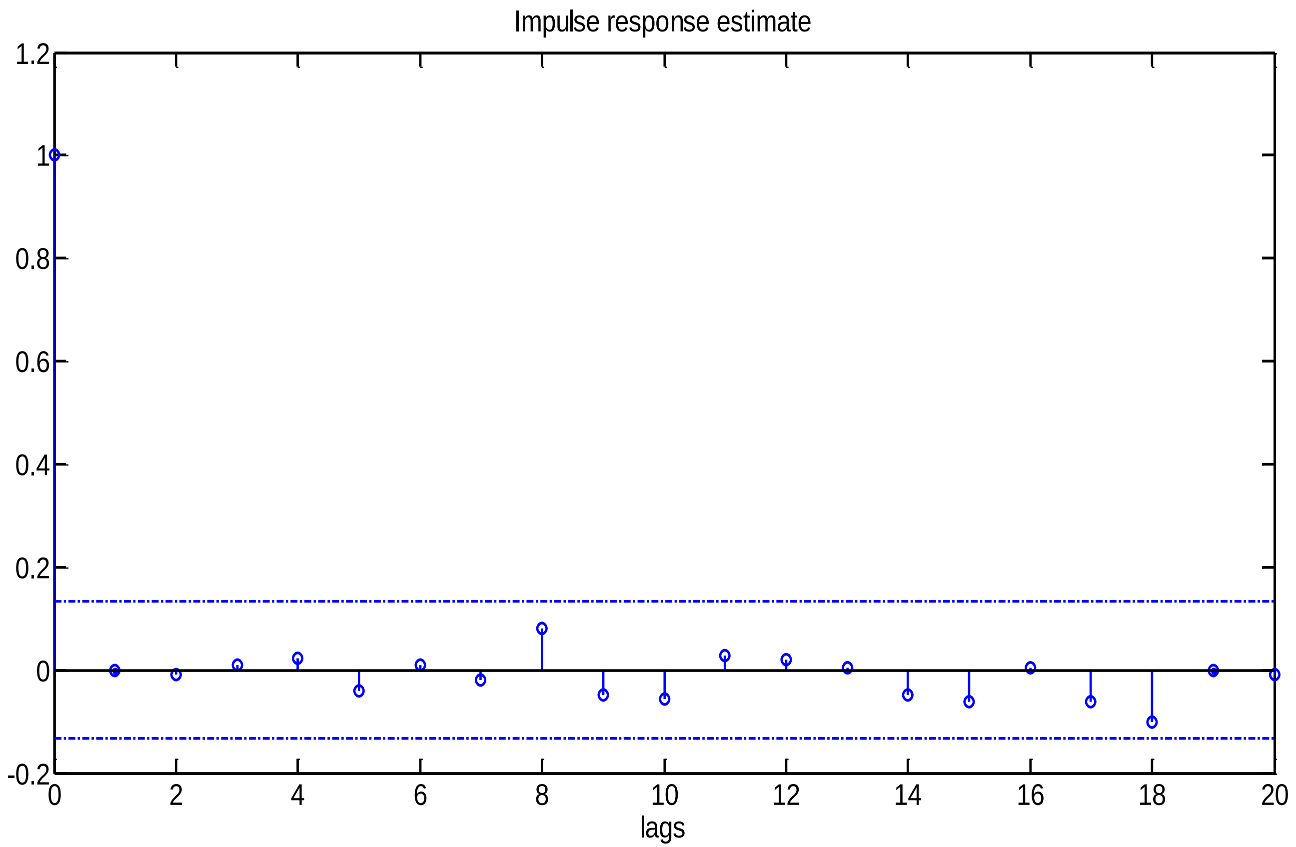
- Developed identified model (ARMAX) is validated with the validation data.
- The percentage fit obtained is 93.83%



