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syms L1 L2 L3 L4 theta1 theta2 theta3 theta_dot_1 theta_dot_2 theta_dot_3 real
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% Creating Transformation matrix and Rotational Matrix
% T10 will transform any vector in frame1 to frame0
% R10 will rotate any vector in frame1 to frame0
% T10 is Transformation matrix of frame1 expressed in frame0
% R10 is Rotational matrix of frame1 expressed in frame0
[T10,R10]=DH(0,0,theta1,L1+L2)
```

T10 =

$$\begin{pmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 & 0 \\ \sin(\theta_1) & \cos(\theta_1) & 0 & 0 \\ 0 & 0 & 1 & L_1+L_2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

R10 =

$$\begin{pmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 \\ \sin(\theta_1) & \cos(\theta_1) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

```
[T21,R21]=DH(90,0,theta2,0)
```

T21 =

$$\begin{pmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \sin(\theta_2) & \cos(\theta_2) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

R21 =

$$\begin{pmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 \\ 0 & 0 & -1 \\ \sin(\theta_2) & \cos(\theta_2) & 0 \end{pmatrix}$$

```
[T32,R32]=DH(0,L3,theta3,0)
```

T32 =

$$\begin{pmatrix} \cos(\theta_3) & -\sin(\theta_3) & 0 & L_3 \\ \sin(\theta_3) & \cos(\theta_3) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

R32 =

$$\begin{pmatrix} \cos(\theta_3) & -\sin(\theta_3) & 0 \\ \sin(\theta_3) & \cos(\theta_3) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

```
[T43,R43]=DH(0,L4,0,0)
```

T43 =

$$\begin{pmatrix} 1 & 0 & 0 & L_4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

R43 =

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

```
w00=[0;0;0]; %intializing w and v , w=0 and v=0 because they are the fixed frame
v00=[0;0;0];
```

```
% wab means omeaga of frame 'a' written in frame of 'b'
% vab means velocity of frame 'a' written in frame of 'b'
[w11,v11] = omega_and_vel_next(R10',w00,[0;0;theta_dot_1],v00,[0;0;L1+L2]);
[w22,v22] = omega_and_vel_next(R21',w11,[0;0;theta_dot_2],v11,[0;0;0]);
[w33,v33] = omega_and_vel_next(R32',w22,[0;0;theta_dot_3],v22,[L3;0;0]);
[w44,v44] = omega_and_vel_next(R43',w33,[0;0;0],v33,[L4;0;0]);
% w44 means omeaga of frame '4' written in frame of '4'
% v44 means velocity of frame '4' written in frame of '4'
```

```
w11=simplify(w11)
```

w11 =

$$\begin{pmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{pmatrix}$$

```
v11=simplify(v11)
```

v11 =

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

```
w22=simplify(w22)
```

w22 =

$$\begin{pmatrix} \dot{\theta}_1 \sin(\theta_2) \\ \dot{\theta}_1 \cos(\theta_2) \\ \dot{\theta}_2 \end{pmatrix}$$

```
v22=simplify(v22)
```

v22 =

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

```
w33=simplify(w33)
```

w33 =

$$\begin{pmatrix} \dot{\theta}_1 \sin(\theta_2+\theta_3) \\ \dot{\theta}_1 \cos(\theta_2+\theta_3) \\ \dot{\theta}_2+\dot{\theta}_3 \end{pmatrix}$$

```
v33=simplify(v33)
```

v33 =

$$\begin{pmatrix} L_3 \dot{\theta}_2 \sin(\theta_3) \\ L_3 \dot{\theta}_2 \cos(\theta_3) \\ -L_3 \dot{\theta}_1 \cos(\theta_2) \end{pmatrix}$$

```
w44=simplify(w44)
```

w44 =

$$\begin{pmatrix} \dot{\theta}_1 \sin(\theta_2+\theta_3) \\ \dot{\theta}_1 \cos(\theta_2+\theta_3) \\ \dot{\theta}_2+\dot{\theta}_3 \end{pmatrix}$$

```
v44=simplify(v44)
```

v44 =

$$\begin{pmatrix} L_3 \dot{\theta}_2 \sin(\theta_3) \\ L_4 \left(\dot{\theta}_2+\dot{\theta}_3\right) + L_3 \dot{\theta}_2 \cos(\theta_3) \\ -L_4 \dot{\theta}_1 \cos(\theta_2+\theta_3) - L_3 \dot{\theta}_1 \cos(\theta_2) \end{pmatrix}$$

