EE352 - Advanced Control System

Term Project

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1 Aim

Given a unity feedback system where

$$\mathbf{G}(s) = \frac{k}{s(s+1)(s+4)}$$

design a passive lag-lead compensator using bode diagrams to yield a 13.25% overshoot, a peak time of 2 seconds and $K_v = 12$. Also, compare the uncompensated, lag-compensated, and completely compensated system.

2 Process

First, we observe that to obtain $K_v = 12$, we require,

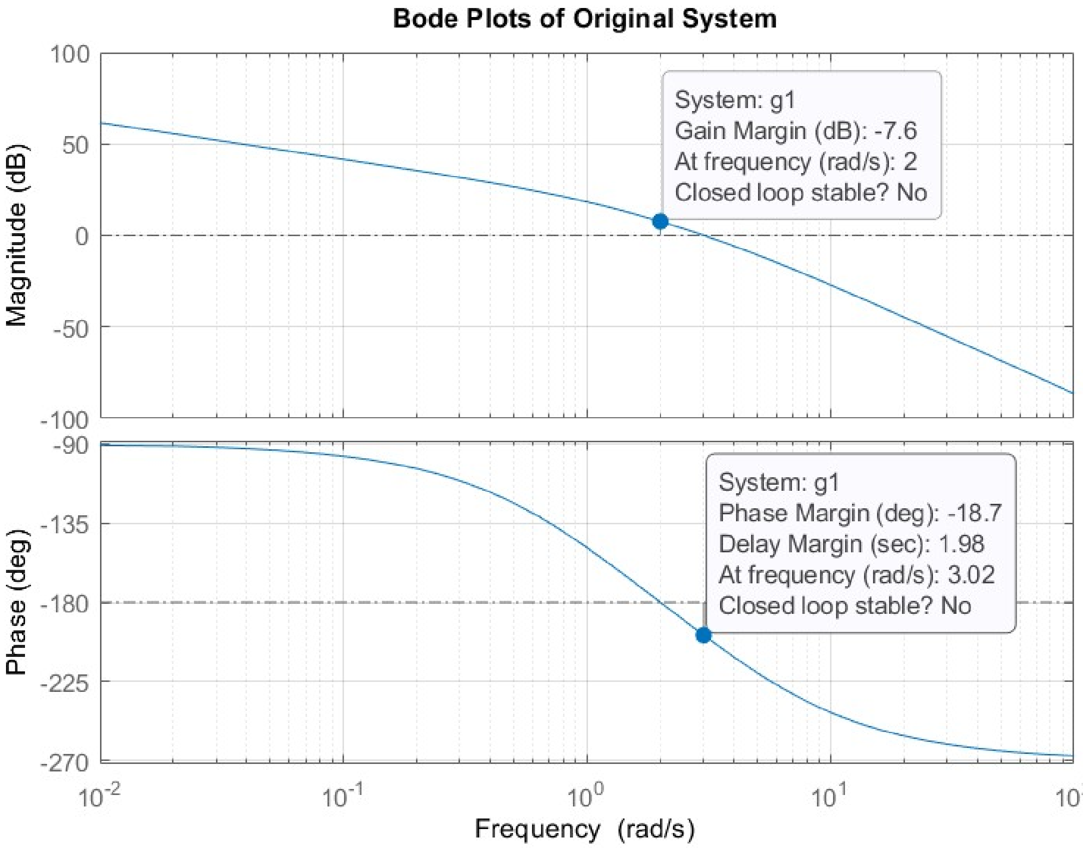
$$\lim_{s \to 0} sG(s) = 12$$

$$\lim_{s \to 0} \frac{sk}{s(s+1)(s+4)} = 12$$

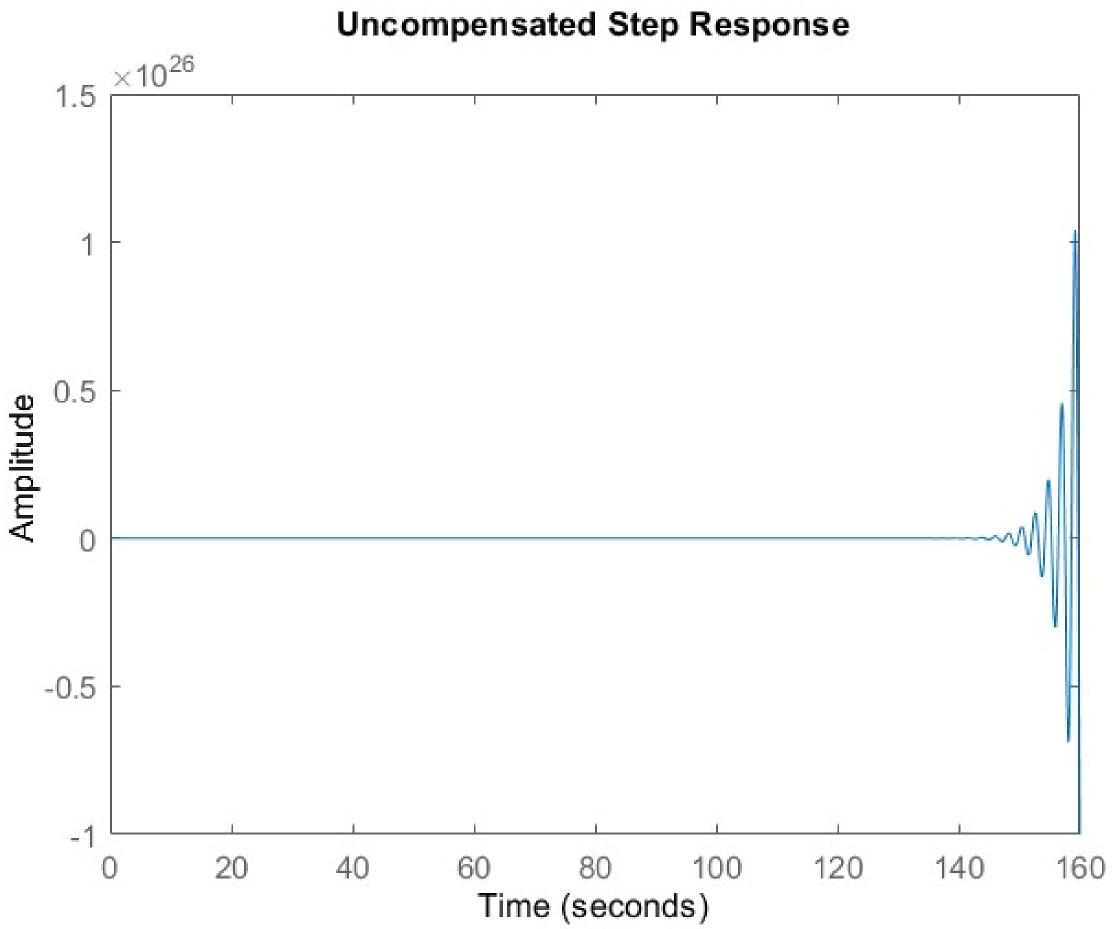
On solving the above equation, we get,

$$k = 48$$

where k is the gain.



Now, we obtain the Bode plot of the uncompensated system. From it, we get the gain margin = -7.6 dB and phase margin = $-18.7°$, both of which are negative, indicating that the system is unstable. This can also be noted by the step response of the system increasing to infinity.



Since we are provided with an overshoot of 13.25%, if we take the second-order approximation, we can find the damping ratio according to the formula:

$$\%\text{overshoot} = \exp\left(\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right)$$

where ζ is the damping ratio of the system, which comes out to be 0.54.