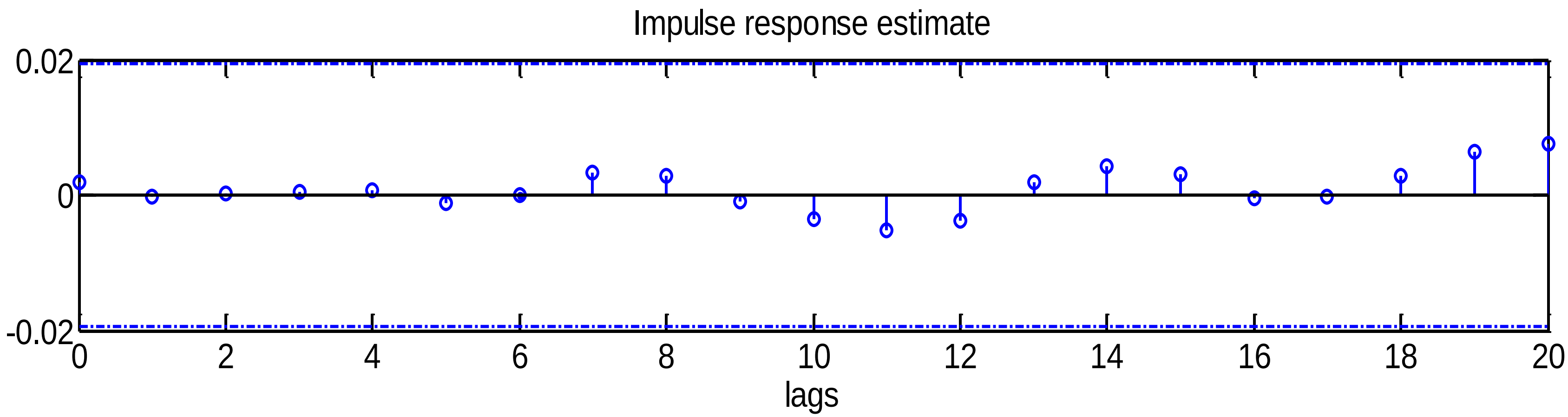
- Analysis of residuals/innovations, whose mean is zero.

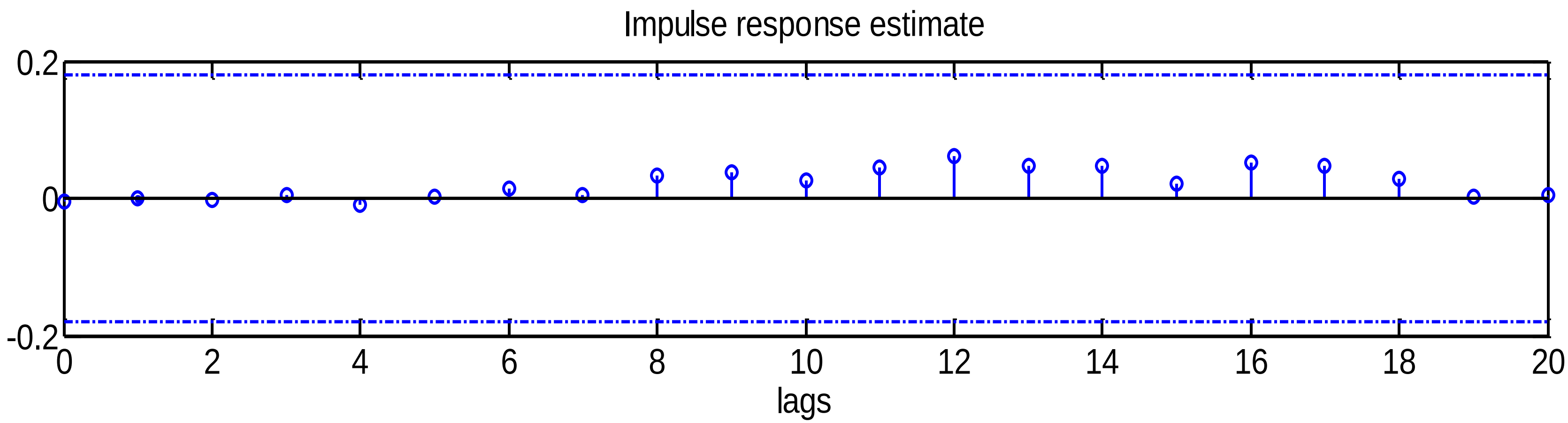


- Figure showing autocorrelation of innovation sequence with 0.99% confidence interval



- Cross-correlation between input signal sent to the plant and innovation sequence obtained





- Identified ARMAX model obtained from linear plant model :

ARMAX model: A(z)y(t) = B(z)u(t) + C(z)e(t)

