```
syms L1 L2 L3 L4 theta1 theta2 theta3 theta4 theta_dot_1 theta_dot_2 theta_dot_3 theta_dot_4 real
```

```
[T10,R10]=DH(0,0,theta1,0)
```

T10 =

$$\begin{pmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 & 0 \\ \sin(\theta_1) & \cos(\theta_1) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

R10 =

$$\begin{pmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 \\ \sin(\theta_1) & \cos(\theta_1) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

```
[T21,R21]=DH(0,1,theta2+pi/2,0)
```

T21 =

$$\begin{pmatrix} \cos\left(\theta_2+\frac{\pi}{2}\right) & -\sin\left(\theta_2+\frac{\pi}{2}\right) & 0 & 1 \\ \sin\left(\theta_2+\frac{\pi}{2}\right) & \cos\left(\theta_2+\frac{\pi}{2}\right) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

R21 =

$$\begin{pmatrix} \cos\left(\theta_2+\frac{\pi}{2}\right) & -\sin\left(\theta_2+\frac{\pi}{2}\right) & 0 \\ \sin\left(\theta_2+\frac{\pi}{2}\right) & \cos\left(\theta_2+\frac{\pi}{2}\right) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

```
[T32,R32]=DH(45,0,theta3-pi/2,sqrt(2))
```

T32 =

$$\begin{pmatrix} \cos\left(\theta_3-\frac{\pi}{2}\right) & -\sin\left(\theta_3-\frac{\pi}{2}\right) & 0 & 0 \\ \sigma_1 & \sigma_2 & -\frac{\sqrt{2}}{2} & -1 \\ \sigma_1 & \sigma_2 & \frac{\sqrt{2}}{2} & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

where

$$\sigma_1 = \frac{\sqrt{2}\,\sin\left(\theta_3-\frac{\pi}{2}\right)}{2}$$

$$\sigma_2 = \frac{\sqrt{2}\,\cos\left(\theta_3-\frac{\pi}{2}\right)}{2}$$

R32 =

$$\begin{pmatrix} \cos\left(\theta_3-\frac{\pi}{2}\right) & -\sin\left(\theta_3-\frac{\pi}{2}\right) & 0 \\ \frac{\sqrt{2}\,\sin\left(\theta_3-\frac{\pi}{2}\right)}{2} & \frac{\sqrt{2}\,\cos\left(\theta_3-\frac{\pi}{2}\right)}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}\,\sin\left(\theta_3-\frac{\pi}{2}\right)}{2} & \frac{\sqrt{2}\,\cos\left(\theta_3-\frac{\pi}{2}\right)}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}$$

```
[T43,R43]=DH(0,sqrt(2),theta4,0)
```

T43 =

$$\begin{pmatrix} \cos(\theta_4) & -\sin(\theta_4) & 0 & \sqrt{2} \\ \sin(\theta_4) & \cos(\theta_4) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

R43 =

$$\begin{pmatrix} \cos(\theta_4) & -\sin(\theta_4) & 0 \\ \sin(\theta_4) & \cos(\theta_4) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

```
w00=[0;0;0]; %intializing w and V , w=0 and v=0 because they are the fixed frame
v00=[0;0;0];
```

```
% wab means omeaga of frame 'a' written in frame of 'b'
% vab means velocity of frame 'a' written in frame of 'b'
[w11,v11] = omega_and_vel_next(R10',w00,[0;0;theta_dot_1],v00,[0;0;0]);
[w22,v22] = omega_and_vel_next(R21',w11,[0;0;theta_dot_2],v11,[1;0;0]);
[w33,v33] = omega_and_vel_next(R32',w22,[0;0;theta_dot_3],v22,[0;-1;1]);
[w44,v44] = omega_and_vel_next(R43',w33,[0;0;theta_dot_4],v33,[sqrt(2);0;0]);
```

```
w11=simplify(w11);
v11=simplify(v11);
w22=simplify(w22);
v22=simplify(v22);
w33=simplify(w33);
v33=simplify(v33);
w44=simplify(w44) % w44 means omeaga of frame '4' written in frame of '4'
```