Now, we can find the required phase margin from the equation:

$$\Phi_m = 90 - \tan^{-1}\left(\frac{\sqrt{-2s^2+\sqrt{1+4s^4}}}{2s}\right)$$

$$\Phi_m = \tan^{-1}\left(\frac{2s}{\sqrt{-2s^2+\sqrt{1+4s^4}}}\right)$$

Which equates required margin as:

$$\Phi_{m,req} = 54.94^\circ \approx 55^\circ$$

Now, we will also find out the closed-loop bandwidth from the equation:

$$\omega_{BW} = \frac{\pi}{T_p\sqrt{1-\xi^2}}\sqrt{(1-2\xi^2)^+ + \sqrt{4\xi^4-4\xi^2+2}}$$

On putting the values, we obtain closed loop Bandwidth = 2.29 rad/s. Now we will proceed to Compensator Design. First, we choose a new phase margin frequency near the closed-loop bandwidth. Let us take $\omega = 1.9$ rad/s as the new phase margin frequency. From the Bode plot of the uncompensated system, we find the phase to be -177.65 degrees at this frequency.
The phase margin at this frequency is $\phi_{\text{m}} = 180° - 177.65° = 2.35°$. We need to increase this to 55°.

Now, assuming the lag compensator to have a $-5°$ contribution, we obtain,

$$\phi_{\text{lead}} = (\phi_{\text{m-req}} - \phi_{\text{m}}) + 5 = 57.65°.$$

So, we need to design a lead compensator that adds 57.65° phase.
Now let the lag compensator be $= \frac{1}{\alpha}\left(\frac{s+1/T_1}{s+1/(\alpha T_1)}\right)$ and the lead compensator be $\frac{1}{\beta}\left(\frac{s+1/T_2}{s+1/(\beta T_2)}\right)$ where $\alpha = 1/\beta$.
Since, the lead compensator adds a maximum of 57.65° at the phase margin frequency, we set $\omega_{\max} = 1.9$ rad/s, $\phi_{\max} = 57.65°$.
We know,
$\phi_{\max} = \tan^{-1}\left(\frac{1-\beta}{2\sqrt{\beta}}\right)$, so for $\phi_{\max} = 57.65°$, we get $\beta = 0.084$. Since $\alpha = 1/\beta$, we get $\alpha = 11.9$.

Now, we want the lag compensator to stabilize the system and keep the gain of 48 required for $k_v = 12$ but have minimal effect on the phase response at the phase margin frequency. So, we choose the lag compensator's higher break frequency to be 1 decade below the new phase-margin frequency. So, $1/T_1 = 0.18$ rad/s, hence the lag compensator is

$$G_{\text{lag}}(s) = \frac{1}{11.9}\left(\frac{s+0.19}{s+0.159}\right).$$

And the lag-compensated system is

$$G_{\text{lag-comp}}(s) = \frac{4.034s + 0.7664}{s^4 + 5.016s^3 + 4.08s^2 + 0.0636s}.$$

Now, for the lead compensator, we already found $\beta = 0.084$ and $\omega_{\max} = 1.9$ rad/s. So, from the formula, we obtain

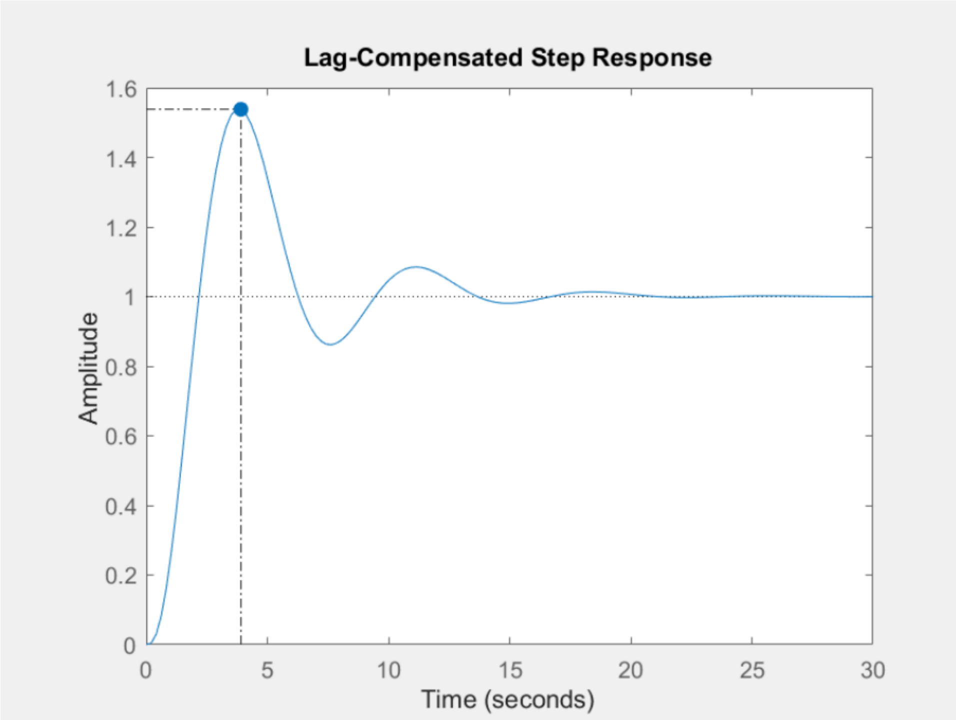
$$\omega_{\max} = \frac{1}{T_2\sqrt{\beta}}, \quad \text{so} \quad \frac{1}{T_2} = \omega_{\max}\sqrt{\beta} = 0.55 \text{ rad/s}.$$

