09.00 # Theory of Transformation.

10.00 2D

Matrix & Linear

11.00 Affine

general geometric

12.00

2D

01.00

Eucliden Transformaton

02.00

03.00

Riged Shear

04.00

05.00

3D transformation.

06.00

07.00

Aribitrary Line

08.00 Fundamental

Affine transformation

08.00

09.00 Orient Preserving transformation

10.00

11.00

12.00 Quaternin transformation

01.00

02.00

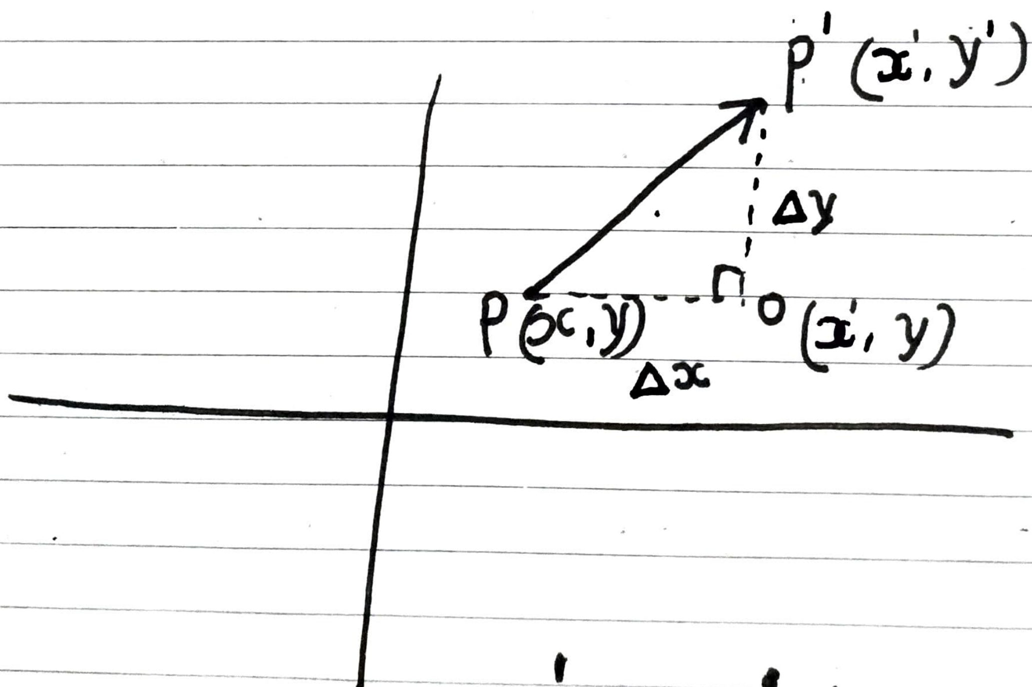
03.00

04.00

05.00

06.00

07.00



08.00

$$x' = x + \Delta x$$

$$y' = y + \Delta y$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} dx \\ dy \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} dx \\ dy \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = I_2 \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} dx \\ dy \end{bmatrix}$$

$$\begin{array}{cc} & \begin{array}{cccccc} C_1 & C_2 & C_3 & C_4 & & C_n \end{array} \\ \begin{array}{c} R_1 \\ R_2 \\ R_3 \\ R_4 \\ \\ R_m \end{array} & \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & a_{24} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & a_{34} & \cdots & a_{3n} \\ a_{4} & & & & & \\ \vdots & & & & & \\ a_{m1} & a_{m2} & a_{m3} & a_{m4} & \cdots & a_{mn} \end{bmatrix} m \times n \end{array}$$

$$
\begin{array}{c}
\\
R_1 \\ R_2 \\ R_3 \\ \\ \\ R_n
\end{array}
\begin{array}{c}
\begin{array}{ccccc} R_1' & R_2' & R_3' & & R_m' \end{array} \\
\left[\begin{array}{ccccc}
a_{11} & a_{21} & a_{31} & \cdots & a_{m1} \\
a_{12} & a_{22} & a_{32} & \cdots & a_{m2} \\
a_{13} & a_{23} & a_{33} & \cdots & a_{m3} \\
a_{14} & a_{24} & a_{34} & \cdots & a_{m4} \\
\vdots & \vdots & \vdots & & \vdots \\
a_{1n} & a_{2n} & a_{3n} & \cdots & a_{mn}
\end{array}\right]_{n \times m}
\end{array}
$$

Transpos

Convert Raw $\rightarrow$ Column

and

Column $\rightarrow$ Raw

$$
\left[\quad\quad\right]^{T}_{m \times n} = \left[\quad\quad\right]_{n \times m}
$$

$$[x', y']^T = I_2 [x \;\; y]^T + [dx \;\; dy]^T$$

2D Translation Transformation.

Scaling Transformation

Let O be an origin

Let O' be a point (h, k)

w.r.t O

Let P be a point (x_0, y_0)

w.r.t. O

if origin $O \longrightarrow O'$

then co-ordinates of P

w.r.t O' is $(x_0 - h, y_0 - k)$

$$x' = S_x x$$

$$y' = \cancel{y} \cdot S_y y$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

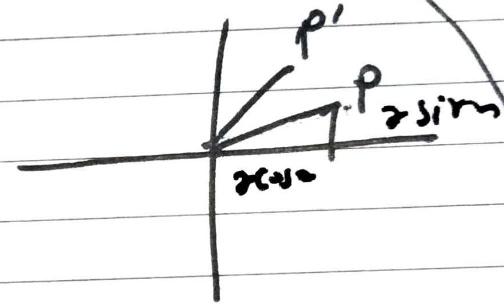
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} x' & y' \end{bmatrix}^T = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \begin{bmatrix} x & y \end{bmatrix}^T$$

Rotation

$$x' = f(x, y, \text{trig}(\theta))$$

$$y' = g(x, y \ \ \text{trig}(\theta))$$



$$x' = r\cos\theta$$

$$y' = r\sin\theta$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = [r] \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} r & 0 \\ 0 & r \end{bmatrix} \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$$

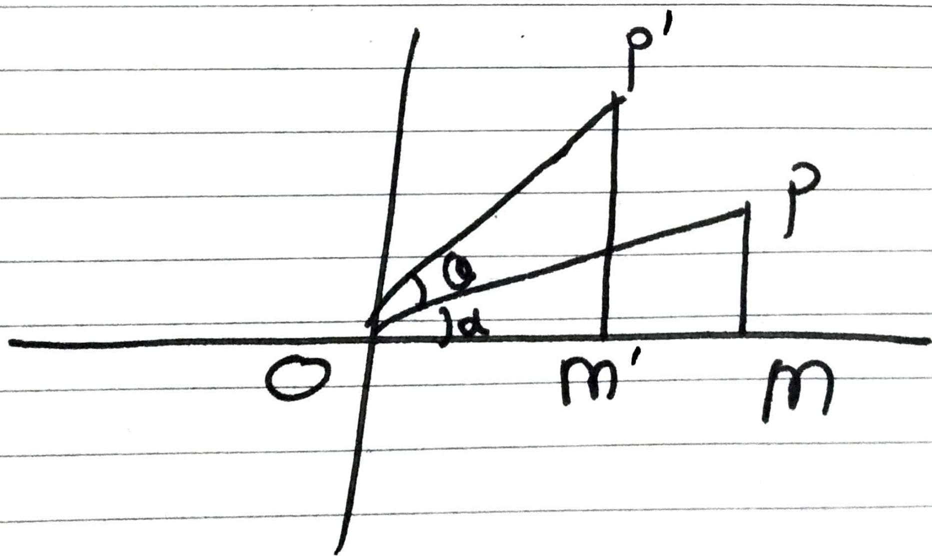
$$x' = \gamma x \cos(\alpha+\theta)$$

$$y' = \gamma y \sin(\alpha+\theta)$$

$$= \gamma \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x\cos(\alpha+\theta) \\ y \sin(\alpha+\theta) \end{bmatrix}$$

$$= \begin{bmatrix} \gamma & 0 \\ 0 & \gamma \end{bmatrix} \begin{bmatrix} x\,(\cos\alpha\cos\theta - \sin\alpha\sin \\ y\,(\cos\alpha\sin\theta + \sin\alpha\cos \end{bmatrix}$$

$$= \begin{bmatrix} \gamma & 0 \\ 0 & \gamma \end{bmatrix} \begin{bmatrix} x\cos\alpha\cos\theta - x\sin\alpha\sin\theta \\ y\cos\alpha\sin\theta + y\sin\alpha\cos\theta \end{bmatrix}$$