A(z) = 1 - 2.483 z^-1 + 2.288 z^-2 - 0.8049 z^-3

B1(z) = -0.693 z^-1 + 0.9769 z^-2 - 0.2844 z^-3

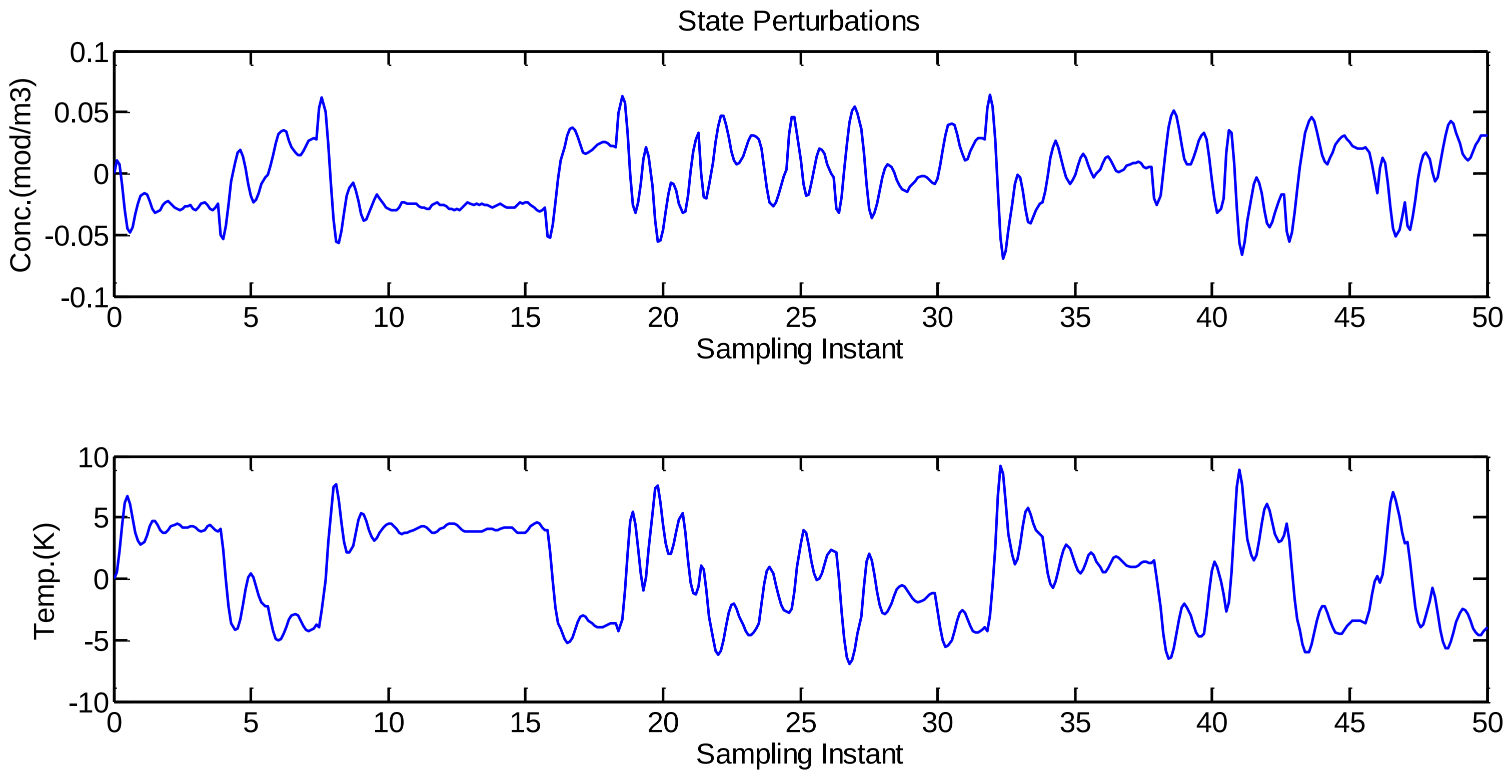
B2(z) = -1.641 z^-1 + 11.87 z^-2 - 10.23 z^-3

