# Cellular Automata Simulation of the 2D Ising Model

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## Contents

**Abstract**

Cellular Automata (CA) are discrete, computational models consisting of a regular grid of cells, each in one of a finite number of states. In this project, the two–dimensional Ising model is implemented as a stochastic cellular automaton to investigate magnetic phase transitions. Each lattice site acts as a binary cell (-1 for down spin and +1 for up spin), whose evolution depends on nearest–neighbor interactions(spin-spin coupling) and temperature. Macroscopic observables such as magnetization, susceptibility, internal energy, and specific heat are computed. The automaton successfully reproduces ferro-paramagnetic phase transition, hysteresis, and ordered pattern formations.

# 1 Introduction

Cellular Automata are spatially extended dynamical systems where simple local rules generate complex global behavior. Applications include biological growth, fluid dynamics, traffic flow, and magnetism.

In the Ising model:

- Cells are binary: -1 or 1
- Interactions are local: Von Neumann neighborhood
- Updates are probabilistic: Stochastic Updates
- Emergent order appears

This project studies phase transitions and pattern formation using an Ising CA.

# 2 Ising Model

## 2.1 Cellular Automata Formulation

Lattice

A 2D square lattice:

$$82 \times 82 \ = N_x \times N_y = N$$

with periodic boundary conditions.

### 2.1.1 Cell States

Each automaton cell:

$$s_{i,j} = \pm 1$$

represents spin orientation.

### 2.1.2 Neighborhood

Von Neumann neighborhood: 4 neighbours

$$(i \pm 1, j), (i, j \pm 1)$$

## 2.2 Update Rules

### 2.2.1 Hamiltonian

$$H = -\frac{1}{2}\sum_{\langle i,j\rangle} J_{i,j}\sigma_i\sigma_j + g\mu_B B\sum_i \sigma_i$$

where $J_{i,j}$ (coupling strength)=$J$ for Von Neumann neighbours and 0 othherwise; $\sigma_i$ are pauli matrices representing spin ($s_i$=$\sigma_i\hat{z}$), g is Landé g-factor, $\mu_B$ is Bohr magneton; and B=$|B|\hat{z}$ is externally applied Magnetic field

### 2.2.2 Energy Change

$s_i = \pm 1$,
$s_{i,j}$ is the spin of (i,j) site in the lattice.
If $s_{i,j}$ flips to $-s_{i,j}$:

$$\Delta E = Js_{i,j}\sum_n s_n - 2g\mu_B B s_{i,j}$$

where $\sum_n s_n$ is the sum of all 4 neighbour spins.

### 2.2.3 Glauber Probability

$$P(\Delta E, T) = \frac{1}{1 + e^{\Delta E/(k_B T)}}$$

With this probability a spin $s_i$ flips to $-s_i$.

## 2.3 Algorithm

1. Initialize random lattice (grid with cells: -1 or 1)
2. Select random cell
3. Compute $\Delta E$ (For Von Neumann neighbours)
4. Flip with Glauber probability
5. Repeat this for N times, which constitute a sweep. We can have enough number of sweeps to get to the equilibrium.
6. Measure observables

# 3 Measured Observables

## 3.1 Magnetization

$$M = \frac{1}{N}\sum s_i \quad ; \quad \langle |M| \rangle$$

## 3.2 Magnetic Susceptibility

$$\chi = \frac{N}{k_B T}(\langle M^2 \rangle - \langle M \rangle^2)$$

### 3.3 Energy

$$E = \langle H \rangle$$

### 3.4 Specific Heat

$$C_v = \frac{1}{N k_B T^2} (\langle E^2 \rangle - \langle E \rangle^2)$$

## 4 Spin Configuration Snapshots

### 4.1 Ferromagnetic Phase ($J > 0,\ T < T_c$)

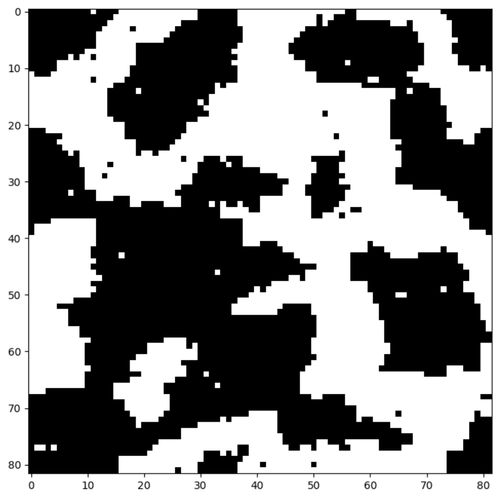


Figure 1: Ferromagnetic CA snapshot.

Large aligned domains emerge via local cooperation.