$OM = r\cos\alpha$

$PM = r\sin\alpha$

$x' = r\cos(\alpha+\theta)$

08.00

$r(\cos\alpha\cos\theta - \sin\alpha\sin\theta)$

09.00

$r\cos\alpha\cos\theta - r\sin\alpha\sin\theta)$

10.00

$x\cos\theta - y\sin\theta$

11.00

12.00

$y' = r\sin(\alpha+\theta)$

01.00

$= r(\cos\alpha\sin\theta + \sin\alpha\cos\theta)$

02.00

$= r\cos\alpha\sin\theta + r\sin\alpha\cos\theta$

03.00

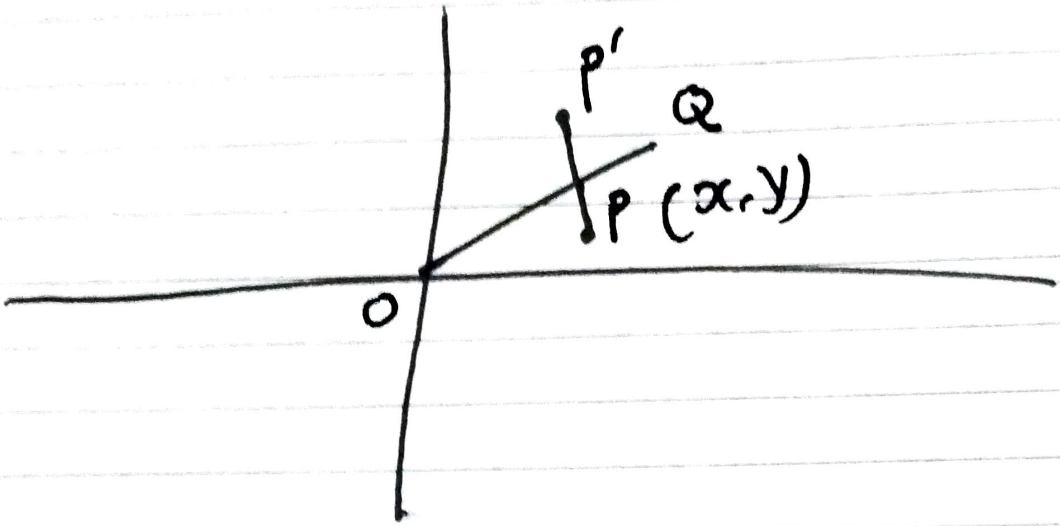
$= x\sin\theta + y\cos\theta$

04.00

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

05.00

06.00



$$\begin{bmatrix} \cos(-\theta) & -\sin(-\theta) \\ \sin(-\theta) & \cos(-\theta) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Scaling Rotation, Reflection

$$\begin{bmatrix}x'\\y'\end{bmatrix} = M_{2\times2}\begin{bmatrix}x\\y\end{bmatrix}$$

Translation.

$$\begin{bmatrix}x'\\y'\end{bmatrix} = I_{2\times2}\begin{bmatrix}x\\y\end{bmatrix} + M_{2\times2}$$

$$\boxed{\begin{aligned} P' &= M_{2\times2}\ P \\ P' &= M_{2\times2}\cdot P + D_{2\times1} \end{aligned}}$$

$$\boxed{P'_{m\times 1} = M_{m\times m}\cdot P_{m\times 1}}$$

Linear Transformation of $P_{m\times 1}$ in $P'_{m\times 1}$

Linear

$$P'_{n\times 1} = M_{n\times n}\, P_{n\times 1}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & & & & \\ a_{31} & & & & \\ \vdots & & & & \\ a_{n1} & & & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix}$$

$$x'_1 = a_{11}x_1 + a_{12}x_2 + \dots a_{1n}x_n$$

Affine

$$P'_{n\times 1} = M_{n\times n}\, P_{n\times 1} + D_{n\times 1}$$

$$\begin{bmatrix} x'_1 \\ x'_2 \\ \vdots \\ x'_n \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & & & \\ \vdots & & & \\ a_{n1} & & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_n \end{bmatrix}$$

Three times Linear Tranf

$$P'_{m\times 1} = M_{m\times m} P_{m\times 1}$$

$$P''_{m\times 1} = M_{m\times m} P'_{m\times 1}$$

$$= M_{m\times m} (M_{m\times m} P_{m\times 1})$$

$$P'''_{m\times 1} = M_{m\times m} (M_{m\times m} (M_{m\times m} P_{m\times 1}))$$

Three time Affine transform

$$P'_{n\times 1} = (M_{n\times n} P_{n\times 1}) + D_{n\times 1}$$

$$P''_{n\times 1} = M_{n\times n} (P'_{n\times 1}) + D_{n\times 1}$$

$$= M_{n\times n} (M_{n\times n} P_{n\times 1} + D_{n\times 1}) + D_{n\times 1}$$

$$= M_{n\times n} M_{n\times n} P_{n\times 1} + M_{n\times n} D_{n\times 1} + D_{n\times 1}$$

$$P'''_{n\times 1} = M_{n\times 1} (P''_{n\times 1}) + D_{n\times 1}$$

$$= M_{n\times 1} M_{n\times 1} M_{n\times 1} P_{n\times 1} + M_{n\times n} M_{n\times n} D_{n\times 1} + M_{n\times n} D_{n\times 1} + D_{n\times 1}$$

Let $[x_1\ x_2\ x_3 \ldots x_n]^T$ be a Point Let c be any Real number then $[x_1\ x_2\ x_3 \ldots x_n c]$ is an homogomus Coordinate of Point P