w44 =

$$\begin{pmatrix} -\frac{\sqrt{2}\,\cos(\theta_3+\theta_4)\,(\dot{\theta}_1+\dot{\theta}_2)}{2} \\ \frac{\sqrt{2}\,\sin(\theta_3+\theta_4)\,(\dot{\theta}_1+\dot{\theta}_2)}{2} \\ \dot{\theta}_3+\dot{\theta}_4+\frac{\sqrt{2}\,(\dot{\theta}_1+\dot{\theta}_2)}{2} \end{pmatrix}$$

```
v44=simplify(v44) % v44 means velocity of frame '4' written in frame of '4'
```

v44 =

$$\begin{pmatrix} \frac{\cos(\theta_4)\,\sigma_2}{2}+\sin(\theta_4)\,\sigma_1 \\ \cos(\theta_4)\,\sigma_1-\frac{\sin(\theta_4)\,\sigma_2}{2} \\ \frac{\sqrt{2}\,\dot{\theta}_1\sin(\theta_2)}{2}-\dot{\theta}_2\sin(\theta_3)-\dot{\theta}_1\sin(\theta_3) \end{pmatrix}$$

where

$$\sigma_1 = \dot{\theta}_1+\dot{\theta}_2+\sqrt{2}\,\dot{\theta}_3+\dot{\theta}_1\cos(\theta_3)+\dot{\theta}_2\cos(\theta_3)+\dot{\theta}_1\cos(\theta_2)\cos(\theta_3)-\frac{\sqrt{2}\,\dot{\theta}_1\sin(\theta_2)\sin(\theta_3)}{2}$$

$$\sigma_2 = 2\,\dot{\theta}_1\sin(\theta_3)+2\,\dot{\theta}_2\sin(\theta_3)+2\,\dot{\theta}_1\cos(\theta_2)\sin(\theta_3)+\sqrt{2}\,\dot{\theta}_1\cos(\theta_3)\sin(\theta_2)$$

```
v40=simplify(R10*R21*R32*R43*v44) % v40 means velocity of frame '4' written in frame of '0'
```

v40 =

$$\begin{pmatrix} \frac{\dot{\theta}_3\sigma_1}{2}-\frac{\dot{\theta}_2\sigma_2}{2}-\frac{\dot{\theta}_3\sigma_2}{2}-\frac{\dot{\theta}_1\sigma_1}{2}-\frac{\dot{\theta}_2\sigma_1}{2}-\frac{\dot{\theta}_1\sigma_2}{2}-\dot{\theta}_1\sin(\theta_1+\theta_2)-\dot{\theta}_2\sin(\theta_1+\theta_2)-\dot{\theta}_1\sin(\theta_1)-\frac{\sqrt{2}\,\dot{\theta}_1\sigma_2}{2}-\frac{\sqrt{2}\,\dot{\theta}_2\sigma_2}{2}-\frac{\sqrt{2}\,\dot{\theta}_3\sigma_2}{2}+\frac{\sqrt{2}\,\dot{\theta}_1\sigma_1}{2}+\frac{\sqrt{2}\,\dot{\theta}_2\sigma_1}{2}-\frac{\sqrt{2}\,\dot{\theta}_3\sigma_1}{2} \\ \frac{\dot{\theta}_1\sigma_3}{2}+\frac{\dot{\theta}_2\sigma_3}{2}-\frac{\dot{\theta}_3\sigma_3}{2}+\dot{\theta}_1\cos(\theta_1+\theta_2)+\dot{\theta}_2\cos(\theta_1+\theta_2)+\dot{\theta}_1\cos(\theta_1)+\frac{\dot{\theta}_1\sigma_4}{2}+\frac{\dot{\theta}_2\sigma_4}{2}+\frac{\dot{\theta}_3\sigma_4}{2}+\frac{\sqrt{2}\,\dot{\theta}_1\sigma_4}{2}+\frac{\sqrt{2}\,\dot{\theta}_2\sigma_4}{2}+\frac{\sqrt{2}\,\dot{\theta}_3\sigma_4}{2}-\frac{\sqrt{2}\,\dot{\theta}_1\sigma_3}{2}-\frac{\sqrt{2}\,\dot{\theta}_2\sigma_3}{2}+\frac{\sqrt{2}\,\dot{\theta}_3\sigma_3}{2} \\ \dot{\theta}_3\sin(\theta_3) \end{pmatrix}$$