C(z) = 1 - 1.21 z^-1 + 0.2659 z^-2 - 0.05553 z^-3

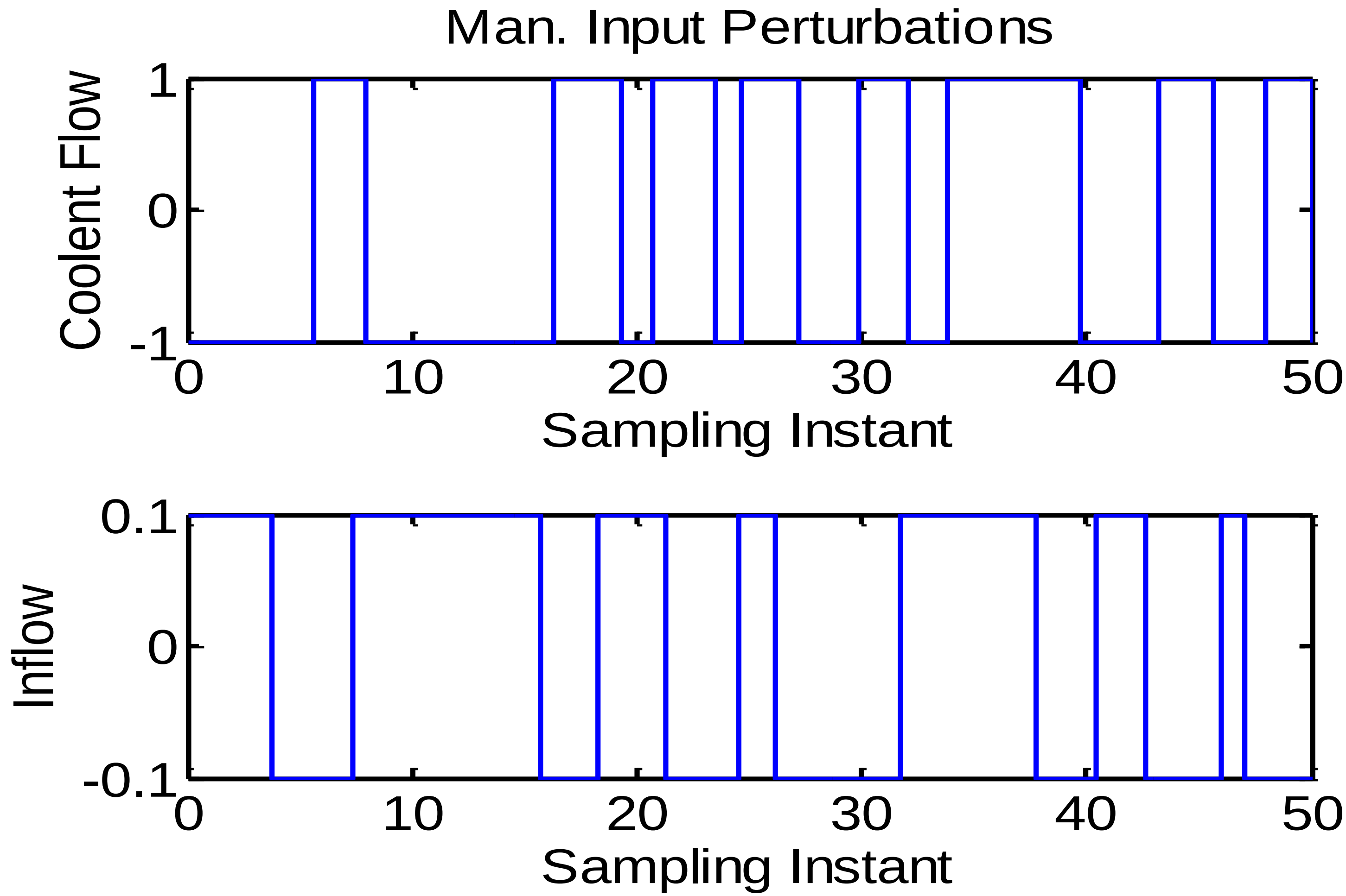
Sample time: 0.1 seconds

Part 2b: Identification of ARMAX model by using Nonlinear plant model

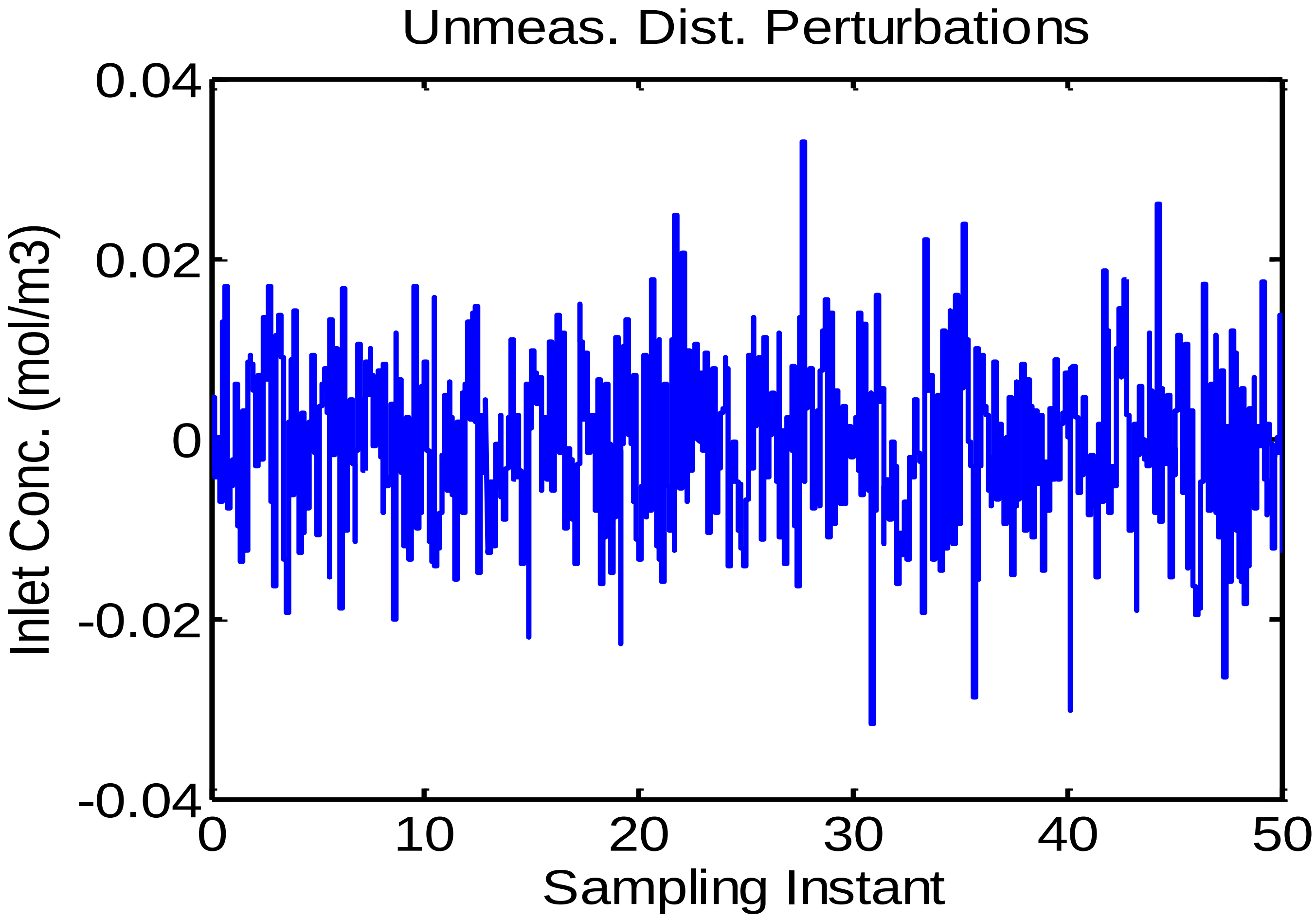
- State perturbations obtained from nonlinear plant model