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} dx \\ dy \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ c' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ c \end{bmatrix} + \begin{bmatrix} dx \\ dy \\ dc \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = M_{2\times 2} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} dx \\ dy \end{bmatrix}$$

$$\begin{bmatrix} a_{11} & a_{21} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \times \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \times$$

$$\begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix}$$

$$\cancel{S_x \cos\theta} \qquad \cancel{S_x \cos\theta}$$

$$S_x \cdot \cos\theta \quad -S_y \overset{\sin\theta}{\cancel{\cos\theta}}.$$

$$S_x \sin\theta \quad S_y \cos\theta$$

$$= \begin{bmatrix} S_x \cos\theta & -S_y \sin\theta \\ S_x \sin\theta & S_y \cos\theta \end{bmatrix} \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix}$$

$$= S_x \cos\theta \cos\phi + -S_y \sin\theta \sin\phi \quad \sim$$

$$- S_x \sin\phi \cos\theta \quad - S_y \sin\theta \cos\phi$$

$$S_x \sin\theta \cos\phi + S_y \cos\theta \sin\phi \quad \sim$$

$$- S_x \sin\theta \sin\phi + S_y \cos\theta \cos\phi$$

$$\begin{bmatrix} S_x\cos\theta\cos\phi - S_y\sin\theta\sin\phi & -S_{\cancel{x}\,y}\cos\theta\sin\phi - S_{y\,x}\sin\theta \ldots \\ S_{x\,y}\sin\theta\cos\phi + S_{\cancel{y}\,x}\cos\theta\sin\phi & S_y\cos\theta\cos\phi - S_x\sin\theta\sin\ldots \end{bmatrix}$$

above answer is correct

In hear go Right to Left

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$a_{11} + a_{22} = S_x\cos\theta\cos\phi - S_y\sin\theta\sin\phi + S_y\cos\theta\cos\phi - S_x\sin\theta\sin\phi$$

$$S_x(\cos\theta\cos\phi - \sin\theta\sin\phi) + S_y(-\cos\theta\cos\phi - \sin\theta\sin\phi)$$

$$S_x(\cos(\theta-\phi)) - S_y(\cos(\theta-\phi))$$

$$\boxed{a_{11} + a_{22} \quad (S_x - S_y)(\cos(\theta-\phi))}$$

$$S_x \cos\theta \cos\phi - S_y \sin\theta \sin\phi + S_x \sin\theta \sin\phi - S_y \cos\theta \cos\phi$$

$$S_x (\cos\theta \cos\phi + \sin\phi \sin\theta) + S_y (-\sin\theta \sin\phi - \cos\theta \cos\phi)$$

$$S_x (\cos(\theta+\phi)) - S_y (\cos(\theta+\phi))$$

$$\boxed{a_{11} - a_{22} \quad (S_x - S_y)(\cos(\theta+\phi))}$$

$$\boxed{a_{12} + a_{21} = (S_y - S_x) \overset{\sin}{\cancel{\cos}} (\theta - \phi)}$$

$$\boxed{a_{12} - a_{21} = (-S_x - S_y)(\sin(\theta+\phi))}$$

$$\boxed{\begin{aligned} \tan(\theta+\phi) &= \frac{a_{21} - a_{12}}{a_{11} + a_{22}} \\ \tan(\theta-\phi) &= \frac{a_{12} + a_{21}}{a_{22} - a_{11}} \end{aligned}}$$

Let R is Point in 2D Plane P is any arbitrary Point in Plane and by Apply function ($F(P) = P'$) new Point is reaceving.

$$P = (x, y)$$

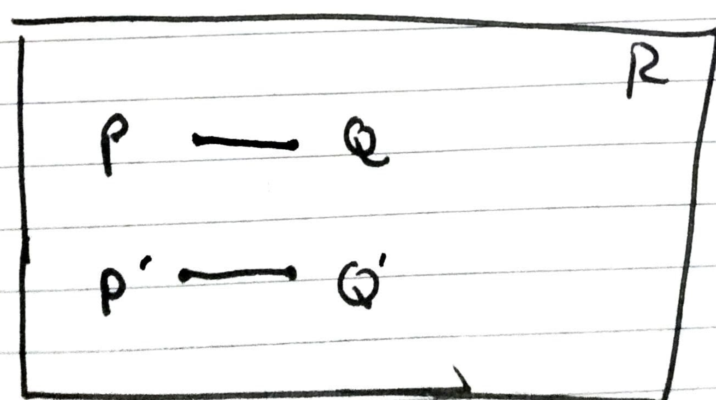
$$F(P) = (F(x), F(y))$$

$$F(P) = P'$$

if transformation $\Leftrightarrow$

$$d(P, Q) = d(P', Q')$$

is distance Preservesing. transformation



- Scaling is not distance Preserving.

Oritantion Preserveing // Reverseing

P Q R are any three non-coliner Points

P

R

Q

~~PQR~~

P → Q → R Clockwise

F is uclidabl transformation.
oritantion Preserving.

F(P), F(Q), F(R) Results are also Clockwise

~~Oritan~~ oriantation Reverseing

F(P), F(Q), F(R) Results are Counter Clockwise

* If F is Ulicadian oritamition. Presenving. then it is call Rigit F

| |. of three Point

$$\begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ 1 & 1 & 1 \end{vmatrix}$$

if it gives ~~Positive~~ Negative Result then Viwer sitting on z^+ direction Shows Cot Clockwise

if it gives ~~Negative~~ Positive Result then Viwer Sitting on z^+ direction Shows Counter Clock wise

if it gives zero then these Points are co-linear

Let P Q R any three Points in x-y Plane

Let F be Regit angle Preserving transformation on P Q R

P Q R

F(P) F(Q) F(R)

are on same angle after the transformation

$\angle P$ $\angle Q$ $\angle R$ is same as $\angle P'$, $\angle Q'$ $\angle R'$

translation & Rotation are Rigid and angle preserving

$\boxed{\text{Reflection is not angle preserving}}$

if Rigid $\Rightarrow$ translation
rotation

uclidian $\Rightarrow$ translation
rotation
Reflection
} Cascade of three

$\boxed{\text{Rigid} \subset \text{uclidian} \subset \text{Afine}}$

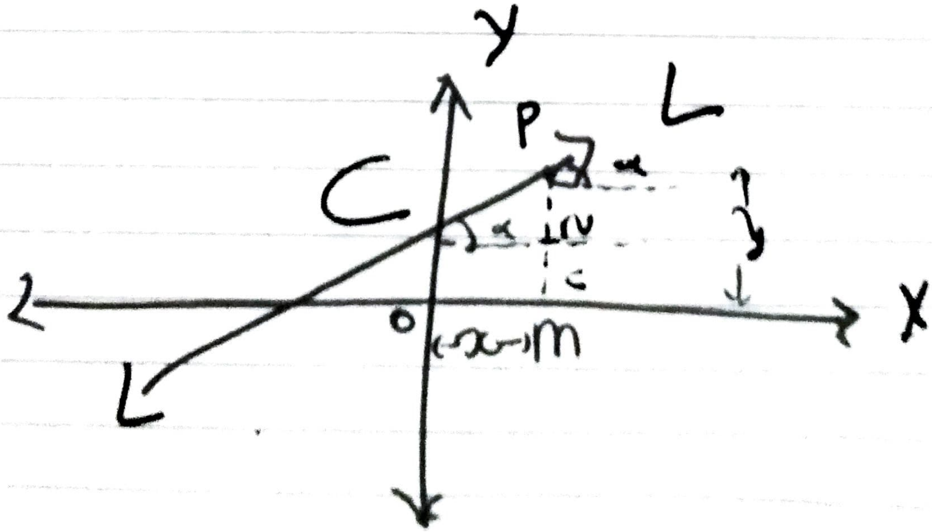
P, Q & R Three Points where R at origin.

$$\begin{vmatrix} x_1 & x_2 & 0 \\ y_1 & y_2 & 0 \\ 1 & 1 & 1 \end{vmatrix}$$

$$x_1(y_2) - x_2(y_1)$$

$$x_1 y_2 - x_2 y_1$$

Eq of Line



$$PM = PN + NM$$

$$y = PN + c$$

$$\boxed{y = x\tan\alpha + c}$$

$$\tan\alpha = \frac{PN}{CN}$$

$$PN = CN \tan\alpha$$
$$= OM \tan\alpha$$
$$\boxed{PN = x\tan\alpha}$$

where $\tan\alpha = m$

$$\boxed{y = mx + c}$$

This is Eq of 2D straight Line.