where

$$\sigma_1 = \sin(\theta_1+\theta_2-\theta_3)$$

$$\sigma_2 = \sin(\theta_1+\theta_2+\theta_3)$$

$$\sigma_3 = \cos(\theta_1+\theta_2-\theta_3)$$

$$\sigma_4 = \cos(\theta_1+\theta_2+\theta_3)$$

```
w40=simplify(R10*R21*R32*R43*w44) % w40 means velocity of frame '4' written in frame of '0'
```

w40 =

$$\begin{pmatrix} \frac{\sqrt{2}\,\cos(\theta_1+\theta_2)\,(\dot{\theta}_3+\dot{\theta}_4)}{2} \\ \frac{\sqrt{2}\,\sin(\theta_1+\theta_2)\,(\dot{\theta}_3+\dot{\theta}_4)}{2} \\ \dot{\theta}_1+\dot{\theta}_2+\frac{\sqrt{2}\,\dot{\theta}_3}{2}+\frac{\sqrt{2}\,\dot{\theta}_4}{2} \end{pmatrix}$$

```
J1=subs(v40, [theta_dot_1 ,theta_dot_2 ,theta_dot_3,theta_dot_4], [1,0,0,0]);
J2=subs(v40, [theta_dot_1 ,theta_dot_2 ,theta_dot_3,theta_dot_4], [0,1,0,0]);
J3=subs(v40, [theta_dot_1 ,theta_dot_2 ,theta_dot_3,theta_dot_4], [0,0,1,0]);
J4=subs(v40, [theta_dot_1 ,theta_dot_2 ,theta_dot_3,theta_dot_4], [0,0,0,1]);
Jv0=[J1,J2,J3,J4];
Jv0=simplify(Jv0) % linear velocity Jacobian written in frame of 0
```

Jv0 =

$$\begin{pmatrix} \sigma_1-\frac{\sigma_5}{2}-\sin(\theta_1+\theta_2)-\sin(\theta_1)-\frac{\sigma_6}{2}-\sigma_2 & \sigma_1-\frac{\sigma_5}{2}-\sin(\theta_1+\theta_2)-\frac{\sigma_6}{2}-\sigma_2 & \sin(\theta_1)\sin(\theta_2)\sin(\theta_3)-\cos(\theta_1)\cos(\theta_2)\sin(\theta_3)-\sqrt{2}\,\cos(\theta_1)\cos(\theta_3)\sin(\theta_2)-\sqrt{2}\,\cos(\theta_2)\cos(\theta_3)\sin(\theta_1) & 0 \\ \frac{\sigma_8}{2}+\frac{\sigma_7}{2}+\cos(\theta_1+\theta_2)+\cos(\theta_1)-\sigma_3+\sigma_4 & \frac{\sigma_8}{2}+\frac{\sigma_7}{2}+\cos(\theta_1+\theta_2)-\sigma_3+\sigma_4 & \sqrt{2}\,\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)-\cos(\theta_2)\sin(\theta_1)\sin(\theta_3)-\cos(\theta_1)\sin(\theta_2)\sin(\theta_3)-\sqrt{2}\,\cos(\theta_3)\sin(\theta_1)\sin(\theta_2) & 0 \\ 0 & 0 & \sin(\theta_3) & 0 \end{pmatrix}$$

where

$$\sigma_1 = \frac{\sqrt{2}\,\sigma_5}{2}$$

$$\sigma_2 = \frac{\sqrt{2}\,\sigma_6}{2}$$

$$\sigma_3 = \frac{\sqrt{2}\,\sigma_7}{2}$$

$$\sigma_4 = \frac{\sqrt{2}\,\sigma_8}{2}$$

$$\sigma_5 = \sin(\theta_1+\theta_2-\theta_3)$$

$$\sigma_6 = \sin(\theta_1+\theta_2+\theta_3)$$

$$\sigma_7 = \cos(\theta_1+\theta_2-\theta_3)$$

$$\sigma_8 = \cos(\theta_1+\theta_2+\theta_3)$$

```
check = simplify(v40-Jv0*[theta_dot_1 ;theta_dot_2 ;theta_dot_3;theta_dot_4]) %verifies v=J*theta_dot
```