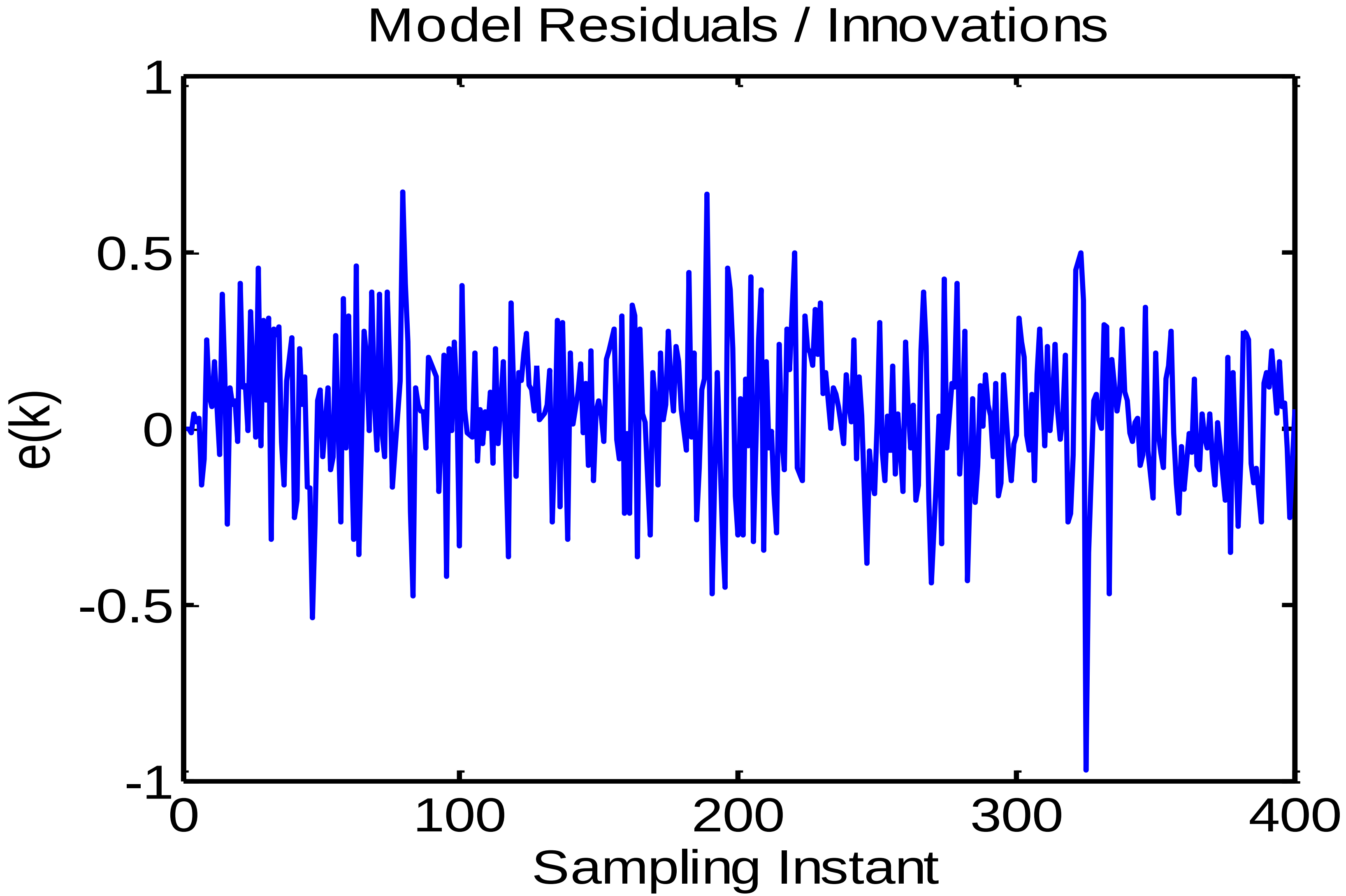
- Manipulated input perturbations given to non-linear plant model



