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Tetrad Formalism and Reference Frames in General Relativity

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Abstract—This review is devoted to problems of defining the reference frames in the tetrad formalism of General Relativity. Tetrads are the expansion coefficients of components of an orthogonal basis over the differentials of a coordinate space. The Hamiltonian cosmological perturbation theory is presented in terms of these invariant differential forms. This theory does not contain the double counting of the spatial metric determinant in contrast to the conventional Lifshits–Bardeen perturbation theory. We explicitly write out the Lorentz transformations of the orthogonal-basis components from the cosmic microwave background (CMB) reference frame to the laboratory frame, moving with a constant velocity relative to the CMB frame. Possible observational consequences of the Hamiltonian cosmological perturbation theory are discussed, in particular, the quantum anomaly of geometric interval and the shift of the origin, as a source of the CMB anisotropy, in the course of the universe evolution.

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CONTENTS

1. INTRODUCTION

Since Newtonian times, theoretical physics has been dealing with the laws of nature referred to as equations of motion and some initial data needed to solve these equations unambiguously. The initial data are to be measured by a set of devices identified with a reference frame. The guiding principle of modern physical theories is to define the transformation groups of reference frames (treated as manifolds of initial data) that preserve the equations of motion. The Galilei group in Newtonian mechanics and the Lorentz–Poincaré group in Special Relativity (SR) can serve as such examples.

Thus, it is important to establish a sense of the group of general coordinate transformations in General Relativity (GR) [1]. The group of general coordinate transformations was assumed by A. Einstein to be a direct generalization of the Poincaré group, in which inertial reference frames are substituted by "nonuniformly moving reference frames" [2].

However, such an interpretation of general coordinate transformations was then revised within the framework of a new geometric principle (developed by H. Weyl and V.A. Fock) of constructing physical theories, namely, the gauge symmetry principle. Whereupon, GR stimulated the discovery of gauge symmetry.[1]

Fock noted that the group of general coordinate transformations in GR took quite a different physical role than the Poincaré group in SR [6]. Fock's idea was then developed further by Yang and Mills [7, 8], Utiyama [9], Kibble [10], and others. They substantiated the modern understanding of the role of gauge transformations, in particular, general coordinate transformations in GR. According to this understanding, the local gauge symmetry significantly differs from the symmetry of reference frames, which is associated with a set of initial data and integrals of motion. While additional symmetry of reference frames leads to new initial data; i.e., to an increase in its number, the observation of local gauge symmetry results in new conditions imposed on the initial data, thereby decreasing its number.[2] As was shown in [14], these consequences, following the invariance of field theory with respect to gauge transformations of coordinates and field functions, were formulated in Hilbert's report "Foundations of Physics" [15, 16], presented November 20, 1915, to the Göttingen Mathematical Society. The GR action was first written out in this report, and the GR equations were derived by variation of the action. In addition, Hilbert first formulated the theorem that was later referred to as the second Noether theorem [17–19]. This theorem leads to the interpretation of general coordinate transformations as gauge ones and, therefore, to all the consequences concerning both a decrease in the number of independent degrees of freedom [20] and the appearance of constraints imposed on initial data [14].

It can be said that principles of modern physical theories were first formulated in Hilbert's paper "Foundations of Physics" [15, 16]. According to these principles, the basic notions of classical physics, namely, action functional, symmetry of initial data, and equations of motion, should be completed with "geometric notions" concerning invariant interval, gauge symmetry, and constraint equations for initial data, respectively. These principles include the Dirac's definition of observables [11], in particular, initial data varying under transformations of reference frames, as invariants of gauge transformations. For gauge-invariant observables to be defined and, therefore, the gauge arbitrariness to be removed in solutions of equations, gauge (i.e., general coordinate) transformations must be separated from transformations of reference frames.

The problem of separating general coordinate transformations from relativistic transformations of reference frames in GR was solved by Fock [21], by introducing an orthogonal basis and physical quantities forming a Lorentz group representation. The Maurer–Cartan linear differential forms serve as such quantities and describe the motion of orthogonal bases in the physical space of events. The components of a tetrad defined as the square root of a metric are expansion coefficients of the Maurer–Cartan forms in coordinate differentials. The Maurer–Cartan differential forms, being invariants of general coordinate transformations, serve as measurable geometric quantities in the physical space. In this case, the integrable noninvariant coor-

[1] Since 1918, Weyl [3] had been trying to find geometric principles to formulate electrodynamics. Weyl's attempts were crowned with success [4] only after the discovery of quantum mechanics. The redefinition of quantum-mechanical momentum operator in the presence of an electromagnetic field was the starting point of Weyl's geometric principle of electrodynamics (presently known as a local gauge symmetry principle). Such a redefinition was given by Fock in 1926 [5].

[2] For example, see the relationship between gauge transformations and relativistic transformations of reference frames in quantum electrodynamics [11–13].

dinate differentials are treated as auxiliary mathematical quantities similar to electromagnetic potentials.

A great number of papers were devoted to the choice of reference frame in GR (see, for example, monograph [22] and references therein). In this review, we consider only the reference frames used for describing the evolution of metric and fields in the form of the Hamiltonian dynamics [20, 23, 24]. This class of reference frames was defined by Dirac [23] as a bundle of four-dimensional coordinate manifold of spacetime into a set of nonoverlapping spacelike hypersurfaces. The group of general coordinate transformations conserving such a bundle was found by Zel'manov, who referred to it as a group of kinemetric transformation; this group involves the reparametrization subgroup of coordinate time. This means that the coordinate time, which is not invariant with respect to gauges, is not observable. Therefore, one encounters the problem of setting physical gauge-invariant variables and observables (including time, energy, and interaction potentials) in GR for a specific reference frame [26–28]. A method of solving similar problems in gauge field theory was outlined by Dirac [11–13] as the Hamiltonian reduction. The method involves the calculation of action and other quantities on a constraint surface in order to separate out the dynamical essence of the theory from extra variables and gauge arbitrariness.

This review is devoted solely to the Hamiltonian reduction in GR problems, which is treated as a method of defining physical gauge-invariant variables and observables in a specific reference frame. These observables are to be used for constructing quantum gravity defined as the first and second quantization of energy constraint. We will compare the gauge-invariant method of describing field dynamics [26–28] to some noninvariant methods, similar to island universe [20] or so-called global-time theory [29] assuming the coordinate time is not observable.

The contents of the paper are as follows. In Section 2, we consider the notion of reference frame in mechanics and the Hamiltonian reduction of relativistic mechanics with constraints onto an equivalent system without constraints. Section 3 is devoted to the Hamiltonian reduction in GR in the homogeneous approximation. In Section 4, we present a review of papers (Dirac, Zel'manov, Lichnerowicz *et al.*) devoted to defining reference frames in GR in order to formulate the Hamiltonian reduction in terms of gauge-invariant observables in a specific reference frame and to apply these results to topical problems of modern cosmology. Reference frames in GR are treated in Section 5 in the tetrad formalism to construct the Hamiltonian cosmological perturbation theory and its Lorentz transformations. We here use the system of units in which $c = \hbar = 1$.

2. REFERENCE FRAMES IN MECHANICS

2.1. Newtonian Mechanics

First we'll review the initial notions by using a simple example of one-dimensional Newtonian mechanics, defined by the action functional

$$S_L = \int dt L(X(t), dX(t)/dt),$$
$$L(X(t), dX(t)/dt) = \left[\frac{dX(t)}{dt}\right]^2 \frac{m}{2}. \qquad (1)$$

Here, $x(t)$ is the variable describing a particle trajectory, t is the time coordinate, and m is the mass treated as a fundamental parameter of the theory. Variation of action (1), $\delta S_L = 0$, under given boundary conditions $\delta X(t_0) = \delta X(t_1) = 0$, yields the equation of motion

$$m\frac{d^2X(t)}{dt^2} = 0. \qquad (2)$$

The general solution

$$X(t) = X_I + \frac{P_I}{m}(t - t_I) \qquad (3)$$

of this equation depends on the initial data $X(t_I) = X_I$ and $dX(t_I)/dt_I = P_I/m$ given at the time t_I. The initial data are to be measured by a set of physical devices (in this example, by a ruler and a watch relative to a fixed spacetime point) and to be associated with a reference frame. The reference frames that move with constant relative velocities are referred to as inertial frames. The transformation $X \mapsto \tilde{X} = X + X_g + v_g(t - t_I)$ turns a fixed reference frame with its origin at the point $X(t_I) = X_I$ into the reference frame moving with the velocity v_g and with its origin at $X_g(t_I) = X_I + X_g$. This transformation group for reference frames in Newtonian mechanics is referred to as the Galilei group. Equations (2) are independent of initial data and, therefore, reference frame. The independence of the equations, treated as the laws of nature, on initial data is referred to as the relativity principle [30–32].

In the Hamiltonian approach, action (1) takes the form

$$S_H = \int dt\left[P(t)\frac{dX(t)}{dt} - H\right]. \qquad (4)$$

Here, $P(t)$ is the momentum, $\{P, X\}$ is the phase space,

$$H(P) = \frac{P^2}{2m} \qquad (5)$$

is the Hamiltonian function, and its value on a trajectory is the energy $E = H(P_I)$. Variation of action (4) yields the first-order equations

$$P(t) = m\frac{dX(t)}{dt}, \quad \frac{dP(t)}{dt} = 0, \qquad (6)$$

rather than second-order equations (2). According to Newtonian mechanics, all observers in different reference frames use the same absolute time t.

2.2. Special Relativity As a Model of General Relativity

2.2.1. Relativistic mechanics as a consequence of electrodynamics. As was shown above, the notion of the spatial coordinates X_i (i = 1, 2, 3) in Newtonian mechanics as dynamical variables is clearly separated from absolute time t, treated as an evolution parameter.

Relativistic mechanics is based on the Faraday–Maxwell electrodynamics symmetry group found by Lorentz and Poincaré. The time $t = X_{(0)}$ and spatial coordinates $X_{(i)}$ (i = 1, 2, 3) are treated in this group as a unified space of events or the Minkowski spacetime $X_{(\alpha)}$ ($\alpha = 0, i$) with the scalar product of any pair of vectors ($A_{(\alpha)}B_{(\alpha)} = A_{(0)}B_{(0)}B - A_{(i)}B_{(i)}$) [32].

Relativistic particles are described in SR by the action

$$S_{\mathrm{SR}} = -m\int d\tau\sqrt{\left(\frac{dX_{(\alpha)}}{d\tau}\right)^2}. \tag{7}$$

This action is invariant with respect to the transformations $\bar{X}_{(\alpha)} = X_{(\alpha)_p} + \Lambda_{(\alpha)(\beta)}X_{(\beta)}$ of the Poincaré group, which is the transformation group of reference frames. Its subgroup of rotations $\Lambda_{(\alpha)(\beta)}X_{(\beta)}$ is referred to as the Lorentz group. Fixing the indices (0), (i) in this space of events $[X_{(0)}|X_{(i)}]$ implies the choice of a specific Lorentz reference frame.

It should be noted that Special Relativity contains a new symmetry with respect to the transformations that do not change the initial data; namely, action (7) is invariant with respect to the reparametrization of the coordinate evolution parameter

$$\tau \longrightarrow \tilde{\tau} = \tilde{\tau}(\tau). \tag{8}$$

This results in originating constraints between the variables. This transformation group is referred to as a gauge group, while the quantities invariant with respect to gauge transformations are called observables. The geometric time interval

$$s(\tau) = \int_0^\tau d\tilde{\tau}\sqrt{\left(\frac{dX_{(\alpha)}}{d\tilde{\tau}}\right)^2} \tag{9}$$

on the world line of a particle in the space of events $X_{(\alpha)}$ can be taken as such an observable that is invariant with respect to the time reparametrization. This interval is measured by a comoving observer. The time variable of the space of events $X_{(0)}$ is the time measured by an external observer.

The goal of the theory is to solve the equations describing trajectories in the space of events in terms of gauge invariants.

There are the two methods of describing relativistic particles in literature. The first one, based on constraint-free systems, was proposed by Poincaré and Einstein in 1904–1905 [33, 34]. The second method was formulated by Hilbert and Einstein in 1915 in terms of systems with constraints [1, 15].

2.2.2. Poincaré–Einstein dynamics of a relativistic particle. Any reference frame in SR is defined by a unit timelike vector $l_{(\mu)}$, with $l^2_{(\mu)} = l^2_{(0)} - l^2_{(i)} = 1$, which will be referred to as a time axis. These vectors form a complete set of the Lorentz reference frames. The time in each frame is defined in Minkowski space $X_{(\mu)}$ as a scalar product of the time axis vector and the coordinate: $X_{(0)} = l_{(\mu)}X_{(\mu)}$. The spatial coordinates are defined on the three-dimensional hypersurface $X^{\perp}_{(\mu)} = X_{(\mu)} - l_{(\mu)}(l_{(\nu)}X_{(\nu)})$ perpendicular to the time axis.

Without loss of generality, the time axis can be chosen in the form $l_{(\mu)}$ = (1, 0, 0, 0) defining the observer rest reference frame. After solving the equations, arbitrary Lorentz frame can be introduced. Taking out the factor $dX_{(0)}/d\tau$ from the radicand in Eq. (7), we arrive at the action integral in the initial formulation of SR (1905):

$$\begin{aligned} S_{\mathrm{SR:1905}} &= -m\int d\tau\frac{dX_{(0)}}{d\tau}\sqrt{1-\left[\frac{dX_{(i)}}{dX_{(0)}}\right]^2} \\ &= -m\int dX_{(0)}\sqrt{1-\left[\frac{dX_{(i)}}{dX_{(0)}}\right]^2}. \end{aligned} \tag{10}$$

Expressing the momentum in terms of the velocity $V_{(i)} = dX_{(i)}/dX_{(0)}$ entering into the variation of the Lagrangian $L = -m\sqrt{1-V^2_{(i)}}$,

$$P_{(i)} = \partial L/\partial(V_{(i)}) = mV_{(i)}/\sqrt{1-V^2_{(i)}}, \tag{11}$$

one can obtain the Hamiltonian function

$$H(P_{(i)}) = P_{(i)}V_{(i)} - L = \sqrt{m^2 + P^2_{(i)}(X_{(0)})} \tag{12}$$

and rewrite action (10) in the Hamiltonian form

$$S_{\mathrm{SR:1905}} = \int dX_{(0)}\left[P_{(i)}\frac{dX_{(i)}}{dX_{(0)}} - H(P_{(i)})\right]. \tag{13}$$

The energy is defined as the Hamiltonian function on the trajectory: $H(P_{I(i)}) = \sqrt{m^2 + P^2_{I(i)}}$. The famous formula $E = mc^2$ (with c = 1) is a consequence of the definition of physical observables from the correspondence to classical mechanics and follows from the low-energy expansion of the Hamiltonian function in powers of dynamical variables:

$$H(P_{(i)}) = \sqrt{m^2 + P^2_{(i)}} = m + \frac{P^2_{(i)}}{2m} + \dots. \tag{14}$$

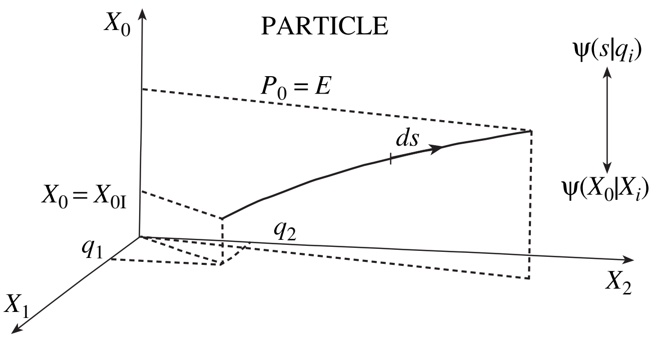

Fig. 1. Motion of an unstable relativistic particle in a world line in the space of events. The motion is completely described by the two Newtonian-like sets of observables, dynamical and geometric. Each has its proper time and wave function Ψ. The two measured life-times of the particle (the time as either a dynamical variable X^0 or a geometric interval s) are interrelated by the equations of motion following from the action of Hilbert-type geometrodynamics rather than by Lorentz transformations.

Variation of action (13) with respect to canonical momenta $P_{(i)}$ and variables $X_{(i)}$ yields, respectively, the velocity in terms of the momenta,

$$V_{(i)} = \frac{P_{(i)}}{\sqrt{m^2 + P_{(i)}^2}}, \tag{15}$$

and the momentum conservation law: $dP_{(i)}/dX_{(0)} = 0$. The solution of these equations determines the particle trajectory in the space of events:

$$X_{(i)}(X_{(0)}) = X_{(i)}(X_{I(0)}) + V_{(i)}[X_{(0)} - X_{I(0)}], \tag{16}$$

where $X_{I(0)}$ is the initial time relative to the observer rest frame.

The transformation to any reference frame is described by the corresponding Lorentz transformation and equivalent to an appropriate choice of a time axis. Every reference frame has its proper time, energy, and momentum. The relationship between dynamical variables and times in different reference frames is treated as the relativity principle formulated, most clearly, by Einstein [34]. According to the Einstein relativity principle, the Lorentz transformations contain extra information on relativistic effects as compared to solutions (16) of the dynamical equations derived by variation of action (13).

Therefore, the appearance of relativistic effects due to the Lorentz kinematic transformations (i.e., transformations of reference frames) means that the Einstein theory significantly differs from Newtonian mechanics. In the latter, all the physical effects are to be deduced from the equations of motion by variational method with due regard to initial data. In this case, the Galilei group in Newtonian mechanics contains nothing beyond solutions of the equations of motion.

The following question arises. Can a relativistic particle theory be formulated such that all physical consequences, including relativistic effects, be described by a variational equation?

We will prove that such a relativistic particle theory can be formulated by perfect analogy to the Hilbert's "Foundations of Physics" [15], i.e., as a geometrodynamics. According to this theory, the description of a physical system is based on action functional, geometric interval, symmetry of reference frames, gauge symmetry, equations of motion, and constraint equations for initial data.

2.2.3. Geometrodynamics of a relativistic particle. According to Hilbert [15], the geometrodynamics of a relativistic particle is based on the two principles: the action [27, 28]

$$\begin{aligned} S_{\mathrm{SR:1915}} &\equiv \int d\tau L_{\mathrm{SR:1915}} \\ &= -\frac{m}{2}\int d\tau e(\tau)\left[\left(\frac{dX_{(\alpha)}}{e(\tau)d\tau}\right)^2 + 1\right] \end{aligned} \tag{17}$$

for the variables $X_{(\alpha)} = [X_{(0)}|X_{(i)}]$ forming the space of events of the moving particle and the geometric interval

$$ds = e(\tau)d\tau \tag{18}$$

in the Riemann one-dimensional space on the world line of the particle in this space (see Fig. 1). Here, $e(\tau)$ is the only metric component, so-called the shift of the coordinate evolution parameter.