check =

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

```
J5=subs(w40, [theta_dot_1 ,theta_dot_2 ,theta_dot_3,theta_dot_4], [1,0,0,0]);
J6=subs(w40, [theta_dot_1 ,theta_dot_2 ,theta_dot_3,theta_dot_4], [0,1,0,0]);
J7=subs(w40, [theta_dot_1 ,theta_dot_2 ,theta_dot_3,theta_dot_4], [0,0,1,0]);
J8=subs(w40, [theta_dot_1 ,theta_dot_2 ,theta_dot_3,theta_dot_4], [0,0,0,1]);
Jw0=[J5,J6,J7,J8];
Jw0=simplify(Jw0) % angular velocity Jacobian written in frame of 0
```

Jw0 =

$$\begin{pmatrix} 0 & 0 & \frac{\sqrt{2}\,\cos(\theta_1+\theta_2)}{2} & \frac{\sqrt{2}\,\cos(\theta_1+\theta_2)}{2} \\ 0 & 0 & \frac{\sqrt{2}\,\sin(\theta_1+\theta_2)}{2} & \frac{\sqrt{2}\,\sin(\theta_1+\theta_2)}{2} \\ 1 & 1 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}$$

```
check = simplify(w40-Jw0*[theta_dot_1 ;theta_dot_2 ;theta_dot_3;theta_dot_4]) %verifies w=J*theta_dot
```

check =

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

```
J0=[Jv0;Jw0] %because [v;w]=J*[theta_dot]
```

J0 =

$$\begin{pmatrix} \sigma_3-\frac{\sigma_7}{2}-\sin(\theta_1+\theta_2)-\sin(\theta_1)-\frac{\sigma_8}{2}-\sigma_4 & \sigma_3-\frac{\sigma_7}{2}-\sin(\theta_1+\theta_2)-\frac{\sigma_8}{2}-\sigma_4 & \sin(\theta_1)\sin(\theta_2)\sin(\theta_3)-\cos(\theta_1)\cos(\theta_2)\sin(\theta_3)-\sqrt{2}\,\cos(\theta_1)\cos(\theta_3)\sin(\theta_2)-\sqrt{2}\,\cos(\theta_2)\cos(\theta_3)\sin(\theta_1) & 0 \\ \frac{\sigma_{10}}{2}+\frac{\sigma_9}{2}+\cos(\theta_1+\theta_2)+\cos(\theta_1)-\sigma_5+\sigma_6 & \frac{\sigma_{10}}{2}+\frac{\sigma_9}{2}+\cos(\theta_1+\theta_2)-\sigma_5+\sigma_6 & \sqrt{2}\,\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)-\cos(\theta_2)\sin(\theta_1)\sin(\theta_3)-\cos(\theta_1)\sin(\theta_2)\sin(\theta_3)-\sqrt{2}\,\cos(\theta_3)\sin(\theta_1)\sin(\theta_2) & 0 \\ 0 & 0 & \sin(\theta_3) & 0 \\ 0 & 0 & \sigma_2 & \sigma_2 \\ 0 & 0 & \sigma_1 & \sigma_1 \\ 1 & 1 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}$$