2D curve : two unknown variables in equation which has true relation for any arbitrary point on that curve is called eq to curve

Line will intersect either one or both

~~or~~
if Both then

$$OM = x \text{ intersect}$$

$$ON = y \text{ intersect}$$

A Line in 2D system can be ~~un~~ ~~the~~.
intersect y in c and any Point P ~~eo~~ except c which made angle α with x axis is unique.

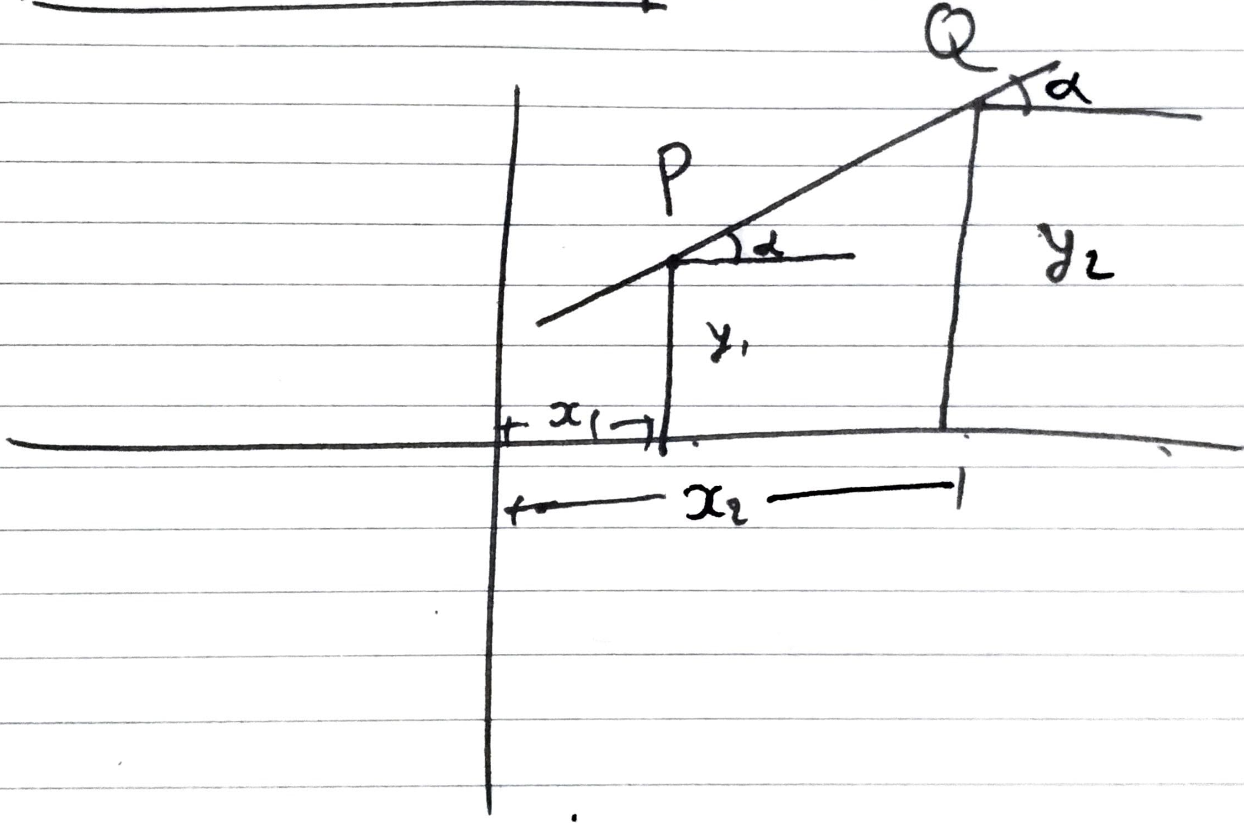
Let $c = y$ intersec

$\alpha =$ angle w.r.t +x axis

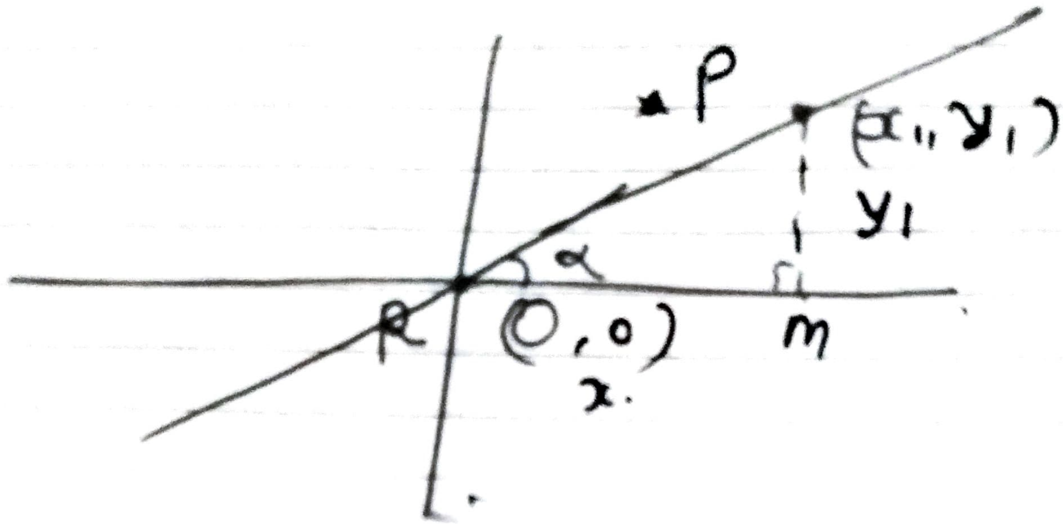
This eq is in term of α and c For any arbitrary point P which is on Line.

angle α is ~~measure~~ measure of slope of line

$$\boxed{\tan\alpha = \text{Slope}}$$



$$\boxed{\tan\alpha = \frac{QM}{PM} = \frac{y_2 - y_1}{x_2 - x_1}}$$



$$\tan\alpha = \frac{y_1}{x_1}$$

$$y = \left(\frac{y_1}{x_1}\right)x + c$$

$\boxed{\text{where } c = 0}$

Line which pass through origin

$$y(x_1) = y_1(x)$$

$$\boxed{y x_1 - y_1 x = 0}$$

Here in above eq.

x is remain Same and y is Variable

eq > 0 $y \uparrow$ $\Rightarrow$ P Q R anticlock

eq < 0 $y \downarrow$ $\Rightarrow$ P Q R clock

P $(x_1, y_1) \to (x_1 - x_3, y_1 - y_3)$ (P-R)

Q $(x_2, y_2) \to (x_2 - x_3, y_2 - y_3)$ (Q-R)

R (x_3, y_3) $(0, 0)$ (R-R)

① $\begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ 1 & 1 & 1 \end{vmatrix}$

$$= x_1(y_2 - y_3) - x_2(y_1 - y_3) + x_3(y_1 - y_2)$$

$$= x_1 y_2 - x_1 y_3 + x_2 y_3 - x_2 y_1 + x_3 y_1 - x_3 y_2 \quad \text{--- (1)}$$

$$\begin{vmatrix} x_1 - x_3 & y_1 - y_3 & 0 \\ x_2 - x_3 & y_2 - y_3 & 0 \\ 1 & 1 & 1 \end{vmatrix}$$

mistake

$$= (x_1 - x_3)(y_2 - y_3) - (y_1 - y_3)(x_2 - x_3)$$

$$= x_1 y_2 - x_1 y_3 - x_3 y_2 + \cancel{x_3 y_3} - y_1 x_2 + y_1 x_3 + y_3 x_2 - \cancel{y_3 x_3}$$