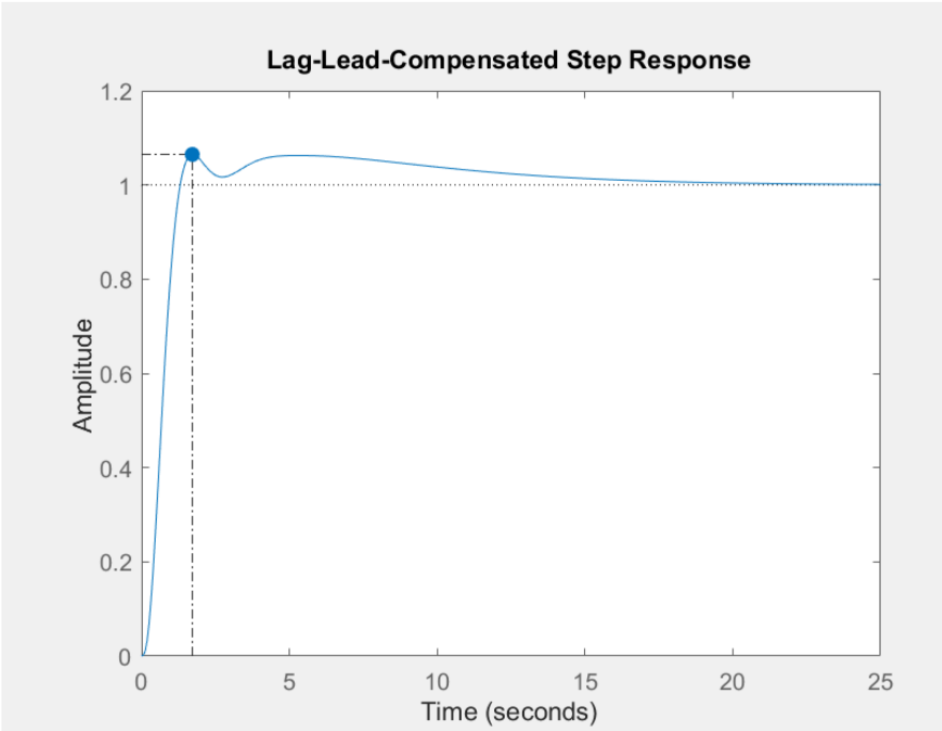
So, the lead compensator is as follows,

$$G_{\text{lead}}(s) = 11.9\left(\frac{s+0.55}{s+6.545}\right).$$

And the completely compensated system is

$$G_{\text{lag-lead-comp}}(s) = \frac{48s^2 + 35.52s + 5.016}{s^5 + 11.56s^4 + 36.91s^3 + 26.76s^2 + 0.4163s}.$$





3 Comparison

From the unity feedback step response of all the systems, we observed,

- The uncompensated system is unstable and increases to infinity.
- The addition of a lag compensator stabilizes the system by increasing gain at lower frequencies and increasing at higher frequencies. From the Bode Plot, we get gain-

margin = 11.8 dB, phase margin = 28 deg, both of which positively indicate a stable system.