## 4.2 Critical Region ($T \approx T_c$)

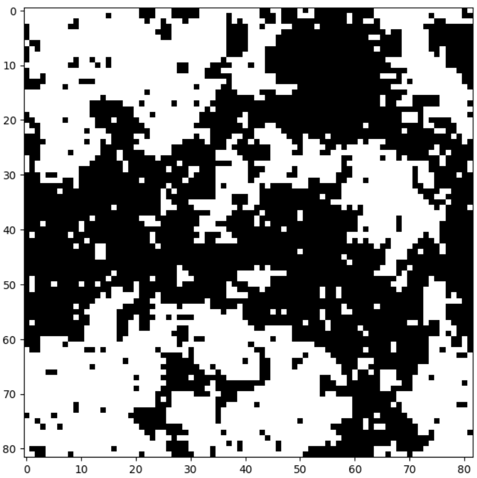

Figure 2: Critical domain structure.

Thermal effects start disrupting the aligned domains.

## 4.3 Paramagnetic Phase ($T > T_c$)

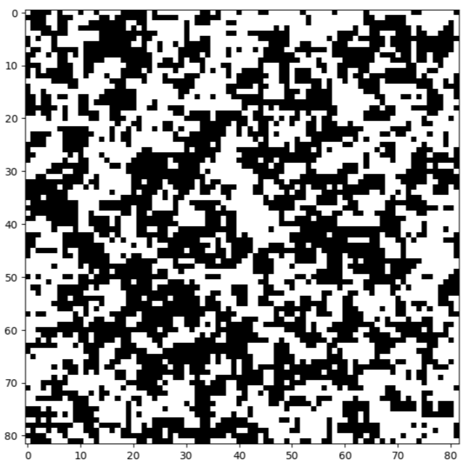

Figure 3: Paramagnetic CA snapshot.

Thermal noise destroys long–range order.

### 4.4 Antiferromagnetic Phase ($J < 0$)


Figure 4: Checkerboard antiferromagnetic order.

Neighbor anti-alignment produces bipartite ordering.

# 5 Results

- Results are obtained in natural units.
  $k_B, g, \mu_B, \hbar = 1$
  J=1 for Ferromagnet and -1 for Anti-ferromagnet.

- These results were obtained by 100 sweeps at each temperature. And 20 sweeps for Hysteresis curve. We can get better results if we increase the sweeps and let the code run for more time.

## 5.1 Magnetization

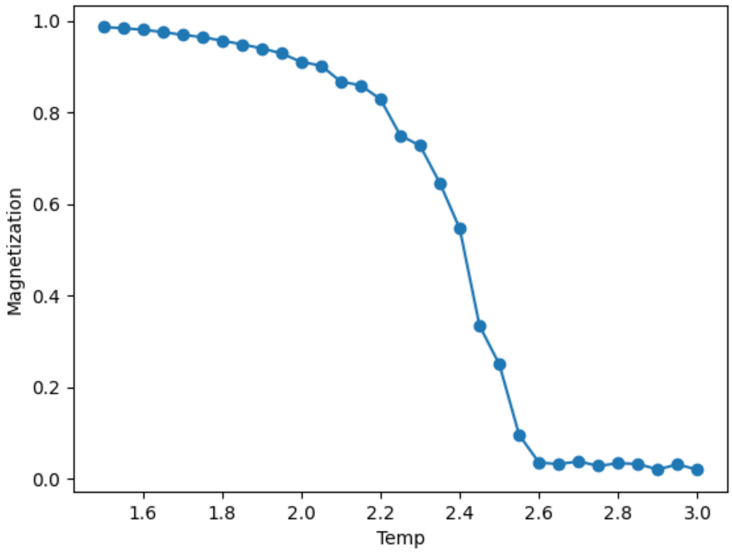


Figure 5: Magnetization vs Temperature.

Ferromagnetic phase(High magnetization) changes to Paramagnetic phase(Low magnetization) under no external field. Sharp drop near $T_c \approx 2.3$ indicates transition.

## 5.2 Magnetic Susceptibility

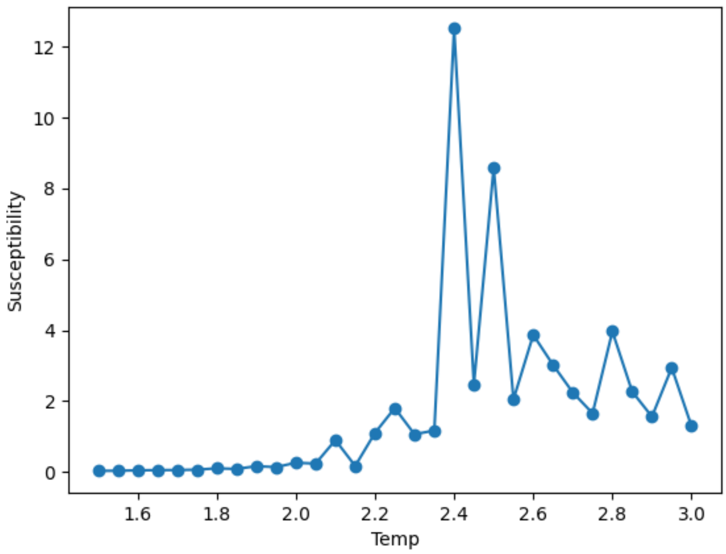


Figure 6: Susceptibility peak at criticality.