where

$$\sigma_1 = \frac{\sqrt{2}\,\sin(\theta_1+\theta_2)}{2}$$

$$\sigma_2 = \frac{\sqrt{2}\,\cos(\theta_1+\theta_2)}{2}$$

$$\sigma_3 = \frac{\sqrt{2}\,\sigma_7}{2}$$

$$\sigma_4 = \frac{\sqrt{2}\,\sigma_8}{2}$$

$$\sigma_5 = \frac{\sqrt{2}\,\sigma_9}{2}$$

$$\sigma_6 = \frac{\sqrt{2}\,\sigma_{10}}{2}$$

$$\sigma_7 = \sin(\theta_1+\theta_2-\theta_3)$$

$$\sigma_8 = \sin(\theta_1+\theta_2+\theta_3)$$

$$\sigma_9 = \cos(\theta_1+\theta_2-\theta_3)$$

$$\sigma_{10} = \cos(\theta_1+\theta_2+\theta_3)$$

```
J1=subs(v44, [theta_dot_1 ,theta_dot_2 ,theta_dot_3,theta_dot_4], [1,0,0,0]);
J2=subs(v44, [theta_dot_1 ,theta_dot_2 ,theta_dot_3,theta_dot_4], [0,1,0,0]);
J3=subs(v44, [theta_dot_1 ,theta_dot_2 ,theta_dot_3,theta_dot_4], [0,0,1,0]);
J4=subs(v44, [theta_dot_1 ,theta_dot_2 ,theta_dot_3,theta_dot_4], [0,0,0,1]);
Jv4=[J1,J2,J3,J4];
Jv4=simplify(Jv4) % linear velocity Jacobian written in frame of 4
```

Jv4 =

$$\begin{pmatrix} \sin(\theta_4)\,\sigma_2+\frac{\cos(\theta_4)\,\sigma_1}{2} & \sin(\theta_3+\theta_4)+\sin(\theta_4) & \sqrt{2}\,\sin(\theta_4) & 0 \\ \cos(\theta_4)\,\sigma_2-\frac{\sin(\theta_4)\,\sigma_1}{2} & \cos(\theta_3+\theta_4)+\cos(\theta_4) & \sqrt{2}\,\cos(\theta_4) & 0 \\ \frac{\sqrt{2}\,\sin(\theta_2)}{2}-\sin(\theta_3) & -\sin(\theta_3) & 0 & 0 \end{pmatrix}$$

where

$$\sigma_1 = 2\sin(\theta_3)+2\cos(\theta_2)\sin(\theta_3)+\sqrt{2}\,\cos(\theta_3)\sin(\theta_2)$$

$$\sigma_2 = \cos(\theta_3)+\cos(\theta_2)\cos(\theta_3)-\frac{\sqrt{2}\,\sin(\theta_2)\sin(\theta_3)}{2}+1$$

```
check = simplify(v44-Jv4*[theta_dot_1 ;theta_dot_2 ;theta_dot_3;theta_dot_4]) %verifies v=J*theta_dot
```

check =

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

```
J5=subs(w44, [theta_dot_1 ,theta_dot_2 ,theta_dot_3,theta_dot_4], [1,0,0,0]);
J6=subs(w44, [theta_dot_1 ,theta_dot_2 ,theta_dot_3,theta_dot_4], [0,1,0,0]);
J7=subs(w44, [theta_dot_1 ,theta_dot_2 ,theta_dot_3,theta_dot_4], [0,0,1,0]);
J8=subs(w44, [theta_dot_1 ,theta_dot_2 ,theta_dot_3,theta_dot_4], [0,0,0,1]);
Jw4=[J5,J6,J7,J8];
Jw4=simplify(Jw4) % angular velocity Jacobian written in frame of 4
```

Jw4 =

$$\begin{pmatrix} -\frac{\sqrt{2}\,\cos(\theta_3+\theta_4)}{2} & -\frac{\sqrt{2}\,\cos(\theta_3+\theta_4)}{2} & 0 & 0 \\ \frac{\sqrt{2}\,\sin(\theta_3+\theta_4)}{2} & \frac{\sqrt{2}\,\sin(\theta_3+\theta_4)}{2} & 0 & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 1 & 1 \end{pmatrix}$$

```
check = simplify(w44-Jw4*[theta_dot_1 ;theta_dot_2 ;theta_dot_3;theta_dot_4]) %verifies w=J*theta_dot
```

check =

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

```
J4=[Jv4;Jw4] %because [v;w]=J*[theta_dot]
```

J4 =

$$\begin{pmatrix} \sin(\theta_4)\,\sigma_2+\frac{\cos(\theta_4)\,\sigma_1}{2} & \sin(\theta_3+\theta_4)+\sin(\theta_4) & \sqrt{2}\,\sin(\theta_4) & 0 \\ \cos(\theta_4)\,\sigma_2-\frac{\sin(\theta_4)\,\sigma_1}{2} & \cos(\theta_3+\theta_4)+\cos(\theta_4) & \sqrt{2}\,\cos(\theta_4) & 0 \\ \frac{\sqrt{2}\,\sin(\theta_2)}{2}-\sin(\theta_3) & -\sin(\theta_3) & 0 & 0 \\ \sigma_3 & \sigma_3 & 0 & 0 \\ \sigma_4 & \sigma_4 & 0 & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 1 & 1 \end{pmatrix}$$