Variation of the action with respect to the function $e(\tau)$ yields the equations of geometrodynamics

$$[e(\tau)d\tau]^2 = dX_{(\alpha)}^2. \tag{19}$$

Solving these equations in $e(\tau)$, we arrive at

$$e(\tau) = \pm\sqrt{\left(\frac{dX_{(\alpha)}}{d\tau}\right)^2}. \tag{20}$$

It is seen that action (17) coincides on these solutions with initial action (7) of the relativistic particle up to a sign. The negative sign of $e(\tau)$ in Eq. (20) implies the change of the mass sign in action (7) for an antiparticle. Equation (19) is referred to as a constraint equation.

For the Hamiltonian relativistic-particle theory with constraints, the corresponding action can be derived from (17) by introducing the canonical momenta $P_\alpha = \partial L_{\mathrm{SR:1915}}/\partial \dot{X}_{(\alpha)}$:

$$S_{\mathrm{SR:1915}} = \int_{\tau_1}^{\tau_2} d\tau\left[-P_{(\alpha)}\frac{dX_{(\alpha)}}{d\tau} + \frac{e(\tau)}{2m}(P_{(\alpha)}^2 - m^2)\right]. \tag{21}$$

Here, the function $e(\tau)$ of the shift of the coordinate evolution parameter τ defines geometric interval (18):

$$ds = e(\tau)d\tau \mapsto s(\tau) = \int_0^\tau d\bar{\tau}e(\bar{\tau}). \quad (22)$$

Action (21) and interval (18) are invariant with respect to the reparametrization of the coordinate evolution parameter τ:

$$\tau \longrightarrow \tilde{\tau} = \tilde{\tau}(\tau). \quad (23)$$

Therefore, SR could be referred to as a one-dimensional GR, with the reparametrization group of coordinate evolution parameter (23) serving as a group of gauge (general coordinate) transformations. The equation for the auxiliary shift function $\delta S_{SR}/\delta e = 0$ determines the energy constraint imposed on the particle momenta $P_{(0)}$, $P_{(i)}$,

$$P_{(0)}^2 - P_{(i)}^2 = m^2, \quad (24)$$

so-called a mass surface equation.

The equations

$$P_{(\alpha)} = m\frac{dX_{(\alpha)}}{ed\tau} \equiv m\frac{dX_{(\alpha)}}{ds}, \quad \frac{dP_{(\alpha)}}{ds} = 0 \quad (25)$$

for the variables $P_{(\alpha)}$, $X_{(\alpha)}$ derived by a variation of action (21) are gauge-invariant. The solution

$$X_{(\alpha)}(s) = X_{I(\alpha)} + \frac{P_{I(\alpha)}}{m}s \quad (26)$$

of these equations in terms of geometric interval (22) is a generalization of solution (3) of the Newtonian equations in the Minkowski space. In this case, the geometric time interval serves as an evolution parameter, while $P_{I(\alpha)}$ and $X_{I(\alpha)}$ are initial data for the four variables at the point $s = 0$:

$$X_{(\alpha)}(s = 0) = X_{I(\alpha)}. \quad (27)$$

These equations contain three new features, as compared to Newtonian mechanics, namely, momentum constraint (24), the time component in solution (26) of the equation of motion, and the initial value $X_{I(\alpha)}$ of time as a variable.

2.2.4. Reduction of geometrodynamics to the Einstein theory (1905). Action (21) and interval (22) were above referred to as geometrodynamics of a particle. The geometrodynamics of a particle is characterized by the two times in every reference frames, namely, the time as a geometric interval measured by an observer on the world line and the time as a dynamical variable measured by an fixed observer.

The physical interpretation of solutions (24) and (26) of geometrodynamics is determined by the choice of a specific Lorentz reference frame $P_\mu = (P_{(0)}, P_{(i)})$, so-called observer rest frame. The solution

$$P_{(0)\pm} = \pm\sqrt{P_{(i)}^2 + m^2} = \pm H \quad (28)$$

of constraint equation (24) in the zero momentum component $P_{(0)}$ in this reference frame is the Hamiltonian function in the spatial dynamical variables $[P_{(i)}, X_{(i)}]$. According to the principle of correspondence to Newtonian mechanic, these variables belong to so-called reduced phase space. The variable $X_{(0)}$ is the evolution time relative to the observer rest frame.

In a given Lorentz reference frame, the time component of solution (26),

$$X_{(0)}(s) - X_{(I0)} = \frac{P_{0\pm}}{m}s \quad (29)$$

of geometrodynamics has no an analogy in Newtonian mechanics. In this case, formula (29) is a pure kinematic relation between the two times noted above, namely, the dynamical variable $X_{(0)}$ and the geometric interval s:

$$s = [X_{(0)} - X_{0I}]\frac{m}{P_{0\pm}}. \quad (30)$$

This equation will be referred to as a geometric ratio of the two times of a relativistic particle, namely, the time $[X_{(0)}]$ as a variable and the time s as an interval.

The substitution of geometric ratio (30) into spatial part

$$X_{(i)}(s) = X_{(Ii)} + \frac{P_{(i)}}{m}s \quad (31)$$

of solution (26) gives the relativistic equation of motion in the reduced phase space $[P_{(i)}, X_{(i)}]$,

$$X_{(i)} = X_{(iI)} + \frac{P_{(i)}}{P_{0+}}[X_{(0)} - X_{(0I)}], \quad (32)$$

with the time $[X_{(0)}]$ as a variable.

Thus, geometrodynamics in a specific reference frame consists of constraint-free “particle dynamics” (32) and “geometry” (31) describing purely relativistic effects by the equations of motion in the same reference frame [27, 28].

2.2.5. What could Einstein not know in 1905? Formula (13) for the action describing a moving particle can be derived by substitution of solution (28) into geometrodynamic action (21). Such a substitution also gives the action for a particle with negative energy (28):

$$S_{SR:1915}\big|_{P_{(0)} = P_{0-}} = \int_{X_{(0)}}^{X_{(0I)}} d\bar{X}_0\left[-P_{(i)}\frac{dX_{(i)}}{d\bar{X}_{(0)}} - \sqrt{P_{(i)}^2 + m^2}\right]. \quad (33)$$

The equations corresponding to this action have the solutions

$$X_i = X_{(iI)} + \frac{m}{P_{0-}}[X_{(0I)} - X_{(0)}(s)] = X_{(iI)} + \frac{m}{P_{0+}}[X_{(0)}(s) - X_{(0I)}]. \quad (34)$$

The problem of negative energy was solved in quantum field theory [19].

It is known that quantum relativistic mechanics is defined as the quantization of energy constraint (24), $(P_0)^2 - (P_i)^2 = m^2$, by substituting the particle momentum $P_\alpha = (P_0, P_i)$ by its operator $\hat{P}_\alpha = -i\partial_\alpha$. The quantization yields the Klein–Gordon–Fock equation for wave function,

$$[(\hat{P}_\alpha)^2 - m^2]\Psi[P_\alpha|X_\alpha] = 0, \tag{35}$$

as a quantum analog of constraint equation (24). This equation has the normalized solution,

$$\begin{aligned}\Psi[P_{(\alpha)}|X_{(\alpha)}] = (2|P_{(0)}|)^{-1/2}[a^+\Psi_{P_{(0)+}}\theta(X_{(0)} - X_{I(0)}) \\ + a^-\Psi^*_{P_{(0)-}}\theta(X_{I(0)} - X_{(0)})],\end{aligned} \tag{36}$$

where θ is the Heaviside step function and the terms with the coefficients a^+ and a^- correspond to two classical solutions of constraint equation (24) with positive and negative energies (28).

Quantum field theory is known to be formulated as the quantization of the coefficients a^+ and a^-, i.e., as the second quantization of relativistic particles [18, 19]. In this case, to exclude negative energies, $-|P_{(0)}|$, and therefore to ensure the stability of quantum systems, the coefficients a^+ and a^- are to be treated as production and annihilation operators, respectively, for particles with positive energy.[3]

This treatment is equivalent to the postulate of the existence of the vacuum as a lowest-energy state in the space of events. The postulate imposes a constraint on the motion of a classical particle in the space of events, namely, the particle with the energy P_{0+} (P_{0-}) moves forward (backward):

$$P_{0+} \longrightarrow X_{I(0)} \le X_{(0)}, \quad P_{0-} \longrightarrow X_{I(0)} \ge X_{(0)}. \tag{37}$$

The following question arises. How does causal quantization (36) with constraint (37) influence geometric interval s (22)?

2.2.6. Quantum anomaly of geometric interval. To answer this question we perform the Lorentz transformation from the rest reference frame to the comoving frame: $[\bar{X}_{(0)}|\bar{X}_{(i)}]$, where $\bar{P}_{(i)} = 0$ and $\bar{P}_{0\pm} = \pm m$.

It follows from (30) and (37) that the time $\bar{X}_{(0)}$ in the comoving frame is related to the geometric interval s by the equation

$$\begin{aligned}s(\bar{X}_{(0)}|\bar{X}_{I(0)}) = (\bar{X}_{(0)} - \bar{X}_{I(0)})\theta(\bar{X}_{(0)} - \bar{X}_{I(0)})\theta(\bar{P}_{(0)}) \\ + (\bar{X}_{I0} - \bar{X}_0)\theta(\bar{X}_{I(0)} - \bar{X}_{(0)})\theta(-\bar{P}_{(0)}).\end{aligned} \tag{38}$$

[3] Moreover, the initial data $X_{I(0)}$ is treated in quantum theory as a point of production or annihilation of a particle.

This expression for the geometric interval s in quantum field theory looks like the Green's causal function of the comoving time:

$$\frac{d^2 s(\bar{X}_{(0)}|\bar{X}_{I(0)})}{d\bar{X}^2_{(0)}} = \delta(\bar{X}_{(0)} - \bar{X}_{I(0)}). \tag{39}$$

Therefore, the positive geometric arrow of time, $s \ge 0$, is a consequence of the postulate of existence of the vacuum as a lowest-energy state, which leads to the existence of the absolute time origin $s = 0$. The positive arrow of time implies the breaking of classical symmetry with respect to the transformation s to $-s$. In contrast to classical symmetry, breaking of symmetry in quantum theory is referred to as a quantum anomaly [36].[4]

Under the assumption of the existence of the vacuum as a physical lowest-energy state, the second quantization of an arbitrary relativistic system leads to the absolute geometric-time origin $s = 0$ in this system. The question on what was before the creation of a relativistic particle, string, or universe has no physical sense for an observer measuring time because time is created together with the quantum relativistic universe as a consequence of universe stability.

2.2.7. How does the invariant reduction differ from the choice of gauge? We now compare the gauge-invariant method of describing field dynamics [27, 28] to the gauge-noninvariant method, assuming that the coordinate time x^0 becomes observed.[5] In the case of SR under consideration, this assumption implies the use of the synchronous gauge $e(x^0) = 1$ in action (21):

$$S_{\rm SR} = \int_{\tau_1}^{\tau_2} d\tau\left[-P_{(\alpha)}\frac{dX_{(\alpha)}}{d\tau} + \frac{e(\tau)}{2m}(P^2_{(\alpha)} - m^2)\right]. \tag{40}$$

This yields a constraint-free theory:

$$\begin{aligned}&S_{\rm SR}|_{[e=1]} \\ &= \int_{\tau_1}^{\tau_2} d\tau\left[P_{(i)}\frac{dX_{(i)}}{d\tau} - P_{(0)}\frac{dX_{(0)}}{d\tau} - \frac{1}{2m}(-P^2_{(0)} + P^2_{(i)} + m^2)\right].\end{aligned} \tag{41}$$

From the viewpoint of quantization, Eq. (41) describes an unstable system because it contains the variable $X_{(0)}$ making a negative contribution to the energy $E = (-P^2_{(0)} + P^2_{(i)} + m^2)/2m$, which is defined in the interval ($-\infty < E < \infty$). The particle action on the three-dimensional hypersurface defined by the condi-

[4] The anomaly associated with Dirac fields also follows from the vacuum existence postulate. This fact was first pointed out by Jordan [35] and then rediscovered by a lot of authors [36]. The vacuum existence postulate is verified by a number of experimental effects, in particular, anomalous decays of pseudoscalar bound states (neutral pion and parapositronium) into two photons.

[5] It is the assumption of the coordinate time x^0 as an observable in SR that is used in the island universe [20] and in so-called global-time theory [29].

tion $P_{(0)} = 0$ (the similar constraint in GR is referred to as a minimal surface [20]) coincides with the Newtonian action

$$S_{\rm SR}\big|_{[e=1,\,P_{(0)}=0]} = S_{\rm Newton} = \int_{\tau_1}^{\tau_2} d\tau\left[P_{(i)}\dot{X}_{(i)} - \frac{P_{(i)}^2}{2m}\right] \quad (42)$$

up to a constant factor, as well as with the Einstein action (13) in the nonrelativistic limit, where the time $X_{(0)}$ in the rest frame coincides with the time interval s. Theory (41) under the constraint $P_{(0)}^2 = P_{(i)}^2 + m^2$ reduces to SR because the gauge symmetry is restored.

2.2.8. Relativity as a principle of gauge symmetry. Poincaré [33] and Einstein [34] found that, in contrast to classical mechanics, just the two observers are needed to completely describe the motion of a relativistic particle: the first is at rest, and the second moves with the particle. For example, every Einstein observer measures its proper lifetime of an unstable particle. Therefore, time is a relative quantity.

Einstein described this time relativity as a pure kinematic effect by using the Lorentz transformations from a fixed reference frame to a moving one.

As was shown above there exists a geometrodynamic generalization of the Poincaré–Einstein dynamics to the gauge theory with constraint (21) that allows us to describe this two-time relativity as a consequence of the equations of motion rather than the Lorentz kinematic transformations. This geometrodynamic description defines the new two-time relativity as a ratio of the dynamical particle evolution parameter X_0 to geometric interval s (22).

We now illustrate this inference with a mini-universe. In this case, purely relativistic effects can't be described kinematically by transformations of Lorentz-type variables.

3. RELATIVISTIC QUANTUM MINI-UNIVERSE

3.1. Homogeneous Approximation of General Relativity

As shown in the preceding section, relativistic effects in variational equations can be dynamically described within the framework of SR formulated by analogy to the Hilbert's variational description of GR [15]. According to Hilbert, GR geometrodynamics is based on the two basic notions: the action

$$S_{GR} = \int d^4x\sqrt{-g}\left[-\frac{\varphi_0^2}{6}R(g) + \mathcal{L}_{\rm matter}\right], \qquad \varphi_0^2 = \frac{3}{8\pi}M_{\rm Planck}^2 \quad (43)$$

in the variables of a field space of events and the geometric interval of the Riemannian coordinate manifold

$$ds = g_{\mu\nu}dx^\mu dx^\nu. \quad (44)$$

Both action (43) and interval (44) are invariant with respect to the general coordinate transformations

$$x^\mu \longrightarrow \tilde{x}^\mu = \tilde{x}^\mu(x^0, x^1, x^2, x^3). \quad (45)$$

They serve as a generalization of the action considered above and as an interval for a relativistic particle, invariant with respect to the reparametrization group of coordinate time. In the case of the homogeneous approximation,

$$ds = a^2(x^0)[N_0^2(x^0)(dx^0)^2 - (dx^i)^2], \quad (46)$$

the GR action reduces to the Hamiltonian cosmology action [39–41]:

$$S_{\rm cosmic-1915} = \int dx^0\left[-P_\varphi\partial_0\varphi + N_0\left(\frac{P_\varphi^2}{4V_0} - \rho_0(\varphi)V_0\right)\right]. \quad (47)$$

Here, V_0 is the volume and the matter energy density is approximated by the homogeneous density $\rho_0(\varphi)$ dependent only on the scale factor

$$\varphi(x^0) = \varphi_0 a(x^0). \quad (48)$$

In the homogeneous approximation, the GR equations of motion, as well as the conformal time interval

$$d\eta = N_0(x^0)dx^0, \quad \eta = \int_0^{x^0} d\bar{x}^0 N_0(\bar{x}^0), \quad (49)$$

are invariant with respect to reparametrization of the coordinate parameter, $x^0 \longrightarrow \bar{x}^0 = \bar{x}^0(x^0)$, which serve as gauge transformations. As shown above, the reparametrization group of the coordinate parameter means that one of the variables (here, the only variable is φ) is identified with time as a variable, while the momentum P_φ is taken as the corresponding Hamiltonian function, with its value on the equations of motion becoming the energy of events.

3.2. Hilbert Variational Principle

Variation of action (47) with respect to the lapse function N_0, $\delta S_{\rm cosmic}/\delta N_0 = 0$, yields the energy constraint equation

$$P_\varphi^2 = E^2(\varphi), \quad (50)$$

where

$$E(\varphi) = 2V_0\sqrt{\rho_0(\varphi)} \quad (51)$$

is treated by a terrestrial observer as the universe energy.

Equation (50) gives two values, positive and negative, of universe energy:

$$P_\varphi^\pm = \pm E(\varphi) = \pm 2V_0\sqrt{\rho_0(\varphi)}. \quad (52)$$

Variation of action (47) with respect to momentum P_φ, $\delta S_{\rm cosmic}/\delta P_\varphi = 0$, yields a geometric relation between the two times in the form of the Friedmann differential equation:

$$P_\varphi^{\pm} = 2V_0\frac{d\varphi}{d\eta} \equiv 2V_0\varphi' = \pm 2V_0\sqrt{\rho_0(\varphi)}. \tag{53}$$

The integral solution

$$\eta(\varphi_I|\varphi) = 2V_0\int\limits_{\varphi_I}^{\varphi}\frac{d\tilde{\varphi}}{P_\varphi^{\pm}(\tilde{\varphi})} = \pm\int\limits_{\varphi_I}^{\varphi}\frac{d\tilde{\varphi}}{\sqrt{\rho_0(\tilde{\varphi})}} \tag{54}$$

of this equation is referred to as the Hubble law [37, 38].

3.3. Reduction of Action

The reduction of Hilbert action (47) implies the calculation of its values on solutions (52) of constraint equation (50):

$$\begin{aligned} S_{\text{cosmic-1915}}\big|_{P_\varphi = P_\varphi^{\pm}} &= S^{\pm}_{\text{cosmic-1905}} \\ &= \mp 2V_0\int d\varphi\sqrt{\rho_0(\varphi)}. \end{aligned} \tag{55}$$

Reduced action (55) leads to equations not containing a geometric interval. To solve the initial geometrodynamics completely, we should supplement the reduced theory with relation (54) between the geometric interval and time as a variable. As will be shown below, relation (54) describes classical cosmology, i.e., the Hubble law, and is an auxiliary relation to the Wheeler–De Witt quantum cosmology, provided that this cosmology is defined as a quantization of constraint equation (50). The quantization is defined by the substitution of variables by operators, $P_\varphi \longrightarrow \hat{P}_\varphi = -id/d\varphi$, acting on the Wheeler–De Witt wave function Ψ [39]:

$$\hat{P}_\varphi^2\Psi - E^2(\varphi)\Psi = 0. \tag{56}$$

We will show below that both classical [37] and quantum [39–41] cosmologies follow from the Hilbert geometrodynamics. This allows us to combine them to settle their troubles, namely, the quantization of classical cosmology and the description of the Hubble law in quantum cosmology.