E P 0 8

$$\begin{vmatrix} x_1 - x_3 & x_2 - x_3 & 0 \\ y_1 - y_3 & y_2 - y_3 & 0 \\ 1 & 1 & 1 \end{vmatrix}$$

$$(x_1 - x_3)(y_2 - y_3) - (x_2 - x_3)(y_1 - y_3)$$

$$= x_1 y_2 - x_1 y_3 - x_3 y_2 + \cancel{x_3 y_3} - y_1 x_2 + y_1 x_3 + y_3 x_2 - \cancel{y_3 x_3}$$

$$= x_1 y_2 - x_1 y_3 - x_3 y_2 - y_1 x_2 + y_1 x_3 + y_3 x_2$$

$$R(x_3, y_3) \longrightarrow R'(x_3 + dx, y_3 + dy)$$

$$P(x_1, y_1) \longrightarrow P'(x_1 + dx, y_1 + dy)$$

$$Q(x_2, y_2) \longrightarrow Q'(x_2 + dx, y_2 + dy)$$

BEFORE	AFTER
$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	P.T.O

$$D = \begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ 1 & 1 & 1 \end{vmatrix} \begin{matrix} = 0 \\ > 0 \\ < 0 \end{matrix}$$

$$= \sqrt{(x_2 + dx - x_1 - dx)^2 + (y_2 + dy - y_1 - dy)^2}$$

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\begin{vmatrix} x_1 + dx & x_2 + dx & x_3 + dx \\ y_1 + dy & y_2 + dy & y_3 + dy \\ 1 & 1 & 1 \end{vmatrix}$$

This is distance Preserving.

Now for ~~oartion~~ & orientation Preserving

$$= \frac{dx}{dx} \begin{vmatrix} x_1 + dx & x_2 + dx & x_3 + dx \\ y_1 + dy & y_2 + dy & y_3 + dy \\ 1 & 1 & 1 \end{vmatrix}$$

$$= \frac{1}{dx} \begin{vmatrix} x_1 + dx & x_2 + dx & x_3 + dx \\ y_1 + dy & y_2 + dy & y_3 + dy \\ dx & dx & dx \end{vmatrix}$$

$$= \frac{1}{dx}\begin{vmatrix} x_1 & x_2 & x_3 \\ y_1+dy & y_2+dy & y_3+dy \\ dx & dx & dx \end{vmatrix} \quad R_1 = R_1 - R_3$$

$$= \begin{vmatrix} x_1 & x_2 & x_3 \\ y_1+dy & y_2+dy & y_3+dy \\ \cancel{dx}\,1 & \cancel{dx}\,1 & 1 \end{vmatrix}.$$

$$= \frac{dy}{dy}\begin{vmatrix} x_1 & x_2 & x_3 \\ y_1+dy & y_2+dy & y_3+dy \\ 1 & 1 & 1 \end{vmatrix}.$$

$$= \frac{1}{dy}\begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ dy & dy & dy \end{vmatrix}. \quad R_2 = R_2 - R_3$$

$$= \frac{\cancel{dy}}{\cancel{dy}}\begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ 1 & 1 & 1 \end{vmatrix}$$

Here D BEFORE and AFTER is same

Hence Orientation Preserving.

Theorem: Rotation is Rigid Transformation

08.00

$$(x_1, y_1) \to \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$$

09.00

10.00

$$x_1\ P' \begin{bmatrix} x_1\cos\theta - y_1\sin\theta \\ x_1\sin\theta + y_1\cos\theta \end{bmatrix}$$

11.00

12.00

$$(x_2, y_2) \to \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$$

01.00

02.00

$$Q' \begin{bmatrix} x_2\cos\theta - y_2\sin\theta \\ x_2\sin\theta + y_2\,\cancel{\sin}\cos\theta \end{bmatrix}$$

03.00

04.00

$$\boxed{d(P, Q) = d(P', Q')}$$

05.00

06.00

$$P(x_1, y_1) = P'(x_1\cos\theta - y_1\sin\theta,\ x_1\sin\theta + y_1\cos\theta)$$

07.00

08.00

$$Q(x_2, y_2) = Q'(x_2\cos\theta - y_2\sin\theta,\ x_2\sin\theta + y_2\cos\theta)$$

$$d(P,Q) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \quad \cdots \text{①}$$

08.00

09.00

$$d(P', Q') = \sqrt{(x_1\cos\theta - y_1\sin\theta - x_2\cos\theta - y_2\sin\theta)^2 + (x_1\sin\theta + y_1\cos\theta - x_2\sin\theta - y_2\sin\theta)^2}$$

10.00

11.00

12.00

by simplification it is proved

01.00

02.00

$$= \sqrt{(x_2 - x_1)^2 (\cos^2\theta + \sin^2\theta) + (y_2 - y_1)^2 (\sin^2\theta + \cos^2\theta)}$$

03.00

04.00

$$= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

05.00

06.00

$$\boxed{d(P', Q') = d(P, Q)}$$

07.00

08.00

$$P(x_1, y_1) \rightarrow P' = \begin{pmatrix} x_1 \cos\theta - y_1 \sin\theta, \\ x_1 \sin\theta + y_1 \cos\theta \end{pmatrix}$$

$$Q(x_2, y_2) \rightarrow Q' \begin{pmatrix} x_2 \cos\theta - y_2 \sin\theta, \\ x_2 \sin\theta + y_2 \cos\theta \end{pmatrix}$$

$$R(x_3, y_3) \rightarrow R' \begin{pmatrix} x_3 \cos\theta - y_3 \sin\theta, \\ x_3 \sin\theta + y_3 \cos\theta \end{pmatrix}$$

Before

$$D = \begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ 1 & 1 & 1 \end{vmatrix}$$

After

$$D = \begin{vmatrix} x_1 \cos\theta - y_1 \sin\theta & x_2 \cos\theta - y_2 \sin\theta & x_3 \cos\theta - y_3 \sin\theta \\ x_1 \sin\theta + y_1 \cos\theta & x_2 \sin\theta + y_2 \cos\theta & x_3 \sin\theta + y_3 \cos\theta \\ 1 & 1 & 1 \end{vmatrix}$$

$$= \frac{1}{\cos\theta \sin\theta} \begin{vmatrix} x_1\cos^2\theta - y_1\cos\theta\sin\theta & x_2\cos^2\theta - y_2\cos\theta\sin\theta & x_3\cos^2\theta - y_3\cos\theta\sin\theta \\ x_1\sin^2\theta + y_1\cos\theta\sin\theta & x_2\sin^2\theta + y_2\cos\theta\sin\theta & x_3\sin^2\theta + y_3\cos\theta\sin\theta \\ 1 & 1 & 1 \end{vmatrix}$$

R_1 multiply by $\cos\theta$
R_2 multiply by $\sin\theta$

$$R_1 = R_1 + R_2$$

$$R_2 = R_1 + R_2$$

by this simplification we get

$$D = \begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ 1 & 1 & 1 \end{vmatrix}$$