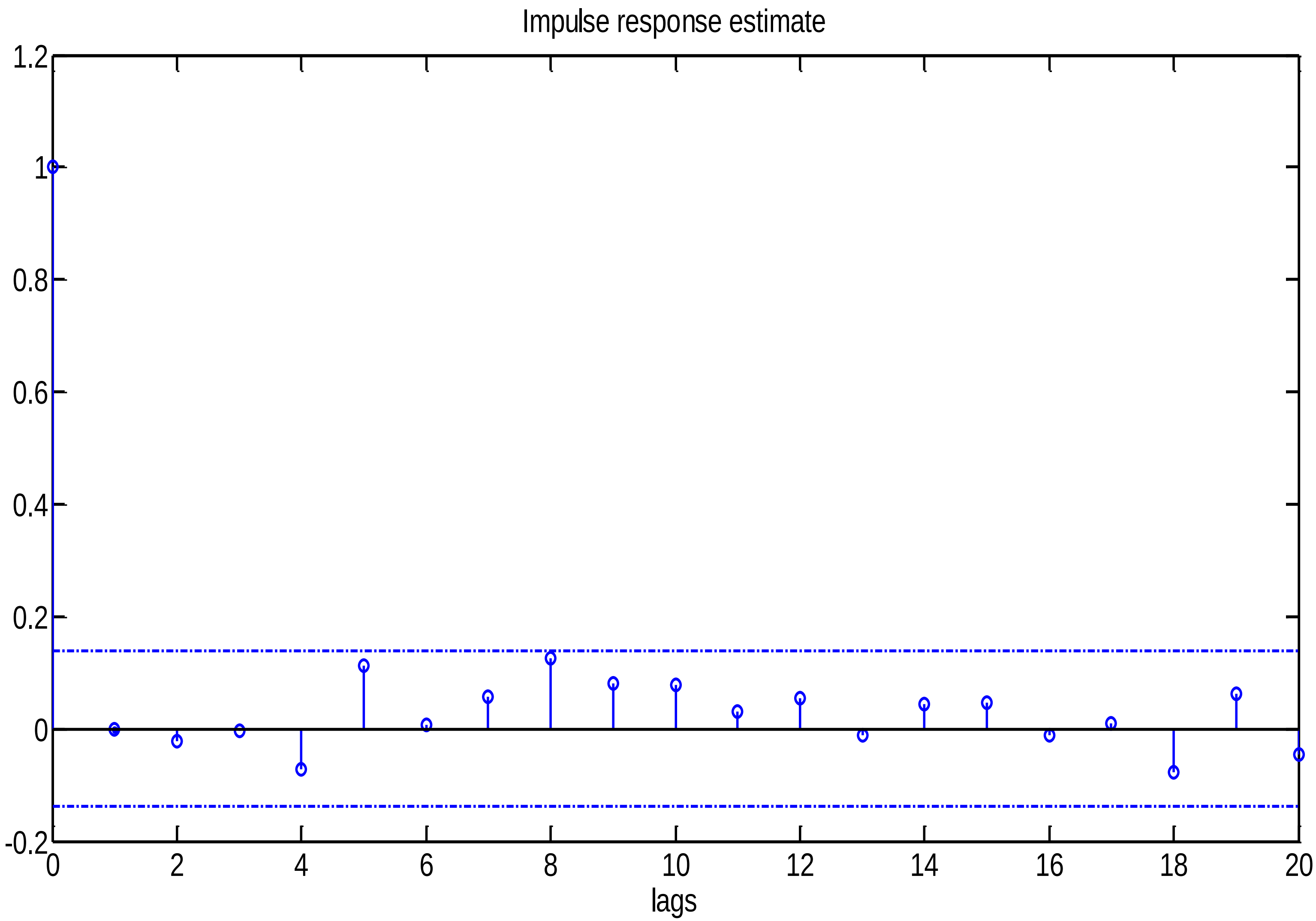
- Perturbations in unmeasured disturbance variable