3.4. Friedmann Equation and Classical Cosmology

We now consider universe evolution equation (53),

$$\varphi_0^2 a'^2 = \rho_0(a) \tag{57}$$

in a Euclidean space, where $a = \varphi/\varphi_0$. The universe is filled with homogeneous matter with the density ρ_0 conformally dependent on the scale $a(\eta)$:

$$\rho_0(a) = \rho_{\text{rigid}}a^{-2} + \rho_{\text{rad}} + \rho_{\text{M}}a + \rho_\Lambda a^4. \tag{58}$$

Here, ρ_{rigid} is the contribution of the equation of rigid state, $\rho_{\text{rigid}} = p_{\text{rigid}}$. The densities ρ_{rad}, ρ_{M}, and ρ_Λ describe the contributions of radiation, baryon matter, and Λ-term, respectively.

For each of these states, Eq. (57) can be solved in terms of the conformal time η, under the initial conditions $a(\eta_0) = 1$ and $a'(\eta_0) = H_0$:

$$\begin{aligned} a_{\text{rigid}}(\eta) &= \sqrt{1 - 2H_0 r}, \quad a_{\text{rad}}(\eta) = 1 - H_0 r, \\ a_{\text{M}}(\eta) &= \left[1 - \frac{1}{2}H_0 r\right]^2, \quad a_\Lambda(\eta) = \frac{1}{1 + H_0 r}, \end{aligned} \tag{59}$$

where $r = \eta_0 - \eta$. The conformal time $d\eta$ is defined in observational cosmology as the instant of time at which the photon is radiated by an atom at a cosmic object moving with the velocity $c = 1$ in a geodetic line on the world cone:

$$(ds)^2 = a^2(\eta)[(d\eta)^2 - (dr)^2] = 0, \tag{60}$$

where $r = \sqrt{x_1^2 + x_2^2 + x_3^2}$ is the radius. This allows us to find the relation of the distance

$$r(\eta) = \int\limits_{\eta}^{\eta_0} d\tilde{\eta} \equiv \eta_0 - \eta, \tag{61}$$

covered by a photon to its conformal time $\eta_0 - \eta$. Here, η_0 is the present-day conformal time of the photon measured by a terrestrial observer with $a(\eta_0) = 1$, and η is the time instant at which the photon is radiated by an atom at the distance r from the Earth. Therefore, η is the difference between the present-day conformal time η_0 and the time of a photon flight to the Earth, coinciding with the distance. Equation (61) gives

$$\eta = \eta_0 - r. \tag{62}$$

In observational cosmology, density (58) can be expressed in terms of the present-day critical density $\rho_{\text{cr}} = \varphi'^2 \equiv \varphi^2(\varphi'/\varphi)^2 = \varphi_0^2 H_0^2$,

$$\begin{aligned} \rho_0(a) &= \rho_{\text{cr}}\Omega(a), \\ \Omega(a) &= \Omega_{\text{rigid}}a^{-2} + \Omega_{\text{rad}} + \Omega_{\text{M}}a + \Omega_\Lambda a^4 \end{aligned} \tag{63}$$

and the relative ones Ω_{rigid}, Ω_{rad}, Ω_{M}, and Ω_Λ, satisfying the condition $\Omega_{\text{rigid}} + \Omega_{\text{rad}} + \Omega_{\text{M}} + \Omega_\Lambda = 1$ [37].

Classical cosmology describes the redshift of the radiation spectrum $E(\eta)$ at a cosmic object relative to the spectrum $E(\eta_0)$ at the Earth. Redshift is defined as the scale factor,

$$\frac{E(\eta)}{E(\eta_0)} = a(\eta) = (1 + z)^{-1} \tag{64}$$

versus the coordinate distance (given by conformal time (62)) to the object.

Taking these relations into account and substituting $a = 1/(1 + z)$ and $\eta = \eta_0 - r$, we can write out scale evolution equation (57) at the light ray geodetic line $dr/d\eta = -1$ in the form

$$\frac{1}{H_0}\frac{dz}{dr} = (1+z)^2\sqrt{\rho_{\rm cr}[\Omega_{\rm rigid}(1+z)^2 + \Omega_{\rm rad} + \Omega_{\rm M}(1+z)^{-1} + \Omega_\Lambda(1+z)^{-4}]},$$

where $H_0 = \sqrt{\rho_{\rm cr}}/\varphi_0$. The solution

$$H_0 r(z) = \int_1^{1+z} \frac{dx}{\sqrt{\Omega_{\rm rigid}x^6 + \Omega_{\rm rad}x^4 + \Omega_{\rm M}x^3 + \Omega_\Lambda}} \quad (65)$$

(coinciding with solution (54)) of this equation determines the coordinate distance as a function of redshift z and gives formulas (59) for every state. Formula (65), being a basis of observational cosmology (for example, see [37]), is used to find the equation of the state of the universe matter from astrophysical red-shift data under the assumption of flat space. We here consider the effect of the quantization on Hubble law (54) in observational cosmology defined by (65), namely, the consequences of the Hamiltonian action reduction and cosmological time interval.

3.5. Quantum Cosmology as a Quantization of Constraints

3.5.1. Primary quantization. The first quantization

$$i[P_\varphi, \varphi] = 1 \quad (66)$$

of the cosmological scale φ means that energy constraint equation (50) turns into Wheeler–De Witt equation (56) for the universe moving in the space of events φ:

$$\partial_\varphi^2\Psi + E^2(\varphi)\Psi = 0. \quad (67)$$

This equation can be derived by variation of the corresponding Klein–Gordon-like field theory,

$$S_{\rm U} = \int d\varphi\frac{1}{2}[(\partial_\varphi\Psi)^2 - E^2(\varphi)\Psi^2] \equiv \int d\varphi L_{\rm U}, \quad (68)$$

which will be referred to as the field theory of universes.

The negative energy in solutions (52) means that the energy of the relativistic system under consideration has no minimum; therefore, an arbitrary small interaction causes this system to be unstable. In the quantum field theory resulting from the second quantization of the Wheeler–De Witt equations for Ψ, a system can be made stable if the vacuum is postulated as a lowest-energy state. As shown in the preceding section concerning the second quantization of a particle, such a vacuum originates if the creation of the universe with negative energy is treated as the annihilation of the universe with positive energy.

3.5.2. Secondary quantization. By introducing the canonical momenta $P_\Psi = \partial L_{\rm U}/\partial(\partial_\varphi\Psi)$, we obtain an action in the Hamiltonian form:

$$S_{\rm U} = \int d\varphi\{P_\Psi\partial_\varphi\Psi - H_{\rm U}\}, \quad (69)$$

where

$$H_{\rm U} = \frac{1}{2}[P_\Psi^2 + E^2(\varphi)\Psi^2] \quad (70)$$

is the Hamiltonian. The definition of the energy $E(\varphi)$ of the universe makes it possible to write out the Hamiltonian $H_{\rm U}$ as the product of $E(\varphi)$ and the universe number,

$$N_{\rm U} = A^+A^-, \quad (71)$$

$$H_{\rm U} = E(\varphi)\frac{1}{2}[A^+A^- + A^-A^+] = E(\varphi)\left[N_{\rm U} - \frac{1}{2}\right], \quad (72)$$

where the holomorphic variables

$$\Psi = \frac{1}{\sqrt{2E(\varphi)}}\{A^{(+)} + A^{(-)}\},$$
$$P_\Psi = i\sqrt{\frac{E(\varphi)}{2}}\{A^{(+)} - A^{(-)}\} \quad (73)$$

are introduced [42]. To exclude negative energies, A^- should be postulated as the annihilation operator for the universe with positive energy; this implies the existence of the vacuum as a lowest-energy state:

$$A^{(-)}|0\rangle_A = 0. \quad (74)$$

However, the universe number $N_{\rm U} = A^+A^-$ is not conserved, because the energy $E(\varphi)$ depends on φ. It is just the φ-dependence of $E(\varphi)$ that leads to an extra term in the action written in terms of the holomorphic variables in functional space:

$$P_\Psi\partial_\varphi\Psi$$
$$= \left[\frac{i}{2}(A_q^+\partial_\varphi A^- - A^+\partial_\varphi A^-) - \frac{i}{2}(A^+A^+ - AA)\Delta(\varphi)\right], \quad (75)$$

where

$$\Delta(\varphi) = \frac{\partial_\varphi E(\varphi)}{2E(\varphi)}. \quad (76)$$

The last term in Eq. (75) describes the cosmological creation of the universe from the vacuum if $\partial_\varphi E(\varphi) \neq 0$.

3.5.3. Bogoliubov transformation and universe creation. To define the vacuum and a set of conserved numbers, so-called integrals of motion, we can use the Bogoliubov transformations [43] of the variables (A^+, A^-) (similar to the case of cosmological particle creation [42]),

$$A^+ = \alpha B^+ + \beta^* B^-, \quad A^- = \alpha^* B^- + \beta A^+$$
$$(|\alpha|^2 - |\beta|^2 = 1), \quad (77)$$

so the corresponding equations, expressed in terms of the universes (A^+, A^-),

$$(i\partial_\varphi + E)A^+ = iA^-\Delta(\varphi),$$
$$(i\partial_\varphi - E)A^- = iA^+\Delta(\varphi), \qquad (78)$$

take the diagonal form in terms of the quasiuniverses (B^+, B^-),

$$(i\partial_\varphi + E_B)B^+ = 0, \quad (i\partial_\varphi - E_B)B^- = 0. \qquad (79)$$

This means that the Bogoliubov transformation coefficients satisfy the Bogoliubov equations

$$(i\partial_\varphi + E)\alpha = i\beta\Delta(\varphi),$$
$$(i\partial_\varphi - E)\beta^* = i\alpha^*\Delta(\varphi). \qquad (80)$$

Introducing the variables r and θ,

$$\alpha = e^{i\theta(\varphi)}\cosh r(\varphi), \quad \beta^* = e^{i\theta(\varphi)}\sinh r(\varphi), \qquad (81)$$

(referred to as a shift parameter and a rotation parameter, respectively), we arrive at the equations

$$(i\partial_\varphi\theta - E(\varphi))\sinh 2r = -\Delta(\varphi)\cosh 2r \sin 2\theta,$$
$$\partial_\varphi r = \Delta(\varphi)\cos 2\theta. \qquad (82)$$

The quasiuniverse energy entering into Eqs. (79) is given by the expression

$$E_B(\varphi) = \frac{E(\varphi) - \partial_\varphi\theta}{\cosh 2r}. \qquad (83)$$

By virtue of Eqs. (79), the quasiuniverse number $\mathcal{N}_B = (B^+B^-)$ is conserved:

$$\frac{d\mathcal{N}_B}{d\varphi} \equiv \frac{d(B^+B^-)}{d\varphi} = 0. \qquad (84)$$

Therefore, we arrive at the definition of the vacuum as a state of quasiuniverses in the form

$$B^-|0\rangle_U = 0. \qquad (85)$$

The number of universes created from the Bogoliubov vacuum can be found by averaging operator (71) of the universe number (wheelerons) over the Bogoliubov vacuum. Therefore, this number is proportional to the Bogoliubov coefficient squared defined in Eq. (77):

$$N_U(\varphi) = {}_U\langle A^+A^-\rangle_U \equiv |\beta|^2. \qquad (86)$$

This quantity can be referred to as a functional of universe distribution $N_U(\varphi)$, whereas the quantity

$$R_U(\varphi) = i(\alpha^*\beta^* - \alpha\beta) \equiv {}_U\langle P_\Psi\Psi\rangle_U$$
$$= -\sinh 2r \sin 2\theta \qquad (87)$$

can be referred to as a rotation functional, because the variables r and θ are shift and rotation parameters, respectively.

In terms of the functional of universe distribution $N_U(\varphi)$ and the rotation functional $R_U(\varphi)$, the Bogoliubov equations take the form

$$\begin{cases} \dfrac{dN_U}{d\varphi} = \Delta(\varphi)\sqrt{4N_U(N_U+1) - R_U^2} \\ \dfrac{dR_U}{d\varphi} = -2E(\varphi)\sqrt{4N_U(N_U+1) - R_U^2} \end{cases} \qquad (88)$$

under the initial conditions $N_U(\varphi = \varphi_I) = R_U(\varphi = \varphi_I) = 0$.

In the model of the equation of rigid state with the energy $E(\varphi) = Q/\varphi$, functional (86) can be found explicitly because Eqs. (88) can be solved exactly. In this case, the distribution function takes the form

$$N_U = \frac{1}{4Q^2 - 1}\sin^2\left[\sqrt{Q^2 - \frac{1}{4}}\ln\frac{\varphi}{\varphi_I}\right] \neq 0, \qquad (89)$$

where $\varphi = \varphi_I\sqrt{1 + 2H_I\eta}$, where φ_I and $H_I = \varphi_I'/\varphi_I = Q/(2V_0\varphi_I^2)$ are initial data.

3.5.4. Quantum anomaly of conformal time. The vacuum existence postulate restricts the motion of the universe in the field space of events and means that the universe moves forward ($\varphi > \varphi_I$) or backward ($\varphi < \varphi_I$) if the energy of events is positive ($P_\varphi \geq 0$) or negative ($P_\varphi \leq 0$), respectively, where φ_I are initial data. In quantum theory, the quantity φ_I is considered as a creation point of the universe with positive energy $P_\varphi \geq 0$ or as an annihilation point of the anti-universe with negative energy $P_\varphi \leq 0$. We can assume that the singular point $\varphi = 0$ belongs to the anti-universe: $P_\varphi < 0$. A universe with positive energy of events has no the cosmological singularity $\varphi = 0$.

According to the vacuum existence postulate, solution (54) takes the form

$$\eta(\varphi_I, \varphi_0) = \theta(P_\varphi)\int_{\varphi_I}^{\varphi_0}\frac{d\varphi}{\sqrt{\rho_0(\varphi)}}\theta(\varphi_0 - \varphi_I)$$
$$+ \theta(-P_\varphi)\int_{\varphi_0}^{\varphi_I}\frac{d\varphi}{\sqrt{\rho_0(\varphi)}}\theta(\varphi_I - \varphi_0) \geq 0, \qquad (90)$$

where θ is the Heaviside step function. Therefore, this postulate leads to conformal time (90) being positive both for the universe ($P_\varphi > 0$, $\varphi > \varphi_I$) and for the anti-universe ($P_\varphi < 0$, $\varphi_I < \varphi$); i.e., it results in the conformal arrow of time (90) $\eta > 0$.

3.6 Levi-Civita Transformations and Hubble Law

The conformal time η and corresponding momentum Π can be introduced, respectively, as a new field variable and a proper energy of geometric space of events if the Levi-Civita canonical transformation [27, 44], $(P_\varphi|\varphi) \longrightarrow (\Pi|\eta)$, is used to turn energy constraint (50) into the new canonical momentum Π. Let us consider this transformation for the universe filled with photons with $\rho_0(\varphi)$ = const. In this case, the transformation takes the form $P_\varphi = \pm 2\sqrt{\Pi V_0}$ and $\varphi = \pm(1/2)\sqrt{\Pi/V_0}\eta$, with action (47) reducing to $S_c = \int dx^0[-\Pi\partial_0\eta + N_0(\Pi - \rho_0 V_0)]$.

Solving the constraint equation $\Pi - V_0\rho_0 = 0$ means that the nonzero energy $\Pi = V_0\rho_0$ corresponds to the conformal time.

In this case, the reduced action takes the form $S = -V_0\int_0^{\eta_0} d\eta\rho_0 = -V_0\rho_0\eta_0$. In quantum theory, with the geometric energy Π substituted by the operator $\hat{\Pi} = id/d\eta$, the geometric evolution of wave function is determined from the quantum version of the constraint equation $(\hat{\Pi} - V_0\rho_0)\psi_{\text{geometric}}(\eta) = 0$, which has the solution

$$\psi_{\text{geometric}}(\eta) = e^{-iV_0\rho\eta}\theta(\eta). \tag{91}$$

The Hubble evolution $\varphi = \varphi(\eta)$ could be considered as a relativistic effect associated with two complementary methods of describing the relativistic universe, namely, by using either field or geometric wave function, $\exp(-iP_\varphi\varphi)$ or (91).

3.7. Quantization Results

Thus, the unified geometrodynamic formulation of both the theories (SR and GR), which is based on the Hilbert variational principle [15], makes it possible to quantize the cosmological models similarly to the first and second quantization of a relativistic particle. The latter is a basis of the modern quantum field theory [18], which is verified by a great number of high-energy experiments. A similar approach to the quantization within the framework of GR was first formulated by Wheeler and De Witt [39]. They assumed that cosmological time, treated as a variable, be identical with the cosmological scale factor. Moreover, they introduced into GR the notion of a field space of events, in which the relativistic universe moves, by analogy to the notion of Minkowski space. However, the Wheeler–De Witt formulation [39] does not contain time as a geometric interval and, therefore, its scale-factor dependence (interpreted in the Friedmann cosmology as the Hubble law). Thus, as noted above, classical cosmology fails to quantize [37], while quantum cosmology fails to describe the Hubble law [39–41].

In this section, we use the invariant reduction of the Wheeler–De Witt cosmology [44], considered as a relativistic-universe geometrodynamics, to restore the relation of observational cosmology (i.e., the Hubble law) to the first and second quantization of the universe and calculate the distribution of created universes. This reduction allows us to solve a series of problems, namely, Hubble evolution, universe creation from the vacuum, arrow of time, initial data, and elimination of the cosmological singularity, under the assumption that the Hamiltonian is diagonal and the universe is stable. In this case, we attain the physical description of nature that looks like quantum field theory in particle physics. In what follows, we consider a similar invariant reduction in GR in order to define physical observables, quantize gravity, and formulate a low-energy perturbation theory.

4. REFERENCE FRAMES IN GENERAL RELATIVITY IN METRIC FORMALISM

4.1. Dynamics and Geometry of General Relativity

We now present the general gauge-invariant formulation of GR in metric formalism based on the action

$$S[\varphi_0|F] = \int d^4x\sqrt{-g}\left[-\frac{\varphi_0^2}{6}R(g) + \mathcal{L}_{\text{matter}}(\varphi_0|g, f)\right] \tag{92}$$

depending on the fields $F = (g, f)$ and the geometric interval

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu. \tag{93}$$

Here, the Newton constant

$$\varphi_0^2 = \frac{3}{8\pi}M_{\text{Planck}}^2$$

defines a mass scale.