Hence it is prove

For Reflection

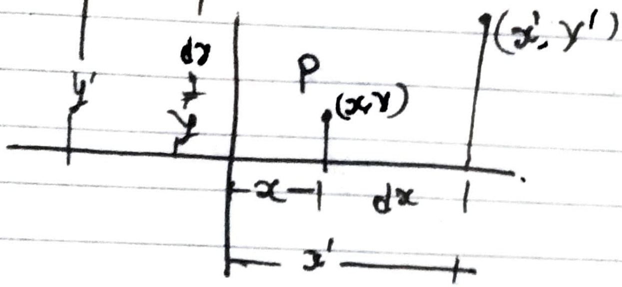
$$P(x_1, y_1) = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Prove $d(P, Q) = d(P', Q')$

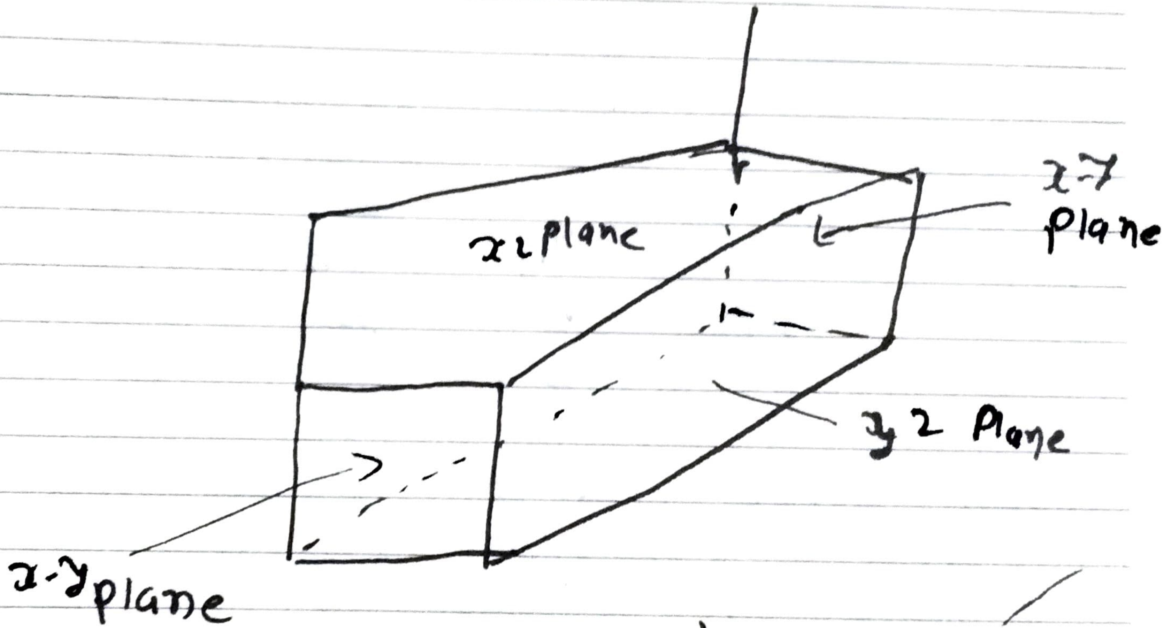
and

$$D = \begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_1 \\ 1 & 1 & 1 \end{vmatrix} \leftarrow \text{Before}$$

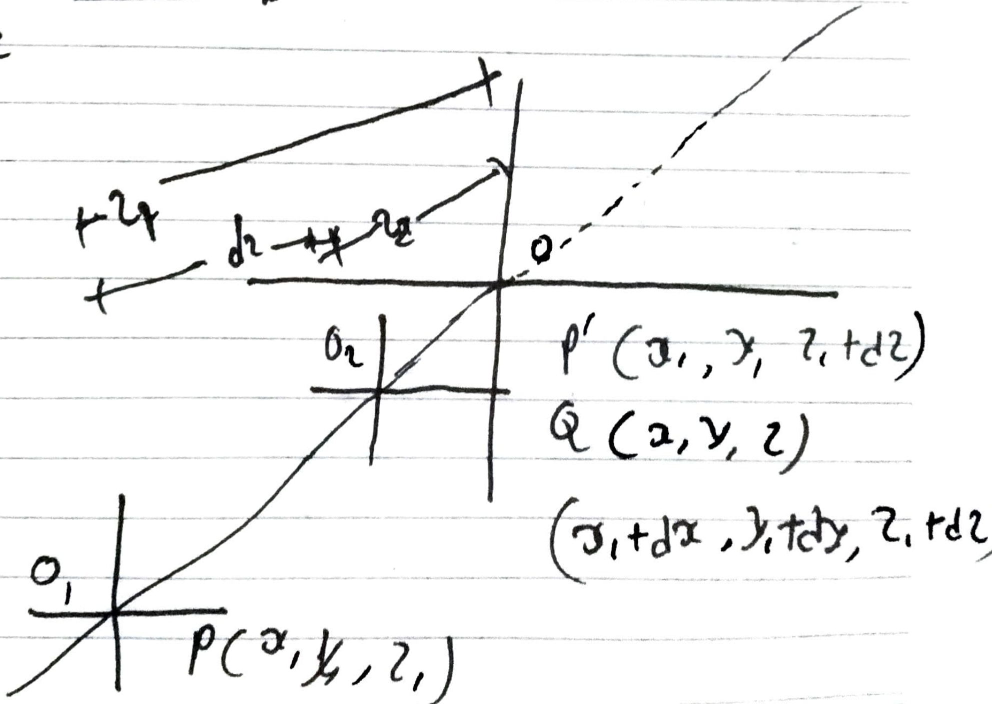
After is Prove same as be Previous Page.



09.00
10.00
11.00
12.00



01.00
02.00
03.00
04.00
05.00
06.00
07.00
08.00



OCT 08

$\forall P, Q \in R^3 \quad \exists (dx, dy, dz) \in R^3$

08.00

$Q.x = P.x + dx$

09.00

$Q.y = P.y + dy$

10.00

$Q.z = P.z + dz$

1.00

$$\begin{matrix} x_1 & x_2 & x_3 & dx \\ y_1 & y_2 & y_3 & dy \\ z_1 & z_2 & z_3 & dz \\ 1 & 1 & 1 & \end{matrix}$$