where

$$\sigma_1 = 2\sin(\theta_3)+2\cos(\theta_2)\sin(\theta_3)+\sqrt{2}\,\cos(\theta_3)\sin(\theta_2)$$

$$\sigma_2 = \cos(\theta_3)+\cos(\theta_2)\cos(\theta_3)-\frac{\sqrt{2}\,\sin(\theta_2)\sin(\theta_3)}{2}+1$$

$$\sigma_3 = -\frac{\sqrt{2}\,\cos(\theta_3+\theta_4)}{2}$$

$$\sigma_4 = \frac{\sqrt{2}\,\sin(\theta_3+\theta_4)}{2}$$

```
Rot=sym(zeros(6,6));
Rot(1:3,1:3)=R10*R21*R32*R43;
Rot(4:6,4:6)=R10*R21*R32*R43;
Rot % matrix that converts J4 to J0
```

Rot =

$$\begin{pmatrix} \sigma_3 & \sigma_2 & \frac{\sqrt{2}\,\sigma_8}{2} & 0 & 0 & 0 \\ \sigma_4 & \sigma_1 & -\frac{\sqrt{2}\,\sigma_7}{2} & 0 & 0 & 0 \\ \sigma_6 & \sigma_5 & \frac{\sqrt{2}}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_3 & \sigma_2 & \frac{\sqrt{2}\,\sigma_8}{2} \\ 0 & 0 & 0 & \sigma_4 & \sigma_1 & -\frac{\sqrt{2}\,\sigma_7}{2} \\ 0 & 0 & 0 & \sigma_6 & \sigma_5 & \frac{\sqrt{2}}{2} \end{pmatrix}$$

where

$$\sigma_1 = -\cos(\theta_4)\left(\sigma_9\sigma_8-\frac{\sqrt{2}\,\sigma_{10}\sigma_7}{2}\right)-\sin(\theta_4)\left(\sigma_{10}\sigma_8+\frac{\sqrt{2}\,\sigma_9\sigma_7}{2}\right)$$

$$\sigma_2 = -\cos(\theta_4)\left(\sigma_9\sigma_7+\frac{\sqrt{2}\,\sigma_{10}\sigma_8}{2}\right)-\sin(\theta_4)\left(\sigma_{10}\sigma_7-\frac{\sqrt{2}\,\sigma_9\sigma_8}{2}\right)$$

$$\sigma_3 = \cos(\theta_4)\left(\sigma_{10}\sigma_7-\frac{\sqrt{2}\,\sigma_9\sigma_8}{2}\right)-\sin(\theta_4)\left(\sigma_9\sigma_7+\frac{\sqrt{2}\,\sigma_{10}\sigma_8}{2}\right)$$

$$\sigma_4 = \cos(\theta_4)\left(\sigma_{10}\sigma_8+\frac{\sqrt{2}\,\sigma_9\sigma_7}{2}\right)-\sin(\theta_4)\left(\sigma_9\sigma_8-\frac{\sqrt{2}\,\sigma_{10}\sigma_7}{2}\right)$$

$$\sigma_5 = \frac{\sqrt{2}\,\cos(\theta_4)\,\sigma_{10}}{2}-\frac{\sqrt{2}\,\sin(\theta_4)\,\sigma_9}{2}$$

$$\sigma_6 = \frac{\sqrt{2}\,\cos(\theta_4)\,\sigma_9}{2}+\frac{\sqrt{2}\,\sigma_{10}\sin(\theta_4)}{2}$$

$$\sigma_7 = \cos(\theta_1)\cos\left(\theta_2+\frac{\pi}{2}\right)-\sin(\theta_1)\sin\left(\theta_2+\frac{\pi}{2}\right)$$

$$\sigma_8 = \cos(\theta_1)\sin\left(\theta_2+\frac{\pi}{2}\right)+\cos\left(\theta_2+\frac{\pi}{2}\right)\sin(\theta_1)$$