Action (92) and interval (93) are invariant with respect to the general coordinate transformations

$$x^\mu \longrightarrow \tilde{x}^\mu = \tilde{x}^\mu(x^0, x^1, x^2, x^3). \tag{94}$$

If these transformations are treated as gauges and the corresponding four equations for $g_{0\alpha}$ are considered as constraint equations similar to the Gauss equations in electrodynamics, then the coordinates x^μ can not be taken as measurable quantities. This was illustrated above with the similar formulation of SR. To define truly measurable coordinates one should solve constraint equations defined in a specific reference frame.

Reference frames, in relativistic theory, are associated with corresponding sets of devices measuring initial data [22]. As noted above, the choice of a reference frame means variable, coordinates, and equations of motion should be subdivided into space and time components. In this case, the number of variables entering into the action is preserved. In the Hamiltonian approach, time components of metric tensor serve, as a rule, as Lagrange multipliers. Variation of the action with respect to these multipliers yields the above-mentioned constraint equations for the initial data.

4.2. Dirac Hamiltonian Approach

Until now, most Hamiltonian approaches to GR were based on the Dirac–ADM formulation [23, 24]. According to this formulation, the choice of a reference frame is a classification of coordinates and variables such that interval (93) takes the form

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu$$
$$= [\sqrt{\gamma}N_d dx^0]^2 - \gamma_{ij}(dx^i + N^i dx^0)(dx^j + N^j dx^0). \quad (95)$$

Here, γ_{ij} is the metric of the three-dimensional hyperspace embedded into the four-dimensional manifold, N^j is the three-dimensional origin shift vector depending on time, and N_d is the Dirac lapse function [23] (see a more detailed consideration in monograph [22]). In terms of these variables, action (92) takes the form

$$S[\varphi_0|F] = \int d^4x[\mathbf{K}(\varphi_0|g) - \mathbf{P}(\varphi_0|g) + \mathbf{S}(\varphi_0|g) + \sqrt{-g}\mathcal{L}_{\text{matter}}(\varphi_0|g, f)]. \quad (96)$$

Here,

$$\begin{cases} \mathbf{K}(\varphi_0|g) = N_d\dfrac{\varphi_0^2}{6}[\mathrm{Tr}\,\mathrm{D}^2 - (\mathrm{Tr}\,\mathrm{D})^2] \\ = P_\gamma^{ij}\mathrm{D}_{ij} - N_d\dfrac{6}{\varphi_0^2}\Big[\mathrm{Tr}\,P_\gamma^2 - \dfrac{1}{2}(\mathrm{Tr}\,P_\gamma)^2\Big] \\ \mathbf{P}(\varphi_0|g) = N_d\dfrac{\varphi_0^2}{6}|\gamma|R^{(3)} \\ \mathbf{S}(\varphi_0|g) = \dfrac{\varphi_0^2}{3}[\partial_0\mathrm{D} - \partial_k(N^k\mathrm{D})] - \dfrac{\varphi_0^2}{3}\partial_i(\sqrt{\gamma}\partial^i(\sqrt{\gamma}N_d)) \end{cases} \quad (97)$$

are kinetic, potential, and surface terms, respectively;

$$\mathrm{D}_{ij} = \frac{1}{2N_d}(\partial_0\gamma_{ij} - \nabla_i N_j - \nabla_j N_i)$$
$$= \frac{1}{2N_d}(\hat{\partial}_0\gamma_{ij} - \gamma_{il}\partial_j N^l - \gamma_{jl}\partial_i N^l) \quad (98)$$

are the covariant rates of change of the metric (i.e., second quadratic forms); and

$$P_{\gamma i}^k = \frac{\varphi_0^2}{6}[\mathrm{D}_i^k - \delta_i^k\,\mathrm{Sp}\mathrm{D}]$$
$$\left(P_{\gamma k}^k \equiv \mathrm{Tr}\,P_\gamma = -\frac{\varphi_0^2}{3}\mathrm{Sp}\mathrm{D}\right) \quad (99)$$

are the corresponding momenta of the spatial metric. These momenta are defined with the use of the Legendre transformation $zD^2/2 = PD - z^{-1}P^2/2$ in the **K**-term in (97). The quantity $R^{(3)}$ is the three-dimensional curvature.

Because of GR symmetry (92) with respect to transformations (94), treated as gauges, the theory has extra, purely gauge, degrees of freedom. These degrees of freedom can be eliminated by using gauge transformations. The condition that decreases the number of fields and metric components with the help of a symmetry group of interval and action is referred to as a gauge. The transversality conditions, $\partial_j(|\gamma|^{1/3}\gamma^{ij}) \simeq 0$, and the Dirac gauge [23]

$$\mathrm{Tr}\,P_\gamma = -\frac{\varphi_0^2}{3}\mathrm{Tr}\,\mathrm{D}$$
$$\equiv -\frac{\gamma_0^2}{3N_d\sqrt{\gamma}}[\partial_0\sqrt{\gamma} - \partial_k(\sqrt{\gamma}N^k)] \simeq 0, \quad (100)$$

defining the minimal spatial hypersurface, both can serve as examples.

The Hamiltonian form of action follows from Eqs. (97) and (98) [23]:

$$S_h[\varphi_0|F] = \int_{x_1^0}^{x_2^0} dx^0\left\{\int d^3x\left[\sum_F P_F\partial_0 F + \mathscr{C} - N_d\mathscr{H}_t(\varphi_0|F)\right]\right\}. \quad (101)$$

Here, $\sum_F P_F\partial_0 F = P_\gamma^{ij}\partial_0\gamma_{ij} + \sum_f P_f\partial_0 f$;

$$\mathscr{C} = N^k T_k^0(\varphi_0|F) + C_0\mathrm{D} + C_i\partial_j(|\gamma|^{1/3}\gamma^{ij}) \quad (102)$$

is the constraint sum with the Lagrange multipliers N^k, C_0, and C_i;

$$\mathscr{H}_t(\varphi_0|F) = |\gamma|T_0^0(\varphi_0|F) \quad (103)$$

is the Dirac Hamiltonian; and

$$\begin{cases} T_0^0(\varphi_0|F) = \dfrac{6}{|\gamma|\varphi_0^2}\Big[\mathrm{Tr}\,P_\gamma^2 - \dfrac{1}{2}(\mathrm{Tr}\,P_\gamma)^2\Big] \\ + \dfrac{\varphi_0^2}{6}R^{(3)} + T_{0\,\text{matter}}^0 \\ T_k^0(\varphi_0|F) = 2\nabla_j P_{\gamma k}^j + T_{k\,\text{matter}}^0 \\ = \dfrac{\varphi_0^2}{3}[\nabla_j\mathrm{D}_k^j - \partial_k\mathrm{Tr}\,\mathrm{D}] + T_{k\,\text{matter}}^0 \end{cases} \quad (104)$$

are the energy–momentum tensor components. The identification of these tensors with observables means that the observable evolution parameter is identified with noninvariant coordinate time x^0. In this case, as was shown in [20], the perturbation theory is based on the Euclidean metric $N_d = 1$, $\gamma_{ij} = \delta_{ij}$ and can be described by the effective action

$$S[\varphi_0|F]\big|_{\sqrt{g^{(3)}}N_d \simeq 1} = S_{\text{Bjern}}$$
$$= \int_{x_1^0}^{x_2^0} dx^0\left\{\int d^3x\left[\sum_F P_F\partial_0 F + \mathscr{C} - \sqrt{\gamma}T_0^0(\varphi_0|F)\right]\right\}. \quad (105)$$

This action, first written by Bjorn [29], results from GR action in so-called synchronous reference frame

$$\sqrt{\gamma}N_d \simeq 1. \quad (106)$$

In fact, this condition concerning the synchronous reference frame should be considered, by Dirac [26], as a gauge that can only be used in equations of motion as a weak equality. Substituting gauge (106) into action (101), we arrive at a new theory (105), which is not equivalent to initial one (101) and could coincide with it only in a nonrelativistic limit. This was illustrated with the GR absolute time in Section 2.2.7. The conventional perturbation theory based on the Euclidean metric $N_d = 1$, $\gamma_{ij} = \delta_{ij}$, and theory (105) contains no energy constraints. Therefore, they can not solve the problems mentioned above concerning the initial data, energy localization, arrow of time, and elimination of cosmological singularity. It is well known [20] that the Dirac GR formula considered above encountered just these problems. Moreover, minimal surface condition (100) contradicts to the experimental observations on the distance-dependence of redshift (i.e., the Hubble law) provided that this dependence is related to the dilation proportional to the quadratic form: $\mathrm{Sp}D \sim H_0 \neq 0$, where H_0 is the Hubble constant.

These problems can be solved by using the experience of solving the problems mentioned above of relativistic mechanics and cosmology, as shown in [27, 28, 45–47]. This experience is based on the following subjects: the determination of gauge symmetry of GR reference frames, where the Hamiltonian formula (101) is defined; the definition of invariant evolution parameters; and the invariant reduction of the theory in a specific reference frame.

4.3. Gauge Symmetry of Reference Frames in General Relativity

In the reference frame considered above, the gauge group of the Hamiltonian approach is the diffeomorphism group of metric parameterization (95) [22, 25]:

$$x^0 \longrightarrow \tilde{x}^0 = \tilde{x}^0(x^0),$$

$$x_i \longrightarrow \tilde{x}_i = \tilde{x}_i(x^0, x_1, x_2, x_3), \quad (107)$$

$$\tilde{N} = N\frac{dx^0}{d\tilde{x}^0}, \quad \tilde{N}^k = N^i\frac{\partial\tilde{x}^k}{\partial x_i}\frac{dx^0}{d\tilde{x}^0} - \frac{\partial\tilde{x}^k}{\partial x_i}\frac{\partial x^i}{\partial\tilde{x}^0}. \quad (108)$$

This transformation group conserves the set of hypersurfaces x^0 = const and is referred to as a kinematic subgroup [22, 42] of the group of general coordinate transformations (94). The group of kinemetric transformations contains the reparametrization of coordinate time (107).

4.4. Class of Functions of Dynamical Variables

There are the two physical consequences of GR invariance with respect to the reparametrization of coordinate time (107). Firstly, such an invariance means that no physical instruments exist which could measure the coordinate time x^0. If this invariance is treated as a gauge principle, then (as illustrated above with SR) the Dirac Hamiltonian considered as a sum of constraints, as well as coordinate evolution parameter x^0, is not a gauge invariant. As shown in [27, 28, 45–47], the invariance with respect to the reparametrization of coordinate time in GR means that reference frame (95) should be predetermined by choosing two invariant observable times, namely, one as a variable and the other as an interval. The SR example considered in the preceding section shows that the homogeneous degree of freedom makes it possible to introduce in GR a gauge-invariant dynamical evolution parameter, i.e., time as a variable.

Secondly, the function class on which the parameters of transformations of physical variables are defined should coincide with the function class on which the physical variables are defined. This means that physical variables in reference frame (95) include the class of homogeneous functions (107) depending only on coordinate time $f(x^0)$. In the perturbation theory, these homogeneous degrees of freedom can be treated as zero Fourier harmonics, which are picked out by averaging over spatial volume $V_0 = \int d^3x$.

In particular, the logarithm of spatial metric determinant $\log\gamma(x^0, x^i)$ can be written out as the sum

$$\log\gamma(x^0, x^i) \equiv \langle\log\gamma\rangle(x^0) + \overline{\log\gamma}(x^0, x^i). \quad (109)$$

Here,

$$\langle\log\gamma\rangle(x^0) = \frac{1}{V_0}\int_{V_0} d^3x \log\gamma(x^0, x^i) \quad (110)$$

is the averaging over spatial volume $V_0 = \int d^3x$ (i.e., a zero Fourier harmonic), and $\overline{\log\gamma} = \log\gamma - \langle\log\gamma\rangle$ is the deviation satisfying the strong condition

$$\int d^3x\overline{\log\gamma} \equiv \int d^3x[\log\gamma - \langle\log\gamma\rangle] \equiv 0, \quad (111)$$

with accordance with (110).

The similar zero Fourier harmonic also enters into the spur of canonical momenta of metric components:

$$\mathrm{Tr}P_\gamma = \langle\mathrm{Tr}P_\gamma\rangle + \overline{\mathrm{Tr}P_\gamma}, \quad (112)$$

where

$$P^k_{\gamma i} = \frac{\varphi_0^2}{6}[D^k_i - \delta^k_i\mathrm{Tr}D] \quad \left(\mathrm{Tr}P_\gamma = -\frac{\varphi_0^2}{3}\mathrm{Sp}\,D\right). \quad (113)$$

It is known that, as a rule, the infrared spectrum $\langle P_\gamma\rangle$ in gauge theories is nonperturbative, while deviations from it $\bar{P}_\gamma$ are to be calculated by perturbation theory.

4.5. Universe Evolution As a Zero Mode Solution of the Constraint Equation

The separation of actual dynamical variables from nondynamical ones is a crucial stage of the extraction of the physical information adequate to symmetry principles, as well as the variational equations including both equations of motion and constraint equations. This goal can be realized by two methods, namely, the conventional method of imposing gauge conditions [23] (type of Dirac's minimal surface condition (100)) and the method of explicitly solving the variational equations with constraints. The latter was developed by Dirac, who considered the Gauss equations of quantum electrodynamics as an example [11].

The explicit solution of the variational constraint equations, similar to the Gauss conditions in electrodynamics or the conditions $T_k^0(\varphi_0|F) = 0$ in GR, makes it possible to allow for the dynamics in the function class that is defined by the gauge transformation group and by the condition of finite energy density. For example, the Gauss condition $\partial_1 E(x^0, x^1) = 0$ in the Schwinger two-dimensional electrodynamics was shown in [48] to have the explicit, nontrivial, homogeneous solution $E(x^0, x^1) = E_0(x^0)$, so-called zero mode, which determines the topology and mass spectrum of the theory.

A similar, nontrivial, homogeneous solution of the Gauss equation

$$\frac{\delta S_{\rm GR}}{\delta N^k} \equiv T_k^0 = 2\nabla_i P^i_{\gamma k} = 0 \tag{114}$$

also exists in GR, where $S_{\rm GR}$ is metric action (92) in GR.

As is seen, the zero Fourier component of metric momentum (112)

$$[P^i_{\gamma k}]_{\rm particular} = \frac{1}{3}\delta^i_k\langle {\rm Tr} P_\gamma\rangle \neq 0 \tag{115}$$

arises as a particular solution of the Gauss equation

$$T_k^0 = \frac{2}{3}\nabla_i[\delta^i_k\langle {\rm Tr} P_\gamma\rangle] = \frac{2}{3}\partial_k\langle {\rm Tr} P_\gamma\rangle = 0. \tag{116}$$

The existence of such a homogeneous solution of the Gauss equation in GR follows from the invariance of the equation

$$T_k^0(\varphi_0|F) = T_k^0(\varphi|\bar{F}) = 0 \tag{117}$$

with respect to scale transformations of the metric and all the fields, with a scale defined in the class of homogeneous functions

$$^{(n)}F = {}^{(n)}\bar{F}a^n(x^0), \quad g_{\mu\nu} = \bar{g}_{\mu\nu}a^2(x^0). \tag{118}$$

Here, (n) is the conformal weight and $\varphi(x^0) = \varphi_0 a(x^0)$ is the running mass scale.

If all the variables, except for the scalar factor $a(x^0)$, are ignored, then the tensor $T_k^0(\varphi|\bar{F})$ is identically equal to zero. Therefore, the scale factor $a(x^0)$ also can be referred to as a zero mode solution of constraint equation (117).

As was shown in Section 4, it is the factor $a(x^0)$ that is unambiguously related to cosmological evolution and to the time as a dynamical variable [45]. To avoid double counting of variables in the conventional cosmological perturbation theory [49] the strong condition $\int d^3x\bar{D} \equiv 0$ must be imposed, where D is determined by Eq. (98) conserving the number of GR variables [47].

In what follows we show that the allowance for the zero mode of the Gauss condition $T_k^0(\varphi|\bar{F}) = 0$ makes it possible to combine the three GR branches: noninvariant island-universe theory [20] (in which the cosmological scale factor is identically equal to unity and the symmetry with respect to the reparametrization of the coordinate time is violated by its global synchronization), Hamiltonian cosmology [41] (in which the cosmological scale factor is treated as a dynamical variable), and cosmological perturbation theory (in which the variable similar to the cosmological scale factor is allowed for two times [49–51]).

4.6. Separation of Zero Mode and Reduction of Action

Using transformation (118) we rewrite action (92) in the form

$$S[\varphi_0|F] = S[\varphi|\bar{F}] - \int dx^0 \frac{(\partial_0\varphi)^2}{N_0}. \tag{119}$$

Here, $S_h[\varphi|\bar{F}]$ coincides with Dirac action (101), the fields F are substituted by the fields $\bar{F}$ with running masses φ, the quantity

$$\varphi(x^0) = \varphi_0 a(x^0) \tag{120}$$

is the running mass scale, and the equation

$$N_0(x^0)^{-1} = V_0^{-1}\int_{V_0} d^3x N_d^{-1}(x^0, x^i) \equiv \langle N_d^{-1}\rangle \tag{121}$$

is the averaging of the inverse lapse function N_d^{-1} over spatial volume $V_0 = \int d^3x$. Therefore, the lapse function N_0 determines the geometric time ζ

$$d\zeta = N_0(x^0)dx^0, \quad \zeta(x^0) = \int^{x^0} d\bar{x}^0 N_0(\bar{x}^0), \tag{122}$$

in reference frame (152).

The Hamiltonian action (up to a total derivative) takes the form

$$S_h[\varphi_0|F] = S_h[\varphi|\bar{F}] - \int dx^0\left[P_\varphi\partial_0\varphi - N_0\frac{P_\varphi^2}{4V_0}\right], \quad (123)$$

where the canonical momentum

$$P_\varphi = 2V_0\varphi' \equiv 2V_0 d\varphi/d\zeta$$

defines the energy in the space $[\varphi|\bar{F}]$ of events.