Homogeneous matrix

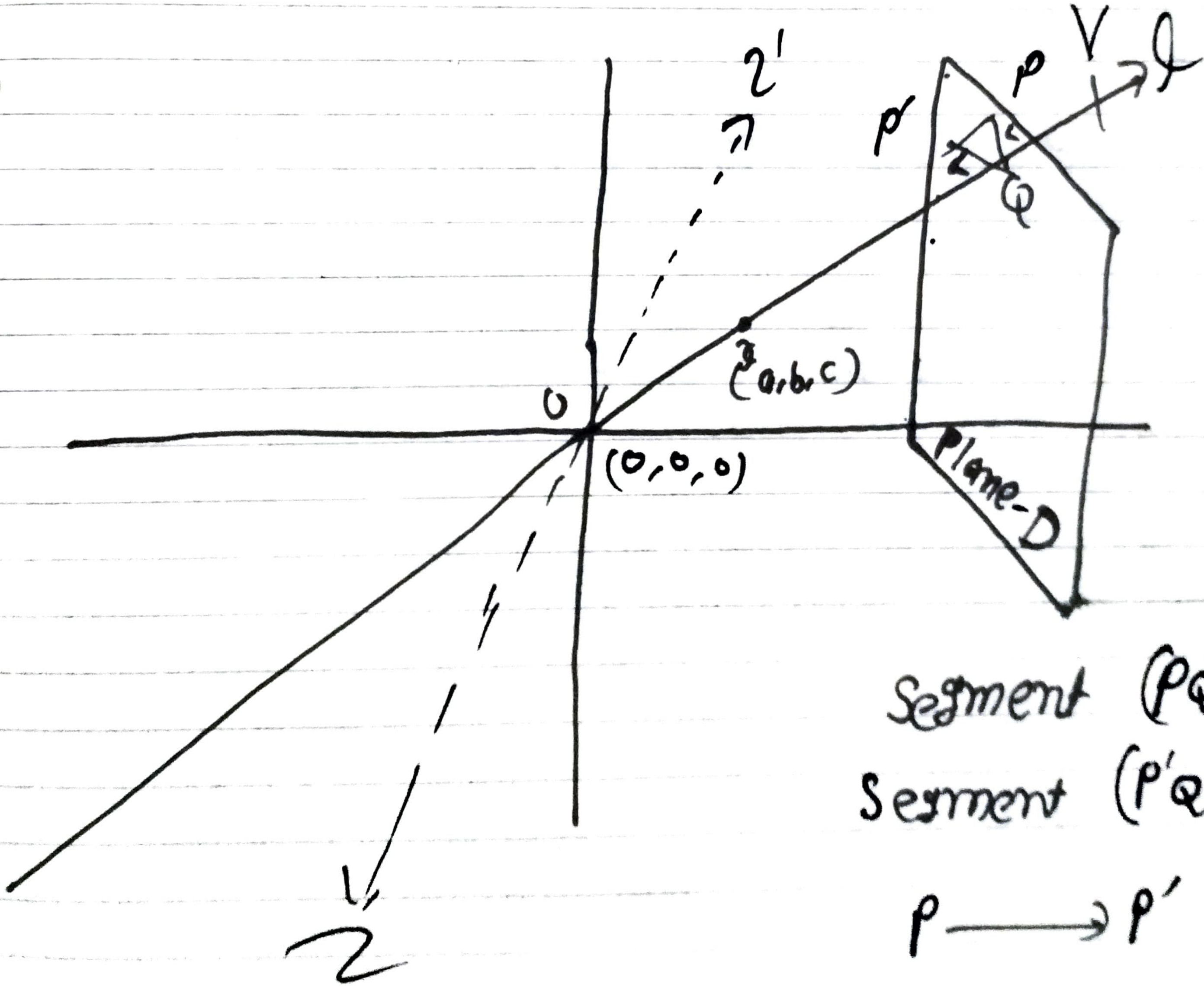
$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & dx \\ 0 & 1 & 0 & dy \\ 0 & 0 & 1 & dz \\ & & & \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ \end{bmatrix}$$

$$\begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Scaling matrix

$P(x, y, z)$

3D rotation



Segment $(PQ) = L$

Segment $(P'Q) = L$

$P \longrightarrow P'$

Charatarizing 3D roation

Let us

Consider 3D carti system Seteled at O

Let X (a, b, c) be any arbitrary point in space which is not origin

Let line L be a Line Passig through the origin. and Point X

Let P be any arbitory Point in Space.

We have to define rotation of P with line L as an axis of rotation.

Case - I

P is on Line L

Point will not move From its Position during the rotation

[Case-2]

P is not on Line L

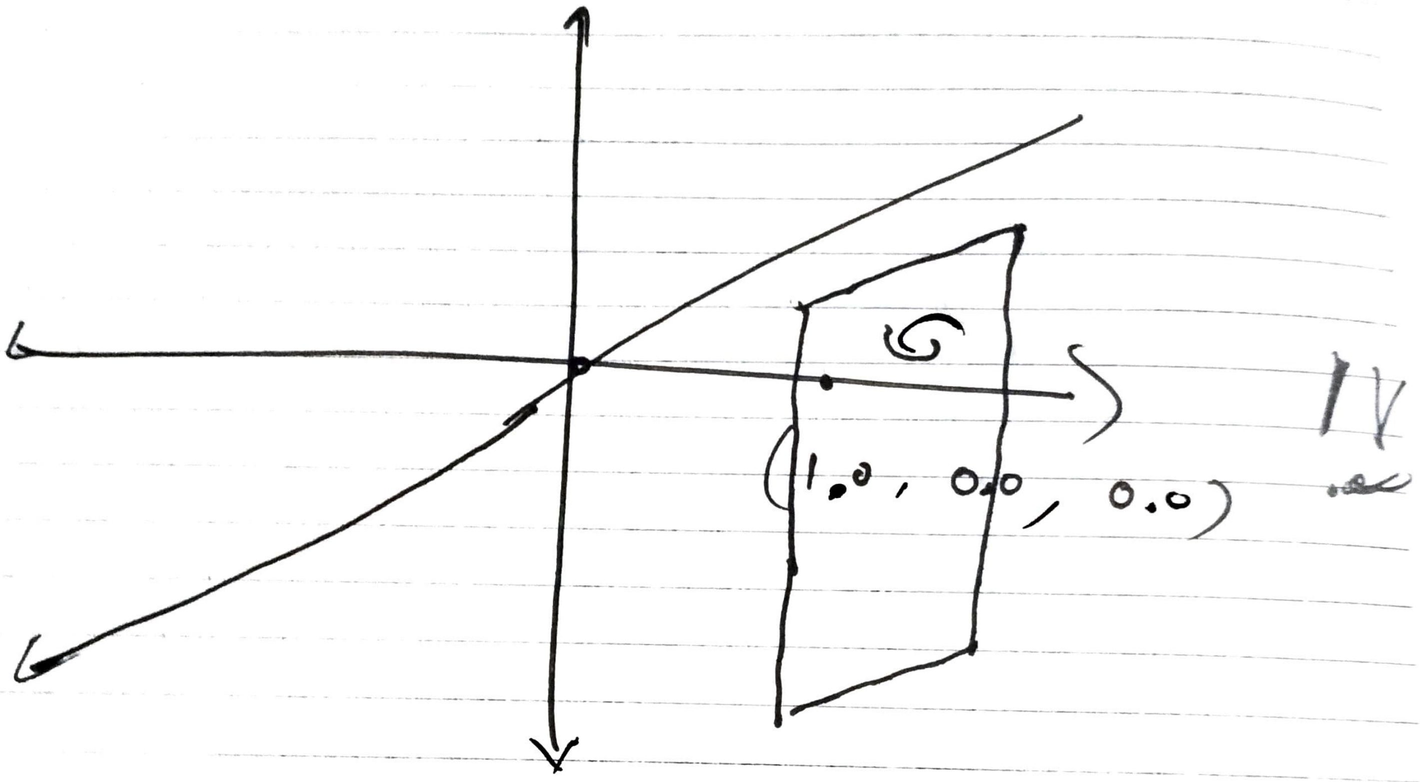
drop a Perpendicular from Point P to line L and Consider a D Plane intersecting which intersect Line at Q and contains Perpendicular QP in it.

Now, Consider observer V on L who is situated at its Positive end. and is at Long enough distance to be able to view segment (QP) entirely.

V observing to origin.