- From step response, we notice steady error as 0,
  However, % Overshoot = 53.8% and T_p = 3.91s, so to improve transient characteristics we need a lead compensator.

- Addition of both lag and lead compensators causes us to reach the desired specifications. From the Bode Plot, we get gain margin = 16.2 dB and phase margin = 63.6 deg.

- From the step response, we get % Overshoot = 6.47% and T_p = 1.72 sec, both of which are within given requirements. Hence, the designed compensator is correct.

4 Code

```
s=tf('s');
G=1/(s*(s+1)*(s+4));

sG=s*G; % Create sG(s).
sG=minreal(sG); % Cancel common factors.
Kv = 12;
K=dcgain(Kv/sG); % Solve for K
g1=(K)*G;

%In order to meet the steady-state error requirement, Kv = 12, then the
%value of K is 48. (lim sG(s) = 12 as s tends to 0)
figure("Name","Response of Uncompensated System")
T0=feedback(g1,1); % Find T0(s).
step(T0) % Generate closed-loop uncompensated step response.
title('Uncompensated Step Response') % Add title to uncompensated step response.

%We can see the system is unstable and increases to infinity. This is
%because gain margin is negative (seen from bode plot)
%From the given question requirements, we find the damping ratio from the
%percentage overshoot of 13.25, which comes out to be 0.54, and from this
%damping ratio we get the required phase margin of 55° .
%Also, from the given peak time of 2 seconds and the damping ratio we just
%found, we calculate the closed loop bandwidth of 2.29 rad/s
%Now we choose a new phase-margin frequency near the closed loop
%bandwidth. Let us take w=1.9 rad/s as new phase-margin frequency.
%The uncompensated system has a phase of -177.65° at this frequency.
%Assuming the lag compensator to have a -5° contribution, we can find that
%the required contribution of lead compensator from => (-180+55) = -177.65-5+x,
%=> x=57.65°. So, the contribution of lead compensator is 57.65°
%Let us choose the lag compensator's higher break frequency to be 1 decade
%below the new phase-margin frequency, at 0.19 rad/s.
%We can find the Beta of lead compensator from
%tan(57.65)=(1-beta)/(2*(beta)^0.5)), which results in beta=0.084
%Hence alpha of lag compensator=1/beta=11.9
%So the Lag compensator is as follows:-
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%So the Lag compensator is as follows:-

Lag=(1/11.9)*((s+0.19)/(s+0.0159));
figure("Name","Response of Lag compensated System")
G1=g1*Lag;
T1=feedback(G1,1); % Find T1(s).
step(T1) % Generate closed-loop lag-compensated step response.
title('Lag-Compensated Step Response') % Add title to lag-compensated step response.

%Now for Lead compensator, we have beta=0.084, So, the lower break
%frequency = 1.9*(0.084)^0.5, which results in 0.55\

%So the Lead compensator is as follows: -
Lead=11.9*(s+0.55)/(s+6.545);
figure("Name","Response of Completely compensated System")
G2=g1*Lag*Lead;
T2=feedback(G2,1); % Find T2(s).
step(T2) % Generate closed-loop lag-lead-compensated step response.
title('Lag-Lead-Compensated Step Response') % Add title to lag-lead-compensated step response.
figure("Name","Bode Plots",'Position', [350, 60, 700, 500]);
bode(g1,g1*Lag,g1*Lag*Lead);
grid on;
ax = findall(gcf,'type','axes');
legend(ax(3),"Original","Lag Compensated","Lag Lead Compensated",'Location', 'northeast');
%ax(3) is axes for Magnitude Plot
legend(ax(2),"Original","Lag Compensated","Lag Lead Compensated",'Location', 'northeast');
%ax(2) is axes for Phase Plot
title("Bode Plots of Systems")
```