The energy constraint $\frac{\delta S[\varphi|\bar{F}]}{\delta N_d} = 0$ yields the equation

$$\frac{N_0^2 P_\varphi^2}{N_d^2} = \mathcal{H}_t(\varphi|\bar{F}), \quad (124)$$

where $\mathcal{H}_t(\varphi|\bar{F}) = |\gamma| T_0^0(\varphi|\bar{F})$ coincides in form with expression (103) after the corresponding change of variables $(\varphi_0|F) \longrightarrow (\varphi|\bar{F})$. This equation has the solution

$$N_d = N_0\frac{\langle\sqrt{\mathcal{H}_t}\rangle}{\sqrt{\mathcal{H}_t}}, \quad (125)$$

$$P_{\varphi(\pm)} = \pm 2V_0\varphi' = \pm 2V_0\langle\sqrt{\mathcal{H}_t}\rangle. \quad (126)$$

Substituting this solution into action (123), we arrive at the relativistic-universe action

$$S_\pm[\varphi_I|\varphi_0]\big|_{\text{energy constraint}}$$
$$= \int_{\varphi_I}^{\varphi_0} d\varphi\left\{\int d^3x\left[\sum_{\bar{F}}\bar{P}_F\partial_\varphi\bar{F} + \bar{\mathcal{C}} \pm 2\sqrt{\mathcal{H}_t(\varphi|\bar{F})}\right]\right\} \quad (127)$$

moving in the field space of events $[\varphi|\bar{F}]$, where $\bar{\mathcal{C}} = \mathcal{C}/\partial_0\varphi$, φ_I is the point of universe creation or annihilation. Equations (127) unambiguously determine all the fields $\bar{F}(\varphi, x^i)$ as functions of the dynamical evolution parameter φ and initial data of these fields at the time instant $\varphi(\zeta = 0) = \varphi_I$ of universe creation. The geometric time can be found from Eq. (126) [27]:

$$\zeta(\varphi_0|\varphi_I) = \theta(\varphi_0 - \varphi_I)\int_{\varphi_I}^{\varphi_0}\frac{d\varphi}{\langle\sqrt{\mathcal{H}_t}\rangle}$$
$$+ \theta(\varphi_I - \varphi_0)\int_{\varphi_0}^{\varphi_I}\frac{d\varphi}{\langle\sqrt{\mathcal{H}_t}\rangle} \geq 0. \quad (128)$$

We here allow for the quantum anomaly of geometric interval, which follows from the stability condition for the quantum system. This was illustrated above with SR and cosmology examples. The quantum anomaly implies an absolute origin of geometric time.

4.7. Problem of Initial Data in General Relativity

The initial data φ_I and $H_I = \varphi_I'/\varphi_I$ can be treated as a creation (an annihilation) point of the universe with a positive (negative) energy $P_\varphi \geq 0$ ($P_\varphi \leq 0$).

The initial value problem in GR is a problem of finding the dependence of metric and fields on a coordinate time from the equations invariant with respect to the reparametrization of this time. To solve this problem we pick out the invariant dynamical evolution parameter φ as a scale factor and show that the action contains the invariant homogeneous geometric interval $d\zeta = N_0 dx^0$ corresponding to this parameter. The extraction of the scale factor $a = \varphi/\varphi_0$ ($\varphi_0^2 = M_{\text{Planck}}^2 3/8\pi$) by the conformal transformation $F^{(n)} = a^n\bar{F}^{(n)}$, where n is the conformal weight, allows us to describe the universe motion covering a hypersurface in the field space $[\varphi|\bar{F}]$ of events (Fig. 2).

These equations allow us to unambiguously determine the dependence of the fields $\bar{F}$ on the dynamical evolution parameter φ if the initial data $\bar{F}_I(x^i) = \bar{F}(\varphi_I, x^i)$ and $P_{FI}(x^i) = P_{FI}(\varphi_I, x^i)$ are given. Here, φ_I and $P_{\varphi I}$ are treated as the coordinates of a point of universe creation (or annihilation) with a positive energy $E = P_\varphi \geq 0$ in the phase space.

In what follows we assume that both GR and SR can be interpreted only within the framework of quantum theory with the vacuum as a lowest-energy state. The universe stability condition in the form of this postulate leads to the quantum anomaly and an absolute origin of the global time, $\zeta = 0$. Cosmology involves the problem of defining initial data for the universe creation, $\varphi(\zeta = 0) = \varphi_I$ and $P_\varphi(\zeta = 0) = P_{\varphi I}$. Time, matter, and temperature (as a characteristic of matter motion) are also created together with the universe. For this problem to be solved theoretical quantities should be identified with observable ones in accordance with the correspondence principle for classical theory in flat spacetime.

4.8. Low-Energy Dynamics and Correspondence Principle

Thus, there is a direct analogy [39] between GR and the relativistic particle dynamics considered in Section 2. This analogy can be extended to the definition of observables (similar to time, particle number, and one-particle energy) by using the correspondence of GR to the theory of classical fields $\bar{F}$ in a flat spacetime. In this case, field energy is much less than a huge energy of homogeneous cosmological density $\rho_0(\varphi)$.

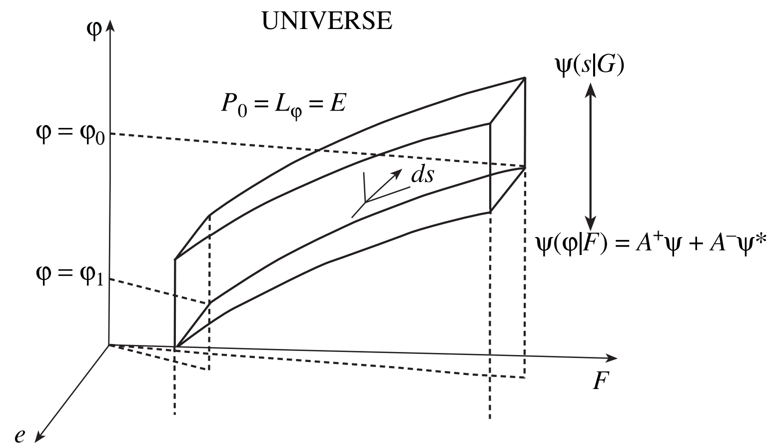

Fig. 2. Universe moving on its hypersurface in the world field space of events. Every observer in the universe has the two sets of measurable quantities, namely, the field set (mass φ, particle number densities $a_q a_q^+$ with quantum numbers q) and the geometric set (time interval η and initial data for Bogoliubov quasiparticle number density $b_q b_q^+$) [54].

The local energy density $T_{t0}^0(\varphi|\bar{F})$ entering into Eq. (103) can be written out as a sum of the homogeneous cosmological density $\rho_0(\varphi)$ and the local density of particle-like perturbations in field theory:

$$T_{t0}^0(\varphi|\bar{F}) = \rho_0(\varphi) + \mathcal{T}_{t0}^0(\varphi|\bar{F}). \quad (129)$$

We then expand the square root $\sqrt{T_{t0}^0}$ in reduced action (127) in powers of the local density,

$$\begin{aligned} &d\varphi 2\sqrt{\rho_0(\varphi) + \mathcal{T}_{t0}^0} \\ &= d\varphi\left[2\sqrt{\rho_0(\varphi)} + \frac{\mathcal{T}_{t0}^0}{\sqrt{\rho_0(\varphi)}}\right] + \ldots, \end{aligned} \quad (130)$$

use the definition $d\eta = d\varphi/\sqrt{\rho_0(\varphi)}$ of the conformal time, which coincides with the geometric time ζ in this approximation, and present reduced action (127) in the form[6]

$$S^{(\pm)}[\varphi_I|\varphi_0]\Big|_{\text{energy constraint}} = S_{\text{cosmic}}^{(\pm)} + S_{\text{field}}^{(\pm)} + \ldots. \quad (131)$$

Here, the first term is cosmological action (47)

$$S_{\text{cosmic}}^{(\pm)}[\varphi_I|\varphi_0] = \mp 2V_0\int_{\varphi_I}^{\varphi_0} d\varphi\sqrt{\rho_0(\varphi)} \quad (132)$$

considered above in Section 2. The second term is the conventional field action written in terms of the conformal time,

$$S_{\text{field}}^{(\pm)} = \int_{\eta_I}^{\eta_0} d\eta\int d^3x\left[\sum_F P_F\partial_\eta F + \bar{\mathcal{C}} - \sqrt{\bar{\gamma}}\mathcal{T}_{t0}^0\right], \quad (133)$$

with the running masses $m(\eta) = a(\eta)m_0$ describing cosmological particle creation from the vacuum [55]. In the limiting case, in which the evolution of the universe and particle creation are ignored ($\varphi \simeq \varphi_0$), action (133) coincides with GR action (105) in the synchronous-time gauge:

$$\begin{aligned} &S_{\text{Bjern}}[x_{\text{synchro}}^0 = \eta, T_{t0\,\text{synchro}}^0 = \mathcal{T}_{t0}^0] \\ &\quad = S_{\text{field}}^{(\pm)}[\varphi(\eta) = \varphi_0]. \end{aligned} \quad (134)$$

As is seen from the comparison of invariant action (133) with action (105) in the synchronous-time gauge, the conformal time η serves as world observable time, just so the geometric interval in Newtonian mechanics (42) serves as absolute observable time. Thus, according to the correspondence principle, the measured cosmological time is to be identified with the conformal time $d\eta$ rather than with the Friedmann time $dt = a(\eta)d\eta$.

4.9. Relative Units of Measurement

The identification of measured quantities with conformal ones corresponds to the choice of relative length units. In this case, a homogeneous dilation of all the spatial intervals, $L = aL_c$ implies the dilation of the length units, $l = al_c$, so that a measured interval, defined as the ratio $L_r = aL_c/al_c = L_c/l_c$ of the two expanding quantities, does not vary.

[6] In quantum field theory, the interaction $\sqrt{\rho_0 + \mathcal{T}_{(2)} + \mathcal{T}_I} = \sqrt{\rho_0 + \mathcal{T}_{(2)}} + \mathcal{T}_I/\sqrt{\rho_0 + \mathcal{T}_{(2)}}$ is initially separated. This yields a form factor, which makes it possible to partially eliminate ultraviolet divergences [27].

This yields a cosmological model of the evolution of the universe, in which all measurable quantities are identified with conformal ones. Namely, the conformal time $d\eta$, coordinate distance r, running masses $m = m_0/(1+z)$, and conformal temperature $T_c(z) = T(z)/(1+z)$ are identified with Friedmann time $dt = a(\eta)d\eta$, Friedmann distance $R = r/(1+z)$, constant masses m_0, and conventional temperature $T(z)$, respectively. In this case, the redshift of spectral lines of atoms at cosmic objects,

$$\begin{aligned}\frac{E_{\text{emission}}}{E_0} &= \frac{m_{\text{atom}}(\eta_0 - r)}{m_{\text{atom}}(\eta_0)} \equiv \frac{\varphi(\eta_0 - r)}{\varphi_0} \\ &= a(\eta_0 - r) = \frac{1}{1+z},\end{aligned} \tag{135}$$

is explained by the running masses. In the conformal cosmological model, the conformal observable distance r loses the factor a, in comparison with the Friedmann distance $R = ar$. In this case, the recent distance–redshift data from Ia supernovae [52] was shown in [38] to be compatible with the equation of rigid state

$$p = \rho(\varphi) = \frac{Q^2}{4V_0^2\varphi^2} \tag{136}$$

and with the dependence $a(\eta) = \sqrt{1 + 2H_0(\eta - \eta_0)}$ of the scale factor on the conformal (i.e., observable) time. It is the dependence that is used to explain the chemical evolution of the universe [53].

The evolution of the universe in relative units was shown to differ from that in absolute units of Standard Cosmology. In fact, the temperature evolution of the expanding universe in relative units looks like the mass evolution of a cold universe with a constant temperature of cosmic background radiation [38, 54, 55].

There is one more argument in favor of relative units, namely, the large deficit of luminous masses, $M/M_L \geq 10$ (where M_L is the mass of luminous matter), in all superclusters with a mass of $M \geq 10^{15} M_\odot$ and a size of $R \gtrsim 5$ Mpc [56]. In this case, the Newtonian velocity is less than the cosmic one by an order of magnitude. In terms of relative units, this deficit can be caused [56] by galaxy deceleration in the course of the cosmological evolution of galaxy masses (135).

The evolution of the universe as a mechanism of cosmic-object deceleration in a central gravitational field appears to be proposed by Einstein and Strauss in 1945 [57]. This idea was then developed in terms of conformal variables and coordinates in [56], where the cosmological evolution was shown to cause an energy decrease and the formation of galaxy and its clusters because of the capture of cosmic objects by central gravitational field. In fact, the definition of total particle energy, $E(\eta) = \dfrac{p^2}{2m_0 a(\eta)} - \dfrac{\alpha m}{r}$ (where p is the momentum and $\alpha = GM_\odot m = r_g m/2$ is the Newton constant), follows from the definition of the conformal one-particle energy with the running mass $m(\eta) = a(\eta)m_0$ and from Newtonian potential. Therefore, energy is not conserved in contrast to the energy of a particle with constant mass in Newtonian mechanics. The scale factor $a(\eta)$ increases as the energy varies from positive to negative values. The energy goes to zero at a time $\eta = \eta_I$, which is the time of galaxy capture described in [56]. The galaxy capture could be a crucial argument to obviate one of fundamental troubles of the modern theory of galaxy formation: how does a system of unbound particles with positive energy turn into a system of bound particles with negative energy?

On the other hand, cosmic evolution allows us to find the range of applicability of the Newtonian mechanics when describing galaxies. The range is determined by the critical radius $R_{\text{cr}} \simeq 10^{20}$ cm $(M/M_\odot)^{1/3}$, at which the Newtonian orbital velocity in absolute units coincides with the Hubble velocity of galactic dilation relative to the center of the supercluster containing the galaxy. According to [56], the Newtonian dependence of the orbital velocity on the distance to the supercluster center,

$$v_{\text{Newtonian orbital}}(R_{\text{circle}}) = \sqrt{\frac{r_g}{2R_{\text{circle}}}},$$

for circular trajectories with $\dot{R}_{\text{circle}} = \ddot{R}_{\text{circle}} = 0$ ($R_{\text{circle}} = a(t)r$, $dt = a(\eta)d\eta$) should be substituted by the dependence

$$v_{\text{Cosmic orbital}}(R_{\text{circle}}) = \sqrt{\frac{r_g}{2R_{\text{circle}}} + \gamma(R_{\text{circle}}H)^2}, \tag{137}$$

provided that cosmic evolution is taken into account. Here, $\gamma = \left[2 - \left(\frac{3}{2}\Omega_{\text{matter}} + 3\Omega_\Lambda\right)\right]$. In the cosmology model with cosmological quantities identified with conformal ones, we have

$$\gamma(\Omega_{\text{rigid}} = 1) = 2.$$

In this case, the deficit of visible matter decreases in contrast to the Standard Cosmology, in which the quantity γ is negative,

$$\gamma(\Omega_\Lambda = 0.7, \Omega_{\text{matter}} = 0.3) \simeq -1/2,$$

and much more dark matter is required. Moreover, as is seen from Eq. (137), in the conventional model with absolute units of measurement the orbital velocity could be imaginary.

4.10. Universe Creation

Thus, to define the initial data of the creation of the universe we consider the universe evolution model (with relative units of measurement), in which the kinetic energy of a slightly self-interacting scalar field can serve as a primordial vacuum dark matter in an

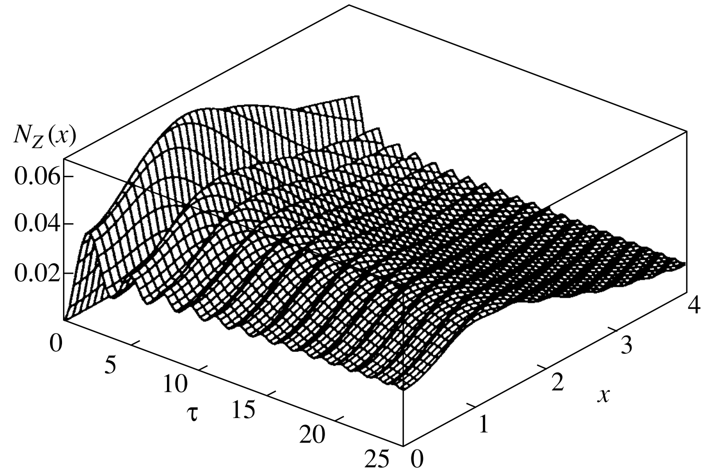

Fig. 3. Longitudinal $N_Z(x)$ components of the boson distribution function versus the dimensionless time $\tau = 2\eta H_I$ and dimensionless momentum $x = q/M_I$ under the initial condition $M_I = H_I$ ($\gamma_v = 1$) (according to [54, 55, 58]).

empty universe.[7] This yields the evolution law in the form of the equation of rigid state (136).

The equation of state (136) is the most singular one. Therefore, it is natural to assume that the universe is governed by this state at instant of creation. There were no particles, time, or temperature before the instant of the creation of the universe. We remind that temperature of a system is a characteristic of collisions and scattering of its particles. If particles are absent, the temperature is also absent.

We will describe the quantum universe creation in GR in the low-energy approximation considered in Section 4.8, when the action is divided into the parts describing universes by Eq. (47) (or (132)) and particles by Eq. (133).

In this approximation, the second quantization with the postulate of the vacuum as a lowest-energy state is a general method of describing the cosmological creation of both universe (47) and particles (133) [42].

In the case of the equation of rigid state (136), the energy takes the form $E(\varphi) = Q/\varphi$, where the parameter Q coincides with the integral of motion $Q = 2V_0\varphi_I^2 H_I = 2V_0\varphi_0^2 H_0$, which is referred to as a scale charge.

If the scale charge of the vacuum is equal to zero, the two universes with opposite scale charges $\pm|Q|$ are created simultaneously. In terms of the Standard Cosmology, they correspond to expanding (+) and contracting (–) universes:

$$\varphi_{\pm Q}^2 = \varphi_I^2 \pm \frac{Q}{2V_0}\eta. \qquad (138)$$

[7] In present-day literature, the vacuum dark matter is referred to as a quintessence [38].

Vacuum postulate (85) can exclude negative energies but it can't forbid the Hubble parameter to be negative.

In the model with the event energy taking the form $E(\varphi) = Q/\varphi$, the universe distribution function N_U given by Eq. (86) can be found since Eqs. (88) can be solved exactly. In this case,

$$N_U = \frac{1}{4Q^2 - 1}\sin^2\left[\sqrt{Q^2 - \frac{1}{4}}\ln\frac{\varphi}{\varphi_I}\right] \neq 0. \qquad (139)$$

In what follows, we will mainly consider the case of expanding universe.

4.11. Matter Creation from the Vacuum

As was shown in [54, 55, 58] within the framework of the conformal cosmological model considered above, there is an intense resonance vector–boson production from the vacuum in the early universe when its horizon $H_I^{-1} = a_I^2 H_0^{-1}$ coincides with the Compton wavelength $M_I^{-1} = a_I^{-1} M_W^{-1}$ of W- and Z-bosons. This effect is due to these bosons being the only Standard Model particles having the zero–mass production singularity. The initial data for the boson pair production, $a_I^2 = [H_0/M_W]^{2/3} = 10^{-29}$, follow from the uncertainty relation $\Delta E \Delta\eta = 1$ for the boson pair production with the energy $2M_I$ during the universe lifetime $(2H_I)^{-1}$. This corresponds to the redshift $z_I \simeq \sqrt{10^{29}}$. According to calculations, the boson density in units of the initial Hubble parameter reaches equilibrium very rapidly, relativistic bosons make a dominant contribution (see Fig. 3) [54, 55], and the boson temperature is equal to

$$T_{\text{boson}} \sim (M_I^2 H_I)^{1/3} = (M_{W0}^2 H_0)^{1/3} \sim 3\text{K}, \qquad (140)$$

which is an integral of motion of the supernova evolution [52]. The cosmic background radiation temperature succeeds to the temperature of decaying bosons.