Now, rotate (QP) in Plane D around Point Q such that ~~it~~ rota appear counter clock wise to the define observer.

let (QP') be the final Position of (QP) in Plane D So the angle between (QP) and (QP') in Counter Clock wise sence w.r.t defined observer is the angle of rotation.



$$R_x(\theta) \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$ For y

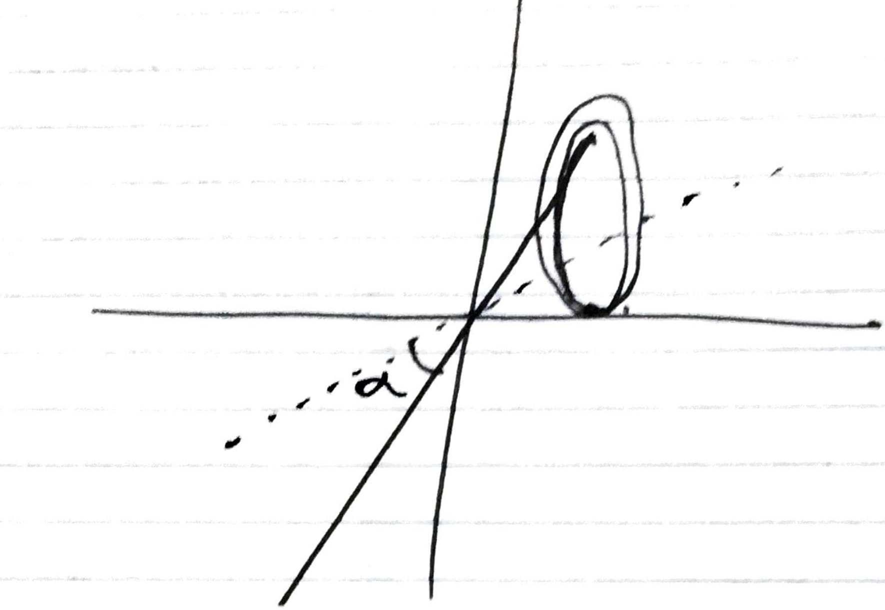
$$R_y\theta = \begin{bmatrix} \cos\theta & 0 & -\sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ \sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$R_z\theta \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

08.00

09.00

10.00



11.00

12.00

01.00

02.00 ~~R_θ $-\alpha$~~ $R_y(-\alpha)\ R_z(\theta)\ R_y(\alpha)$ $\begin{bmatrix} x \\ y \\ 0 \\ 1 \end{bmatrix}$

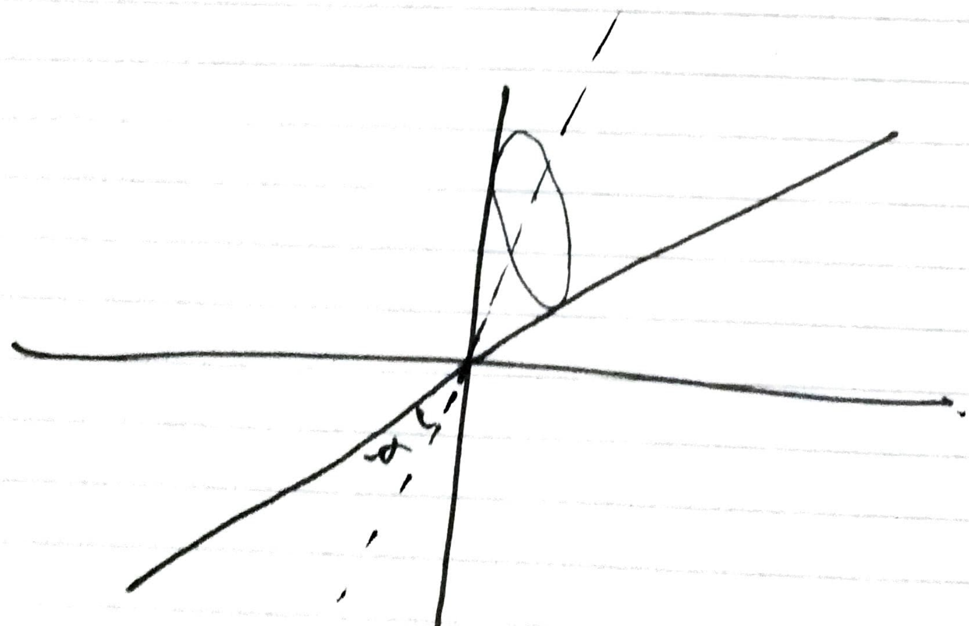
03.00

04.00

05.00

06.00

07.00



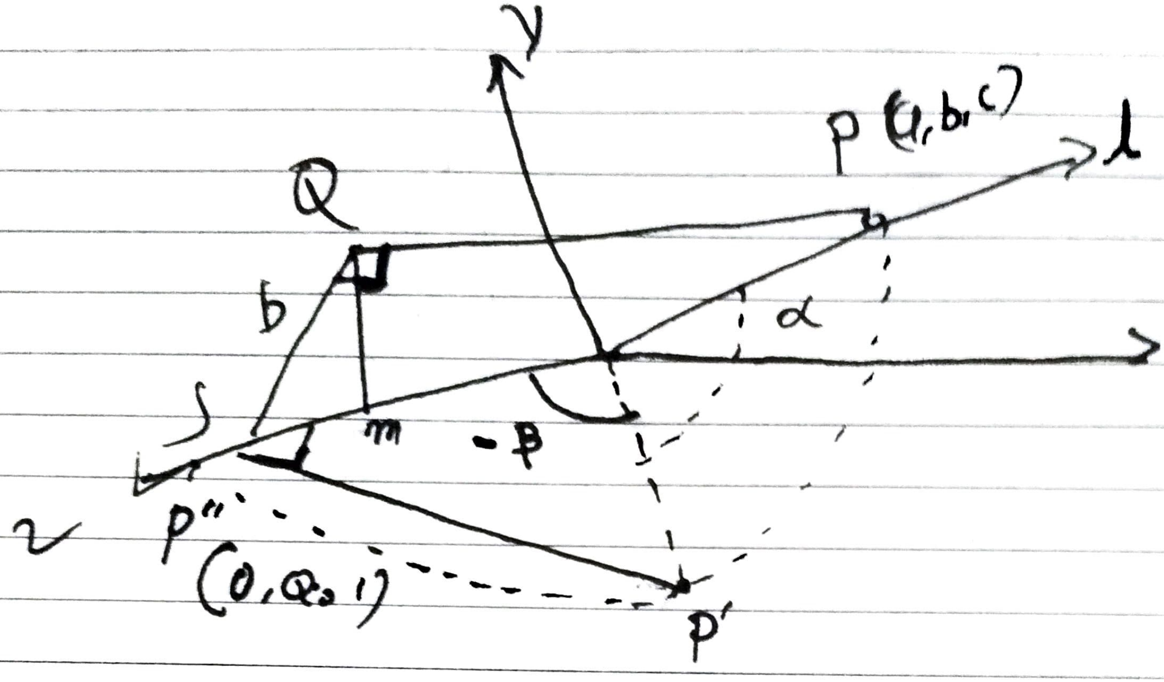
08.00

$$\cancel{R_y(-\alpha)\ R_z(\theta)\ R_y(\alpha)}.$$

$$R_y(\alpha)\ R_z(\theta)\ R_y(-\alpha)\begin{bmatrix} x \\ y \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos\alpha & 0 & -\sin\alpha & 0 \\ 0 & 1 & 0 & 0 \\ \sin\alpha & 0 & \cos\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} +\cos\alpha & 0 & \sin\alpha & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\alpha & 0 & +\cos\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ 0 \\ 1 \end{bmatrix}$$



$yz = x$ Perpendicular axis

$zx = y$

$xy = z$

$$R_y(-\alpha)\, R_y(\beta)\, R_z(\theta)\, R_y(-\beta)\, R_x(\alpha) \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$l(PQ)$ Distance from $xz = a$

$l(OS)$ Distance from $xy = c$

$l(QM)$ Distance from $x_2 = b$

$l(OP') = l(OQ) = d$

$l(SP') = b$

$l(OS) = c$

$$d = \sqrt{b^2 + c^2}.$$