```
v40=simplify(R10*R21*R32*R43*v44) % v40 means velocity of frame '4' written in frame of '0'
```

v40 =

$$\begin{pmatrix} L_3 \dot{\theta}_2 \cos(\theta_2+\theta_3) \cos(\theta_1) \sin(\theta_3) - \dot{\theta}_1 \sin(\theta_1)\, \sigma_1 - \sin(\theta_2+\theta_3) \cos(\theta_1)\, \sigma_2 \\ \dot{\theta}_1 \cos(\theta_1)\, \sigma_1 - \sin(\theta_2+\theta_3) \sin(\theta_1)\, \sigma_2 + L_3 \dot{\theta}_2 \cos(\theta_2+\theta_3) \sin(\theta_1) \sin(\theta_3) \\ L_4 \dot{\theta}_2 \cos(\theta_2+\theta_3) + L_4 \dot{\theta}_3 \cos(\theta_2+\theta_3) + L_3 \dot{\theta}_2 \cos(\theta_2) \end{pmatrix}$$

where

$$\sigma_1 = L_4 \cos(\theta_2+\theta_3) + L_3 \cos(\theta_2)$$

$$\sigma_2 = L_4 \dot{\theta}_2 + L_4 \dot{\theta}_3 + L_3 \dot{\theta}_2 \cos(\theta_3)$$

```
w40=simplify(R10*R21*R32*R43*w44) % w40 means omeaga of frame '4' written in frame of '0'
```

w40 =

$$\begin{pmatrix} \sin(\theta_1) \left(\dot{\theta}_2+\dot{\theta}_3\right) \\ -\cos(\theta_1) \left(\dot{\theta}_2+\dot{\theta}_3\right) \\ \dot{\theta}_1 \end{pmatrix}$$

```
q40=simplify(T10*T21*T32*T43*eye(4)) % q40 is Transformation matrix of frame4 expressed in frame0
```

q40 =

$$\begin{pmatrix} \cos(\theta_2+\theta_3) \cos(\theta_1) & -\sin(\theta_2+\theta_3) \cos(\theta_1) & \sin(\theta_1) & \cos(\theta_1)\, \sigma_1 \\ \cos(\theta_2+\theta_3) \sin(\theta_1) & -\sin(\theta_2+\theta_3) \sin(\theta_1) & -\cos(\theta_1) & \sin(\theta_1)\, \sigma_1 \\ \sin(\theta_2+\theta_3) & \cos(\theta_2+\theta_3) & 0 & L_1+L_2+L_4 \sin(\theta_2+\theta_3)+L_3 \sin(\theta_2) \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

where

$$\sigma_1 = L_4 \cos(\theta_2+\theta_3) + L_3 \cos(\theta_2)$$

```
q=q40(1:3,4) % q is position vector of frame4 wrt frame0 expressed in frame0
```

q =

$$\begin{pmatrix} \cos(\theta_1) \left(L_4 \cos(\theta_2+\theta_3) + L_3 \cos(\theta_2)\right) \\ \sin(\theta_1) \left(L_4 \cos(\theta_2+\theta_3) + L_3 \cos(\theta_2)\right) \\ L_1+L_2+L_4 \sin(\theta_2+\theta_3)+L_3 \sin(\theta_2) \end{pmatrix}$$

```
%verify v44
q_differentiate40 = diff(q,theta1)*theta_dot_1+diff(q,theta2)*theta_dot_2+diff(q,theta3)*theta_dot_3;
q_differentiate44 = simplify((R10*R21*R32*R43)'*(q_differentiate40)); %q_differentiate40 is velocity found by direct differentiation of position vector
simplify(q_differentiate44-v44) % verifies our v44 is correct and thus v40 is also correct
```

ans =

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

```
%verify w44
r=q40(1:3,1:3); % r is rotational matrix of frame4 expressed in frame0
rdot=simplify(diff(r,theta1)*theta_dot_1+diff(r,theta2)*theta_dot_2+diff(r,theta3)*theta_dot_3);
omega40=simplify(rdot*r')
```

omega40 =

$$\begin{pmatrix} 0 & -\dot{\theta}_1 & -\cos(\theta_1) \left(\dot{\theta}_2+\dot{\theta}_3\right) \\ \dot{\theta}_1 & 0 & -\sin(\theta_1) \left(\dot{\theta}_2+\dot{\theta}_3\right) \\ \cos(\theta_1) \left(\dot{\theta}_2+\dot{\theta}_3\right) & \sin(\theta_1) \left(\dot{\theta}_2+\dot{\theta}_3\right) & 0 \end{pmatrix}$$

```
omega40=simplify(invert_skew(omega40)) %omega40 is angular velocity found by direct method
```

omega40 =

$$\begin{pmatrix} \sin(\theta_1) \left(\dot{\theta}_2+\dot{\theta}_3\right) \\ -\cos(\theta_1) \left(\dot{\theta}_2+\dot{\theta}_3\right) \\ \dot{\theta}_1 \end{pmatrix}$$

```
omega44=simplify((R10*R21*R32*R43)'*omega40)
```

omega44 =

$$\begin{pmatrix} \dot{\theta}_1 \sin(\theta_2+\theta_3) \\ \dot{\theta}_1 \cos(\theta_2+\theta_3) \\ \dot{\theta}_2+\dot{\theta}_3 \end{pmatrix}$$

```
simplify(omega44-w44) % verifies our w44 is correct and thus w40 is also correct
```

ans =

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$