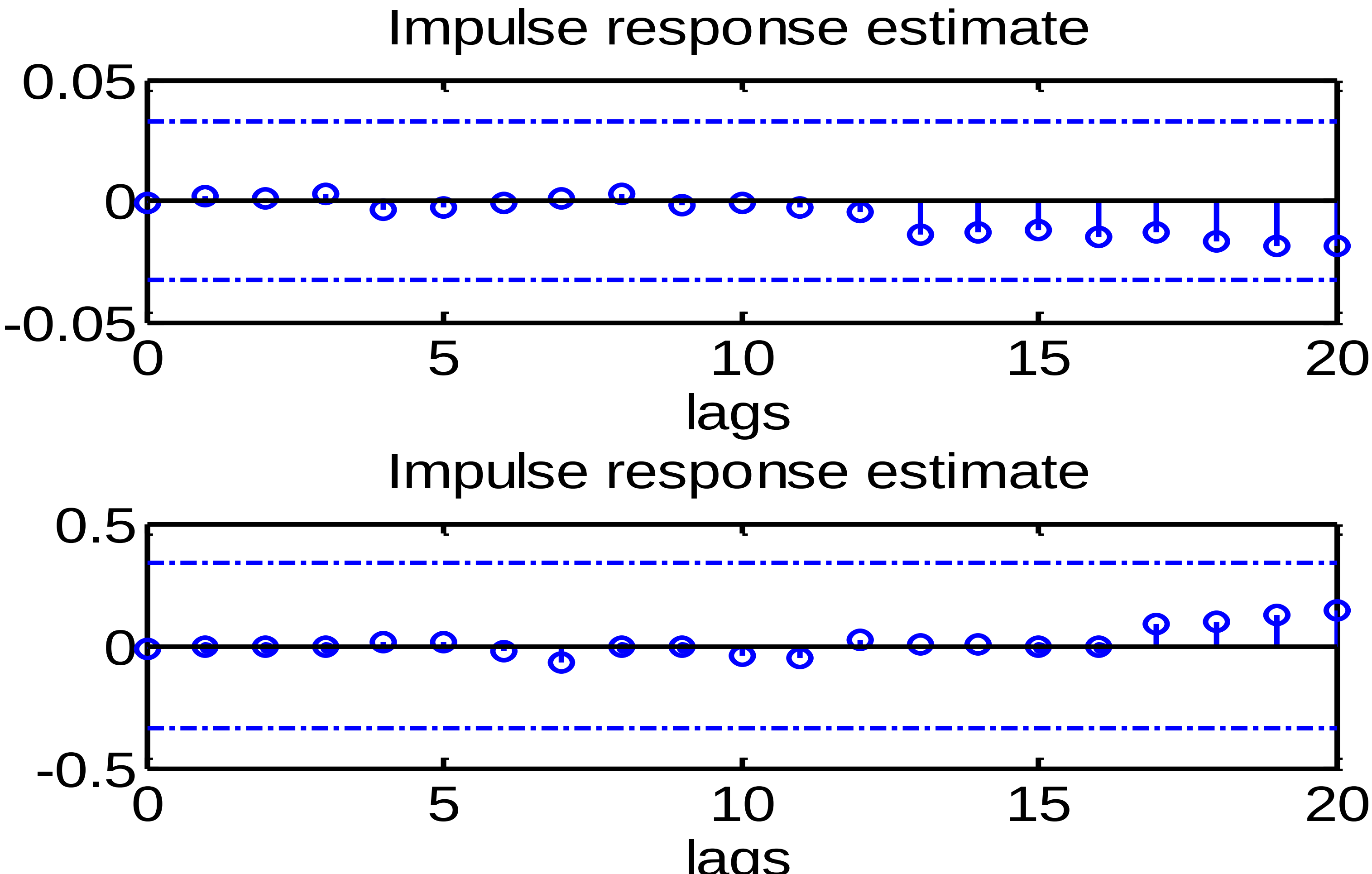
- Analysis of residuals/innovations, whose mean is zero.