The boson lifetime [55]

$$\tau_W = 2H_I\eta_W \simeq \left(\frac{2}{\alpha_g}\right)^{2/3} \simeq 16, \qquad \tau_Z \sim 2^{2/3}\tau_W \sim 25 \tag{141}$$

determines the present-day baryon density

$$\Omega_b \sim \alpha_g = \alpha_{\rm QED}/\sin^2\theta_W \sim 0.03. \tag{142}$$

The ratio of the created matter density $\rho_v(\eta_I)$ to the primordial scalar–field (quintessence) density $\rho_{\rm cr}(\eta_I) = H_I^2\varphi_I^2$ is extremely small:

$$\frac{\rho_{\rm v}(\eta_I)}{\rho_{\rm cr}(\eta_I)} \sim \frac{M_I^2}{\varphi_I^2} = \frac{M_W^2}{\varphi_0^2} \sim 10^{-34}. \tag{143}$$

Transverse bosons produce a baryon asymmetry of the universe because they cause the polarization of the Dirac left-fermion vacuum according to the Standard Model selection rules [59]: the difference of baryon and lepton numbers is conserved, but their sum is not. The superweak interaction [60], observed experimentally and responsible for the CP-violation with the constant $X_{\rm CP} \sim 10^{-9}$, makes the baryon asymmetry of the universe freeze up at the density

$$\rho_b(\eta = \eta_L) \simeq 10^{-9} \times 10^{-34}\rho_{\rm cr}(\eta = \eta_L). \tag{144}$$

The consequent evolution in the cold universe repeats the well-known hot-universe scenario [53] because such evolution is determined by the conformally invariant mass-temperature ratio m/T. The baryon asymmetry increases with mass, while the primordial quintessence density varies inversely with the mass squared. Because of this, the present-day baryon density, estimated from Eqs. (141), (143), and (144),

$$\Omega_{\rm b}(\eta_0) = \left[\frac{\varphi_0}{\varphi_L}\right]^3 \times 10^{-43} \sim 10^{43}\left[\frac{\eta_I}{\eta_L}\right]^{3/2} \times 10^{-43} \sim \left[\frac{\alpha_{\rm QED}}{\sin^2\theta_W}\right] \sim 0.03 \tag{145}$$

is in close agreement with experimental data.

Thus, there is a reparametrization–invariant description of the Dirac Hamiltonian formulation of GR and cosmology within the framework of metric formalism.

4.12. Results

The reparametrization-invariant description of reference frames in GR yields the following results.

(i) A zero–mode solution of the GR variational equation $T_k^0(\varphi_0|F) = 0$ is proved to exist in the class of functions defined by the gauge group of the Hamiltonian approach and by the finiteness condition for energy density.

(ii) The zero mode of the Gauss condition $T_k^0 = 0$ is identified with the cosmological scale factor. This makes it possible to join the two theories: the nonivariant theory of the island universe [20] (where the cosmological scale factor is identically equal to unity, and the symmetry with respect to the reparametrization of the coordinate time is violated by global synchronization) and Hamiltonian cosmology [41] (where the cosmological scale factor is considered as a dynamical variable).

(iii) The identification of the cosmological scale factor and its canonical momentum with the invariant evolution parameter and the universe energy, respectively, together with the postulate of the quantum theoretical vacuum as the lowest-energy state, allow us to define an absolute origin of the geometric interval s. The vacuum postulate eliminates the cosmological singularity of the universe if its reparametrization-invariant energy of events is positive [28].

(iv) The problem of universe homogeneity is resolved by defining the cosmological equations as exact GR equations averaging over spatial volume in a specific reference frame [54, 55].

(v) The reparametrization-invariant evolution parameter φ given in a specific reference frame and the absolute origin $\zeta = 0$ of the geometric interval determine the following initial data for the GR field variables $F(\varphi, x)$: $F_I = F(\varphi_I, x)$, where $\varphi_I = \varphi(\zeta)|_{\zeta=0}$.

(vi) The theoretical description in terms of the Standard Model [54, 55], when applied to the creation of matter in the universe, its energy balance, and a series of cosmological observations and astrophysical data, testifies to the existence of an initial value of the scale factor φ_I, which corresponds to the redshift $z^2 \sim 10^{29}$.

However, the metric formalism does not define inertial reference frames interrelated by Lorentz transformations. Such transformations can be defined only in the tetrad formalism of GR.

5. REFERENCE FRAMES IN TETRAD FORMALISM

5.1. Action, Linear Forms, and Symmetry

Tetrad formalism is required in GR to separate general covariant transformations from transformations of reference frames.

We consider Hilbert–Einstein action (92) in the space defined by the interval

$$ds^2 = \eta^{\alpha\beta}\omega_{(\alpha)}\omega_{(\beta)} \equiv g_{\mu\nu}dx^\mu dx^\nu, \tag{146}$$

$$\eta^{ab} = \mathrm{diag}(1 - 1 - 1 - 1), \tag{147}$$

Here,

$$\omega_{(\alpha)} = e_{(\alpha)\mu}dx^{\mu} \tag{148}$$

are the Cartan linear forms [20, 21], and the coefficients $e_{(\alpha)\mu}$ of expansion over differentials of a coordinate space are referred to as tetrads. These forms allow us to include fermions and other fields f of the Standard Model,

$$S[\varphi_0|F = e, f] = S_{\rm GR}[\varphi_0|e] + S_{\rm SM}[\varphi_0|F] \tag{149}$$

and to separate gauge transformations from transformations of reference frames [61].

The Cartan forms $\omega_{(\alpha)}$ are invariant with respect to the general coordinate transformations

$$x^{\mu} \longrightarrow \tilde{x}^{\mu} = \tilde{x}^{\mu}(x^0, x^1, x^2, x^3), \tag{150}$$

which are treated as gauges. The forms transform as a vector representation of Lorentz group treated as a reference frame transformation group. An example of such transformation is

$$\begin{cases} \bar{\omega}_{(0)} = \dfrac{\omega_{(0)} - V\omega_{(1)}}{\sqrt{1-V^2}} \\ \bar{\omega}_{(1)} = \dfrac{\omega_{(1)} - V\omega_{(0)}}{\sqrt{1-V^2}} \\ \bar{\omega}_{(2)} = \omega_{(2)} \\ \bar{\omega}_{(3)} = \omega_{(3)}. \end{cases} \tag{151}$$

It is known that transformations of reference frames do not change the number of variables, while gauge transformations result in the constraints decreasing this number [23].

5.2. Reference Frame

The choice of a Lorentz reference frame in GR implies the fixation of Lorentz indices (α) in Cartan forms (148) and the subdivision of their components into timelike $\omega_{(0)}$ and spacelike $\omega_{(\alpha)}$ components.

The Hamiltonian dynamics is formulated in a specific Lorentz reference frame in terms of the Cartan forms [22–24]:

$$\omega_{(0)} = Ndx^0, \tag{152}$$

$$\omega_{(a)} = e_{(a)i}(dx^i + N^i dx^0). \tag{153}$$

Here, the triads $e_{(a)i}$ form the spatial metric

$$\gamma_{ij} = e_{(a)i}e_{(a)j}, \quad \gamma^{ij} = e^j_{(a)}e^i_{(a)}.$$

According to Dirac [23], the spatial metric determinant can be extracted by taking out the factor ψ^2 from the triads

$$e_{(a)i} = \psi^2\mathbf{e}_{(a)i}, \quad \det|\mathbf{e}| = 1, \tag{154}$$

$$N = N_d\psi^6. \tag{155}$$

The symmetric parameterization $\mathbf{w}_{(a)} = \mathbf{e}_{(a)i}dx^i$ of Cartan forms can be used as a nonlinear realization of equi-affine symmetry [61].

The Dirac parameterization defines a set of hypersurfaces $x^0 = \text{const}$ with a unit vector $\nu^{\alpha} = (1/N, -N^k/N)$ perpendicular to the hypersurfaces. The second form (extrinsic curvature) is defined by the equation

$$\begin{aligned} \pi_{(a)i} &= \frac{1}{N_d}[(\partial_0 - N^l\partial_l)e_{(a)i} - e_{(a)l}\partial_i N^l] \\ &= \psi^2[v_{(a)i} + 2\mathbf{e}_{(a)i}v_{\psi}], \end{aligned} \tag{156}$$

where the velocities

$$v_{(a)i} = \frac{1}{N_d}\left[(\partial_0 - N^l\partial_l)\mathbf{e}_{(a)i} + \frac{1}{3}\mathbf{e}_{(a)i}\partial_l N^l - \mathbf{e}_{(a)l}\partial_i N^l\right] \tag{157}$$

and

$$v_{\psi} = \frac{1}{N_d}\left[(\partial_0 - N^l\partial_l)\ln\psi - \frac{1}{6}\partial_l N^l\right] \tag{158}$$

indicate that this hypersurface is embedded into a four-dimensional spacetime.

After performing transformation (154), one can write out the intrinsic three-dimensional scalar curvature ${}^{(3)}R(e)$ in the form

$${}^{(3)}R(e) = \frac{1}{\psi^4}{}^{(3)}R(\mathbf{e}) + \frac{8}{\psi^5}\Delta\psi, \tag{159}$$

where ${}^{(3)}R(\mathbf{e})$ is the curvature expressed in terms of the tetrads $\mathbf{e}_{(a)i}$: $\Delta\psi = \partial_i(\mathbf{e}^i_{(a)}\mathbf{e}^j_{(a)}\partial_j\psi)$.

5.3. Invariant Evolution Parameter

Thus, GR and cosmology provide a series of arguments in favor of the identification of the GR invariant evolution parameter with the cosmological scale factor. This factor can be introduced into theory (149) by the conformal transformation of all the fields with the conformal weight (n), $F^{(n)} = a^n\bar{F}^{(n)}$, including the Cartan forms

$$\omega_{(\alpha)} = a\bar{\omega}_{(\alpha)}, \quad \psi^2 = a\bar{\psi}^2. \tag{160}$$

In general, such a transformation is used as a definition of the cosmological perturbation theory [50, 51]. In the gauge-invariant Hamiltonian approach to GR considered in the preceding section, the scale factor should be treated as a dynamical variable used as an invariant evolution parameter. To define a complete set of Hamiltonian canonical momenta the number of variables of the theory should be restored by eliminating the zero Fourier harmonic from the spatial metric determinant with the help of a constraint condition. To find this condition, we perform transformation (160) of action (149) and obtain the equation

$$S[\varphi_0|F] = S[\varphi|\bar{F}] - V_0\int dx^0\frac{(\partial_0\varphi)^2}{N_0}. \qquad (161)$$

Here, $V_0 = \int d^3x$ is the volume of the Dirac coordinate space. The averaged lapse function $N_0(x^0)$,

$$N_0(x^0)^{-1} = V_0^{-1}\int\limits_{V_0} d^3x\bar{N}_d^{-1}(x^0, x^i) \equiv \langle\bar{N}_d^{-1}\rangle \qquad (162)$$

determines the geometric time ζ,

$$d\zeta = N_0(x^0)dx^0, \qquad (163)$$

and $S[\varphi|\bar{F}]$ is the sum of the GR action

$$\begin{aligned} &S_{GR}[\varphi|e] \\ &= \int dx^0d^3x(\mathbf{K}[\varphi|e] - \mathbf{P}[\varphi|e] - \mathbf{S}[\varphi|e]) \end{aligned} \qquad (164)$$

and SM action (149), with all masses, including the Planck mass, scaled by the factor $\varphi = \varphi_0 a$. Action (164) is the sum of the kinetic $\mathbf{K}$, potential $\mathbf{P}$, and quasisurface $\mathbf{S}$ terms:

$$\mathbf{K}[\varphi|e] = \overline{N_d}\varphi^2\left[-4\overline{v_\psi}^2 + \frac{v_{(ab)}^2}{6}\right], \qquad (165)$$

$$\mathbf{P}[\varphi|e] = \frac{\overline{N_d}\varphi^2\psi^{12}}{6}{}^{(3)}R(e), \qquad (166)$$

$$\begin{aligned} \mathbf{S}[\varphi|e] &= 2\varphi^2[\partial_0\overline{v_\psi} - \partial_l(N^l\overline{v_\psi})] \\ &- \frac{\varphi^2}{3}\partial_j[\psi^2\partial^j(\psi^6 N_d)]. \end{aligned} \qquad (167)$$

Here, we used definitions (157), (158), $v_{(ab)} = \frac{1}{2}(\mathbf{e}_{(a)i}v^i_{(b)} + \mathbf{e}_{(b)i}v^i_{(a)})$, and

$$\overline{v_\psi} = \overline{N_d^{-1}}\left[(\partial_0 - N^l\partial_l)\log\bar{\psi} - \frac{1}{6}\partial_l N^l\right], \qquad (168)$$

which is the velocity of the logarithm of spatial determinant

$$\log\psi^2 = \log a(x^0) + \log\bar{\psi}^2. \qquad (169)$$

After removing scale factor (160), the part of action (163) that contains the spatial determinant takes the form

$$\begin{aligned} L_{\rm SD} &= -\int d^3x N_d[4\varphi^2(\overline{v_\psi})^2 + 4\varphi v_\varphi\overline{v_\psi} + (v_\varphi)^2] \\ &= \int d^3x\mathcal{L}_{\rm SD}. \end{aligned} \qquad (170)$$

Here, the first term is due to the kinetic part $\mathbf{K}[\varphi|e]$; the second term is derived from the quasisurface part $\mathbf{S}[\varphi|e]$, which is not a total derivative because of time-dependence of the scale factor; and the third term is the quadratic action for the scale factor.

The scale factor is a dynamical variable, therefore its canonical momentum can be obtained by variation of Lagrangian (170) with respect to the time derivative $\partial_0\varphi$:

$$\begin{aligned} P_\varphi &\equiv \frac{\partial L_{\rm SD}}{\partial(\partial_0\varphi)} \\ &= -\int d^3x[4\varphi v_\psi + 2v_\varphi] \equiv -[4\varphi V_\psi + 2V_\varphi]. \end{aligned} \qquad (171)$$

In this case, the average of the canonical momentum of the spatial determinant takes the form

$$\begin{aligned} P_\psi &\equiv -\int d^3x\frac{\partial\mathcal{L}_{\rm SD}}{\partial(\partial_0\log\bar{\psi})} \\ &= -\int d^3x\bar{p}_\psi = -2\varphi[4\varphi V_\psi + 2V_\varphi], \end{aligned} \qquad (172)$$

where $V_\varphi = \int d^3x v_\varphi$, $V_\psi = \int d^3x v_\psi$, and $\varphi' = d\varphi/d\zeta$. It is easy to prove that the velocities V_φ and V_ψ can not be expressed in terms of canonical momenta P_φ and P_ψ because, Eqs. (171) and (172) form a degenerate system having no solutions.

This means that action (170) is singular, because of the double counting of the spatial determinant.

To avoid double counting, the field variable $\log\bar{\psi}$ in Eq. (169) should be defined in the class of functions under strong constraint conditions

$$\int\limits_{V_0} d^3x\log\bar{\psi} \equiv 0, \quad \int\limits_{V_0} d^3x\overline{v_\psi} \equiv 0. \qquad (173)$$

These conditions mean that the scale factor and its velocities must be orthogonal to the logarithm of spatial determinant $\log\bar{\psi}$.

Under these conditions, action (161) takes the form

$$\begin{aligned} S[\varphi_0|F] = \int dx^0\Bigg\{ &V_0\varphi(x^0)\partial_0\left(\frac{\partial_0\varphi}{N_0}\right) \\ &+ \int d^3x(K[\varphi|\bar{e}] - \mathbf{P}[\varphi|\bar{e}] + \mathcal{L}_{\rm SM}(\varphi|F))\Bigg\}, \end{aligned} \qquad (174)$$

where $\mathcal{L}_{\rm SM}$ is the SM Lagrangian density. The quantities N_0, $\mathbf{K}[\varphi|\bar{e}]$, and $\mathbf{P}[\varphi|\bar{e}]$ are defined by Eqs. (121), (165), and (166), respectively.

This separation allows us to formulate the Hamiltonian approach to the theory under consideration.

5.4. Hamiltonian and Constraints

The equations derived from action (164) have no uniquely defined solutions for metric components, which depend on initial data and gauge. For gauge ambiguity to be excluded the coordinates should be fixed by imposing gauge conditions allowed by the equations of motion.

Following Dirac [23], we fix the physical coordinates by imposing the transversality condition

$$\partial_i \mathbf{e}^i_{(a)} = 0 \tag{175}$$

and the minimal-surface condition [23]

$$\overline{v_\psi} = 0, \tag{176}$$

where $\overline{v_\psi}$ is given by Eq. (168).

All the equations considered above can be derived by variation of the Hamiltonian action

$$\begin{aligned} S = \int dx^0 \Bigg[&- P_\varphi \partial_0 \varphi + N_0 \frac{P_\varphi^2}{4V_0} \\ &+ \int d^3x \left(\sum_F P_F \partial_0 F + \mathcal{C} - N_d \mathcal{H}_t(\varphi|\bar{F}) \right) \Bigg], \end{aligned} \tag{177}$$

where P_F is the set of the field momenta p_ψ, $p^i_{(b)}$, and p_f, defined as

$$\begin{aligned} p_\psi &= \frac{\partial K[\varphi|e]}{\partial(\partial_0 \ln \overline{\psi})} = -8\varphi^2 \overline{v_\psi}, \\ p^i_{(a)} &= \frac{\partial \mathbf{K}[\varphi|e]}{\partial(\partial_0 \mathbf{e}_{(a)i})}. \end{aligned} \tag{178}$$

The quantity

$$\mathcal{C} = N^i T^0_i + C_0 p_\psi + C_{(a)} \partial_k \mathbf{e}^k_{(a)} \tag{179}$$

is the sum of constraints (175) and (176), where N_d, N^i, C_0, and $C_{(a)}$ are the Lagrange multipliers. In Eq. (177),

$$\begin{aligned} \mathcal{H}_t(\varphi|\bar{F}) &= \left[\frac{6 p_{(ab)} p_{(ab)}}{\varphi^2} - \frac{16}{\varphi^2} p_\psi^2 \right] \\ &+ \frac{\varphi^2 \overline{\psi}^7}{6} [{}^{(3)}R(\mathbf{e}) \overline{\psi} + 8 \Delta \overline{\psi}] + \overline{\psi}^{12} \overline{T}^0_{0(\mathrm{SM})}, \end{aligned} \tag{180}$$

$$\begin{aligned} T^0_{kt} &= -p_\psi \partial_k \overline{\psi} + \frac{1}{6} \partial_k (p_\psi \overline{\psi}) \\ &+ p^i_{(b)} \partial_k \mathbf{e}_{i(b)} + \overline{T}^0_{k(\mathrm{SM})}, \end{aligned} \tag{181}$$

where $\overline{T}^0_{0(\mathrm{SM})}$ and $\overline{T}^0_{k(\mathrm{SM})}$ are the energy–momentum tensor components of the matter fields $\bar{F}$ and

$$p_{(ab)} = \frac{1}{2} [p^i_{(a)} \mathbf{e}_{(b)i} + p^i_{(b)} \mathbf{e}_{(a)i}]. \tag{182}$$

Variation of action (177) with respect to the Lagrange multipliers N^i, C_0, and $C_{(a)}$ yields the first-class constraints

$$T^0_{kt} = 0 \tag{183}$$

and the second-class constraints

$$p_\psi = 0, \quad \partial_k \mathbf{e}^k_{(a)} = 0, \tag{184}$$

as the minimal-surface and transversality conditions, respectively.