$$\sigma_9 = \sin\left(\theta_3-\frac{\pi}{2}\right)$$

$$\sigma_{10} = \cos\left(\theta_3-\frac{\pi}{2}\right)$$

```
simplify(J0-(Rot*J4)) % verifies that J0 == [[R 0];[0 R]] * J4
```

ans =

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

```
max_rankk = min(size(J0)) % maximum possible rank or highest degree of maneuverability
```

max_rankk = 4

```
q40=simplify(T10*T21*T32*T43*eye(4));
q=q40(1:3,4); %position of frame 4 wrt frame 0
```

```
%verify v44
q_differentiate40 = diff(q,theta1)*theta_dot_1+diff(q,theta2)*theta_dot_2+diff(q,theta3)*theta_dot_3+diff(q,theta4)*theta_dot_4;
q_differentiate44 = simplify((R10*R21*R32*R43)'*(q_differentiate40));
simplify(q_differentiate44-v44) % verifies our v44 is correct and thus v40 is also correct
```

ans =

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

```
%verify w44
r=q40(1:3,1:3); % rotational matrix to convert from frame 4 to frame 0
rdot=simplify(diff(r,theta1)*theta_dot_1+diff(r,theta2)*theta_dot_2+diff(r,theta3)*theta_dot_3+diff(r,theta4)*theta_dot_4);
omega40=simplify(rdot*r');
omega40=simplify(invert_skew(omega40)); %maps the skew omega to omega vector
omega44=simplify((R10*R21*R32*R43)'*omega40);
simplify(omega44-w44) % verifies our w44 is correct and thus w40 is also correct
```

ans =

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

```
eqn = det(Jv0(1:3,1:3))==0;
S = solve(eqn,[theta1,theta2,theta3,theta4],'ReturnConditions',true,'Real',true);
subs(S.conditions,S.parameters,[theta1,theta2,theta3,theta4]) % workspace boundary characterizing condition
```

ans =

$$\theta_1 \in \mathbb{R} \wedge \theta_2 \in \mathbb{R} \wedge \theta_3 \in \mathbb{R} \wedge \theta_4 \in \mathbb{R} \wedge \left(\frac{\theta_3}{\pi} \in \mathbb{Z} \vee \sin(\theta_2)+\cos(\theta_3)\sin(\theta_2)+\sqrt{2}\,\cos(\theta_2)\sin(\theta_3)=0\right)$$

```
J0_eval_1=subs(J0,[theta1,theta2,theta3,theta4],[0,0,0,0]) %evaluating J0 at workspace boundary
```

J0_eval_1 =

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 3 & 2 & \sqrt{2} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ 0 & 0 & 0 & 0 \\ 1 & 1 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}$$

```
rank(J0_eval_1) % rank of J0 at worksapce boundary
```

ans = 3

```
J0_eval_2=subs(J0,[theta1,theta2,theta3,theta4],[0,0,pi/2,0]) %evaluating J0 inside workspace boundary
```

J0_eval_2 =

$$\begin{pmatrix} -\sqrt{2} & -\sqrt{2} & -1 & 0 \\ 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ 0 & 0 & 0 & 0 \\ 1 & 1 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}$$

```
rank(J0_eval_2) % rank of J0 inside worksapce boundary
```

ans = 4

```
max_rank=min(size(J0)) % maximum possible rank
```

max_rank = 4

```
J4_eval_1=subs(J4,[theta1,theta2,theta3,theta4],[0,0,0,0]) %evaluating J4 at workspace boundary
```

J4_eval_1 =

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 3 & 2 & \sqrt{2} & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 1 & 1 \end{pmatrix}$$

```
rank(J4_eval_1) % rank of J4 at worksapce boundary
```

ans = 3

```
J4_eval_2=subs(J4,[theta1,theta2,theta3,theta4],[0,0,pi/2,0]) %evaluating J4 inside workspace boundary
```

J4_eval_2 =

$$\begin{pmatrix} 2 & 1 & 0 & 0 \\ 1 & 1 & \sqrt{2} & 0 \\ -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 1 & 1 \end{pmatrix}$$

```
rank(J4_eval_2) % rank of J4 inside worksapce boundary
```

ans = 4

```
max_rank=min(size(J4)) % maximum possible rank
```

max_rank = 4