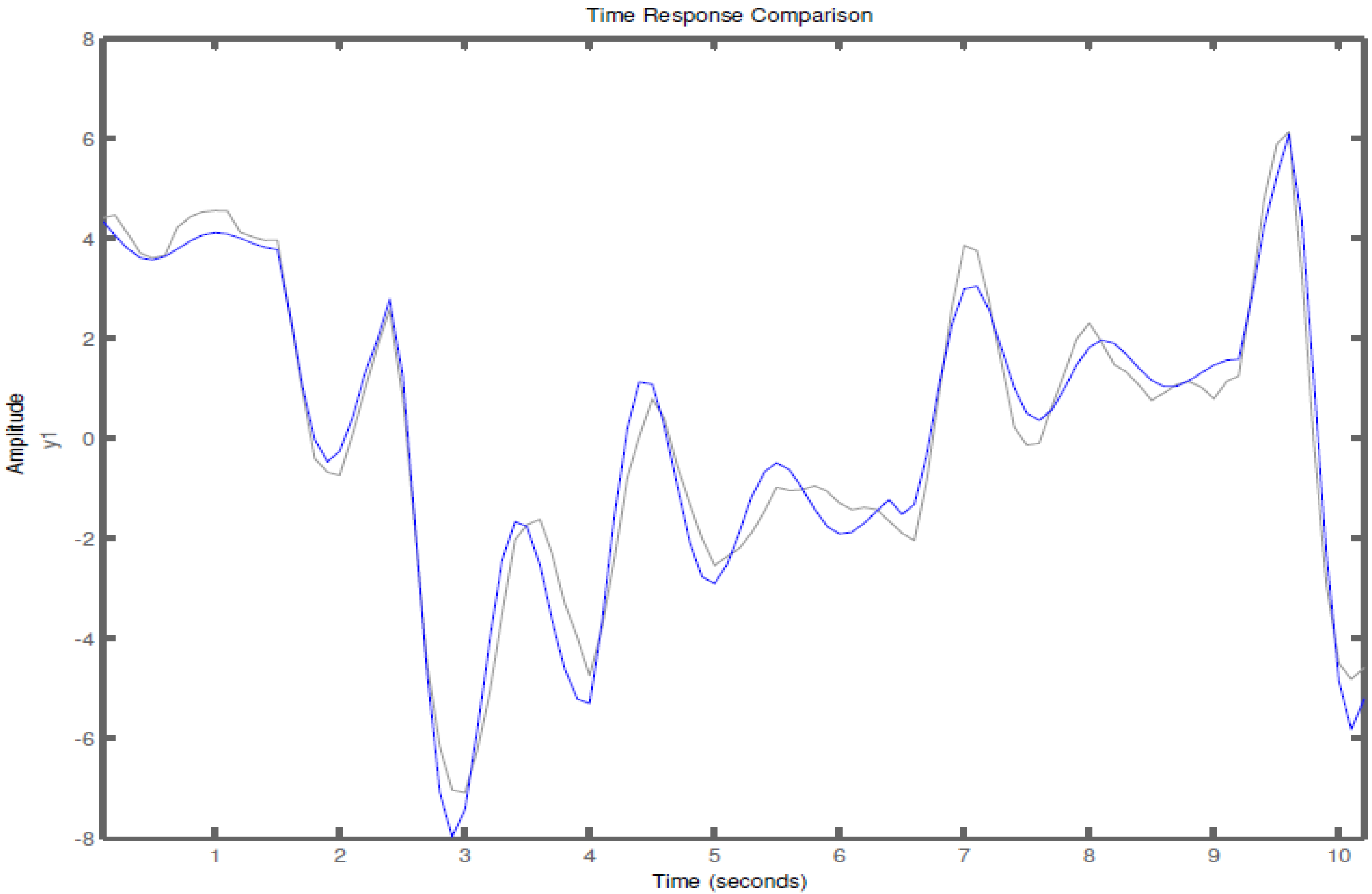
- Figure showing autocorrelation of innovation sequence with 0.99% confidence interval



- Cross-correlation between input signal sent to the plant and innovation sequence obtained



- Developed identified model is validated with the validation data.
- The percentage fit obtained is 81.46%



- Identified ARMAX model obtained from nonlinear plant model :

- Discrete-time ARMAX model: $A(z)y(t) = B(z)u(t) + C(z)e(t)$

$A(z) = 1 - 1.335 z^{-1} + 0.6024 z^{-2} + 0.1205 z^{-3}$

$B1(z) = -0.7162 z^{-1} + 0.1802 z^{-2} + 0.0395 z^{-3}$

$B2(z) = -1.346 z^{-1} + 8.343 z^{-2} + 3.29 z^{-3}$

$C(z) = 1 - 0.125 z^{-1} - 0.2446 z^{-2} - 0.1722 z^{-3}$

Sample time: 0.1 seconds