## 5.3 Energy

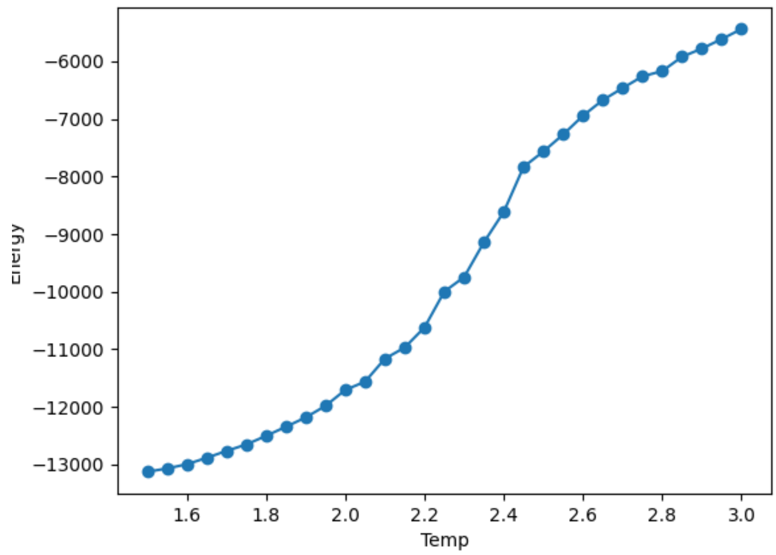


Figure 7: Energy vs Temperature.

## 5.4 Specific Heat

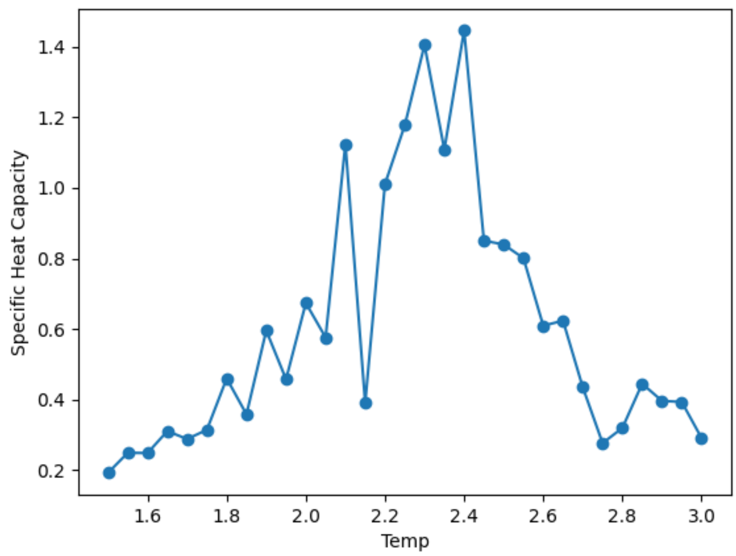


Figure 8: Specific heat peak near $T_c$.

### 5.5 Hysteresis Loop

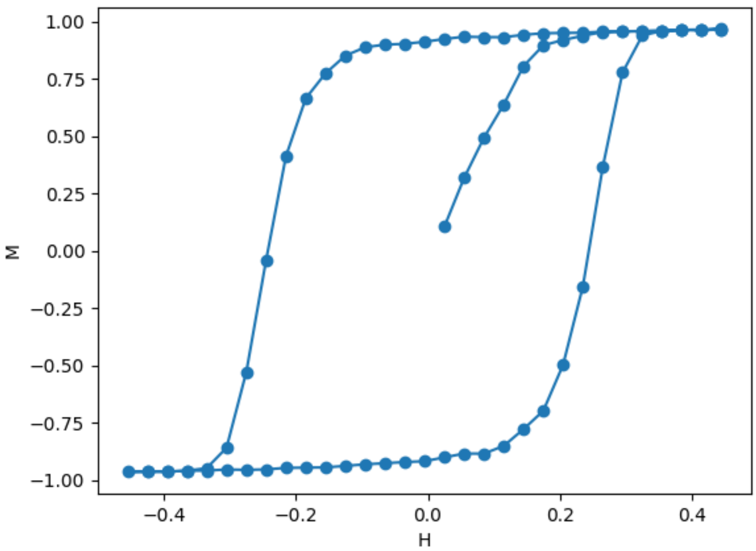


Figure 9: Magnetization vs External Field.

The simulated hysteresis loop demonstrates magnetic memory, showing non-zero magnetization at zero applied field (remanence/retention) and a finite coercive field required to reverse magnetization. This confirms ferromagnetic behavior with energy loss during cyclic magnetization.

## 6 Conclusion

The Ising cellular automaton demonstrates how simple stochastic local rules generate complex macroscopic physics including phase transitions, hysteresis, and pattern formation. This project highlights the power of CA in modeling emergent phenomena in statistical systems.

## 7 References


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