5.5. Equations of Exact Theory

The time lapse function is determined by the equation

$$\overline{N}_d \frac{\delta S[\varphi_0|F]}{\delta \overline{N}_d} = 0, \tag{185}$$

which serves as an energy constraint and takes the form

$$\frac{\varphi'^2}{\mathcal{N}} = \mathcal{N} \mathcal{H}_t. \tag{186}$$

Here, $\mathcal{N} = \overline{N_d}/N_0$ is the reparametrization-invariant part of function of the running time (121) satisfying the condition $\langle \mathcal{N}^{-1} \rangle = 1$, and the quantity $\mathcal{H}_t$ is given by Eq. (180).

The averaging of Eq. (185) over the volume V_0 yields the Friedmann-type equation of the exact theory,

$$\left\langle \overline{N}_d \frac{\delta S[\varphi_0|F]}{\delta \overline{N}_d} \right\rangle = 0 \Rrightarrow \varphi'^2 = \rho_t, \tag{187}$$

where

$$\rho_t \equiv \langle \mathcal{N} \mathcal{H}_t \rangle \tag{188}$$

is the total generator of evolution relative to time ζ (163) for all the dynamical variables, with the exception of the scale factor. Thus, in the Hamiltonian approach with the extracted scale factor, energy constraint equation (185) is subdivided into local (189) and global (187) parts.

Substituting (187) into (185), we arrive at the equation

$$\frac{\langle \mathcal{N} \mathcal{H}_t \rangle}{\mathcal{N}} = \mathcal{N} \mathcal{H}_t, \tag{189}$$

which unambiguously determines the local part of the lapse function

$$\mathcal{N} = \frac{\langle \sqrt{\mathcal{H}_t} \rangle}{\sqrt{\mathcal{H}_t}}. \tag{190}$$

Substituting this solution into global constraint equation (187), we reduce it to the form

$$\varphi'^2 = \langle\sqrt{\mathcal{H}_t}\rangle^2 \equiv \rho_t. \tag{191}$$

Thus, after isolating the evolution parameter as a dynamical variable, the energy constraint equation is concerned with only the zero Fourier harmonic. This completely corresponds to the diffeomorphism group of the Hamiltonian approach [22–24]. For this reason, the quantum gravity, if defined as quantization of a global constraint, is significantly simplified and, in essence, reduced to the quantum cosmology considered in Section 3. On the other hand, there is only the local energy constraint $\mathcal{H}_t = 0$ in the conventional Hamiltonian approach, in which the scale factor is not isolated [20]. This contradicts the diffeomorphism group of the Hamiltonian approach and complicates considerable the quantization problem considered as quantization of a constraint.

The second equation of the Friedmann cosmology is derived by variation of action (174) with respect to the scale factor φ:

$$\varphi\frac{\delta S[\varphi_0|F]}{\delta\varphi} = 0 \Rrightarrow 2\varphi\varphi'' = \rho_t - 3p_t, \tag{192}$$

where

$$3p_t \equiv \langle 3\mathbf{K}[\varphi|\bar{e}] - \mathbf{P}[\varphi|\bar{e}] + 2\mathbf{S}[\varphi|\bar{e}] + \mathcal{N}\bar{\psi}^{12}\overline{T^k_{k(\mathrm{SM})}}\rangle \tag{193}$$

is the precise pressure of all the fields including the SM fields. The relationship

$$2\varphi'^2 + 2\varphi\varphi' \equiv (\varphi^2)'' = 3(\rho_t - p_t) \equiv \langle\hat{\mathbf{A}}_t\mathcal{N}\rangle \tag{194}$$

follows from Eqs. (187) and (192), where the expression

$$\hat{\mathbf{A}}_t\mathcal{N} \equiv 4\mathbf{P}[\varphi|\bar{e}] - 2\mathbf{S}[\varphi|\bar{e}] + \bar{\psi}^{12}(3\overline{T^0_{0(\mathrm{SM})}} - \overline{T^k_{k(\mathrm{SM})}})\mathcal{N} \tag{195}$$

determines the equation for $\log\bar{\psi}$:

$$\frac{1}{2}\frac{\delta S[\varphi_0|F]}{\delta\log\bar{\psi}} = 0 \Rrightarrow \hat{\mathbf{A}}_t\mathcal{N} = \langle\hat{\mathbf{A}}_t\mathcal{N}\rangle. \tag{196}$$

Equations (189) and (196) in the infinite-volume limit $V_0 \longrightarrow \infty$ reduce to the zero–density and zero–pressure equations, $\mathcal{H}_t = 0$ and $\hat{\mathbf{A}}_t\mathcal{N} = 0$, because the averages $\langle\mathcal{H}\rangle$ and $\langle\hat{\mathbf{A}}_t\mathcal{N}\rangle$ tend to zero in this limit.

5.6. Resolution of Constraint Equation and Reduction of Action

The Hamiltonian form of global constraint equation (191),

$$\frac{P_\varphi^2}{4V_0^2} = \langle\sqrt{\mathcal{H}_t}\rangle^2 \tag{197}$$

has the two solutions

$$P_{\varphi(\pm)} = \pm 2V_0\langle\sqrt{\mathcal{H}_t}\rangle = 2V_0\varphi' \tag{198}$$

corresponding to positive and negative energies of events.

Substitution of solutions (198) into action (177) yields reduced action (127), considered above in the metric approach,

$$S_{(\pm)}[\varphi_I|\varphi_0]\big|_{\text{energy constraint}} = \int_{\varphi_I}^{\varphi_0} d\varphi\left\{\int d^3x\left[\sum_F P_F\partial_\varphi F + \bar{\mathcal{C}} \mp 2\sqrt{\mathcal{H}_t(\varphi)}\right]\right\}, \tag{199}$$

which is similar to reduced action (33) in GR. Here, $\bar{\mathcal{C}} = \mathcal{C}/\partial_0\varphi$ and the scale factor φ serves as a dynamical evolution parameter in the space of events $[\varphi|F]$. It is easy to verify that the variation of this action with respect to $\log\bar{\psi}$ yields Eq. (196), with $\mathcal{N}$ determined by Eq. (190). Action (199) describes the field evolution in terms of the red-shift parameter related to the scale factor φ by the equation $\varphi = \varphi_0/(1 + z)$.

5.7. Geometric Sector

Solutions of the equations derived from (177) in terms of geometric time (122) define geometric interval given by Eqs. (9), (152), and (153), $ds^2 = \omega_{(0)}^2 - \omega_{(a)}^2$, with the Cartan forms

$$\omega_{(0)}(\zeta, x) = a(\zeta)\bar{\psi}^6\mathcal{N}d\zeta, \tag{200}$$

$$\omega_{(a)}(\zeta, x) = a(\zeta)\bar{\psi}^2(\mathbf{e}_{(a)i}dx^i + \mathcal{N}_{(a)}d\zeta). \tag{201}$$

In this case, the Dirac minimal-surface and transversality conditions given by Eq. (184) are to be imposed on the metric components [23]:

$$\partial_\zeta[\bar{\psi}^6] = \partial_{(b)}[\bar{\psi}^6\mathcal{N}_{(b)}], \quad \partial_i\mathbf{e}^i_{(a)} \simeq 0 \quad (\det|\mathbf{e}| = 1). \tag{202}$$

We remind that invariant geometric time ζ (122) is determined by the Friedmann-type equations of exact theory (187):

$$\frac{d\varphi}{d\zeta} \equiv \varphi' = \pm\langle\sqrt{\mathcal{H}_t}\rangle. \tag{203}$$

The solution

$$\zeta(\varphi_0|\varphi_I) = \int_{\varphi_I}^{\varphi_0}\frac{d\varphi}{\langle\sqrt{\mathcal{H}_t}\rangle} \tag{204}$$

of Eq. (203) can be treated as a relativistic relation between the evolution parameter φ and geometric time (122). Equations (192) and (203) describe the Friedmann-like universe evolution (under no assumption of homogeneity) as a pure relativistic effect in the GR field space of events. This effect is similar to the Lorentz time dilation in SR. In this case, all fields $\bar{F}^{(n)} = a(\zeta)^n F^{(n)}$ are to be considered in the space

$$\overline{\omega_{(0)}}(\zeta, x) = \bar{\psi}^6 \mathcal{N} d\zeta, \tag{205}$$

$$\overline{\omega_{(a)}}(\zeta, x) = \bar{\psi}^2(\mathbf{e}_{(a)i}dx^i + \mathcal{N}_{(a)}d\zeta) \tag{206}$$

with mass multiplied by a scale factor $a(\zeta)$: $\bar{m} = a(\zeta)m$.

The variables $\bar{\bar{F}}^{(n)} = (\bar{\psi}^2)^n \bar{F}^{(n)}$ defined in the space with a basis

$$\overline{\overline{\omega_{(0)}}}(\zeta, x) = \bar{\psi}^4 \mathcal{N} d\zeta, \tag{207}$$

$$\overline{\overline{\omega_{(a)}}}(\zeta, x) = (\mathbf{e}_{(a)i}dx^i + \mathcal{N}_{(a)}d\zeta), \tag{208}$$

with mass multiplied by the scale $a(\zeta)\bar{\psi}^2$, i.e., $\bar{\bar{m}} = a(\zeta)\bar{\psi}^2 m$, have been also considered [63–65]. In what follows, we will refer to the fields $\bar{\bar{F}}$ as Lichnerowicz variables. In general, it is convenient to introduce energy density (180),

$$\begin{aligned}\mathcal{H}_t &= \left[\frac{6p_{(ab)}p_{(ab)}}{\varphi^2} - \frac{16}{\varphi^2}p_\psi^2\right]\\ &+ \frac{\varphi^2\bar{\psi}^7}{6}[{}^{(3)}R(\mathbf{e})\bar{\psi} + 8\Delta\bar{\psi}] + \bar{\psi}^{4y}\bar{\bar{T}}^0_{0(\mathrm{SM})}\end{aligned} \tag{209}$$

and pressure density (193),

$$\begin{aligned}&3p_t\\ &\equiv \langle 3\mathbf{K}[\varphi|\bar{e}] - \mathbf{P}[\varphi|\bar{e}] + 2\mathbf{S}[\varphi|\bar{e}] + \mathcal{N}\bar{\psi}^{4y}\bar{\bar{T}}^k_{k(\mathrm{SM})}\rangle,\end{aligned} \tag{210}$$

with an arbitrary weight y, referred to as a Lichnerowicz index. Here, we use the Hamiltonian form of notations (165)–(167):

$$\mathbf{K}[\varphi|e] = \mathcal{N}\left[\frac{6p_{(ab)}p_{(ab)}}{\varphi^2} - \frac{16}{\varphi^2}p_\psi^2\right], \tag{211}$$

$$\mathbf{P}[\varphi|e] = \mathcal{N}\frac{\varphi^2\bar{\psi}^7}{6}[{}^{(3)}R(\mathbf{e})\bar{\psi} + 8\Delta\bar{\psi}], \tag{212}$$

$$\begin{aligned}\mathbf{S}[\varphi|e] &= -4[\partial_\zeta p_\psi - \partial_l(N^l p_\psi)]\\ &- \frac{\varphi^2}{3}\partial_j[\bar{\psi}^2\partial^j(\bar{\psi}^6\mathcal{N})].\end{aligned} \tag{213}$$

In this case, the scalar sector $\mathcal{N}$, $\bar{\psi}$, and $\partial_\zeta[\bar{\psi}^6] = \partial_{(b)}[\bar{\psi}^6\mathcal{N}_{(b)}]$ is defined by the equations

$$N_d\frac{\delta S[\varphi]}{\delta N_d} = 0 \;|\!\!\Rightarrow \mathcal{N} = \frac{\langle\sqrt{\mathcal{H}_t}\rangle}{\sqrt{\mathcal{H}_t}}, \tag{214}$$

$$\psi\frac{\delta S[\varphi]}{2\delta\psi} = 0 \;|\!\!\Rightarrow \hat{\mathbf{A}}_t\mathcal{N} = \langle\hat{\mathbf{A}}_t\mathcal{N}\rangle, \tag{215}$$

where

$$\begin{aligned}\hat{\mathbf{A}}_t\mathcal{N} &\equiv 4\mathbf{P}[\varphi|\bar{e}] - 2\mathbf{S}[\varphi|\bar{e}]\\ &+ \bar{\psi}^{4y}\left(3\bar{\bar{T}}^0_{0(\mathrm{SM})} - \bar{\bar{T}}^k_{k(\mathrm{SM})}\right)\mathcal{N}.\end{aligned} \tag{216}$$

5.8. Hamiltonian Cosmological Perturbation Theory

We now consider the expansion of Cartan forms (200) and (201),

$$\omega_{(0)} = a(\eta)(1 + \bar{\Phi})d\eta, \tag{217}$$

$$\begin{aligned}\omega_{(a)} &= a(\eta)(1 - \bar{\Psi})(dx^i_{(a)} + h^{(TT)}_{(a)i}dx^i\\ &+ \partial_{(a)}\nu d\eta + N^{(T)}_{(a)}d\eta)\end{aligned} \tag{218}$$

by the perturbation theory, where $h^{(TT)}_{(a)a} = 0$. The transversality condition $\partial_i h^{(TT)}_{(a)i} = 0$ and minimal-surface condition $p_\psi = 0$ (202) are imposed. The equation

$$\Delta\nu = \frac{3}{4}\bar{\Phi}'_h \tag{219}$$

gives the longitudinal component $\partial_{(a)}\nu$ of origin shift vector (222).

To avoid double counting in the cosmological perturbation theory, we define cosmological perturbations of the metric components,

$$\mathcal{N} = (1 + \bar{\Phi} + 3\bar{\Psi}), \quad \bar{\psi}^2 = (1 - \bar{\Psi}), \tag{220}$$

$$\mathbf{e}_{(a)i} = \delta_{(a)i} + h^{(TT)}_{(a)i}, \tag{221}$$

$$N_{(a)} = \partial_{(a)}\nu + N^T_{(a)}, \quad \partial_{(a)}N^T_{(a)} = 0, \tag{222}$$

in the class of functions with the nonzero Fourier harmonics

$$\tilde{\Phi}(k) = \int d^3x\bar{\Phi}(x)e^{ikx}, \tag{223}$$

satisfying the strong conditions

$$\begin{aligned}\langle\Psi\rangle &= \int d^3x\Psi(\eta, x_i) \equiv 0,\\ \langle P_\Psi\rangle &= \int d^3x\partial\mathcal{L}/\partial\dot{\Psi}(\eta, x_i) \equiv 0.\end{aligned} \tag{224}$$

The energy–momentum tensor components can be also expanded:

$$T^0_{0\mathrm{sm}} = \rho_s + \delta\bar{T}^0_0, \quad T^k_{k\mathrm{sm}} = 3p_s + \delta\bar{T}^k_k, \tag{225}$$

where ρ_s and p_s are the density and pressure, respectively. Under the minimal-surface condition $p_\psi = 0$, the first-order terms of expressions (211)–(213) are

$$\mathbf{K}^{(1)} = 0, \quad \mathbf{P}^{(1)} = -\frac{2\varphi^2}{3}\Delta\bar{\Phi}, \quad \mathbf{S}^{(1)} = \frac{\varphi^2}{3}\Delta\bar{\Psi}. \quad (226)$$

Under the conditions $\partial_i T_{ik}^{(TT)} = 0$, $T_{ii}^{(TT)} = 0$, and $\partial_k T_k^{0(T)} = 0$, the equations for the vector and tensor components take the form

$$\begin{aligned} T_k^{0(T)} &= -\frac{\varphi^2}{12}\Delta N_k^T, \\ \bar{T}_{ik}^{TT} &= \frac{\varphi^2}{12}\left[-\Delta h_{ik}^{(TT)} + \frac{(\varphi^2 h_{ik}^{TT\prime})'}{\varphi^2}\right]. \end{aligned} \quad (227)$$

They coincide with the equations of the cosmological perturbation theory [49, 50].

In this case, the scalar sector, $\mathcal{N} = 1 + \Phi$ and $\bar{\psi} = 1 - \Psi/2$, defined by Eqs. (214), (215) in view of Eqs. (226) takes the form

$$t_0^0 = [\Delta - \alpha_\psi h^2]\Psi - 2h^2\Phi, \quad (228)$$

$$[t_0^0 + t_k^k] = -\beta h^2\Psi + (\Delta - \alpha_\phi h^2)\Phi, \quad (229)$$

where $t_0^0 = \delta\tilde{T}_0^0 3/(2\varphi^2)$ and

$$\begin{aligned} \alpha_\phi &= 5 + \gamma^2, \quad \alpha_\psi = 6 - 2y, \\ \beta &= 15 - 2y, \quad \gamma^2 = \frac{p_s}{\rho_s}, \quad h^2 = \frac{3\rho_s}{2\varphi^2}. \end{aligned} \quad (230)$$

The solutions of Eqs. (228) and (229) in terms of Fourier components can be written out in the form

$$\begin{aligned} \tilde{\Psi} &= -\left[\frac{1-\gamma_\Psi}{k^2+\mu_1^2} + \frac{\gamma_\Psi}{k^2-\mu_2^2}\right]\widetilde{t_{00}} \\ &+ \left[\frac{\zeta_\Psi}{k^2+\mu_1^2} - \frac{\zeta_\Psi}{k^2-\mu_2^2}\right]\widetilde{t_{kk}}, \end{aligned} \quad (231)$$

$$\begin{aligned} \tilde{\Phi} &= -\left[\frac{1-\gamma_\Phi}{k^2+\mu_1^2} + \frac{\gamma_\Phi}{k^2-\mu_2^2}\right]\widetilde{t_{00}} \\ &+ \left[\frac{1-\zeta_\Phi}{k^2+\mu_1^2} + \frac{\zeta_\Phi}{k^2-\mu_2^2}\right]\widetilde{t_{kk}}, \end{aligned} \quad (232)$$

where

$$\begin{aligned} \mu_1^2 = h^2[&\sqrt{(2\beta - \alpha_\psi\alpha_\phi) + (\alpha_\psi + \alpha_\phi)^2/4} \\ &+ (\alpha_\psi + \alpha_\phi)/2], \end{aligned} \quad (233)$$

$$\begin{aligned} \mu_2^2 = h^2[&\sqrt{(2\beta - \alpha_\psi\alpha_\phi) + (\alpha_\psi + \alpha_\phi)^2/4} \\ &- (\alpha_\psi + \alpha_\phi)/2], \end{aligned} \quad (234)$$

$$\gamma_\Phi = \frac{\mu_2^2 + (\alpha_\psi - 2)h^2}{\mu_1^2 + \mu_2^2}, \quad \gamma_\Psi = \frac{\mu_2^2 + (\alpha_\phi - \beta)h^2}{\mu_1^2 + \mu_2^2}, \quad (235)$$

$$\zeta_\Phi = \frac{\mu_2^2 + \alpha_\psi h^2}{\mu_1^2 + \mu_2^2}, \quad \zeta_\Psi = \frac{\beta h^2}{\mu_1^2 + \mu_2^2}. \quad (236)$$

The Hamiltonian version of the cosmological perturbation theory, with the cosmological factor isolated from action, does not require its convergence to be proved because the perturbations are given in a different class of functions (with nonzero Fourier harmonics) than the cosmological dynamics described by exact equations (188) and (192).

Let us compare the Hamiltonian cosmological perturbation theory described by Eqs. (228) and (229) with the standard one [49, 50], in which the zero Fourier harmonic of the logarithm of the spatial metric determinant is taken into account twice: as a cosmological scale factor and as a spatial potential of the metric.

Equations (228) and (229) basically differ from the equations

$$\begin{aligned} -3\mathcal{H}(\mathcal{H}\Phi + \Psi') + \Delta\Psi &= 4\pi G\delta T_0^0, \\ 3[(2\mathcal{H}' + \mathcal{H}^2)\Phi + \mathcal{H}\Phi' + \Psi'' + 2\mathcal{H}\Psi'] \\ + \Delta(\Phi - \Psi) &= 4\pi G\delta T_i^i \end{aligned} \quad (237)$$

of the standard cosmological perturbation theory (where $4\pi G = 3/(2\varphi^2)$, $\mathcal{H} = a'/a$, and $\Delta = \partial_i^2$, see Eq. (4.15) in [51]) in the absence of time derivatives of the metric components. These derivatives are responsible for the primordial temperature perturbation spectrum of cosmic microwave background (CMB) in the inflation model [51, 62]. The time derivatives of Φ and Ψ are eliminated by imposing strong conditions (224) to avoid double counting. In this case, scalar metric components serving as dynamical variables turn into conventional potentials similar to the Coulomb electrodynamic potential. Thus, the explanation of the primordial temperature perturbation spectrum of CMB in the inflation model [51, 62] is based on double counting, which makes it impossible to construct the Hamiltonian formalism and quantum theory.

On the other hand, it is the Hamiltonian quantum theory that allows us, as is shown in Section 4, to treat CMB and its primordial temperature perturbation spectrum as a result of decays of primordial *W*- and *Z*-bosons created from the Bogoliubov stable vacuum at the time when the boson mass was equal to the Hubble constant. In this case, the equations for longitudinal components of *W*- and *Z*-bosons [55] are close to Eqs. (237) of the inflation model, which describe the primordial temperature perturbation spectrum.

5.9. Central Gravitational Fields

In the Newtonian case, when p_s, $\rho_s \ll \varphi^2 k^2$, solutions of Eqs. (228) and (229) coincide with those of the conventional perturbation theory with time-depending masses:

$$\tilde{\Psi} = -\frac{4\pi G}{k^2}\tilde{T}_0^0, \quad \tilde{\Phi} = -\frac{4\pi G}{k^2}[\tilde{T}_0^0 + \tilde{T}_k^k],$$
$$G = \frac{3}{8\pi\varphi^2}. \tag{238}$$

Let us consider the central gravitational field of a unit mass with

$$\bar{T}_0^0 = M[\delta^3(x) - V_0^{-1}], \quad \bar{T}_{kk} = 0. \tag{239}$$

Here, by definition, $\varphi = \varphi_0 a$, $M = M_0 a$, and $\varphi_0 = \sqrt{3/8\pi G}$. Solutions (238) can be written in the conventional form

$$\bar{\Psi}(x) = \bar{\Phi}(x) = \frac{3}{4\pi\varphi^2}\int d^3x \frac{T_{00}}{|y-x|}\bigg|_{T_{00} = M\delta^3(x)} = \frac{r_g}{r}, \tag{240}$$

where $r = \sqrt{x_1^2 + x_2^2 + x_3^2}$ and

$$r_g = \frac{3M}{4\pi\varphi^2} = 2GM. \tag{241}$$

As follows from Eq. (219), the shift vector N^i is

$$N^i = \left(\frac{3r'_g}{4}\right)\frac{x^i}{r}. \tag{242}$$

Substituting Eqs. (240) and (250) into the conformal integral, we arrive at the equation

$$ds_c^2 = \left(1 - \frac{r_g}{r}\right)d\eta^2 - \left(1 + \frac{r_g}{r}\right)\left(dx_i + \frac{3}{2}\frac{x^i}{r}r'_g d\eta\right)^2, \tag{243}$$

where $ds_c^2 = ds^2/a^2(\eta)$.

5.10. Cosmological Generalization of Newtonian Potential

In the case of the equation of the rigid state $\gamma^2 = 1$, we have

$$\alpha_\phi = 6, \quad \alpha_\psi = 6 - 2y, \quad \beta = 15 - 2y,$$
$$h^2 = 3\rho_s/(2\varphi^2) = (3/2)H_0^2(1+z)^4 \tag{244}$$

instead of Eqs. (230); hence, Eqs. (233) and (234) take the form

$$\mu_{1,2}^2 = (3/2)H_0^2(1+z)^4[\sqrt{26 + (y-2)^2} \pm (6-y)]. \tag{245}$$

Here, $y = 3, 1$ is the Lichnerowicz index, and γ_Ψ and γ_Φ are given by Eqs. (235).

In the case of (239), we obtain the solutions that reduce in the coordinate representation to the Yukawa potentials with Jeans-like oscillations:

$$\Psi = -\frac{r_g}{r}[(1-\gamma_\Psi)e^{-\mu_1(z)r} - \gamma_\Psi\cos\mu_2(z)r], \tag{246}$$

$$\Phi = -\frac{r_g}{r}[(1-\gamma_\Phi)e^{-\mu_1(z)r} - \gamma_\Phi\cos\mu_2(z)r]. \tag{247}$$

In the case of a point mass

$$\bar{T}_0^0 = \sum_J M_J\left[\delta^3(x-y_J) - \frac{1}{V_0}\right], \tag{248}$$

solutions (231) and (232) take the form

$$\bar{\Psi}(x) = -\sum_J \frac{r_{gJ}}{r_J}[(1-\gamma_\Psi)e^{-\mu_1(z)r_J} - \gamma_\Psi\cos\mu_2(z)r_J],$$
$$\bar{\Phi}(x) = -\sum_J \frac{r_{gJ}}{r_J}[(1-\gamma_\Phi)e^{-\mu_1(z)r_J} - \gamma_\Phi\cos\mu_2(z)r_J], \tag{249}$$

where $r_{gJ} = 2GM_J$, $r_J = |x - y_J|$, and $\mu_{1,2}^{(y=1,3)}(z)$ is given by Eq. (245). The minimal-surface condition determines the shift of the source coordinate in the course of evolution:

$$N^i = \sum_J \frac{3r'_{gI}}{4}\frac{(x-y_J)^i}{|x-y_J|} \times [(1-\gamma_\Psi)e^{-\mu_1(z)r_J} - \gamma_\Psi\cos\mu_2(z)r_J]. \tag{250}$$

The conformal integral squared

$$ds_c^2 = (1+\bar{\Phi})d\eta^2 - (1-\bar{\Psi})(dx^i + N^i d\eta)^2 \tag{251}$$

yields the equation for photon momentum,

$$p_\mu p_\nu g^{\mu\nu} \simeq (p_0 + N^i p^i)^2(1-\bar{\Phi}) - p_j^2(1+\bar{\Psi}) = 0, \tag{252}$$

which gives

$$p_0 \simeq -N^i p^i + (1 + \bar{\Phi} + \bar{\Psi})|p|, \quad |p| = \sqrt{p_i^2}. \tag{253}$$

Finally, the relative value of spatial fluctuations of photon energy, expressed in terms of the metric components (the potentials $\bar{\Phi}$, $\bar{\Psi}$ and the shift vector N^i), is

$$\frac{p_0 - |p|}{|p|} = -(N^i n^i - \bar{\Phi} - \bar{\Psi}), \quad n^i = \frac{p_i}{|p|}. \tag{254}$$

Thus, the spatial anisotropy of photon energy fluctuations in the photon flow is due to minimal-surface condition (250)

5.11. Cosmological Generalization of Schwarzschild Solution

In order to isolate the Laplace operator $\Delta = \partial_{(a)}\partial_{(a)}$ from Eqs. (189) and (196), we substitute

$$\mathcal{N}_\psi = \psi^7 \mathcal{N} \tag{255}$$

into them:

$$\hat{\mathbf{A}}_t\mathcal{N} = \langle \hat{\mathbf{A}}_t\mathcal{N}\rangle, \quad \mathcal{N}\mathcal{H}_t = \frac{\langle \mathcal{N}\mathcal{H}_t\rangle}{\mathcal{N}},$$

where

$$\hat{\mathbf{A}}_t\mathcal{N} \equiv 4\mathbf{P} - \mathbf{S} + \psi^5\mathcal{N}_\psi(3T_0^0 - T_k^k), \tag{256}$$

$$\mathcal{N}\mathcal{H}_t \equiv \mathbf{P} + \mathbf{K} + \psi^5\mathcal{N}_\psi T_0^0. \tag{257}$$

As a result, the expressions for **P** and **S** take the form

$$\mathbf{P} = \frac{4\varphi^2}{3}\mathcal{N}_\psi\Delta\psi + \frac{\varphi^2}{6}\mathcal{N}_\psi R(\mathbf{e}), \tag{258}$$

$$\mathbf{S} = \frac{\varphi^2}{3}(\mathcal{N}_\psi\Delta\psi - \psi\Delta\mathcal{N}_\psi). \tag{259}$$

If **P** and **S** are equal to zero, then

$$\Delta\mathcal{N}_\psi = 0, \quad \Delta\psi = 0, \quad R(\mathbf{e}) = 0. \tag{260}$$

These equations have the solutions

$$\psi = 1 + \frac{r_g(\zeta)}{4r}, \tag{261}$$

$$\mathcal{N}_\psi = 1 - \frac{r_g(\zeta)}{4r}, \tag{262}$$

where $r_g(\zeta) = M3/4\pi\varphi^2$ is the gravitational radius.

It is easy to verify that the conventional Schwarzschild metric in the vacuum, $T_\nu^\mu = 0$, can be treated as a solution of the Einstein equations under the assumption of mass independent of the geometric time: $r_g(\zeta) \sim r_g(\zeta_0) = \text{const}$. The generalization of the Schwarzschild metric to the conformally flat metric can be written in terms of Cartan forms (152) and (153):

$$\omega_{(0)} = \frac{\mathcal{N}_\psi}{\psi}d\zeta, \tag{263}$$

$$\omega_{(r)} = \psi^2\left(dr + \frac{V'}{r^2\psi^6}d\zeta\right), \tag{264}$$

$$\omega_{(\theta)} = \psi^2 r^2 d\theta, \tag{265}$$

$$\omega_{(\varphi)} = \psi^2 r^2 \sin\theta d\varphi. \tag{266}$$

Here,

$$V(r, \zeta) = \int^r d\bar{r}\bar{r}^2\psi^6(\bar{r}, \zeta) \tag{267}$$

gives the radial component of the shift vector satisfying the minimal-surface condition

$$\bar{p}_\psi \simeq 0. \tag{268}$$

Thus, the Schwarzschild solution is an approximation similar to the Coulomb potential in electrodynamics, when both source motion and field dynamics are disregarded.

5.12. Transformation of Reference Frames

The Lorentz invariance of GR in a specific reference frame can be treated similar to the Lorentz invariance of electrodynamic equations in SR [12]. Therefore, to perform the transformation to a new reference frame one should return to the initial variables and interval

$$ds^2 = \omega_{(0)}^2 - \omega_{(a)}^2 \tag{269}$$

and choose new variables and linear forms $\bar{\omega}_{(0)}$ and $\bar{\omega}_{(a)}$ related to the initial forms by the Lorentz transformations

$$\omega_{(0)}(\zeta, x_1, x_2, x_3) = \frac{1}{\sqrt{1-v^2}}\bar{\omega}_{(0)}(\bar{\zeta}, \bar{x}_1, x_2, x_3) + \frac{v}{\sqrt{1-v^2}}\bar{\omega}_{(1)}(\bar{\zeta}, \bar{x}_1, x_2, x_3),$$

$$\omega_{(1)}(\zeta, x_1, x_2, x_3) = \frac{1}{\sqrt{1-v^2}}\bar{\omega}_{(1)}(\bar{\zeta}, \bar{x}_1, x_2, x_3) + \frac{v}{\sqrt{1-v^2}}\bar{\omega}_{(0)}(\bar{\zeta}, \bar{x}_1, x_2, x_3).$$

Then, the Hamiltonian description of the Dirac variables and coordinates is to be formulated; in this case, the dependence of the new forms $\bar{\omega}$ on the spacetime coordinates $(\bar{\zeta}, \bar{x})$ should coincide with the dependence of ω on (ζ, x).

6. CONCLUSIONS

The main goal of theoretical physics is to establish several physical principles to explain all observable effects just as a few Euclidean axioms and logical laws make it possible to prove many geometry theorems.

In modern physics, symmetry properties serve as such fundamental principles. The following statement of Weyl [66] is worth reminding: *{The conclusion having become the key principle of modern mathematics is as follows: every time you deal with an object* $\sum$ *having a structure, you should try to find its automorphism group; i.e., the group of the transformations unaffecting structure relationships of the object.}* From this viewpoint, transformations of reference frames form an automorphism group in mechanics, while the equations of motion derived by variation of action are invariant structure relationships.

General Relativity first introduced new geometric principles into the theory of dynamical systems. The principles were explicitly formulated by Hilbert in "Foundations of Physics" [15], where the action functional, the symmetry of reference frames, and the equations of motion were complemented by the definition of geometric interval, the gauge symmetry, and the constraint equations. This stimulated the establishment of the gauge symmetry of Maxwell equations [4] and, finally, the development of modern unified gauge theory.

As shown in this review, the same geometric formulation of SR makes it possible to describe relativistic effects by dynamical variational equations rather than Lorentz kinemetric transformations. Moreover, the notions of "Foundations of Physics" [15] help us understand the history of SR. It should be noted that the modern quantum field theory [18, 19] is nothing but the first and second quantization of the energy constraint equation referred to as a mass surface of relativistic particle.

In the spirit of the unified geometric formulation of both SR and GR with the use of the Hilbert variational principle, the following question arises. Is it possible to adopt modern quantum-field methods (based on relativistic dynamics in a specific reference frame) to the Hilbert formulation of GR? If this was the case, then GR would be quantized by analogy to the first and second quantization in SR. Almost all notions of such a quantization of GR turned out to be formulated by a number of authors. They used relativistic and gauge symmetries to define physical gauge-invariant variables and observables of GR in a specific reference frame, as well as to construct a GR action. These notions are listed below.

(i) The difference between transformations of reference frames and gauge transformations in GR was first pointed out by Fock in 1929 [21]. He introduced a gauge-invariant orthogonal basis depending on the reference frame.

(ii) A specific reference frame corresponding to the quantization of GR was defined by Dirac in 1958 [23].

(iii) A group of gauge transformations leaving the geometric interval invariant in a kinematic reference frame was considered by Zel'manov in 1976 [25]. The group contains a subgroup of transformations of a coordinate evolution parameter and requires that the gauge-invariant time be introduced as a variable of field space of events.

(iv) The time as a variable was identified with a cosmological scale factor by Wheeler and De Witt in 1967 [39]. They introduced into GR the notion of a field space of events. The relativistic universe moves in this space on a world hypersurface characterized by a geometric interval measured by a comoving observer, with this space similar to the Minkowski space in SR.

(v) The transformation that isolates the cosmological scale factor (treated in GR as an independent variable [27]) from all field variables was found by Lichnerowicz in 1944 [63].

(vi) To define gauge-invariant physical measurable quantities (time, one-particle energy, particle number, number of universes, etc.) the Hamiltonian reduction [27, 28] and the correspondence principle (formulated by Einstein [34] in SR to define particle energy) were employed. Expanding the reduced action in physical fields as suggested by Einstein, we obtain the Wheeler–De Witt cosmology [39] as the first-order approximation. The second-order approximation yields the conventional field theory in the conformally flat spacetime. In this case, gauge-invariant observables coincide with conformal ones or relative coordinates and time, introduced by Friedmann. In terms of these variables, the universe is at rest and all mass increases with the cosmological factor [37, 38].

(vii) The quantum gravity is defined by Wheeler and De Witt as the first and second quantization of energy constraint in a specific reference frame [39]. This completely accommodates to the modern quantum field theory [18, 19] verified by the huge amount of high-energy physics data.

We demonstrated that the quantum gravity considered above makes it possible to resolve a series of problems of modern cosmology: Hubble evolution, universe and matter creation from the vacuum, evolution and horizon of the universe, arrow of time, initial data, and cosmological singularity. We listed a number of theoretical and experimental arguments in favor of this quantum gravity, which allows us to answer the topical questions concerning origin of dark energy and dark matter, the universe and matter creation, and the matter evolution in the universe.

In this case, the primordial temperature perturbation spectrum of the CMB temperature in the inflation model was shown to be explained by the double counting of the scale factor. This makes it impossible to consistently deduce the Hamiltonian formalism and quantum theory. On the other hand, it is the Hamiltonian quantum theory considered in Section 4 that allows us to describe CMB and its primordial temperature perturbation spectrum as a product of decays of primordial *W*- and *Z*-bosons. The bosons were created from the Bogoliubov stable vacuum when their mass coincided with the Hubble parameter. In this case, the equations for longitudinal components of *W*- and *Z*-bosons [55] are similar to Eqs. (237) of the inflation model, which describe the primordial temperature perturbation spectrum.

The version described above of the GR quantization by extracting the scale factor reveals hidden scale symmetry of GR and the Standard Model. Namely, physical laws, when treated as equations of motion, are independent of measurement units, as well as values of initial data in a specific reference frame. The Planck mass, as a fundamental parameter, disappears from equations of motion and emerges as the present-day value of a dynamical variable evolved from a possible initial data. For this reason, GR is mathematically equivalent to the theory of conformal scalar field [67]. The scale symmetry indicates the possible geometrization of Higgs particle (which need not be introduced together with Higgs potential). Moreover, such a geometrization could predict some physical effects for experimental verification with modern high-energy particle accelerators [68].

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